CS & IT ENGINEERING

Theory of Computation

Finite Automata

Lecture No. 06

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TOPICS TO BE COVERED

01 DFA [$\delta : Q \times \Sigma \rightarrow Q$]

02 Understanding DFA

03 Construction of DFA

04

05

→ Model_I

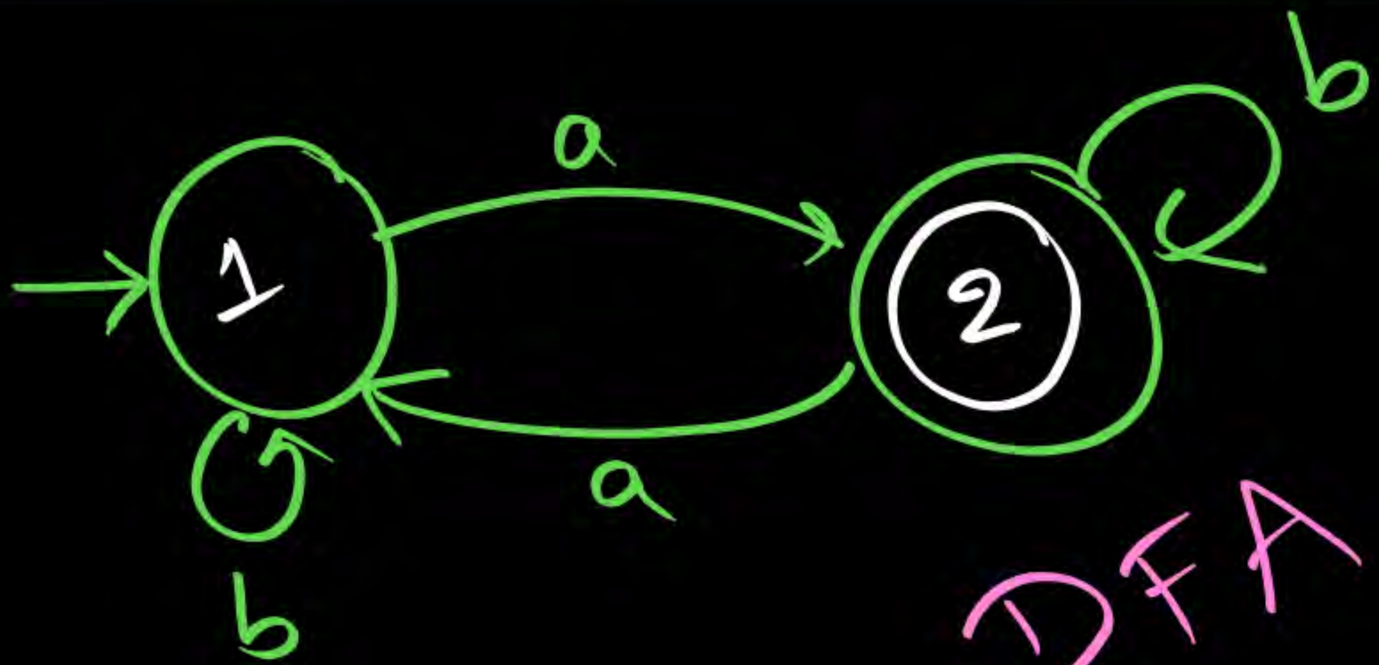
→ Model_II

→ Model_III

What is DFA ?

Input (w) $\rightarrow$ **DFA accepts L**

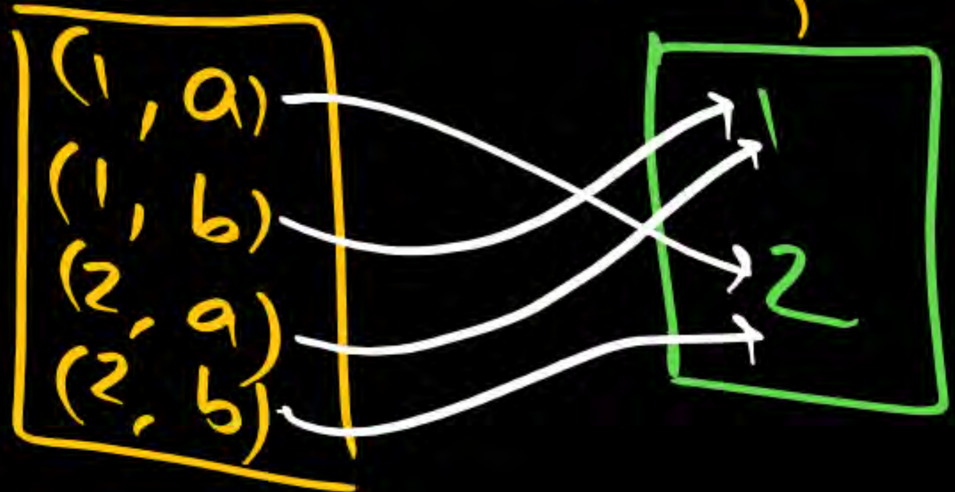
- If $w \in L$, exactly one path exist that halts at final state
- If $w \notin L$, exactly one path exist that halts at nonfinal



DFA

$L = \{a, ab, ba, \ldots\}$

$Q \times \Sigma \longrightarrow Q$

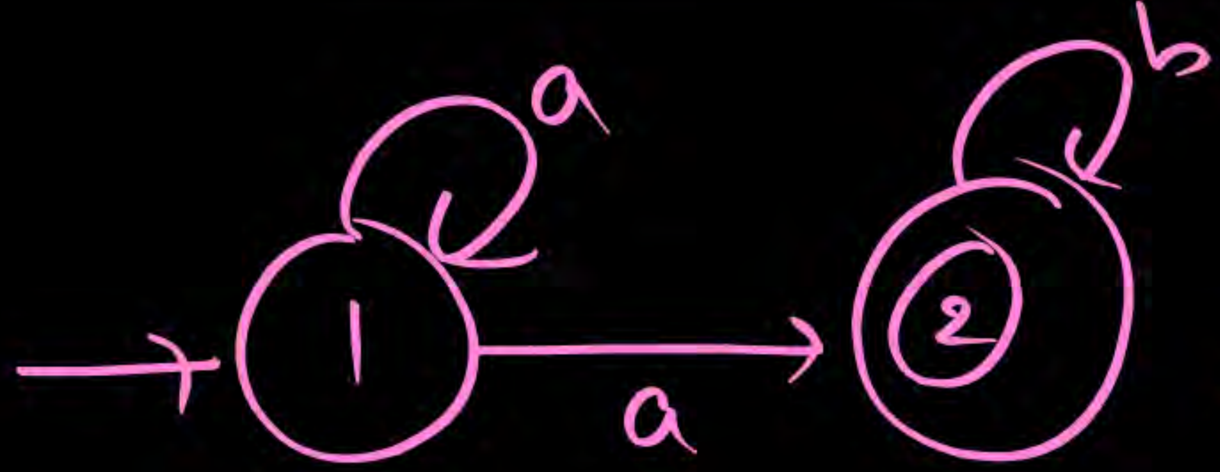


start

~~ε~~	1 Halt
✓ a	$1 \xrightarrow{a} 2$ Halt
~~b~~	$1 \xrightarrow{b} 1$ Halt
~~aa~~	$1 \xrightarrow{a} 2 \xrightarrow{a} 1$ Halt
✓ ab	1
✓ ba	1
~~bb~~	1
⋮	

$Q \times \Sigma = \{1, 2\} \times \{a, b\}$

$= \{(1,a), (1,b), (2,a), (2,b)\}$



Is it DFA ?

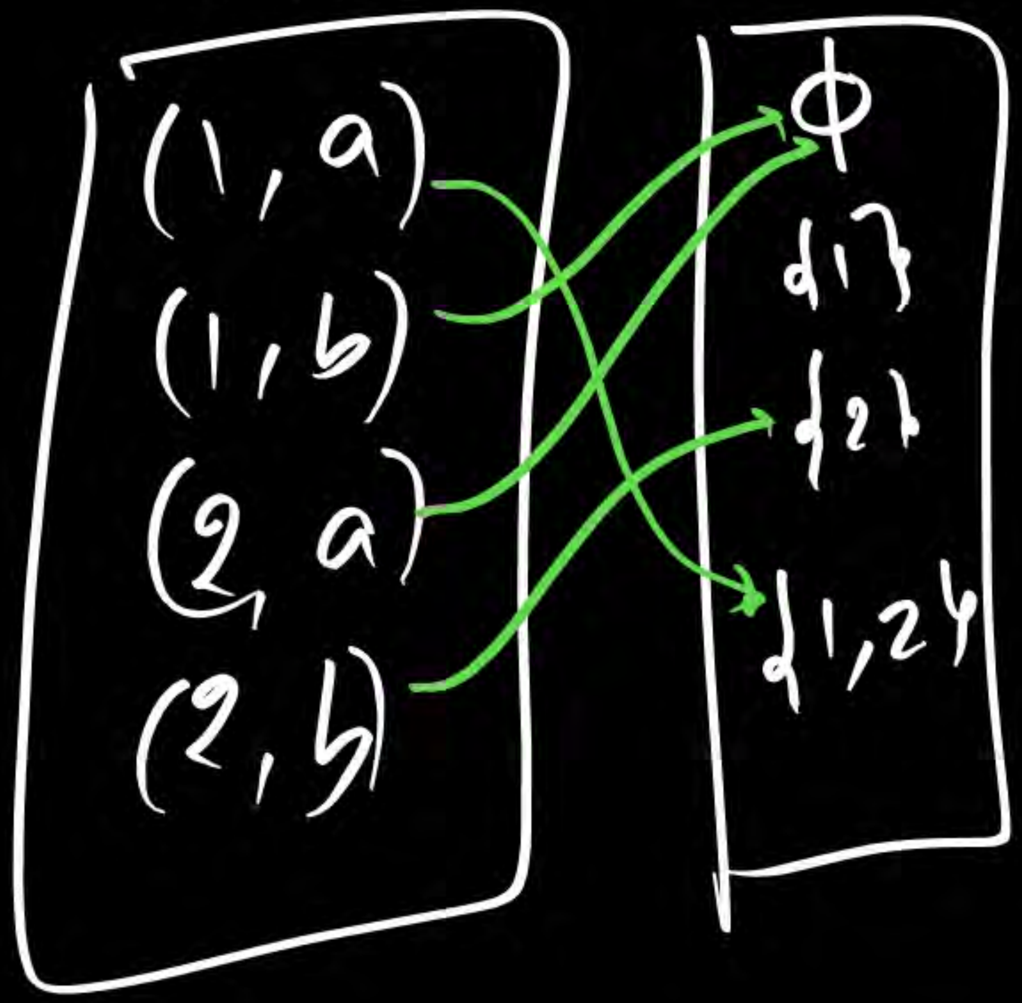
$\delta(1,a) = 1 \text{ or } 2$

$\delta(1,a) \Rightarrow 2$ transition

$\delta(1,b) \Rightarrow$ no transition

$\delta(2,a) \Rightarrow$ no transition

$Q \times \Sigma \rightarrow 2^Q$



DFA definitions :

I) $\delta : Q \times \Sigma \rightarrow Q$

II) From every state, for every i/p symbol, exactly one transition to next state

III) $\forall w \in \Sigma^*$ $\Rightarrow$ exactly one path exist
(every string)

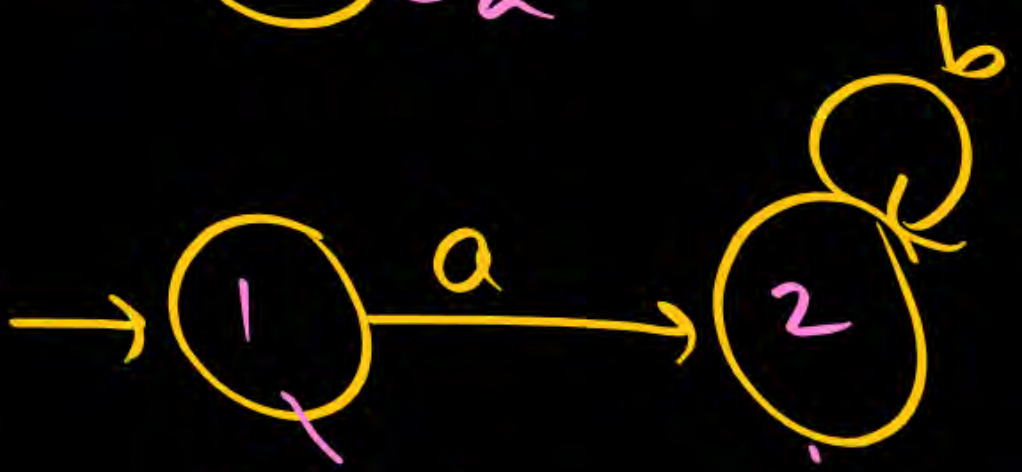
[valid string $\Rightarrow$ halts at final
Invalid $\Rightarrow$ " " non final]

Identify correct DFAs $\Sigma = \{a,b\}$

~~1~~

~~2~~

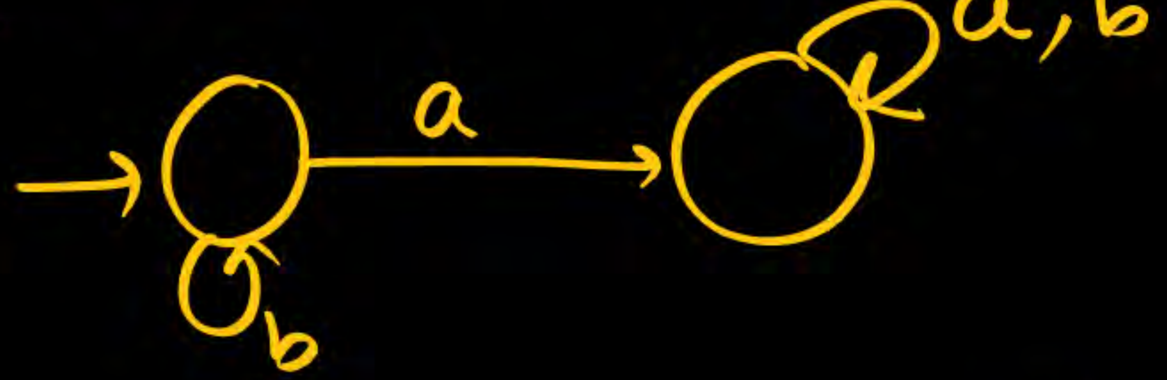
~~3~~



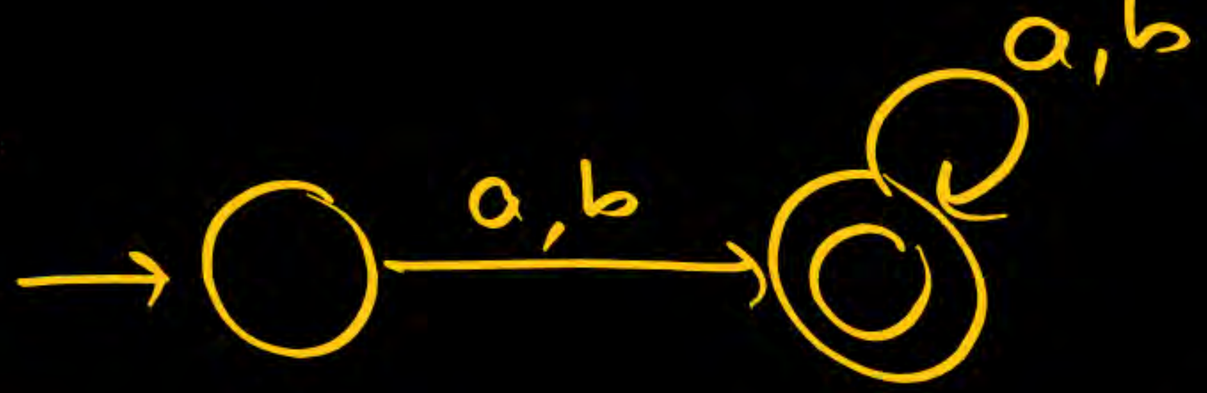
$\delta(1,b) = X$

$\delta(2,a) = X$

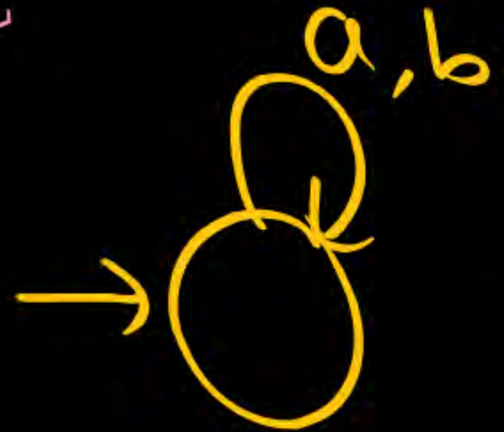
~~4~~



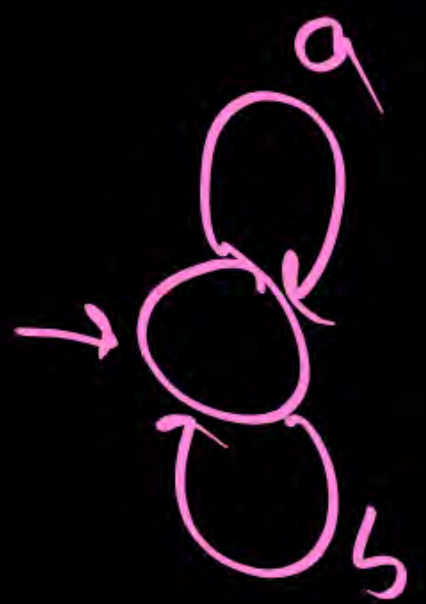
~~5~~



~~6~~



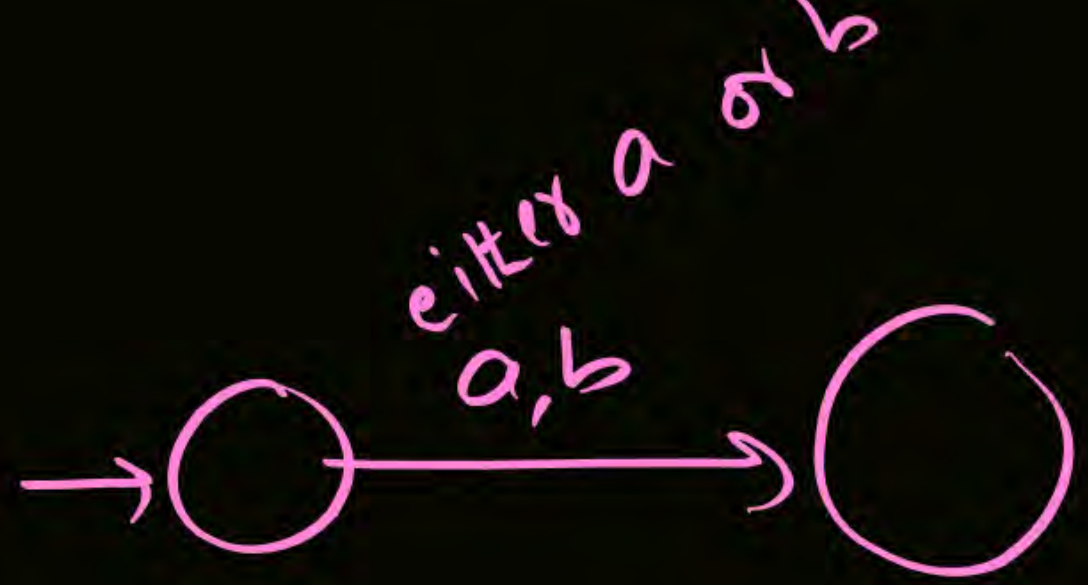
OR

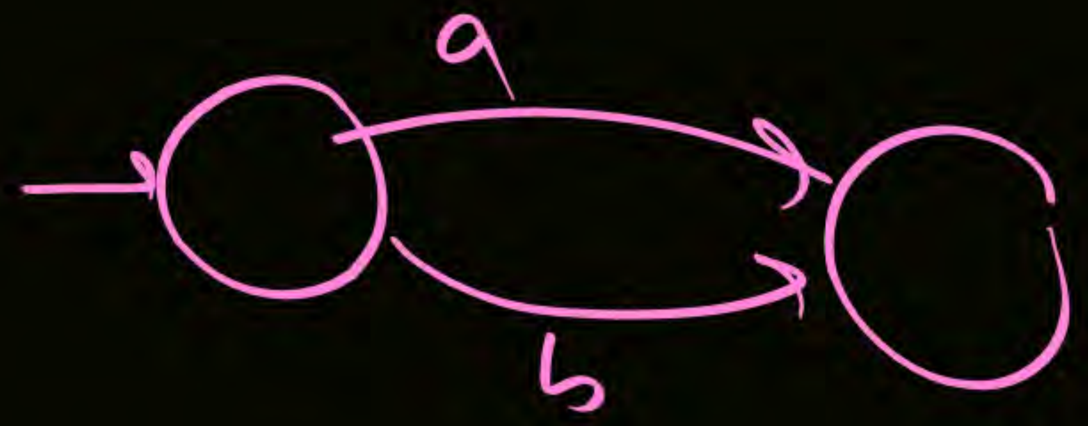


$F \subseteq Q$

F is set of final states

- zero final
- 1 final
- 2 finals
- ⋮





Note: $|Q| = n$

$|\Sigma| = K$

How many transitions in DFA ?

$$= nK$$

$$= |Q \times \Sigma|$$

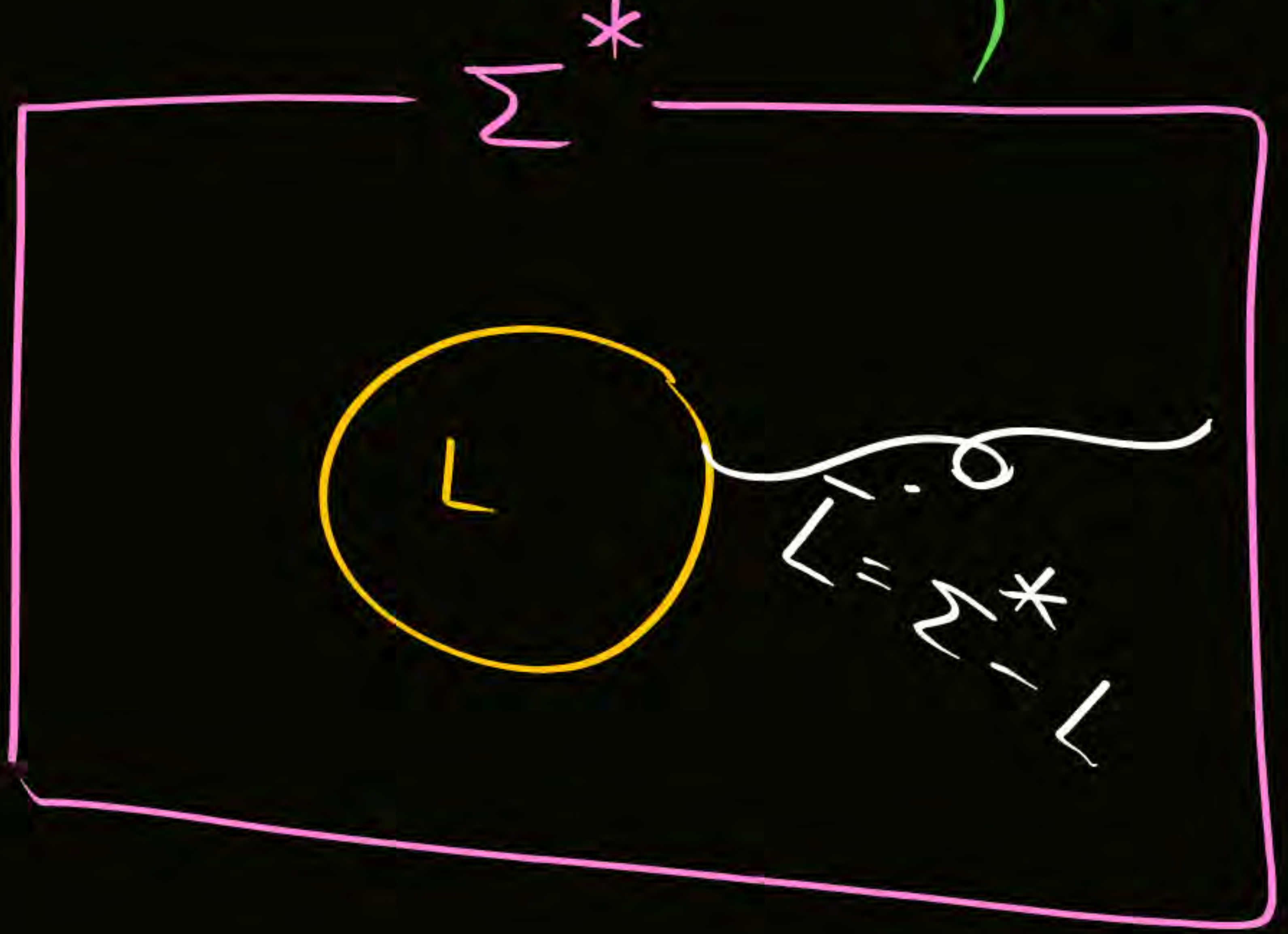
$$= |Q| \times |\Sigma|$$

$$\overline{L} = \Sigma^* - L$$

$$\overline{L} = \text{Universal set} - L$$

$$\left\{\begin{array}{l} \text{I) } L \cup \overline{L} = \Sigma^* \\ \text{II) } L \cap \overline{L} = \phi \end{array}\right.$$

$$w \in L \iff w \notin \overline{L} \qquad w \notin L \iff w \in \overline{L}$$



Model - I [special problems]

Complement to each other

$L_1 = \phi = \{\ \}$

$L_2 = \Sigma^*$

universal set

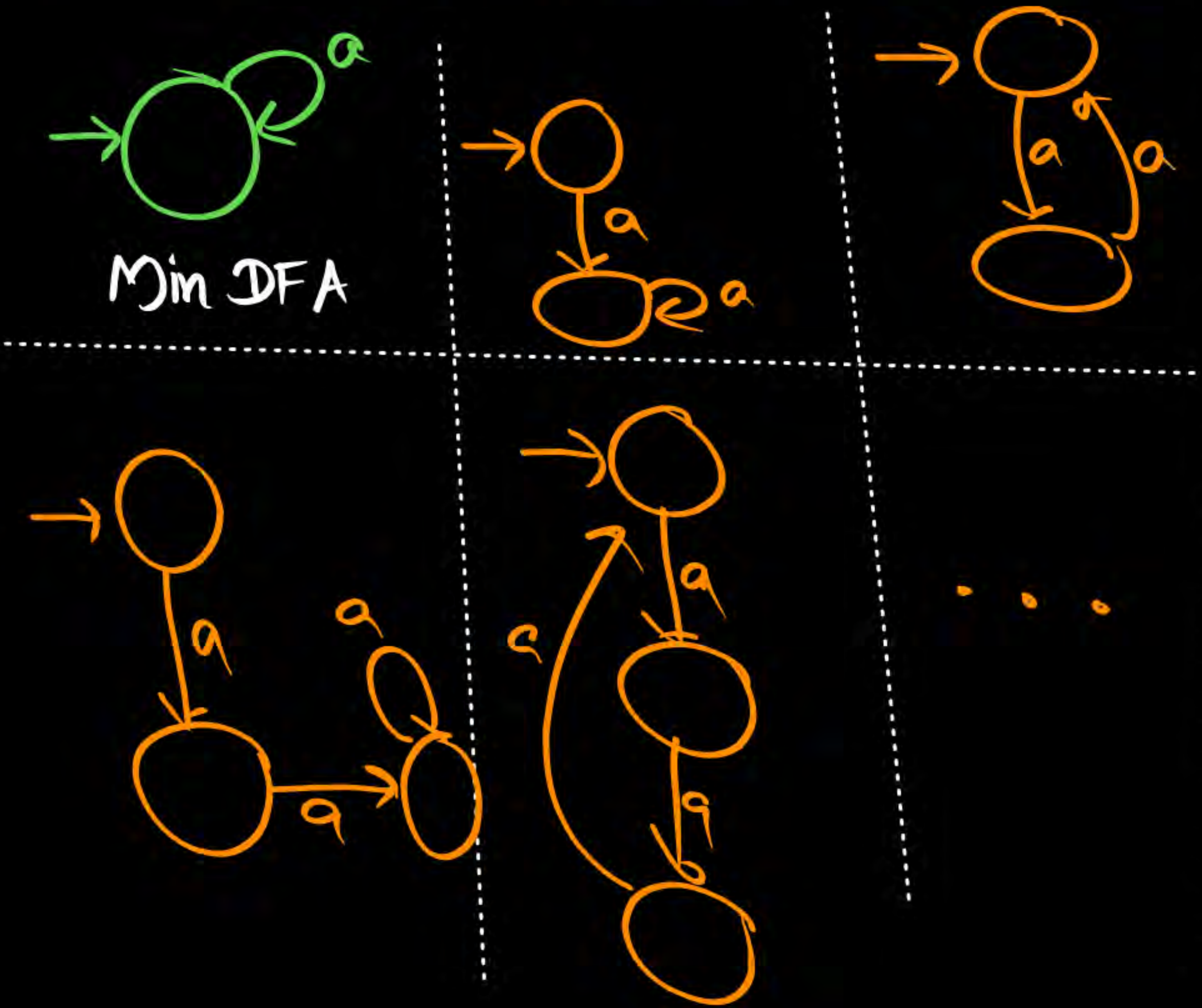
$L_3 = \{\varepsilon\}$

$L_4 = \Sigma^+$

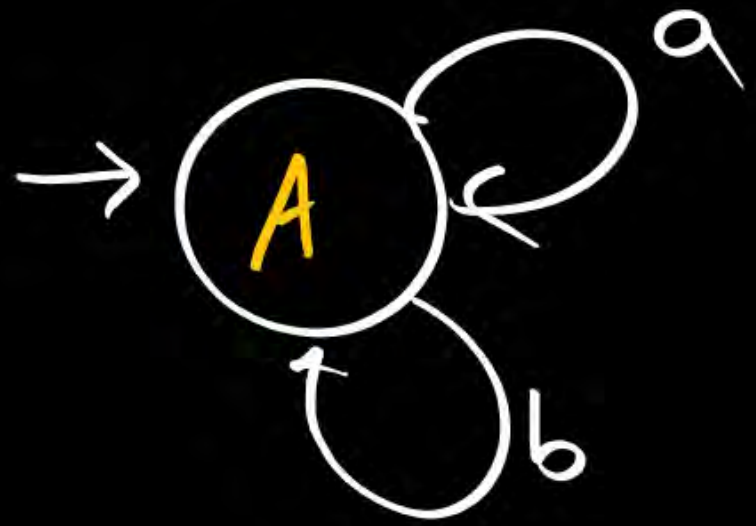
Complement to each other

Model-I [Special]

① $L = \{ \}$ over $\Sigma = \{a\}$



② $L = \{ \}$ over $\Sigma = \{a, b\}$



ε : A

a : $A \xrightarrow{a} A$

b : $A \xrightarrow{b} A$

Note :

I) For every regular language, No.of DFAs = Infinite

II) " " " " , No.of Min DFAs = 1
(unique)
(only one)

Minimum DFA
Minimized DFA
Minimal DFA
DFA with minimum no.of states

III) If DFA has $\underbrace{\text{only non final states}}_{\text{(no final state)}}$

then L (DFA) is $\underline{\ \phi = \{\}\ }$

IV) If DFA has only final states,
(no non-final)

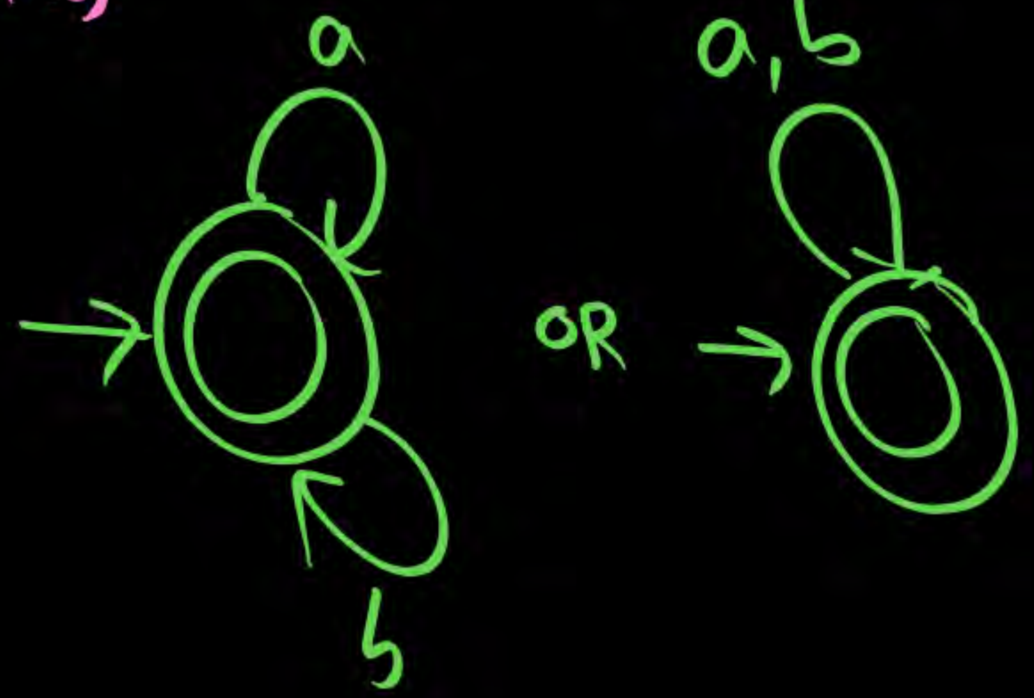
then L(DFA) is $\underline{\ \Sigma^*\ }$

③ $L = \Sigma^*$ over $\Sigma = \{a\}$

$= a^*$

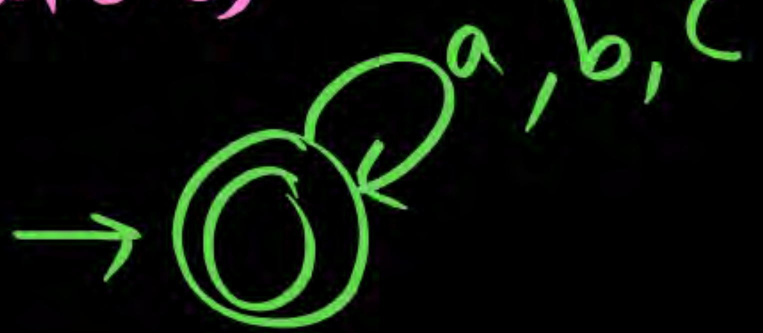
④ $L = \Sigma^*$ over $\Sigma = \{a, b\}$

$= (a+b)^*$



⑤ $L = \Sigma^*$ over $\Sigma = \{a, b, c\}$

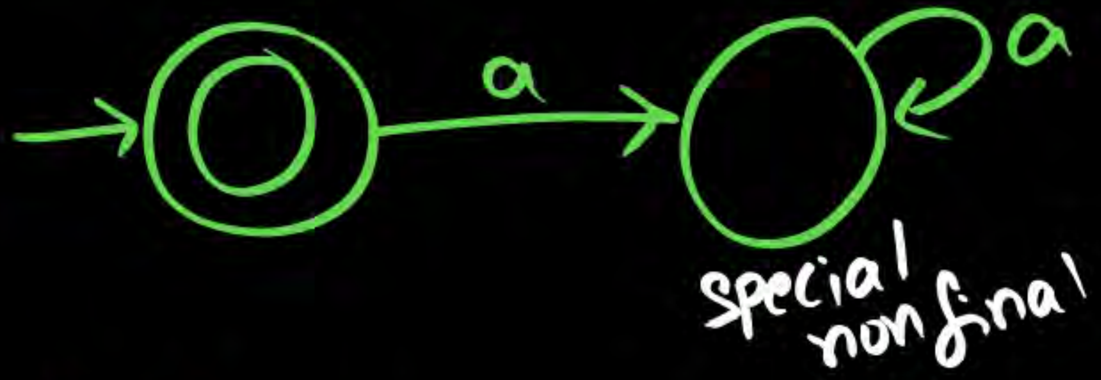
$= (a+b+c)^*$



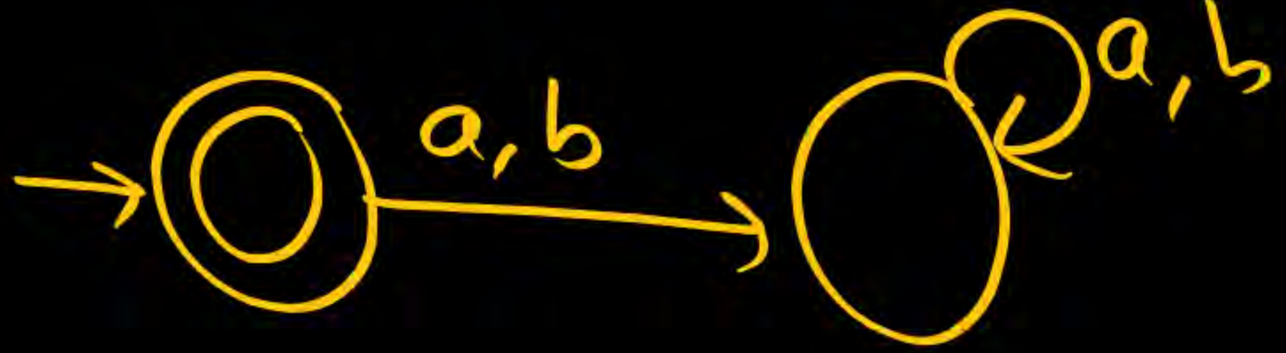
⑥ $L = \Sigma^*$ over $\Sigma = \{\#, @\}$

$= (\# + @)^*$

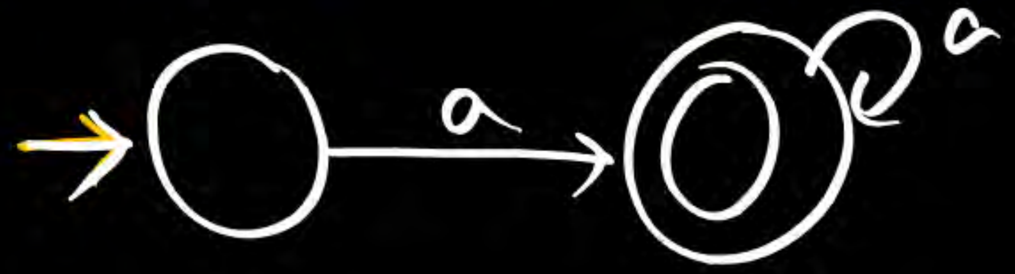
(7) $L = \{\varepsilon\}$ over $\Sigma = \{a\}$



(8) $L = \{\varepsilon\}$ over $\Sigma = \{a, b\}$



(9) $L = \Sigma^+$ over $\Sigma = \{a\}$



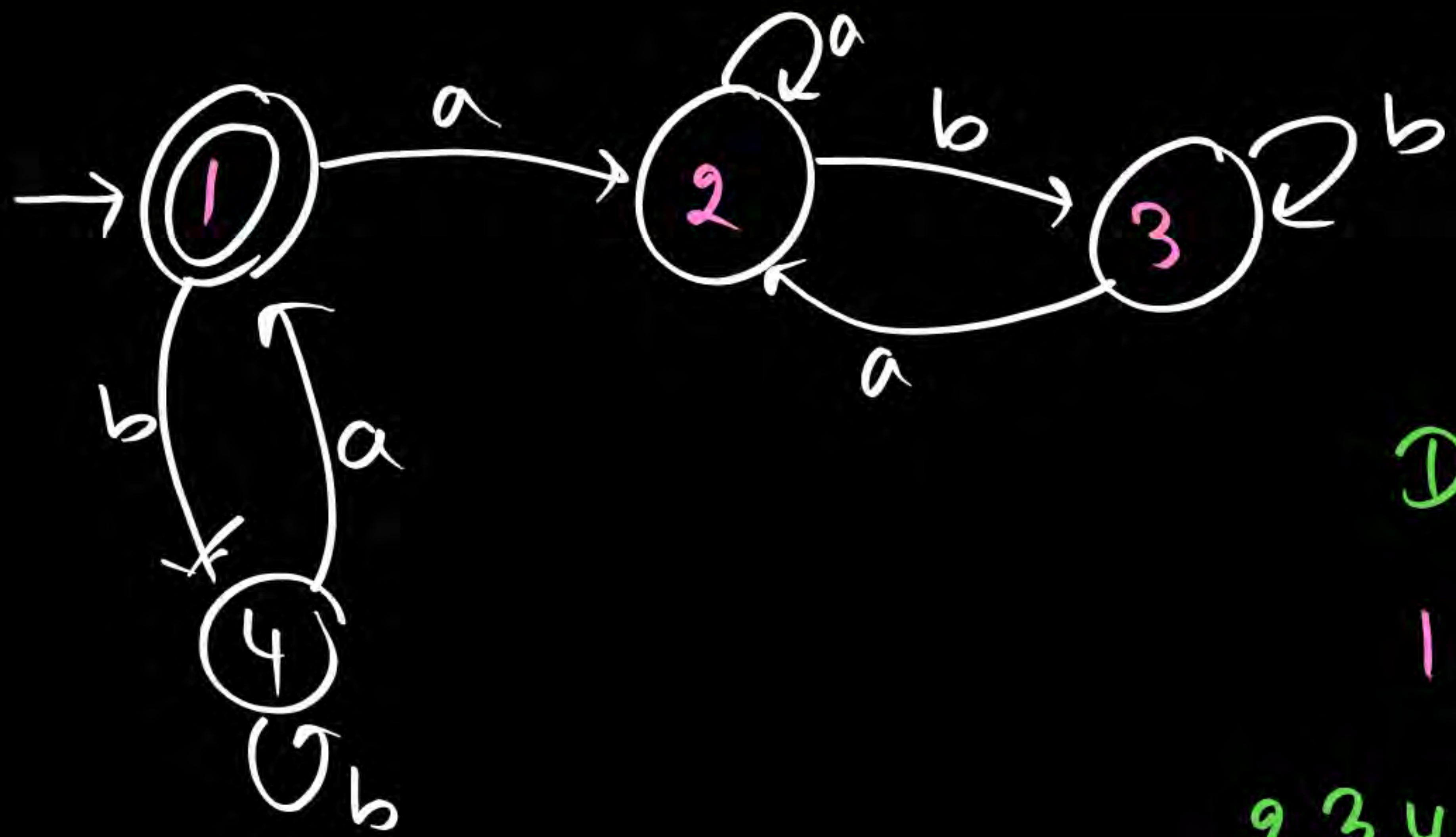
(10) $L = \Sigma^+$ over $\Sigma = \{a, b\}$

Special Nonfinal

From this nonfinal there is no path to final state

Dead state

Trap state



DFA ✓

1 is final

2, 3, 4 are nonfinals

2 & 3 are dead state

Note:

If L has min DFA with $\boxed{n}$ states

then $\bar{L}$ has min DFA with $\underline{\;n\;}$ states.

$$\begin{tikzcd}
L & \bar{L} \\
\text{min DFA} \arrow[r, bend left, "f \leftrightarrow nf"] & \text{min DFA} \arrow[l, bend left, "f \leftrightarrow nf"]
\end{tikzcd}$$

Model - II [Length based Problems]

① $L_1 = \{w \mid w \in \{a,b\}^*, |w| = 2\} = (a+b)^2$

② $L_2 = \{w \mid w \in \{a,b\}^*, |w| = 3\} = (a+b)^3$

③ $L_3 = \{w \mid w \in a^*, |w| = \boxed{2}\} = aa$

④ $L_4 = \{w \mid w \in a^*, |w| = \boxed{3}\} = aaa \Rightarrow 5$ states

⑤ $L_5 = \{w \mid w \in a^*, |w| \le 2\} = \varepsilon + a + aa = (\varepsilon + a)^2$

*** ⑥ $L_6 = \{w \mid w \in (a+b)^*, |w| \le 2\} = (\varepsilon + a + b)^2$

⑦ $L_7 = \{w \mid w \in a^*, |w| \ge 2\} = aaa^* = aa^+ = a^+a = a^*aa = aa^*a$

⑧ $L_8 = \{w \mid w \in (a+b)^*, |w| \ge 2\} = (a+b)^2(a+b)^*$

Model - II [Length based problems]

① $L_1 = \{w \mid w \in \{a,b\}^*, |w| = \boxed{2}\} = (a+b)^2$

= Set of all 2 length strings

$= \{aa, ab, ba, bb\}$

$= aa+ab+ba+bb$

$= (a+b).(a+b)$

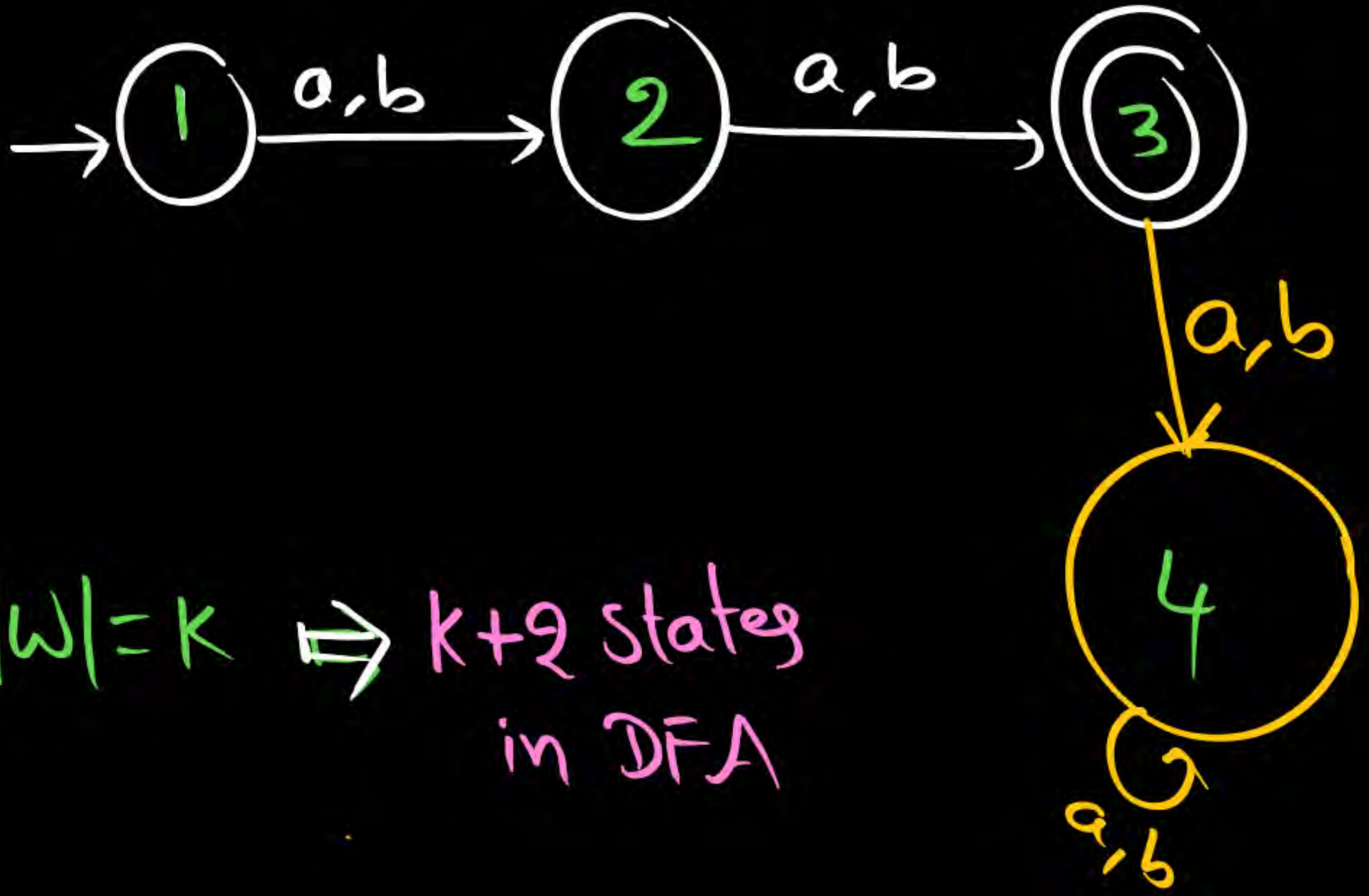
$= (a+b)^2$

× ε : 1

× a : $1 \xrightarrow{a} 2$

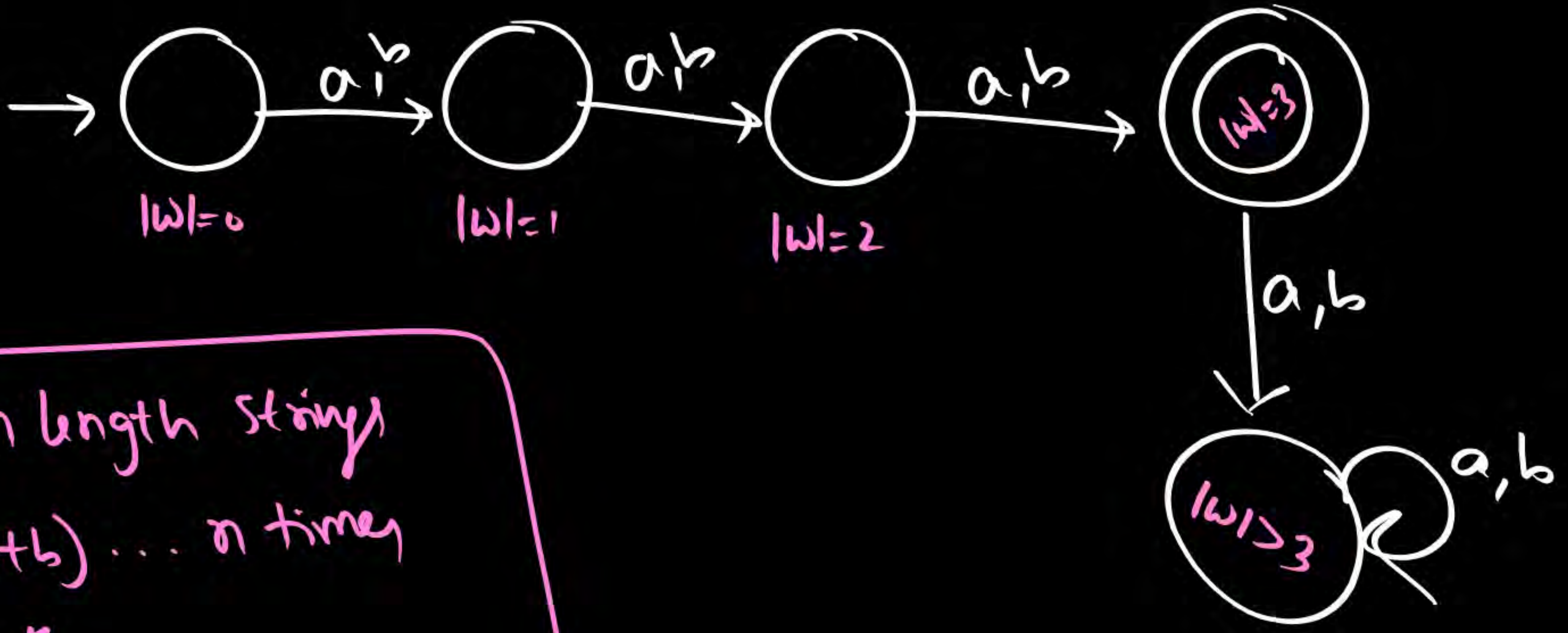
× b : $1 \xrightarrow{b} 2$

✓ aa : $1 \xrightarrow{a} 2 \xrightarrow{a} 3$



Note : $|w| = K \Rightarrow K+2$ states in DFA

(2) $L = (a+b)^3$



Exactly n length strings

$(a+b).(a+b) \dots n$ times

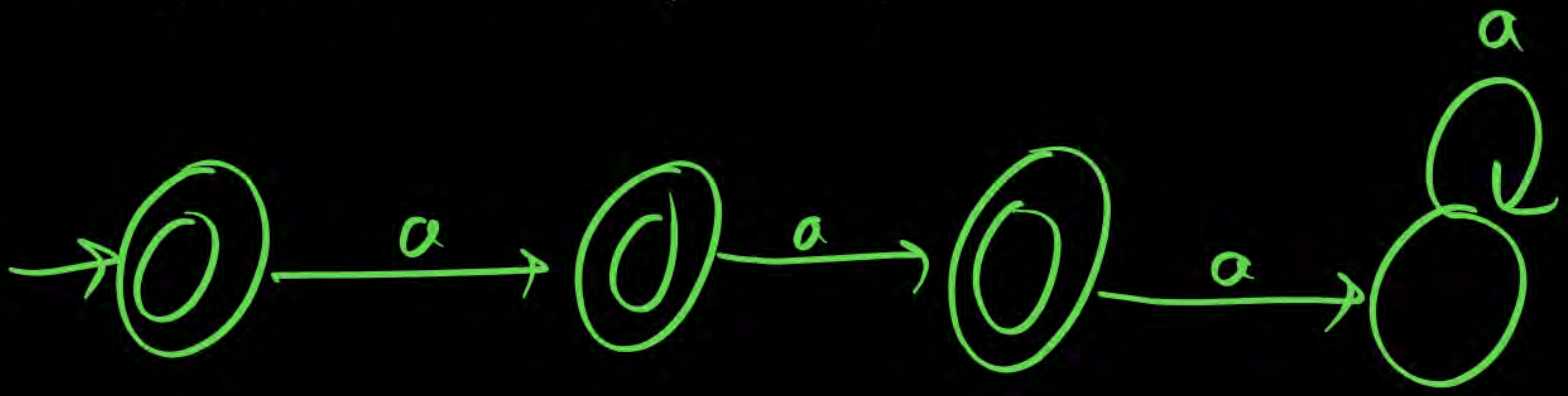
$\Downarrow$

$n+2$ states in DFA

(5) $\underbrace{|w| \leq 2}_{\text{Length of string is almost 2}}, \quad w \in a^*$

$$|w| = \left.\begin{matrix} 0 \\ 1 \\ 2 \end{matrix}\right\} \leq 2$$

$$L = \varepsilon + a + aa = (\varepsilon + a)^2$$

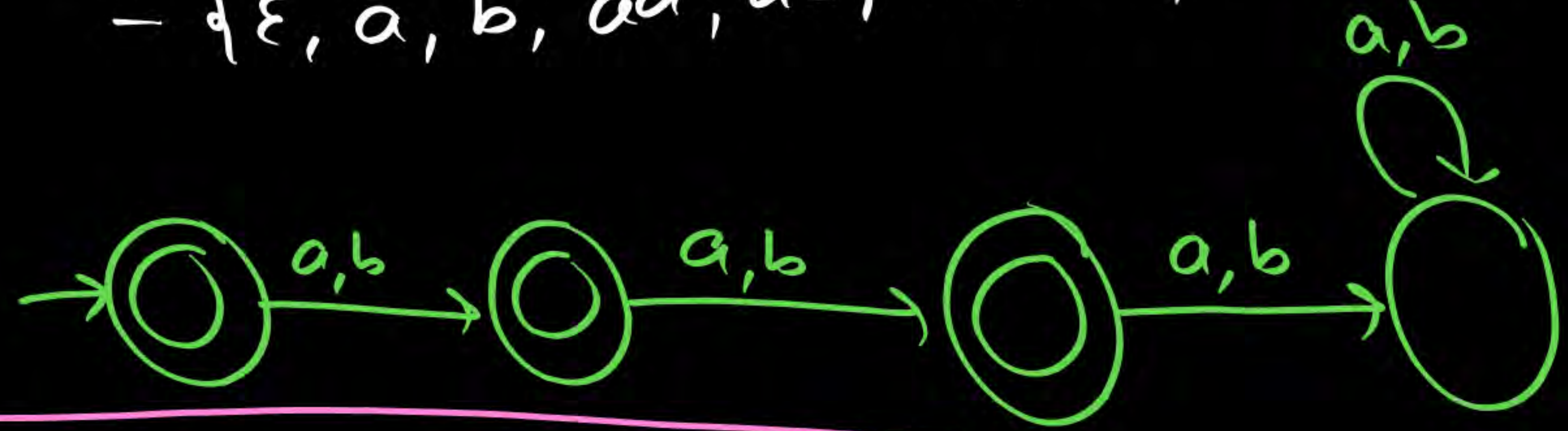


⑥ $|w| \leq \boxed{2}$, $w \in \{a,b\}^*$

$$L = (\varepsilon + a + b)^2$$

$$= \{\varepsilon, a, b, aa, ab, ba, bb\}$$

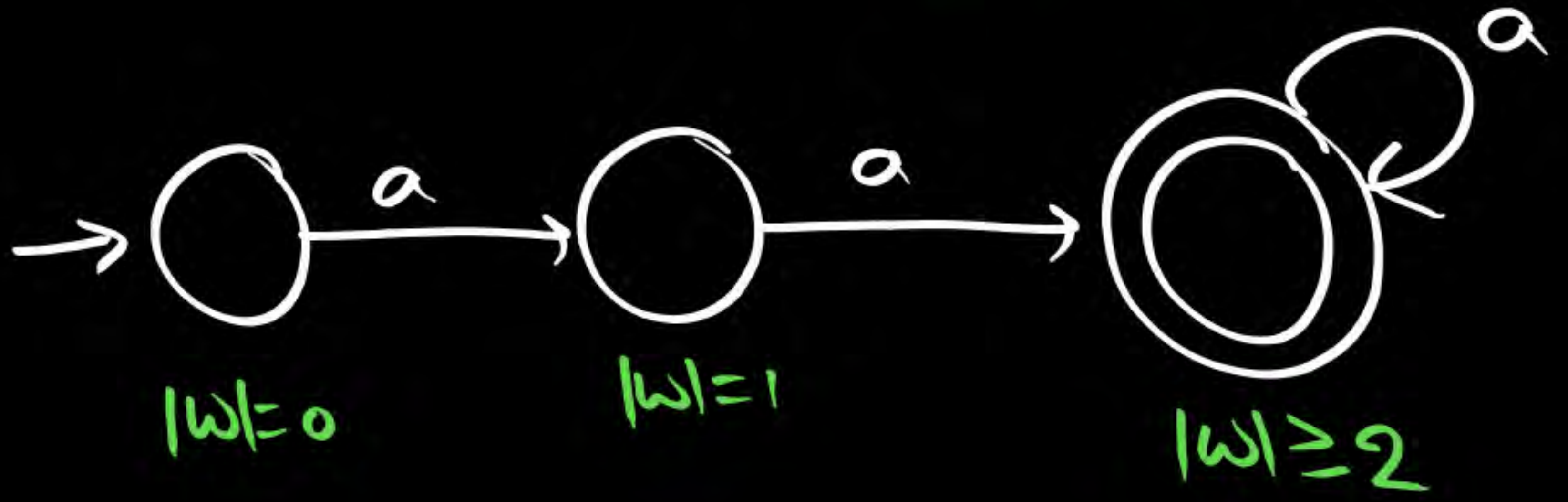
$|w| = 0$ ✓
$= 1$ ✓
$= 2$ ✓
$= 3$ ×
$= 4$ ×
⋮
×



If $|w| \leq n \Rightarrow n+2$ states
If $|w| = n \Rightarrow n+2$ states

(7) $L = \{w \mid w \in a^*, |w| \geq 2\}$

Length of string is atleast 2



$|w| = 0$ ×

$= 1$ ×

$= 2$ ✓

$= 3$ ✓

$= 4$ ✓

⋮

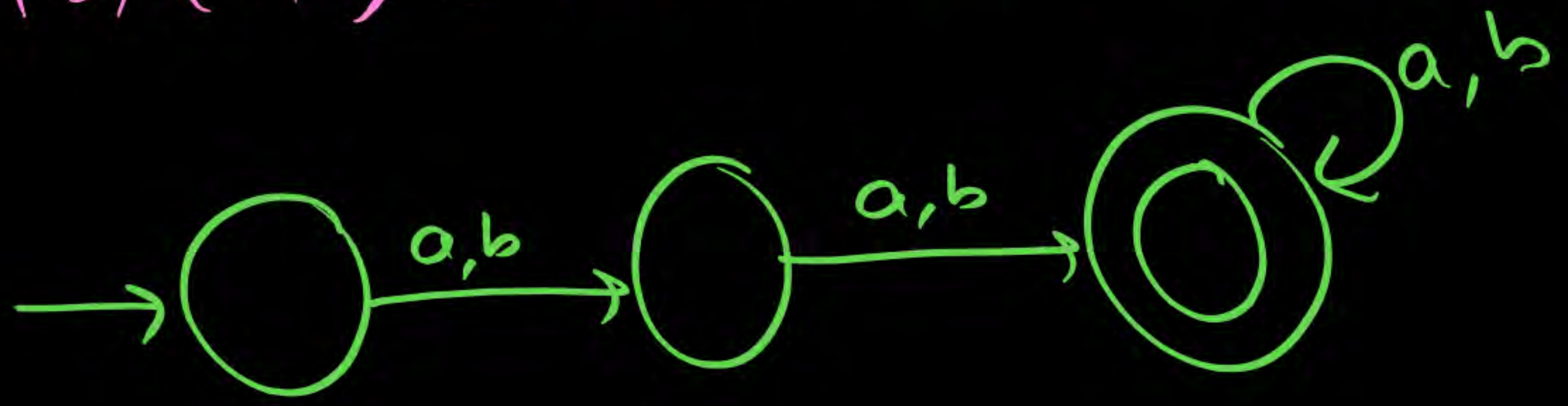
Note: If $|w| \geq n \Rightarrow n+1$ states in DFA

(8) $\{w \mid w \in \{a,b\}^*, |w| \geq \boxed{2}\}$.

$$L = (a+b)^2 (a+b)^* = x^2 x^* = x x^+ = x^+ x = x^+ x^+$$

Assume

$x = a+b$



Exactly n length

At most n length

Dead state is required $\Rightarrow (n+1) + 1$ Dead

At least n length

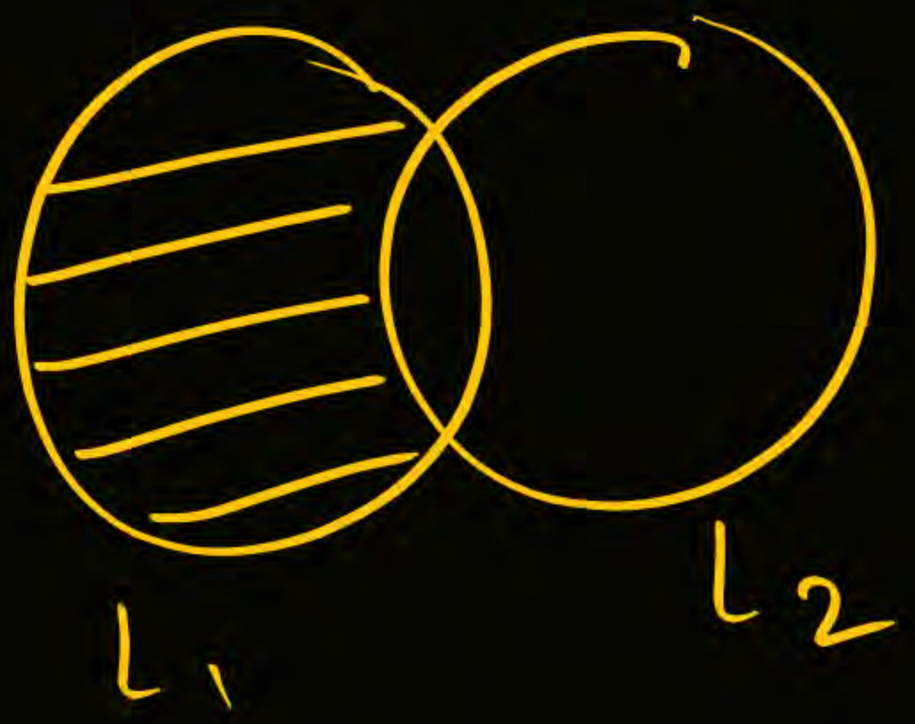
Dead state is not required $\Rightarrow n+1$ states

$$L = \{ w \mid |w| = 1024 \, , \, w \in \{a, b, c\}^* \}$$

$\Downarrow$

1026 states in DFA

$$\boxed{L_1 - L_2 = L_1 \cap \overline{L_2}}$$



FA

represent L

$L = \{w_1, w_2, \ldots\}$

Halts at final

$= \phi$

not a string

empty set

empty expression

Valid string

$w \in L$

Summary

Model-III [No. of Symbols]

① $\{w \mid w \in \{a,b\}^*, \underbrace{n_a(w) = 2}_{\text{No. of a's in } w \text{ is } 2}\}$

② $\{w \mid w \in \{a,b\}^*, n_a(w) \le 2\}$

③ $\{w \mid w \in \{a,b\}^*, n_a(w) \ge 2\}$

4 state ⇐ ④ $\{w \mid w \in a^*, \underbrace{n_a}_{|w|}(w) = 2\}$

4 State ⇐ ⑤ $\{w \mid w \in a^*, \underbrace{n_a(w)}_{|w|} \le 2\}$

3 State ⇐ ⑥ $\{w \mid w \in a^*, \underbrace{n_a(w)}_{|w|} \ge 2\}$

$w = aaa$

$|w| = 3$

$n_a(w) = 3$

$\underbrace{|w| = n_a(w)}_{\text{same}}$


THANK YOU
GW
SOLDIERS !