Branch : CSE/IT **Batch : Hinglish**

Discrete Mathematics II

Set Theory

DPP-02

[MCQ]

1. Suppose A, B, C and D are subsets of U (the universe) with A as a subset of B and C as subset of D i.e $A \subseteq B$ and $C \subseteq D$, then consider the following statements

I. $A \cap C \subseteq B \cap D$

II. $A \cup C \subseteq B \cup D$

Which of the following is correct options?

(a) Only I is true

(b) Only II is true

(c) Neither I nor II is true

(d) Both I and II are true

[NAT]

2. Let A = {1, 2, 3, … 15}. How many subsets of A contains all of the odd integers in A?

[MCQ]

3. Determine whether each of the following statements is true of false. For each false statement, given a counterexample.

(a) If A and B are infinite sets, then $A \cap B$ is infinite.

(b) If B is infinite and $A \subseteq B$, then A is infinite.

(c) If $A \subseteq B$ with B finite, then A is finite.

(d) If $A \subseteq B$ with A finite, then B is finite.

[NAT]

4. Let U be a given universe with $A, B \subseteq U$, $A \cap B = \phi$, $|A| = 12$, and $|B| = 10$. If seven elements are selected from $A \cup B$, what is the probability the selection contains four elements from A and three from B ?

[MCQ]

5. Let $A, B \subseteq \mathbf{R}$, where $A = \{x |\ x^2 - 7x = -12\}$ and $B = \{x |\ x^2 - x = 6\}$. Determine $A \cup B$ and $A \cap B$.

(a) $A \cup B = \{5\}$ and $A \cap B = \{-2, 3, 4\}$

(b) $A \cup B = \{3\}$ and $A \cap B = \{-2, 3, 4\}$

(c) $A \cup B = \{-2, 3, 4\}$ and $A \cap B = \{3\}$

(d) $A \cup B = \{2, 3, 4\}$ and $A \cap B = \{5\}$

Answer Key

1. **(d)**
2. **(128)**
3. **(a)**
4. **(0.3483)**
5. **(c)**

Hints and Solutions

1. (d)

I. $A \cap C \subseteq B \cap D$, is True.

Let a be an arbitrary element of $A \cap C$, so $a \in A \cap C$ then $a \in A \subseteq B$, so $a \in B$ and $a \in C \subseteq D$, so $a \in D$. That concludes that $a \in B$ and $a \in D$, therefore by definition $a \in B \cap D$. If follows that every element of $A \cap C$ belongs to $B \cap D$, which by definition means $A \cap C \subseteq B \cap D$.

II. $A \cup C \subseteq B \cup D$, is True.

If a is an arbitrary element that belongs to $A \cup C$ then it definitely belongs to $B \cup D$ as $A \subseteq B$ and $C \subseteq D$.

2. (128)

In the given set A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}

There are 8 odd integers. For all odd integer we have choices whether to include it or not with the 7 even integers in the set.

Therefore possiblities = $2^7 = 128$.

3. (c)

(a) False: Let $A = \{0,1,2,3,\ldots\}, B = \{0,-1,-2,\ldots\}$. Then A, B are infinite but $|A \cap B| = |\{0\}| = 1$

(b) False: Let $A = \{1,2\}$ and $B = Z^+$.

(c) True

(d) False: Let $A = \{1,2\}$ and $B = Z^+$.

4. (0.3483)

Since $|A \cap B| = 0, |A \cup B| = 12 + 10 = 22$. There are $\binom{22}{7}$ ways to select seven elements from $A \cup B$.

Among these selections $\binom{12}{4}\binom{10}{3}$ contain four elements from A and three from B. Consequently, the probability sought here is

$\binom{12}{4}\binom{10}{3} / \binom{22}{7} = (495)(120)/(170,544) \doteq 0.3483$.

5. (–2, 3, 4)

$x^2 - 7x = -12 \Rightarrow x^2 - 7x + 12 = 0 \Rightarrow (x-4)(x-3) = 0 \Rightarrow x = 4, x = 3.$

$x^2 - x = 6 \Rightarrow x^2 - x - 6 = 0 \Rightarrow (x-3)(x+2) = 0 \Rightarrow x = 3, x = -2.$

Consequently, $A \cap B = \{3\}$ and $A \cup B = \{-2,3,4\}$.