CS & IT ENGINEERING

DISCRETE MATHS
SET THEORY

Lecture No. 08

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TOPICS

01 TYPES OF RELATION

02 NUMBER OF RELATION

3 OPERATION ON RELATION

Relation:

$A = \{\ \}$

$B = \{\ \}$

$A \times B$

$\downarrow$

$\boxed{r_1 : a = b}$

$\downarrow$

Relation: $\{(2,2), (3,3)\}$.

$A = \{1, 2, 3\}$ $B = \{2, 3\}$.

$A \times B = \{(1,2)\ (1,3)\ (2,2)\ (2,3)\ (3,2)\ (3,3)\}$.

$\downarrow$

$\boxed{a > b}$

$\downarrow$

$R_2 : \{(3,2)\}$ ✓

Relation :

Subset of crossproduct of 2 sets.

→ conditions on A×B, result we are getting is called Relation.

$A \times B$

$\downarrow$

$\boxed{\text{Cd.}}$

$\downarrow$

→ { }

→ { ..... }

$A = \{1, 2, 3\}$

Total no. of subsets:

$$2^3 \left\{ \begin{array}{l} \phi \\ \{1\} \\ \{2\} \\ \{3\} \\ \{1\,2\} \\ \{1\,3\} \\ \{2\,3\} \\ \{1\,2\,3\} \end{array} \right.$$

1	2	3
0	0	0
1	0	0
0	1	0

2^3

$|A| = 3$ $|B| = 2$

$A = \{1, 2, 3\}$ $B = \{2, 3\}$.

$|A \times B| = 3 \cdot 2 = 6$.

$A \times B = \{ \ldots\ldots \}$ Total elements $= 6$

$\downarrow$

$\boxed{}$

Total relations = Total no. of subsets.

ϕ

$\{ \quad \}$

$\{$

$\vdots$

$A \times B$.

$|A| = m$ $|B| = n$.

$|A \times B| = m \cdot n$.

Total relations $= 2^{m \cdot n}$.

*: $|A| \rightarrow$ nonempty set.

$|A| = n$.

$|A \times A| = n^2$.

Total Relations $= 2^{n^2}$.

$$A = \{1, 2\}$$

$$A \times A = \{(1,1)\,(1,2)\,(2,1)\,(2,2)\}$$

$$2^4 \left\{ \begin{array}{l} \phi \\ \{(1,1)\} \leftarrow \\ \{(1,2)\} \leftarrow \\ \{(2,1)\} \\ \{(1,1)(1,2)\} \leftarrow \\ \vdots \end{array} \right.$$

Symmetric :

$(a, b) \in R \rightarrow (b, a) \in R$

$A = \{1, 2, 3\}$

$A \times A = \{ \dots\dots\dots \}$

Conditions

$(a, b) \in R \rightarrow (b, a) \in R$

Check

Symm

Result: $\{ \dots \}$

Symmetric :

$$\forall a \forall b \{(a, b) \in R \rightarrow (b, a) \in R\}$$

$R_2 : \{ (1, 2) \}$ X. not symm.

$$(a, b) \in R \rightarrow (b, a) \in R$$

$$\underbrace{(1, 2) \in R}_{T} \rightarrow \underbrace{(2, 1) \in R}_{F}$$

$a = 1$

$b = 2$

Relation1: $\{ (1, 2) (2, 1) \}$ ✓

Symmetric ✓

$$(a, b) \in R \rightarrow (b, a) \in R$$

$$\underbrace{(1, 2) \in R}_{T} \rightarrow \underbrace{(2, 1) \in R}_{T}$$

$a = 1$

$b = 2$

$R_3 = \{ (1,1)\ (1,2)\ (2,1) \}$ → symmetric.

$\forall a \forall b\ (a,b) \in R \to (b,a) \in R$.

$\underbrace{(1,1) \in R}_{T} \to \underbrace{(1,1) \in R}_{T}$

$a = 1$
$b = 1$

$(a,b) \in R \to (b,a) \in R$.

$\underbrace{(1,2) \in R}_{T} \to \underbrace{(2,1) \in R}_{T}$.

$a = 1$
$b = 2$.

if (a,b) is present then (b,a) must be present.

$R = \{ \quad \}$ Symmetric

$\underbrace{\underbrace{(a, b) \in R}_{F} \longrightarrow}_{\text{True}} (b, a) \in R$

- → doesnot have problem with same elements.
- → Demands flipping.

$$R_1 = \{ (1,1)(2,2)\boxed{(2,3)} \} \quad \times$$

$$R_2 = \{ (1\,2)(2\,1)\boxed{(2\,3)(3\,2)} \} \quad \checkmark$$

$$R_3 = \{ (\underline{1\,1})(\underline{2\,2})(\underline{3\,3}) \} \quad \checkmark$$

$A = \{1, 2, 3\}$

$A \times A = \{\ldots\ldots\}$ 9 elements.

Total Relations: 2^{n^2}

$\{\ \}$ ✓

$\{11\ 22\ 33\}$ ✓

$\{11\ 22\ 33\ (12)\ (21)\}$ ✓

$\{12\}$ ✗

$\{21\}$ ✗

Total no. of symmetric Relations:

$|A| = n = 3$

$A \times A$:

(1, 1)	(1, 2)	(1, 3)
(2, 1)	(2, 2)	(2, 3)
(3, 1)	(3, 2)	(3, 3)

$\rightarrow$ n elements

$\rightarrow 2^n$

Total elements $= n^2$

Diagonal $= n$.

Non diagonal $= n^2 - n$.

boxes/family $= \dfrac{n^2 - n}{2}$

2^n

$\downarrow$

1^n

RnS

$n \quad \dfrac{n^2-n}{2}$

Diago. boxes

$2^n \quad 2^{\frac{n^2-n}{2}}$

$A = \{1, 2, 3\}$. $|A| = n = 3$

$|A \times A| = n^2$.

$A \times A = \{ (11)\ (22)\ (33)\ \boxed{(12)\ (21)}\ \boxed{(13)\ (31)}\ \boxed{(23)\ (32)} \}$.

2 choices 2 2 choices 2 choices 2 choices 2 choices

$\longleftarrow n \longrightarrow$ | $\longleftarrow n^2 - n \longrightarrow$

2^n.

boxes $= \dfrac{n^2 - n}{2}$.

$\rightarrow 2^n \cdot 2^{\frac{n^2-n}{2}}$

$$\{ (11)\ (22) \} \quad \text{Total subsets} = \{ \phi \} \rightarrow 00$$

2 choices. 2 choices.

	11	22
$\{\phi\} \rightarrow$	0	0
$\{\{11\}\} \rightarrow$	1	0
$\{\{22\}\} \rightarrow$	0	1.
$\{\{11\ \ 22\}\} \rightarrow$	1,	1.

$$\{ 11\ \ 22\ \ 33 \} \quad 2^3.$$

(each element: 0/1)

Reflexive:

$$\forall a \in A \, (aRa)$$

OR

$$\forall a \in A \quad (a, a) \in R$$

$A = \{1, 2, 3\}$

$A \times A : \{ \ldots \ldots \}$

$R_1 = \{ (1,1) \, (1,2) \} \rightarrow$ not reflexive.

$A = \{1, 2, 3\}$

$\forall a \in A \, (a, a) \in R$

$(1, 1) \in R$ ✓

$R_2 = \{ \boxed{(11)\ (22)\ (33)}\ (12) \}$ → Reflexive

$\forall a \in A$. $(a, a) \in R$.

$A = \{1, 2, 3\}$.

$(1,1) \in R$.

$(2,2) \in R$.

$(3,3) \in R$.

$R_3 = \{\ \}$ ✗.

$R_4 = \{(11)(22)(12)\}$ ✗.

$R_5 = \{(12)\}$ ✗

$R_6 = \{(21)\}$ ✗.

$R_7 = \{(13)\}$

$R_8 = \{(11)(22)(33)(12)(21)\}$ ✓

$A = \{1, 2, 3\}$

Total elements $= n^2$

Diag $\to n$.

non diag $\to n^2 - n$

Tot

$A \times A = \begin{bmatrix} (11) & (12) & (13) \\ (21) & (22) & (23) \\ (31) & (32) & (33) \end{bmatrix} \to$ present

1 choice

Total reflexive.

$= 2^{n^2-n}$

R_1 .... (12)

R_2 .... (21)

R_3 .... (12)(21)

Diagonal.

boxes.

$RnS : 1^{n} \; 2^{\frac{n^2-n}{2}}$

present.

(11) (22) (33)

$x1$: 12 21

$x2$: 13 31

$x3$: 23 32.

$Rns = \{ \boxed{11\ 22\ 33} \ \ \boxed{12\ 21} \}$ ✓

Reflexive.

Symmetric.

$2^n \cdot 2^{\frac{n^2-n}{2}}$

2^{n^2-n}

$R \cap S$

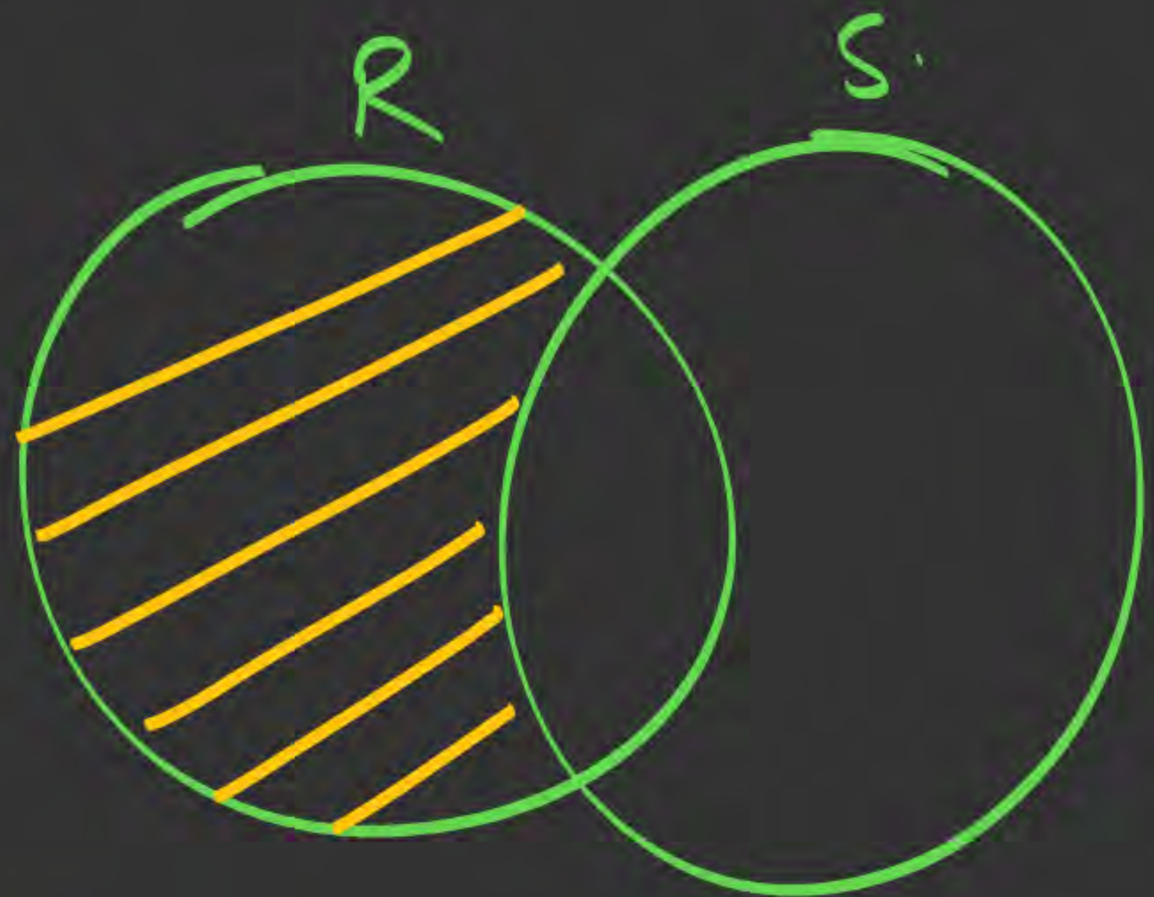
$2^{\frac{n^2-n}{2}}$

$(R \cap \overline{S}) = R - (R \cap S)$

$(S \cap \overline{R}) = S - (R \cap S)$

$|R \cup S| = |R| + |S| - |R \cap S|$

$= 2^{n^2-n} + 2^n \cdot 2^{\frac{n^2-n}{2}} - 2^{\frac{n^2-n}{2}}$



$$R \cap \overline{S} = R - (R \cap S)$$

$$|H| = 40 \quad |E| = 30$$

$$|H \cap E| = 10$$

$$H \cap \overline{E} = H - |H \cap E|$$

A: → nonempty set:

A × A

$R_1 \to$ symmetric. $\{(1,2)(2,1)\}$ ✓

$R_2 \to$ symmetric. $\{(1,3)(3,1)\}$ ✓

$R_1 \cup R_2$:

$R_1 \cap R_2$:

Assume:

Union is not symm.

$R_1 \cup R_2 = \{(a,b) \ldots (b,a)\}$

R_1

$(a,b)(b,a)$

R_2:

	Union	Intersection
R_1, R_2 symmetric.	✓	✓
R_1, R_2 are reflexive.	✓	✓

$R_1 = \{ \boxed{} \ldots$ $R_2 = \{ \boxed{} \ldots$

R_1, R_2 are reflexive relation

$R_1 \cup R_2$:

$R_1 \cap R_2$:


THANK YOU GW
SOLDIERS !