

# CS & IT ENGINEERING

Theory of Computation  
Push Down Automata



Lecture No. 01



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01

Revision of Regulars

02

CFL

03

04

05



# Regular Languages

- ① Regular Expression
- ② FA
- ③ RG : LLG / RLG
- ④ Regulars & Non Regulars
- ⑤ closure properties



# Regular Expressions:



$$\begin{aligned} ① & (a+b)^* \\ &= (a^*+b^*)^* \\ &= (a^*+b^*)^+ \\ &= (a^++b^+)^* \\ &= (a+b)^++\epsilon \\ &= (a^*b^*)^* \\ &= (a^*b^*)^+ \\ &= (b^*a^*)^* \\ &= (b^*a^*)^+ \\ &= b^*(ab^*)^* \\ &= a^*(ba^*)^* \\ &= (b^*a)^*b^* \\ &= (a^*b)^*a^* \end{aligned}$$

$$\begin{aligned} ② & aX \\ ③ & abX \\ ④ & aaaX \\ ⑤ & Xa \\ ⑥ & Xab \\ ⑦ & Xaaa \\ ⑧ & XaX \\ ⑨ & XabX \\ ⑩ & XaaaX \\ ⑪ & (a+b)^2 = x^2 \\ ⑫ & (\epsilon + a + b)^2 \\ ⑬ & (a+b)^2 X \end{aligned}$$

$$\begin{aligned} ⑭ & a^*b^* \\ ⑮ & b^*a^* \\ ⑯ & (aa)^* \\ ⑰ & a(aa)^* \\ &= (aa)^*a \\ ⑱ & b^*ab^*a^*b^* \\ ⑲ & b^*(a+\epsilon)b^*(a+\epsilon)b^* \\ ⑳ & XaXaX \\ ㉑ & [(a+b)^2]^* \\ ㉒ & (x^2)^*x \end{aligned}$$

$$\begin{aligned} ㉓ & (b^*ab^*ab^*)^* + b^* \\ &= b^*(b^*ab^*ab^*)^* \\ &= (b^*ab^*ab^*)^*b^* \\ &= \underline{b^*}(b^*ab^*ab^*)^*\underline{b^*} \\ &= (b^*ab^*a)^*b^* \\ &= b^*(ab^*ab^*)^* \\ ㉔ & b^*(b^*ab^*ab^*)^*b^*ab^* \\ ㉕ & (b^*ab^*ab^*ab^*)^* + b^* \end{aligned}$$

$$\begin{aligned} ㉖ & \phi^0 = \epsilon \\ ㉗ & \phi^* = \epsilon \\ ㉘ & aa^* = a^+ \\ ㉙ & a^* \\ &= (a^*)^* \\ &= (a^*)^+ \\ &= (a^+)^* \\ &= a^++\epsilon \\ &= (a^*)^{100} \end{aligned}$$

$$\begin{aligned} ㉚ & a^+ \\ &= aa^* \\ &= a^*a \\ &= aa^*a^* \\ &= a^*aa^* \\ &= (a^*)^{100}a \\ &= a^+a^* \\ &= a^*a^+ \end{aligned}$$

$$\begin{aligned} X &= (a+b)^* \\ x &= (a+b) \end{aligned}$$



①  $(a+b)^*$



$S \rightarrow aS \mid bS \mid \epsilon$

$S \rightarrow Sa \mid Sb \mid \epsilon$

② RegExp

DFA  
NFA  
LLG  
RLG

③

④

⑤



30 languages

Page 1  $\Rightarrow$  Reg Exp

Page 2 & 3  $\Rightarrow$  DFA  
NFA

Page 4 & 5  $\Rightarrow$  LLG  
RLG

Page 6 & 7  $\Rightarrow$  Reg & Non Reg

Page 8  $\Rightarrow$  closure properties

Max 10 pages



# Equivalence classes [Myhill-Nerode equivalence classes]



I) No. of states in min DFA for Reg = No. of equivalence classes for Reg

→ Find no. of equivalence class for  $L = ab(a+b)^*$ .  
Regular lang.

= 4 equivalence classes

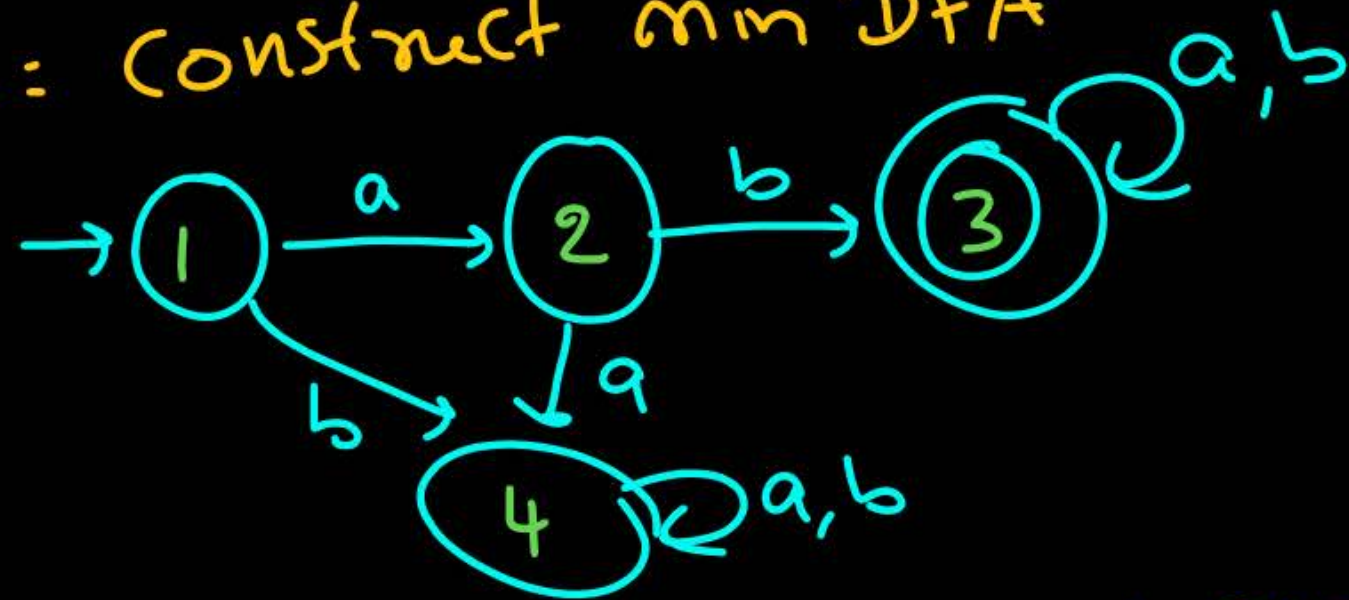
[1] =  $\epsilon$

[2] =  $a$

[3] =  $ab(a+b)^*$

[4] =  $(b+aa)(a+b)^*$

→ Step 1: Construct min DFA



→ Step 2: Each state represent one equivalence class.

$$[1] \cup [2] \cup \underbrace{[3]} \cup [4] = (a+b)^*$$

$$[1] \cap [2] = \emptyset$$

$$[1] \cap [3] = \emptyset$$

$$[1] \cap [4] = \emptyset$$

$$[2] \cap [3] = \emptyset$$

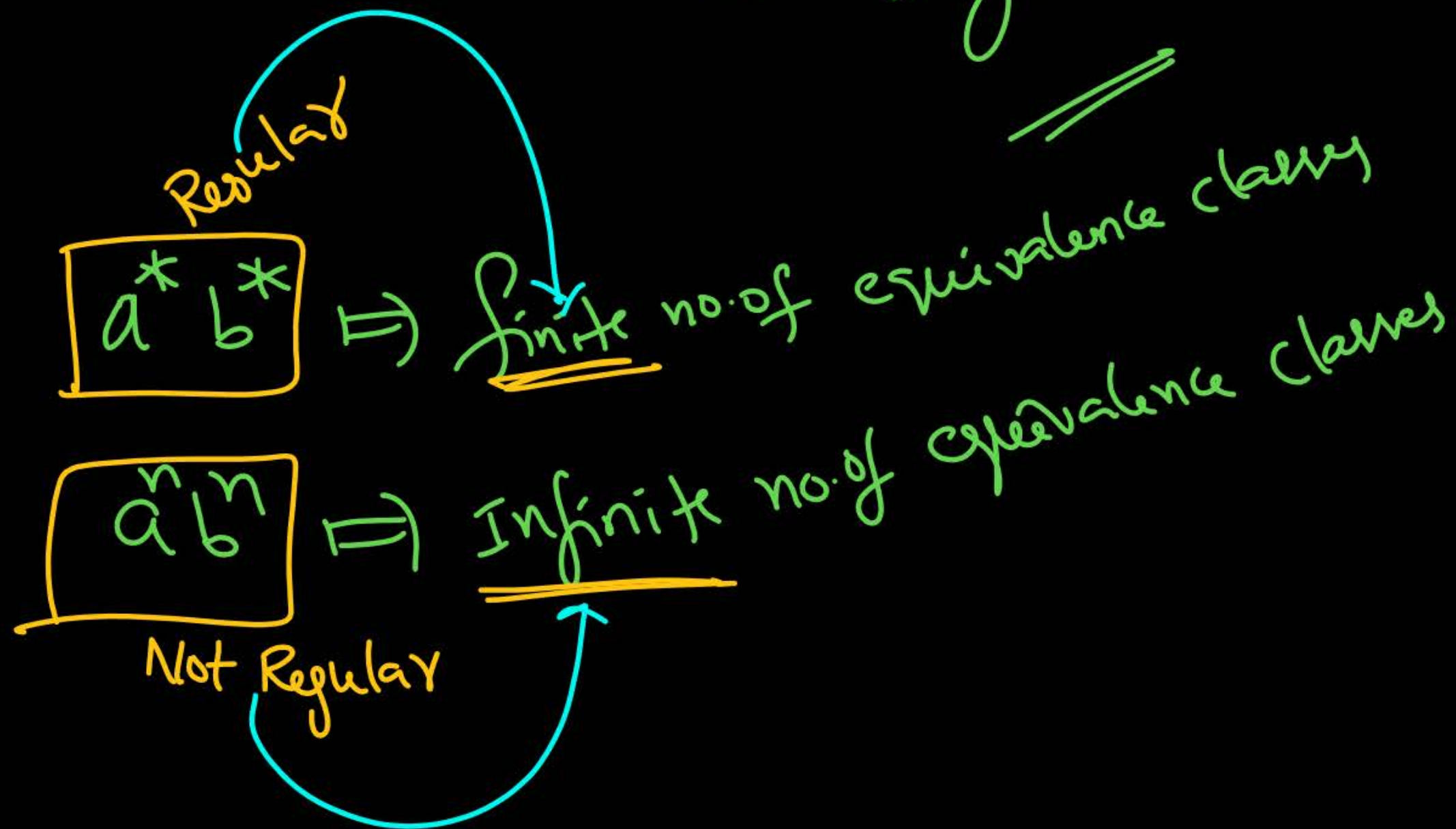
$$[2] \cap [4] = \emptyset$$

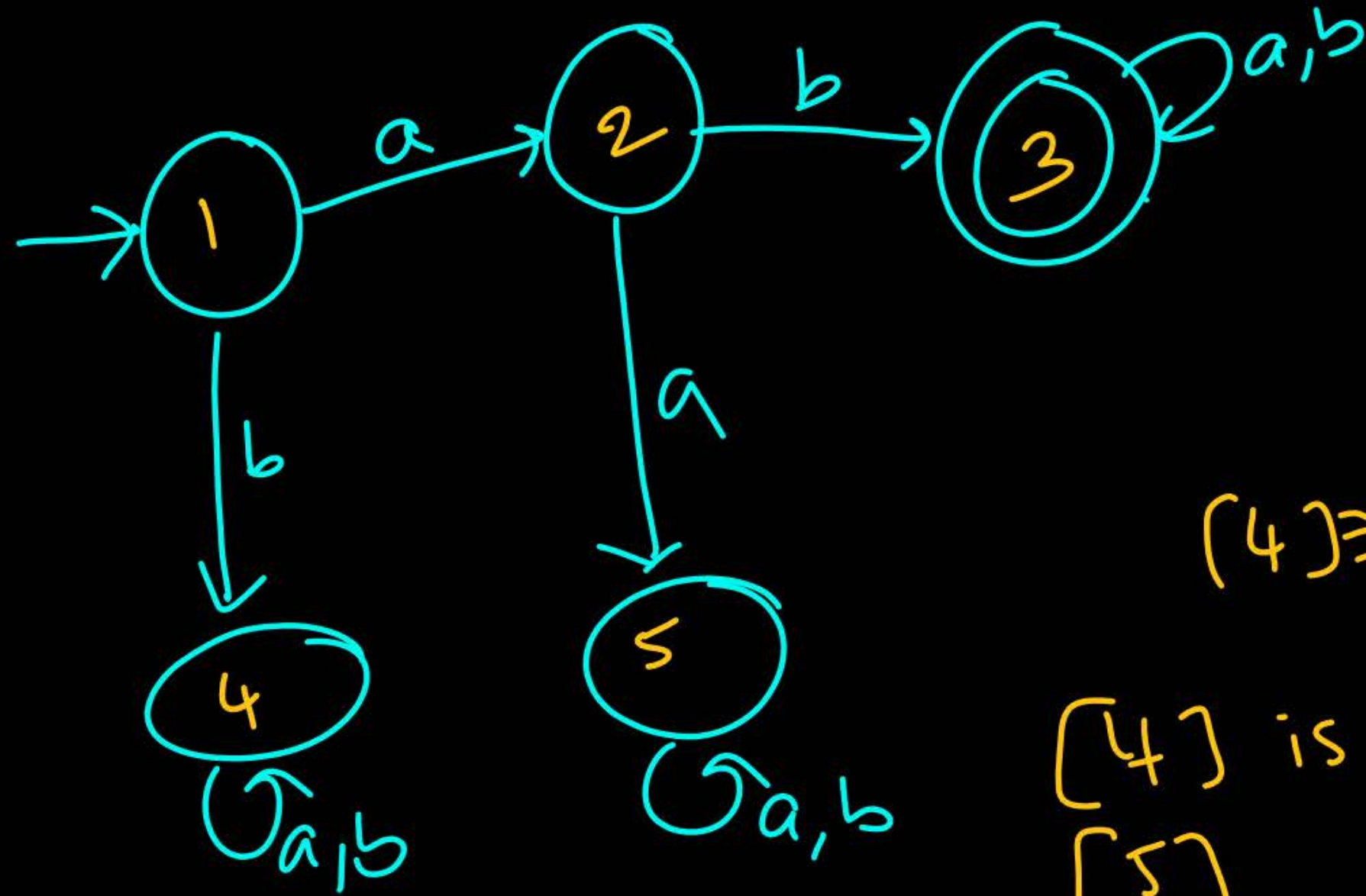
$$[3] \cap [4] = \emptyset$$



## II) No. of equivalence classes for Non regular languages

Infinite





$$[4] \cong [5] \cong [45]$$

$[4]$  is not equivalence class  
 $[5]$  " "  
 $[45]$  is equivalence class





$X$  and  $Y$  are 2 different equivalence classes

iff

Some string  $\in \Sigma^*$   $\rightarrow$

one of  $Xw$  and  $Yw$   
is valid and other is  
 $\in L$   $\notin L$



Learn

will

Limitation

Apply

will

no limitation



# Context Free Languages:



CFLs

Regulars

$a^*b^*$

$(a+b)^*$

$a^*b^*$

$a^*$

$L_1$

$L_2$

$b^*$

$\phi$   
 $\{a^n | n < 10\}$  finite  
 $\{a^n b^n | n < 100\}$  languages

$(\epsilon + a + b)^{100}$

Inf & Regs

$a^n b^n$

$a^n b^{2n}$

$a^{2n} b^n$

$L_3$

$a^n b^n a^n$

Not CFLs

$L_4$

Non Regulars

# Context Free Languages:



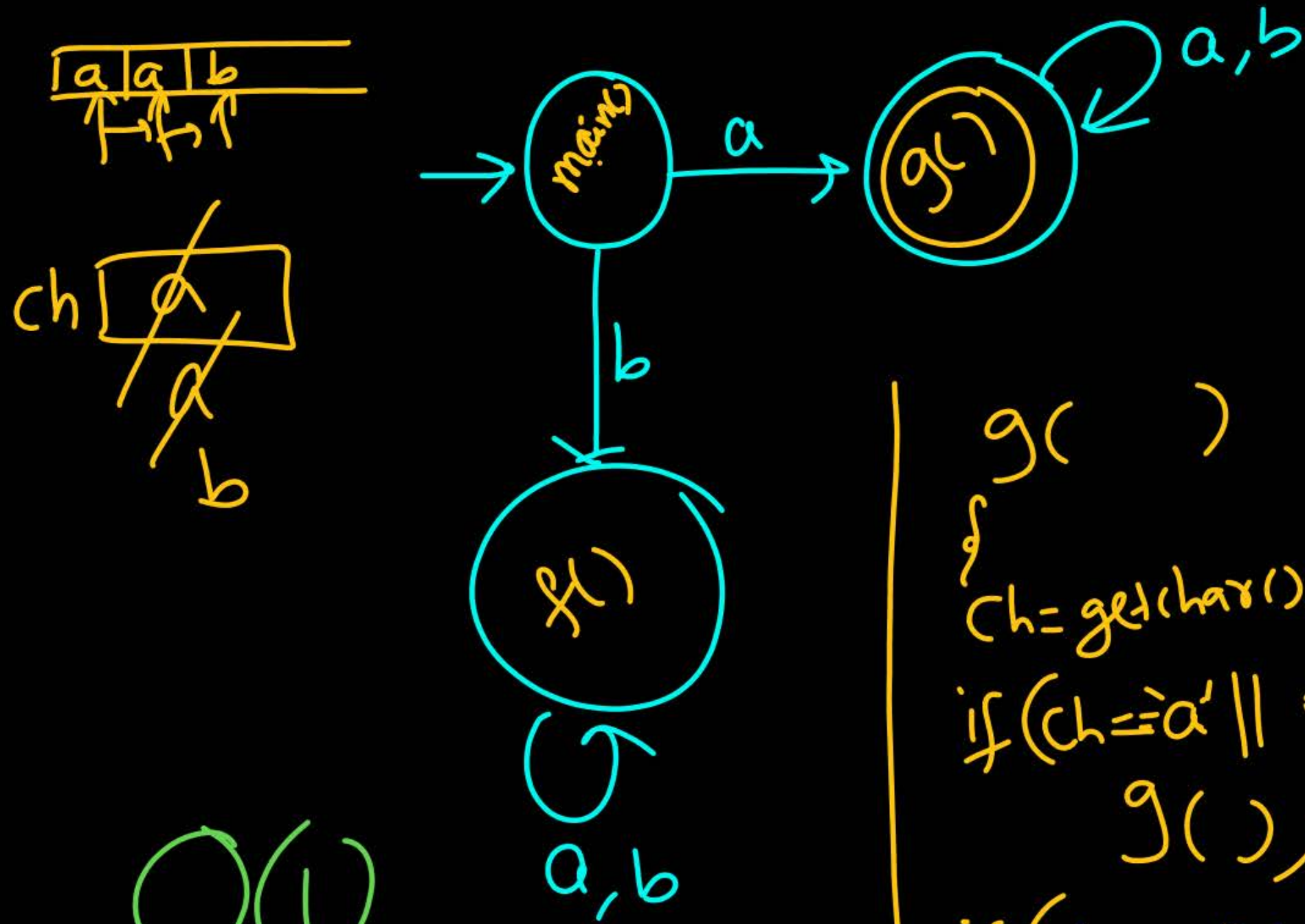
CFLs

Regulars

→ Using 1 stack,  
we can solve  
any CFL.

$O(1)$  space  
→ Every regular  
do not require  
memory to  
solve





$O(1)$   
Space

```

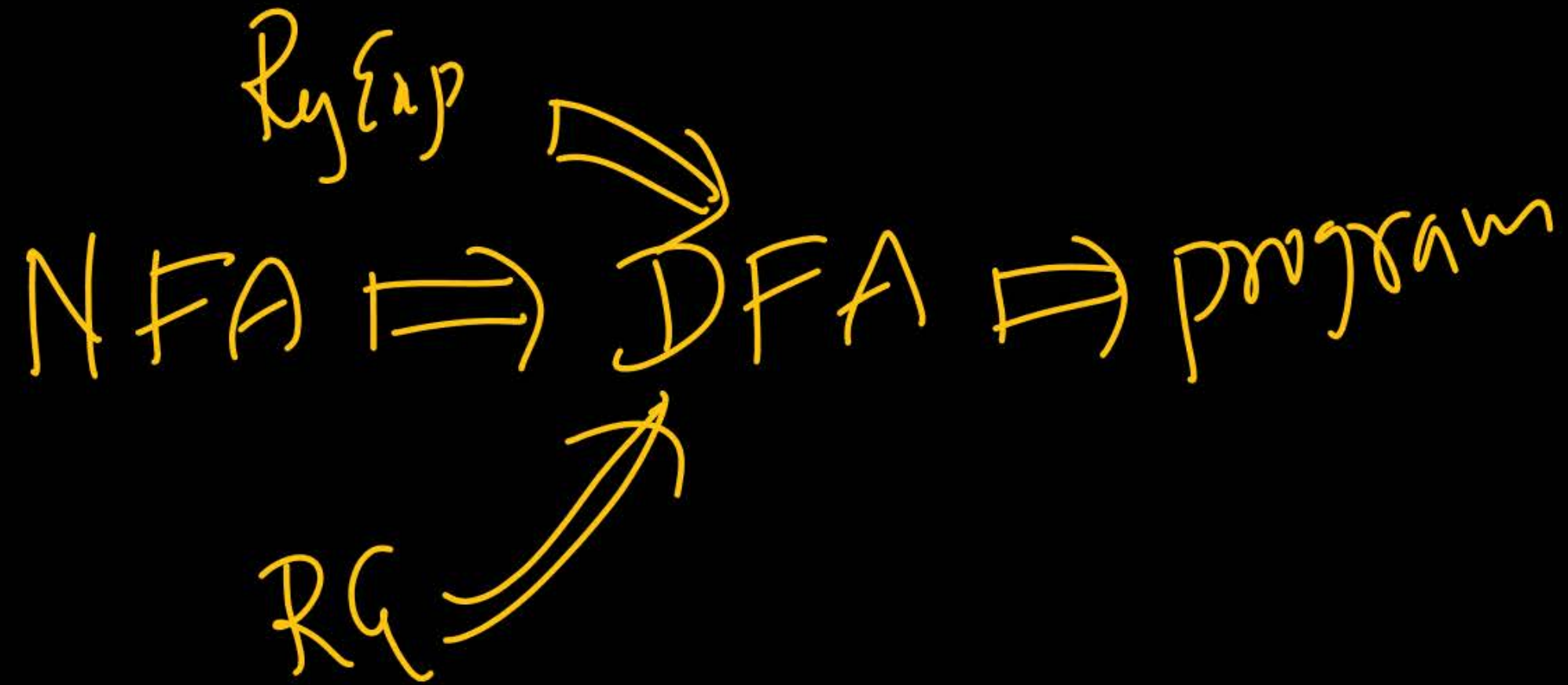
g()
{
    ch = getchar();
    if (ch == 'a' || ch == 'b')
        g();
    if (ch == '\n')
        "Accept"
}

```

```

char ch;
main()
{
    ch = getchar();
    if (ch == 'a')
        g();
    if (ch == 'b')
        f();
    if (ch == '\n')
        "Not Accept"
}

```





I)  $L_1$  is finite language  
(Reg) (CFL)

II)  $L_2$  is Regular but not finite (Inf & Reg)  
(CFL)

III)  $L_3$  is CFL but not regular (CFL & not reg)

IV)  $L_4$  is not CFL  
(not reg)  
(not finite)

$L$  is not reg & not CFL



$L$  is not CFL



- I) Every Finite Language is Regular.
- II) Every Regular Language is CFL.
- III) CFL may or may not be regular.

```
main ( )
{
    int x;
    x = 10;
}
```

Context Free  $\Rightarrow$  Syntax  
(structure)

```
function ( )
{
    Datatype Id;
    Id = Integer Constant;
}
```

context sensitive  $\Rightarrow$  Semantic  
(meaning)  
(type)  
checks type in every  
Statement



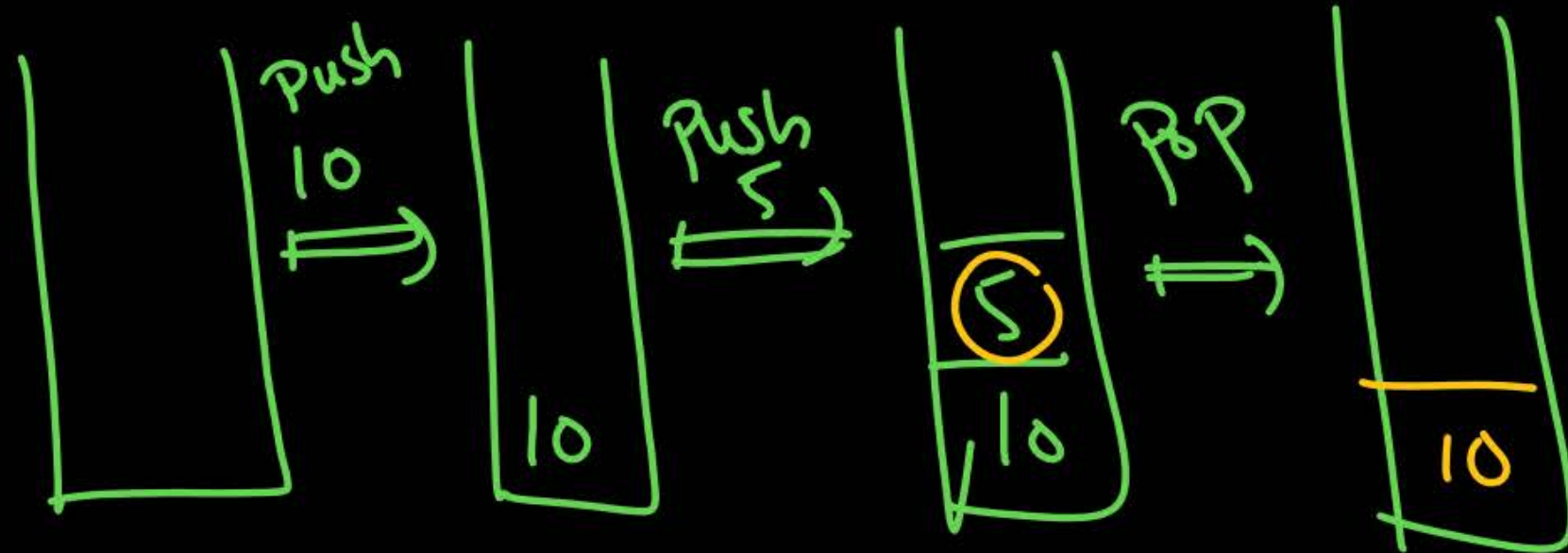
<sup>Regular Language</sup>  
 Words  $\Rightarrow$  <sup>CFL</sup> Structure  $\Rightarrow$  <sup>CSL</sup> meaning

Tokens  $\Rightarrow$  Syntax  $\Rightarrow$  Semantic

words  $\Rightarrow$  sentences  $\Rightarrow$  English language  
 meaning

# Stack [LIFO]

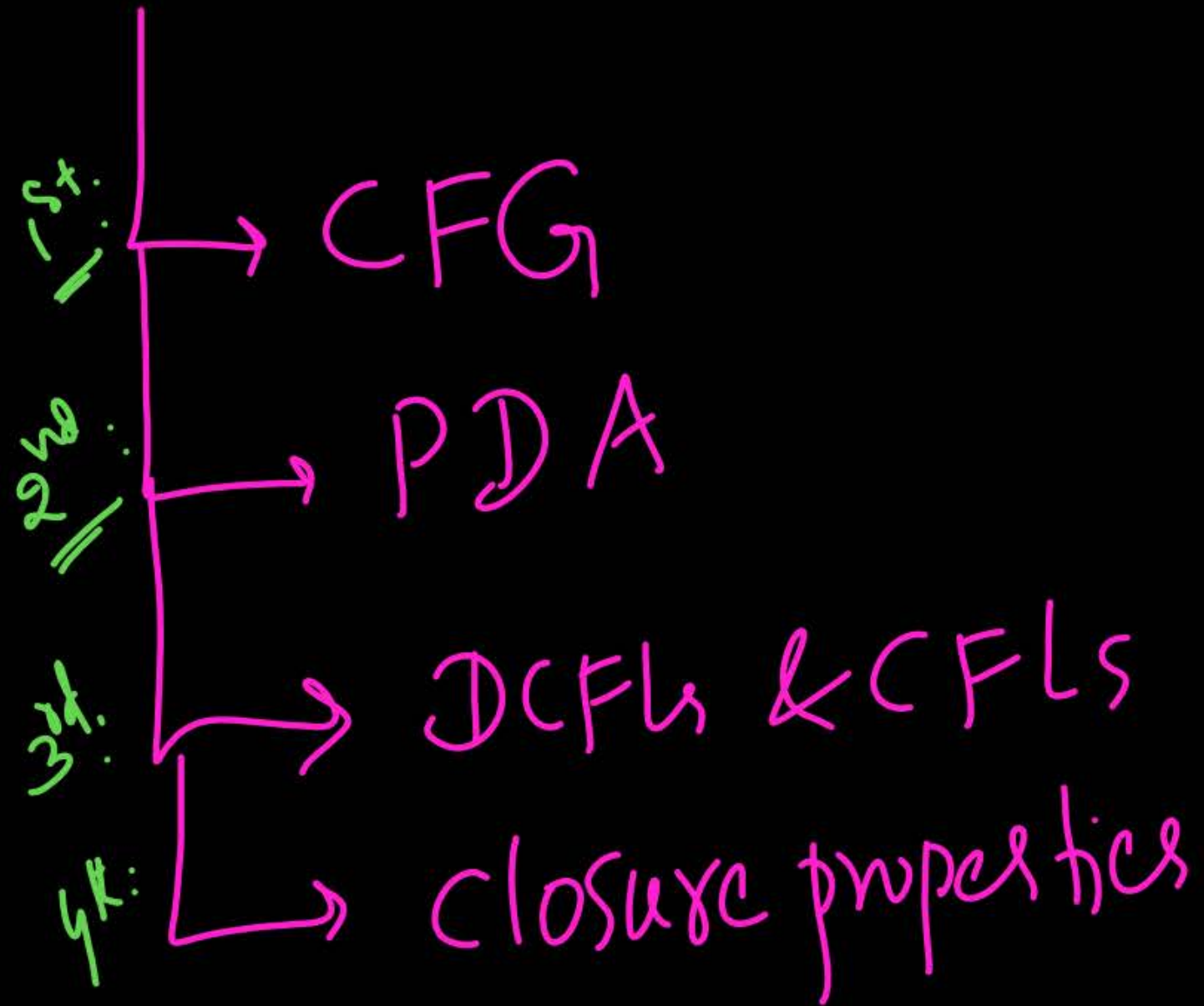
- It is data structure
- Push & Pop
- Insertion & Deletion can happen only at one end





# CFL

- Every regular application
  - Messages
  - Call history
  - Web pages history
  - DFS
  - Balanced parentheses
  - palindromes
- Recursion
  - Activation Record
  - Calling Sequence (Call Tree)



11Am - 1Pm  
morning

→ Sat & sun



→ Regulars Revision ✓  
→ CFLs ?

