ENGINEERING MATHEMATICS

ALL BRANCHES

Special Types of Matrices

DPP-03 DISCUSSION

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Question 1

IF $A^T = A^{-1}$, where A is a real matrix, then A is

A Normal

B Symmetric

C Hermitian

D Orthogonal

$A^T = A^{-1}$

$AA^T = AA^{-1}$ (Pre multiply by A)

$\boxed{AA^T = I}$

Question 2

Match the items in columns I and II.

	Column I		Column II
P	Singular matrix	1.	Determinant is not defined
Q.	Non-square matrix	2.	Determinant is always one
R.	Real symmetric	3.	Determinant is zero
S.	Orthogonal matrix	4.	Eigenvalues are always real
		5.	Eigenvalues are not defined

Continue...

A P – 3, Q – 1, R – 4, S – 2

B P – 2, Q – 3, R – 4, S – 1

C P – 3, Q – 2, R – 5, S – 4

D P – 3, Q – 4, R – 2, S – 1

$$AA^T = I$$

$$|AA^T| = |I|$$

$$|A|.|A^T| = 1$$

$$|A||A| = 1$$

$$|A|^2 = 1$$

$$|A| = \pm 1$$

Question 3

[A] is a square matrix which is neither symmetric nor skew-symmetric and $[A]^T$ is its transpose. The sum and difference of these matrices are defined as $[S] = [A] + [A]^T$ and $[D] = [A] - [A]^T$, respectively. Which of the following statements is true?

A. Both [S] and [D] are symmetric

B. Both [S] and [D] are skew-symmetric

C. [S] is skew-symmetric and [D] is symmetric

D. [S] is symmetric and [D] is skew-symmetric

We know, $[S] = A + A^T$

$$[S]^T = (A + A^T)^T = A^T + (A^T)^T = A^T + A$$

$$[S]^T = [S] \qquad \therefore [S] \text{ is symmetric.}$$

We Know, $[D] = A - A^T$

$$[D]^T = (A - A^T)^T = A^T - (A^T)^T = A^T - A = -(A - A^T)$$

$$[D]^T = -[D] \qquad \therefore [D] \text{ is skew-symmetric.}$$

Question 4

If A and B are square matrices of the same order such that $AB = A$ and $BA = B$, then A and B are both

A Singular

B Idempotent

C Involutory

D None of these

Given $AB = A$ and $BA = B$

$A^2 = AA = (AB)A = A(BA) = AB = A$

$A^2 = A$

$B^2 = BB = (BA)B = B(AB) = BA = B$

$B^2 = B$

Thus A and B are idempotent.

Question 5

The matrix, $A = \begin{bmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{bmatrix}$ is

A. Idempotent

B. Involutory

C. Singular

D. None of these

$$A^2 = \begin{bmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$\therefore A^2 = I$ hence it is involutary.

Question 6

Every diagonal elements of a Skew- Hermitian matrix is

A Purely real

B 0 ✓

C Purely imaginary ✓

D 1

$(\bar{A})^T = -A$ {Skew Hermitian}

- $A \rightarrow a_{ij}$

For diagonal element,
- $(\bar{A})^T \rightarrow \bar{a}_{ji}$

$i = j$

$\bar{a}_{ji} = -a_{ij}$

$\bar{a}_{ii} + a_{ii} = 0$

This is possible when $a_{ii} = 0$ or purely imaginary

Question 7

If A is Hermitian, then iA is

A. Symmetric

B. Skew-symmetric

C. Hermitian

D. Skew-Hermitian

$A = (\bar{A})^T$ {Hermitian matrix}

Now, $(\overline{iA})^T = (-i\bar{A})^T = -i(\bar{A})^T$

$= -iA$

$(\overline{iA})^T = -iA$

$\therefore$ iA is skew-Hermitian.

Question 8

Every diagonal elements of a Skew- symmetric matrix is

A 1

B 0

C Purely real

D None of these

$A^T = -A$ {Skew-symmetric}

let $A = a_{ij}$

$A^T = a_{ji}$

$a_{ji} = -a_{ij}$

For diagonal elements $i = j$

$a_{ii} = -a_{ii}$

$2a_{ii} = 0$

$\boxed{a_{ii} = 0}$

Question 9

The matrix, $A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ -\frac{i}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$ is

A. Orthogonal $AA^T = I$

B. Idempotent $A^2 = I$

C. Unitary $AA^* = I$

D. None of these

AA^*

$= \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ -\frac{i}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ -\frac{i}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$

$\begin{bmatrix} \frac{1}{2} + \frac{1}{2} & \frac{i}{\sqrt{2}} - \frac{i}{\sqrt{2}} \\ -\frac{i}{\sqrt{2}} + \frac{i}{\sqrt{2}} & \frac{1}{2} + \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\boxed{AA^* = I}$ $\therefore$ A is unitary matrix.

Question 10

If A and B are non zero square matrices, then AB=0 implies

$AA^T = I \quad BB^T = I$

A. A and B are orthogonal

B. A and B are singular ✓

C. B is singular ✗

D. A is singular ✗

$A \neq 0,\ B \neq 0$

$AB = 0$

$|AB| = |0|$

$|A||B| = 0$

Case I :- $|A| = 0 \ ; \ |B| \neq 0$ $\quad A = 0$

Case II :- $|A| \neq 0 \ ; \ |B| = 0$ $\quad B = 0$

~~Case~~ III :- $|A| = 0 \ ; \ |B| = 0$ then only A and $B \neq 0$

$A^{-1}AB = A^{-1}0$

$\boxed{B = 0}$

$ABB^{-1} = 0B^{-1}$

$\boxed{A = 0}$

Thank you

GW Soldiers !