CS & IT ENGINEERING

DISCRETE MATHS
SET THEORY

Lecture No. 11

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TOPICS

01 Partial order relation

02 Poset /toset

3 lattice

equivalence Relation.

Relation is called equivalence Relation.

(RST)
- * Reflexive
- * Symmetric.
- * Transitive.

$R_1 = \{(a,b) \mid a+b = \text{even}\} \rightarrow$ (equivalence relation)

R: $a\dot{R}a$ $\quad a+a = \text{even}$

$2a = \text{even}$ ✓

Sy: $aRb \rightarrow bRa$. ✓

$a+b = \text{even} \rightarrow b+a = \text{even}$.

T: $aRb \wedge bRc \rightarrow aRc$

$a+b = \text{even} \wedge b+c = \text{even} \rightarrow a+c = \text{even}$.

$(1,3) \in R \wedge (3,5) \in R \rightarrow (1,5) \in R$.

$1+3 = \text{even} \wedge 3+5 = \text{even} \rightarrow 1+5 = \text{even}$.

$(2,4) \in R \wedge (4,6) \in R \rightarrow (2,6) \in R$.

$2+4 = \text{even} \wedge 4+6 = \text{even} \rightarrow 2+6 = \text{even}$.

$R_2 : \{(a, b) \mid a \equiv b \pmod 4\}$ → equivalence relation.

$a \equiv b \pmod 4$

a, b are having same remainder w.r.t 4.

R: aRa $\quad a \equiv a \pmod 4$

$1 \equiv 1 \pmod 4$

$15 \equiv 15 \pmod 4$

T:

$aRb \wedge bRc \rightarrow aRc.$

$\underline{a} \equiv \underline{b} \pmod 4 \wedge \underline{b} \equiv \underline{c} \pmod 4 \rightarrow \underline{a} \equiv \underline{c} \pmod 4$

$0 \equiv 4 \pmod 4 \wedge 4 \equiv 8 \pmod 4$

$\rightarrow 0 \equiv 8 \pmod 4$

Sy: $aRb \rightarrow bRa.$ ✓

$a \equiv b \pmod 4 \rightarrow b \equiv a \pmod 4$

$A \rightarrow$ non empty set.

$R_1 \rightarrow$ equivalence.

$R_2 \rightarrow$ equivalence.

$R_1 \cup R_2 \rightarrow$ need not be an equivalence relation.

$R_1 \cap R_2 \rightarrow$ equivalence relation.

$$\begin{array}{lccc} R_1 \rightarrow & R & S & T \\ & \cap & \cap & \cap \\ R_2 \rightarrow & R & S & T \\ \hline & R & Sy & T \end{array}$$

$$\begin{array}{lccc} R_1 \rightarrow & R & S & T \\ & \cup & \cup & \cup \\ R_2 \rightarrow & R & S & T \\ \hline & R & Sy & \times \end{array}$$

$R_1 = \{(1\,2)\}$ ✓

$R_2 = \{(2\,1)\}$ ✓

$R_1 \cup R_2 \rightarrow$ not transitive

$= \{(1\,2)(2\,1)\}$

R_1, R_2	$\cup$	$\cap$
Transitive	×	✓

$^{\text{u}}22$

$R_1 = \{(11)(22)(33)(12)(21)\}$ (RST)

$R_2 = \{(11)(22)(33)(23)(32)\}$ (RST)

$R_1 \cup R_2 = \{(11)(22)(33)(12)(21)(23)(32)\}$ → not equivalence relation.

$$\boxed{(13) \notin R_1 \cup R_2}$$

A : non empty set.

A x A.

check.

Result: { }
Relation.

equivalence relation.

* equivalence Relation creates partion on a set.

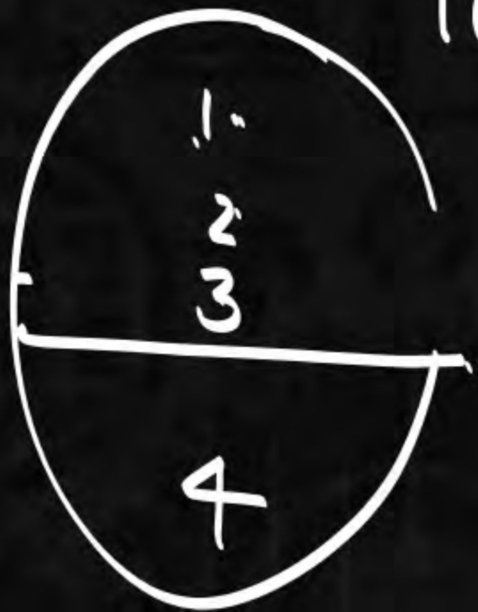
$A = \{1, 2, 3, 4\}$.

$R_1 = \{ (1,2)\ (1,3) \}$.

partion:

$(1,2) \in R \quad \underline{\underline{1R2}}$

$1R3$



aRb, a & b will go in same partions.

$c \not R d$, c & d will go to diff partions.

equivalence relation creates partition on a set.

$A = \{1, 2, 3\}$.

$1R2 \quad 2R1$

$R_1 = \{(1\,1)\,(2\,2)\,(3\,3)\,(1\,2)\,(2\,1)\}$.

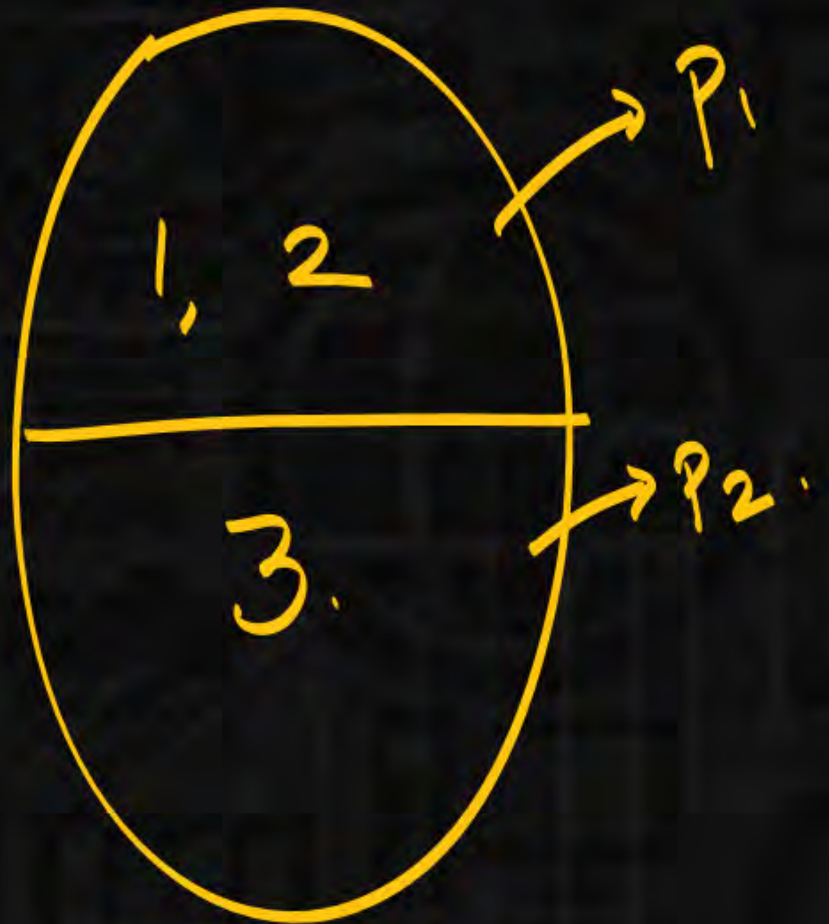
equivalence Relation ✓.

$(1,2) \in R$.

$1R2$

$2R1. \quad (2,1) \in R$.

$1 \cancel{R} 3$



set: z.

$R: \{ (a, b) \mid a + b = \text{even} \}$.

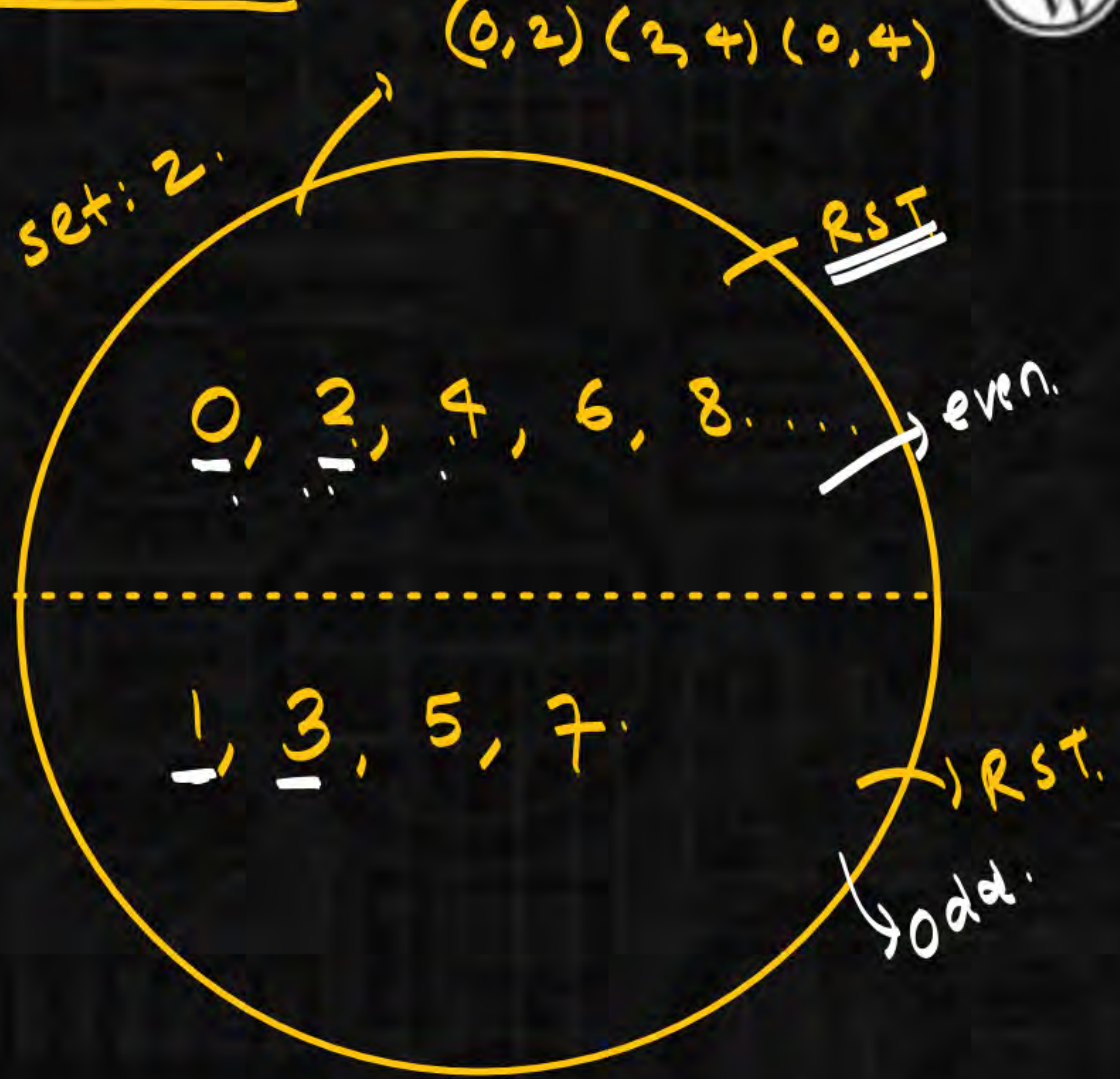
↳ equivalence Relation.

→ equivalence relation creates partion on a set.

$(2,4) \in R$ $\quad 2R4 \quad 2+4=\text{even}$.

$(2,3) \notin R$ $\quad 2R3 \quad 2+3=\text{odd}$.

$(1,2) \notin R$ $\quad 1+2=\text{odd}$.



$R_1 : \{ (a,b) \mid a \equiv b \pmod{4} \}$ $\; : \mathbb{Z}$

$0 \equiv 4 \pmod{4} \quad (0,4) \in R$

$0 \equiv 8 \pmod{4} \quad (0,8) \in R$

$1 \equiv 5 \pmod{4} \quad (1,5) \in R$

[0]	0, 4, 8	RST
[1]	1, 5, 9	RST
[2]	2, 6, 10	
[3]	3, 7, 11	

mod 10

$A \rightarrow$ nonempty set.

$R \rightarrow$ equivalence Relation.

equivalence relation creates $\underbrace{\text{partions}}_{\downarrow \text{ equivalence class}}$ on a set.

A_1, A_2, A_3 are equivalence class

1) $A_1 \cup A_2 \cup A_3 = A$.

2) $A_1 \cap A_2 \cap A_3 = \phi$.

$a_1 \not R b_1$

$a_1 \not R c_1$

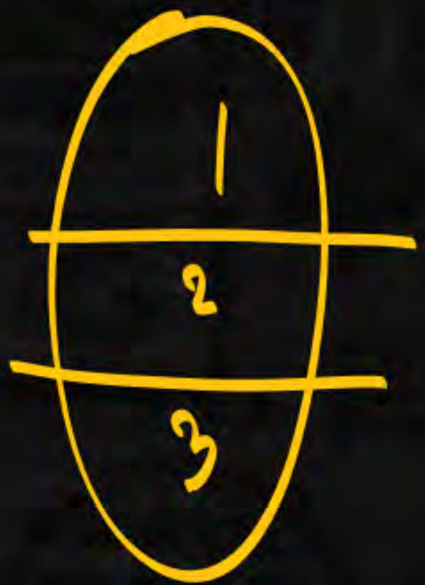
$b_1 \not R c_1$

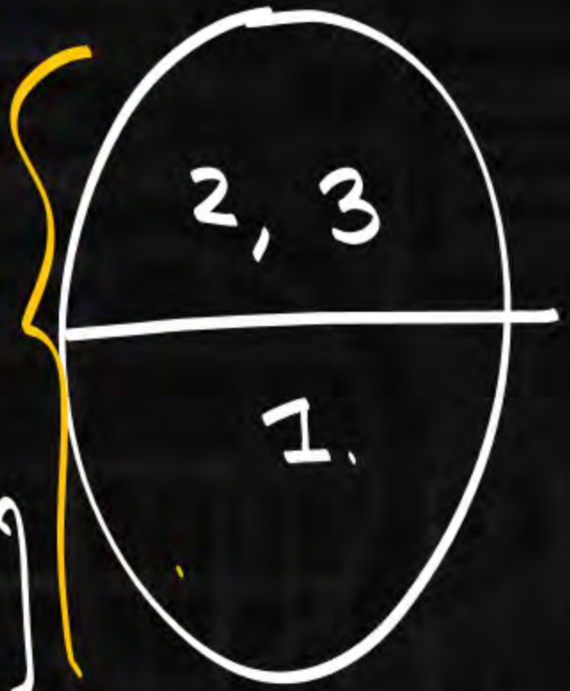
$R_1 = \{(11)\ (22)\ (33)\ (12)\ (21)\}$ → EQR.

$R_2 : \{(11)\ (22)\ (33)\ (23)\ (32)\}$

EQR

11 22 33





$R_3 = \{(11)(22)(33)(13)(31)\}$

EQR

Total no. of equivalence Relations = Total no of partions.

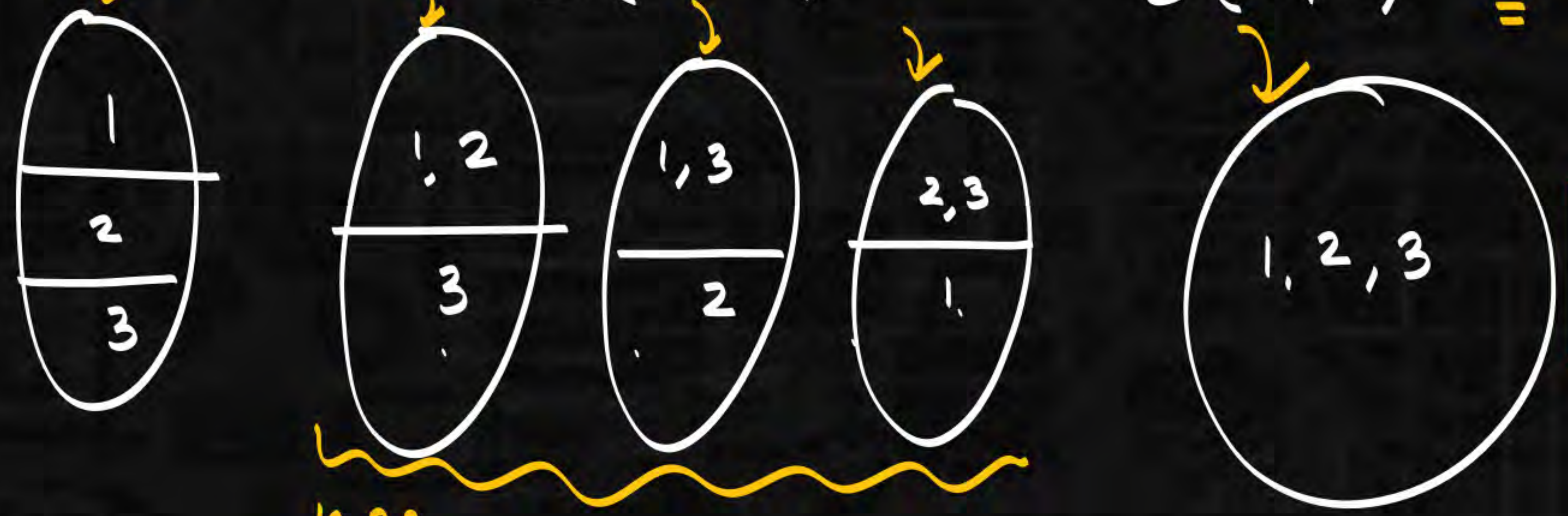
$$= S(3,1) + S(3,2) + S(3,3)$$

$$= \sum_{n=1}^{m} S(m,n)$$

$$\boxed{B(m) = \sum_{n=1}^{m} S(m,n)}$$

$A = \{1, 2, 3\}$

$S(3,3)$ $\qquad$ $S(3,2)$ $\qquad$ $S(3,1)$



n 2 2
3 3
1 2
2 1

$f: A \to B$ Total onto $= 6.$

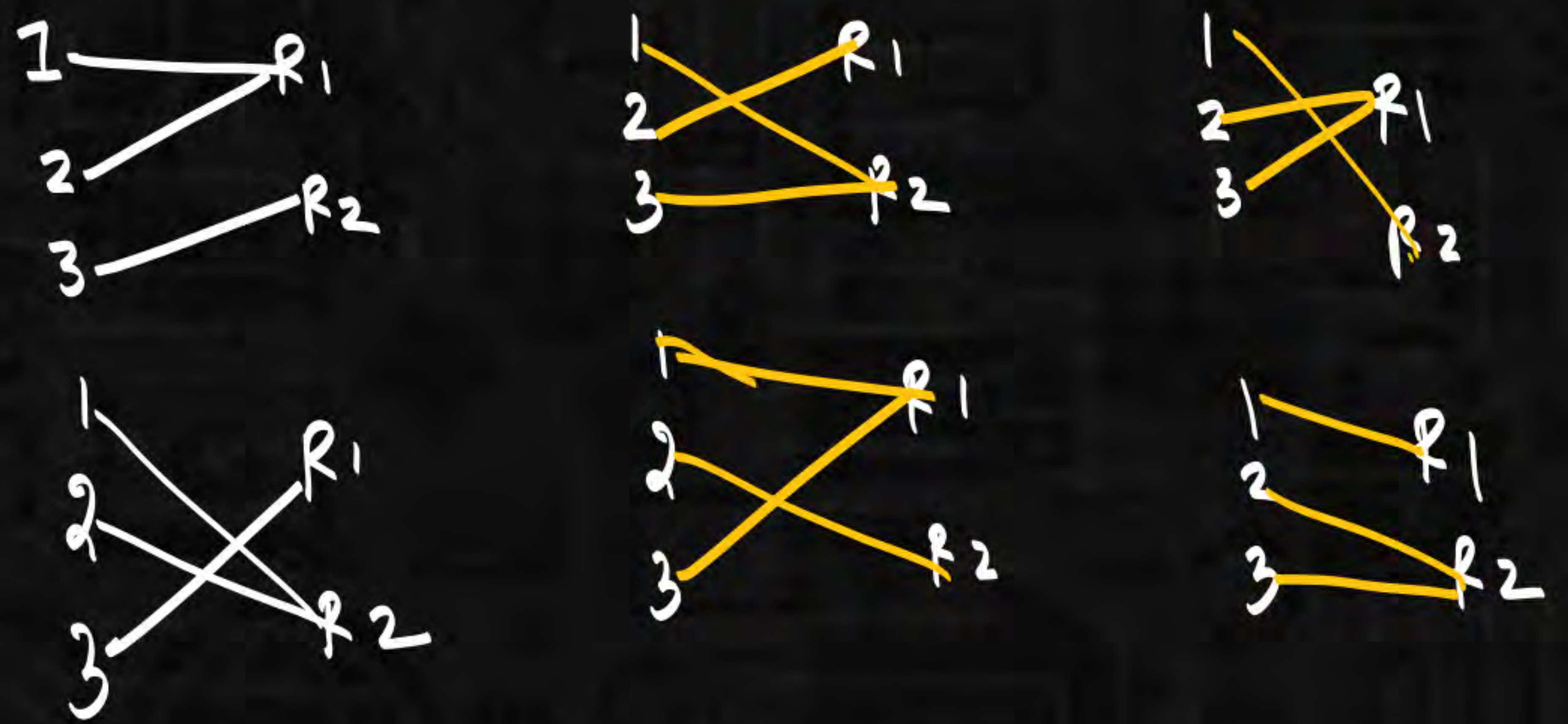
onto :

$|A| = 3 \quad |B| = 2$

$S(3, 2)$

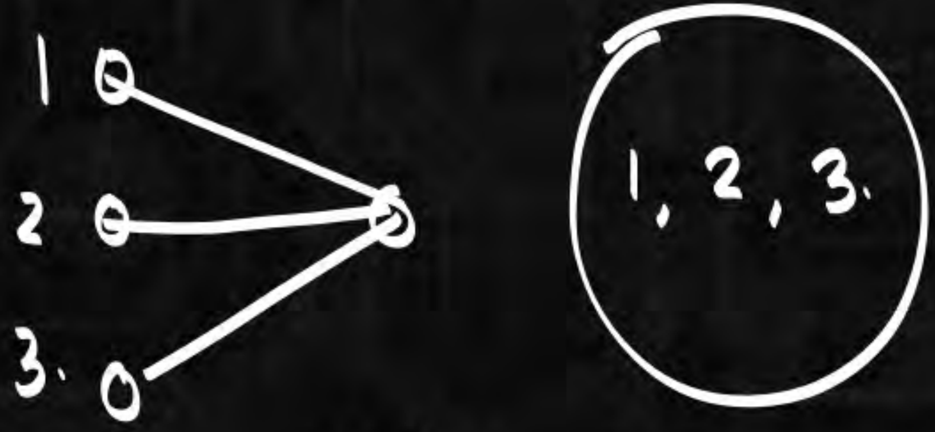
$3 \downarrow$ diff

$2 \downarrow$ identians



$f: A \to B.$

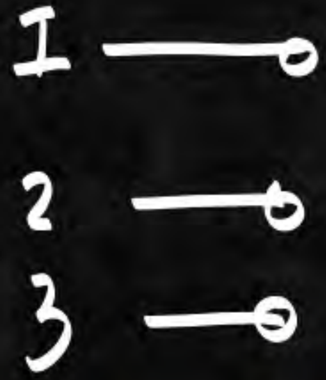
$|A| = 3 \quad |B| = 1.$

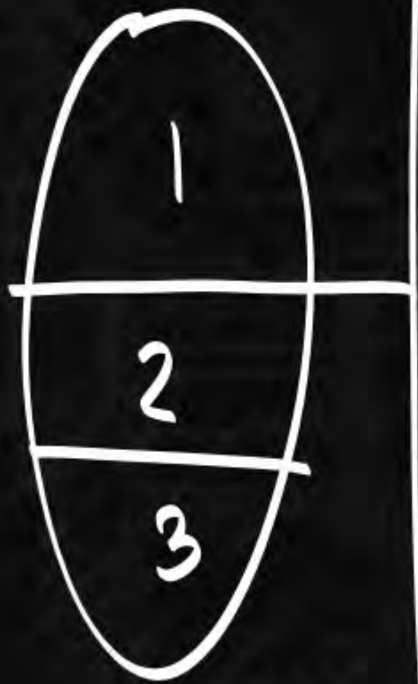


$S(3,1)$

$f: A \to B$

$|A| = 3 \quad |B| = 3$



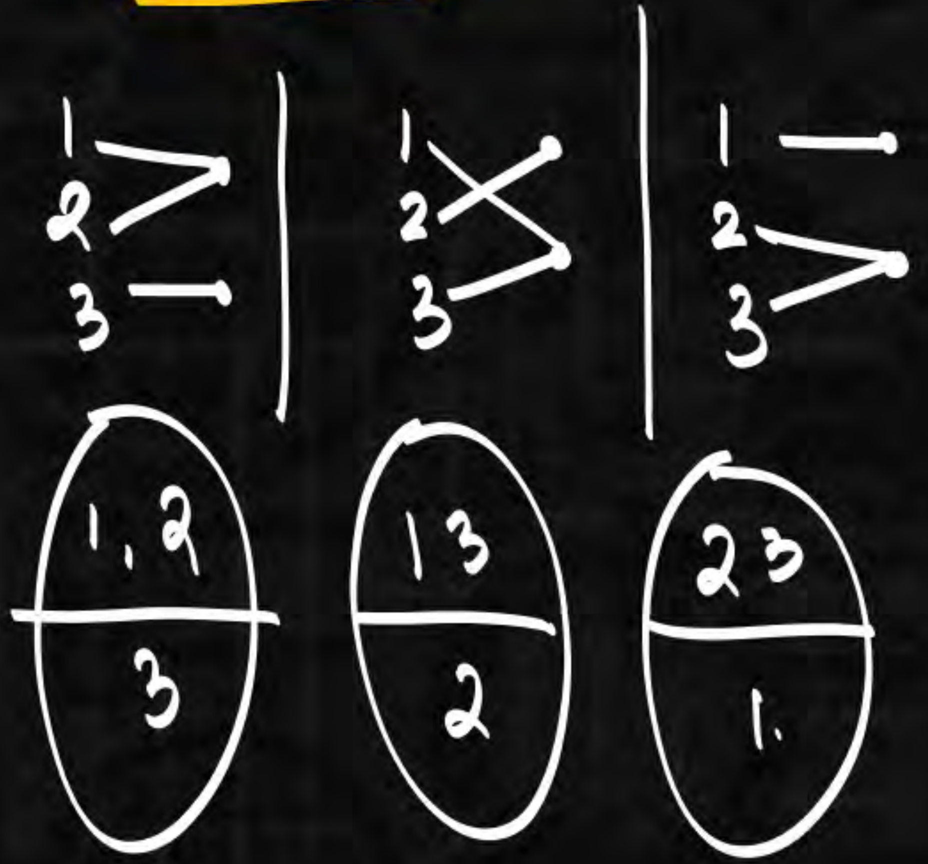


$S(3,3)$

$f: A \to B$

$|A| = 3 \quad |B| = 2.$

$S(3,2)$



$$\begin{array}{ccc} b & & c \\ a & a+b & a+b+c. \end{array}$$

$$\begin{array}{llllll}
B_0 & 1. & & & & \\
B_1 & 1 & 2. & & & \\
B_2 & 2 & 3 & 5. & & \\
B_3 & 5 & 7 & 10 & 15 & \\
B_4 \rightarrow & 15 & 20 & 27 & 37 & 52. \\
B_5 \rightarrow & 52 & \ldots & & &
\end{array}$$

$B_3 = 5.$

$B_4 = 15$


THANK YOU GW SOLDIERS !