Branch : CSE/IT Batch : Hinglish

Discrete Mathematics II

Set Theory

DPP-04

[NAT]

1. Consider a set $x = \{1, 2, 3, 4, 5\}$. The number of symmetric relations for the given set are?

[MCQ]

2. Consider the cross product of a set A= {1,2,3}, set B = $\{x, y, z\}$ and set C. The resultant cross product is ϕ. Then the elements of set C is?

(a) {a, b} (b) {0}

(c) ϕ (d) None of these

[MCQ]

3. If the number of reflexive relations for a set is 64 then what is the cardinality of the set?

(a) 3 (b) 4

(c) 6 (d) 16

[MCQ]

4. Consider the following relations R_1 and R_2 on set A = {a, b, c, d}

R_1: {(a, a), (a, b), (a, c), (b, c), (c, a)}

R2: {(a, a), (a, b), (a, c), (b, c), (b, a), (c, b), (c, a)}

Choose the correct statement from the following:

(a) R_1 is a symmetric relation but not R_2.

(b) R_2 is a symmetric relation but not R_1.

(c) Both R_1 and R_2 are symmetric.

(d) Neither R_1 nor R_2 is symmetric.

[MCQ]

5. Consider the following relation:

I. Relation R_1 = "Has the same birthday" defined on the set of people.

II. Relation R_2 = "Has the same absolute value" defined on the set of real number.

III. Relation R_3 = "Congruence module n(≡)" defined on the set of integers.

Choose the correct statement regarding the given relations.

(a) Only R_1 and R_2 are equivalence relations.

(b) Only R_2 and R_3 are equivalence relations.

(c) Only R_1 and R_3 are equivalence relations.

(d) All R_1, R_2 and R_3 are equivalence relations.

[NAT]

6. Consider the given statements:

I: Every reflexive relation is always symmetric.

II: "Is a subset of" is a transitive relation defined on a power set of sets.

III. The inverse of a transitive relation is a transitive relation.

The number of incorrect statements are?

[NAT]

7. For a set a with cardinality 7, what is the total number of reflexive and symmetric relations?

Answer Key

1. **(32768)**
2. **(c)**
3. **(a)**
4. **(b)**
5. **(d)**
6. **(0)**
7. **(2097152)**

Hints and Solutions

1. (32768)

The number of symmetric relations $\Rightarrow 2^n \cdot 2^{\frac{n^2-n}{2}}$ or $2^{n(n+1)/2}$.

Therefore, $2^{5(5+1)/2} \Rightarrow 2^{\frac{5*6}{2}}$

$\Rightarrow 2^{3\times5} = 2^{15} = 32768$

2. (c)

If the cross product of given sets is empty or ϕ then atleast one of the set involved in the cross product is empty.

3. (a)

The total number of reflexive relations = 2^{n^2-2}

Given, the total number of reflexive relations = 64

Therefore, $2^{n^2-n} = 64$

$2^{n^2-n} = 2^6$

Substituting n = 3 we get

$2^{(3)^2-3} \Rightarrow 2^{9-3} = 2^6 = 64.$

The cardinality of set = 3.

4. (b)

R_1: {(a, a), (a, b), (a, c), (b, c), (c, a)} is not symmetric because (b, a) and (c, b) should be a part of R to make it symmetric.

R_2: {(a, a), (a, b), (a, c) (b, c), (c, a) (b, a), (c, b)}is a symmetric relation. It is R_1 relation including (b, a) and (c, b).

5. (d)

All the given relations are equivalence relations.

R_1 = 'Has the same birthday'

R_2 = 'Has the same absolute value'

R_3 = "Congruence module n(≡)" defined on the set of integers.

6. (0)

All the given statement are correct.

I. Every reflexive relations is also a symmetric relation.

II. If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$, therefore "Is a subset is transitive relation".

III. The inverse of a transitive relations is also transitive for example "is less than" is a transitive relation, then the inverse "Is greater than" is also a transitive relation.

7. (2097152)

Total number of of reflexive and symmetric relations for a set of n elements $\Rightarrow 2^{\frac{n^2-n}{2}}$

Here, cardinality = 7 = n

$\therefore 2^{\frac{7^2-7}{2}} \Rightarrow 2^{\frac{49-7}{2}}$

$2^{\frac{42}{2}} = 2^{21}$

$= 2097152.$