CS & IT ENGINEERING

Theory of Computation

Finite Automata

Lecture No. 11

By- DEVA Sir

TOPICS TO BE COVERED

01 NFA without ε moves

02 NFA with ε moves

03

04

05

NFA

(w) Input

$\delta: Q \times \Sigma \to 2^Q$

NFA

without ϵ moves

From every state, For every state, Zero or more transitions

If $w \in L$, atleast 1 path that halts at final

If $w \notin L$, there is no path halts at final (Either no path or every path halts at nonfinal)

w

$\delta: Q \times \Sigma \to Q$

DFA

From every state, For every i/p symbol, exactly 1 transition

If $w \in L$, Exactly 1 path halts at final

If $w \notin L$, Exactly 1 path halts at nonfinal

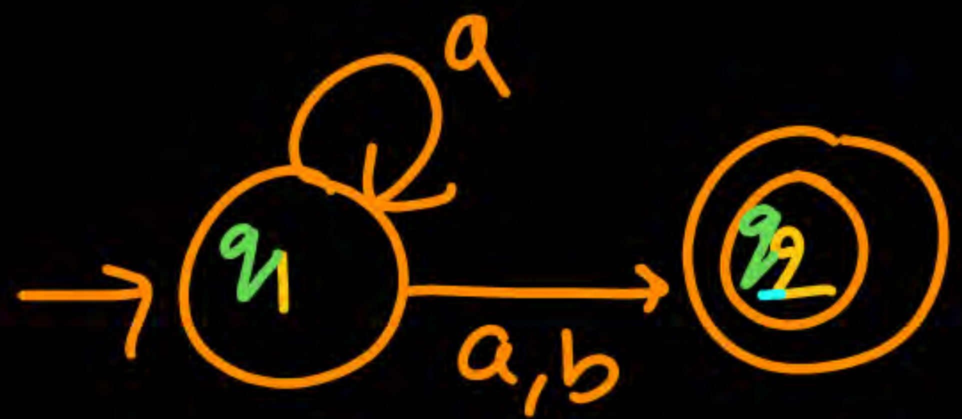
I) Every DFA is NFA.

II) NFA may or may not DFA

III) Min DFA need not be min NFA

IV) For any Regular language:

$$n(\text{Min DFA}) \geq n(\text{min NFA})$$



$\varepsilon : q_1$

$a : q_1 \xrightarrow{a} q_1$

$\boxed{q_1 \xrightarrow{a} \underset{\text{final}}{q_2}}$

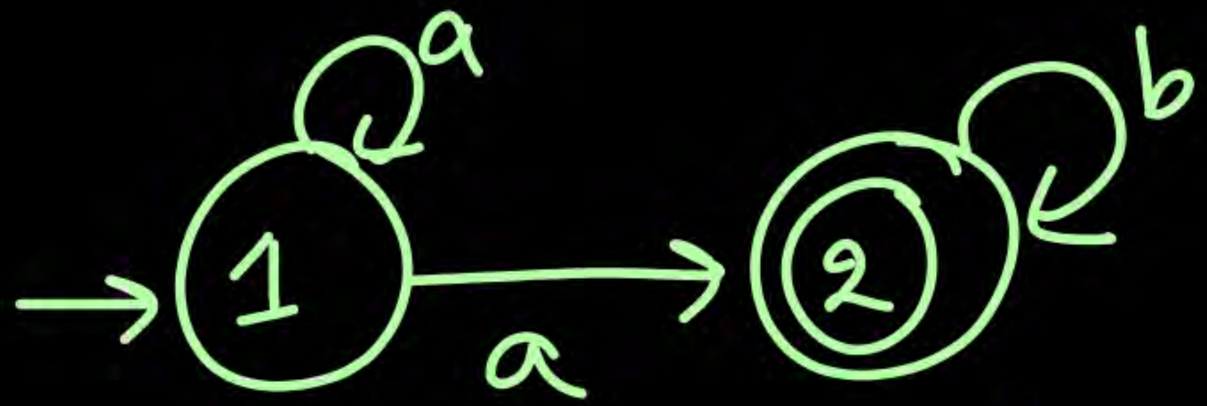
atleast 1 path halts at final

$q_1 \xrightarrow{a} q_1 \xrightarrow{b} q_2$

$q_1 \xrightarrow{a} q_2 \xrightarrow{b} \times$

	No. of paths	Valid/Invalid
ε :	1	Invalid
a	2	valid
b	1	valid
aa	2	valid
ab	1	valid
ba	0	Invalid
bb	0	Invalid
⋮		

Is there any path ?.



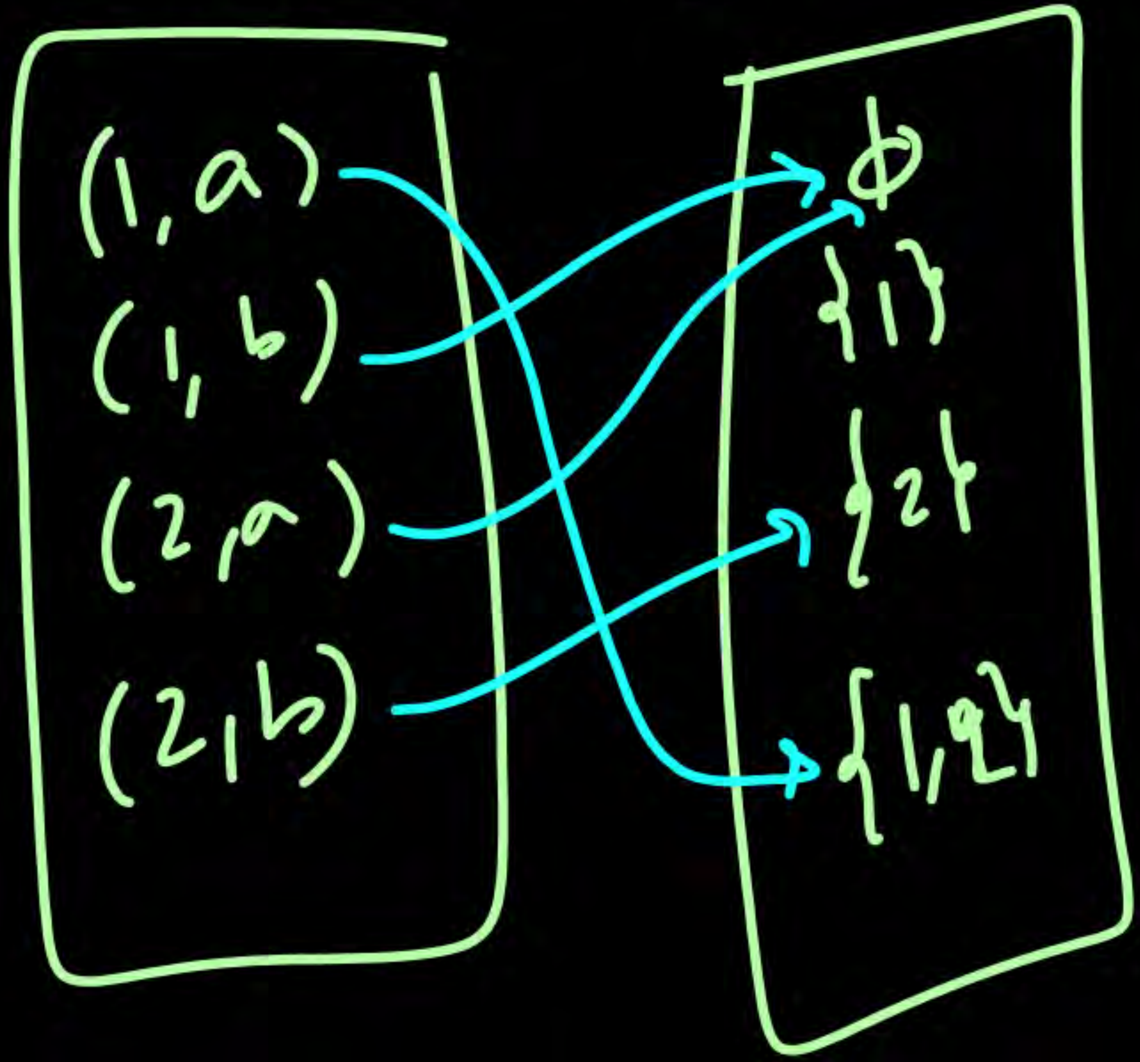
$$\delta_{NFA} : Q \times \Sigma \to 2^{Q}$$

$$\delta(1, a) = \{1, 2\}$$

$$\delta(1, b) = \{\ \}$$

$$\delta(2, a) = \{\ \}$$

$$\delta(2, b) = \{2\}$$



$Q = \{1, 2, 3\}$

$2^{Q} = \{ \phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\} \}$

2^{Q} = power set (Q)

= Set of all subset of Q

$= \{ X \mid X \subseteq Q \}$

$|Q| = n$

$\Rightarrow |P(Q)| = 2^{n}$

DFA

$$A \xrightarrow{a} ?$$

$\delta(A, a) =$ only one

NFA

$$A \xrightarrow{a} ?$$

$\delta(A, a) =$ no transition

or

one transition

or

2 transitions

.

.

.

① $L = \{\ \}$ over $\Sigma = \{a, b\}$

Regular Exp:

$$R = \phi$$

min DFA:

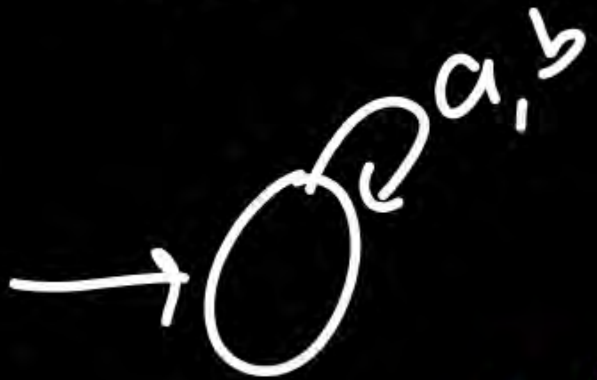
1 state

min NFA:

i) 

ii) 

iii) 

iv) 

1 state

No. of min NFAs = 4

Note:

In general

For every Regular Set,

i) No. of DFAs = Infinite

ii) No. of min DFAs = 1

iii) No. of NFAs = Infinite

iv) No. of min NFAs = 1 or more

② $L = \Sigma^*$ over $\Sigma = \{a, b\}$

Regular Exp:

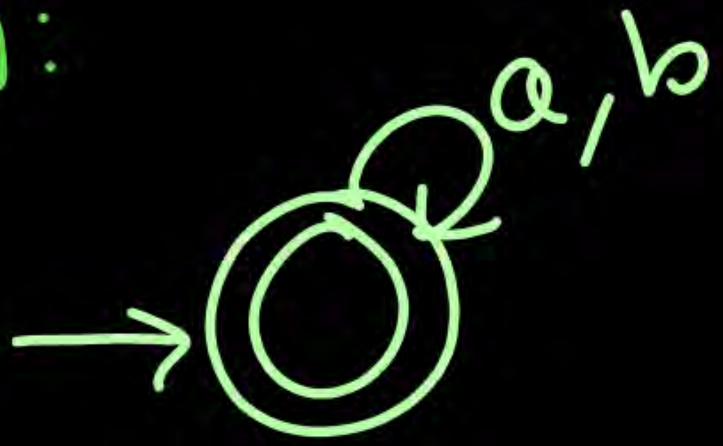
$$R = (a+b)^*$$

$$= (a^*b^*)^* = (b^*a^*)^*$$

min DFA:

1 state

min NFA:



1 state

only 1 min NFA for $L = \Sigma^*$

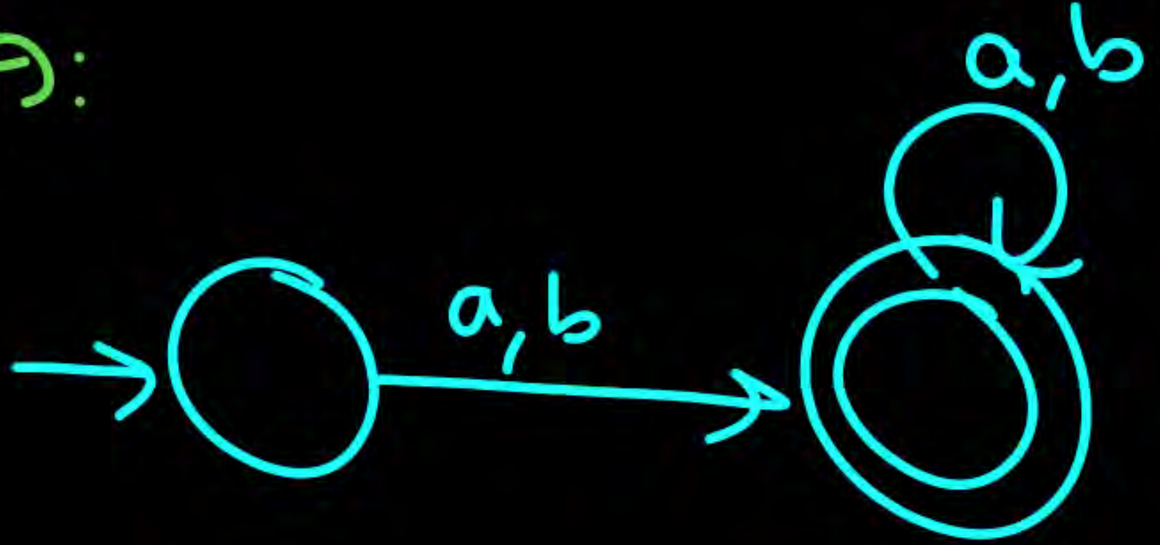
③ $L = \Sigma^{+}$ over $\Sigma = \{a, b\}$

Regular Exp:

$$R = (a+b)^{+} = (a+b).(a+b)^{*}$$

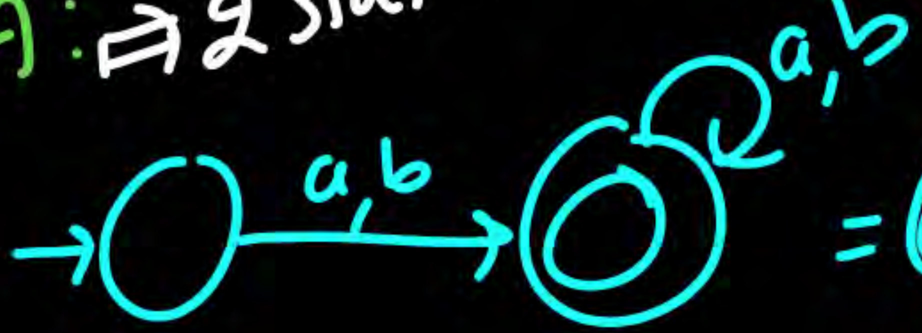
$$= (a+b)^{*}.(a+b)$$

min DFA:

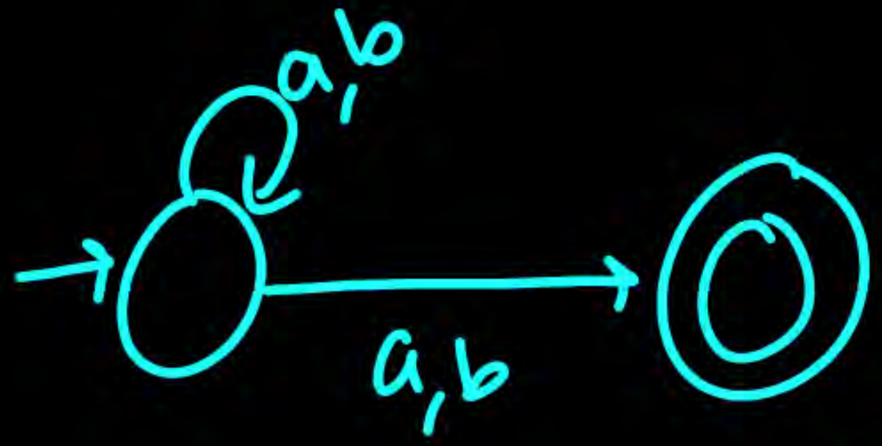


2 States

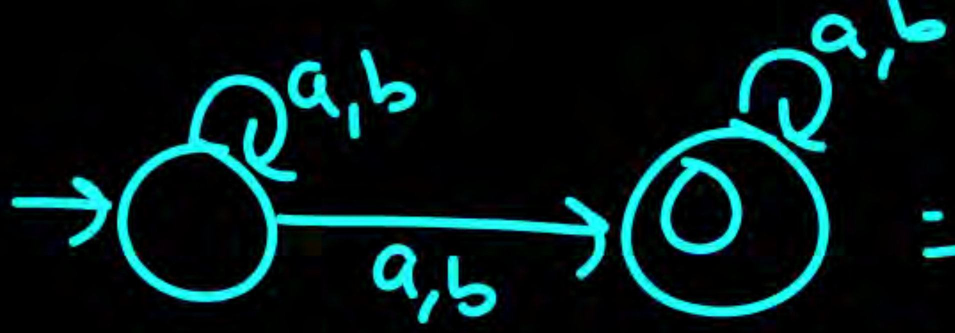
min NFA: $\Rightarrow$ 2 states



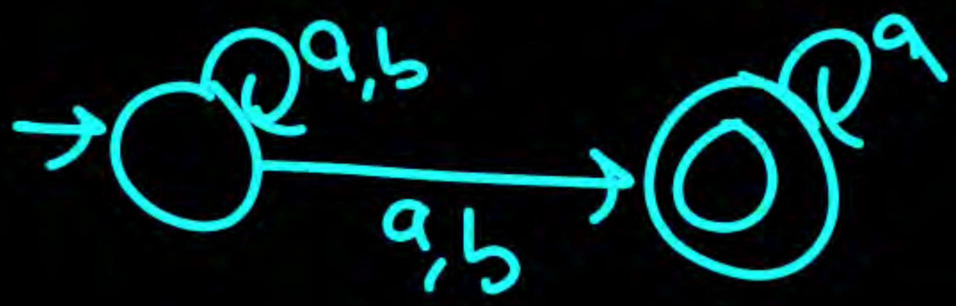
$= (a+b)(a+b)^{*}$



$= (a+b)^{*}(a+b)$



$= (a+b)^{+}$



⋮

$$(a+b)^*(a+b)a^* = (a+b)^+$$

$$(a+b)^*(a+b)b^* = (a+b)^+$$

$$a^*(a+b)(a+b)^* = (a+b)^+$$

$$b^*(a+b)(a+b)^* = (a+b)^+$$

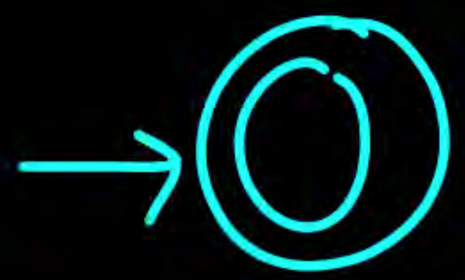
** (4) $L = \{\varepsilon\}$ over $\Sigma = \{a, b\}$

Regular Exp:

$$R = \varepsilon$$

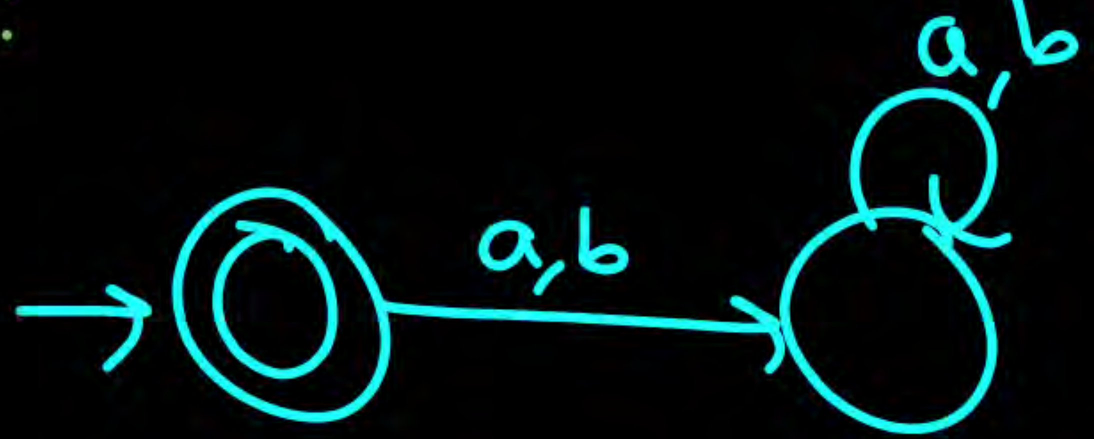
$$= \phi^* = \varepsilon^* = \varepsilon^+$$

Min NFA:



1 state

Min DFA:



2 state

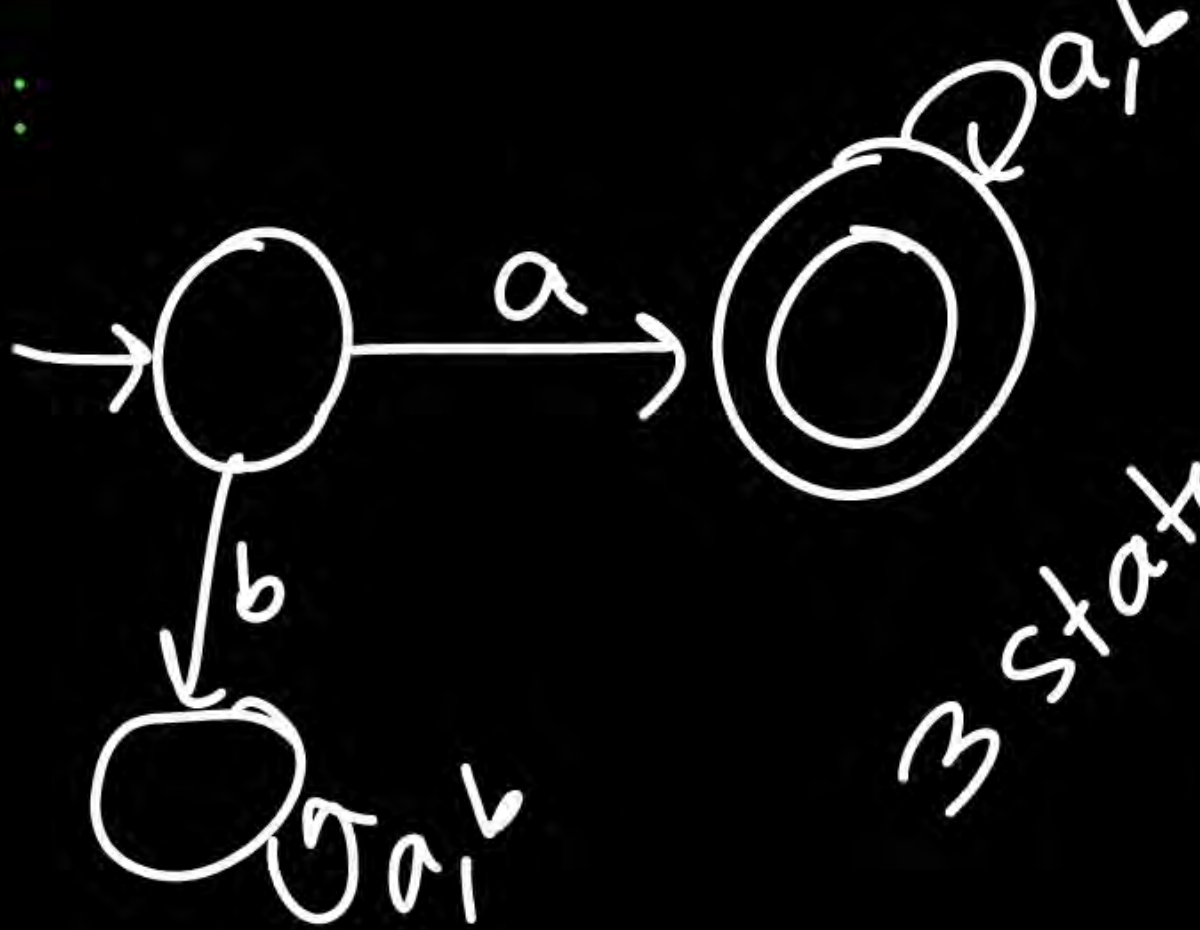
⑤ $L = \{w \mid w \in \{a,b\}^*,\ w \text{ starts with 'a'}\}$

Regular Exp:

$$R = a(a+b)^*$$

$$= (ab^*)^+$$

min DFA:

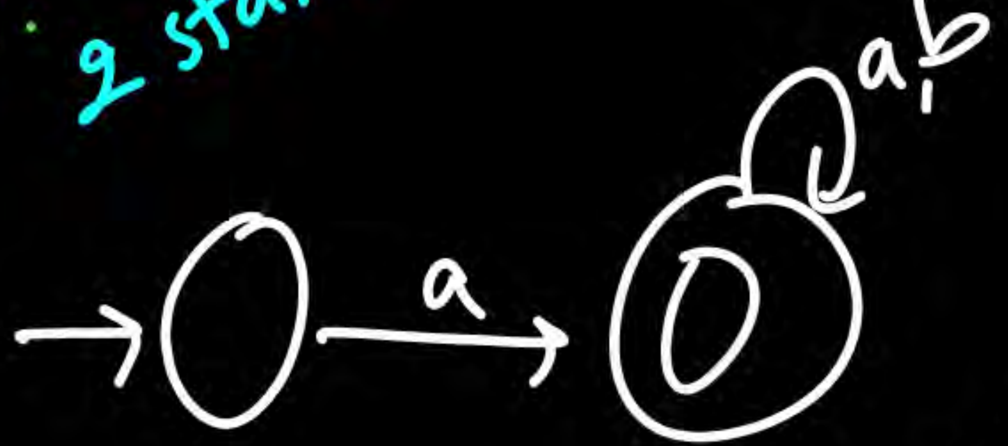


3 states

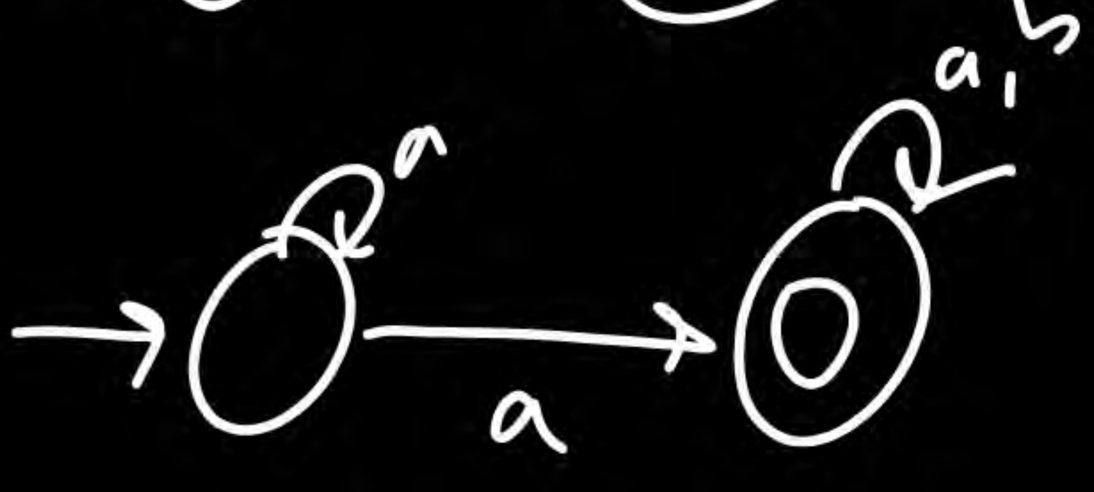
NFA: 2 states

min

i)



ii)



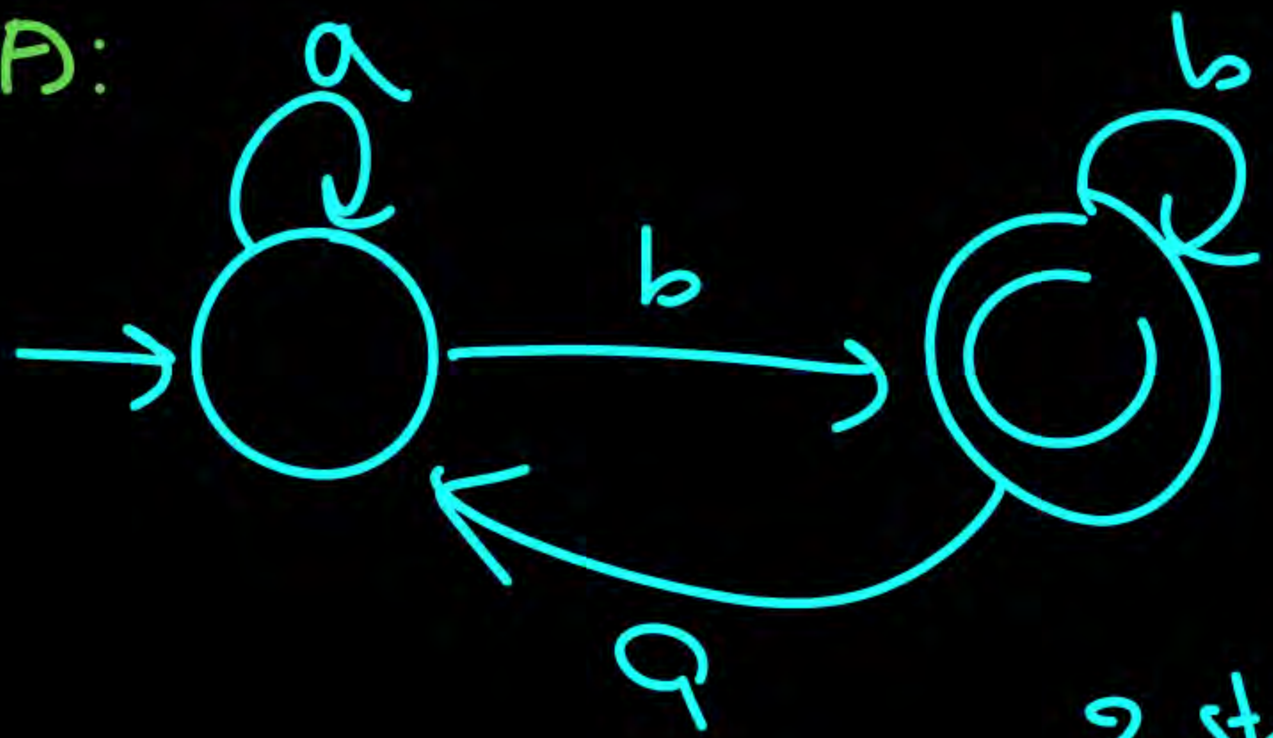
⑥ $L = \{ w \mid w \in \{a,b\}^*, w \text{ ends with } b \}$

Regular Exp:

$$R = (a+b)^* b$$

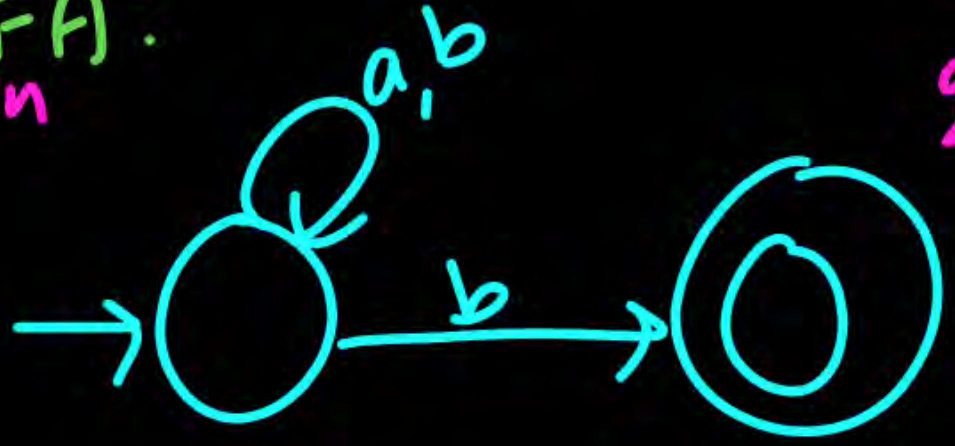
$$= (a^* b)^+$$

min DFA:



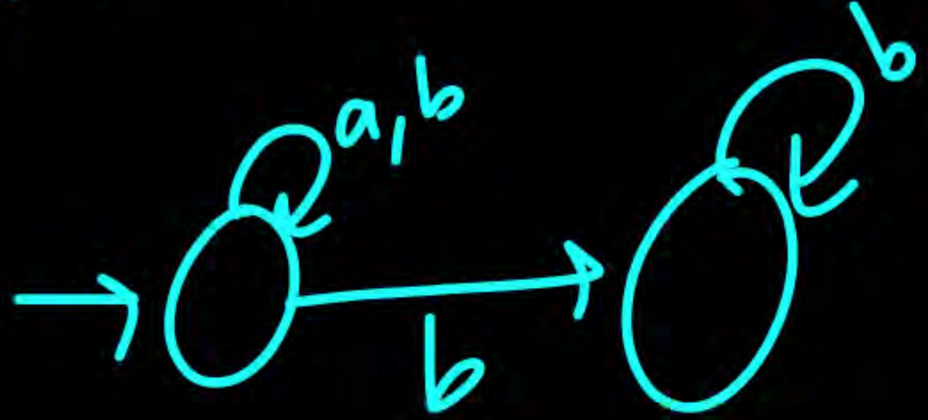
2 states

NFA: min

i) 

2 states

ii) min DFA

iii) 

⋮

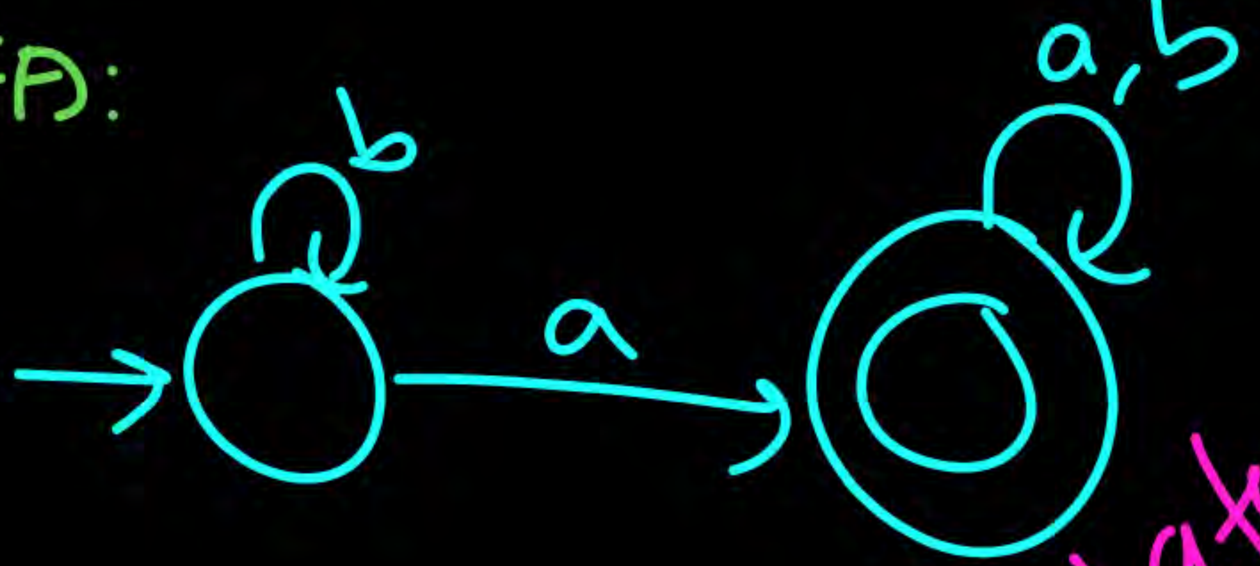
(7) $L = \{w \mid w \in \{a,b\}^*,\ w \text{ contains 'a'}\}$

Regular Exp:

$$R = \Sigma^* a \Sigma^*$$

$$= b^* a (a+b)^*$$

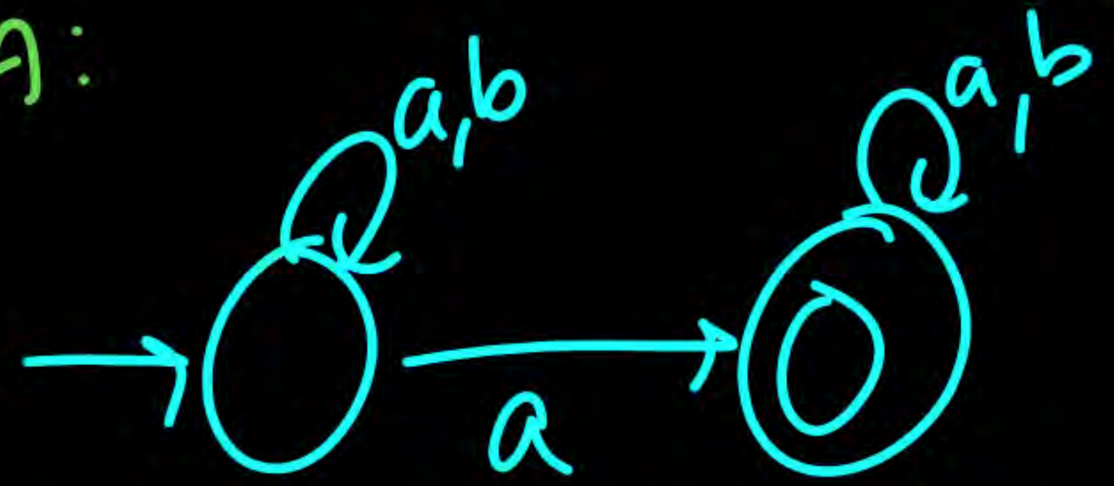
DFA:



2 states

NFA:

i)



ii)

⋮

2 states

8) $L = \{ w \mid w \in \{a,b\}^*, |w| = 2 \}$

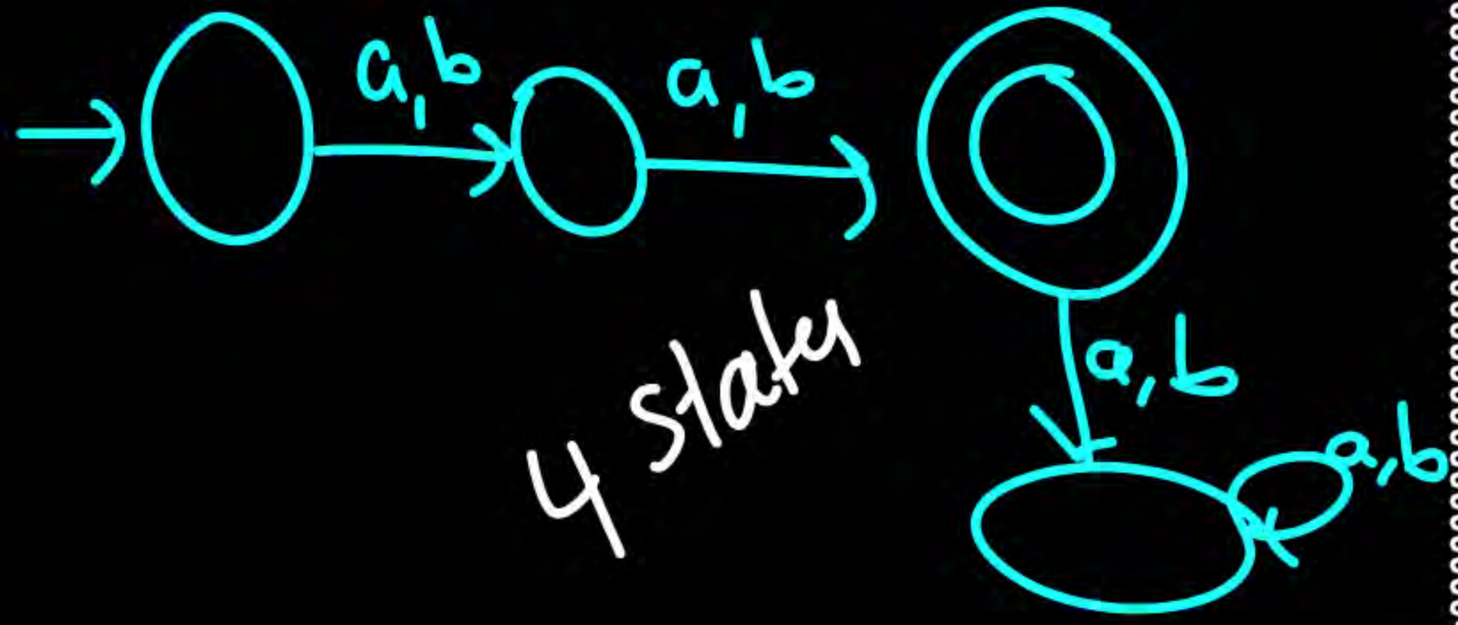
Regular Exp:

$$= (a+b)^2$$

NFA:

3 states

DFA:



4 states

If $|w| = K$

then i) $n(\text{min DFA}) = K+2$

ii) $n(\text{min NFA}) = K+1$

(9) $L = \{ w \mid w \in \{a,b\}^*, |w| \le 2 \}$

Regular Exp:

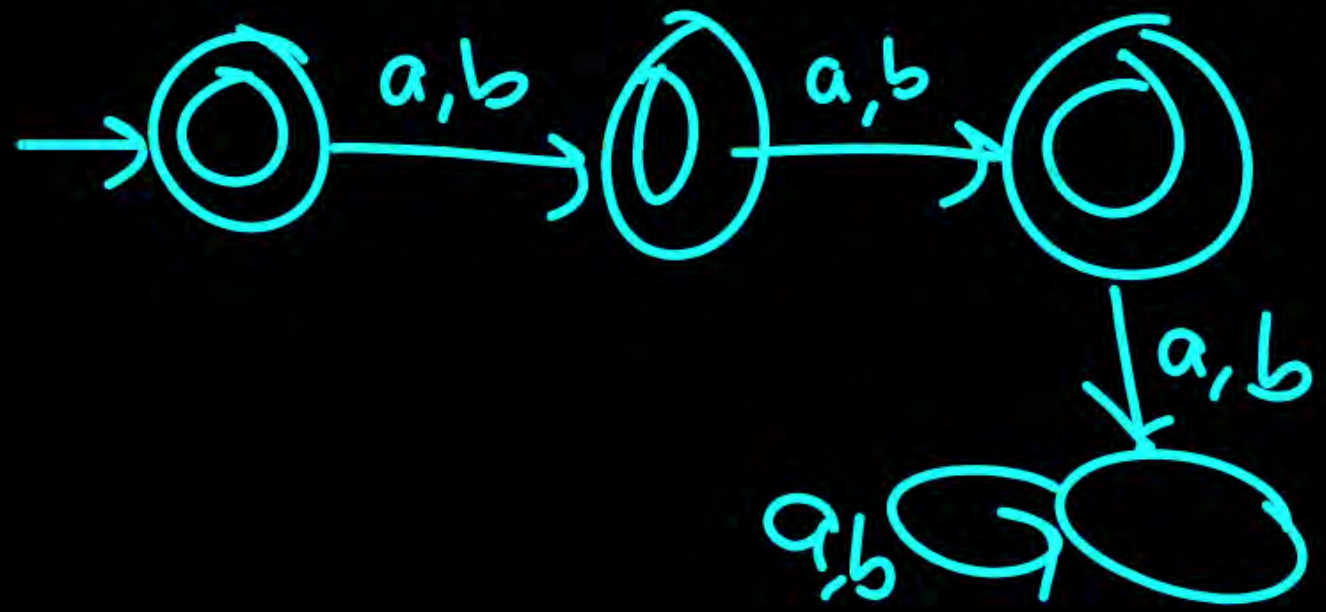
$= (\epsilon + a + b)^2$

NFA:

3 States

DFA:

4 state



If $|w| \le K$ then

i) $n(\text{Min DFA}) = K+2$

ii) $n(\text{Min NFA}) = K+1$

(10) $L = \{ w \mid w \in \{a,b\}^*, |w| \geq 2 \}$

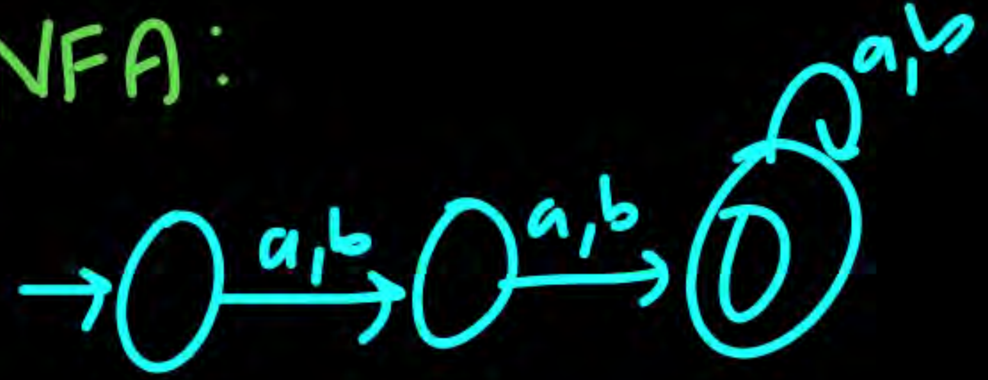
Regular Exp:

$= (a+b)^2 (a+b)^*$

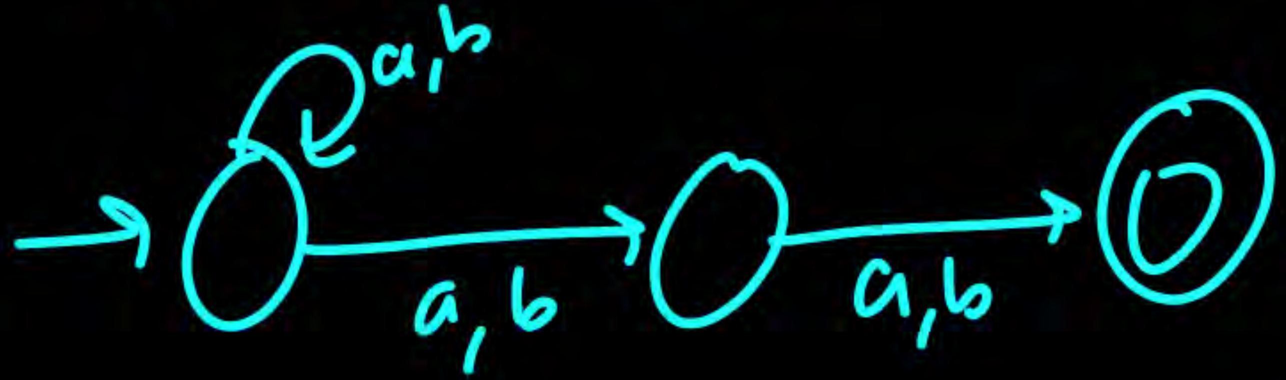
$= (a+b)^* (a+b)^2$

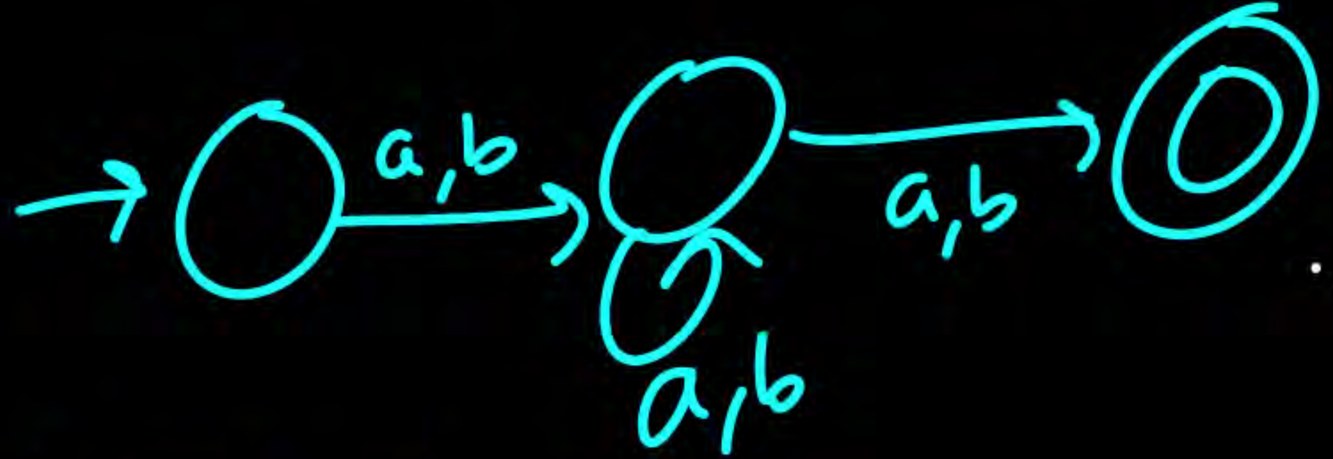
$= (a+b)(a+b)^*(a+b)$

NFA:



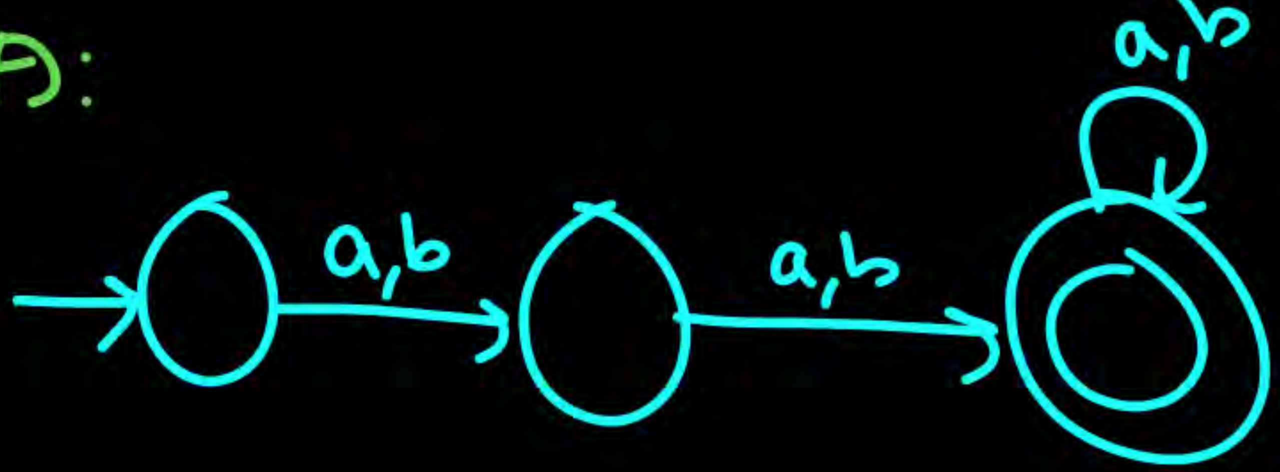
3 States





...

DFA:

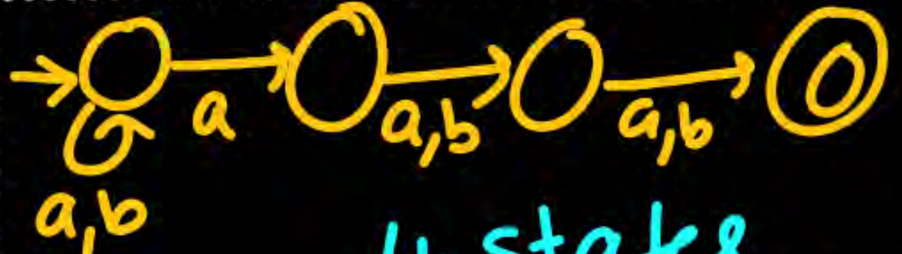
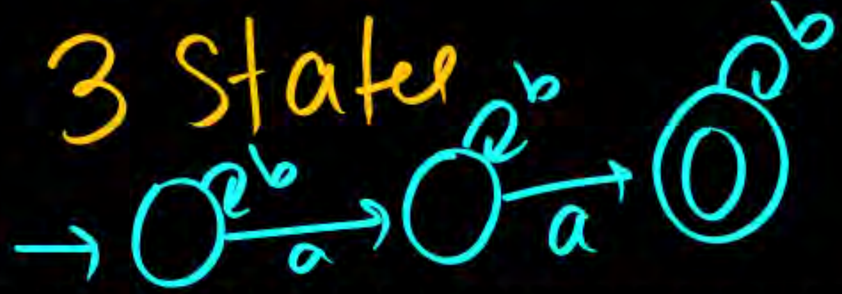


3 States

If $|w| \geq K$ then

i) $n(\text{Min DFA}) = K+1$

ii) $n(\text{Min NFA}) = K+1$

	Min DFA	Min NFA
(11) $L = (a+b)^2 a (a+b)^*$	 5 states	4 states
(12) $L = (a+b)^* a (a+b)^2$	8 States	 4 states
(13) $b^* a b^* a b^*$	4 states	3 States 
(14) $b^*(a+\epsilon) b^* (a+\epsilon) b^*$	4 states	3 states
(15) $\Sigma^* a \Sigma^* a \Sigma^*$	3 States	3 states

Note: $w \in \{a,b\}^*$

		Min DFA	min NFA
I)	$\lvert w \rvert = K \Rightarrow$	$K+2$	$K+1$
II)	$\lvert w \rvert \le K \Rightarrow$	$K+2$	$K+1$
III)	$\lvert w \rvert \ge K \Rightarrow$	$K+1$	$K+1$
IV)	$\#_a(w) = K \Rightarrow$	$K+2$	$K+1$
V)	$\#_a(w) \le K \Rightarrow$	$K+2$	$K+1$
VI)	$\#_a(w) \ge K \Rightarrow$	$K+1$	$K+1$
VII	K^{th} symbol is 'a'	$K+2$	$K+1$
VIII	K^{th} symbol from end is 'a'	2^K	$K+1$

Note: I K^{th} symbol from begin is 'a' $\Sigma = \{a, b\}$

i) $n(\text{Min DFA}) = K + 2$

ii) $n(\text{Min NFA}) = K + 1$

II K^{th} symbol from end is 'a'

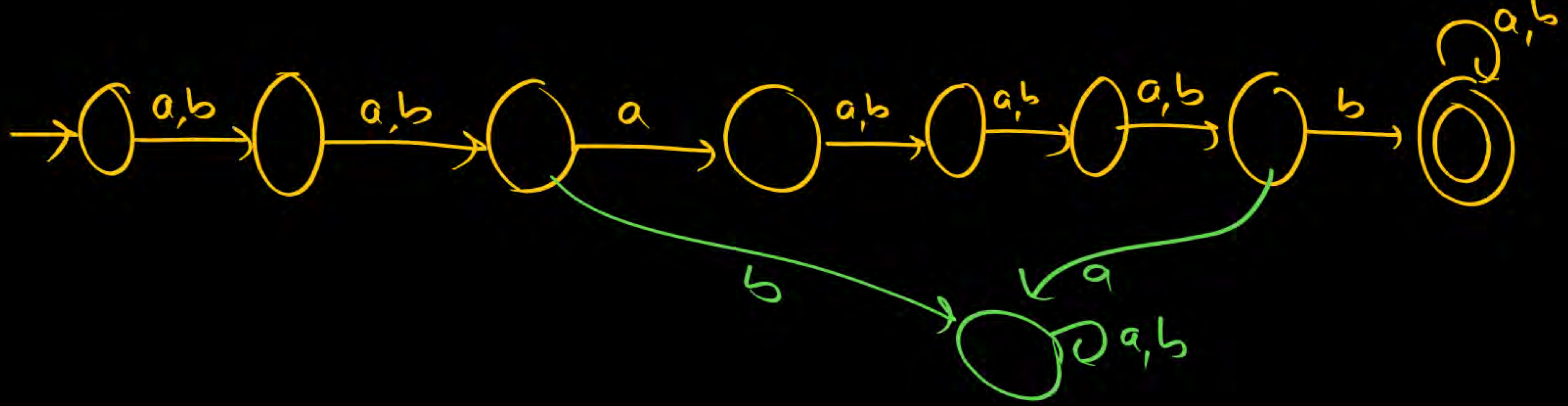
i) $n(\text{Min DFA}) = 2^K$

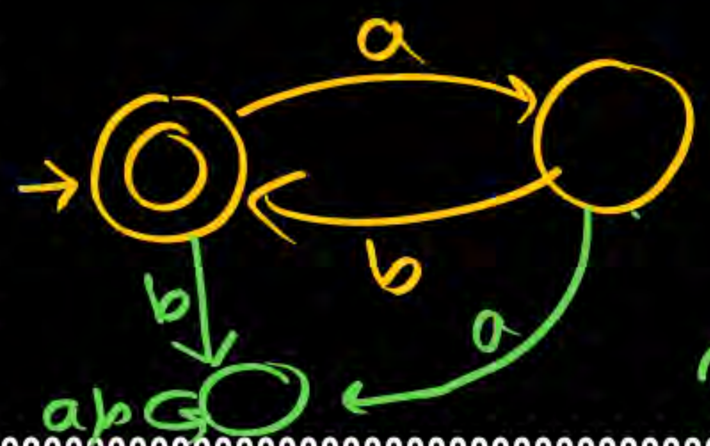
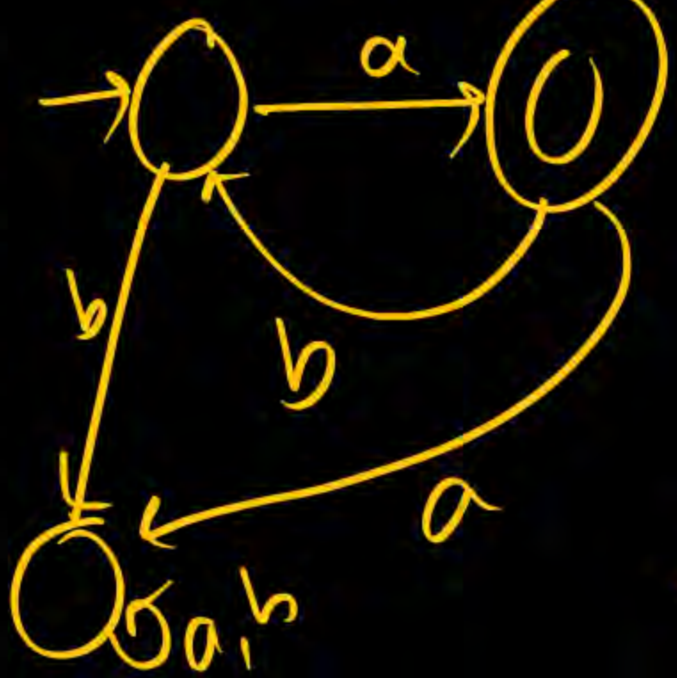
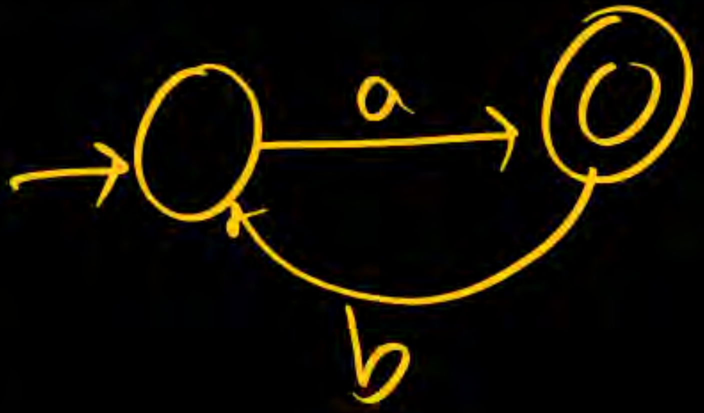
ii) $n(\text{Min NFA}) = K + 1$

	Min DFA	Min NFA
(16) $L = aaa(a+b)^*$	5	4
(17) $L = (a+b)^* aaa$	4	4
(18) $L = (a+b)^* aaa (a+b)^*$	4	4
(19) $L = (a+b)^2 a (a+b)^3 b (a+b)^*$	9	8
(20) $L = \underbrace{\underbrace{(a+b)}_{1} \overset{2^{nd}}{\underbrace{a}_{1}} \underbrace{(a+b)^7}_{7} \overset{10^{th}}{\underbrace{a}_{1}}}_{10 \text{ length min}} (a+b)^*$	12	11

$$L = (a+b)^2 a (a+b)^3 b (a+b)^*$$

$$= \left\{ w_1\, x\, w_2\, y\, w_3 \;\middle|\; \begin{array}{l} w_1, x, w_2, y, w_3 \in (a+b)^*, \\ x = a,\ y = b, \\ |w_1| = 2,\ |w_2| = 3 \end{array} \right\}$$



	Min DFA	Min NFA
(21) $L = (ab)^*$ $= \{\varepsilon, ab, abab, (ab)^3, \dots\}$	 3 states	 2 states
(22) $L = (ba)^*$	3 states	2 states
(23) $L = a(ba)^*$ $= \{\boxed{a}, aba, ababa, \dots\}$ $= (ab)^* a$	 3 states	 2 states

$$\underbrace{a(a+b)^*}_{\text{starting with 'a'}} \neq \underbrace{a(ba)^*}_{}$$

~~L = set of all strings starting with 'a'~~

$$\underbrace{a(ba)^*}_{\text{some strings starting with 'a'}} \subset \underbrace{a(a+b)^*}_{\text{All strings starting with 'a'}}$$

NFA without ε moves	NFA with ε moves
$\delta: Q \times \Sigma \rightarrow 2^Q$	$\delta: Q \times \Sigma \cup \{\varepsilon\} \rightarrow 2^Q$

not "NFA without ε moves"

NFA with ε moves

$\bigcirc \longrightarrow \bigcirc$

Transition (move)

$\bigcirc \xrightarrow{\epsilon} \bigcirc$

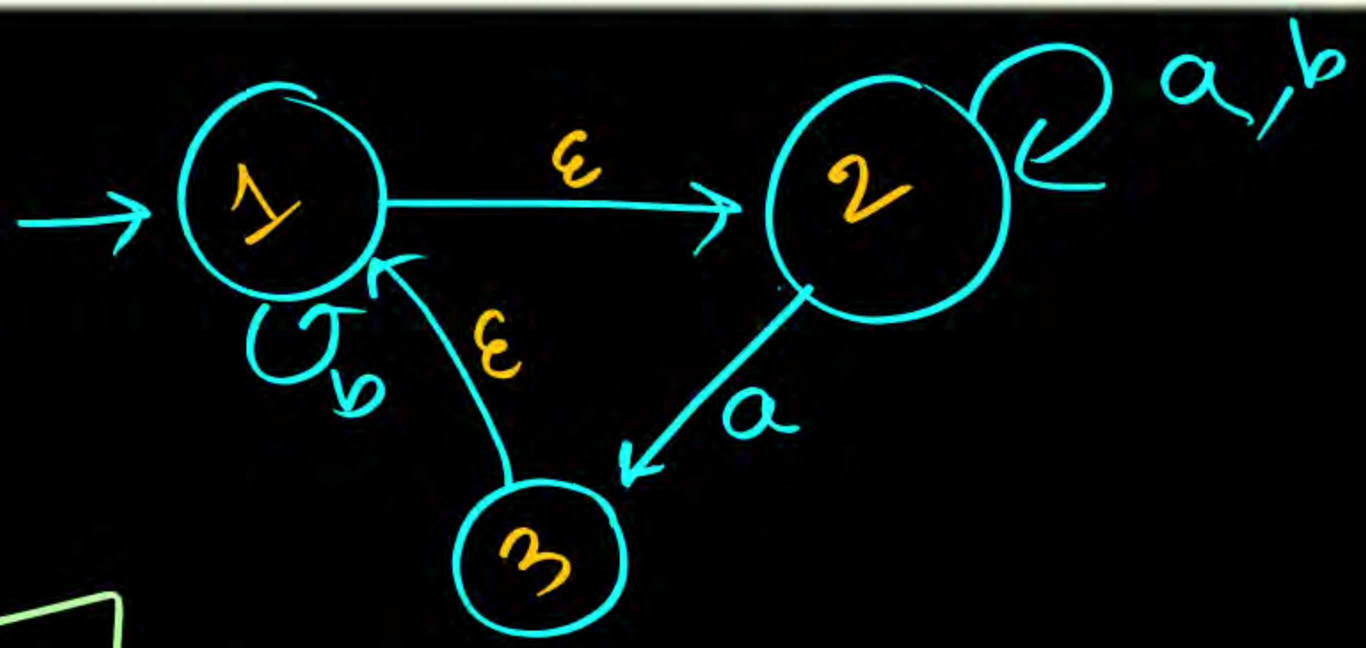
$|\epsilon| = 0$

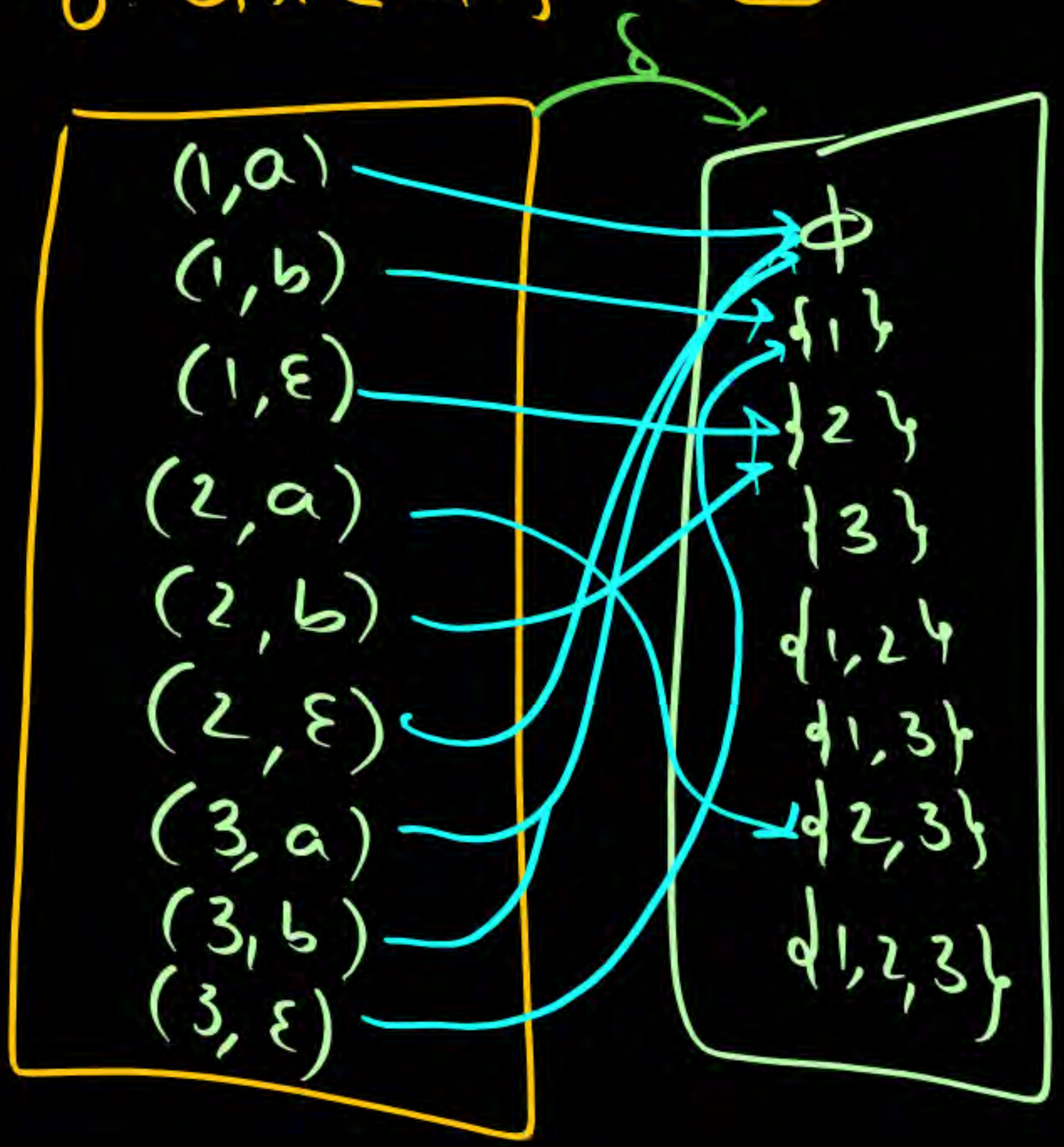
Transition with no i/p symbol

Transition with Zero i/p "

Transition with ϵ

$\delta : Q \times \Sigma \cup \{\varepsilon\} \longrightarrow 2^{Q}$





not DFA

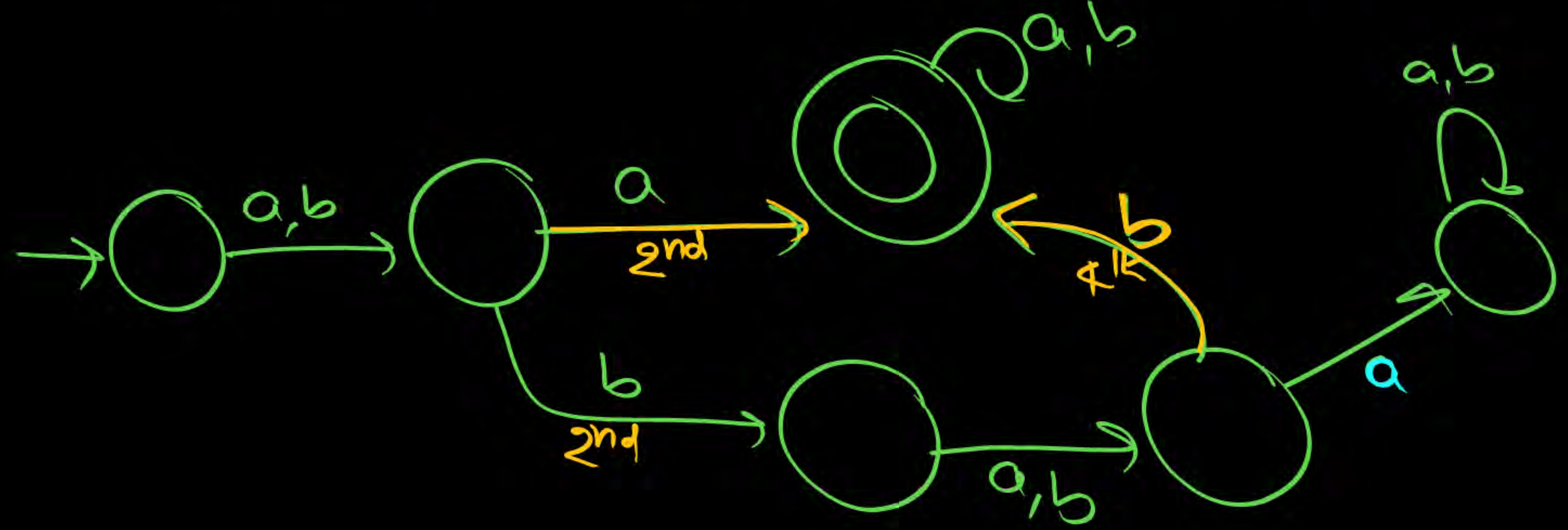
not "NFA without ε move"

It is NFA.

Doubt 1:

$$L = \underbrace{(a+b)\,a\,(a+b)^*}_{2^{nd}\text{ is 'a'}} + \underbrace{(a+b)^3\,b\,(a+b)^*}_{4^{th}\text{ is 'b'}}$$

$$= (a+b)\,a\,(a+b)^* + (a+b)b(a+b)b(a+b)^*$$



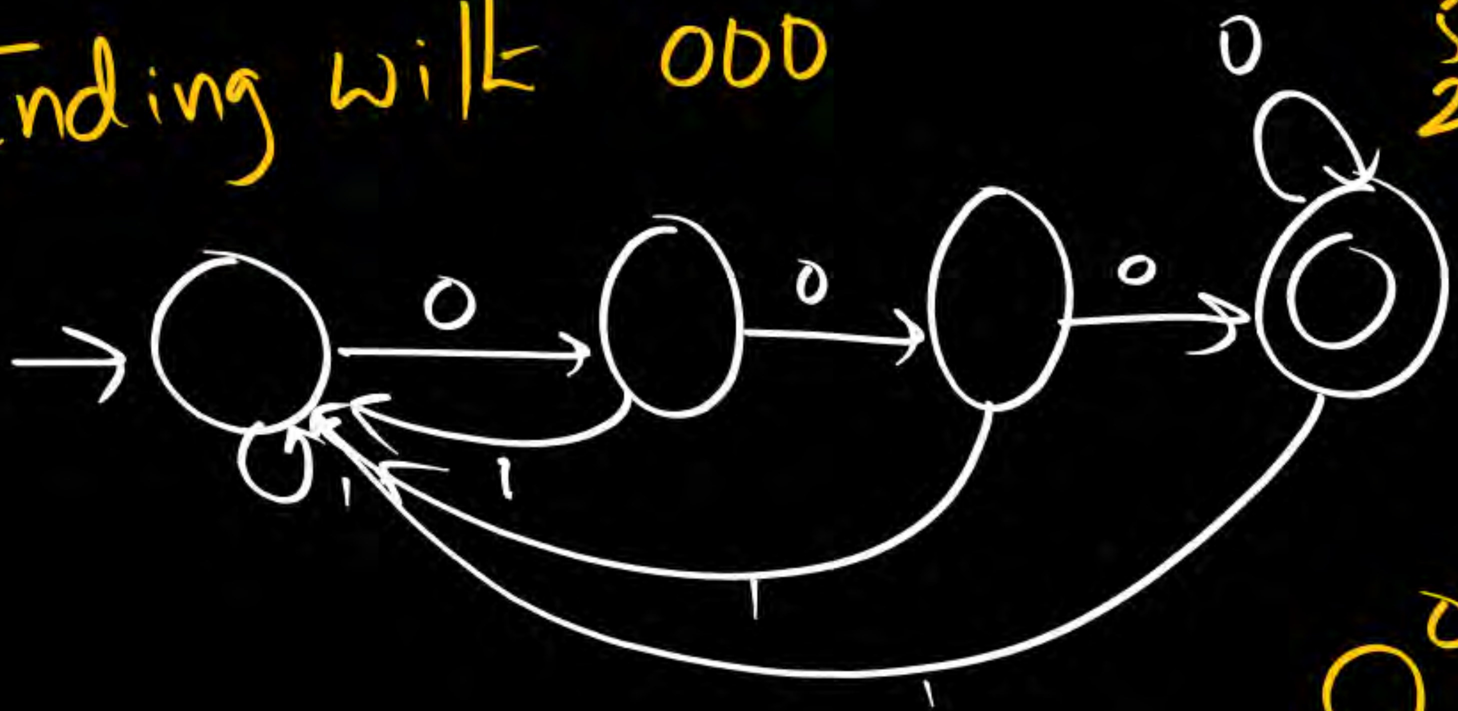
Doubt 2:

H.W. 1

Special problem

$L = \{w \mid w \in \{0,1\}^*, \ Dec(w) \text{ is div by } 8\}$

Method 1: Ending with "000"



$2^5 \ 2^4 \ 2^3 \ \boxed{2^2 \ 2^1 \ 2^0}$

0

0 0

$\underbrace{0\ 0\ 0}$

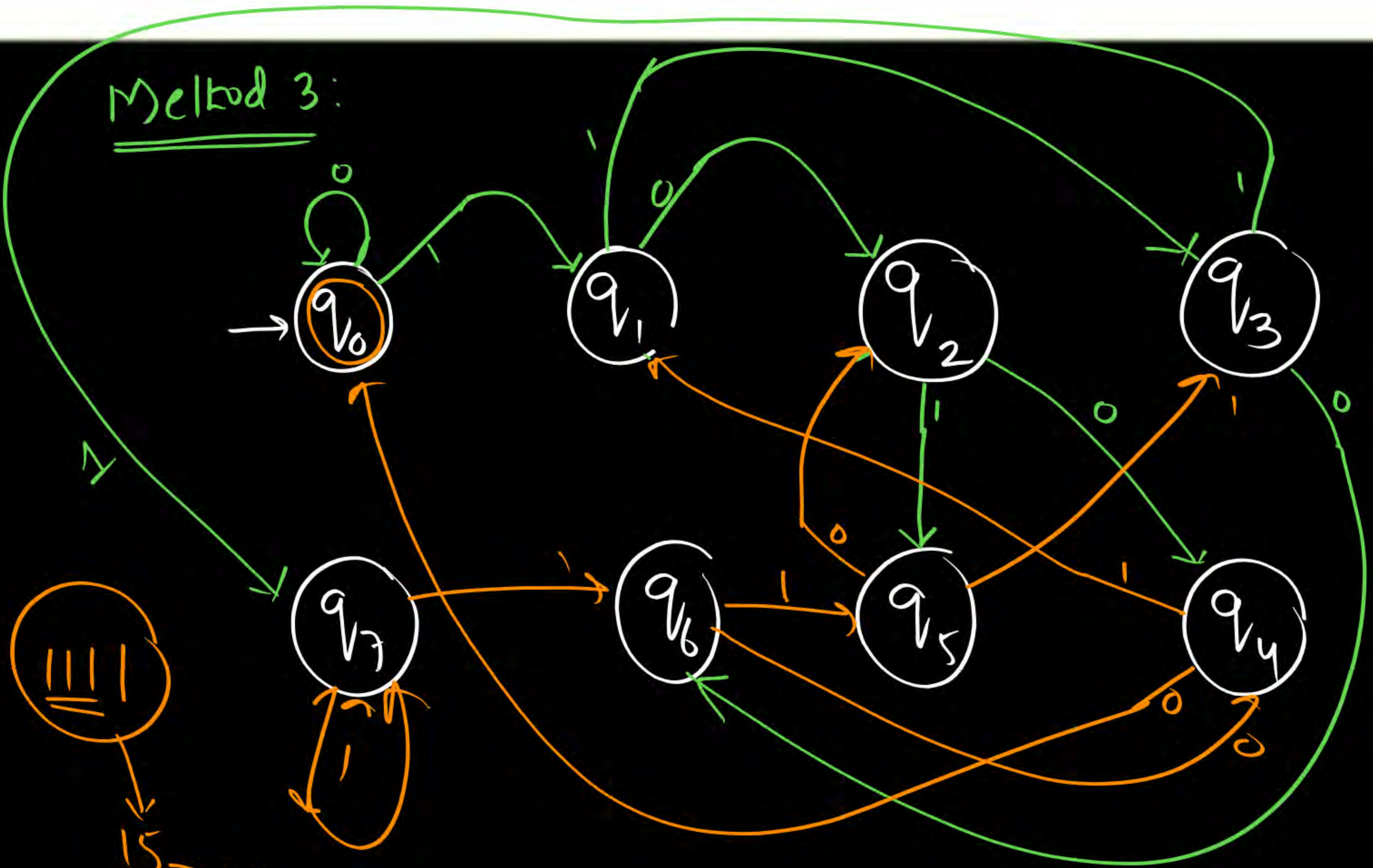
No 1 should appear in last 3 bits.

Method 2 :

	0	1
$\rightarrow {}^{*}q_0$	q_0	q_1
q_1	q_2	q_3
q_2	q_4	q_5
q_3	q_6	q_7
q_4	q_0	q_1
q_5	q_2	q_3
q_6	q_4	q_5
q_7	q_6	q_7

$\xRightarrow{\text{Minimization of DFA}}$ $\Rightarrow$ 4 states

Method 3:



1111

15 → q_7

Bin	Dec	Rem
0	0	q_0
1	1	q_1
10	2	q_2
11	3	q_3
100	4	q_4
101	5	q_5
110	6	q_6
111	7	q_7
1000	8	q_0
1001	9	q_1
1010	10	q_2

Summary

→ DFA, NFA, RgExp

→ DFA Vs NFA

&

NF with ε-move


THANK YOU
GW
SOLDIERS !