**Branch : CSE/IT** **Batch : Hinglish**

# Discrete Mathematics II

## Set Theory

**DPP-06**

**[NAT]**

**1.** Consider the following statement about relations on a set A, where | A | = n. How many of the following statements are TRUE?

**I.** If R is a relation on A and $|R| \geq n$, then R is reflexive.

**II.** If $R_1$, $R_2$ are relations on A and $R_2 \supseteq R_1$, then $R_1$ is reflexive $\Rightarrow$ $R_2$ is reflexive

**III.** If $R_1$, $R_2$ are relations on A and $R_2 \supseteq R_1$, then $R_1$ is symmetric $\Rightarrow$ $R_2$ is symmetric

**IV.** If $R_1$, $R_2$ are relations on A and $R_2 \supseteq R_1$, then $R_1$ is anti-symmetric $\Rightarrow$ $R_2$ is anti-symmetric

**V.** If $R_1$, $R_2$ are relations on A and $R_2 \supseteq R_1$, then $R_1$ is transitive $\Rightarrow$ $R_2$ is transitive

**VI.** If $R_1$, $R_2$ are relations on A and $R_2 \supseteq R_1$, then $R_2$ is reflexive $\Rightarrow$ $R_1$ is reflexive

**VII.** If $R_1$, $R_2$ are relations on A and $R_2 \supseteq R_1$, then $R_2$ is symmetric $\Rightarrow$ $R_1$ is symmetric

**VIII.** If $R_1$, $R_2$ are relations on A and $R_2 \supseteq R_1$, then $R_2$ is anti-symmetric $\Rightarrow$ $R_1$ is anti-symmetric

**IX.** If $R_1$, $R_2$ are relations on A and $R_2 \supseteq R_1$, then $R_2$ is transitive $\Rightarrow$ $R_1$ is transitive

**X.** If R is an equivalence relation A, then $n \leq |R| \leq n^2$

**[MSQ]**

**2.** Consider A = {w, x, y, z}, then which of the following options is/are correct for the given set?

(a) The number of symmetric relations is $2^{10}$

(b) The number of symmetric relations which contain (x, y) is $2^9$

(c) The number of relations which is anti-symmetric and contain (x, y) is $2^4 \cdot 3^5$

(d) The number of relations which is reflexive, symmetric and anti-symmetric is 1

**[MCQ]**

**3.** Let A be a set with |A| = n, and let R be a relation on A that is antisymmetric, then which of the following option is correct?

(a) The maximum value for | R | is $(n^2 + n)/2$.

(b) The number of anti-symmetric relations have size | R | is $2^{(n^2+n)/2}$.

(c) Both a and b

(d) None of these.

**[MCQ]**

**4.** Let A be a set with |A| = n, and let R be an equivalence relation on A with |R| = r. Which of the following is TURE for the given relation R?

(a) r – n will be always even.

(b) r – n will be always odd.

(c) Both a and b

(d) None of these.

**[NAT]**

**5.** Let A = {1, 2, 3, 4, 5, 6, 7, 8}. How many symmetric relations on A contain exactly four ordered pairs?

# Answer Key

1. (3)
2. (a, b, c, d)
3. (c)
4. (a)
5. (1232)

# Hints and Solutions

**1. (3)**

(a) False: Let A = {1, 2} and R = {(1, 2), (2, 1)}.

(b) (i) Reflexive: True

(ii) Symmetric: False. Let A = {1, 2}, $R_1$ = {(1, 1)}, $R_2$ = {(1, 1), (1, 2)}.

(iii) Antisymmetric and Transitive: False. Let A = {1, 2}, $R_1$ = {(1, 2)}, $R_2$ = {(1, 2), (2, 1)}.

(c) (i) Reflexive: False. Let A = {l, 2}, $R_1$ = {(1, 1)}, $R_2$ = {(1, 1), (2, 2)}.

(ii) Symmetric: False. Let A = {1, 2}, $R_1$ = {(1, 2)}, $R_2$ = {(1, 2), (2, 1)}.

(iii) Antisymmetric: True

(iv) Transitive: False. Let A = {1, 2}, $R_1$ = {(1, 2), (2, 1)}, $R_2$ = {(1, 1), (1, 2), (2, 1), (2, 2)}.

**2. (a, b, c, d)**

**Option a: True.**

The number of symmetric relations is $2^{10}$

**Option b: True.**

The number of symmetric relations which contain (x, y) is $2^9$

**Option c: True.**

The number of relations which is anti-symmetric and contain (x, y) is $2^4 \cdot 3^5$

**Option d: True.**

The number of relations which is reflexive, symmetric and anti-symmetric is 1

**3. (c)**

**Option a:**

There are n ordered pairs of the form (x, x), $x \in A$. For each of the $(n^2 - n)/2$ sets {(x, y), (y, x)} of ordered pairs where $x, y \in A$, $x \neq y$, one element is chosen. This results in a maximum value of $n + (n^2 - n)/2 = (n^2 + n)/2$.

**Option b:**

The number of anti-symmetric relations have size | R | is $2^{(n^2+n)/2}$.

Hence, both option a and b is correct.

**4. (a)**

r – n counts the elements in R of the form (a, b), $a \neq b$. Since R is symmetric, r – n will be always even.

**5. (1232)**

An element of your relation can come from one of the following two ways:

**1.** Choice of an unordered pair {i, j}, which gives you two elements of your relation, namely the ordered pairs (i, j) and (j, i) (to maintain symmetry). The number of unordered pairs to choose from is $\binom{8}{2} = 28$

**2.** A singleton {i}, which gives you only one element of your relation, namely (i, i). The number of singletons to choose from is obviously 8.

One can get 4 elements in the relation in one of the following three ways:

1. Choice of 2 unordered pairs, 0 singletons in $\binom{28}{2}\binom{8}{0}$ ways
2. Choice of 1 unordered pair, 2 singletons in $\binom{28}{1}\binom{8}{2}$ ways
3. Choice of 0 unordered pairs, 4 singletons in $\binom{28}{0}\binom{8}{4}$ ways

So, the total is $\binom{28}{2}\binom{8}{0} + \binom{28}{1}\binom{8}{2} + \binom{28}{0}\binom{8}{4}$

= 1232