Branch : CSE/IT **Batch : Hinglish**

Discrete Mathematics II
Set Theory

DPP-05

[MSQ]

1. A binary relation R on N × N is defined as follows:
(a, b) R (c, d) if a ≥ c AND b ≥ d, then consider the below prepositions:
P: R is reflexive.
Q: R is not symmetric.
S: The inverse of R is transitive relation.
Then which of the given propositional logic is true?
(a) $P \wedge \sim S$ (b) $P \wedge Q$
(c) $\sim P \vee S$ (d) $Q \wedge S$

[MCQ]

2. Consider the below statements:
I. A relation defined on an empty set is not a transitive relation.
II. The number of transitive relations on a given set can be calculated using the formula $2^{\frac{n(n+1)}{2}}$.
III. The complement of a transitive relation need not be transitive.
Then choose the correct options from the following
(a) I is true but II and III are false.
(b) I and II both are true, III is false.
(c) I and II are false, only III is true.
(d) I and II are false, only II is true.

[NAT]

3. Consider the below relations:
R_1: {(a, b), (b, c), (b, b), (a, c), (c, b)} defined on set A = {a, b, c}
R_2: "Is parallel to" defined on set of lines.
The number of above relations that are transitive are?

[MCQ]

4. Consider below given statements and choose the correct combinations from the following
I: The intersection of relation R_1 = "is a biological sibling" on the set of persons and relation R_2 = "is elder to" on the set of persons is also a transitive relation.
II: The union of two transitive relations is also transitive.
(a) I is true, II is false.
(b) I is false, II is true.
(c) Both I and II are false.
(d) Both I and II are true.

[NAT]

5. For the set of 6 elements the number of relations that are only symmetric but not anti-symmetric are ______.

[NAT]

6. The number of given relations that are not transitive are: _______.
I. "Division of " on the set of integers.
II. "Multiple of" on the set of integers
III. "Greatest common divisor" on the set of integers.

Answer Key

1. **(a, b)**
2. **(c)**
3. **(1)**
4. **(a)**
5. **(2097088)**
6. **(0)**

Hints and Solutions

1. (a, b)

- Checking reflexive property:

  $(a, b) R (a, b) \Rightarrow \underset{\text{True}}{a \geq a}$ AND $\underset{\text{True}}{b \geq b}$

  Which is true, therefore reflexive P is true.

- Checking symmetric property:

  $$\begin{array}{ll} (a_1\ b_1) R (b_2, a_2) \Rightarrow & a_1 \geq b_2 \ \ R \ \ b_1 \geq a_2 \\ \text{AND} & \text{AND} \\ (b_2, a_2) R (a_1\ b_1) & (b_2 \geq a_1)\ R\ (a_2 \geq b_1) \end{array}$$

  Therefore, not symmetric. Q is true

- Checking transitivity property:

  (a, b)R(c, d) and (c, d) R (e, f) then (a, b)R(e, f)

  $(a \geq c)$ R $(b \geq d)$ AND $c \geq e$ R $d \geq f$ then

  $(a \geq c)$ R $(b \geq f)$ but we don't know about this.

Therefore, not transitive. S is false.

Note: If a relation is transitive then the inverse of the relation is also transitive.

(a) $P \wedge \sim S \cong$ True $\wedge \sim$(False) = True.

(b) $P \wedge Q \cong$ True $\wedge$ True = True.

(c) $\sim P \vee S \cong \sim$True $\vee$ False = False.

(d) $Q \wedge S \cong$ True $\wedge$ False = False.

2. (c)

I is false. A relation defined on an empty set is always a transitive relation.

II is false. There exists no fixed formula to determine the number of transitive relation on a set.

III is true. The complement of a transitive relation need not be transtitive.

3. (1)

R_1 = {(a, b) (b, c) (b, b) (a, c) (c, b)} is not a transitive relation because (c, b) $\in R_1$, (b, c) $\in R_1$ but (c, c) $\notin R_1$. In order R_1 to be transitive (c, c) $\in R_1$.

R_2 = 'Is parallel to' defined on a set of lines is a transitive relation. Example if line x is parallel to line y and line y is parallel to line z, then line x is also parallel to line z.

4. (a)

In **I**, relation R_1 is transitive, relation R_2 is transitive and the intersection of two transitive relation R_1 and R_2 is also transitive.

The **II** statement is incorrect because the union of two transitive relations need not be transitive.

5. (2097088)

The number of relaiton that are only symmetric but not antisymmetric can be calculated by the formula:

$$2^n\left(2^{\frac{n^2-n}{2}}-1\right)$$

Here n = 6, $2^6\left(2^{\frac{6^2-6}{2}}-1\right) \Rightarrow 2^6\left(2^{\frac{36-6}{2}}-1\right)$

$\Rightarrow 2^6\left(2^{\frac{30}{2}}-1\right) \Rightarrow 2^6\left(2^{15}-1\right) \Rightarrow 2097088$

6. (0)

"Divisor of" on set of integers is a transitive relation "Multiple of" on set of integers is a transitive relation "Greatest common divisor" on the set of integers is a transitive relations.