CS & IT ENGINEERING

Theory of Computation

Finite Automata

Lecture No.03

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TOPICS TO BE COVERED

01 Kleene star, Kleene plus

02 properties of + , ·

03 Simplification of Reg Exps

04 How to write Reg Exp. ?

05

Kleene star :

$$R^*$$

$$R^0 + R^1 + R^2 + R^3 + R^4 + \dots$$

$$a^* \Rightarrow \{\varepsilon, a, aa, aaa, \dots\}$$

$$L(a^*) = \{\varepsilon, a, a^2, a^3, \dots\}$$

$$= \{a^n \mid n \geq 0\}$$

$$R^2 = R.R$$

$$R^3 = R.R.R$$

Language (Set)

1) b^* $\Longrightarrow$ $\{\varepsilon, b, bb, bbb, bbbb, \dots\} = \{b^n \mid n \geq 0\}$

2) $(ab)^*$ $\Longrightarrow$ $\{\varepsilon, ab, abab, ababab, \dots\} = \{(ab)^n \mid n \geq 0\}$

3) $(ba)^*$ $\Longrightarrow$ $\{\varepsilon, ba, baba, bababa, \dots\} = \{(ba)^n \mid n \geq 0\}$

4) $(aa)^*$ $\Longrightarrow$ $\{\varepsilon, a^2, a^4, a^6, a^8, \dots\} = \{a^{2n} \mid n \geq 0\}$

5) $(a+b)^*$ $\Longrightarrow$ $\{\varepsilon, a, b, aa, ab, ba, bb, \dots\} = \{a, b\}^*$

6) $(a+\varepsilon)^*$ $\Longrightarrow$ $\{\varepsilon, a, a^2, a^3, \dots\} = \{a^n \mid n \geq 0\} = a^*$

7) $(b+\varepsilon)^*$ $\Longrightarrow$ b^*

$$\Sigma = \{a, b\} = a + b$$

$$\Sigma^* = \{a, b\}^* = (a+b)^*$$

$$\Sigma^2 = \{a, b\} \cdot \{a, b\}$$

$$= \{aa, ab, ba, bb\}$$

= Set of all 2 length strings

$$\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \cup \ldots$$

$$= \{\varepsilon\} \cup \{a, b\} \cup \{aa, ab, ba, bb\} \cup \ldots$$

$$= \{\varepsilon, a, b, aa, ab, ba, bb, \ldots\}$$

= Set of all strings over Σ = Universal Language

$a+b$

- a ✓
- b ✓
- ab ✗
- ba ✗
- aa ✗
- bb ✗

$(a+b)^*$

- $(a+b)^0 \longrightarrow \varepsilon$
- $(a+b)^1 \longrightarrow a, b$
- $(a+b)^2 = (a+b)\cdot(a+b)$

$= aa + ab + ba + bb$

$$(a+\varepsilon)^* \left\{ \begin{array}{l} (a+\varepsilon)^0 \longrightarrow \varepsilon \\ (a+\varepsilon)^1 \longrightarrow a, \varepsilon \\ (a+\varepsilon)^2 \longrightarrow (\underline{a}+\varepsilon)(\underline{a}+\underline{\varepsilon}) \longrightarrow a^2, a, \varepsilon \\ (a+\varepsilon)^3 \longrightarrow a^3, a^2, a, \varepsilon \end{array} \right.$$

$$\underbrace{\qquad\qquad\qquad\qquad}_{\varepsilon, a, a^2, a^3, a^4, \ldots} = a^*$$

$$(a+b+\varepsilon)^* = (a+b)^*$$

Kleene plus :

$$R^+$$

$$R^1 + R^2 + R^3 + R^4 + \ldots$$

$$a^+ \Longrightarrow \{a, a^2, a^3, a^4, \ldots\} = \{a^n \mid n \geq 1\}$$

$$b^+ \Longrightarrow \{b, b^2, b^3, b^4, \ldots\} = \{b^n \mid n \geq 1\}$$

1) $(a+\varepsilon)^+ \Rightarrow a^* = \{a^n \mid n \geq 0\}$

2) $(b+\varepsilon)^+ \Rightarrow b^*$

3) $(a+b+\varepsilon)^+ \Rightarrow (a+b)^*$

4) $a^+ + \varepsilon \Rightarrow a^+ + a^0 = a^*$

5) $a^* + \boxed{\varepsilon} \Rightarrow a^*$

$(a+\varepsilon)^1 \rightarrow a$ ✓, ε ✓

$(a+\varepsilon)^2 = (a+\varepsilon)(a+\varepsilon) \rightarrow \varepsilon.\varepsilon = \varepsilon,\ a,\ a^2$

$\vdots$

$= \varepsilon, a, a^2, a^3, \ldots$

$= a^*$

$$\boxed{R^* = R^+ + R^0}$$

$$\cancel{R^+ = R^* - \{\varepsilon\}}$$

$$\underline{a}^+ + \boxed{\varepsilon} = \underbrace{a + a^2 + a^3 + \cdots + \varepsilon}$$

$$= a^*$$

	OR	Concatenation
① Identity	ϕ Empty exp.	ε
② Associative	✓ holds	✓ holds
③ commutative	✓ holds	✗ not holds
④ Dominator (Annihilator)	Σ^*	ϕ

(5) Distributive

I) OR over Concatenation

Does not hold

i) Left Distribution

$$\underbrace{a+(b\cdot c)}_{\{a,\ bc\}} \ ?\ \underbrace{(a+b)\cdot(a+c)}_{\{aa,\ ac,\ ba,\ bc\}}$$

ii) Right Distribution

$$\underbrace{(a\cdot b)+c}_{\{ab,\ c\}} \ ?\ \underbrace{(a+c)\cdot(b+c)}_{\{ab,\ ac,\ cb,\ cc\}}$$

OR should not be distributed over Concatenation

II) Concatenation over OR

It holds

i) Left Distribution:

$$a.(b+c) = (a.b)+(a.c)$$

$\{ab, ac\}$ $\{ab, ac\}$

Concatenation is distributed over OR

ii) Right Distribution:

$$(a+b).c = (a.c)+(b.c)$$

$\{ac, bc\}$ $\{ac, bc\}$

$$a \cdot \phi = \phi$$

$$\varepsilon \cdot \phi = \phi$$

$$\phi \cdot \phi = \phi$$

$$\{a\} \cdot \{\varepsilon\} \longrightarrow \{a\}$$

$$\{a\} \cdot \{\ \} \longrightarrow \{\ \}$$

Dominator:

Σ^* is dominator for $+$

$$R + D = D + R = D$$

$$\boxed{R + \Sigma^* = \Sigma^* + R = \Sigma^*}$$

ϕ is dominator for $\cdot$

$$R \cdot D = D \cdot R = D$$

$$\boxed{R \cdot \phi = \phi \cdot R = \phi}$$

$$R \cup D = D$$

What is D in the world?

$$R \cup \underbrace{\Sigma^*}_{\text{every string}}$$

every string

prelem

$$\boxed{R \subseteq \Sigma^*}$$

commutative

$$R_1 + R_2 = R_2 + R_1$$

Holds

Note:

$R_1 = a$

$R_2 = \varepsilon$

$$\boxed{a \cdot \varepsilon = \varepsilon \cdot a = a}$$

$$R_1 \cdot R_2 \neq R_2 \cdot R_1$$

need not be equal

Does not hold

Associative:

$$\underbrace{(R_1 + R_2) + R_3}_{\text{Left Associative}} = \underbrace{R_1 + (R_2 + R_3)}_{\text{Right Associative}}$$

$$(a + b) + c = a + (b + c)$$

$$\{a, b, c\}$$

$$(R_1 \cdot R_2) \cdot R_3 = R_1 \cdot (R_2 \cdot R_3)$$

$$(a \cdot b) c = a \cdot (b \cdot c)$$

$$\{abc\}$$

Identity:
(I)

$$R + I = I + R = R$$

$\underbrace{I = \phi}_{\text{for } +}$

$$\boxed{R + \phi = \phi + R = R}$$

ϕ is identity for "OR" operator

$\underbrace{I = \varepsilon}_{\text{for } \cdot}$

$$R \cdot I = I \cdot R = R$$

$$\boxed{R \cdot \varepsilon = \varepsilon \cdot R = R}$$

ε is identity for concatenation

Simplification :

1) $\phi^* = \epsilon$

2) $\epsilon^* = \epsilon$

3) $\phi^+ = \phi$

4) $\epsilon^+ = \epsilon$

$R^* = R^0 + R^1 + R^2 + \dots$

$\phi^* = \phi^0 + \phi^1 + \phi^2 + \dots$

$= \epsilon$

$\epsilon^* = \epsilon^0 + \epsilon^1 + \epsilon^2 + \dots$

$= \epsilon$

$$\phi^1 = \phi$$

$$\phi^2 = \phi . \phi = \phi$$

$$\phi^3 = \phi . \phi . \phi = \phi$$

$$\phi^0 = \varepsilon$$

$$\varepsilon^0 = \varepsilon$$

$$a^0 = \varepsilon$$

$$R^0 = \varepsilon$$

$$(\text{Any})^0 = \varepsilon$$

5) $(a+\phi)^* = a^*$

6) $(a+\phi)^+ = a^+$

7) $(\varepsilon+\phi)^+ = \varepsilon^+ = \varepsilon$

8) $(\varepsilon+\phi)^* = \varepsilon^* = \varepsilon$

9) $\phi^* . \phi^+ = \varepsilon . \phi = \phi$

10) $\varepsilon^* . \varepsilon^+ = \varepsilon . \varepsilon = \varepsilon$

11) $\phi^* . \varepsilon^+ = \varepsilon . \varepsilon = \varepsilon$

12) $\phi^+ . \varepsilon^* = \phi . \varepsilon = \phi$

13) $a^* . \phi^* = a^* . \varepsilon = a^*$

14) $a^* . \boxed{\phi^+} = a^* . \phi = \phi$

15) $a^* . a = a^+ = a . a^*$

16) $a . a^* = a^+ = a^* . a$

$$\boxed{a^* \cdot a} = a^+$$

$$\varepsilon \cdot a \longrightarrow a$$

$$a \cdot a \longrightarrow a^2$$

$$a^2 \cdot a \longrightarrow a^3$$

$$a^3 \cdot a \longrightarrow a^4$$

17. $a^* + a^* = a^*$

18. $a^* + \boxed{a^+} = a^*$

19. $a^+ + a^+ = a^+$

20. $\boxed{a} + a^+ = a + (a + a^2 + \cdots) = a^+$

21. $\boxed{a} + a^* = a^*$

22. $\boxed{a^{100}} + a^+ = a^+$

23. $a^* + a^+ + a + a^{1000} = a^*$

24) $(a^+)^2 = a^+ . a^+ = a . a^+ = a . a . a^* = a a^* a = a^* a a$

25) $(a^*)^2 = a^* . a^* = a^*$

26) $(a^*)^{100} = a^* . a^* \ldots$ 100 times $= a^*$

27) $a^* a^* a^+ a^* = a^+$

$\varepsilon \; \varepsilon \; a \; \varepsilon \rightarrow a$ min

$\varepsilon \; \varepsilon \; a^2 \; \varepsilon \rightarrow a^2$

$\vdots$

$= a^+$

$(a^+)^2 = a^+ \cdot a^+$

$a . a = a^2$

$a . a^2 = a^3$

$a^2 . a$

$a . a^3 = a^4$

$a . a^4 = a^5$

$\{a^n \mid n \geq 2\}$

$$(a^*)^* = a^*$$

$$(a^*)' = a^*$$

$$a^* \cup \text{Any} = a^*$$

$$\Sigma^* \cup \text{Any} = \Sigma^*$$

$$(a^*)^* = (a^*)^0 + (a^*)^1 + (a^*)^2 + \cdots$$

$$= \varepsilon + a^* + a^* + \cdots$$

$$= a^*$$

$\underbrace{a+\varepsilon}$

min string ?

ε

$\{\varepsilon, a\}$

$\varepsilon \rightarrow$ Zero length

$a \rightarrow$ 1 length

$$a^*\, a^*\, a^*\, a^*\, a^*\, \boxed{a^+}\, a^*\, a^* \rightarrow a^+ \checkmark$$

min ε min ε min ε min ε min ε min a min ε min ε

$$\boxed{a^*} + a^* + a^* + a^* \checkmark + a^* + a^+ + a^* + a^* = a^*$$

$$\Sigma^* \cup \text{Any} = \Sigma^*$$

$$\left(\left(a^{+}\right)^{+}\right)^{+} = a^{+}$$

$$\left(\left(a^{+}\right)'\right)' = a^{+}$$

$$a^* \cdot a^*$$

$$\varepsilon \quad a^* \longrightarrow \boxed{a^*}$$

$$a \cdot a^* \longrightarrow a^+$$

$$a^2 \cdot a^* \longrightarrow aaa^* = \boxed{aa^+}$$

$$\vdots$$

$$\Rightarrow a^*$$

$$\{a^n \mid n \geq 2\}$$

$$\{a^2, a^3, a^4, a^5, \ldots\}$$

$$= a \cdot a^+$$

$$= a^+ \cdot a^+$$

$$= a \cdot a \cdot a^*$$

$$\{a^{2n} \mid n \geq 1\}$$

$$\{a^2, a^4, a^6, \ldots\} = (aa)^+$$

28) $(a^*)^* = (a^*)^0 \cup \boxed{(a^*)^1} \cup (a^*)^2 \cup \cdots = a^*$

29) $(a^*)^+ = (a^*)^1 \cup (a^*)^2 \cup (a^*)^3 \cup \cdots = a^*$

30) $(a^+)^* = (a^+)^0 \cup (a^+)^1 \cup (a^+)^2 \cup \cdots = \varepsilon + a^+ = a^*$

31) $(a^+)^+ = (a^+)^1 \cup (a^+)^2 \cup (a^+)^3 \cup \cdots = a^+$

$$a^{*}.a = a^{+} = a.a^{*}$$

$$(\varepsilon + a + aa + \cdots).\,a$$

$$\varepsilon \;.\; a \longrightarrow a$$

$$a \;.\; a \longrightarrow a^{2}$$

$$a^{2} \;.\; a \longrightarrow a^{3}$$

$$\vdots$$

$$= a^{+} = a^{1} + a^{2} + a^{3} + \cdots$$

Note:

I) $R + \phi = R$

II) $R . \phi = \phi$

III) $R . \varepsilon = R$

IV) $R^0 = \varepsilon$

V) $R + \Sigma^* = \Sigma^*$

VI) $a . a^* = a^+$

VII) $a^* . a = a^+$

VIII) $(a+\varepsilon)^* = a^*$

IX) $(a+\varepsilon)^+ = a^*$

X) $a^+ + \varepsilon = a^*$

$$R + \Sigma^*$$

$$\underline{R} + \underbrace{\left[\Sigma^0 + \Sigma^1 + \Sigma^2 + \Sigma^3 + \cdots\right]}_{\text{Set of all strings}} = \Sigma^*$$

$\boxed{\varepsilon . a} = a$

$\underbrace{\{\varepsilon\} . \{a\}}_{\varepsilon . a = a}$

$\varepsilon . \phi = \phi$

$\{\varepsilon\} . \{\ \} = \{\ \}$

$$(a+\varepsilon)^+ = (a+\varepsilon)^1 + (a+\varepsilon)^2 + (a+\varepsilon)^3 + \dots$$

$$= [a+\varepsilon] + [\varepsilon+a+a^2] + [\varepsilon+a+a^2+a^3) + \dots$$

$$= \varepsilon + a + a^2 + a^3 + \dots$$

$$= a^*$$

$a+\varepsilon$

$$(a+\varepsilon)^2 = \underline{(a+\varepsilon)} \cdot \underline{(a+\varepsilon)} = \varepsilon + a + a^2$$

$$\begin{array}{ccc} \varepsilon & \cdot & \varepsilon \\ \varepsilon & \cdot & a \\ a & \cdot & \varepsilon \\ a & \cdot & a \end{array}$$

$$(a+\varepsilon)^3 = (a+\varepsilon)(a+\varepsilon)(a+\varepsilon)$$

$$= \varepsilon + a + a^2 + a^3$$

$$(a^{+})^2 = \underbrace{a^{+}}\cdot\underbrace{a^{+}}$$

$$\left.\begin{array}{l} a \cdot a \longrightarrow a^2 \\ a \cdot a^2 \longrightarrow a^3 \\ a \cdot a^3 \longrightarrow a^4 \\ \quad\quad\quad\; \vdots \end{array}\right\} = a \cdot a^{+}$$

$$= a^{+} \cdot a$$

$$= a \cdot a \cdot a^{*}$$

$$= a \cdot a^{*} \cdot a$$

$$= a^{*} \cdot a \cdot a$$

Home Work :

Find $\underbrace{\text{minimum string}}_{}$ in the following Expressions.

shortest length string

1) $(ab)^+ aaa(b+ab)$ $\longrightarrow$ min = abaaab

2) $(ab)^* aaa(b+\varepsilon)$ $\longrightarrow$

3) $(a+ab)^+ (bb+aaa)^+ (ab)^*$ $\longrightarrow$

4) $(ab)^+ + \varepsilon$

5) $(ab)(a+b)^*aaa + ba$

6) $[(ab^* + a^+ + ba)ab]^+(a+b)^*$

7) $[((ab)^+a)^*aaa]^+$

8) $[((aba)^+aba)^+ab]^*$

9) $(a^*)^+$

10) $((ab)^* a^+)^+ bb^+ aa^*$

11) $[(ab)^+ aba]^+ aaa$

12) $(aaa^* aba^*)^+$

13) $(a + aa + aaa)^+$

14) $a^+ b^+$

15) $a^* b^*$

16) $a^* b^* c^+ d^*$

17) $(a + b^* + c^+)^+$

18) $(a + b + \varepsilon)^+$

19) $((a+ab)aa + ab)^{+}aaa$

20) $(ab \cdot aaa \cdot a + a^{*})^{+}$

$$a + b^+ \cdot c^*$$

$$a + \left[(b^+) \cdot (c^*) \right]$$

Kleene star , Kleene plus

concatenation

or

Summary

→ Reg Exp basics ✓

<u>Next</u>: How to write Reg Exp ?

$$a^* \cdot a^* \cdot a^+ \cdot a^*$$

$$\left.\begin{array}{l} \varepsilon \cdot \varepsilon \cdot a \cdot \varepsilon \longrightarrow a \\ \varepsilon \; \varepsilon \; a^2 \; \varepsilon \longrightarrow a^2 \\ \varepsilon \; \varepsilon \; a^3 \; \varepsilon \longrightarrow a^3 \\ \vdots \end{array}\right\} = a^+$$

$$(ab)^* + \boxed{\varepsilon} = (ab)^*$$

$$\{\varepsilon, ab, (ab)^2, \ldots\} \cup \{\varepsilon\}$$

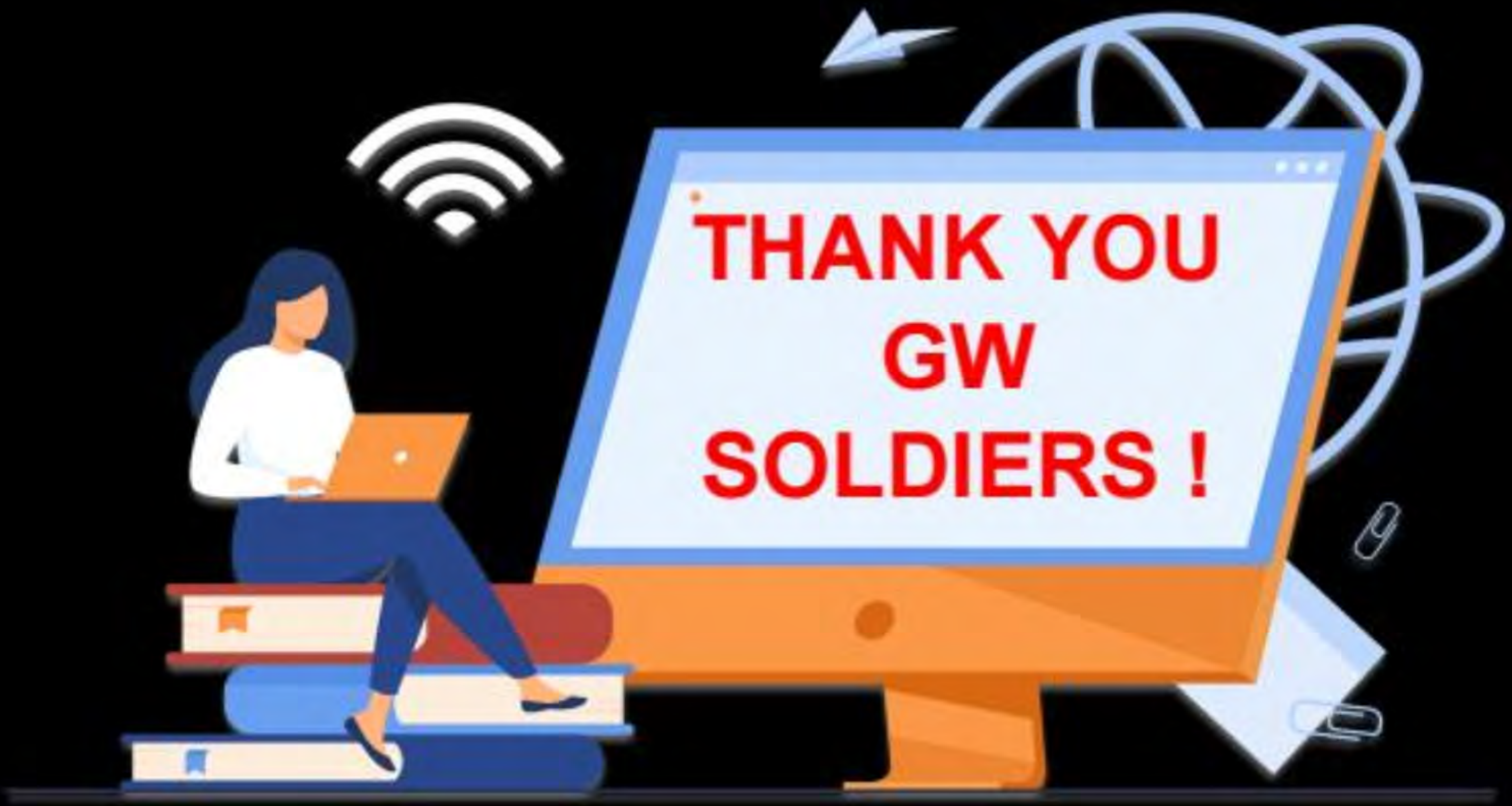
$R^0 \Rightarrow$ Zero occurrences of $R = \varepsilon$

$\phi^0 \Rightarrow \varepsilon$

$a^0 \Rightarrow \varepsilon$

$\varepsilon^0 \Rightarrow \varepsilon$

$(a+b)^0 \Rightarrow \varepsilon$


THANK YOU
GW
SOLDIERS !