CS & IT ENGINEERING

Inference Rule

Lecture No. 03

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TOPICS TO BE COVERED

01 Inference Rule

02 Type 3 Questions in logic

03 Type 3 with Type 1

04 GATE QUESTIONS on 3 &1

05 Practice

The Simplest form of $(p \wedge (\sim r \vee \boxed{q \vee \sim q})) \vee ((r \vee t \vee \sim r) \wedge \sim q)$ is

(a) $p \wedge \sim q$

(b) $p \vee \sim q$ ✓

(c) t

(d) $(p \rightarrow \sim q)$

$(\neg r \vee T) \quad \vee \quad \boxed{T \vee t}$

$p \wedge T \qquad T \wedge \neg q$

$p \quad \vee \quad \neg q$

$A \vee \neg A \equiv T$

$p \wedge T \equiv p$

$p \vee \neg q$

The Simplest form of

$(p \vee (p \wedge q) \vee (p \wedge q \wedge \sim r)) \wedge ((p \wedge r \wedge t) \vee t)$ is

(a) $p \wedge t$

(b) $q \wedge t$

(c) $p \wedge r$

(d) $p \wedge q$

$\underline{p} \vee (\underline{p} \wedge \text{—})$

$p \wedge t$

Which one of the following is NOT equivalent to $p \leftrightarrow q$?

(GATE-15-Set1)

B) $(\neg p \vee q) \wedge (q \to p)$ (B)

(a) $(\sim p \vee q) \wedge (p \vee \sim q)$ True

$p \longleftrightarrow q \equiv (p \to q) \wedge (q \to p)$

$p \to q \quad \wedge (q \to p)$

(b) $\boxed{(\sim p \vee q)} \wedge (q \to p)$ (True)

A) $(\neg p \vee q) \wedge (\neg q \vee p)$

(c) $(\sim p \wedge q) \vee (p \wedge \sim q)$

(d) $(\sim p \wedge \sim q) \vee (p \wedge q)$

P and Q are two propositions. Which of the following logical expressions are equivalent?

I. $P \vee \sim Q$

II. $\sim(\sim P \wedge Q)$ $\equiv P \vee \neg Q$

III. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$

IV. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge Q)$

(GATE-08)

(a) Only I and II

(b) Only I, II and III

(c) Only I, II and IV

(d) All of I, II, III and IV

$(\boxed{P \wedge} Q) \vee (\boxed{P \wedge} \neg Q) \vee (\neg P \wedge \neg Q)$

$P \wedge (\underline{Q \vee \neg Q})$

$\underline{P \wedge T}$

$\underline{P}$

$\underline{P \vee (\neg P \wedge \neg Q)}$

Type-1

valid ?

$(\wedge \quad \wedge \quad) \longrightarrow$ __.

Type-2 :

Solve find out equivalent expression.

$$A \equiv B.$$

$\boxed{\text{premises}} \rightarrow \boxed{\text{conclusion.}}$

premises ↓ Take premises as True

conclusion ↓ Check the conclusion True

I → $\boxed{\text{conclusion}}$ ✓ (I.R)

Check (True)

Check (false) ~~(I.R)~~

Take premises as True check the conclusion

if it is True. (I·R)

if it is false (I ✗ R)

1. premises $\rightarrow$ conclusion OR $\dfrac{\text{premises}}{\therefore \text{conclusion}}$

eg: $A \rightarrow B$ OR $\dfrac{A}{\therefore B}$

2: $\overline{(P_1 \wedge P_2 \wedge P_3 \wedge \quad P_n)}^{\text{True}} \rightarrow Q$ OR

$$\begin{array}{c} P_1 \\ P_2 \\ P_3 \\ \vdots \\ P_n \\ \hline \therefore Q \end{array}$$

propositional stmt which has been written on the left ⟶ is called premises

Arguments

propositional stmt which has been written on right side conclusion

L ∨ R.

→ P1 : $\boxed{\text{mobile phone either in left}}$ or $\boxed{\text{right pocket (T)}}$

→ P2 : mobile phone it is not in left pocket (T)

---

mobile phone it is in right pocket

$$
\begin{array}{l}
P_1 : L \vee R. \\
P_2 : \neg L. \\
\hline
\therefore \; R.
\end{array}
$$

$P_1: L \vee R$

$P_2: \neg L$

$\therefore R$

OR

$$\overbrace{\left((L \vee R) \wedge \neg L\right)}^{T} \rightarrow R \quad \text{Check} \quad (\text{I.R})$$

L	R	$\neg L$	$L \vee R$	$(L \vee R) \wedge \neg L$	$((L \vee R) \wedge \neg L) \rightarrow R$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

$$\overbrace{((L \vee R) \wedge \neg L)}^{T} \rightarrow \overset{T}{R}\ (\text{valid})(\text{tautology})$$

$$((P \vee Q) \wedge \neg P) \longrightarrow Q$$

$$\begin{array}{l} P \vee Q \\ \neg P \\ \hline \therefore Q \end{array}$$

$$(\boxed{(a \rightarrow b)} \vee \boxed{(c \rightarrow d)}) \wedge \boxed{\neg(a \rightarrow b)} \longrightarrow \overset{Q}{\boxed{(c \rightarrow d)}}$$

$$\begin{array}{l} P \rightarrow Q \\ \neg P \\ \hline \therefore Q \end{array}$$

$$\begin{array}{l} \overset{P}{\boxed{(a \rightarrow b)}} \vee \overset{Q}{\boxed{(c \rightarrow d)}} \\ \boxed{\neg(a \rightarrow b)}\ \neg P \\ \hline (c \rightarrow d) \end{array}$$

$$((L \vee R) \wedge \neg L) \to R$$

$R = F$

$\neg L = T$

$L = F$

$$(F \vee F)$$

F

$F \longrightarrow$

True.

$$P \to Q$$

P_1: $\boxed{\text{if you will win the match}}$ then $\boxed{\text{i will give pizza party}}$ (T)

P_2: $\boxed{\text{you won the match. (T}}$)

---

i will give pizza party.

$$\begin{array}{c} P \to Q \\ P \\ \hline \therefore Q \end{array} \quad \Bigg| \quad \begin{array}{l} \text{True} \\ \\ \text{(check)} \end{array}$$

$\cdot$ win $\rightarrow$ pizza (T) $\quad P \rightarrow Q$ (T)

won (T) $\quad P$ (T) $\quad [(P \rightarrow Q) \wedge P] \rightarrow Q$ (valid)

pizza (T) $\quad \therefore Q$

P.M $\rightarrow$ Even (T)

P.M (T)

Even

P	Q	$P \rightarrow Q$
$\boxed{T}$	T	$\boxed{T}$
$\boxed{T}$	F	F
F	T	$\boxed{T}$
F	F	$\boxed{T}$

$$\frac{\begin{array}{c} P \rightarrow Q \\ P \end{array}}{\therefore Q}$$

(may or may not)

win → pizza. $P_1: P \to Q$ (T)

pizza

---

won

$$\frac{\begin{array}{c} P_1: P \to Q \ (T) \\ Q \ (T) \end{array}}{\therefore P}$$

$$\overline{[(P \to Q) \wedge Q]}^{\,T} \to P$$

check $\left\{ \dfrac{\begin{array}{c} P \vee Q \\ \neg P \end{array}}{\therefore Q} \right.$

valid $\left\{ \dfrac{\begin{array}{c} P \to Q \\ P \end{array}}{\therefore Q} \right.$

P.m → Even

Even.

P.m

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

Invalid $\dfrac{\begin{array}{c} P \to Q \\ Q \end{array}}{P}$ ✗

$$\begin{array}{l} P \rightarrow Q \\ P \\ \hline \therefore Q \end{array} \qquad [(P \rightarrow Q) \wedge P] \rightarrow Q$$

$$\begin{array}{l} P \rightarrow Q \\ \neg Q \\ \hline \therefore \neg P \end{array} \equiv \begin{array}{l} \neg Q \rightarrow \neg P \\ \neg Q \\ \hline \therefore \neg P \end{array} \qquad [(P \rightarrow Q) \wedge \neg Q] \rightarrow \neg P$$

$$\begin{array}{l} P \vee Q \\ \underline{\neg P} \\ \therefore Q \end{array} \qquad [(P \vee Q) \wedge \neg P] \longrightarrow Q$$

$$\begin{array}{l} P \rightarrow Q \\ \underline{Q \rightarrow R} \\ \therefore P \rightarrow R \end{array} \qquad [(P \rightarrow Q) \wedge (Q \rightarrow R)] \longrightarrow (P \rightarrow R)$$

$$\frac{P \wedge Q}{\therefore P}$$

$$\overline{(\underline{P}_{T} \wedge \underline{Q}_{T})}^{\,T} \rightarrow P \; (T)$$

Check

$$\frac{P}{\therefore P \vee Q}$$

$$\overset{T}{P} \rightarrow \overline{\underline{P}_{T} \vee Q}_{T}$$

$$\frac{\begin{array}{c} P \vee Q \\ \neg Q \vee R \end{array}}{P \vee R}$$

$$[(P \vee Q) \wedge (\neg Q \vee R)] \rightarrow (P \vee R)$$

1. $$\begin{array}{l} P \to Q \\ P \\ \hline \therefore Q \end{array}$$ (modus ponens)

2. $$\begin{array}{l} P \to Q \\ \neg Q \\ \hline \therefore \neg P \end{array}$$ (modus tollens)

$$\begin{array}{l} \neg Q \to \neg P \\ \neg Q \\ \hline \neg P \end{array}$$

3. $$\begin{array}{l} P \to Q \\ Q \to R \\ \hline P \to R \end{array}$$ (Hypothetical syllogism)

4. $$\begin{array}{l} P \vee Q \\ \neg P \\ \hline \therefore Q \end{array}$$ Disjunctive syllogism

$$\begin{array}{l} P \\ \hline P \vee Q \end{array}$$ Addition

$$\begin{array}{l} P \wedge Q \\ \hline \therefore P \end{array}$$ OR $$\begin{array}{l} P \wedge Q \\ \hline \therefore Q \end{array}$$

(Simplification)

$$\begin{array}{l} P \vee Q \\ \neg Q \vee R \\ \hline P \vee R \end{array}$$ (Resolution)

$$\begin{array}{l} P \to Q \\ Q \\ \hline \therefore P \end{array}$$ Invalid (fallacy)

P
$P \to Q$
$\neg Q \vee m$

$\therefore m$

Check it is valid?

1. P ✓ (T)
2. $P \to Q$ (T) (modus ponens)

Q (T)

(D.S)

3. $\neg Q \vee m$ (T) 4.

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m (valid)

$P \vee Q$
$\neg P$

$\therefore Q$

OR

$\neg P \vee Q$
P

$\therefore Q$

$$\begin{array}{l} P \to Q \\ Q \to m \\ P \\ \hline m \end{array}$$

$$\begin{array}{ll} 1. & P \to Q \\ 2. & Q \to m \\ \hline & P \to m \end{array}$$

(Hypothetical syllogism)

$$\begin{array}{ll} & P \to m \\ 3. & P \\ \hline & \therefore m \end{array}$$

(modus ponens)

$$[(p \vee Q) \wedge \neg Q \wedge (p \rightarrow m) \wedge (\neg m \vee x)] \longrightarrow x.$$

1. $P \vee Q$
2. $\neg Q$
3. $P \longrightarrow m$
4. $\dfrac{\neg m \vee x}{x}$

1. $P \vee Q$

2. $\dfrac{\neg Q}{P}$ (Dist. syllogism)

3. $\dfrac{P \longrightarrow m}{m}$ (modus ponens)

4. $\dfrac{\neg m \vee x}{x}$

$$\begin{array}{l} P \rightarrow m \\ \wedge \\ P \\ \wedge \\ m \rightarrow n \\ \wedge \\ n \rightarrow w \\ \hline w \end{array}$$

$$\begin{array}{l} P \rightarrow m \\ P \\ \hline \end{array}$$

$$\begin{array}{l} m \\ m \rightarrow n \\ \hline \end{array}$$

$$\begin{array}{l} n \\ n \rightarrow w \\ \hline w \end{array}$$

$$\begin{array}{l} m \rightarrow n \\ P \\ n \rightarrow w \\ P \rightarrow m \\ \hline w \end{array}$$

GATE:

if it Rains cricket match will not Played

cricket match was played

There was no rain.

valid

$$\begin{array}{c} R \to \neg P \\ P \\ \hline \therefore \neg R \end{array} \equiv \begin{array}{c} P \to \neg R \\ P \\ \hline \neg R \end{array}$$

2 if it is Rains cricket match will not Played (fallacy)

There was no Rain.

---

cricket match was played

$$\begin{array}{c} R \to \neg P \\ \neg R \\ \hline \therefore P \end{array} \equiv \begin{array}{c} P \to \neg R \\ \neg R \\ \hline P \end{array}$$

Let p, q, r and s be four primitive statements. Consider the following arguments:

P: $[(\sim p \vee q) \wedge (r \rightarrow s) \wedge (p \vee r)] \rightarrow (\sim s \rightarrow q)$

Q: $[(\sim p \wedge q) \wedge [q \rightarrow (p \rightarrow r)]] \rightarrow \sim r$

R: $[[(q \wedge r) \rightarrow p] \wedge (\sim q \vee p)] \rightarrow r$

S: $[p \wedge (p \rightarrow r) \wedge (q \vee \sim r)] \rightarrow q$

Try : Type-3

Type-1

Which of the above arguments are valid?

(GATE - 04)

(a) P and Q only

(b) P and R only

(c) P and S only

(d) P,Q,R and S

The statement formula $\{(a\to c)\wedge(b\to d)\wedge(c\to \sim d)\}\to(\sim a\vee\sim b)$ is

(a) satisfiable but not-valid

(b) valid

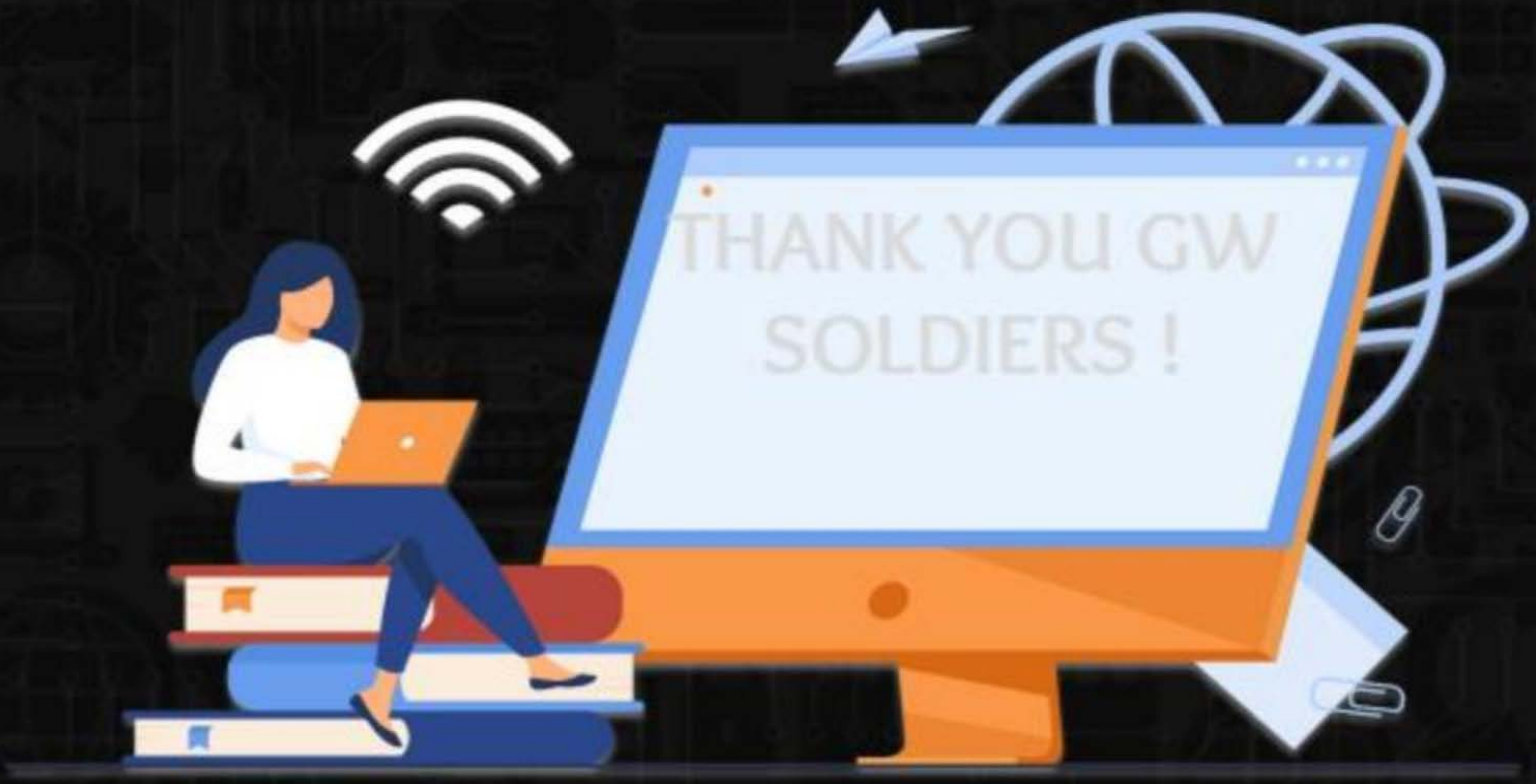
(c) not satisfiable

(d) none of these

$$
\begin{array}{l}
a\to c \\
b\to d \\
c\to \neg d \\
\hline
\neg a\vee\neg b
\end{array}
\qquad
\begin{array}{l}
\equiv \neg a\vee c \\
\equiv \neg b\vee \boxed{d} \\
\equiv \neg c\vee \boxed{\neg d} \\
\hline
\neg b\vee \boxed{\neg c} \\
\neg a\vee \boxed{c} \\
\hline
\neg a\vee\neg b
\end{array}
$$

Resolution

Resolution.


THANK YOU GW
SOLDIERS !