CS & IT ENGINEERING

DISCRETE MATHS
SET THEORY

Lecture No. 10

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TOPICS

01 equivalance relation

02 Partial order relation

3 Poset

Reflenive.	IRReflenive	Symmetric:	Anti	Asy	T
$\forall a \in A$ $(a,a)\in R$.	$\forall a\in A$ $(a,a)\notin R$	$\forall a \forall b\,[(a,b)\in R \to (b,a)\in R]$	$(a,b)\wedge(b,a) \to a=b$	$(a,b)\in R \to (b,a)\notin R$.	$(a,b)\wedge(b,c) \to (a,c)\in R$.
$R=\{\}$ ✗	$R=\{\}$ ✓	$R=\{\}$ ✓	$R=\{\}$ ✓	$R=\{\}$ ✓	$R=\{\}$ ✓
Same elements	→ hates same elements.	→ same ✓ → flipping	same ✓ → no flipping.	same ✗ flipping ✗	same ✓
2^{n^2-n}	2^{n^2-n}	$2^{n}\cdot 2^{\frac{n^2-n}{2}}$	$2^{n}\cdot 3^{\frac{n^2-n}{2}}$	$3^{\frac{n^2-n}{2}}$	—

A = nonemptyset

Z

$A \times A$

$Z \times Z$

$R_1 = \{(1\,2)(2\,1)\}$

$R_1 = \{(a,b) \mid a \leq b\}$

Result:

$\{(\ ), (\ )\}$

Result:

$R_1 : \{(a,b) \mid a+b=3\}$

Sym : ✓

Anti:

Asy:

$(a,b) \in R \rightarrow (b,a) \in R$

$aRb \rightarrow bRa$

$\downarrow$

$a+b=3 \rightarrow b+a=3.$

$(a,b) \in R \rightarrow (b,a) \in R.$

Symmetric : $R_1 : \{(a,b) \mid a+b=3\}$.

$(0,1)$ ✗.

$0+1=3$

$(0,3)$ ✓

$0+3=3$

$(a,b) \in R \longrightarrow (b,a) \in R$.

$a+b=3 \longrightarrow b+a=3$.

$(0,3) \in R \longrightarrow (3,0) \in R$.

$(1,2) \in R \longrightarrow (2,1) \in R$.

$(-1,4) \in R \longrightarrow (4,-1) \in R$.

if $a+b=3$ then $b+a=3$.

$Z : \{ \ldots\ldots 0, \ldots\ldots \}$

$Z \times Z$ ( ) ( ) (

(1,1) (1,2) (2,1)

(1,3) (1,4) ......

$a + b = 3$

$(1,2) \in R \rightarrow (2,1) \in R.$

$(0,3) \in R \rightarrow (3,0) \in R.$

$(-4,1) \in R \rightarrow (1,-4) \in R.$

Check

Result: $\{(1,2)(2,1) \ldots\ldots\}$

Anti : ✗

$(a,b) \in R \wedge (b,a) \in R \rightarrow a = b$

$a + b = 3 \wedge b + a = 3 \rightarrow a = b$

$\underbrace{(1,2) \in R}_{T} \wedge \underbrace{(2,1) \in R}_{T} \rightarrow \underbrace{1 = 2}_{F}$

Asy : ✗

$R = \{ (a,b) \mid a + b = 3 \}$

$R_2 : \{ (a, b) \mid a \equiv b \pmod 4 \}$

Sym:

$(a, b) \in R \rightarrow (b, a) \in R$.

$a \equiv b \pmod 4 \longrightarrow b \equiv a \pmod 4$

$1 \equiv 5 \pmod 4 \longrightarrow 5 \equiv 1 \pmod 4$

$(1, 5) \in R \rightarrow (5, 1) \in R$.

$a \equiv b \pmod 4$

a, b are having same remainder w.r.t 4.

$(1, 5) \in R$. ✓

$\underline{1} \equiv \underline{5} \pmod 4$

$(0, 4) \in R$. ✓

$0 \equiv 4 \pmod 4$

$a \equiv b \pmod 4 \rightarrow b \equiv a \pmod 4$

$0 \equiv 4 \pmod 4 \rightarrow \underline{4} \equiv \underline{0} \pmod 4$

$a = 0 \quad b = 4.$

eg: $\{ (a, b) \mid a + b = \text{even} \}$

Ref: $(a, a) \in R.$ ✓

$\{ (a, a) \mid a + a = \text{even} \}$

$2a = \text{even}$

Sym: $(a, b) \in R \rightarrow (b, a) \in R.$

$\underline{a + b = \text{even}} \rightarrow \underline{b + a = \text{even}}.$

$(1, 3) \in R \longrightarrow (3, 1) \in R$

$(2, 4) \in R \longrightarrow (4, 2) \in R$

Transitive:

$(a, b) \in R \wedge (b, c) \in R \rightarrow (a, c) \in R.$

$a + b = \text{even} \wedge b + c = \text{even} \rightarrow a + c = \text{even}.$

$2 + 4 = \text{even} \wedge 4 + 6 = \text{even} \rightarrow 2 + 6 = \text{even}$

$a = 2 \quad b = 4.$

$$\overset{T}{\boxed{a+b=\text{even} \wedge b+c=\text{even}}} \rightarrow \overset{F}{\overline{a+c=\text{even}}} \qquad T \rightarrow F$$

$$1+3=\text{even} \wedge 3+5=\text{even} \longrightarrow 1+5=\text{even}$$

Z^+ :

$Z^+ \times Z^+$

$(6,3)$ ✗

a divides b

$R = \{(3,6) \ldots\}$

$R = \{(a,b) \mid a|b \}$ ✓

$\underline{R}$: $(a,a) \in R \quad a|a$

$\underline{Sy}$: $(a,b) \in R \rightarrow (b,a) \in R$

$a|b \rightarrow b|a$ ✗

$\underbrace{(3,6) \in R}_{T} \rightarrow \underbrace{(6,3) \notin R}_{F}$

$\underline{Anti}$ $\left\{\begin{array}{l}\rightarrow \text{allows same} \\ \text{no flipping}\end{array}\right\}$

$a|b \wedge b|a \rightarrow a = b$

$2|2 \wedge 2|2 \rightarrow 2 = 2$

$\underline{T}$:

$a|b \wedge b|c \rightarrow a|c$

$Z \times Z$

$R_1 = \{(a,b) \mid a = b+1\}$

$R_2 : \{(a,b) \mid a \leq b\}$

$R_3 : \{(a,b) \mid a+b \leq 3\}$

R ✗

Sy ✓

Ant ✗

Asy ✗

T ✗

$R4 : \{(a,b) \mid \gcd(a,b) = 1\}$

$a+b \leq 3 \wedge b+c \leq 3 \rightarrow a+c \leq 3$

$(3+0) \leq 3 \wedge 0+3 \leq 3 \qquad 3+3 \leq 3\ (F)$

$(3,0) \in R \wedge (0,3) \in R \rightarrow (3,3) \notin R$

$R_1 = \{(a,b) \mid a = b+1\}$.

R: $aRa \quad a = a+1 \; (\times)$

Sy: $aRb \rightarrow bRa$. $(\times)$

$a = b+1 \rightarrow b = a+1$.

$3 = 2+1 \longrightarrow 2 = 3+1$ (False)

$a=3 \cdot b=2$

$(3,2) \in R \rightarrow (2,3) \notin R$.

Anti ✓

Asy ✓

$R_2 : \{(a,b) \mid a \le b\}$

R: $aRa \quad a \le a$ ✓

Sy: $aRb \rightarrow bRa$. ✗

$a \le b \rightarrow b \le a$

$2 \le 3 \rightarrow 3 \le 2$ (F)

$(2,3) \in R \rightarrow (3,2) \notin R$

Anti:

$aRb \wedge bRa \rightarrow a = b$

$a \le b \wedge b \le a \rightarrow a = b$

$2 \le 2 \wedge 2 \le 2 \rightarrow 2 = 2$

$a \le b \wedge b \le a \rightarrow a = b$

$\underbrace{2 \le 3}_{T} \wedge \underbrace{3 \le 2}_{F}$

$T \wedge F$

$F \longrightarrow$

T

Asy ✗

Same element

Transitive ✓

$a \le b \wedge b \le c \rightarrow a \le c$

$(3,3)$

$(0,3)\ (3,0)$

$a+b\leq 3$

$$R=\{(0,3)\ (3,0), \ldots\ldots\}$$

$$\underbrace{(3,0)\in R}_{T} \wedge \underbrace{(0,3)\in R}_{T} \rightarrow (3,3)\in R$$

$\downarrow$

Check $(3,3)$ is present in R or not.

$R_1 : \{ (A, B) \mid A \subseteq B \}$

$R = \{ (G_1, G_2) \mid G_1 \equiv G_2 \}$

R: (A, A) $A \subseteq A$ ✓

Ref: $G R G$ ✓

Sym: $A R B \longrightarrow B R A$. ✗

$\{1\} \subseteq \{1\ 2\} \rightarrow \{1\ 2\} \subseteq \{1\}$

Sym: $G_1 R G_2 \rightarrow G_2 R G_1$. ✓

flipping

Anti ✗
Asy ✗

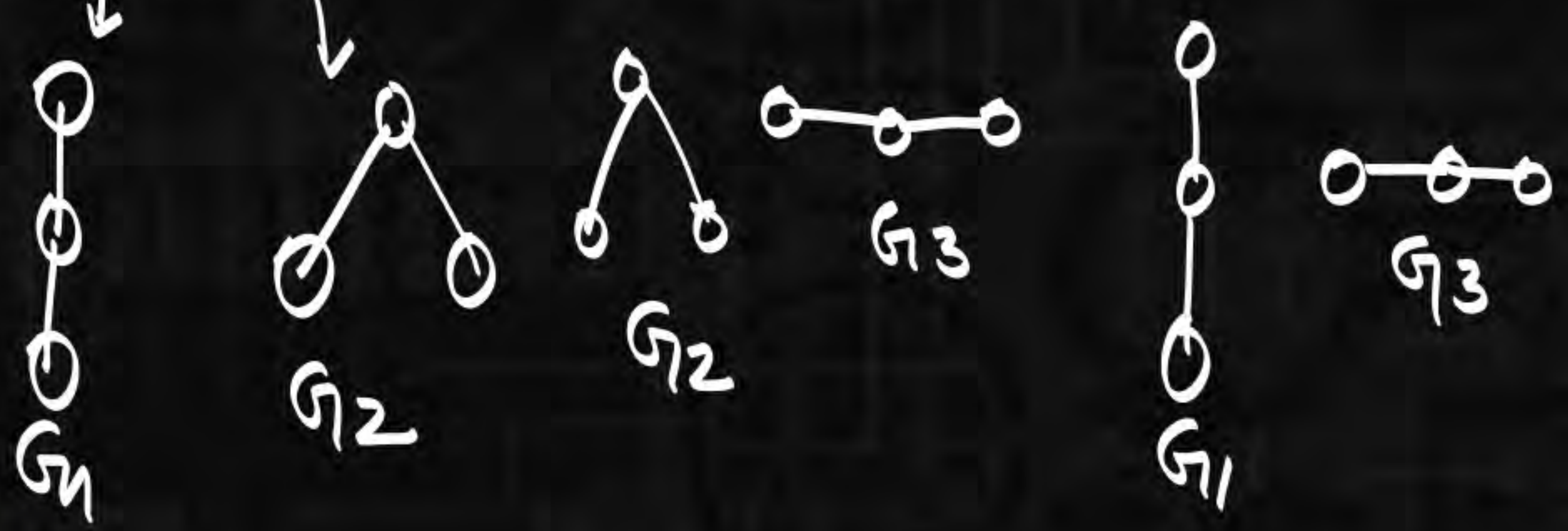
Anti ✓

Asy ✗ $\rightarrow A \subseteq A$

Tr: $A \subseteq B \wedge B \subseteq C \rightarrow A \subseteq C$.

T: $G_1 R G_2 \wedge G_2 R G_3 \rightarrow G_1 R G_3$.

$G_1 \equiv G_2 \wedge G_2 \equiv G_3 \rightarrow G_1 \equiv G_3$.



$ARB \wedge BRA \rightarrow A=B$

$A \subseteq B \wedge B \subseteq A \rightarrow A=B$.

Anti

same set ✓

flipping ✗.

$R_5 = \{ (s_1, s_2) \mid s_1, s_2 \in Q \}$

$01 \, R \, 01$ ✓

$01 \, R \, 011 \rightarrow 011 \, R \, 01$ ✓

Antx Asy.

Transitive $01 \, R \, 011 \wedge 011 \wedge 111 \rightarrow 01 \, R \, 111$

ends with 1.

$Z^2 : (\ (\ )\ (\ )\ (\ )\ )$

Z

$Z^2 \times Z^2$

$Z \times Z$

e_1

$((12)(23))$

e_2

$((23)(34))$

Result

$R_1 : \{ (a\,b) R (c\,d) \mid ad = bc \}$

$\quad\;\; 1\;2 \quad\;\; 3\;4 \qquad 1.4 = 2.3$

Z^2

$Z^2 \times Z^2$:

Sym :

normal $aRb \to bRa$.

L.S R R.S $\to$ R.S R L.S.

$(\overset{1}{a}, \overset{2}{b}) R (\overset{3}{c}, \overset{4}{d}) \longrightarrow (\overset{1}{c}, \overset{2}{d}) R (\overset{3}{a}, \overset{4}{b})$ ✓

$1 \cdot 4 = 2 \cdot 3$

$ad = bc \longrightarrow c \cdot b = d \cdot a$.

$(ab)(cd)$

$(2\,1)(6\,3)$

$ad = bc$

$R_1 = \{ \overbrace{(2,1)(6,3)}^{e_1}, \underbrace{((6,3),(2,1))}_{e_2} \ldots\ldots$

$ad = bc$

$A = \{1, 2\}$

$A^2 = \{(1\,1)(1\,2)$
$(2\,1)(2\,2)\}$

$A^2 \times A^2 = \{(\overset{a\,b}{(1\,1)}, \overset{c\,d}{(1\,1)}), ((1\,1)(1\,2))$
$((1\,1), (2,1)), ((1\,1)(2,2))\}$

$\downarrow$

Result: $(\quad)$

Transitive: normal. $1.4 = 2.3.$

$aRb \wedge bRc \rightarrow aRc.$

$(a,b)R(c,d) \wedge (c,d)R(x,y) \rightarrow (a,b)R(x,y)$

$ad = bc \wedge cy = dx \longrightarrow ay = bx.$ ✓

$\frac{cy}{x} = d.$

$a\left(\frac{\cancel{c}y}{x}\right) = b\cancel{c}$

$ay = bx$

$\{(p,q)\,R\,(r,s) \mid \rightarrow p-s = q-r\}$ Ref (GATE-15)

$1-4 = 2-3$

Sym.

aRa $(p,q)\,R\,(p,q)$

$1-4 = 2-3$

$p-q \neq q-p$

$(p,q)\,R\,(r,s) \rightarrow (r,s)\,R\,(p,q)$ ✓

$p-s = q-r$

$r-q = s-p$

$-(q-r) = -(p-s)$

$q-r = p-s$


THANK YOU GW
SOLDIERS !