CS & IT ENGINEERING

Theory of Computation

Push Down Automata

Lecture No. 6
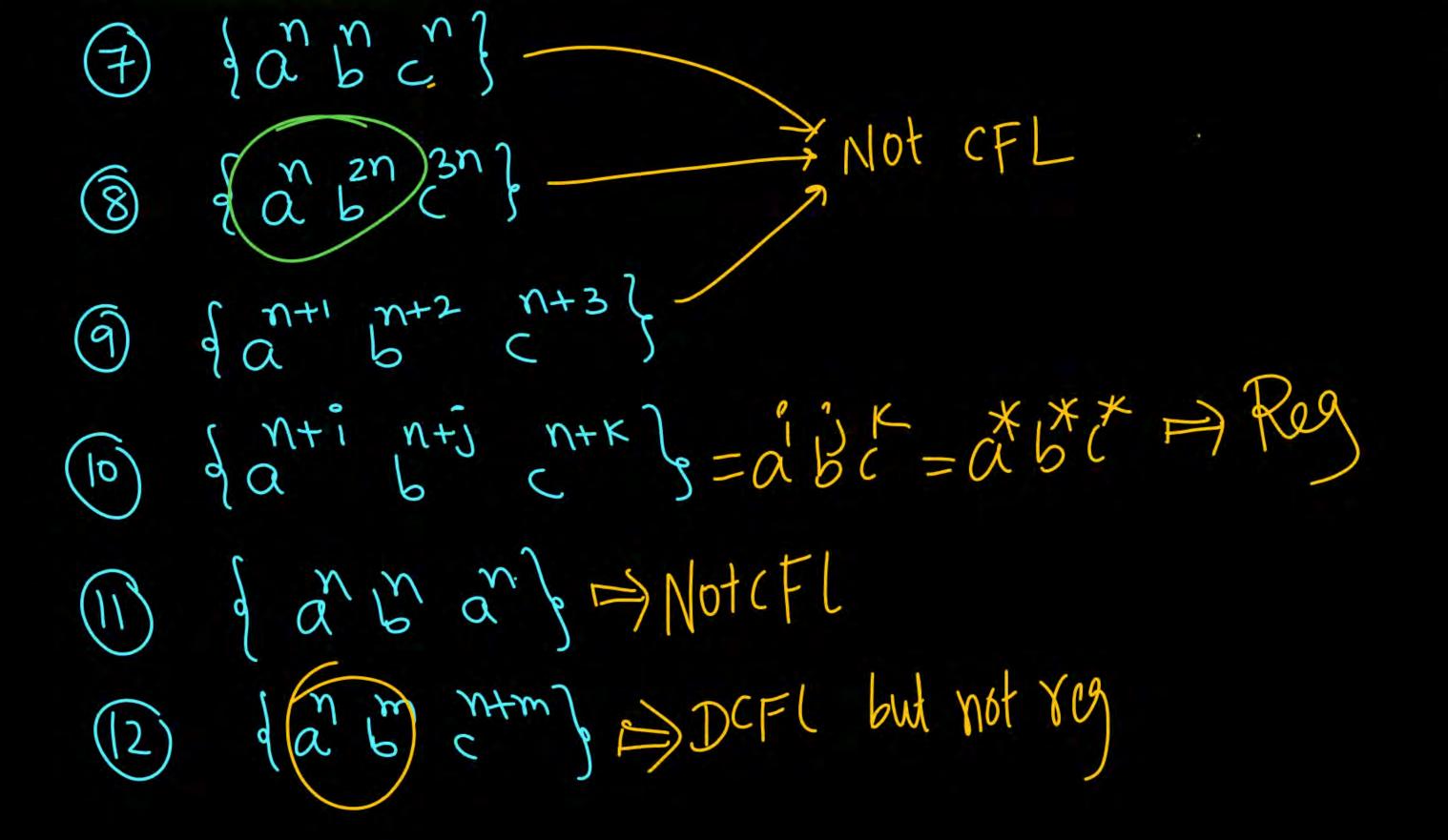
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TOPICS TO BE COVERED

01 Regulars

02 DCFls

03 CFls

04

05

Language

Regular

$\Downarrow$

DCFL

$\Downarrow$

CFL

Non Regular

DCFL

$\Downarrow$

CFL

Not DCFL

CFL

Not CFL

① $\{a^m b^n\} \Rightarrow a^* b^* \Rightarrow$ Reg $\Rightarrow$ DCFL $\Rightarrow$ CFL

② $\{a^n b^n\} \Rightarrow$ DCFL but not reg

③ $\{a^n b^{2n}\}$

④ $\{a^m b^n c^k\} = a^* b^* c^* \Rightarrow$ Reg

⑤ $\{a^n b^n c^*\} \Rightarrow$ DCFL but not reg

⑥ $\{a b^n c^n\}$

⑦ $\{a^n b^n c^n\}$ → Not CFL

⑧ $\{a^n b^{2n} c^{3n}\}$ → Not CFL

⑨ $\{a^{n+1} b^{n+2} c^{n+3}\}$ ↗ Not CFL

⑩ $\{a^{n+i} b^{n+j} c^{n+k}\} = a^i b^j c^k = a^* b^* c^* \Rightarrow$ Reg

⑪ $\{a^n b^n a^n\} \Rightarrow$ Not CFL

⑫ $\{a^n b^m c^{n+m}\} \Rightarrow$ DCFL but not reg

(13) $\{a^{n!}\}$

(14) $\{a^{prime}\}$

(15) $\{a^{n^2}\}$

(16) $\{a^{2^n}\}$

(17) $\{a^{n^n}\}$

$\Rightarrow$ Not regular
$\downarrow$
Not CFL

(18) $\{a^{m^n}\} = a^* \Rightarrow$ Reg

(19) $\{ww \mid w \in \{a,b\}^*\}$ → not CFL

(20) $\{w\#w \mid \quad " \quad \}$ → not CFL

(21) $\{ww^R \mid \quad " \quad \} \Rightarrow$ CFL but not DCFL

(22) $\{w\#w^R \mid \quad " \quad \} \Rightarrow$ DCFL but not reg

(23) $\{\overset{a}{w}\overset{a}{w} \mid w \in a^*\} = (aa)^* \Rightarrow$ Reg $\Rightarrow$ DCFL $\Rightarrow$ CFL

(24) $\{w\#w \mid w \in a^*\} = a^n\#a^n \Rightarrow$ DCFL but not reg

(25) $\{a^n b^n c^k d^k\} \Rightarrow$ DCFL

(26) $\{a^n b^k c^k d^n\}$

Push as | Push bs | Pop bs | Pop as

(27) $\{a^n b^k c^n d^k\} \rightarrow$ not CFL

(28) $\{a^m b^n c^k d^l \mid m+n = k+l\} \rightarrow$ DCFL

Push x's | Pop x's

(29) $\{a^{2n} b^{3n}\} \rightarrow$ DCFL

Push as n as | Every 3 bs Pop 1 a

$$\overline{a_b^n{}_c^K m} \quad \overline{x^n y^K z^m}$$

$$a^n b^K \quad c^n d^K$$

$$\underbrace{a^2 b^1}_{\text{formal}} \quad \underbrace{c^2 d^1}_{\text{actual}}$$

$$f(\text{int } x, \text{int } y, \text{char } ch)$$
from
$\{$

$f(a, b, c);$
actual
a integer b char

$$\{ww \mid w \in a^*\} = (aa)^*$$

$$\varepsilon\ \varepsilon \longrightarrow \varepsilon$$

$$a\ a \longrightarrow a^2$$

$$\{w_1 w_2 \mid w_1, w_2 \in a^*\}$$

$$= a^*$$

$$\text{int } \boxed{temp}^{W};$$

$$\boxed{temp}^{W} = 10;$$

Declaration W

use W

$$\underline{W}_{Decl} \# \frac{W}{use}$$

Not CFL $\Rightarrow$ Not DCFL

Not CFL $\Rightarrow$ Not Reg

(30) $\{a^{n!} b^k\}$ $\longrightarrow$ Not CFL

(31) $\{a^{n^2} b^k\}$

(32) $\{a^n b^n c^{k^2}\}$

(33) $\{w \mid w \in \{0,1\}^*, n_0(w) = n_1(w)\} \longrightarrow$ DCFL

(34) $\{wwx \mid w, x \in \{a,b\}^*\}$

(35) $\{wxw \mid \quad '' \quad \}$

(36) $\{xww \mid \quad '' \quad \}$

(37) $\{ww^R x \mid \quad '' \quad \}$

(38) $\{wxw^R \mid \quad '' \quad \}$

(39) $\{xww^R \mid \quad '' \quad \}$

$\Rightarrow$ Regular
$\Downarrow$
DCFL
$\Downarrow$
CFL

(40) $\{ wwx \mid w, x \in \{a, b\}^+ \}$

(41) $\{ wxw \mid \text{''} \}$

$\Longrightarrow$ Not CFL

(42) $\{ xww \mid \text{''} \}$

(43) $\{ ww^R x \mid \text{''} \}$

$\longrightarrow$ CFL but not DCFL

(44) $\{ wxw^R \mid \text{''} \}$

$\Longrightarrow$ Regular

(45) $\{ xww^R \mid \text{''} \}$

(46) $\{ w w^R x x^R \mid w, x \in \{a, b\}^* \} \longrightarrow$ CFL, not DCFL

push pop push pop

(47) $\{ w w \ x x \mid w, x \in \{a, b\}^* \} \longrightarrow$ not CFL

(48) $\{ w w^R w \mid w \in \{a, b\}^* \}$

(49) $\{ w w w \mid$ " $\}$

(50) $\{ w \# w \# w \mid w \in \{a, b\}^* \}$

$(51) \quad \{w \# w^R \# x \# x^R \mid w, x \in \{a, b\}^*\} \longrightarrow DCFL$

$(52) \quad \{a^n \# a^n \# a^n\} \longrightarrow \text{not CFL}$

$(53) \quad \{a^n \# b^n \# c^n\}$

$(54) \quad \{a^m \# b^n \# c^k\} = a^* \# b^* \# c^* \longrightarrow Reg)$

$(55) \quad \{a^m \# a^n \# a^k\} = a^* \# a^* \# a^*$

(56) $\{a^m b^n c^k \mid \boxed{m=n \ \text{OR} \ m=k}\}$

(57) $\{a^m b^n c^k \mid m<n \ \text{OR} \ m>k\}$

(58) $\{a^m b^n c^k \mid m \neq n \ \text{OR} \ m \leq k\}$

(59) $\{a^m b^n c^k \mid m \neq n \ \text{OR} \ m \neq k\}$

(60) $\{a^m b^n c^k \mid \boxed{\text{if } (m \neq n) \ \text{then } m=k}\} = $ (56)

CFL
but not DCFL

if $(m \neq n)$ then $m = k$

if $\sim p$ then q

$$p \lor q$$

$m = n$ OR $m = k$

(61) $\{a^m b^n \mid m \neq n \text{ OR } m = n\} = a^* b^*$ — Regular

(62) $\{a^m b^n c^k \mid \underbrace{m = n \text{ and } m = k}\} = a^n b^n c^n$ → not CFL

(63) $\{a^m b^n c^k \mid \underset{\text{as a's \& b}}{m \neq n} \text{ AND } \underset{\text{as c's}}{m \leq k}\}$

$m = n = k$

$$\begin{array}{c} = \\ < \\ \leq \\ > \\ \geq \end{array} \qquad \begin{array}{c} = \\ \neq \\ < \\ \leq \\ > \\ \geq \end{array}$$

(64) $\{a^m b^n c^k \mid m \neq n \text{ OR } (m \leq k)\} \longrightarrow$ CFL but not DCFL

$$\{a^m b^n c^k \mid m \neq n\} \cup \{a^m b^n c^k \mid m \leq k\}$$

$\Downarrow$ DPDA

$\Downarrow$ DPDA

Make new PDA

a^*b^*

$wwx \mid w, x \in \{a, b\}^*$

Reg

DCFLs

DCF L
but not reg

$a^n b^n$

$a^n b^{2n}$

$a^m b^n c^{n+m}$

$w \# w^R$

CFLs

CF L
but not DCFL

$ww^R \mid w \in \{a, b\}^*$

$a^m b^n c^k \mid m = n$
OR
$m = k$

not CFL

$a^n b^n c^n$

$a^{n!}$

a^{prime}

$ww \mid w \in \{a, b\}^*$

$w \# w \mid_{=}$

(65) $\overline{\{a^n b^n\}}$ $\longrightarrow$ DCFL

$\hookrightarrow (a+b)^* - \{a^n b^n\}$

(66) $\overline{\{a^n b^n c^n\}}$ $\longrightarrow$ CFL but not DCFL

$= (a+b+c)^* - \{a^n b^n c^n\}$

(67) $\overline{\{ww | w \in \{a,b\}^*\}}$

$= (a+b)^* - \{ww | w \in \{a,b\}^*\}$

H.W.

PDA uses Final State:

①

$a, z_0 | a z_0$
push a

$L = \{\epsilon, a\}$

②

$a, z_0 | z_0$
skip a

$L = a^*$

③

$a, \epsilon | a$
Push a

$L = a^*$

④

$a, z_0 | a z_0$,
it pushes
only 1st a

$b, a / \epsilon$
pop a

$b, a / \epsilon$
pop a

$L = \{ab\}$

⑤

$a, z_0 | a z_0$

$b, a / \epsilon$

$b, z_0 | z_0$

abb^*

$= ab^+$

⑥

$a, a | aa$
$a, z_0 | a z_0$
Push all a's

$b, a | \varepsilon$
POP a

$b, a | \varepsilon$
POP a

$\varepsilon \quad \times$

$a b \checkmark$

$a a b \checkmark$

$a a a b \checkmark$

$a b b \quad \times$

$$\{ a^m b^n \mid m \geq n, \quad m, n \geq 1 \}$$

$\# a's \geq \# b's$

⑦

$a, a | aa$
$a, z_0 | a z_0$, Push all a's

$b, a | \varepsilon$
POP a

$b, a | \varepsilon$
POP a

$\varepsilon \quad \rightarrow m = n = 0$

$a \quad \begin{matrix} m = 1 \\ n = 0 \end{matrix}$

$a a \quad n = 0, m = 2$

$a a a$

$\vdots$

⑥ $\cup \; a^*$

$\{ a^m b^n \mid m > n \geq 1 \} \cup \{ a^* \} = \{ a^m b^n \mid m \geq n \geq 0 \}$

PDA uses Empty Stack :

(If string is valid, show that stack is empty at the end of i/p)

① $\epsilon, z_0 | \epsilon$
Pop z_0

$L = \{\epsilon\}$

② $a, z_0 | \epsilon$

$L = \{a\}$

EX
$a \checkmark$
$aa \times$
$aaa \times$

$\xrightarrow{a}$ 1

③ $a, z_0 | z_0$

$L = \{ \} = \phi$

Not empty

EX
$a \times$
$aa \times$

④

$a, \varepsilon / a$
push a

$b, a / \varepsilon$
pop a

$b, a / \varepsilon$
pop a

Assume
initially
stack is empty

$L = \{ a^n b^n \}$

ε ✓
ab ✓
$aabb$ ✓
$aaabbb$ ✓

*** S

$a, \varepsilon / a$ push a

$b, a / \varepsilon$ pop a

$b, a / \varepsilon$ Pop a

$\varepsilon, a / \varepsilon$ Pop 'a' without reading i/p

Assume initially stack is empty

$\{a^m b^n \mid m \geq n \geq 1\}$

DFA

$a, z_0 / z_0$

ε ✓

a ✓

aa ✓

$| \xrightarrow{a} | \xrightarrow{a} |$

z_0

⑥ $L = a^*$

$\quad \longrightarrow \quad \bigcirc \bigcirc \circlearrowleft \begin{array}{l} a, \varepsilon / a \\ \varepsilon, a / \varepsilon \end{array}$

⑦ $L = (a+b)^*$

$\quad \longrightarrow \quad \bigcirc \bigcirc \circlearrowleft \begin{array}{l} a, \varepsilon / b \\ b, \varepsilon / b \\ \varepsilon, b / \varepsilon \end{array}$

⑧ $L = \{\varepsilon\}$

$\quad \longrightarrow \quad \bigcirc \bigcirc \circlearrowleft a, \varepsilon / a$

⑨ $L = b^*$

$\quad \longrightarrow \quad \bigcirc \bigcirc \circlearrowleft \begin{array}{l} b, \varepsilon / a \\ \varepsilon, a / \varepsilon \end{array}$

→ CFLs & DCFLs

→ PDA using F.S. Vs E.S.