CS & IT
ENGINEERING

Theory of Computation

Push Down Automata

Lecture No. 7

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TOPICS TO BE COVERED

01 Closure properties

02 for Regulars ✓

03 for DCFLs

04 for CFLs

05

Closure properties for DCFLs [Remember closed operations]

closed → CPIFs

~~①~~ $L_1 \cup L_2$

~~②~~ $L_1 \cap L_2$

c ✓③ $\overline{L}$

~~④~~ $L_1 - L_2$

~~⑤~~ $L_1 \cdot L_2$

~~⑥~~ L^{Rev}

~~⑦~~ L^*

~~⑧~~ L^+

~~⑨~~ $L_1 \Delta L_2$

~~⑩~~ Subset(L)

P ✓⑪ prefix(L)

~~⑫~~ suffix(L)

~~⑬~~ Substring(L)

~~⑭~~ f(L)

~~⑮~~ h(L)

~~⑯~~ ε-free h(L)

I ✓⑰ $h^{-1}(L)$

~~⑱~~ L_1 / L_2

~~⑲~~ $L_1 \cup L_2 \cup L_3 \cup \cdots \cup L_k$

~~⑳~~ $L_1 \cap L_2 \cap L_3 \cap \cdots \cap L_k$

~~㉑~~ $L_1 - L_2 - L_3 - \cdots - L_k$

~~㉒~~ $L_1 \cdot L_2 \cdot L_3 \cdot \cdots L_k$

Fs ✓㉓ Finite Subset(L)

~~㉔~~ Finite Substitution(L)

~~㉕~~
~~㉖~~
~~㉗~~
~~㉘~~
~~㉙~~
~~㉚~~

Infinite$(\cup, \cap, -, \cdot, \subseteq, f)$

Closure properties for CFLs [Remember Not closed operations]

$I\,C\,D\,S^2Q\,F_{ID}\,I_{All}$

- ✓ ① $L_1 \cup L_2$
- I ✗ ② $L_1 \cap L_2$
- C ✗ ③ $\overline{L}$
- D ✗ ④ $L_1 - L_2$
- ✓ ⑤ $L_1 \cdot L_2$
- ✓ ⑥ L^{Rev}
- ✓ ⑦ L^*
- ✓ ⑧ L^+

- S ✗ ⑨ $L_1 \Delta L_2$
- S ✗ ⑩ Subset(L)
- ✓ ⑪ prefix(L)
- ✓ ⑫ suffix(L)
- ✓ ⑬ Substring(L)
- ✓ ⑭ f(L)
- ✓ ⑮ h(L)
- ✓ ⑯ ε-free h(L)
- ✓ ⑰ $h^{-1}(L)$

- Q ✗ ⑱ L_1 / L_2
- ✓ ⑲ $L_1 \cup L_2 \cup L_3 \cup \cdots \cup L_k$
- F_{ID} {
  - ✗ ⑳ $L_1 \cap L_2 \cap L_3 \cap \cdots \cap L_k$
  - ✗ ㉑ $L_1 - L_2 - L_3 - \cdots - L_k$
- ✓ ㉒ $L_1 \cdot L_2 \cdot L_3 \cdot \cdots L_k$
- ✓ ㉓ Finite Subset(L)
- ✓ ㉔ Finite Substitution(L)
- I_{All} { ✗ ㉕ ㉖ ㉗ ㉘ ㉙ ㉚ } Infinite ($\cup, \cap, -, \cdot, \subseteq, f$)

① Union for DCFLs

↳ Not closed

$DCFL_1 \cup DCFL_2 \Rightarrow$ Need not be DCFL (Always CFL)

Example 1 :

$DCFL_1 \cup DCFL_2 \Rightarrow$ Not DCFL possible

$$\underset{DCFL}{\{a^n b^n c^*\}} \cup \underset{DCFL}{\{a^* b^n c^n\}} = \underset{\substack{\text{Not DCFL} \\ \text{It is CFL}}}{\{a^m b^n c^k \mid m=n \text{ OR } n=k\}}$$

Example 2 :

$DCFL_1 \cup DCFL_2 \Rightarrow$ DCFL possible

$$\underset{DCFL}{a^*} \cup \underset{DCFL}{a^*} \Rightarrow \underset{DCFL}{a^*}$$

① Union for CFLs

↳ closed

$CFL_1 \cup CFL_2 \Rightarrow$ Always CFL

Algo :

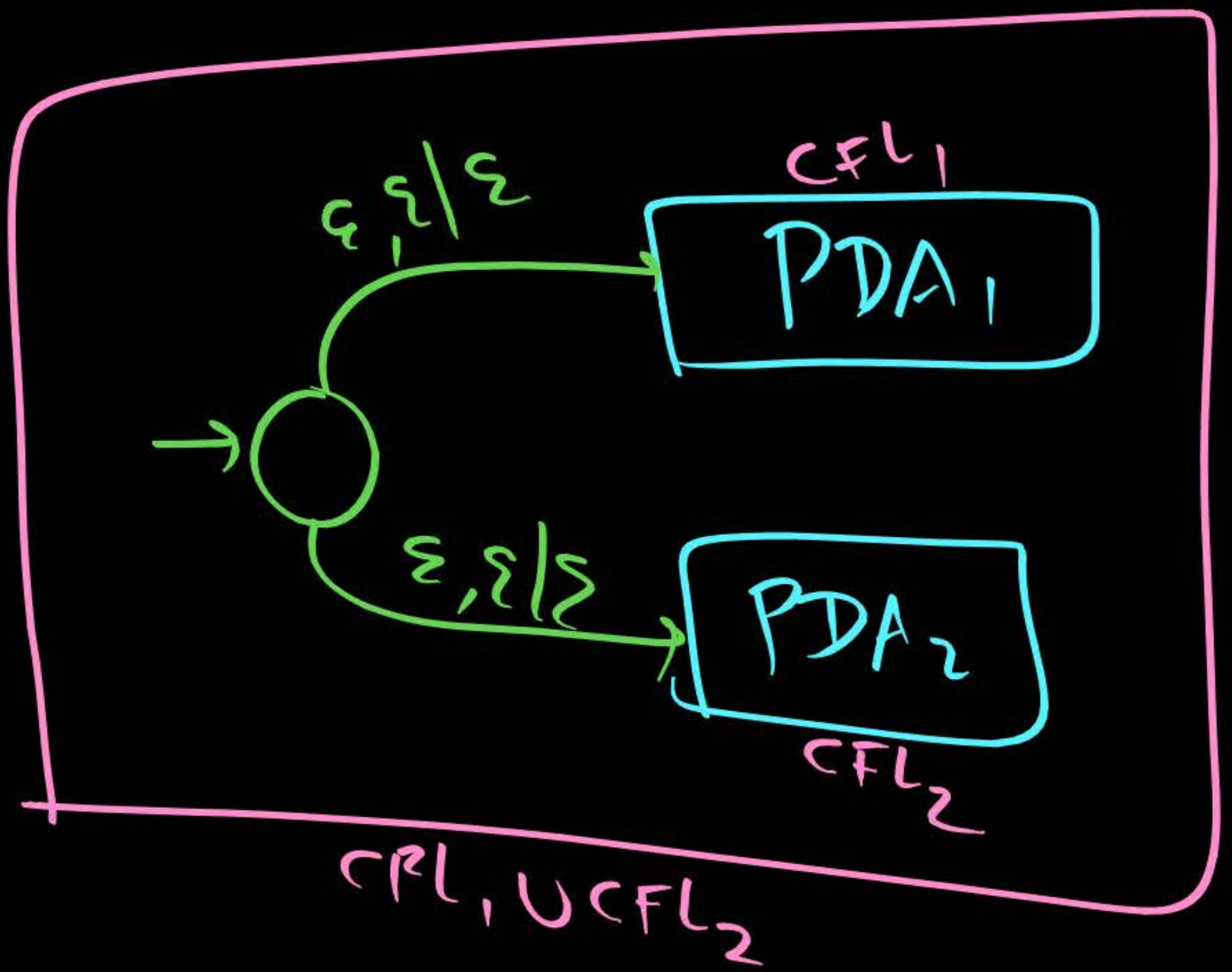
$CFL_1 \Rightarrow$ write CFG_1 with S_1 as start

$CFL_2 \Rightarrow$ " CFG_2 with S_2 as start

⇓

Add new start S and append $S \to S_1 | S_2$

$S \to S_1 | S_2$


CFL_1
$\varepsilon, \varepsilon | \varepsilon$
PDA_1
$\varepsilon, \varepsilon | \varepsilon$
PDA_2
CFL_2
$CFL_1 \cup CFL_2$

① $L_1 = \{a^n b^n\}$ DCFL

$L_2 = \{b^n a^n\}$ DCFL

$$\Rightarrow L_1 \cup L_2 = \{a^n b^n\} \cup \{b^n a^n\}$$

$\boxed{\text{DCFL}}$

② $L_1 = \{a^n b^n\}$

$L_2 = (a+b)^*$

$$\Rightarrow L_1 \cup L_2 = (a+b)^*$$

$\boxed{\text{Regular}}$

③ $L_1 = a^* b^* c^*$

$L_2 = a^n b^n c^+$

$$\Rightarrow L_1 \cup L_2 = L_1 = a^* b^* c^*$$

$\boxed{\text{Regular}}$

② Intersection for DCFLs

↳ Not closed

② Intersection for CFLs

↳ Not closed

$DCFL_1 \cap DCFL_2 \Rightarrow$ May or may not DCFL

(may or may not CFL)

$CFL_1 \cap CFL_2 \Rightarrow$ may or may not CFL.

$$\left.\begin{aligned} L_1 &= a^n b^n c^* \\ L_2 &= a^* b^n c^n \end{aligned}\right\} \Rightarrow L_1 \cap L_2 = \{a^n b^n c^n\}$$

not DCFL

not CFL

(It is CSL)

$$L_1 = a^n b^n c^* \Rightarrow \{\boxed{\epsilon}, ab, c, aabb, cc, \boxed{abc}, aabbc, \boxed{aabbcc}, \ldots\}$$

$$L_2 = a^* b^n c^n \Rightarrow \{\boxed{\epsilon}, a, aa, \boxed{abc}, bbcc, abbcc, \boxed{aabbcc}, \ldots\}$$

$$L_1 \cap L_2 = \{a^n b^n c^n\} = \{\epsilon, abc, aabbcc, \ldots\}$$

①
$$\left.\begin{aligned} L_1 &= \phi \\ L_2 &= \{a^n b^n\} \end{aligned}\right\} \Rightarrow L_1 \cap L_2 = \phi$$

②
$$\left.\begin{aligned} L_1 &= \Sigma^* \\ L_2 &= \{a^n b^n\} \end{aligned}\right\} \Rightarrow L_1 \cap L_2 = L_2 = \{a^n b^n\}$$

③
$$\left.\begin{aligned} L_1 &= \{\boxed{a} b^n c^n\} \\ L_2 &= \{a^n b^n \boxed{c}\} \end{aligned}\right\} \Rightarrow L_1 \cap L_2 = \{abc\}$$

$$L_1 = a\,b^n c^n = \{a, \boxed{abc}, abbcc, \ldots\}$$

$$L_2 = a^n b^n c = \{c, \boxed{abc}, aabbc, \ldots\}$$

$$\boxed{L_1 \cap L_2 = \{abc\}}$$

③ Complement for DCFLs

→ closed

$\overline{\text{DCFL}} \Rightarrow$ Always DCFL

L is DCFL

$\Downarrow$

Construct DPDA

$\Downarrow$ f ↔ nf

modified DPDA

$\Downarrow$

$\overline{L}$ is DCFL

③ Complement for CFLs

→ Not closed

$\overline{\text{CFL}} \Rightarrow$ either CFL or not CFL

$L = \overline{\{a^n b^n c^n\}}$ is CFL

$\Downarrow$

$\overline{L} = \{a^n b^n c^n\}$ is not CFL (It is CSL)

$$\overline{\{a^n b^n\}} = (a+b)^* - \{a^n b^n\}$$

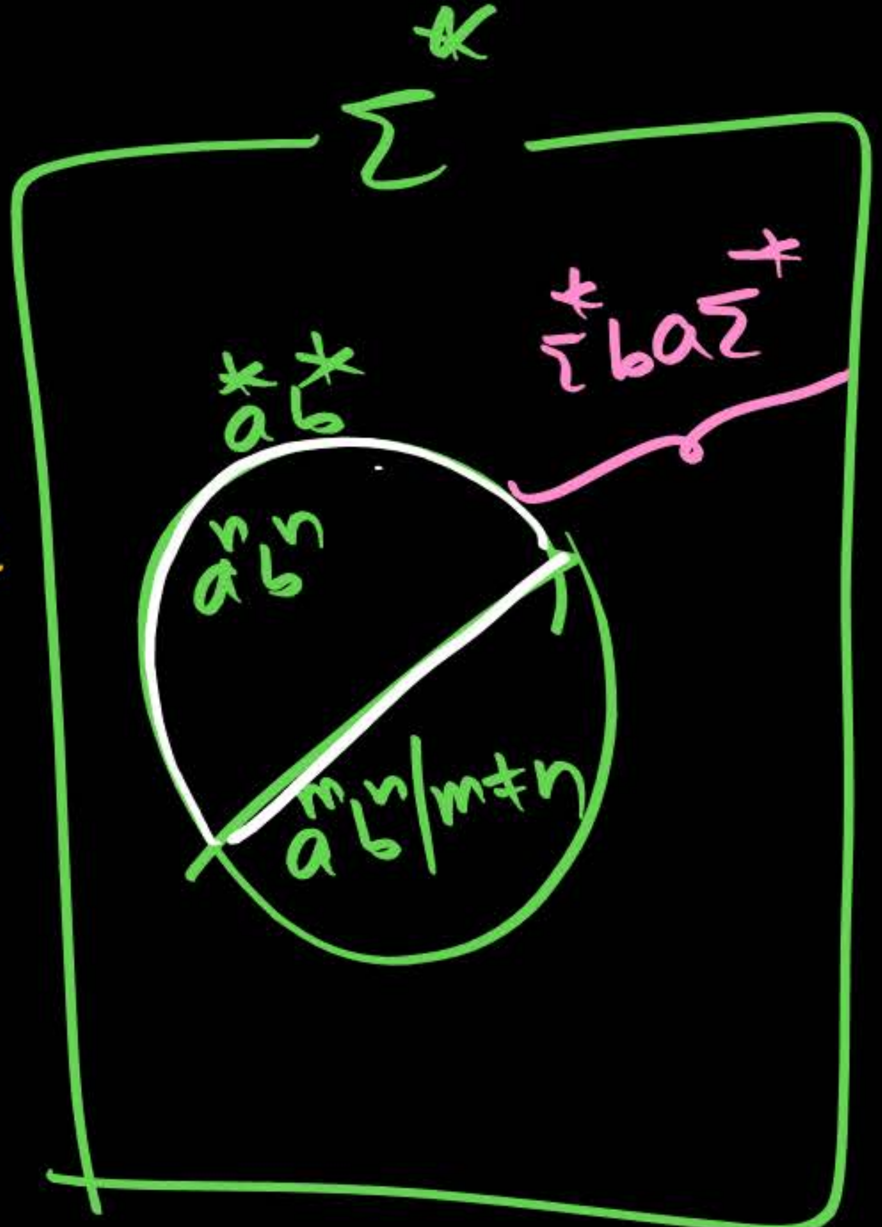
$$= \boxed{\Sigma^* ba \Sigma^* \cup \{a^m b^n \mid m \neq n\}}$$

if the string is not in a^*b^*

If string is in a^*b^*

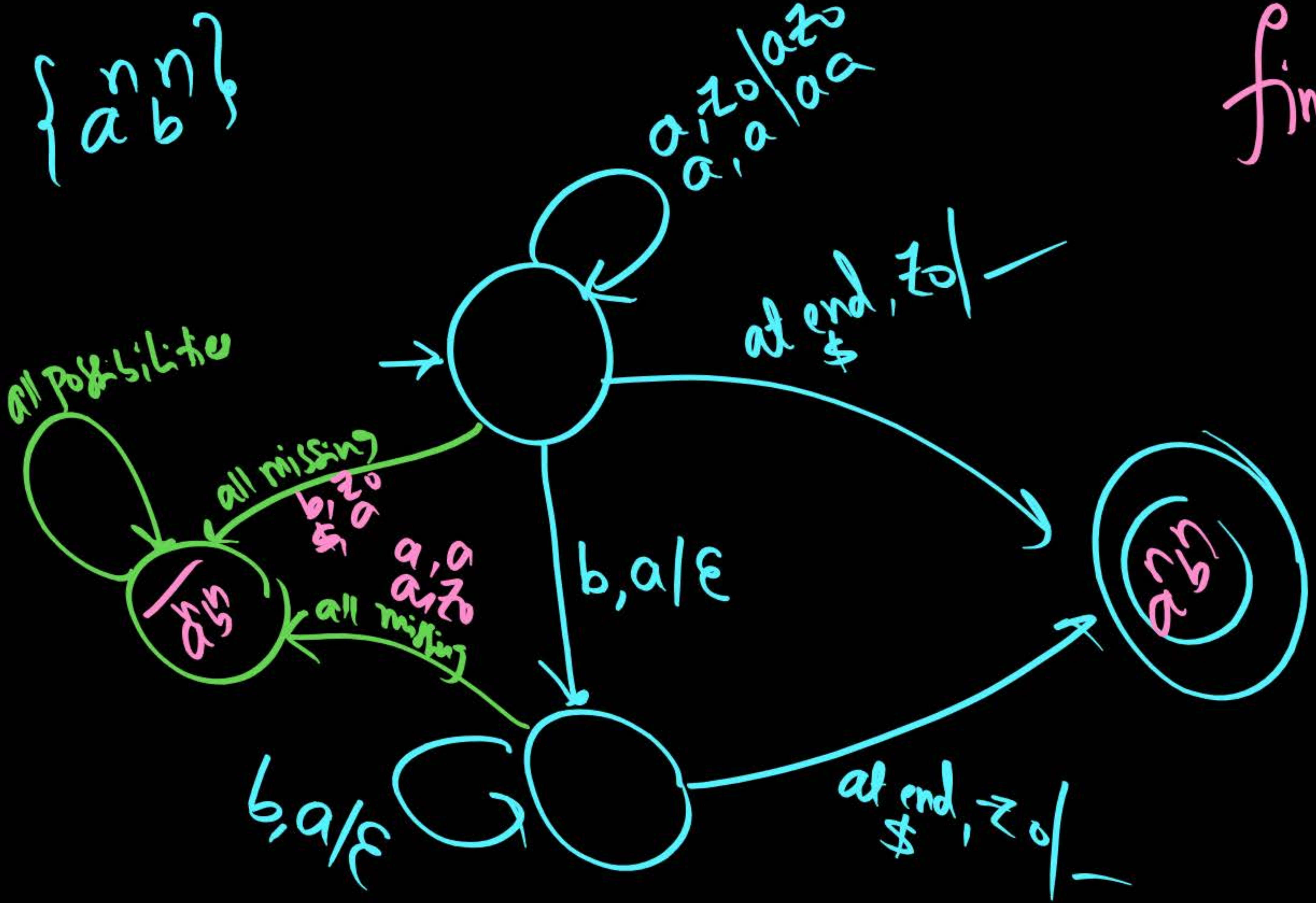
$$L = a^* b^* = \{a^m b^n \mid m = n \text{ OR } m \neq n\}$$

$$\bar{L} = \Sigma^* ba \Sigma^*$$



$\{a^n b^n\}$

final ⟷ nonfinal



$$L = \overline{\{a^n b^n c^n\}} = (a+b+c)^* - \{a^n b^n c^n\}$$

$$= (a+b+c)^* - \{a^m b^n c^k \mid m = n = k\}$$

= CFL but not DCFL

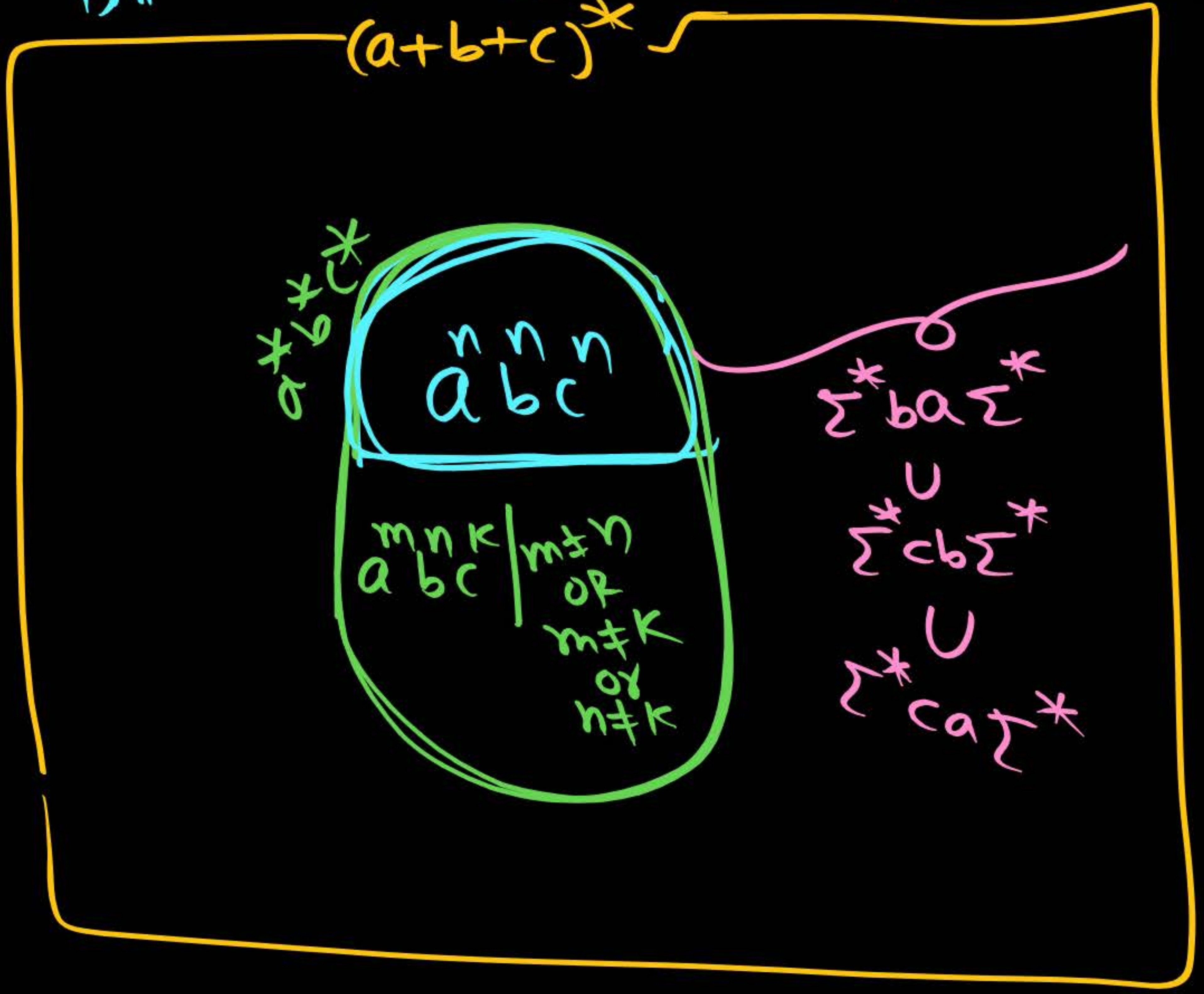
$$\overline{\{a^n b^n c^n\}} = \underset{PDA}{\{a^m b^n c^k \mid m \neq n\}} \cup \underset{PDA}{\{a^m b^n c^k \mid m \neq k\}} \cup \underset{PDA}{\{a^m b^n c^k \mid n \neq k\}} \cup \underset{PDA}{\Sigma^*(ba+cb+ca)\Sigma^*}$$

$\xrightarrow{}$ PDA exist

$$= \{a^m b^n c^k \mid (m \neq n) \vee (m \neq k) \vee (n \neq k)\}$$

$$\cup$$

$$\Sigma^*(ba+ca+cb)\Sigma^*$$



	DCFLs	CFLs
$\cup$	✗	✓
$\cap$	✗	✗
$\overline{L}$	✓	✗

① $\overline{\{a^n b^n\}} \longrightarrow \boxed{\{a^m b^n \mid m \neq n\} \cup \Sigma^* ba \Sigma^*}$ is DCFL

*** ② $\overline{\{a^n b^n c^n\}} \longrightarrow$ CFL but not DCFL

③ $\overline{\{a^* b^*\}} \longrightarrow \Sigma^* ba \Sigma^* \longrightarrow$ Reg

④ $\overline{\{a^m b^n c^k \mid \underbrace{m<n<k}\}} \longrightarrow$ CFL but not DCF

$\longrightarrow \left\{a^m b^n c^k \;\middle|\; \begin{array}{c} m \geq n \\ \text{OR} \\ m \geq k \\ \text{OR} \\ n \geq k \end{array}\right\} \cup \Sigma^* (ba + ca + cb) \Sigma^*$

Note:

I) Complement of a CFL is either CFL or not CFL

(need not be CFL)

(may or may not be CFL)

II) Complement of not CFL is may or may not be CFL

III) complement ~~for~~ CFLs is not closed

Domain

④ Difference

- → Not closed for DCFLs
- → Not closed for CFls

$$L_1 - L_2 = L_1 \cap \overline{L_2}$$

I) $DCFl_1 - DCFl_2 \Rightarrow DCFl_1 \cap \overline{DCFl_2} \Rightarrow DCFl_1 \cap DCFl_3 \Rightarrow$ Need not be DCFL

II) $CFl_1 - CFl_2 \Rightarrow CFl_1 \cap \boxed{\overline{CFl_2}} \Rightarrow$ Need not be CFl

L_1 is DCFL

L_2 is CFL

$$\boxed{L \cup \overline{L} = \Sigma^*}$$

① $L_1 \cup L_2$ is Always CFL

② $L_1 \cap L_2$ is need not be CFL (need not be DCFL)

③ $\overline{L_1} \cup \boxed{\overline{L_2}}$ is need not be CFL

④ $\overline{L_1} \cup L_1$ is Σ^* is Reg

⑤ $\overline{L_2} \cup L_2$ is Σ^* is Reg

$$L_1 \cup L_2 = \overline{\overline{L_1} \cap \overline{L_2}}$$

$L_1 \rightarrow$ DCFL

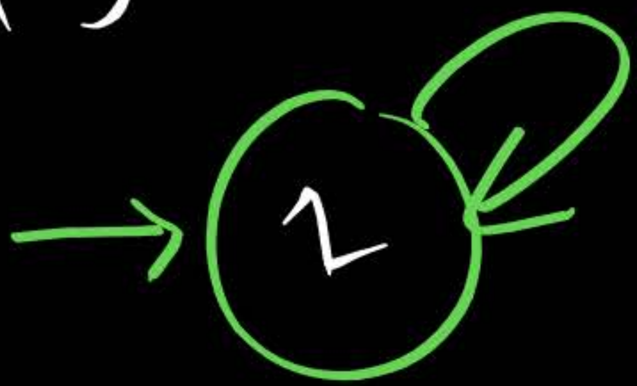
$L_2 \rightarrow$ CFL

$\overline{\overline{L_1} \cap \overline{L_2}}$ is $\underline{\quad \text{CFL} \quad}$

$\Downarrow$

$L_1 \cup L_2$ is $\underline{\underline{\text{CFL}}}$

⑦ $L=(a+b)^*$



$a, \varepsilon | b$ [push b]

$b, \varepsilon | b$ [push b]

$\varepsilon, b | \varepsilon$ [pop b]

ε ✓ (bydef)

a ✓

b ✓

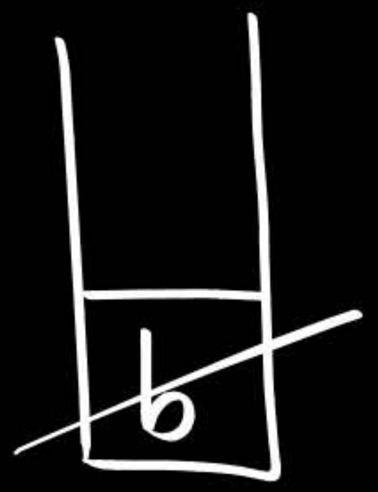
aa ✓

ab ✓

ba ✓

bb ✓

⋮

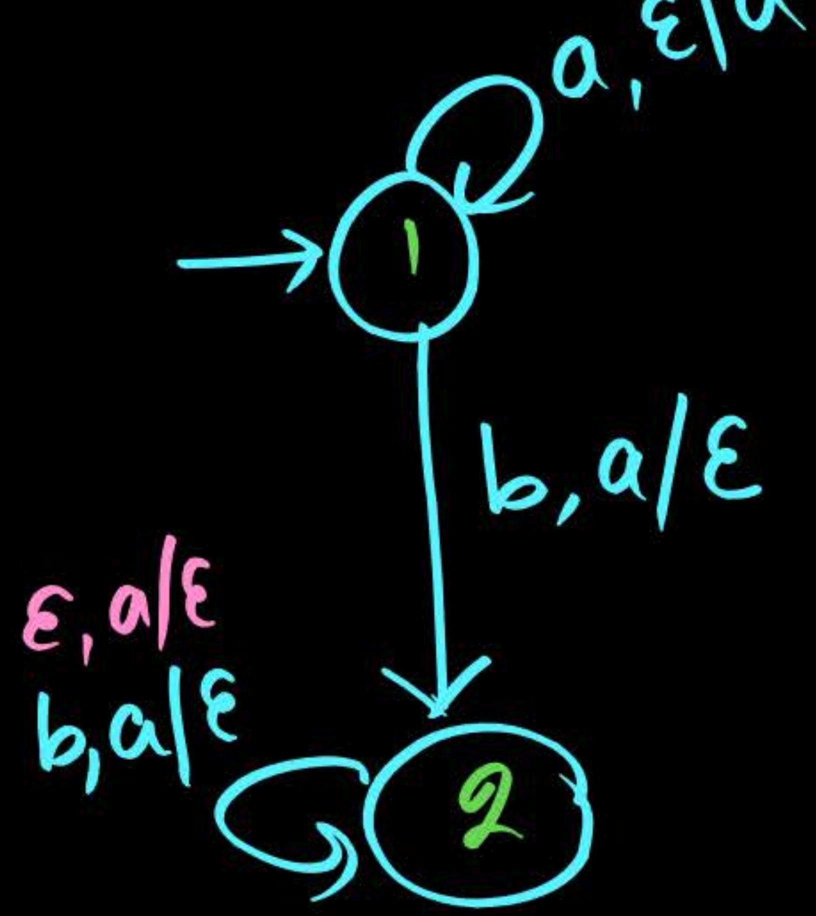


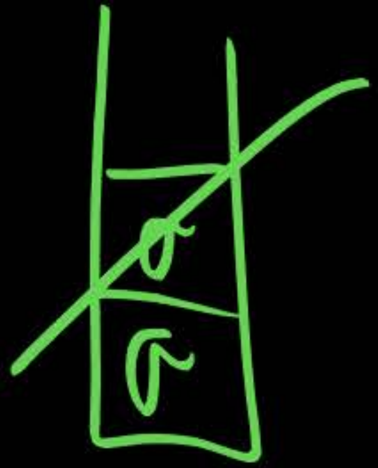
$$1 \xrightarrow[\text{push } b]{a} 1 \xrightarrow[\text{pop } b]{\varepsilon} 1$$

aaεε

⑤

Initially
Stack is empty





ε ✓

ab ✓

a ✗

aab ✓

abb ✗

⋮

$\{a^m b^n \mid m \geq n \geq 1\} \cup \{\epsilon\}$

Summary

↳ closure properties

↳ $\cup, \cap, \bar{L}, -$ ✓

THANK YOU GW SOLDIERS !