# CS & IT ENGINEERING

## Data Structure

**Tree**
**Chapter- 5**
**Lec- 04**

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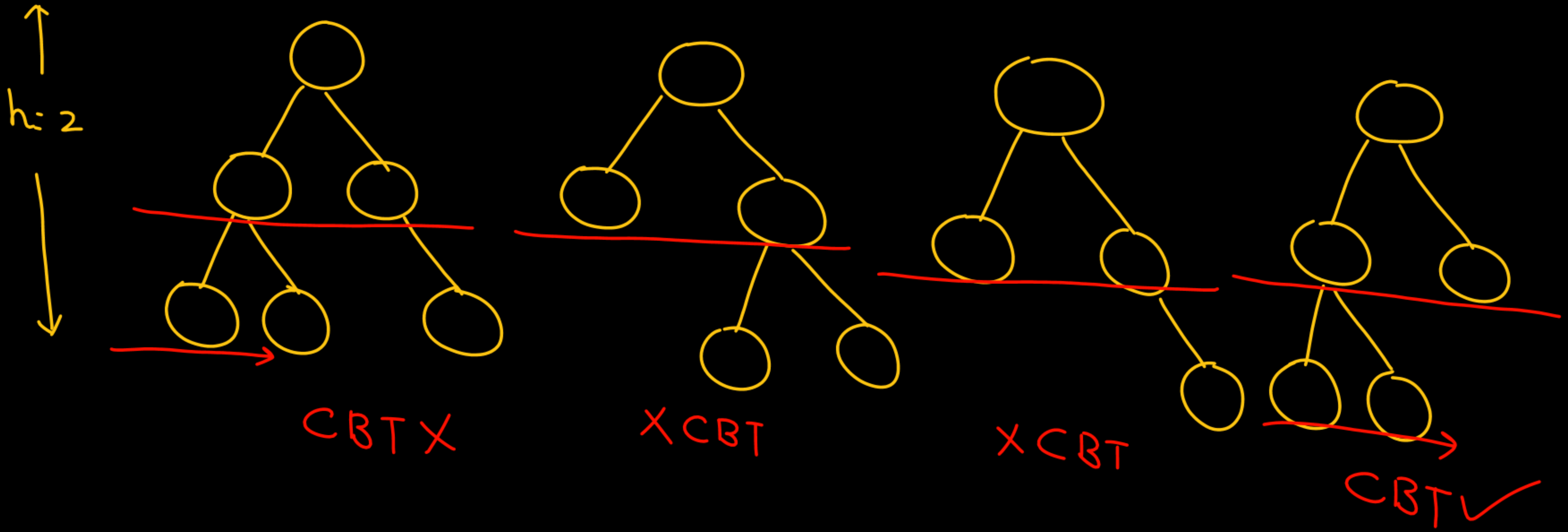
TOPICS TO BE COVERED

- Tree-IV

## Complete binary tree :

CBT is a binary tree which is Full till second last level.

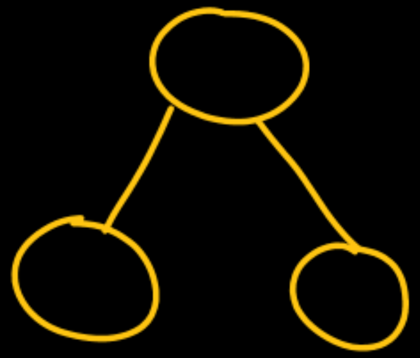
& nodes at last level are filled from left to right.



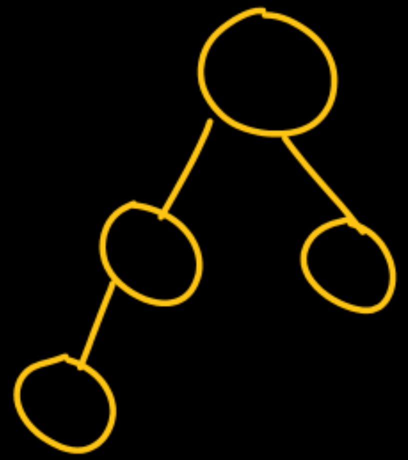
1. Structure of a CBT with 1 node

2. Structure of a CBT with 2 node

3. Structure of a CBT with 3 node



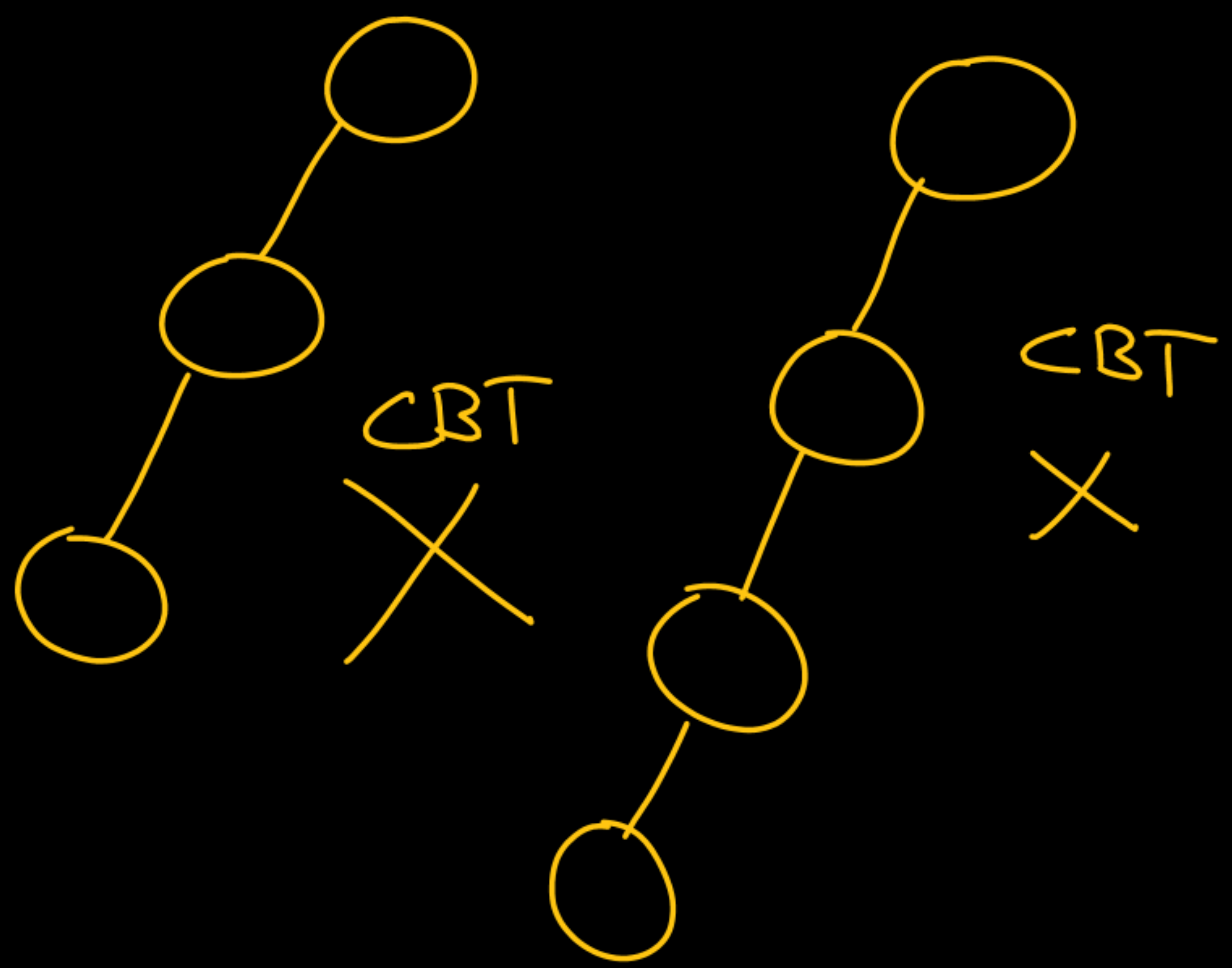
4. Structure of a CBT with 4 node

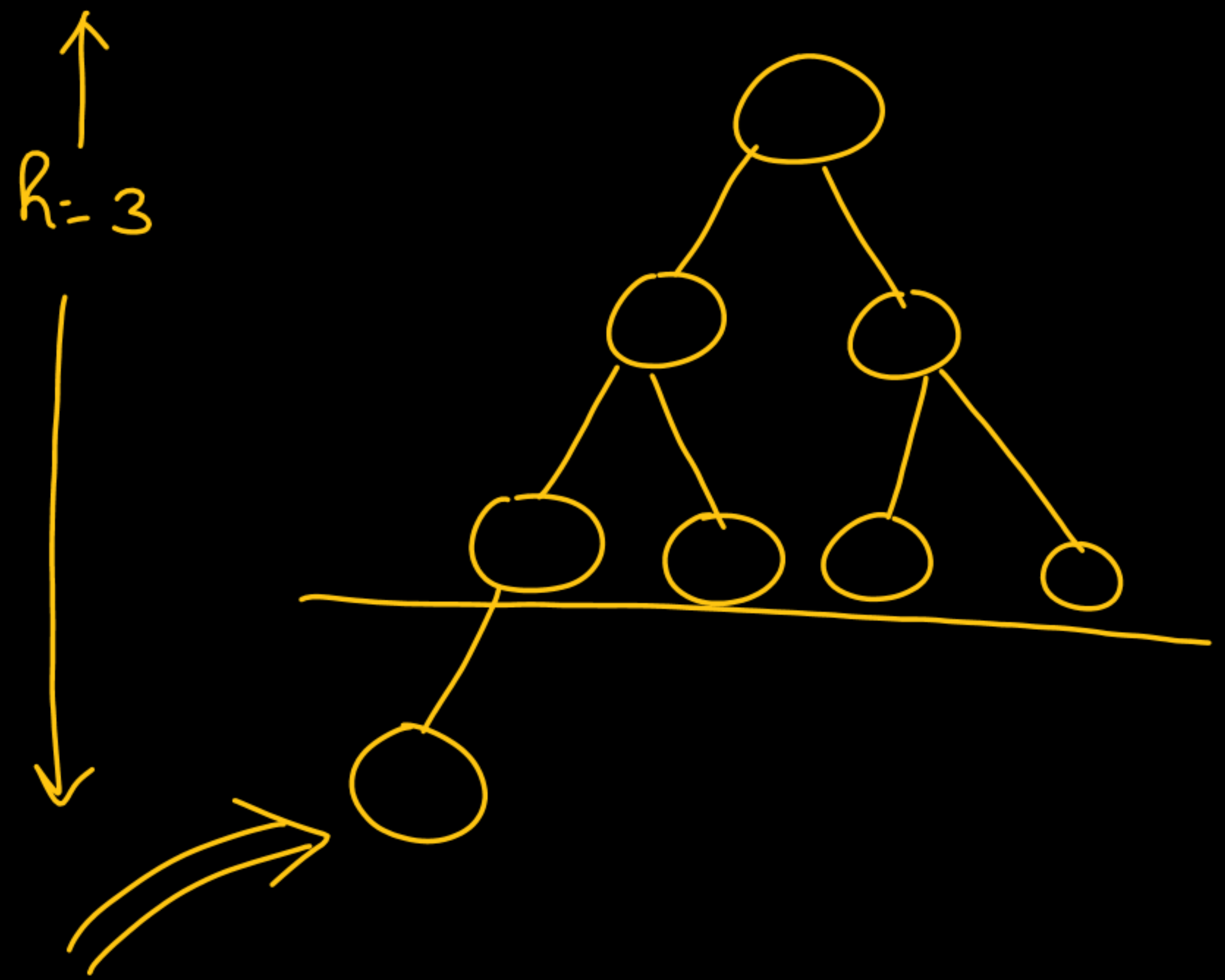


** The structure of a CBT with K nodes is always fix.

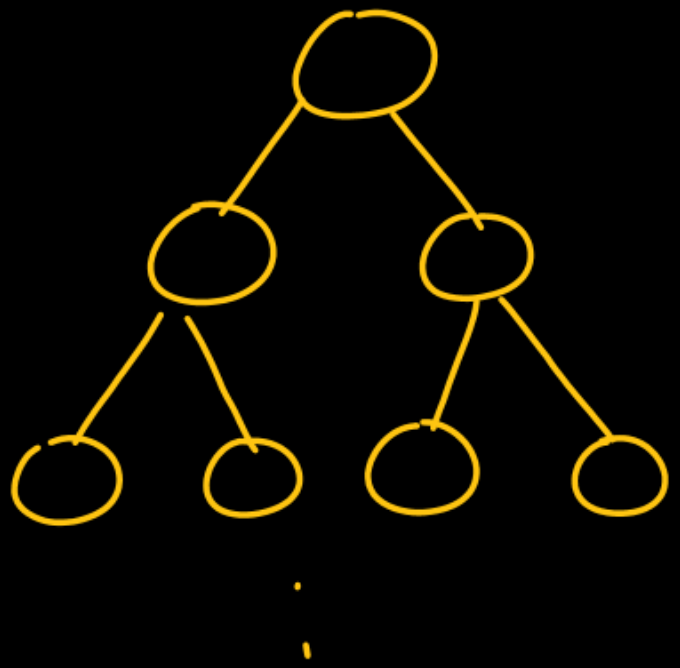
1) Max. no. of nodes in a CBT of height $h$ = $2^{h+1} - 1$

Min no. of nodes in a CBT of height $h$ = ?



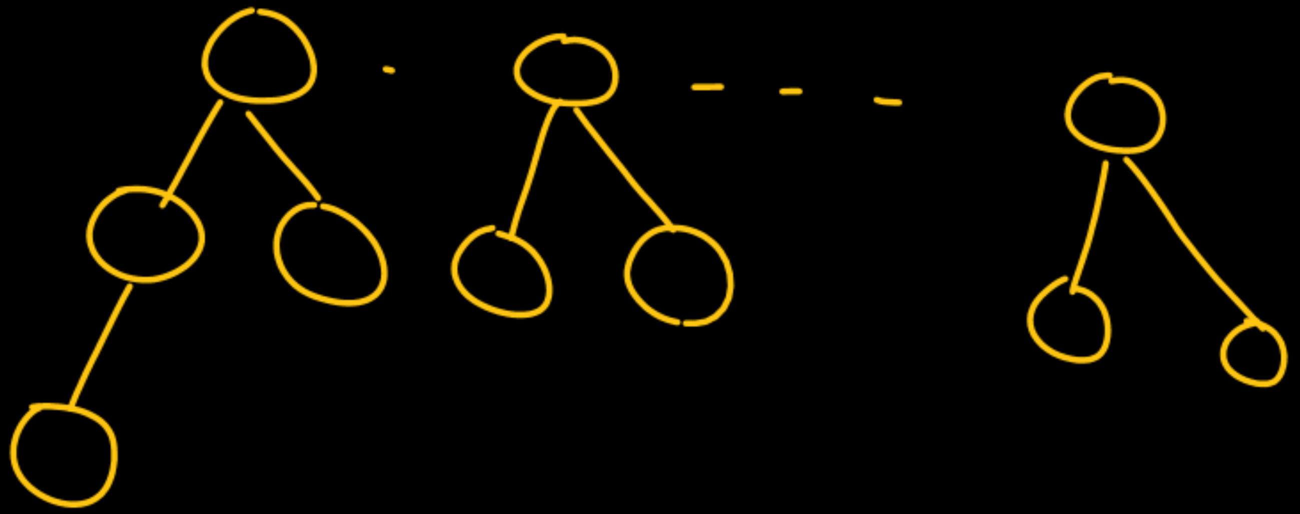

h = 3

| #Nodes | level |
| --- | --- |
| $2^0$ | 0 |
| $2^1$ | 1 |
| $2^2$ | 2 |
| ⋮ | ⋮ |
| $2^{h-1}$ | $h-1$ |
| 1 | $h$ |



$$S = \frac{a(r^n - 1)}{r-1}$$

$$\begin{bmatrix} a = 1 \\ r = 2 \\ n = h \end{bmatrix}$$



$$n_{min} = \left(2^0 + 2^1 + 2^2 + \cdots + 2^{h-1}\right) + 1 = \left(\frac{2^h - 1}{2-1}\right) + 1 = 2^h - \cancel{1} + \cancel{1} = 2^h$$

$$\boxed{2^h \le n \le 2^{h+1} - 1}$$

$x \le 4$ ← max

$x \ge 2$ ← min

$$\boxed{2 \le x \le 4}$$

$$2^h \le n$$

$$\log 2^h \le \log n$$

$$h \log 2 \le \log n$$

$$h \le \frac{\log n}{\log 2}$$

$$\boxed{h \le \log_2 n}$$

$$n \le 2^{h+1} - 1$$

$$(n+1) \le 2^{h+1}$$

$$\log(n+1) \le (h+1)\log 2$$

$$\frac{\log(n+1)}{\log 2} \le h+1$$

$$(h+1) \ge \log_2(n+1)$$

$$\boxed{h \ge \log_2(n+1) - 1}$$

$$\boxed{\log_2(n+1) - 1 \le h \le \log_2(n)}$$

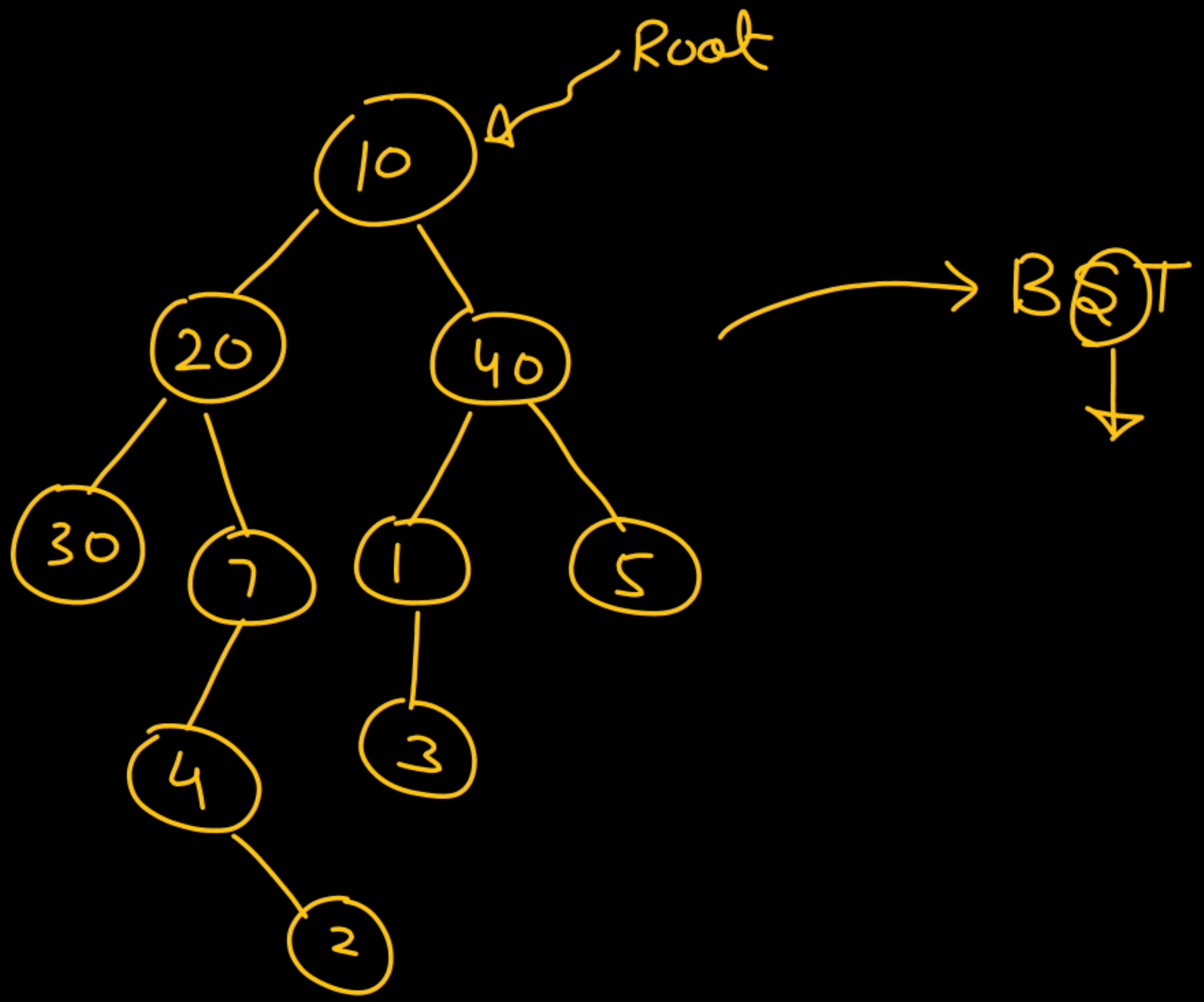
# Binary Search Tree

## Why?

Given, Binary tree and a key and we need to find whether the key is present in the tree or not?

key=100

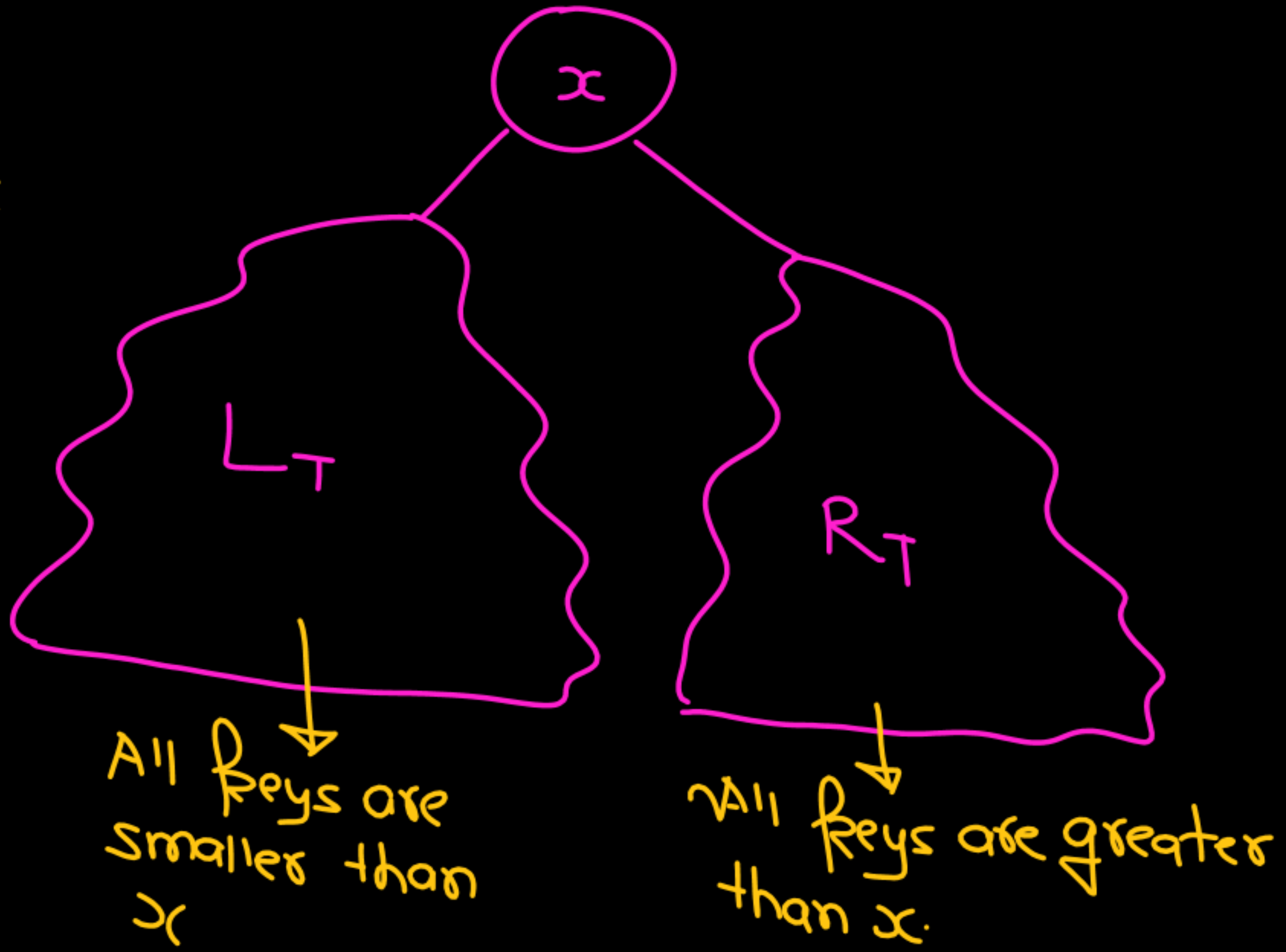
$O(n)$

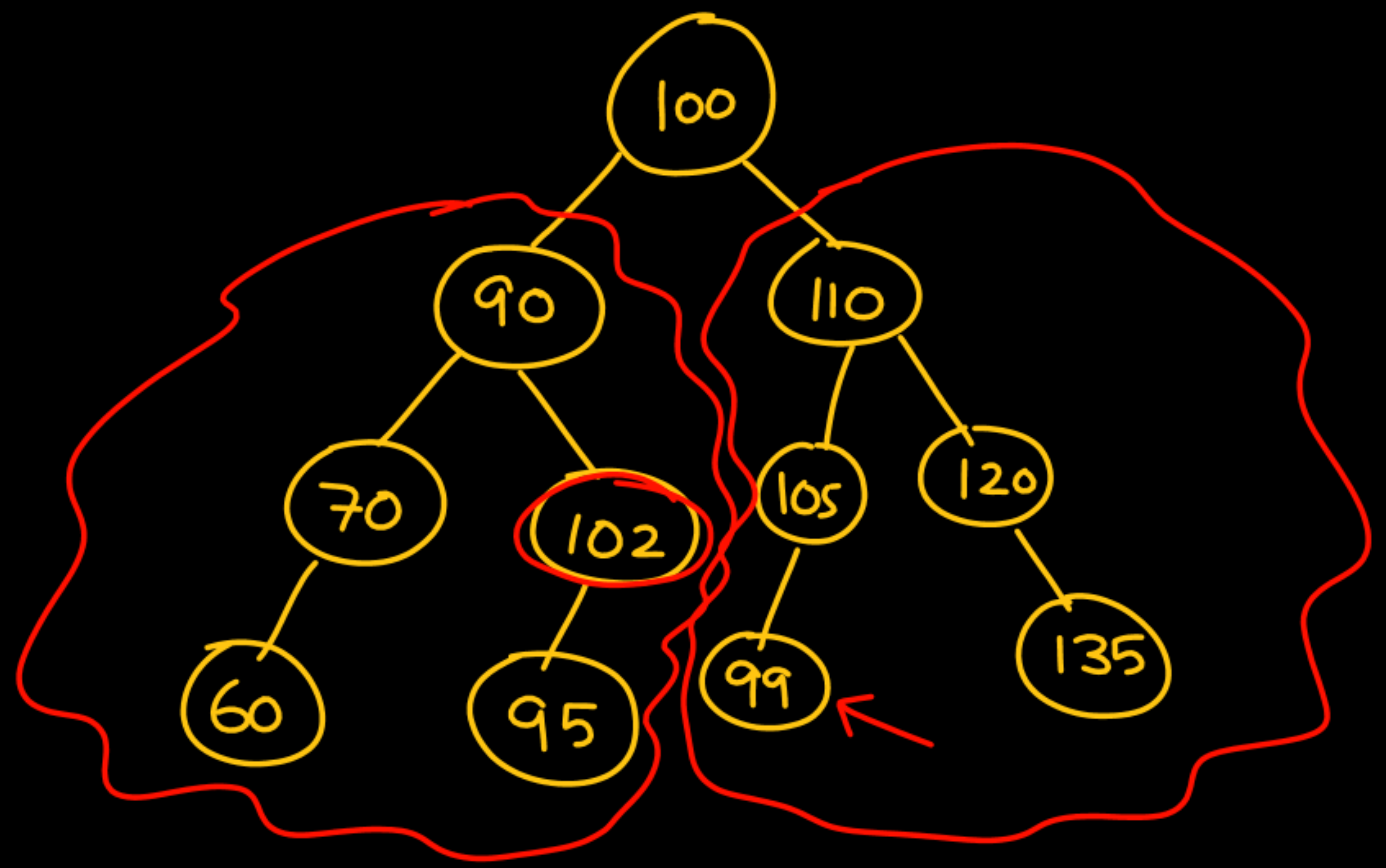


# Binary Search Tree

A binary tree in which $\boxed{\text{every node}}$ satisfies following property:

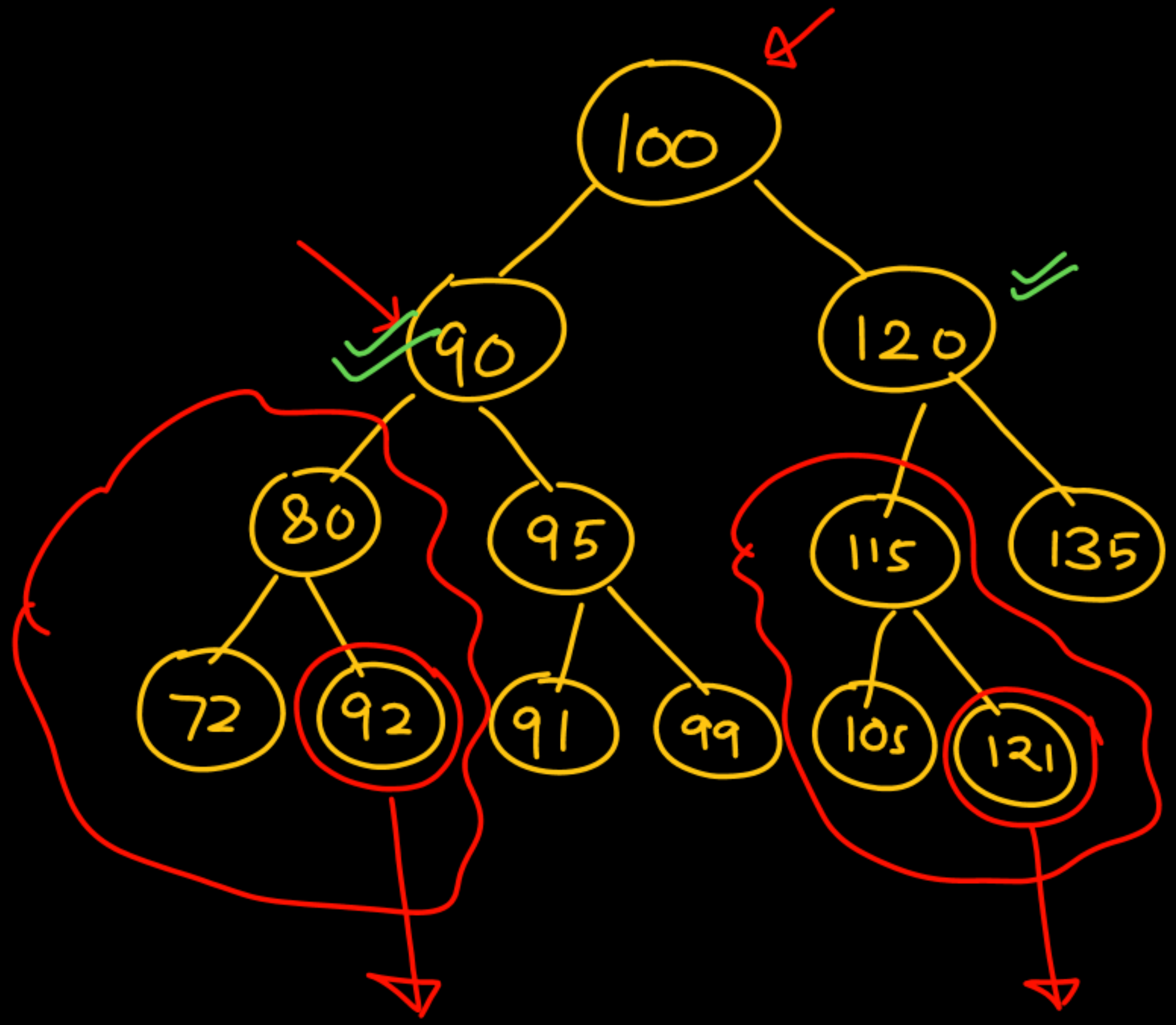
- All the keys in the left subtree of a node are smaller than the node value.
- All the keys in the right subtree of a node are greater than the node value.



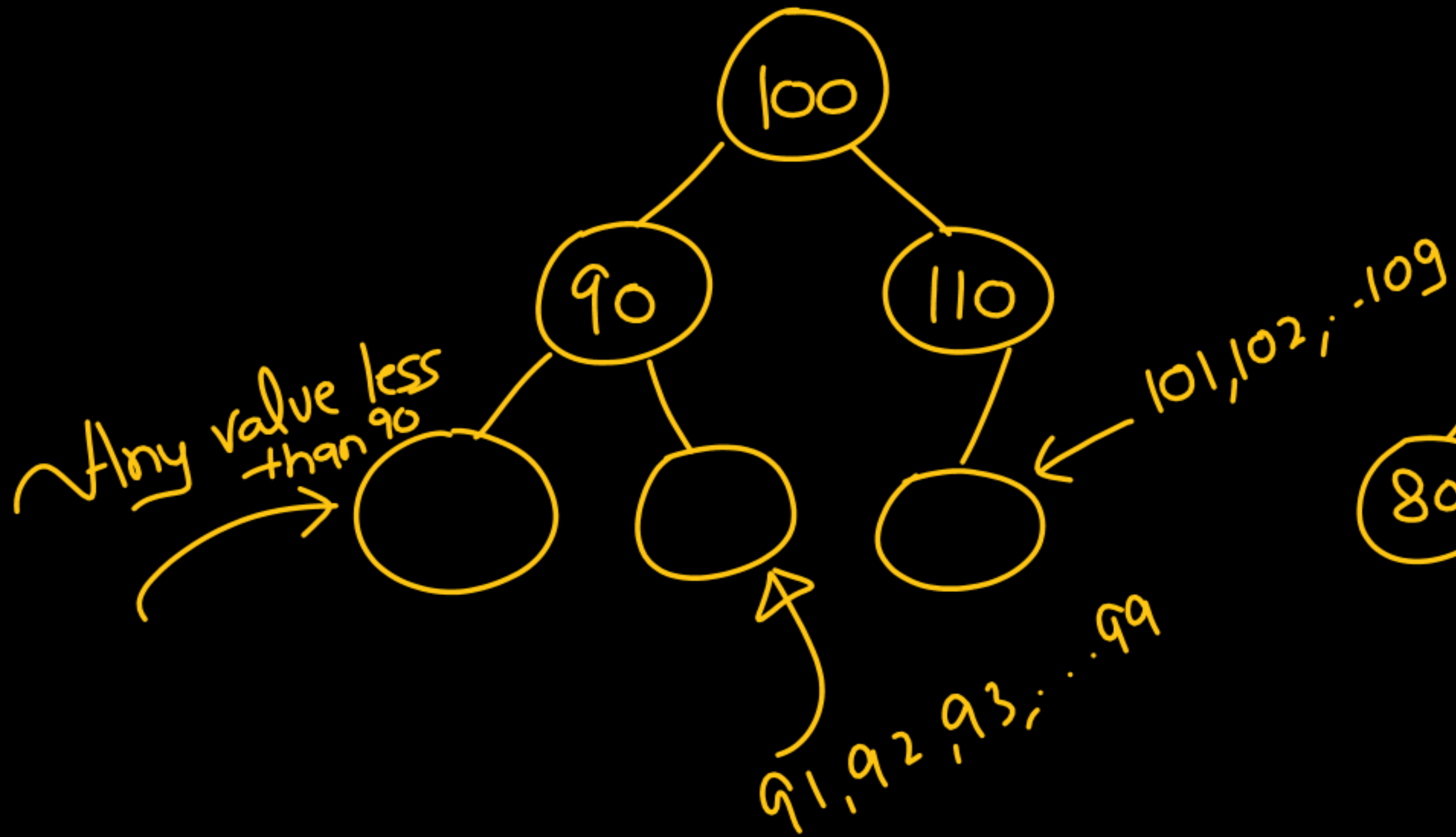


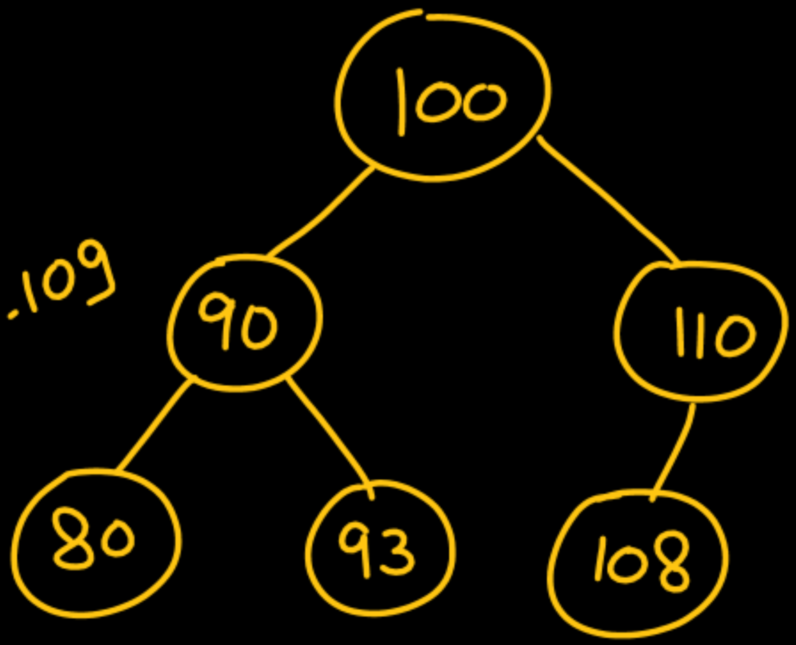
100
90
110
70
102
105
120
60
95
99
135


Is it a BST ?



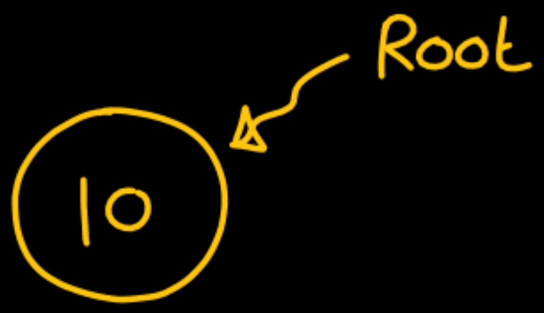
100
90
120
80
95
115
135
72
92
91
99
105
121


100
90
110
Any value less than 90
101,102,...109
91,92,93,...99

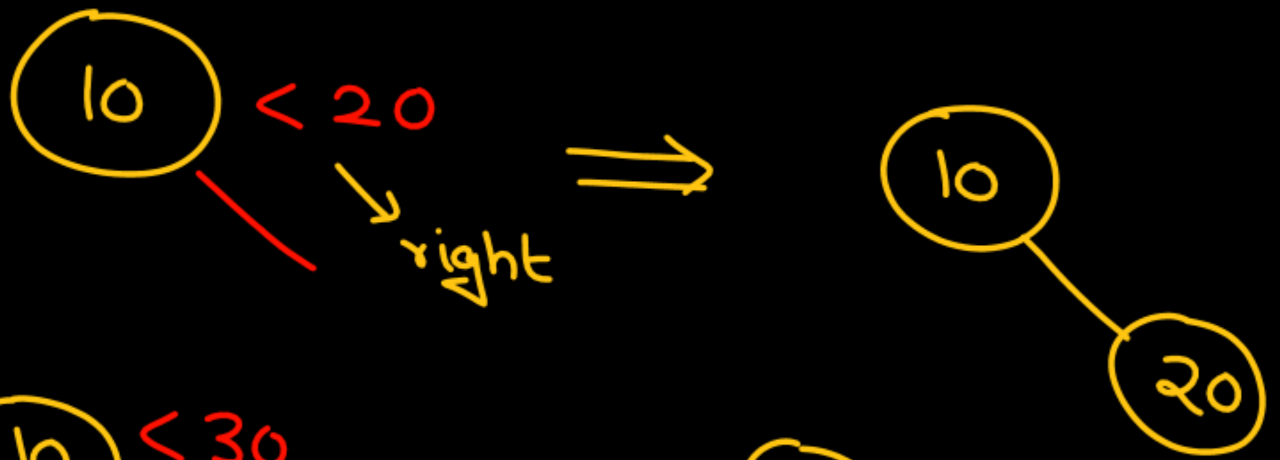

100
90
110
80
93
108

1. Construct a binary search tree by inserting keys 10, 20, 30 in order

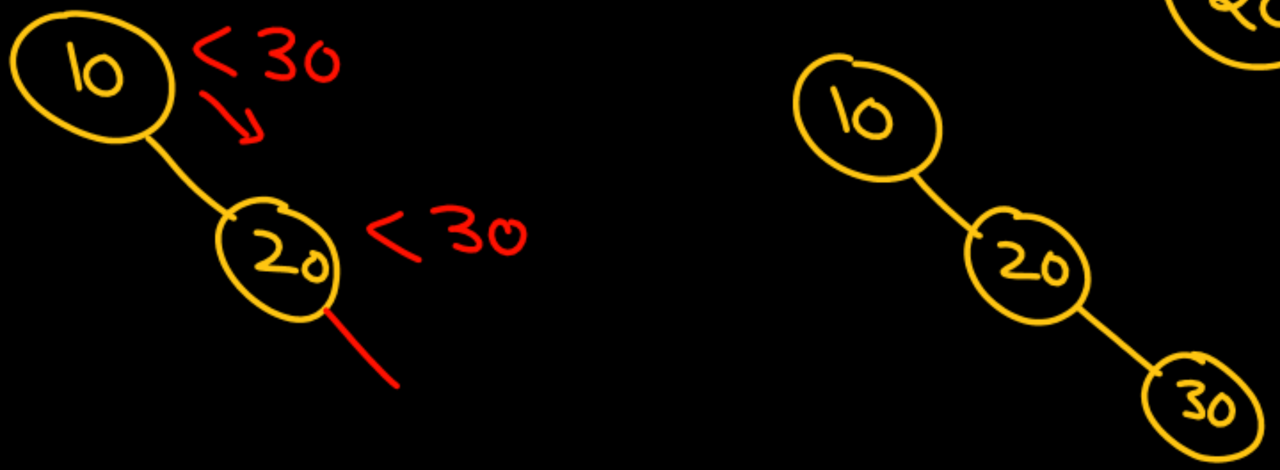
a)



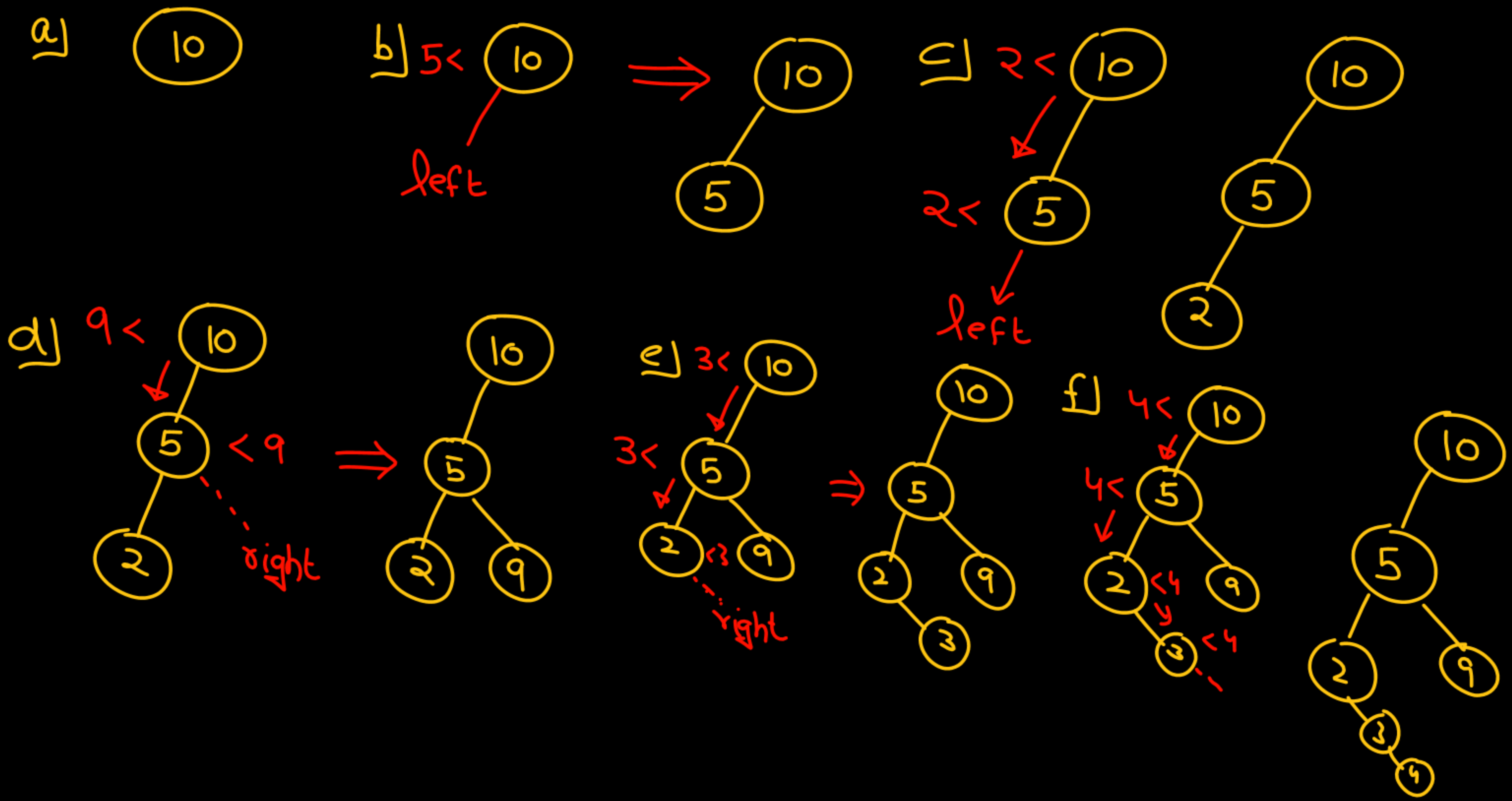

b)




c)



2. Construct binary search tree by inserting keys 10, 5, 2, 9, 3, 4 in order.



No. of BST when insertion order of keys are fixed

Const. BST by inserting keys 10, 20, 30 $\Longrightarrow 1$

Const. BST by inserting keys 10, 5, 2, 9, 3, 4 $\Rightarrow 1$

Const. BST by inserting keys 1,2,3 (in any order)

6 orders possible

(i) 1, 2, 3

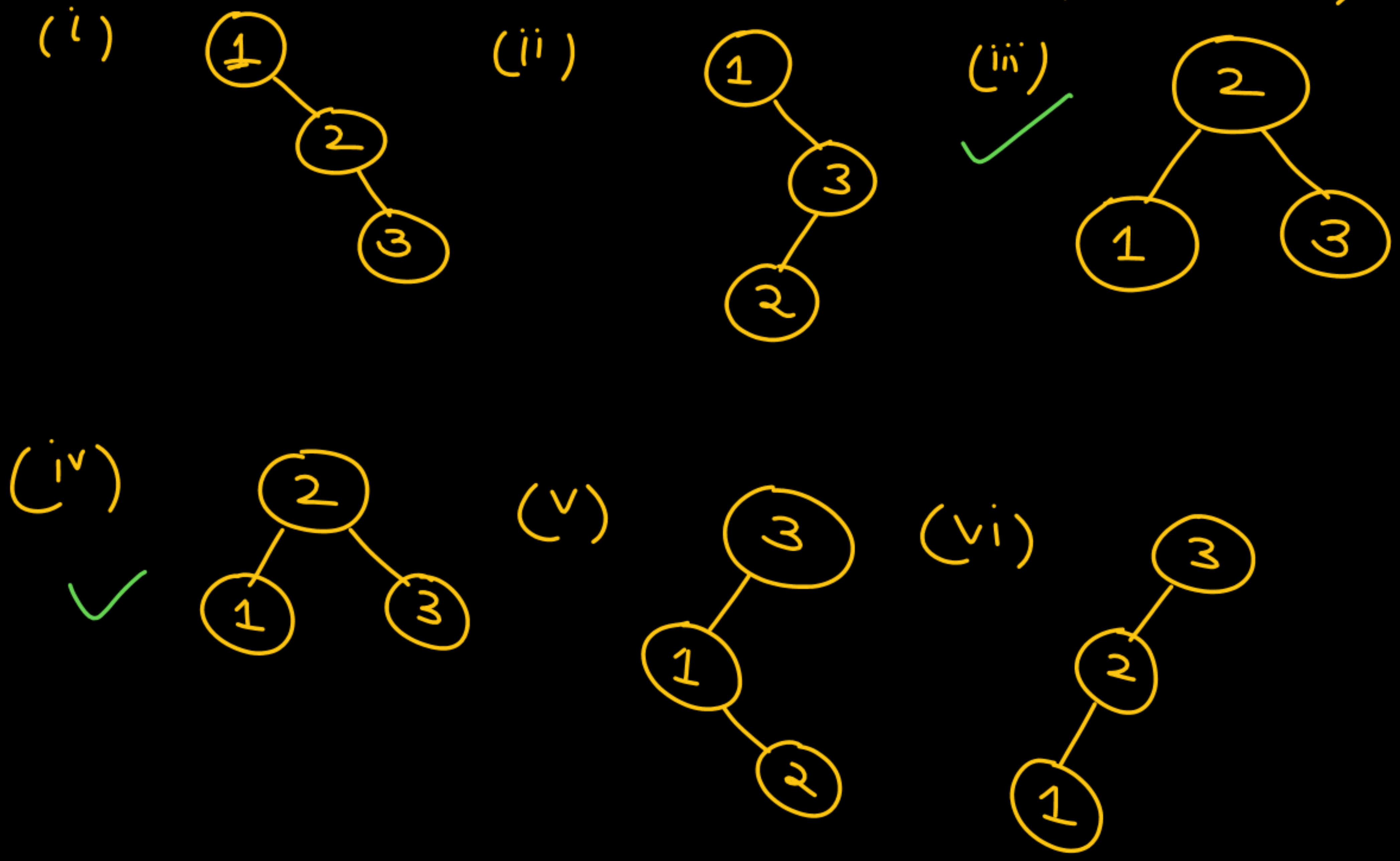
(ii) 1, 3, 2

(iii) 2, 1, 3

(iv) 2, 3, 1

(v) 3, 1, 2

(vi) 3, 2, 1

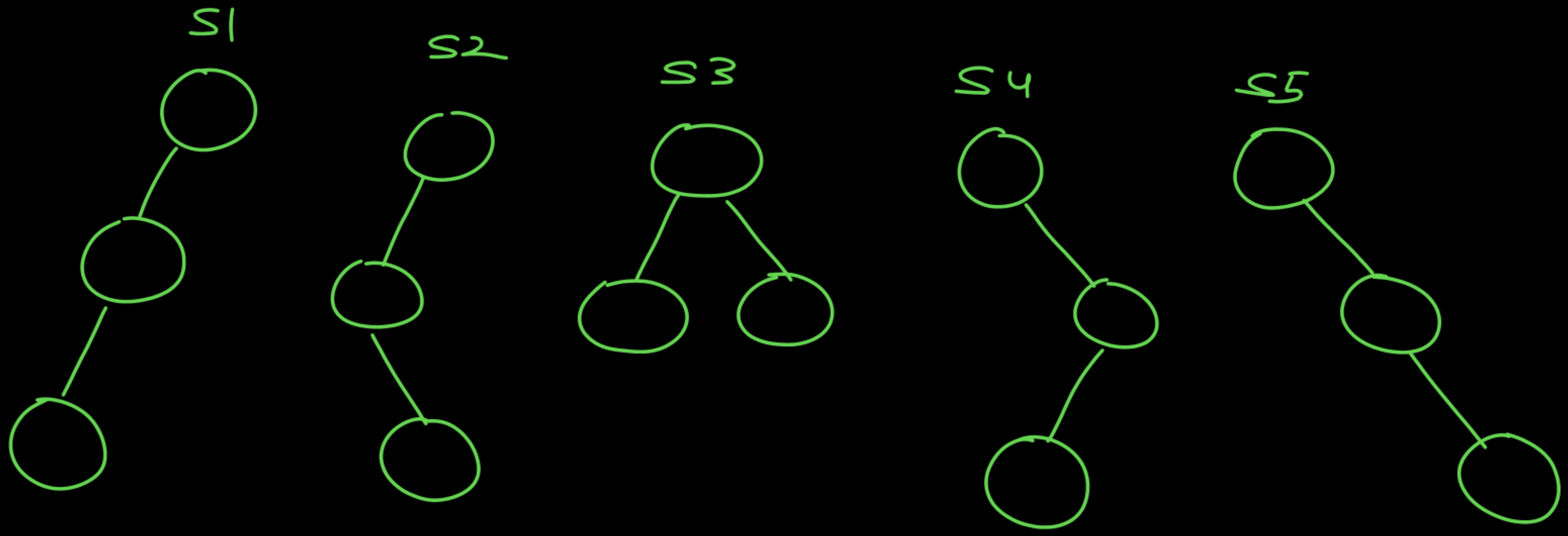


3 keys $\Rightarrow$ 5 BST

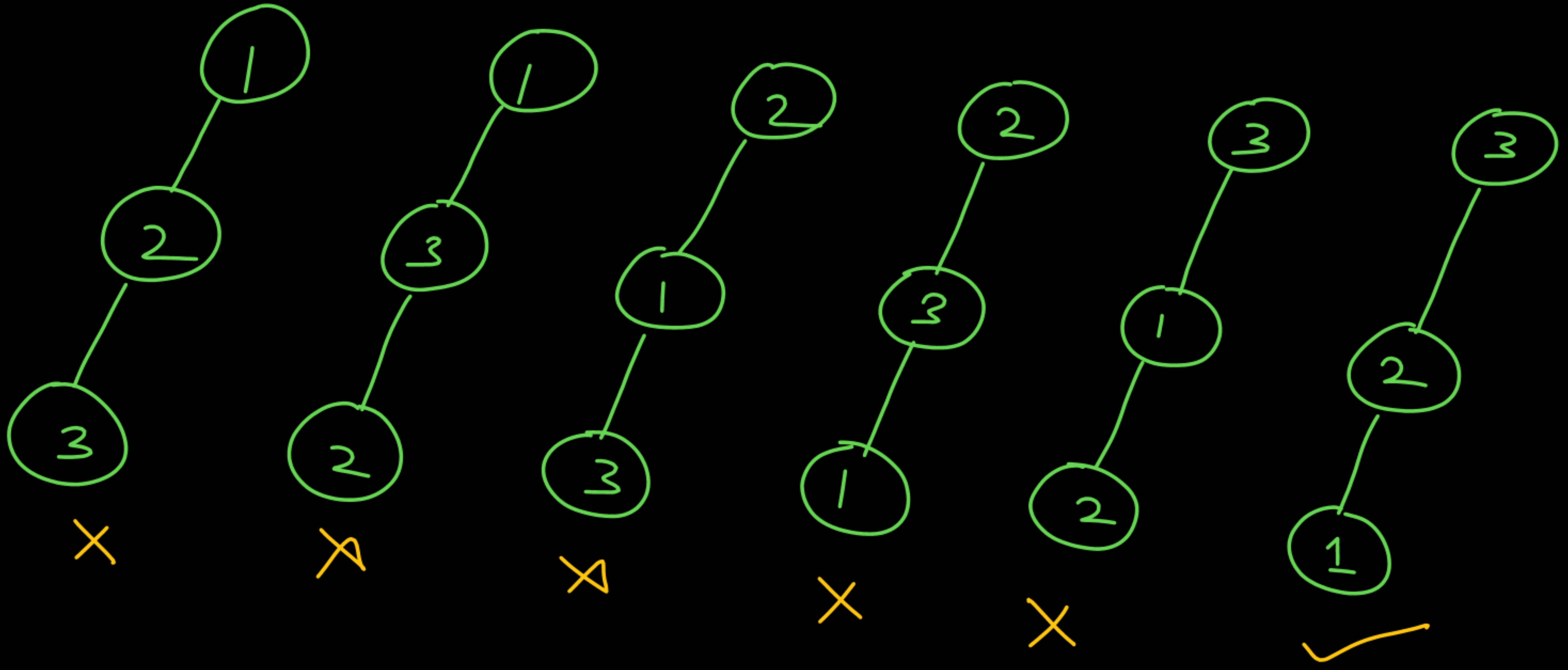
# unlabelled binary tree with 3 nodes = 5

# structure with n nodes = $\dfrac{{}^{2n}C_n}{n+1}$

3 node



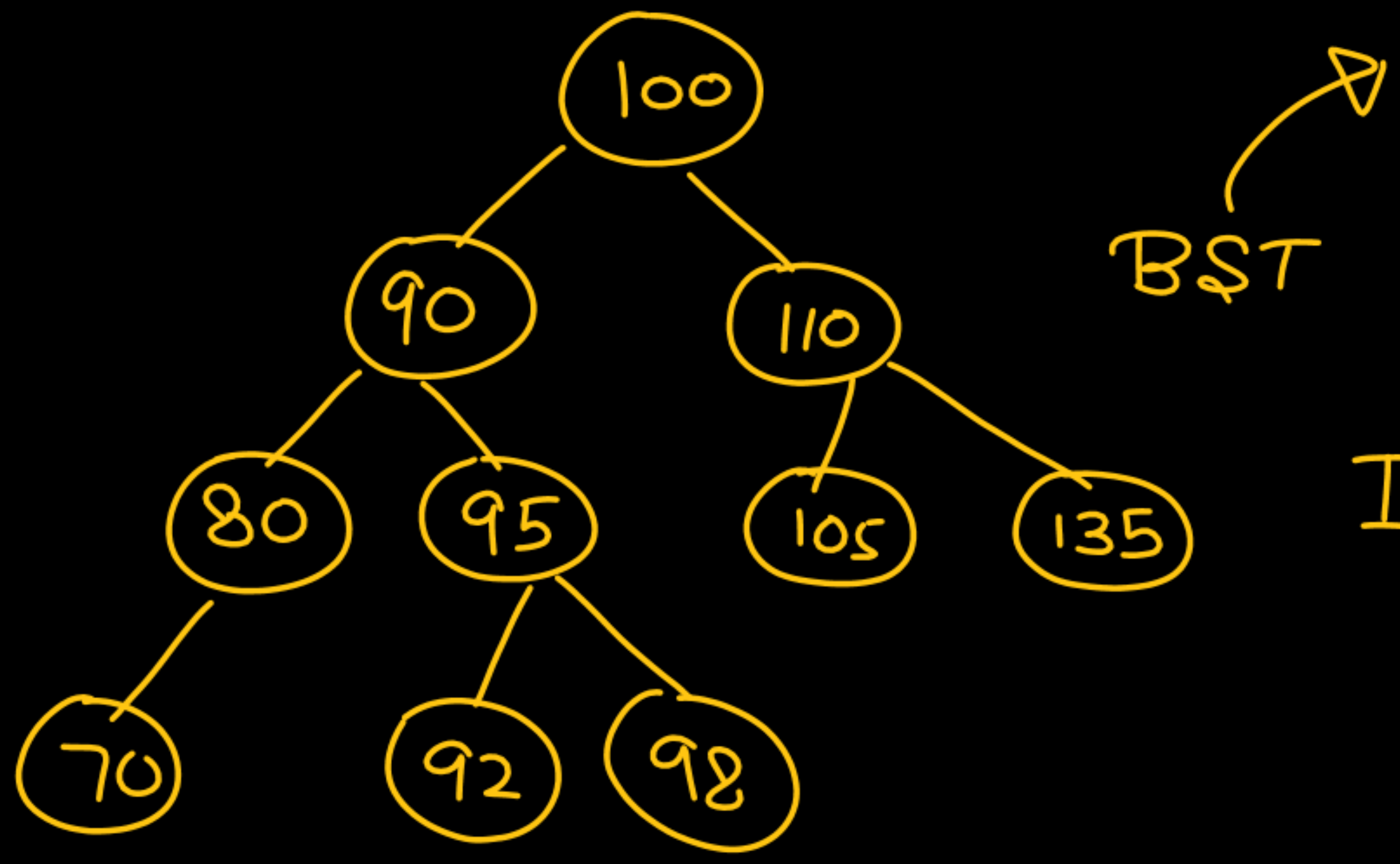
S1



# BST with $n$ keys $= \dfrac{{}^{2n}C_n}{n+1}$

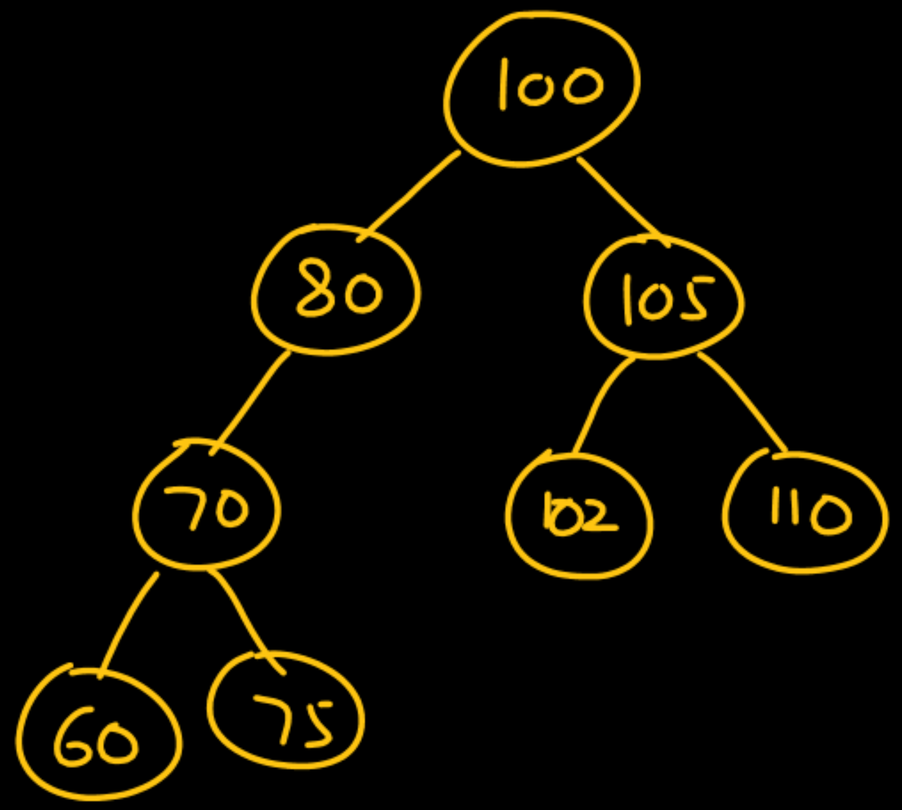
2 ******

Inorder traversal of a BST is always increasing order of keys.




Inorder : 70 80 90 92 95 98 100 105 110 135

Pre : 100 80 70 60 75 105 102 110



Pre : 100 80 70 60 75 105 102 110

Given the preorder of a BST as : 100, 80, 70, 60, 75, 105, 102, 110

the postorder of BST is ____

a)

b)

c)

d)

Pre : 100 80 70 60 75 105 102 110

Given the preorder of a BST as : 100, 80, 70, 60, 75, 105, 102, 110

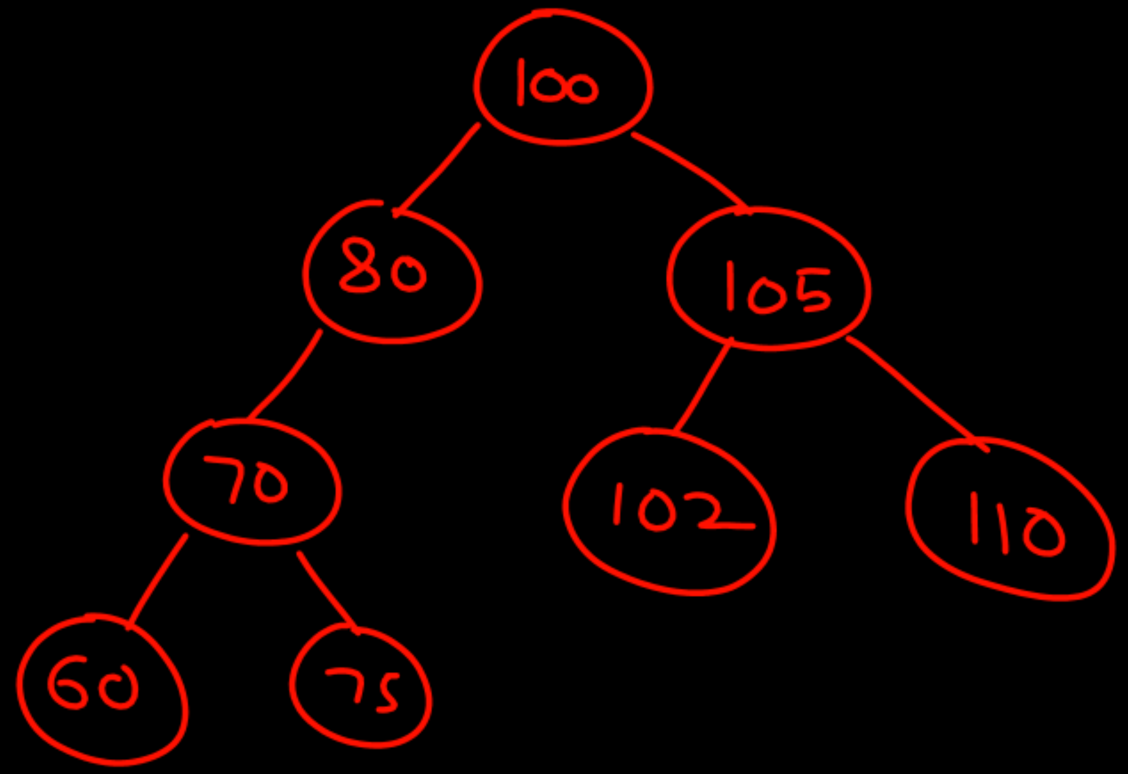
the postorder of BST is ______

In : 60 70 75 80 100 102 105 110

Pre : 100 80 70 60 75 105 102 110

In : 60 70 75 80 100 102 105 110

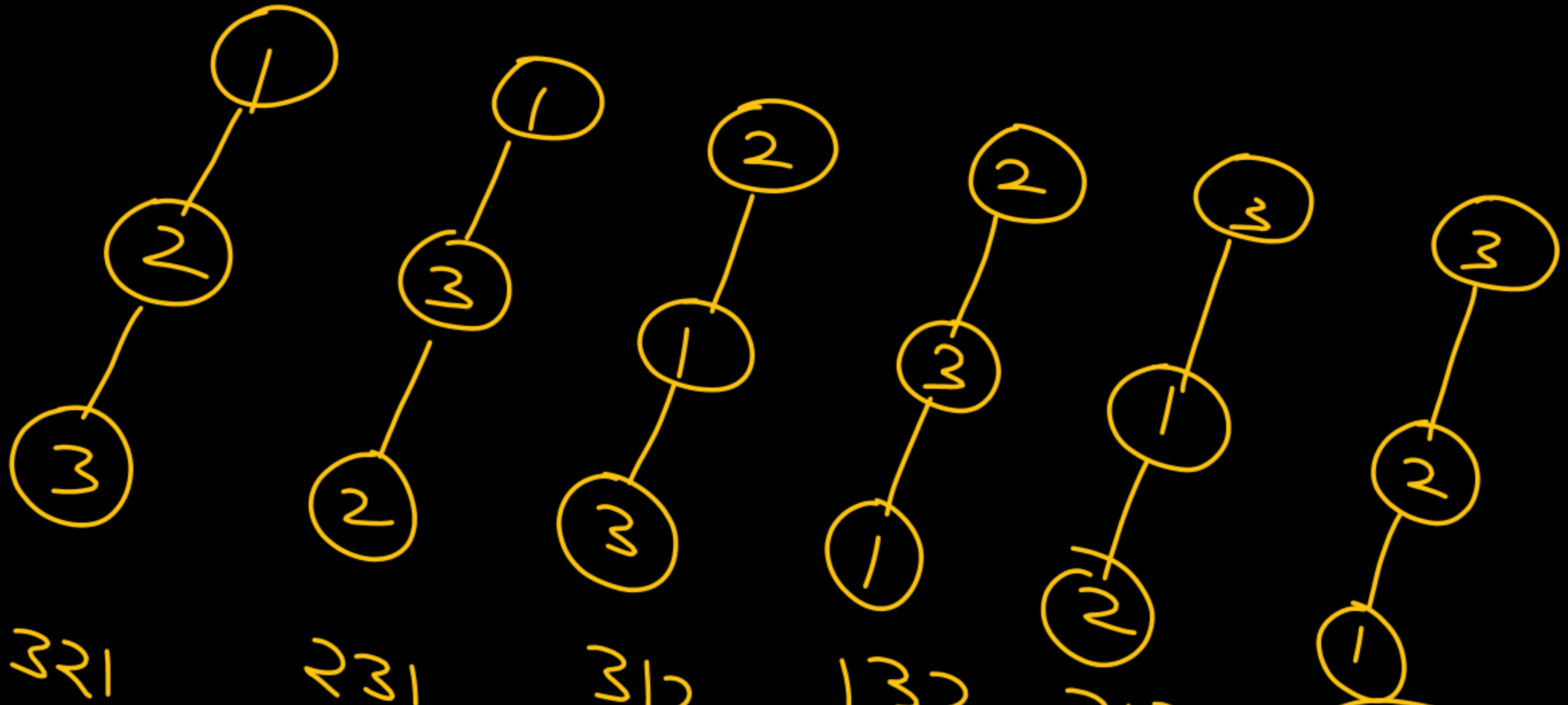
Pre : 100 80 70 60 75 105 102 110



Q BST

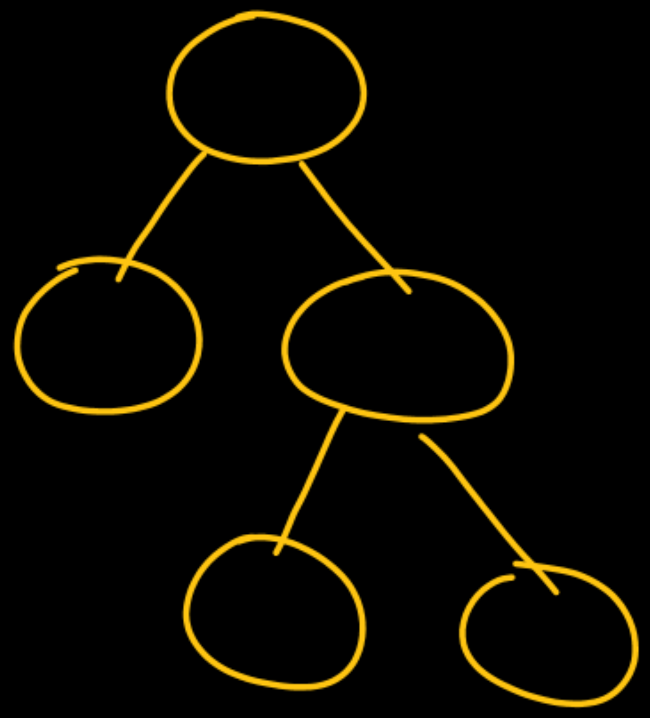
Inorder :

1,2,3 Keys



Inorder: 321 231 312 132 213 123

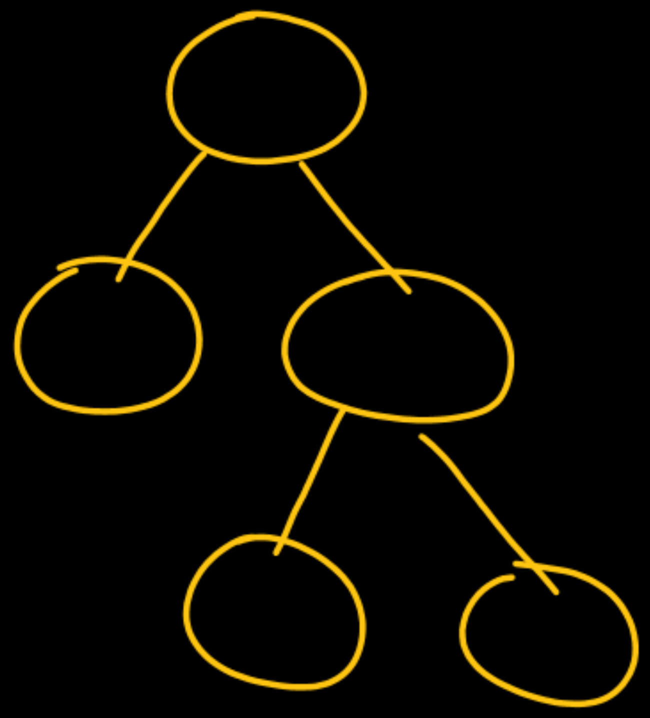
Given a binary tree structure with $n$ nodes (unlabelled)

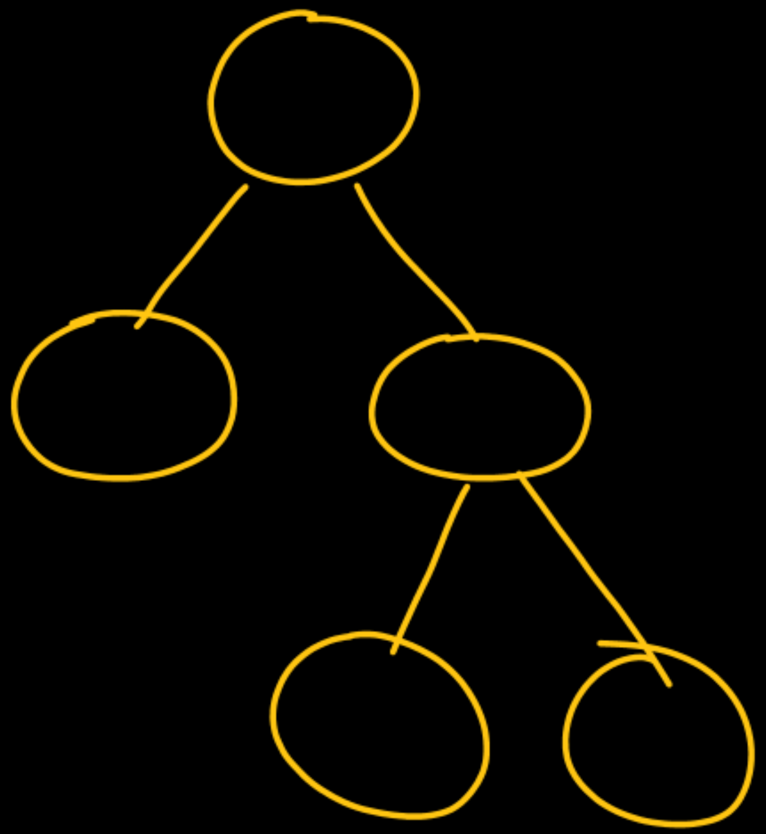


Given a binary tree structure with n nodes (unlabelled)

and n keys are also given

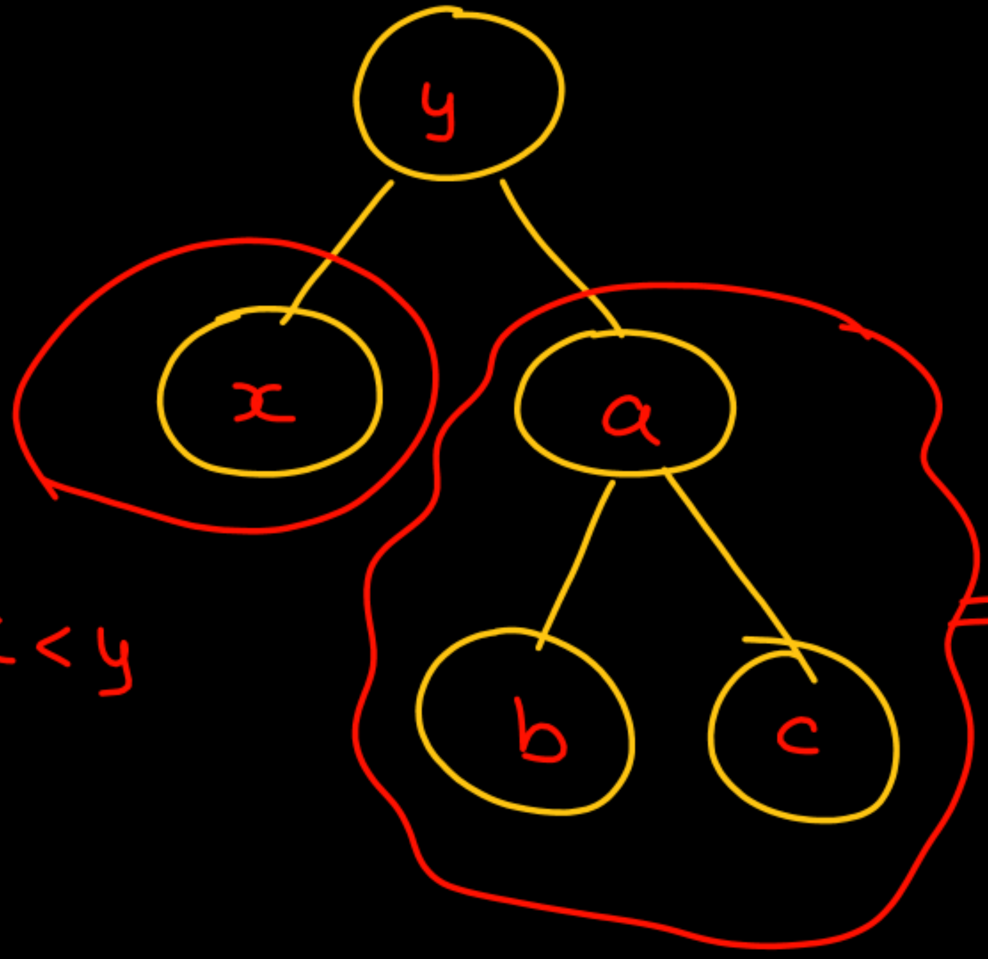
how many BSTs are possible





10, 5, 12, 2, 1

10, 5, 12, 2, 1




$\Rightarrow$ are greater than y

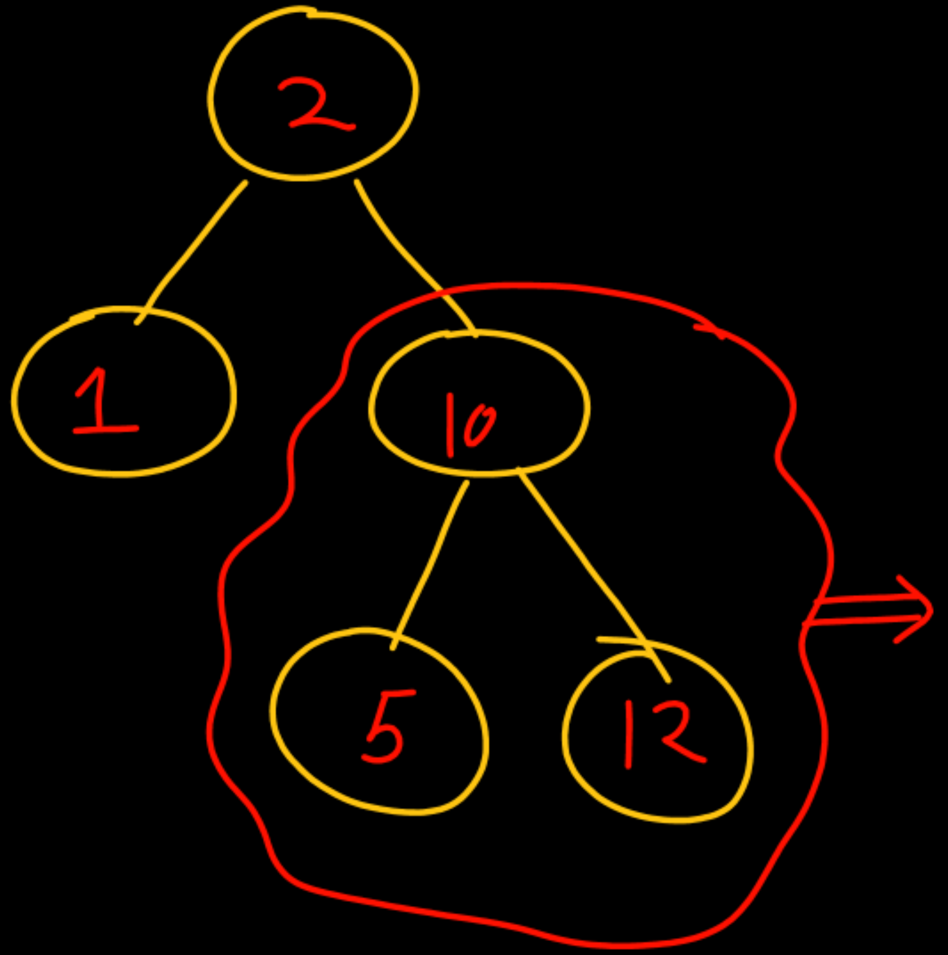
$x < y$

$y < a, b, c$

$x < y < a, b, c$

Min

$10, 5, 12, 2, 1$

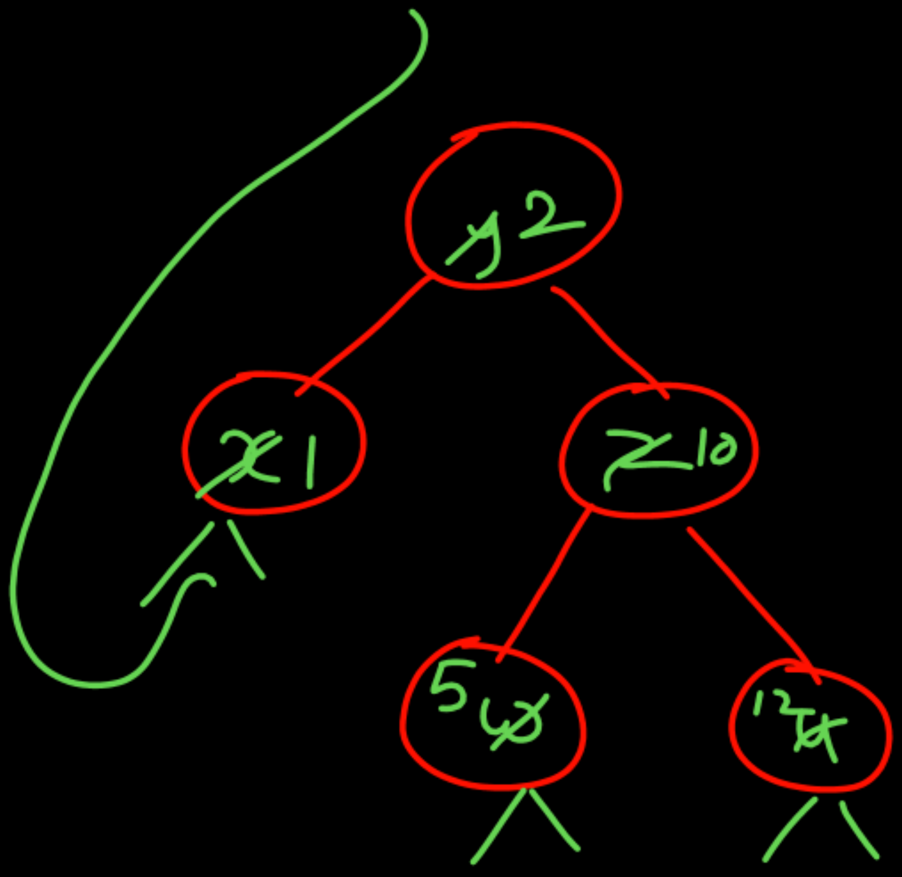



are greater than y

$y < a, b, c$

$x < y < a, b, c$

Min




10, 5, 12, 2, 1

1 2 5 10 12

x, y, w, z, u


THANK YOU GW
SOLDIERS !