# CS & IT ENGINEERING

## DISCRETE MATHS
## SET THEORY

Lecture No.12

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# TOPICS

**01 Partial order relation**

**02** Poset /toset

**3 lattice**

## Partial order relation :

Relation is partial order Relation.

(RAT) Reflexive.
and
Antisymmetric.
and
Transitive.

Set: $Z^+$

$R_1 : \{ (a,b) \mid a|b \} \rightarrow$ partial order Relation.

R: $aRa$. $a|a$. ✓

Anti: $aRb \wedge bRa \rightarrow a=b$.

$a|b \wedge b|a \rightarrow a=b$.

$(3|6) \wedge 6\nmid 3$

$\underbrace{\underbrace{(3|6)}_{T} \wedge \underbrace{6\nmid 3}_{F}}_{F} \rightarrow$

$\underline{\qquad\qquad\qquad}$

T.

T: $a|b \wedge b|c \rightarrow a|c$.

$2|4 \wedge 4|8 \rightarrow 2|8$

$3|6 \wedge 6|9 \rightarrow 3|9$.

$R_2 : \{ (a, b) \mid a \leq b \}$.

$T$: $a \leq b \wedge b \leq c \rightarrow a \leq c$ ✓

$1 \leq 3 \wedge 3 \leq 9 \rightarrow 1 \leq 9$.

$2 \leq 4 \wedge 4 \leq 8 \rightarrow 2 \leq 8$ ✓

R : $aRa$ $\quad a \leq a$ ✓

Anti: $aRb \wedge bRa \rightarrow a = b$ ✓

$a \leq b \wedge b \leq a \rightarrow a = b$.

$2 \leq 2 \wedge 2 \leq 2 \rightarrow 2 = 2$

$a \leq b \wedge b \leq a \rightarrow a = b$.

$\underbrace{3 \leq 6}_{T} \wedge \underbrace{6 \leq 3}_{F}$

Set: { . . . . . . . . . . . . . . . . . . . . . }.

Relation: → partial order Relation (R)

aRb OR bRa

a, b are comparable. Otherwise not comparable.

Set : $Z^+ = \{1, 2, 3, \ldots\ldots\ldots\}$

partial orderset (poset)

Relation : (POR) division $R : |$

partial elements are comparable.

(partial order relation)

3. 6.

$3 R 6$ OR $6 R 3$

T F

$3 | 6$ OR $6 \nmid 3$

3, 6 are comparable

5, 7

$5 \nmid 7$ OR $7 \nmid 5$.

5, 7 are not comparable

Set: Z : $\{\ldots -2, -1, 0, 1, 2 \ldots\}$

Total order set (TOSET)

Relation: $\leq$ (POR) → Total order Relation

all elements are comparable

3, 100

$3 R 100$ OR $100 R 3$

$\underbrace{3 \leq 100}_{T}$ OR $100 \not\leq 3$

3, 100 are comparable.

102, 2

$102 \leq 2$ OR $\underbrace{2 \leq 102}_{T}$

2, 102 are comparable.

Relation → RAT → Partial order Relation → check all elements are comparable.

(Reflexive. Anti Transitive)

set: partial order set (poset)

$\xrightarrow{\text{no}}$ Partial order Relation

$\downarrow$ Yes.

R: Total order Relation

Set: Total order set (TOSET)

(Set, Relation)

$(Z^+, |)$

$(Z, \leq)$

(Set, Relation)

$(P(A), \subseteq)$ poset?

$(\{1\}, \{1,2\})$

R: $ARA$ $\quad A \subseteq A$ ✓

Anti: $A \subseteq B \wedge B \subseteq A \rightarrow A = B$.

$\{1\} \subseteq \{1,2\} \wedge \{1,2\} \subseteq \{1\}$

T: $A \subseteq B \wedge B \subseteq C \rightarrow A \subseteq C$ (True)

$R = \{ (1,1)(1,2)(1,3)(1,4)(1,6)(1,12)$

$(2,2)(2,4)(2,6)(2,12)$

$(3,3)(3,6)(3,12)$

$(4,4)(4,12)$

$(6,6)(6,12)$

$(12,12) \}$

$\equiv$

Set: $\{1, 2, 3, 4, 6, 12\}$.

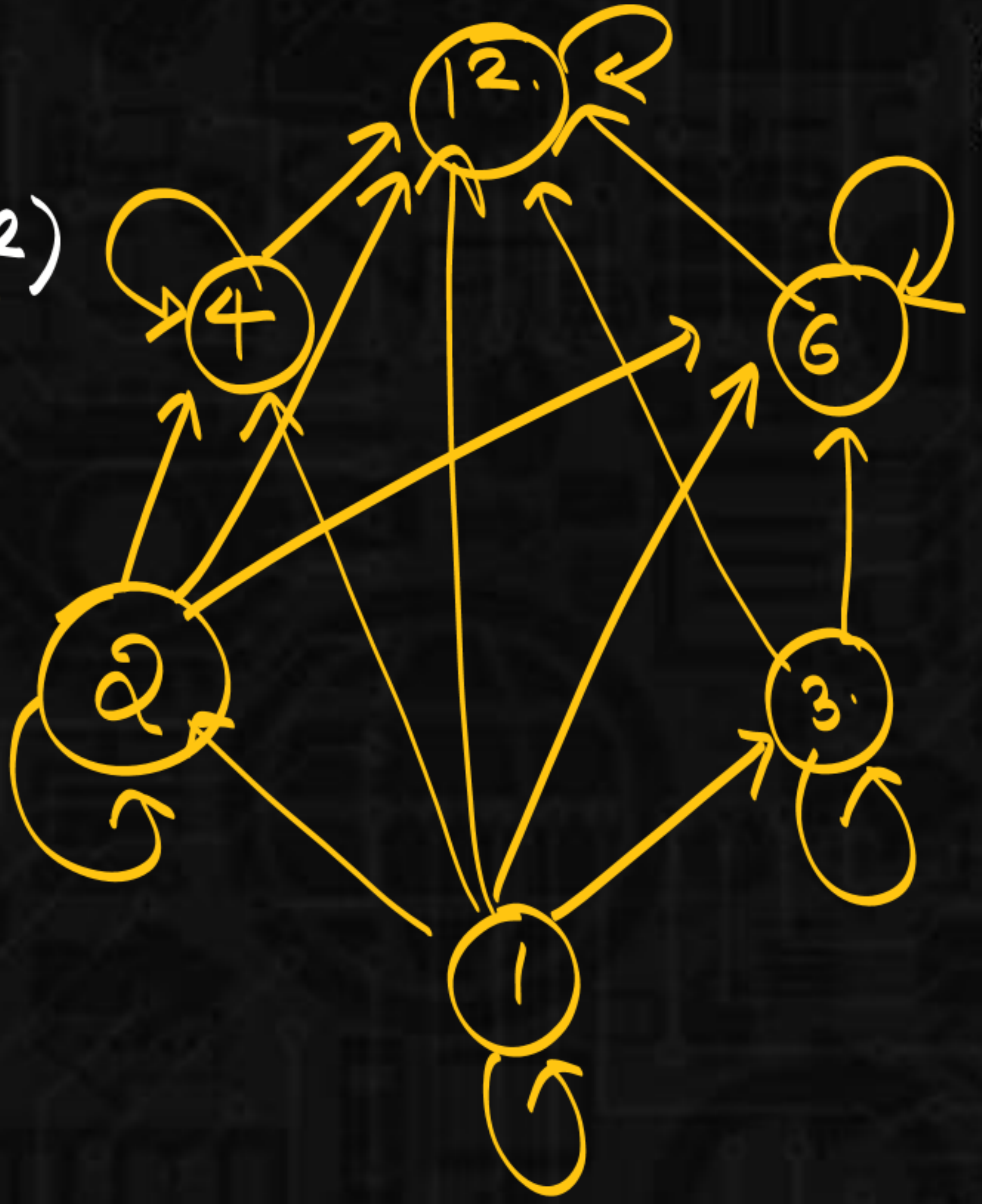
$D_n$: divisors of n

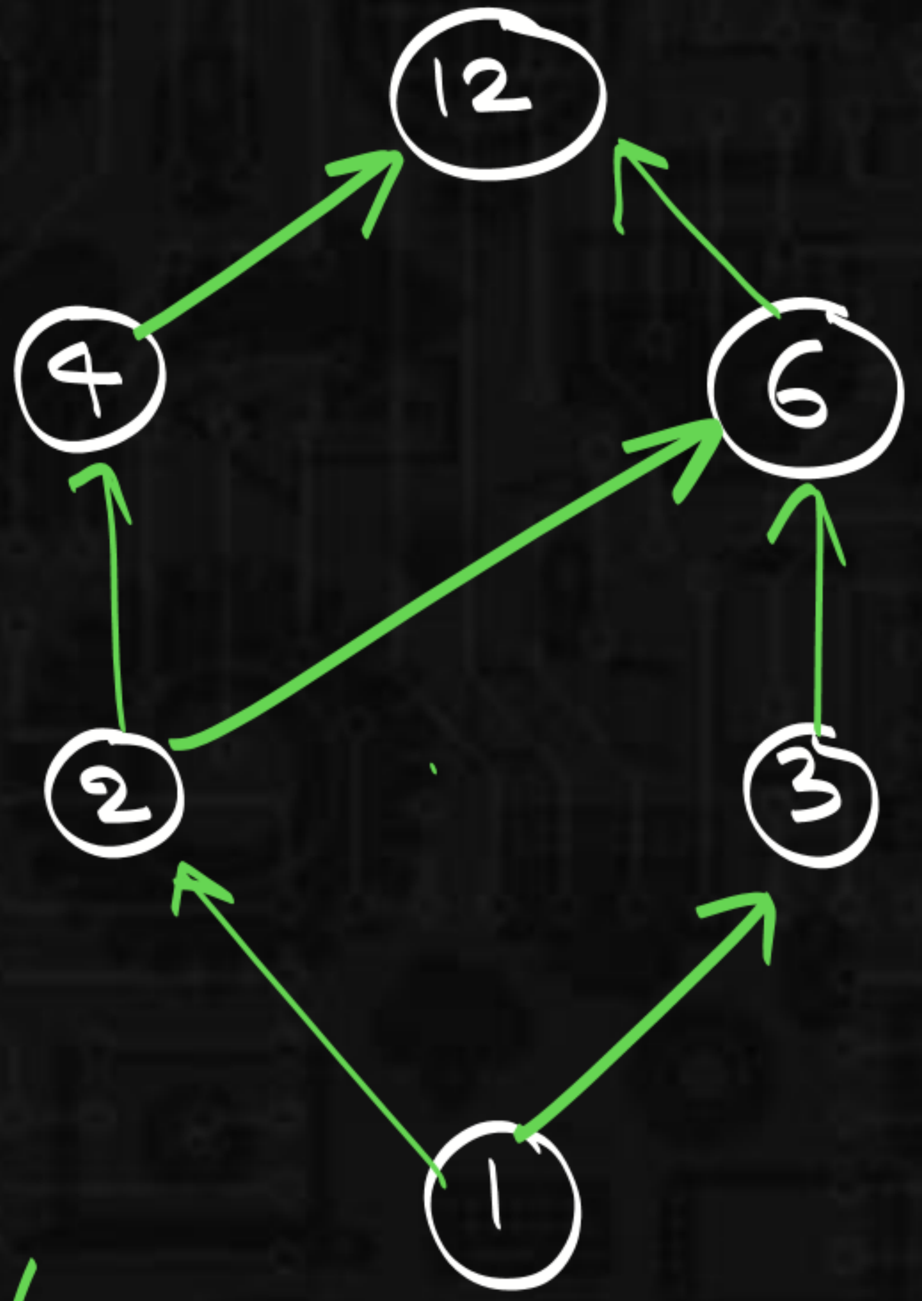
(Set, Relation)

$(\{1,2,3,4,6,12\}, \mid)$

$(D_{12}, \mid)$

$$R = \{ (1,1)(1,2)(1,3)(1,4)(1,6)(1,12)$$
$$(2,2)(2,4)(2,6)(2,12)$$
$$(3,3)(3,6)(3,12)$$
$$(4,4)(4,12)$$
$$(6,6)(6,12)$$
$$(12,12) \}$$

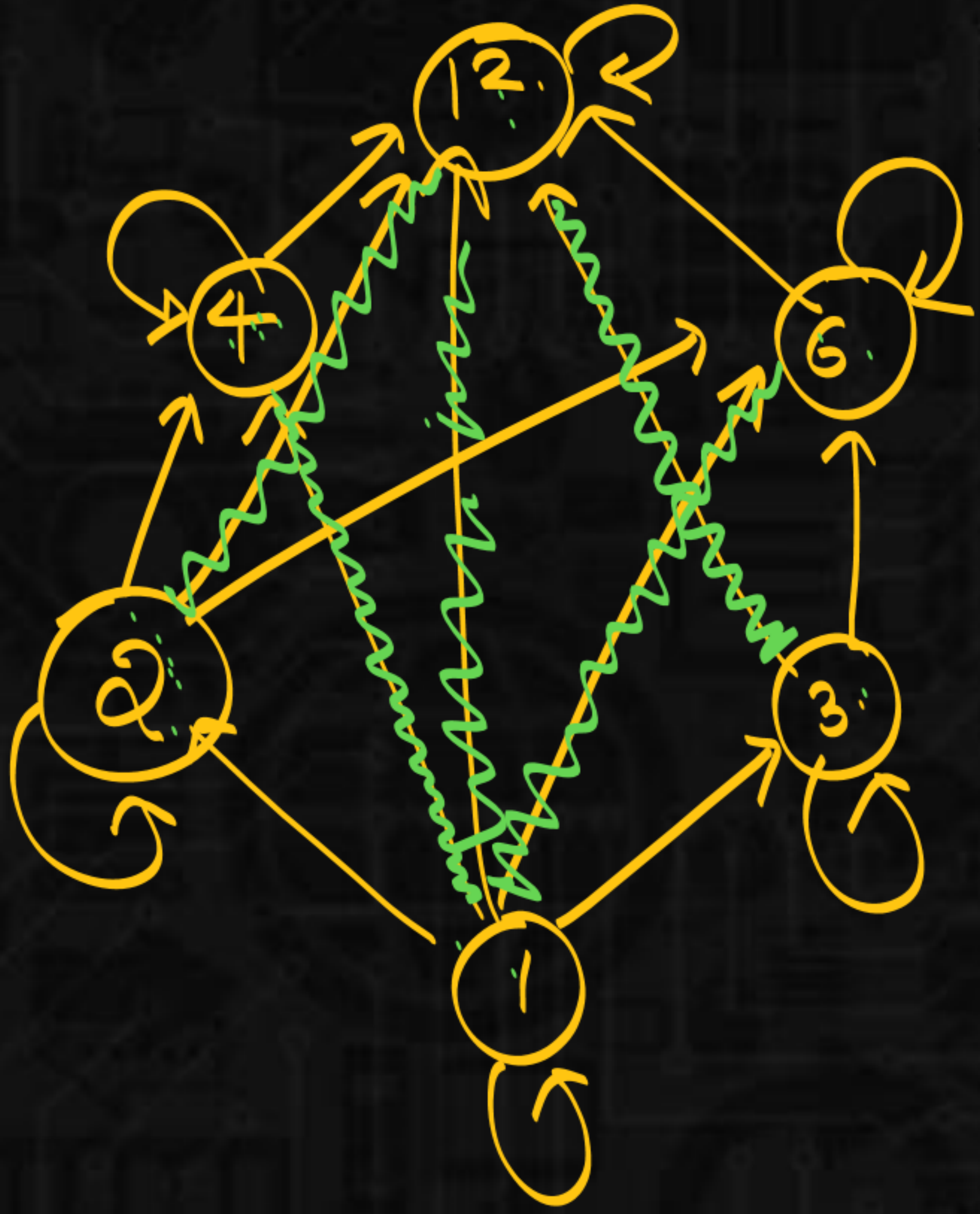


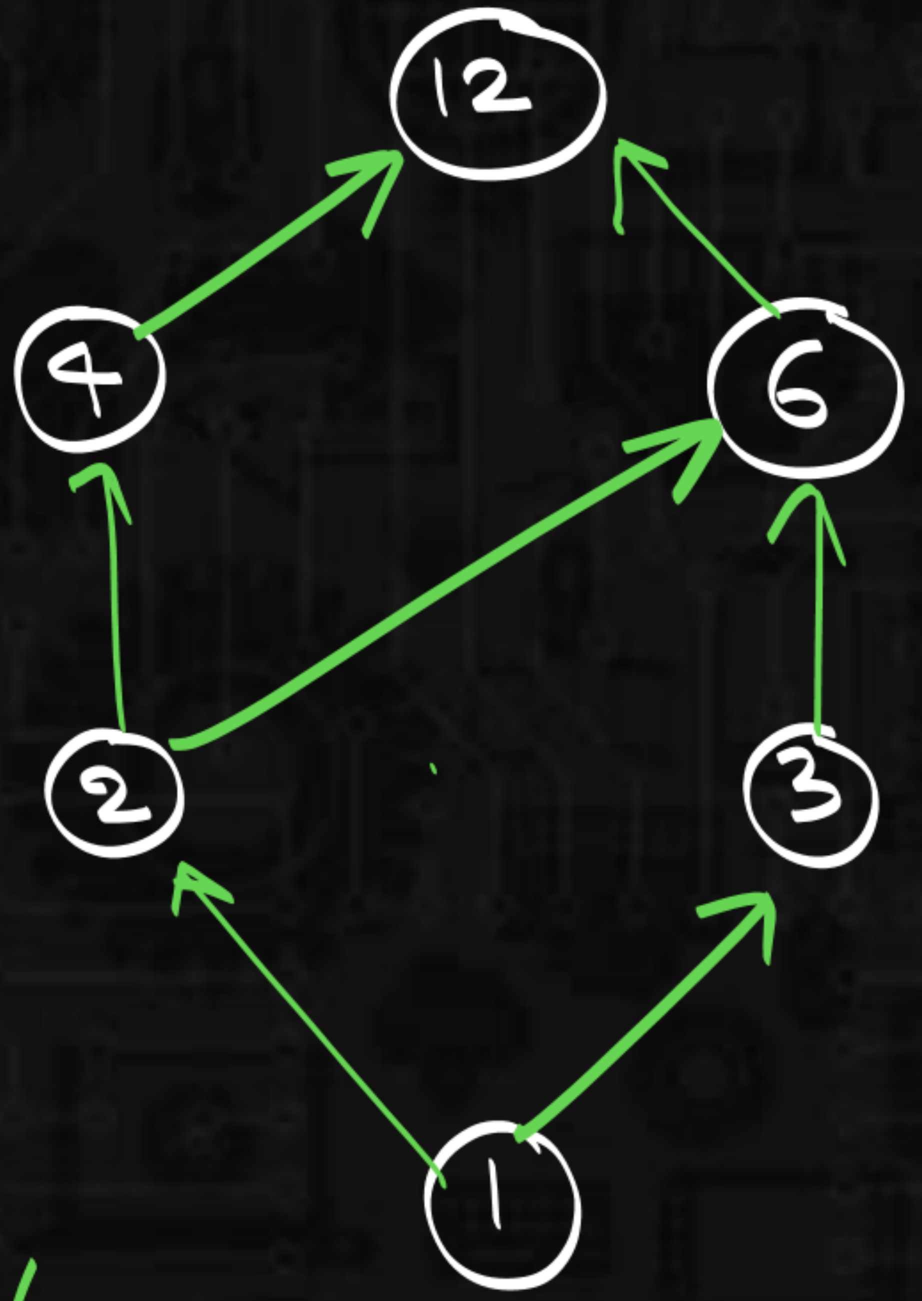



(Hasse diagram)

→ Remove self loop.

→ Remove Transitive arrows.



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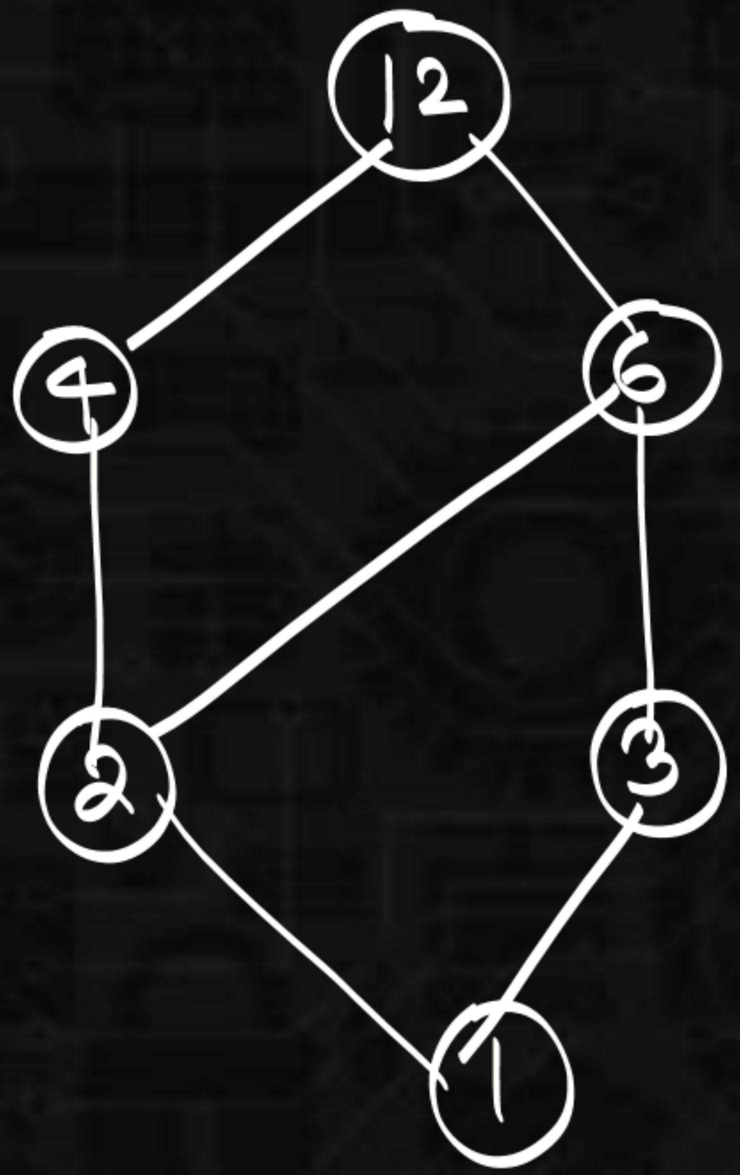


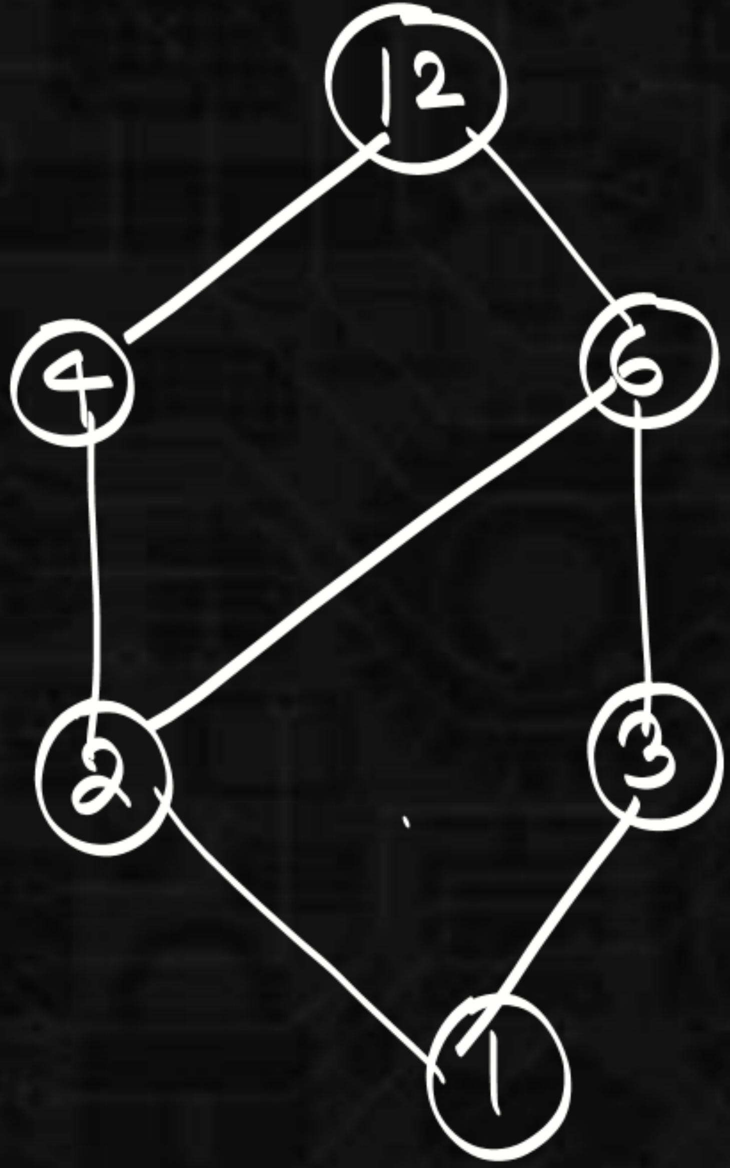

(Hasse diagram)

$R = \{ (1\,1)(1\,2) \ldots \ldots$

$\vdots$

$(12, 12) \}$

$(D_{12}, |)$



$aRb$

**Diagram :**

$$\begin{tikzcd} b \\ a \arrow[u] \end{tikzcd}$$

$$a \to b$$

x

$$\begin{tikzcd} a \arrow[d] \\ b \end{tikzcd}$$

x.

$(D_{24}, |)$

$\downarrow$

$(\{1, 2, 3, 4, 6, 8, 12, 24\}, |)$


$(D_{32}, |)$

$(D_{64}, |)$

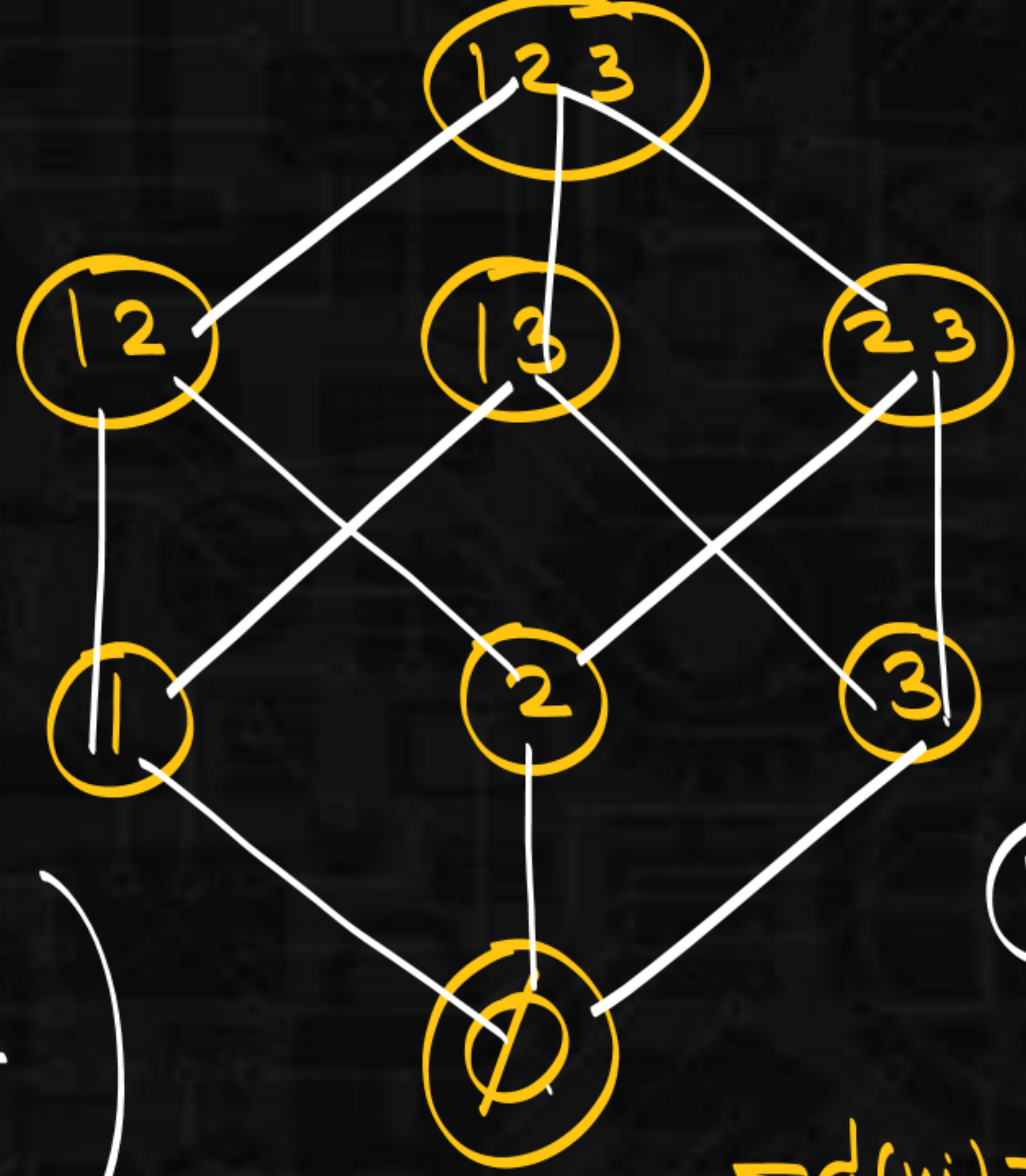
$A = \{1, 2, 3\}$

$(P(A), \subseteq)$

$\downarrow$

(Set, Relation)

$(\{\emptyset, \{1\}, \{2\}, \ldots\}, \subseteq)$





Hypercube:



$\Sigma d(v_i) = 2e.$

$2^n \cdot n = 2e$

$e = n \cdot 2^{n-1}.$

$(Q_3)$

$A = \{1, 2, 3, 4, 5\}$.

$$\left(P(A), \subseteq\right)$$

no. of edges in this.


THANK YOU GW
SOLDIERS !