Q.1 Consider the following statement about relations on a set A, where | A | = n. How many of the following statements are TRUE?

[NAT]

I. If R is a relation on A and $|R| \geq n$, then R is reflexive. (false)

II. If R_1, R_2 are relations on A and $R_2 \supseteq R_1$, then R_1 is reflexive $\Rightarrow R_2$ is reflexive (T)

III. If R_1, R_2 are relations on A and $R_2 \supseteq R_1$, then R_1 is transitive $\Rightarrow R_2$ is (false) transitive

IV. If R_1, R_2 are relations on A and $R_2 \supseteq R_1$, then R_1 is symmetric $\Rightarrow R_2$ is (false) symmetric

V. If R is an equivalence relation A, then $n \leq |R| \leq n^2$ (True)

Ans: 2

I. $|A| = 2$
$A = \{1, 2\}$
$R = \{(1\,2)(2\,1)\}$
$|R| = 2$
$|R| \geq n$
$|R| \geq 2$

II. $R_1 \subseteq R_2$
$\{11\,22\,33\} \subseteq \{11\,22\,33\ldots\}$

III. $R_1 \subseteq R_2$
$\{12\} \subseteq \{12, 21\}$

IV. $R_1 \subseteq R_2$
$\{(a\,b)(b\,a)\} \subseteq \{a\,b\,b\,a \atop \textcircled{c}\textcircled{d}\}$

$$A = \{1, 2, 3\}$$

$$R = \{11 \quad 22 \quad 33\}$$

$$|R| = 3.$$

$$n \leq R \leq n^2.$$

$$R = A \times A$$

$$|R| = n^2$$

Consider A = {w, x, y, z}, then which of the following options is/are correct for the given set? $n = 4$

A. The number of symmetric relations is 2^{10}

$$(T) \quad 2^n \cdot 2^{\frac{n^2-n}{2}} = 2^4 \cdot 2^6 = 2^{10}$$

B. The number of symmetric relations which contain (x, y) is 2^9

$$(T) \quad 2^4 \cdot 2^5$$

C. The number of relations which is anti-symmetric and contain

(x, y) is $2^4 \cdot 3^5$

$$2^n \cdot 3^{\frac{n^2-n}{2}-1} \quad 2^4 \cdot 3^{6\cdot 1} = \quad \frac{n^2-n}{2} \quad \frac{16-4}{2} = \frac{12}{2} = 6.$$

D. The number of relations which is reflexive, symmetric and anti-symmetric is 1

11 22 33

$$\begin{bmatrix} 11 & 12 & 13 \\ 21 & 22 & 23 \\ 31 & 32 & 33 \end{bmatrix}$$

$$boxes = \frac{n^2 - n}{2}$$

$$2^n \quad 2^{\frac{n^2-n}{2} - 1} \quad \underline{(13)} +$$

Q.3 Let A be a set with $|A| = n$, and let R be a relation on A that is antisymmetric, then which of the following option is correct?

(P)(W)

[MCQ]

A. The maximum value for $|R|$ is $(n^2 + n)/2$. ✓

B. The number of anti-symmetric relations have size $|R|$ is $2^{(n^2+n)/2}$

C. Both a and b

D. None of these.

Ans:

$$\underbrace{11 \quad 22 \quad 33}_{n} \quad \underbrace{12 \quad 23 \quad 31}_{\frac{n^2-n}{2}}$$

$$n + \frac{n^2 - n}{2}$$

$$\begin{matrix} 11 & 12 \\ 22 & 23 \\ 33 & 31 \end{matrix}$$

$$= \frac{2n + n^2 - n}{2} = \frac{n(n+1)}{2}$$

( ) ×   ( ) ×   ( ) ×

$$2^n \cdot 3^{\frac{n^2-n}{2}} \Bigg\}$$

$$\boxed{| 1 \quad 2\,2 \quad 3\,3 \,|\, 2 \quad 2\,3 \quad 3\,| }$$

$$n \;+\; n^2 n/2$$
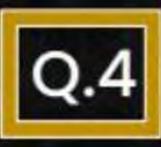
Q.4 Let A be a set with $|A| = n$, and let R be an equivalence relation on A with $|R| = r$. Which of the following is TURE for the given relation R? **[MCQ]**

A. $r - n$ will be always even.

B. $r - n$ will be always odd.

C. Both a and b.

D. None of these.

$A = n.$ $|R| = R.$

$A = \{1, 2, 3\}$ $R - n$

(7)

$n = 3.$ $R_1 = \left\{ \dfrac{(11)(22)(33)}{n} \right\}$

$R_2 = \left\{ \dfrac{(11)(22)(33)}{n} \dfrac{(12)(21)}{+2} \right\}$

$R - n.$

$B - 3 = 0\checkmark$

$(3+2) - 3 = 2$ $R_3 = \left\{ \dfrac{11\ 22\ 33}{n} \dfrac{(12)(21)}{+2} \dfrac{(23)(32)}{+2} \right\}$

$(3+4) - 3 = 4$

Let A = {1, 2, 3, 4, 5, 6, 7, 8}. How many symmetric relations on A contain exactly four ordered pairs?

[NAT]

$$A \times A = \{ \; 11 \; 22 \; 33 \; 44 \; 55 \; 66 \; 77 \; 88 \; \boxed{\quad} \; \boxed{\checkmark} \; \ldots \ldots \boxed{\quad} \}$$

8 same elements.

$$\frac{n^2 - n}{2} = \frac{64 - 8}{2} = \frac{56}{2} = 28 \text{ boxes.}$$

4 O.P:

$$\underbrace{28_{C_2}}_{\text{bones}} \; \underbrace{8_{C_0}}_{\text{same}} + \; 28_{C_1} \times 8_{C_2} + \; 28_{C_0} \times 8_{C_4} = \underline{1232}$$

$$\boxed{()()} \; \boxed{()} \; - \; + \; \boxed{()()} \; ()() \; +^1 \; ()()()()$$

THANK YOU GW SOLDIERS !