CS & IT ENGINEERING

Theory of Computation

Finite Automata

Lecture No. 5

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TOPICS TO BE COVERED

01 Regular Languages

02 Finite Automata

03 DFA

04

05

Regular Languages

- Regular Expression
- Finite Automata
- Regular Grammar

aX

bX

aaX

abX

Xa

Xb

$Xaaa$

Xaa

Xab

XaX

XbX

$XaaX$

$XabX$

$XaaaX$

$|w| = 2$

$|w| \leq 2$

$|w| \geq 2$

$\#_a(w) = 2$

$\#_a(w) \leq 2$

$\#_a(w) \geq 2$

$|w| = \text{even}$

$|w| = \text{odd}$

$n_a(w) = \text{even}$

$n_a(w) = \text{odd}$

Finite Automata

(Finite state machine)

(Finite machine)

$\rightarrow$ It represents a regular language
(accepts)
(recognizes)

FA

FSM

FM

$\overbrace{\text{Automata}}^{\text{one or more m/cs}}$

$\underbrace{\text{Automaton}}_{\text{one M/C}}$

Finite Automata

- By default: **Without o/p** (Acceptors)
  - Input $\rightarrow$ FA
  - DFA (Deterministic FA) $\cong$ NFA (NDFA) Non-deterministic FA
    - without ε moves
    - with ε moves
- **with o/p** (Transducers)
  - I/P $\rightarrow$ FA $\rightarrow$ O/P
  - Moore m/c $\cong$ Mealy m/c

FA

- Definition ?
- Configuration ?
- Representations ?
- DFA Vs NFA

What is FA ?

(w) Input string $\longrightarrow$

Finite Automata accepts L

valid string

If $\boxed{w \in L}$, FA halts at final state

Invalid

If $\boxed{w \notin L}$, FA halts at nonfinal state.

$$L = \underbrace{ab(a+b)^*}_{\text{all strings starts wik ab}}$$

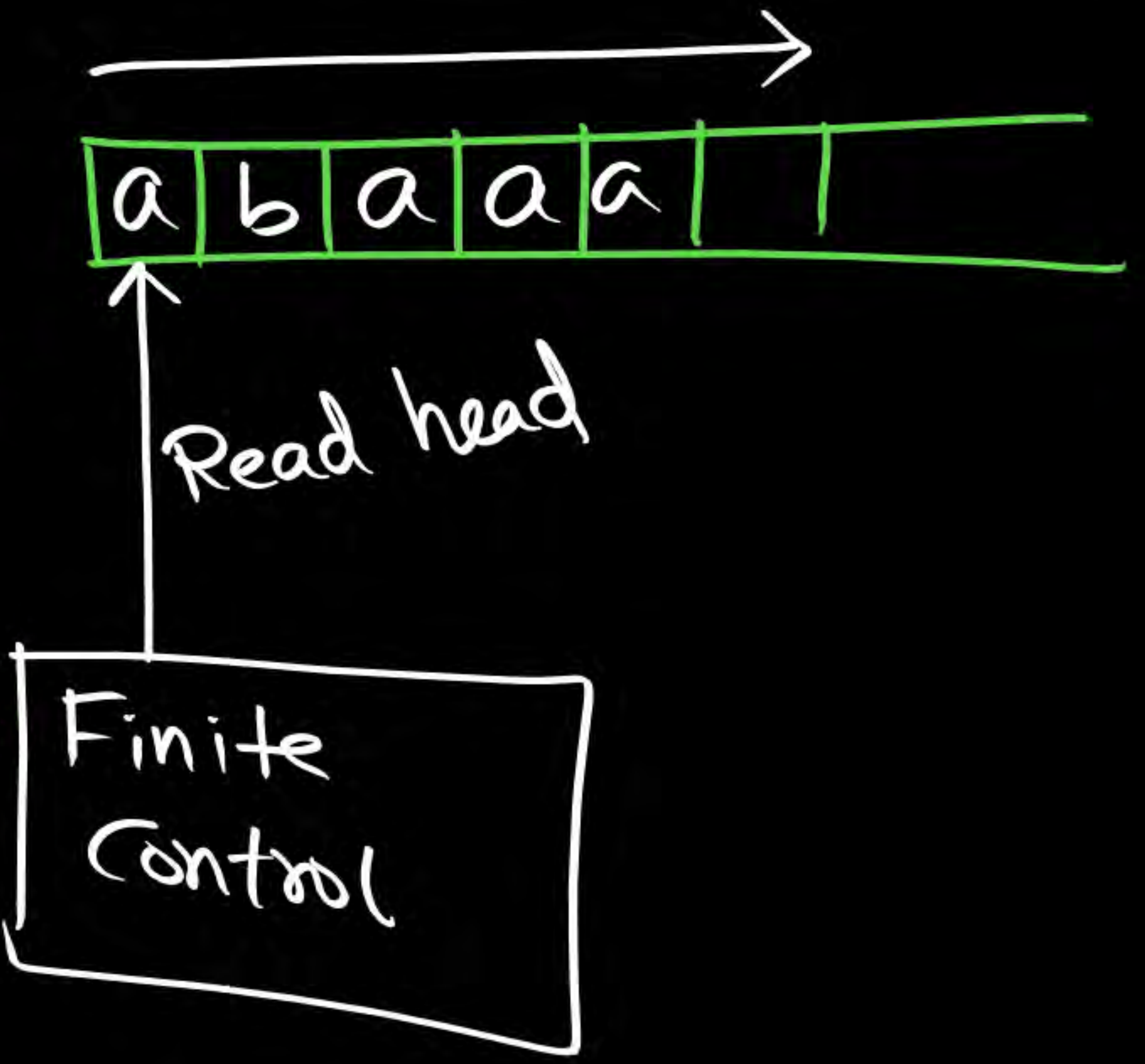
$$\Downarrow$$

$$\Sigma^* = \{\varepsilon, a, b, aa, ab, ba, bb, aaa, aab, aba, abb, baa, bab, bba, bbb, \ldots\}$$

FA Configuration

$$FA = (Q, \Sigma, \delta, q_0, F)$$

- Q → Set of states
- Σ → I/p Alphabet
- δ → Transition Function
- q_0 → Initial state, $q_0 \in Q$
- F → set of finals, $F \subseteq Q$

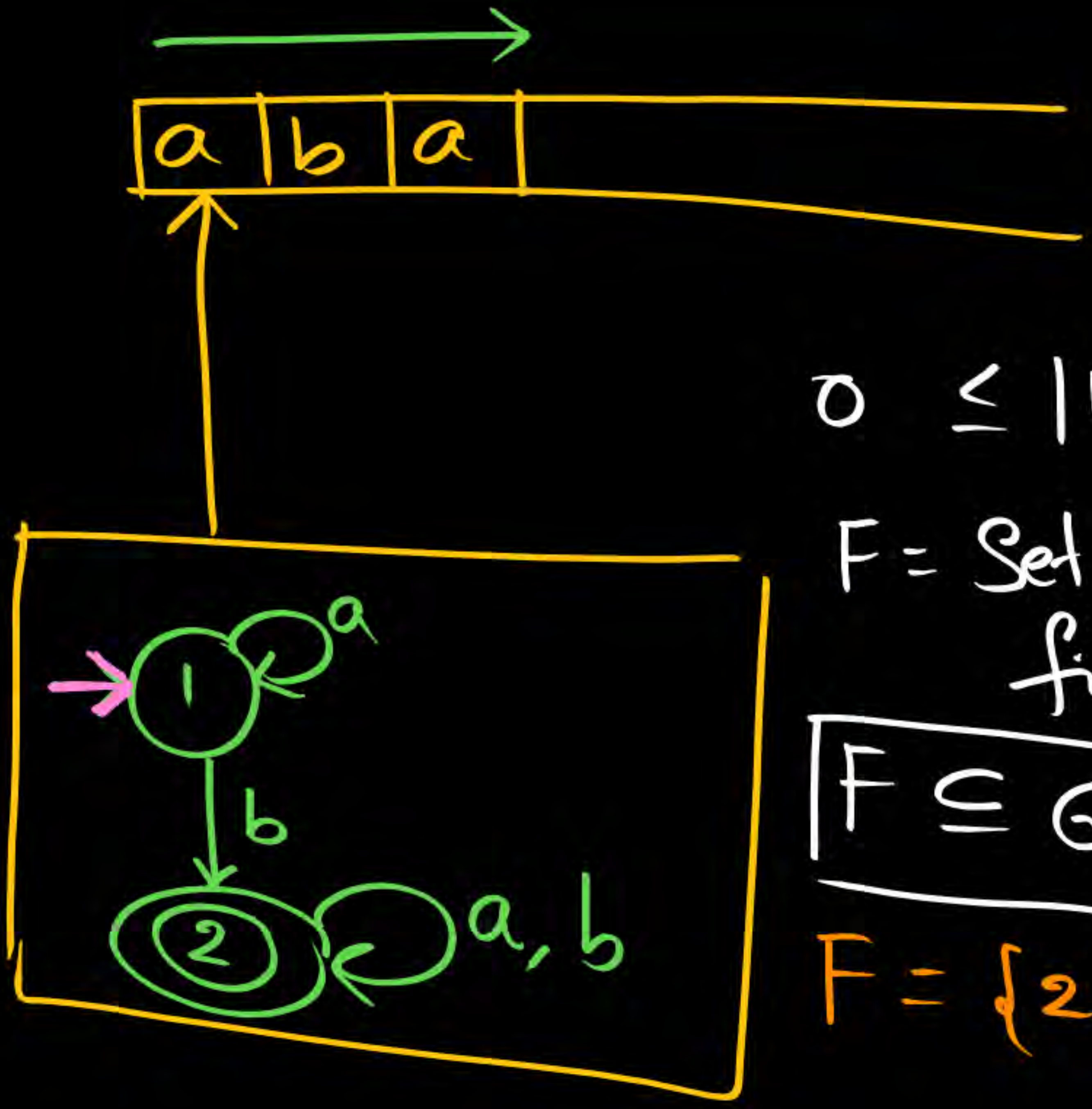


Q = Set of states
= {1, 2}

Σ = Input Alphabet
= Set of i/p symbols
= {a, b}

δ = Transition Function

q_0 = Initial state
= 1



$0 \le |F| \le |Q|$

F = Set of final states

$\boxed{F \subseteq Q}$

F = {2}

FA

- DFA

  $$\delta : Q \times \Sigma \rightarrow Q$$

- NFA
  - without ε moves

    $$\delta : Q \times \Sigma \rightarrow 2^{Q}$$

  - With ε moves

    $$\delta : Q \times \Sigma \cup \{\varepsilon\} \rightarrow 2^{Q}$$

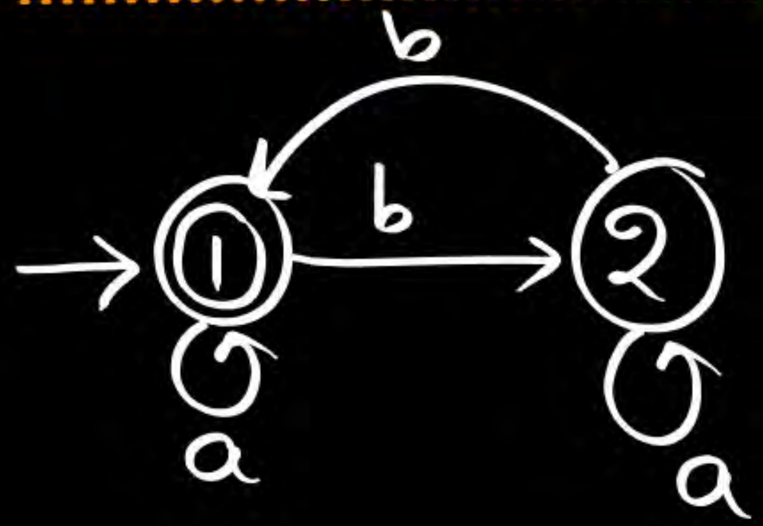
FA Representations

1. State Diagram
2. Transition Table
3. Set / function / Relation

$\delta : Q \times \Sigma \rightarrow Q$

Diagram	Table	Set

Diagram



Initial state

Final state(s) / Non Final

Transition

$Ps \xrightarrow{i} Ns$

Table

DFA

Σ (over columns a, b); rows are Q; 1 is the initial state ($\rightarrow$) and final state ($*$).

δ	a	b
$\rightarrow *1$	1	2
2	2	1

$Q = \{1, 2\}$

$\Sigma = \{a, b\}$

Set

$FA = (\{1,2\}, \{a,b\}, \delta, 1, \{1\})$

I)
$\delta(1,a) = 1$
$\delta(1,b) = 2$
$\delta(2,a) = 2$
$\delta(2,b) = 1$

4 transitions

II) $\delta = \{((1,a),1), ((1,b),2), ((2,a),2), ((2,b),1)\}$

$$Q \times \Sigma \to Q$$

$$\delta(1,a) = 1$$

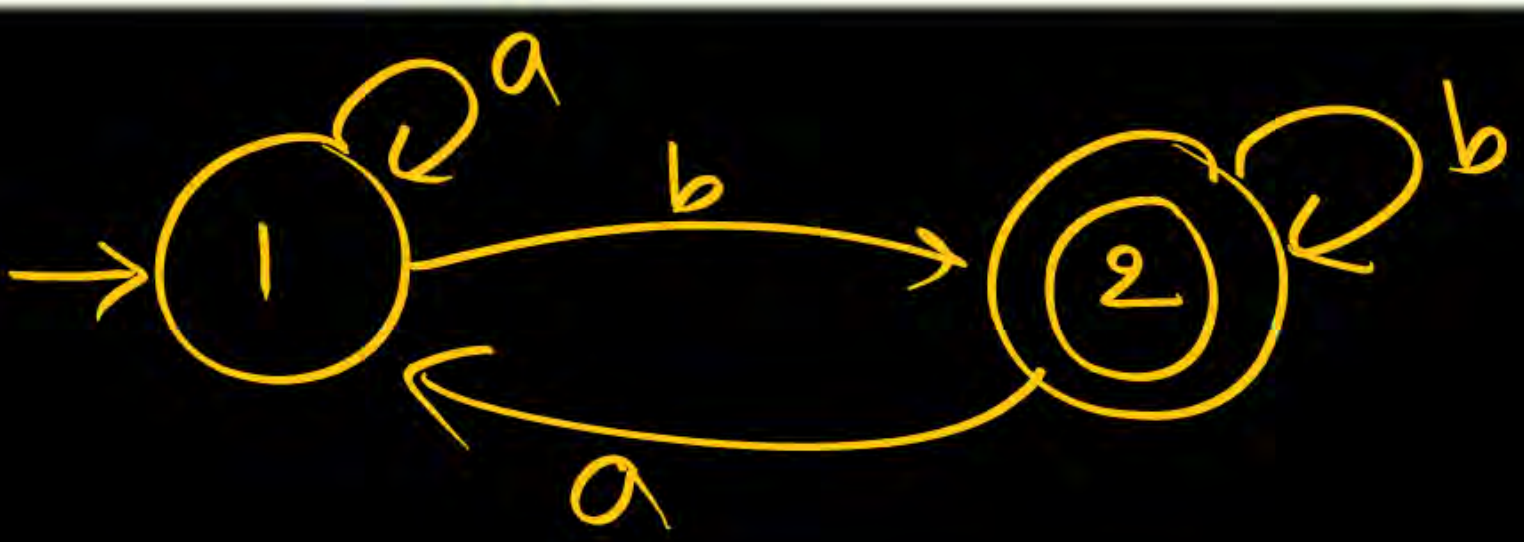
$$\delta = \{ (\underbrace{(1,a)}_{\in Q\times\Sigma}, \underbrace{1}_{\in Q}), ((1,b), 2), \dots \}$$

$$Q \times \Sigma \qquad\qquad Q$$

$$(1,a) \in Q\times\Sigma \longrightarrow 1 \in Q$$

$$(1,b) \in Q\times\Sigma \longrightarrow 2 \in Q$$

L :



$$\Sigma^* = \{ \overset{\times}{\varepsilon}, \overset{\times}{a}, \overset{\checkmark}{b}, \overset{\times}{aa}, \overset{\checkmark}{ab}, \overset{\times}{ba}, \overset{\checkmark}{bb}, \dots \}$$

$$L = ? = (a+b)^* b$$

aba

$$1 \xrightarrow{a} 1 \xrightarrow{b} 2 \xrightarrow{a} \underbrace{1}_{\text{Halting state}}$$

ε : 1 not accepted

$a : 1 \xrightarrow{a} \boxed{1}$ last state, Halting state

$b : 1 \xrightarrow{b} \underbrace{2}_{\text{final}}$

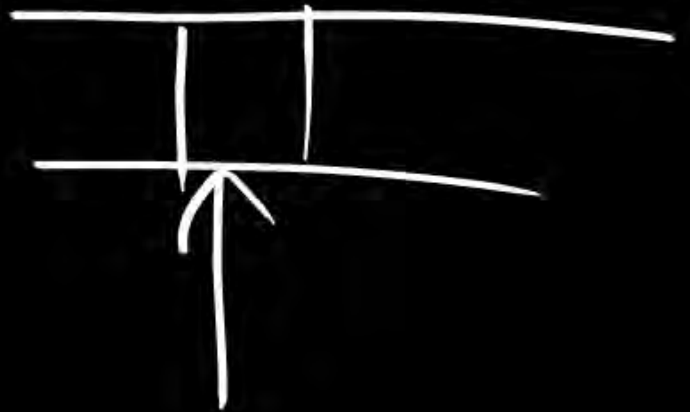
aa :

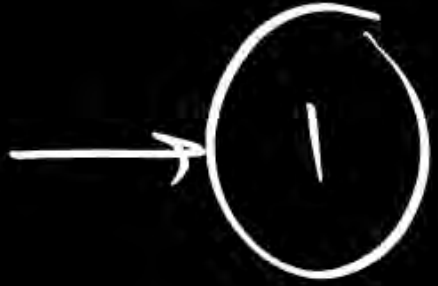
$1 \xrightarrow{a} 1 \xrightarrow{a} 1$

1

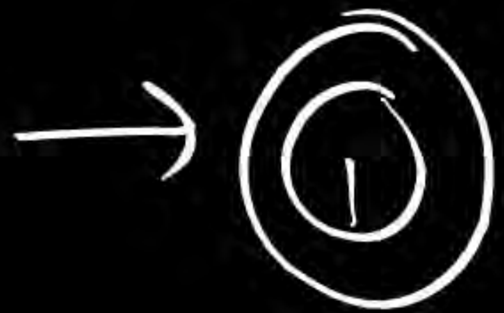
Start

Halt

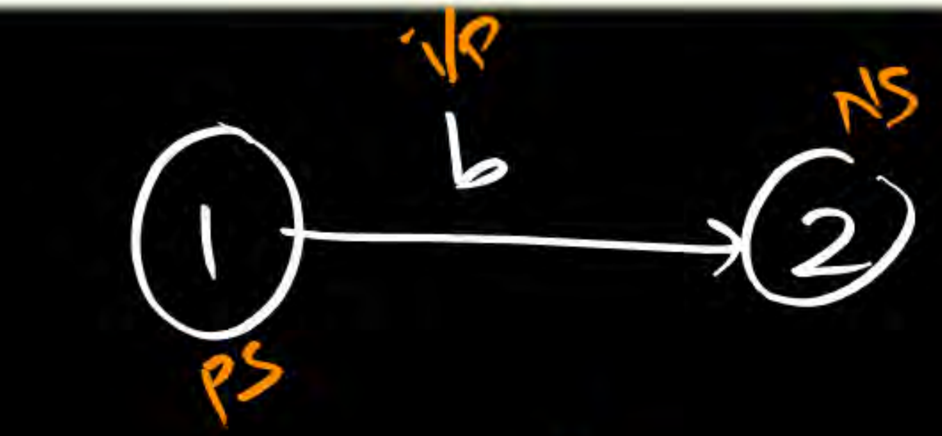




ε not accepted



ε Accepted



	b (i/p)
PS 1	2 (NS)

$$\delta = \{ (\underbrace{\overset{ps}{(1}, \overset{i/p}{b)}}_{\text{Domain}}, \underbrace{\overset{NS}{2}}_{\text{Co-domain}}), \ldots$$

$$Q \times \Sigma \to Q$$

$$a^* (ba)^* a^*$$

$$a^0 (\;\;)^0 a^0$$

$$\epsilon \quad \epsilon \quad \epsilon \longrightarrow \underbrace{\epsilon}_{\text{min string}}$$

$$\Sigma = \{a, b\}$$

symbols

$$|\varepsilon| = 0$$

not symbol

empty string

$$\Sigma = \{\varepsilon, \gamma\}$$

$$|\varepsilon| = 1$$

symbol

ε is not empty string

Reached

Halt

Stop

end

$$L = ab(a+b)^*$$

$$= \text{set of strings}$$

$$= \{abw \mid w \in \{a,b\}^*\}$$

$$ab \longrightarrow \boxed{\text{FA}}$$

$$\varepsilon \longrightarrow \boxed{\text{FA}}$$

ε ✗

a ✗

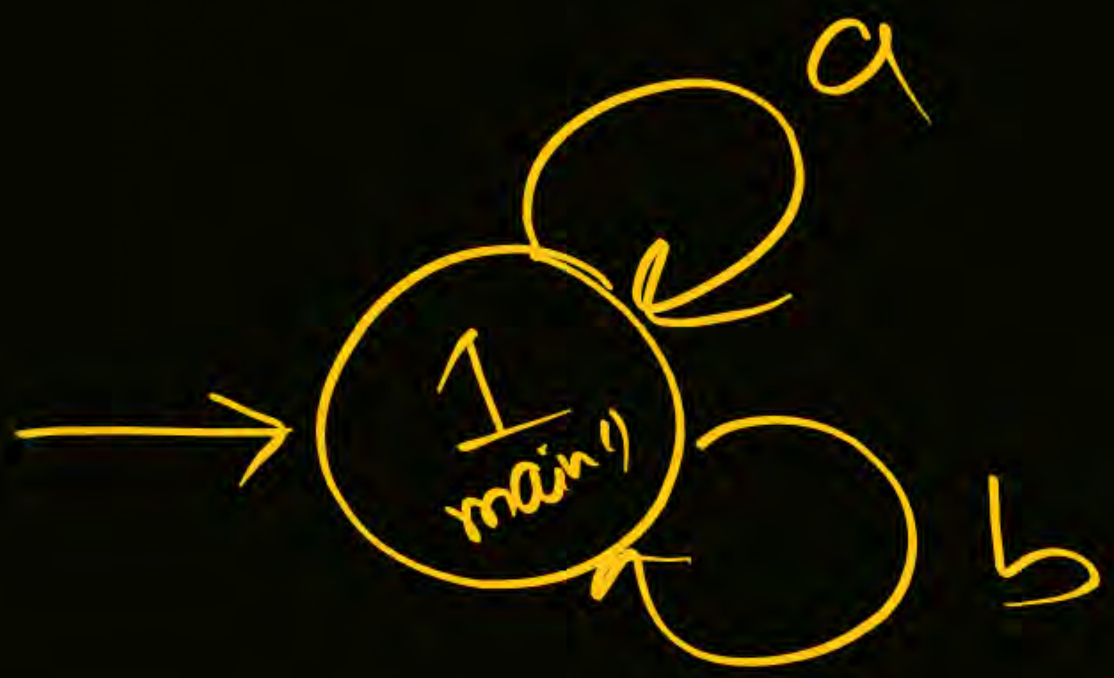
b ✗

aa ✗

ab ✗

ba ✗

bb ✗



$$L = \{ \} = \emptyset$$

$(a+b)^* = \{a,b\}^* = \Sigma^*$

$\Sigma = \{a,b\}$
$= (a+b)$

$\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \cdots$

$(a + b^{*} + c^{+})^{+}$

$\min \to 0 \quad \min \to 1 \quad \min \to 1$

$(a + b^0 + c^1)^1$

$\underbrace{a + \varepsilon + c}$

$\min = \varepsilon$

$(ab)^*$ → ε ✓, a ✗, b ✗, aa ✗, ab ✓, ba ✗, bb ✗, ⋮

$$\underbrace{b^*(ab^*a)^*b^*}_{\downarrow\ \underbrace{aabaa}_{\text{having even a's}}\ \times} \quad \boxed{\text{Subset of}} \quad \boxed{\text{even no. of a's}}$$


THANK YOU
GW
SOLDIERS !