CS & IT ENGINEERING

Theory of Computation

Finite Automata

Lecture No. 7

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TOPICS TO BE COVERED
01 DFA-Model III (No.of Symbols)
02 DFA-Model IV (Length & Remainder)
03 DFA-Model V (Symbols & Remainder)
04 DFA-Model VI (Sequence)
05 DFA-Model VII (Sequence & Number of Symbols)
06 DFA-Model VIII (Multiple Conditions on Symbols)

Model-III [No. of Symbols]

① $\{w \mid w \in \{a,b\}^*, \underbrace{n_a(w) = 2}_{\text{No. of a's in } w \text{ is } 2}\}$

② $\{w \mid w \in \{a,b\}^*, n_a(w) \le 2\}$

③ $\{w \mid w \in \{a,b\}^*, n_a(w) \ge 2\}$

4 states $\Leftarrow$ ④ $\{w \mid w \in a^*, \underbrace{n_a(w)}_{|w|} = 2\}$

4 states $\Leftarrow$ ⑤ $\{w \mid w \in a^*, \underbrace{n_a(w)}_{|w|} \le 2\}$

3 states $\Leftarrow$ ⑥ $\{w \mid w \in a^*, \underbrace{n_a(w)}_{|w|} \ge 2\}$

$w = aaa$

$|w| = 3$

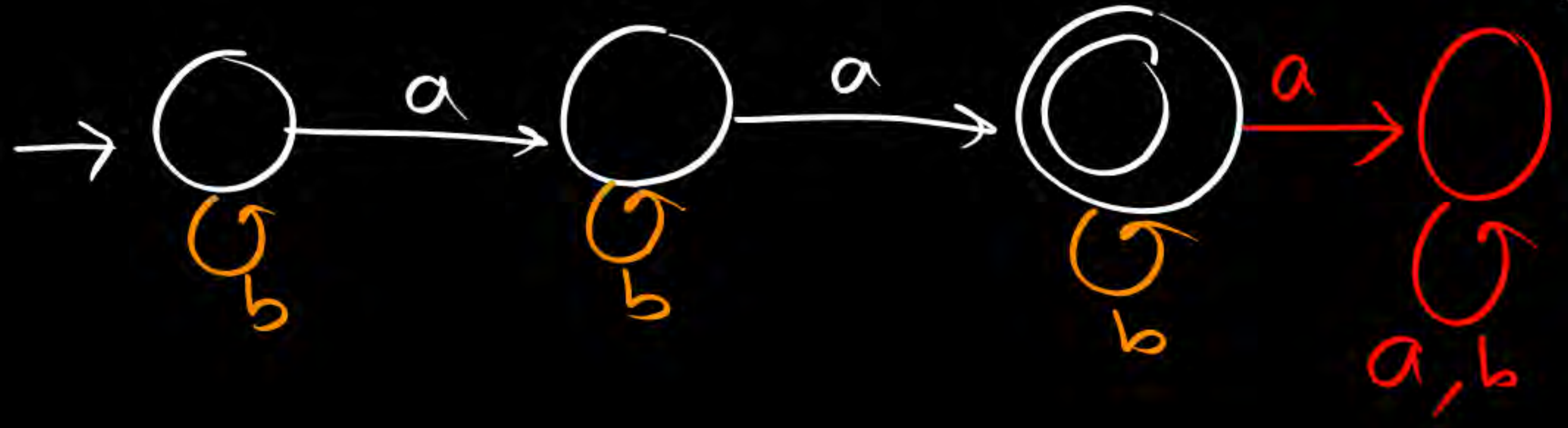
$n_a(w) = 3$

$\underbrace{|w| = n_a(w)}_{\text{same}}$

Model-III

① No. of a's = 2, $w \in \{a, b\}^*$

$b^* a\, b^* a\, b^*$



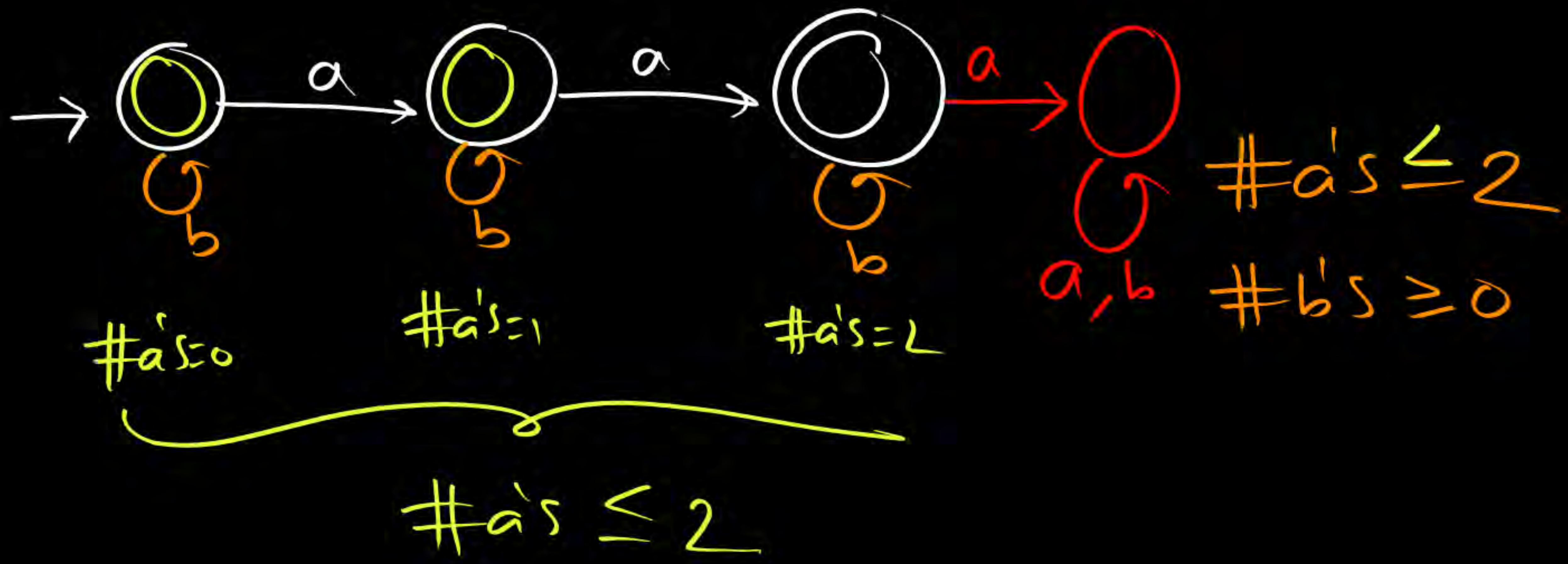
#a's = 2

#b's ≥ 0

Model - III

(2) No. of a's ≤ 2, $w \in \{a,b\}^*$

$$\boxed{b^*(a+\varepsilon)\,b^*\,(a+\varepsilon)\,b^*}$$



Note: $L_1 = \{w \mid w \in \{a,b\}^*,\ n_a(w) = 2\}$

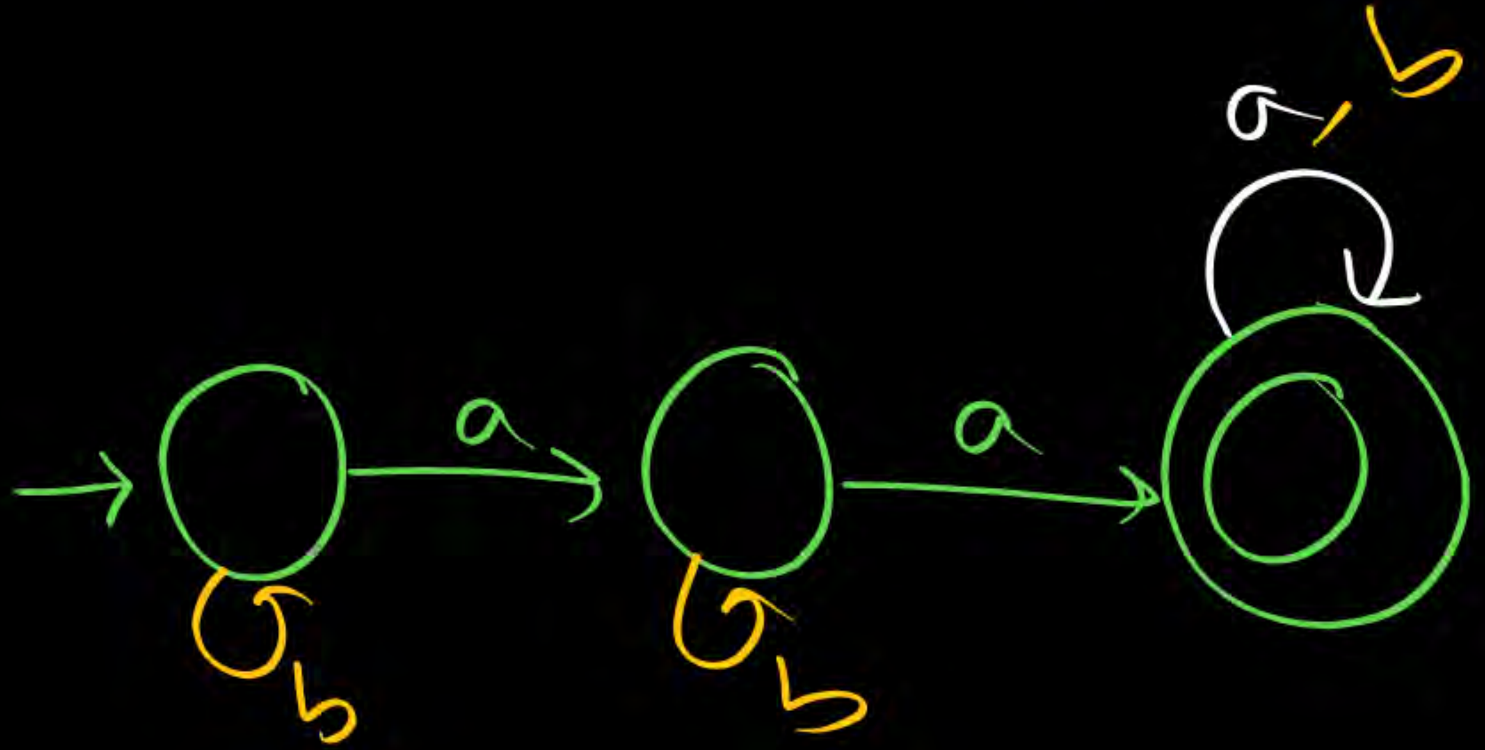
$L_2 = \{w \mid$ " $,\ n_a(w) \le 2\}$

x is no. of final states in DFA that accepts L_1.

y is " " " " L_2:

$$|x - y| = |1 - 3| = 2$$

③ $\#a's \geq 2, \; w \in \{a,b\}^*$



$$\underbrace{(a+b)^* a (a+b)^* a (a+b)^*}$$

Model-4: Length & Remainder (Divisible)

Divisible by 2

0 1

Divisible by 3

Rem = $\boxed{0}$ 1 2

$$\left.\begin{aligned} |W| &= 0 \\ &= 3 \\ &= 6 \\ &= 9 \\ &\vdots \end{aligned}\right\} \Rightarrow \text{rem } 0$$

Model-4 :

complimint to each other

① $\{ w \mid w \in \{a,b\}^*, |w| \text{ is divisible by } 2 \}$

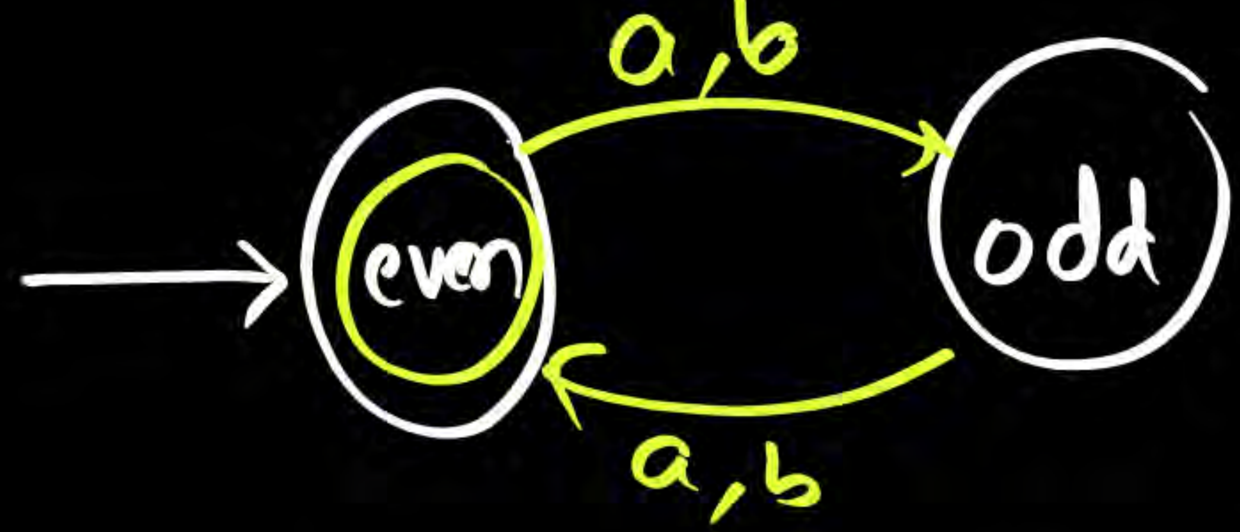
② $\{ w \mid$ " $, |w| \text{ is not divisible by } 2 \}$

③ $\{ w \mid$ " $, |w| \text{ is divisible by } 3 \}$ ⇒ 3 states

④ $\{ w \mid$ " $, |w| = 5n+2 \quad \boxed{n \geq 0} \}$ ⇒ q_2 is final ⇒ 5 States

*** ⑤ $\{ w \mid$ " $, |w| = 3n+5 \quad n \geq 0 \}$

① $\{w \mid w \in \{a,b\}^*, |w| = \text{even}\}$



rem 0

$|w| = 0, 2, 4, 6, 8, \ldots$ } rem 0

rem 1

$|w| = 1, 3, 5, 7, 9, \ldots$ } rem 1

$$[(a+b)^2]^*$$

$|w| = 2n+0$

$|w| = \text{even}$

$= \text{div by } 2$

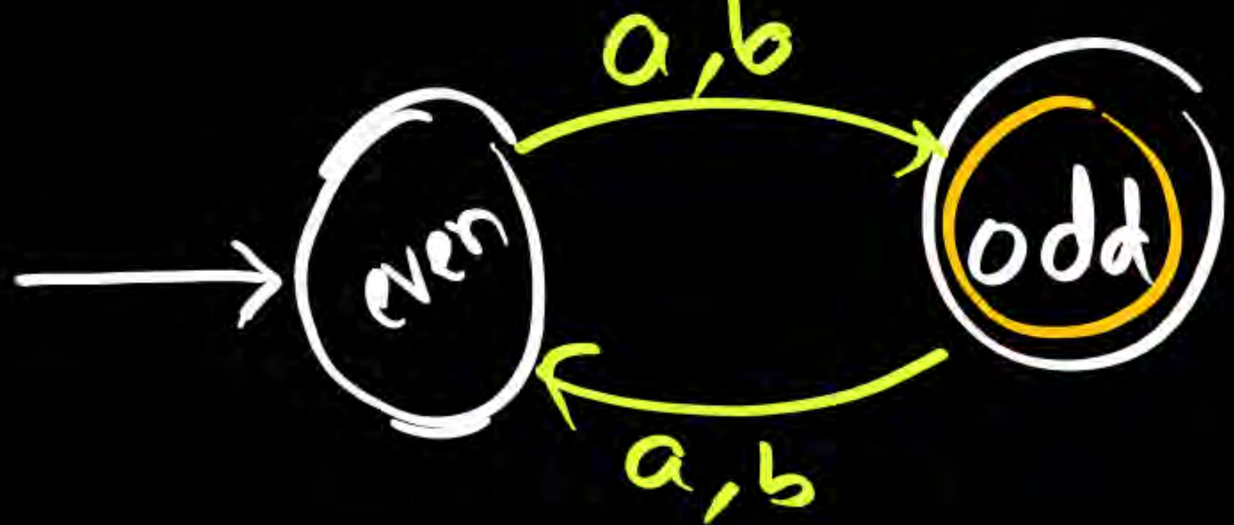
$= \text{multiple of } 2$

$|w| = 0 \bmod 2$

$|w| \equiv_2 0$

$(|w| \% 2) = 0$

(2) $\{w \mid w \in \{a,b\}^*, |w| = \text{odd}\}$



rem 0

$|w| = 0, 2, 4, 6, 8, \ldots$ } rem 0

rem 1

$|w| = 1, 3, 5, 7, 9, \ldots$ } rem 1

$$\underbrace{\left[(a+b)^2\right]^*}_{2n}\ \underbrace{(a+b)}_{1}$$

$2n+1$ length

$|w| = 2n+1$

$|w| = \text{odd}$

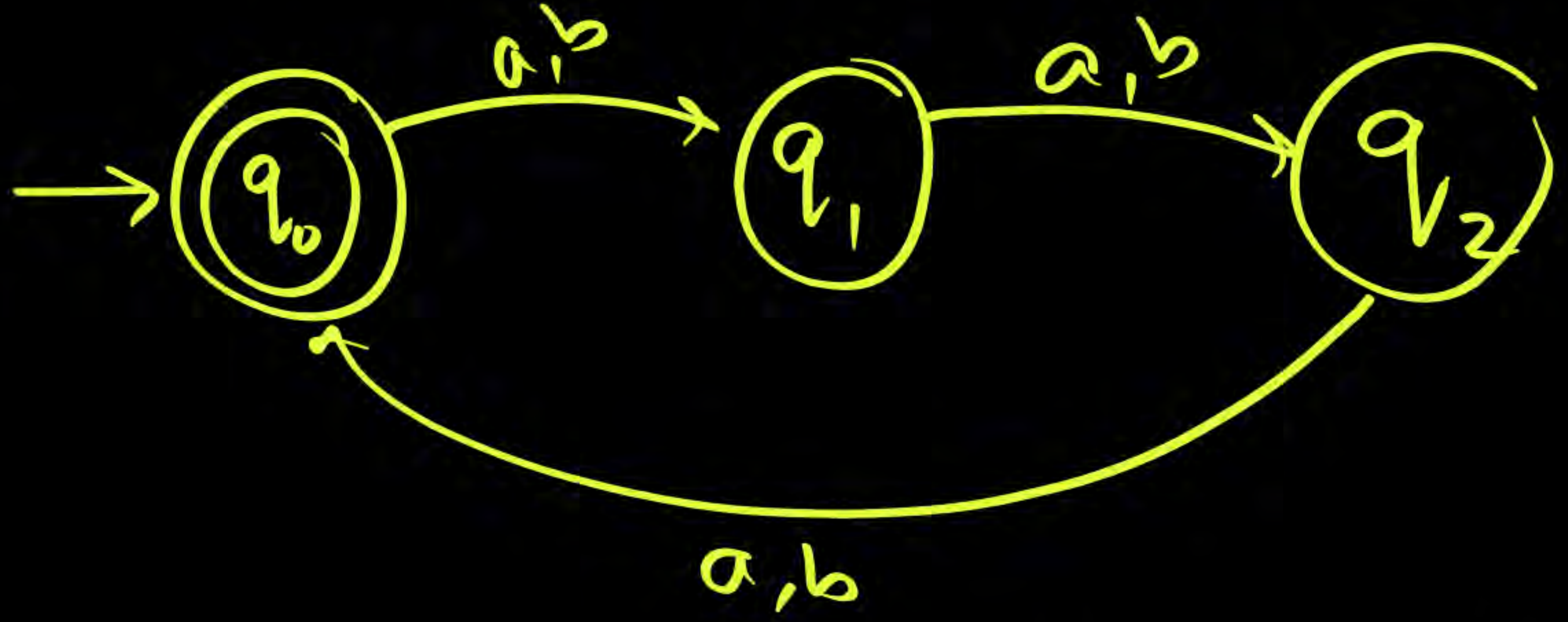
$= \text{not div by } 2$

$|w| = 1 \bmod 2$

$(|w| \% 2) = 1$

$|w| \equiv_2 1$

③ $|w|$ is div by 3 , $w \in \{a,b\}^*$



Note : $L = \{ w \mid w \in \{a,b\}^*, |w| \text{ is divisible by } K \}$

$\hookrightarrow$ No. of states in min DFA = K states

4

5 states

Final state may differ

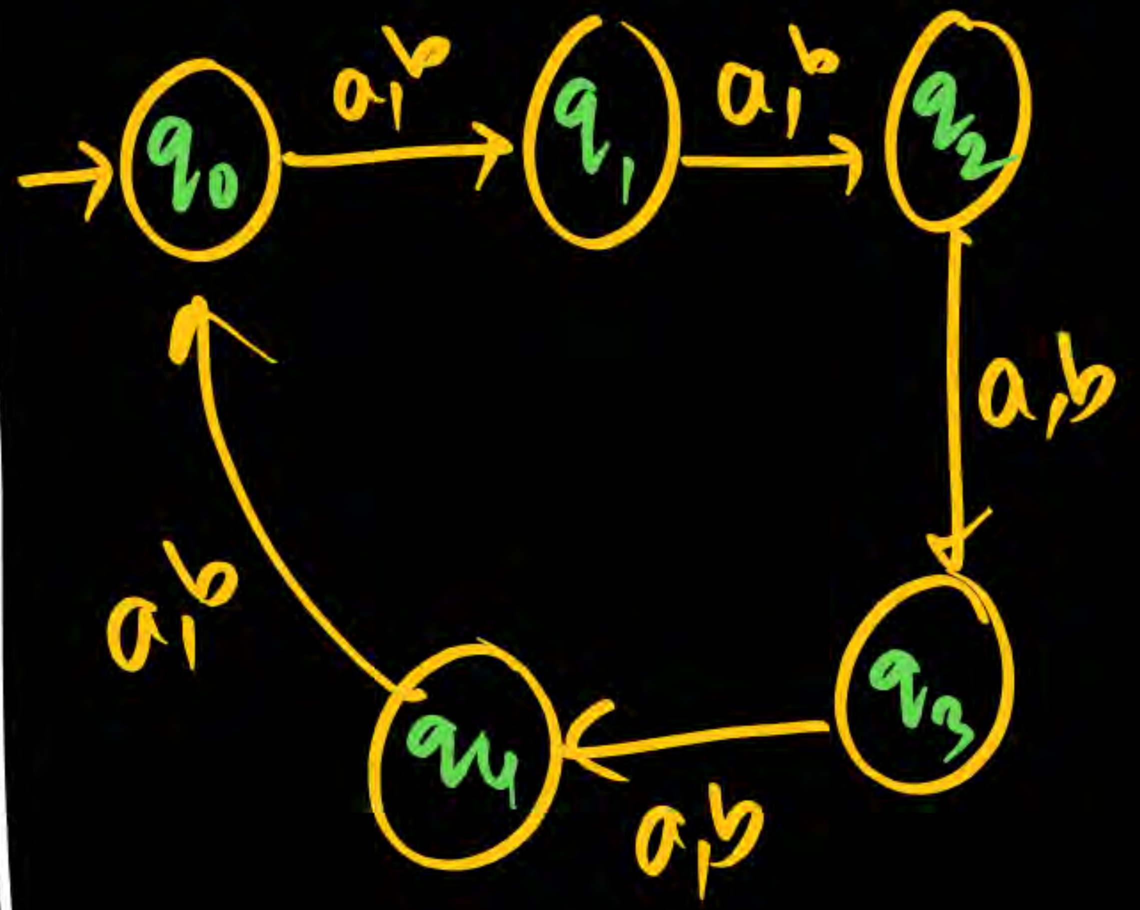
$|w| = 5n \Rightarrow$ rem 0 when divide with 5
→ q_0 is final

$|w| = 5n+1 \Rightarrow$ rem 1
→ q_1 is final

$|w| = 5n+2 \Rightarrow$ rem 2
→ q_2 is final

$|w| = 5n+3 \Rightarrow$ rem 3
→ q_3 is final

$|w| = 5n+4 \Rightarrow$ rem 4
→ q_4 is final



(5)

$$|w| = 3n + 0$$
$$= 3n + 1$$
$$= 3n + 2$$

with $k_1 = 3$ and k_2 the constant term.

$\Rightarrow$ 3 states

If $k_1 > k_2$

$$= 3n + 3 \Rightarrow 4$$
$$= 3n + 4 \Rightarrow 5$$
$$= 3n + 5 \Rightarrow 6$$
$$= 3n + 6 \Rightarrow 7$$
$$= 3n + \text{constant } (k_2) \Rightarrow k_2 + 1 \text{ States}$$

If $k_1 \leq k_2$

$n \geq 0$

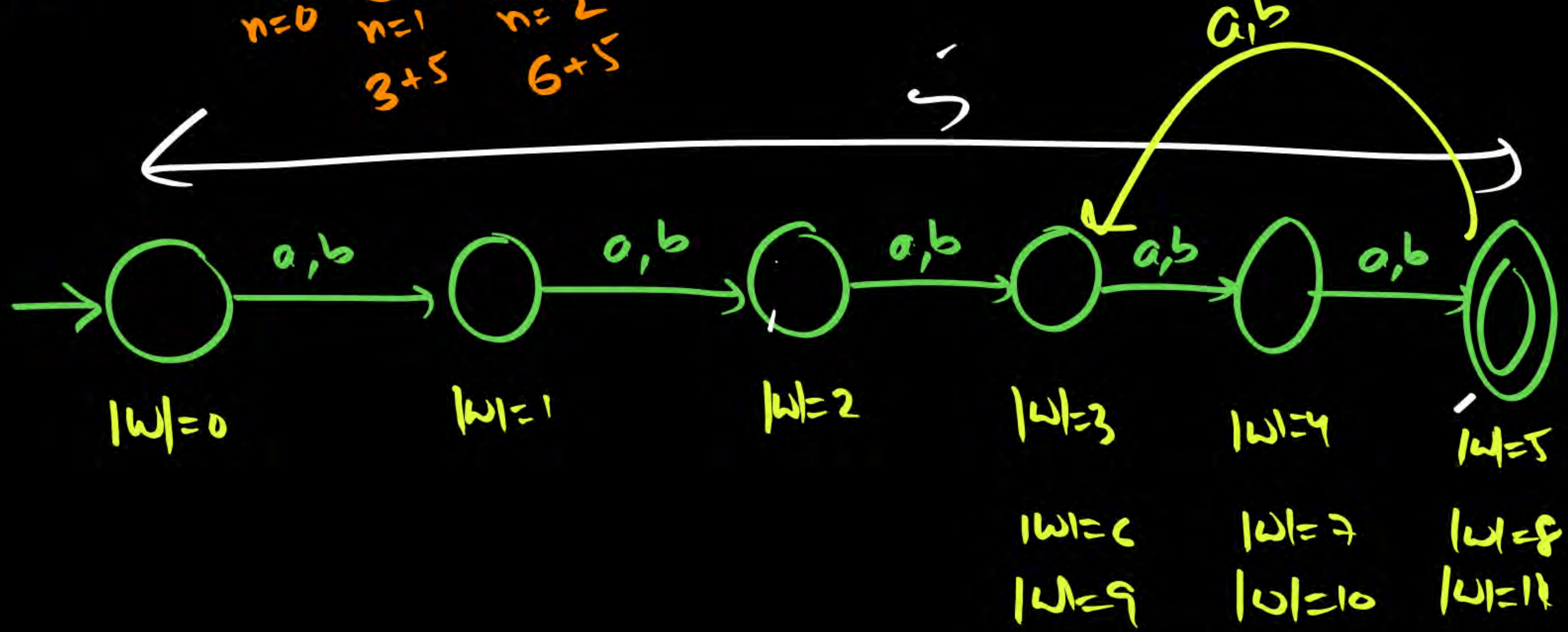
(5) $|w| = 3n + \boxed{5}$ ⟸ $3n+2,\ n \geq 1$

$n \geq 0$ $= \boxed{5}, 8, 11, \ldots$

$\xrightarrow{+3}$ $\xrightarrow{+3}$

$n=0$, $n=1$, $n=2$

$3+5$, $6+5$



$$\underbrace{3n+5}_{n \geq 0} \iff \underbrace{3n+2}_{n \geq 1}$$

min 5   min 5

Note : $L = \{ w \mid w \in \{a,b\}^*, \ |w| = k_1 n + k_2, \ n \geq 0 \}$

I) If $\underbrace{\boxed{k_1} > k_2}_{k_2 \text{ is remainder}}$ $\Longrightarrow$ k_1 states

II) If $\underbrace{k_1 \leq \boxed{k_2}}_{k_2 \text{ is not remainder}}$ $\Longrightarrow$ $k_2 + 1$ states

Note: $L = \{ w \mid w \in \{a,b\}^*, \#_a(w) = k_1 n + k_2, n \geq 0 \}$

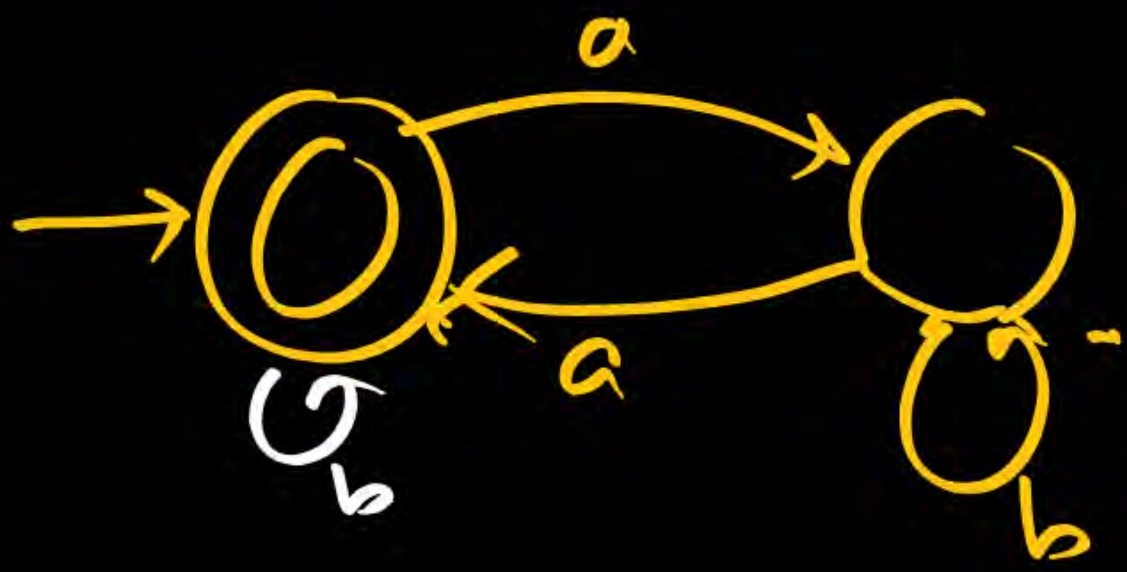
I) If $\underbrace{\boxed{K_1} > K_2}_{\text{K_2 is remainder}} \Longrightarrow K_1$ states

II) If $\underbrace{K_1 \leq \boxed{K_2}}_{\text{K_2 is not remainder}} \Longrightarrow K_2 + 1$ states

Model-V [No. of symbols & Remainder]

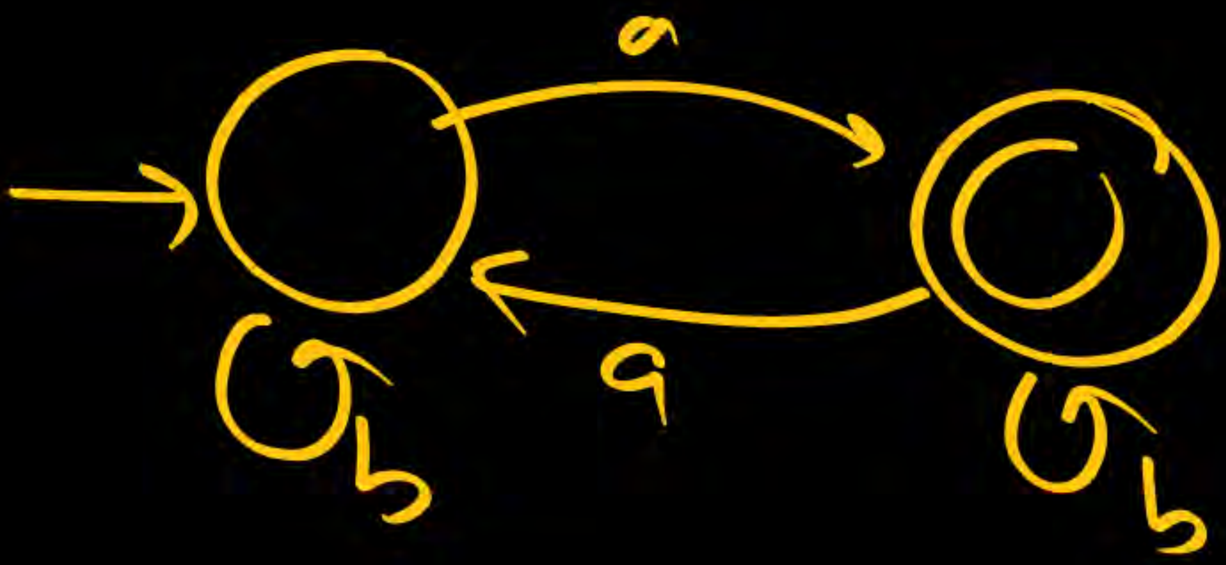
1. $\{w \mid w \in \{a,b\}^*, \ n_a(w) = \text{even}\} \Rightarrow 2$ states
2. $\{w \mid \text{"} , \ n_a(w) = \text{odd}\}$
3. $\{w \mid \text{"} , \ n_a(w) \text{ is div by } 3\} \Rightarrow 3$ states
4. $\{w \mid \text{"} , \ n_a(w) \text{ is not div by } 3\} \Rightarrow 3$ states
5. $\{w \mid \text{"} , \ n_a(w) = 100n + \underbrace{53}_{\text{rem}}, \ n \geq 0\} \Rightarrow 100$ states
6. $\{w \mid \text{"} , \ n_a(w) = 123n + \underbrace{1125}_{\text{not rem}}, \ n \geq 0\} \Rightarrow 1126$ states

1



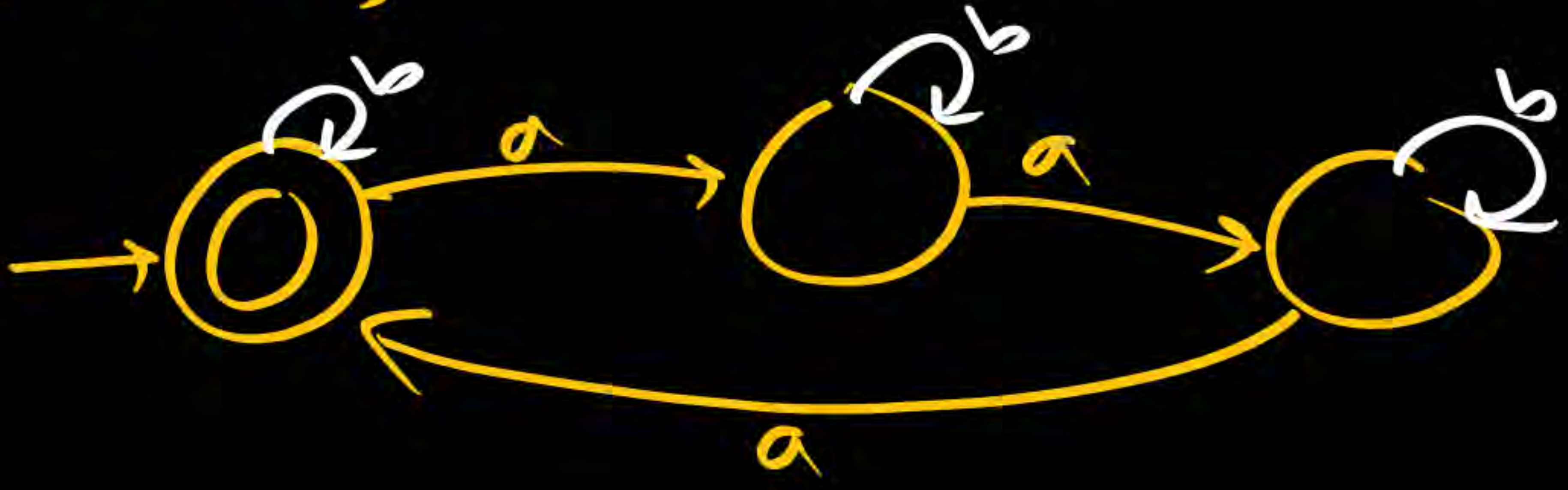
#a's = even

2

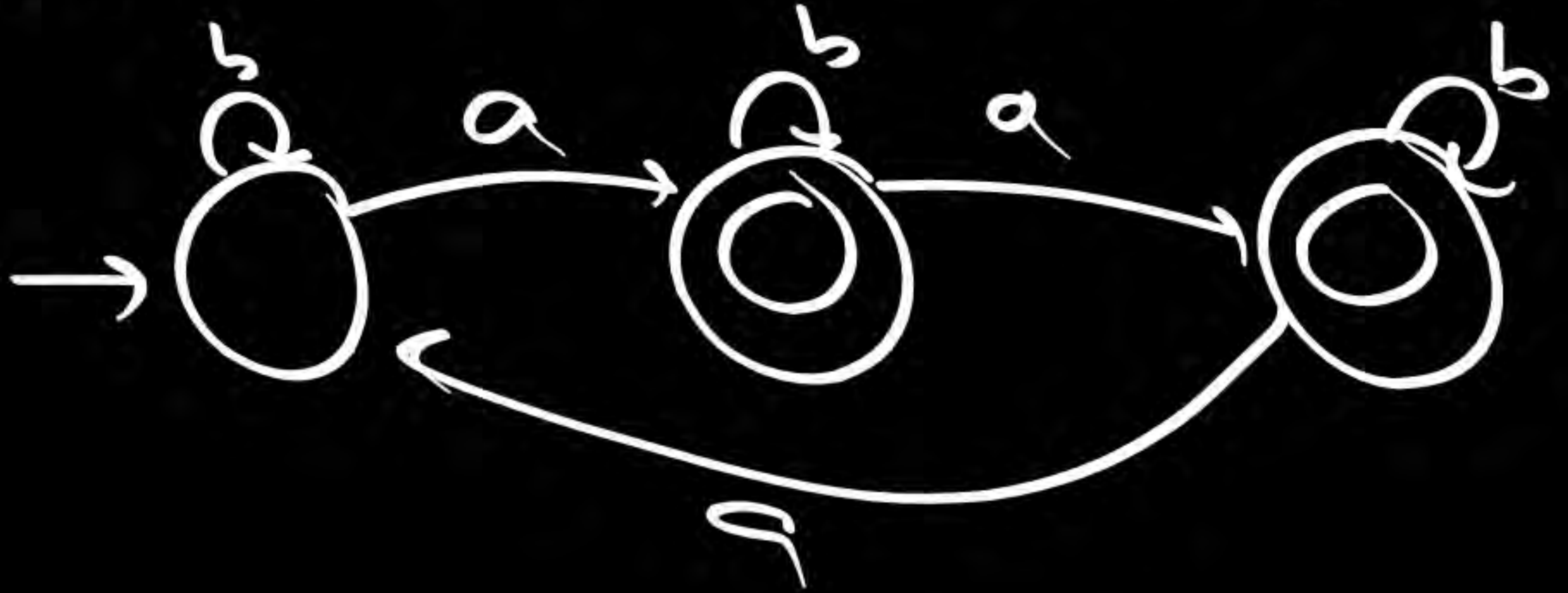


#a's = odd

(3) #a's is div by 3



(4) #a's is not divisible by 3



$[\#_a(\omega)\%3] \neq 0$

$= 1$

$= 2$

Model-6 : [Sequence]

① $a^+ b^+$

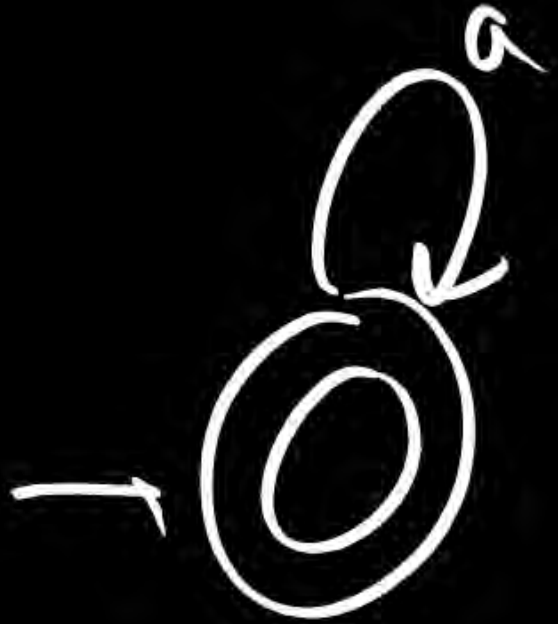
② $a^+ b^*$

③ $a^* b^+$

④ $a^* b^*$

After a's, b's will come

a's followed by b's



$\Rightarrow a^*$

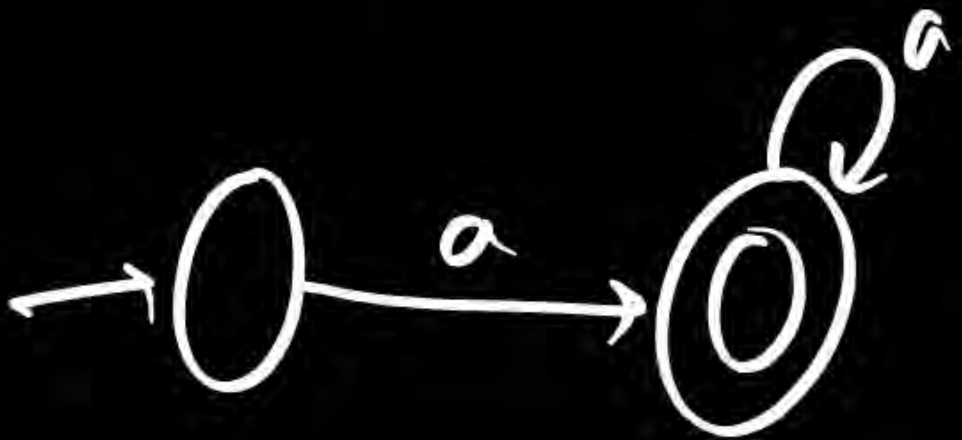
Zero or more a's

ϵ ✓

a ✓

a^2 ✓

⋮



$\Rightarrow a^+$

One or more a's

a ✓

a^2 ✓

a^3 ✓

⋮

$aa^* = a^+$

① $L = \{a^m b^n \mid m \geq 1, n \geq 1\} = a^+ b^+$

4 states

$= \{\underbrace{ab}_{\text{min}}, aab, abb, \dots\}$

$= aa^*bb^*$

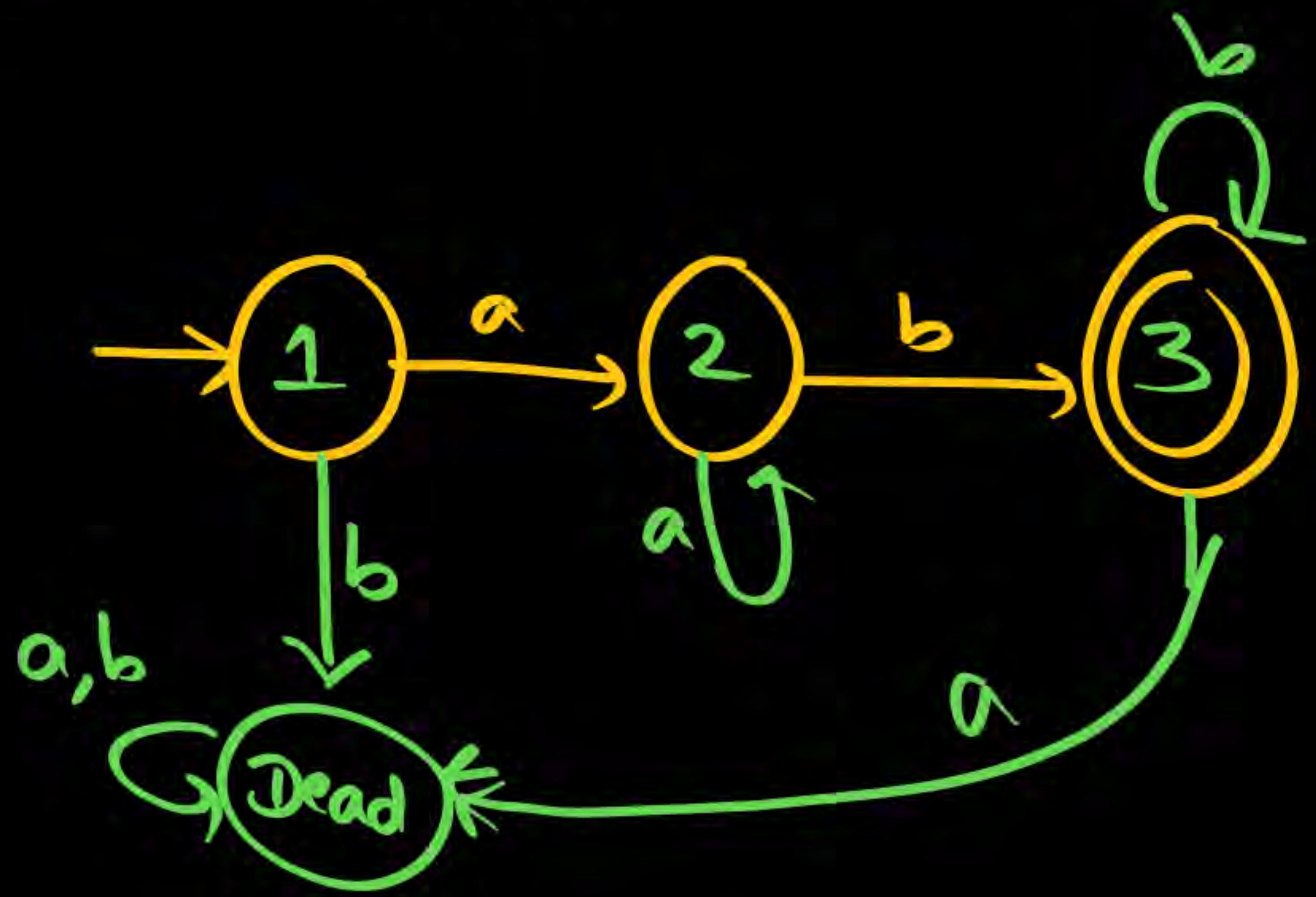
$= a^*abb^*$

$= a^*ab^*b$

$= (a^+)^+(b^+)^+$

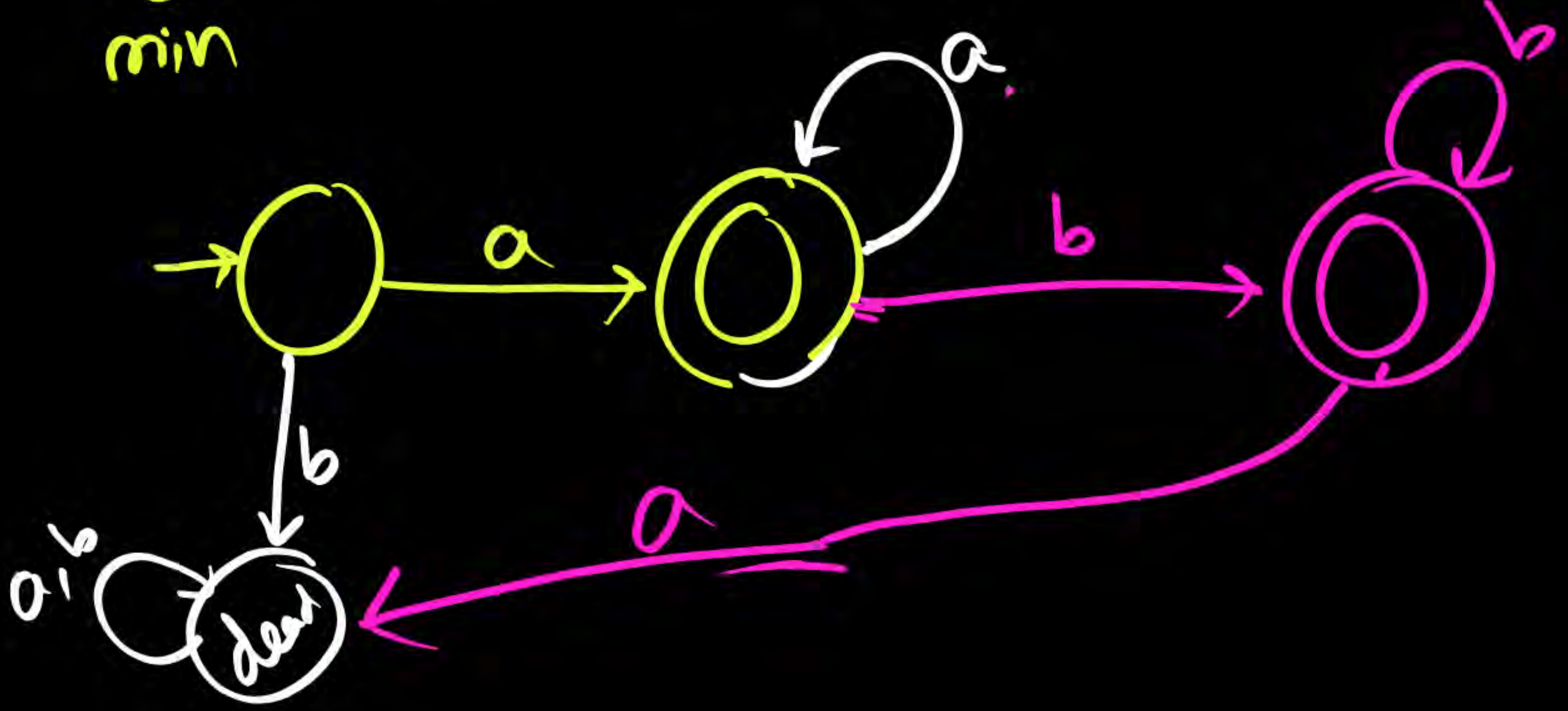
a's will never appear after b's

b's will never before a's

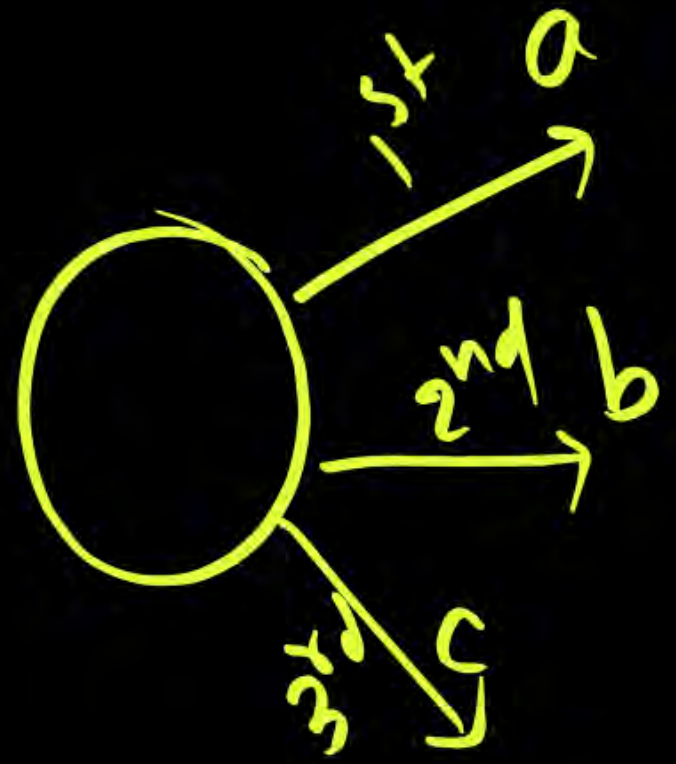


② $\{a^m b^n \mid m \geq 1, n \geq 0\} = a^+ b^*$

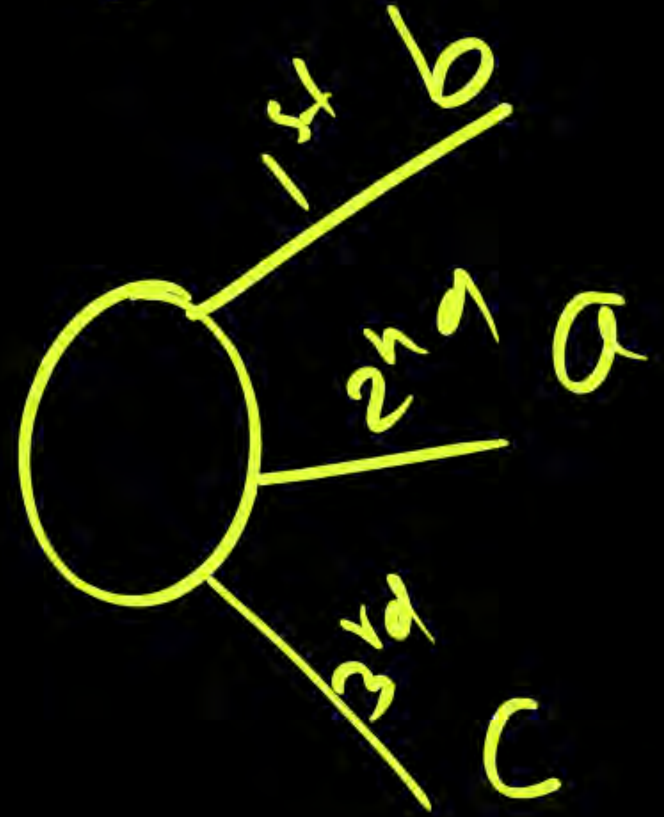
$= \{\underbrace{a}_{\text{min}}, aa, ab, aab, abb, \ldots\}$



$a^* \; b^* \; c^*$



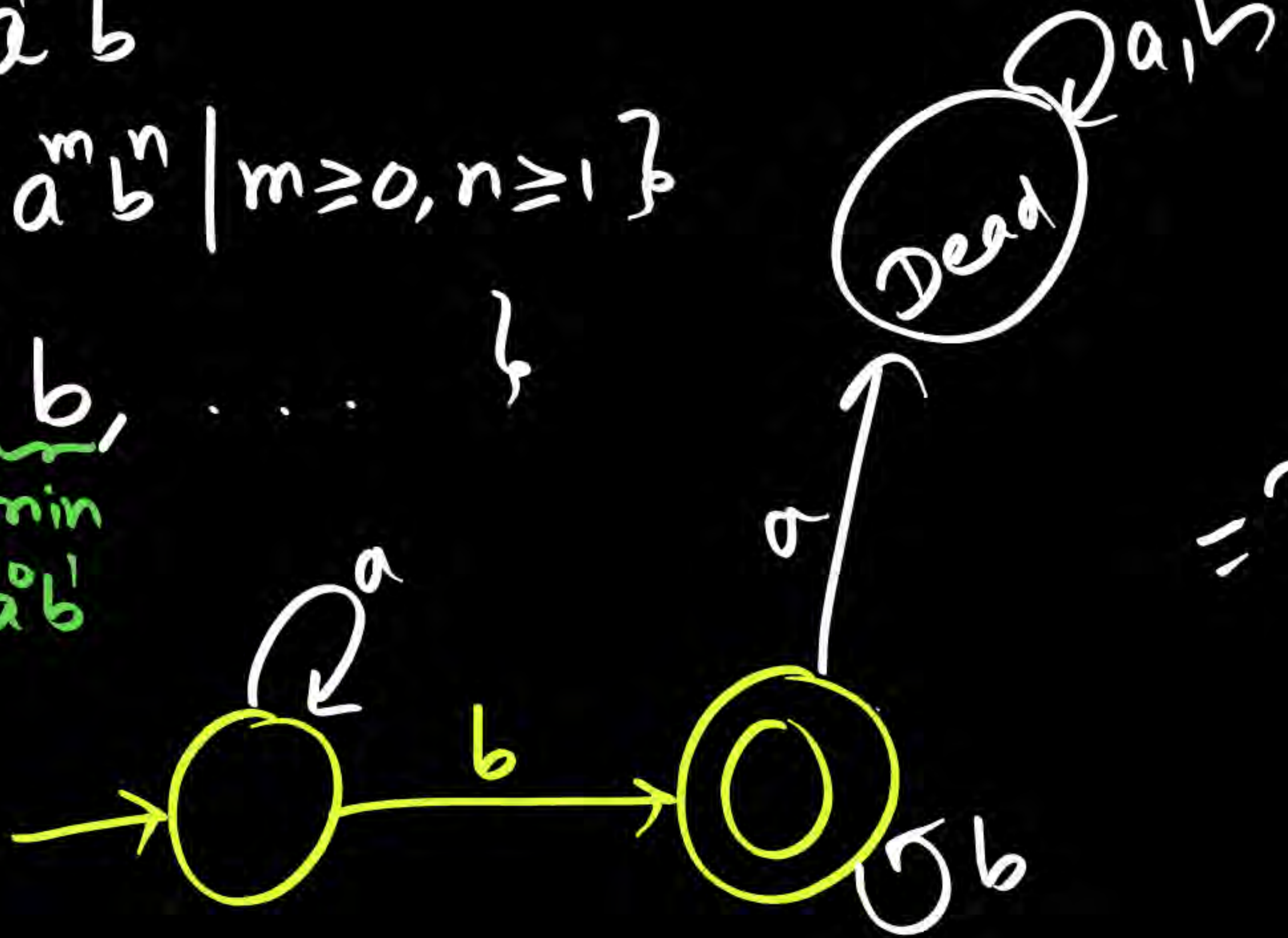
$\underline{b}^* \; \underline{a}^* \; c^*$



③ $L = a^* b^+$

$= \{ a^m b^n \mid m \geq 0, n \geq 1 \}$

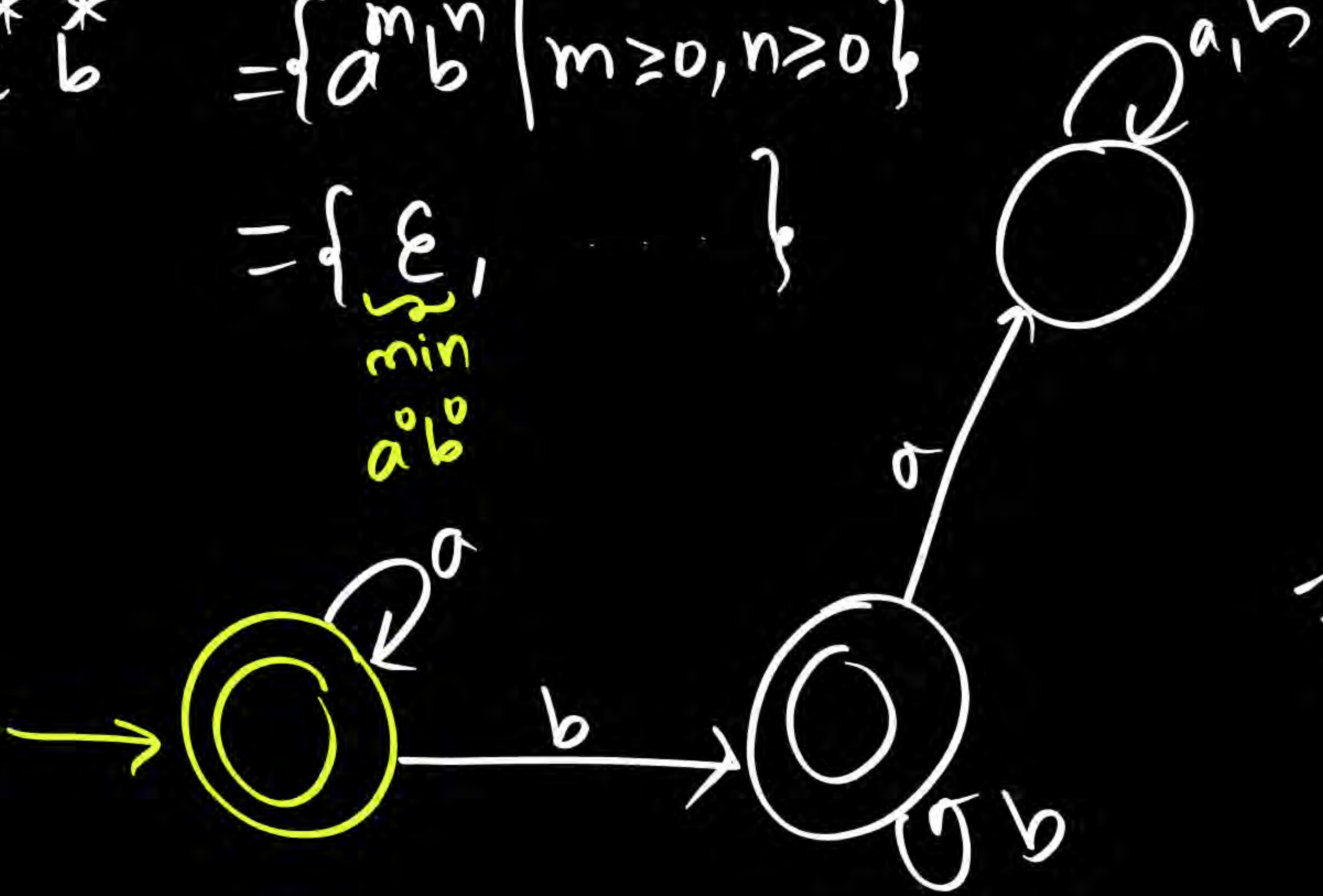
$= \{ \underbrace{b}_{\substack{\text{min} \\ a^0 b^1}}, \ \ldots \ \}$



$= 3$ states

④ $a^* b^* = \{a^m b^n \mid m \geq 0, n \geq 0\}$

$= \{\underbrace{\varepsilon}_{\substack{\text{min} \\ a^0 b^0}}, \ldots\ldots \}$



$= 3$ states

H.W.

(5) $L = b^* a^*$

(6) $L = b^+ a^*$

(7) $L = b^* a^+$

(8) $L = b^* a^*$

$\Sigma = \{a, b\}$

(9) $L = a^+ b^+ c^+$

(10) $L = b^+ a^+ c^+$

(11) $L = c^+ a^+ b^+$

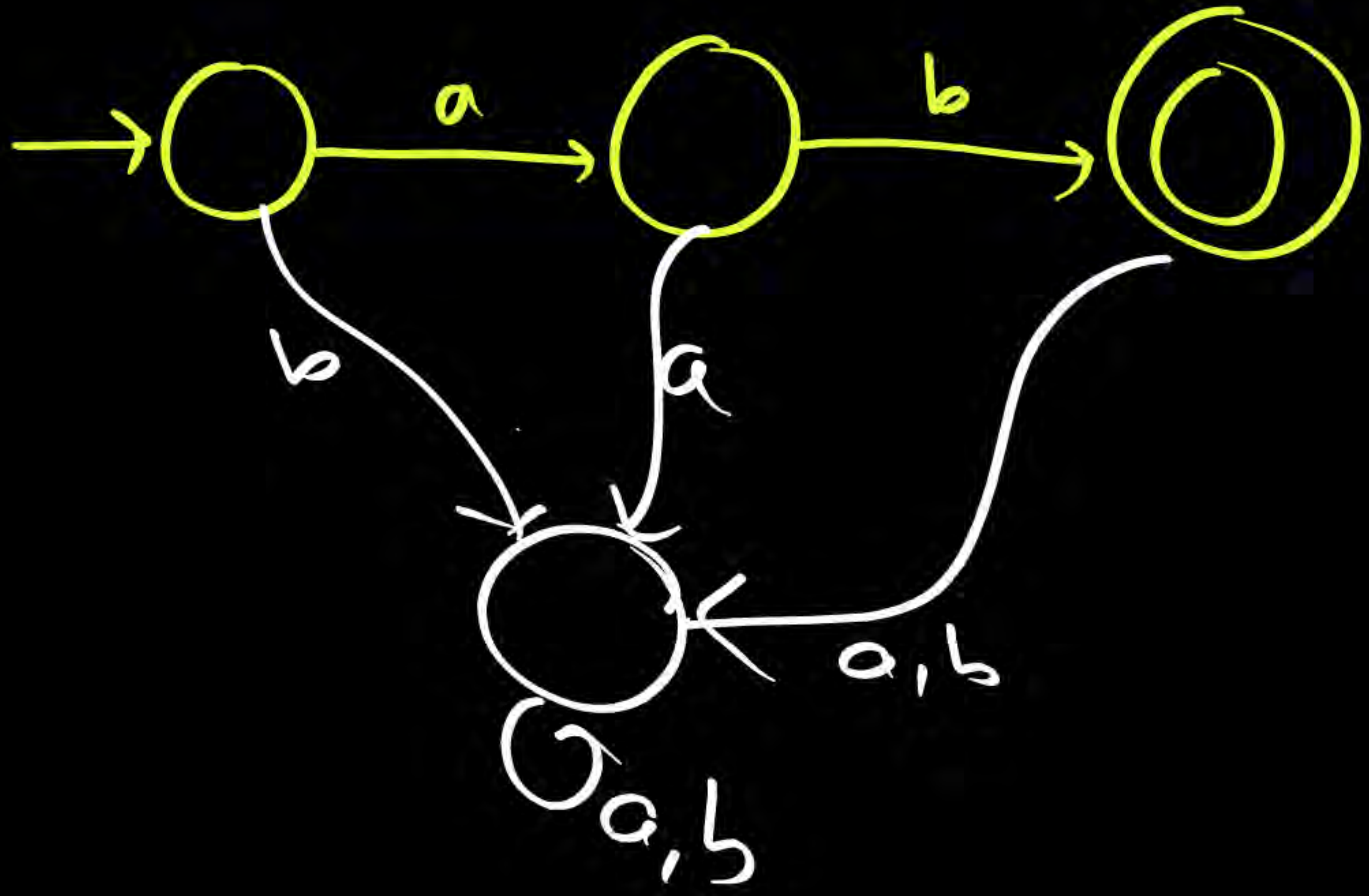
$\Sigma = \{a, b, c\}$

Model-7 : [Sequence, No. of symbols]

1. $\{a^m b^n \mid m=1, n=1\}$
2. $\{a^m b^n \mid m=1, n\geq 1\}$
3. $\{a^m b^n \mid m\geq 1, n\geq 1\}$
4. $\{a^m b^n \mid m\leq 1, n=1\}$
5. $\{a^m b^n \mid m\leq 1, n\leq 1\}$
6. $\{a^m b^n \mid m=\text{even}, n=1\}$
7. $\{a^m b^n \mid m=\text{odd}, n=1\}$
8. $\{a^m b^n \mid m=1, n=\text{even}\}$
9. $\{a^m b^n \mid m=\text{even}, n=\text{even}\}$
10. $\{a^m b^n \mid m=\text{even}, n=\text{odd}\}$

H.W.

① $\{a^m b^n \mid m=1, n=1\} = \{ab\}$



② $\{a^m b^n \mid m=1, n\geq 1\} = \{\underbrace{ab}_{\text{min}}, \ldots\}$

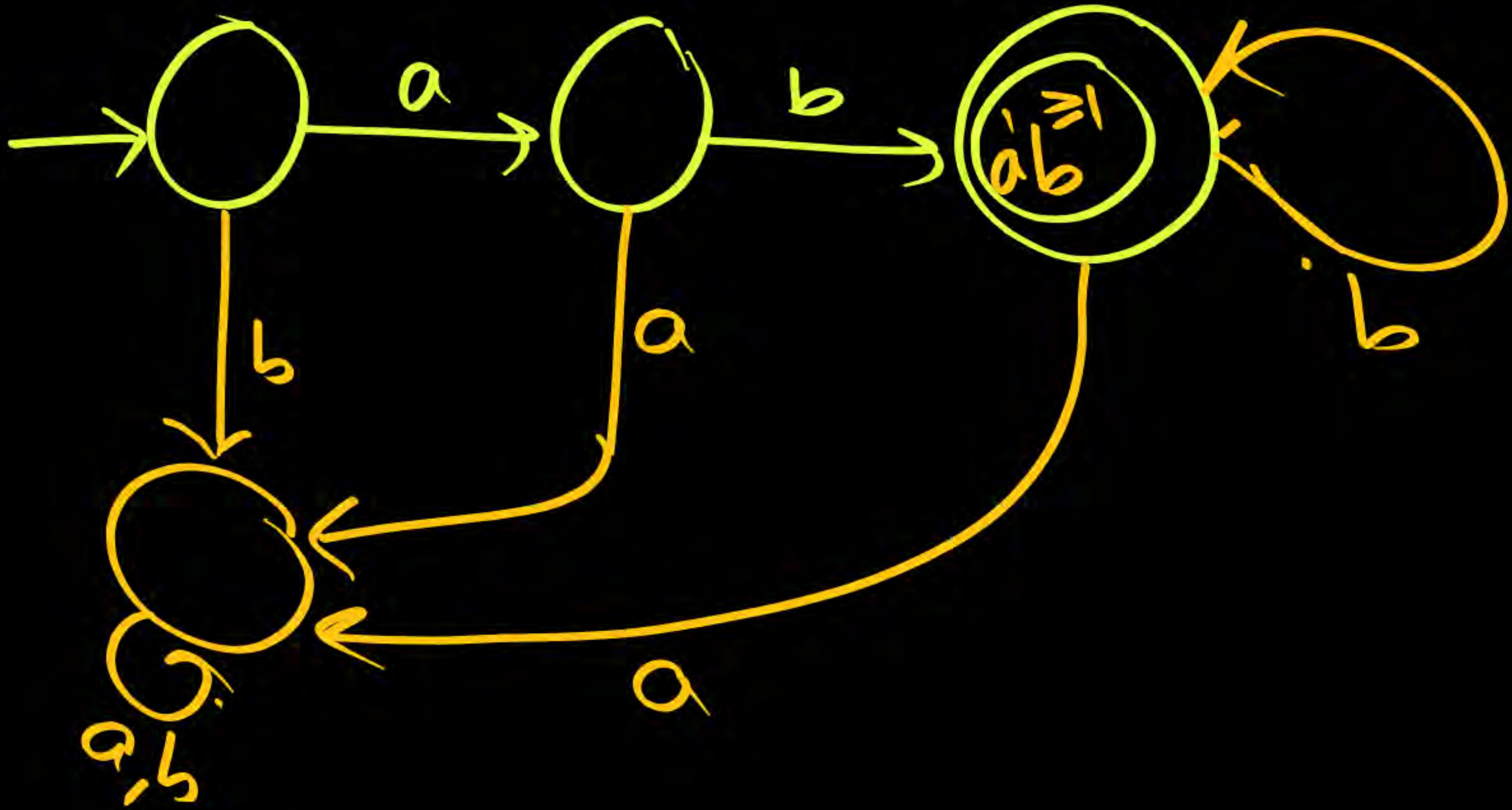
$$\boxed{L = ab^+}$$

$\cancel{\epsilon}$

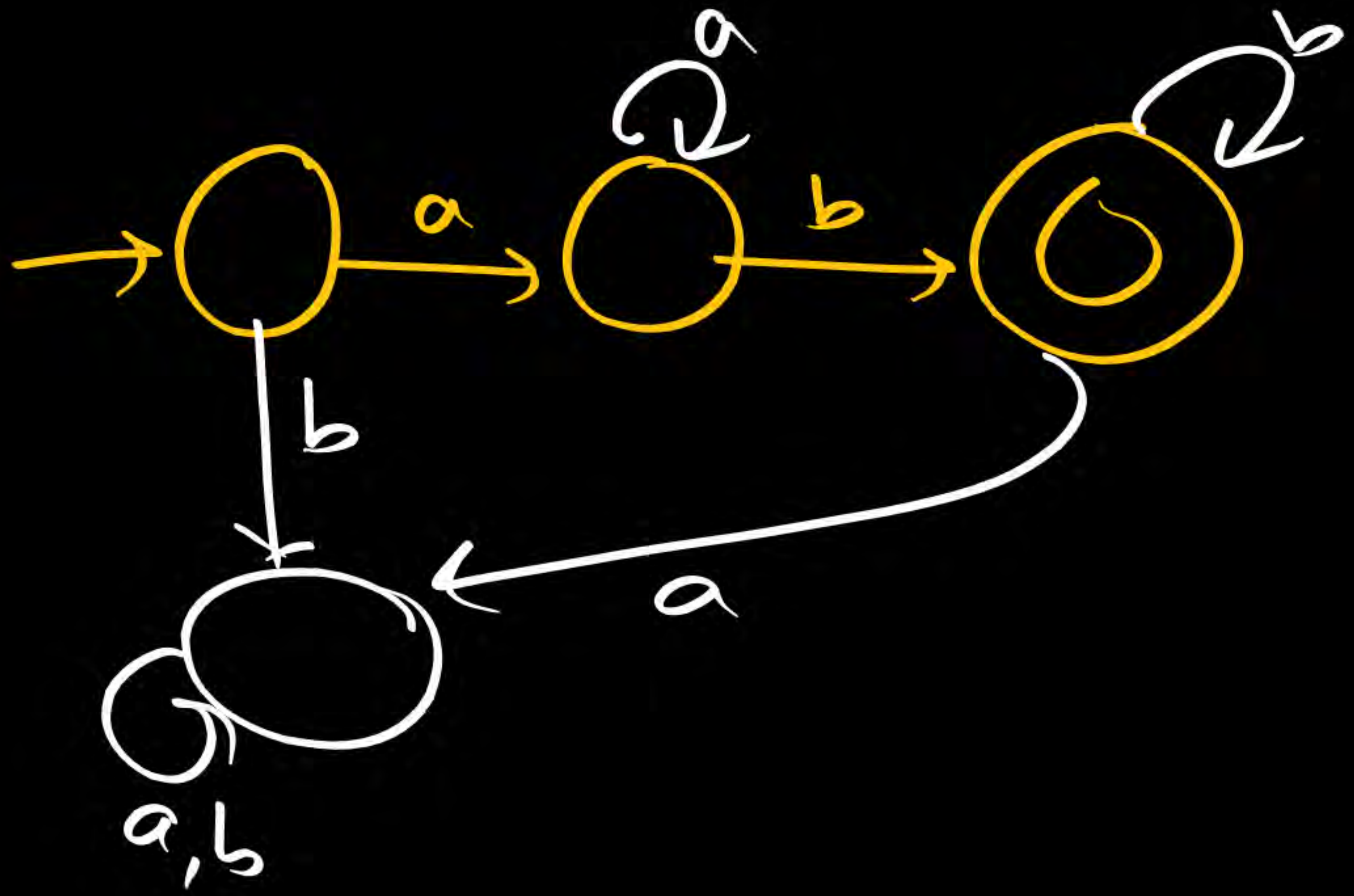
$\cancel{a}$

$\cancel{b}$

ab ✓

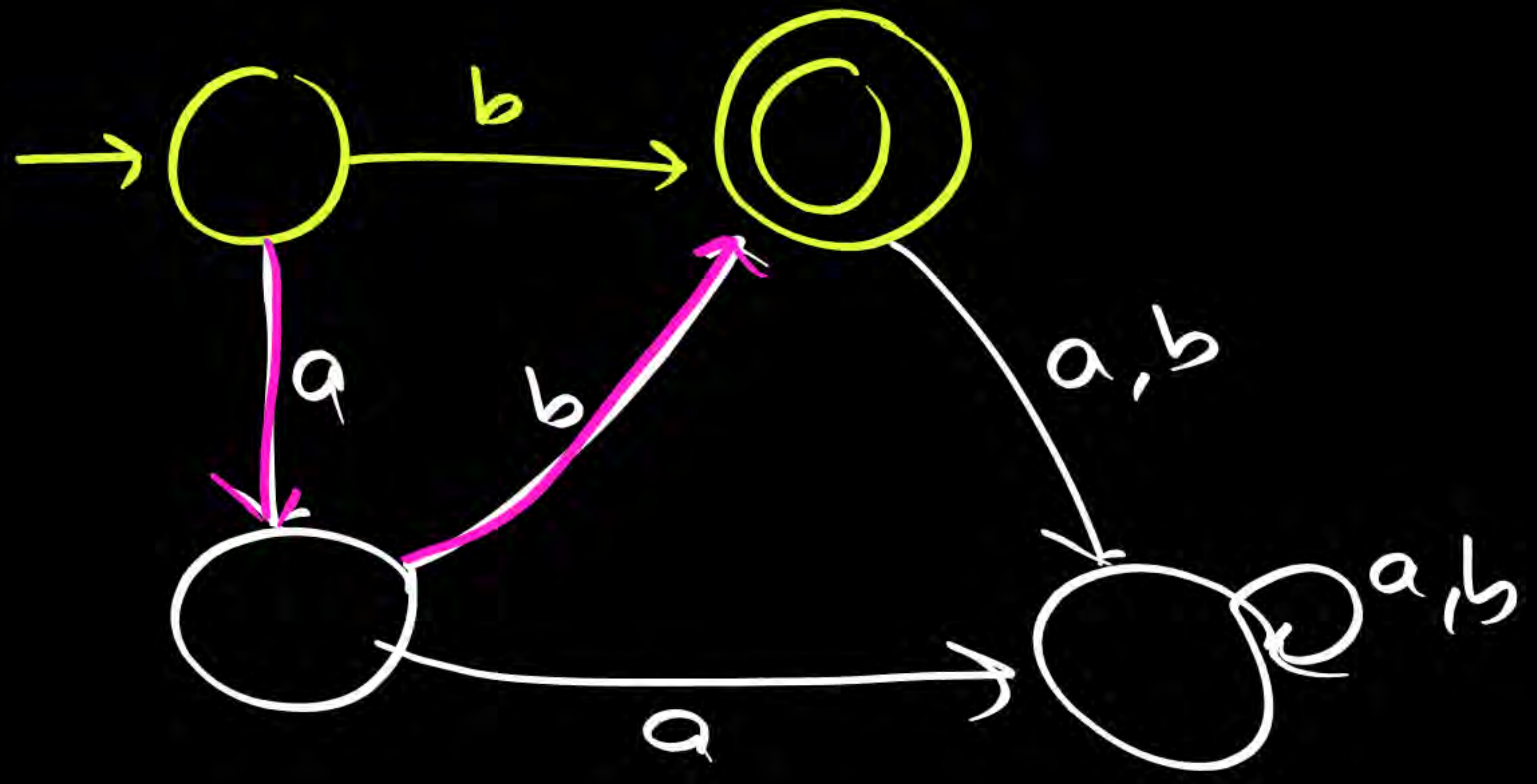


③ $L = \{a^m b^n \mid m \geq 1, n \geq 1\} = a^+ b^+$



④ $\{a^m b^n \mid m \le 1, n = 1\} = (\varepsilon + a)b = b + ab$

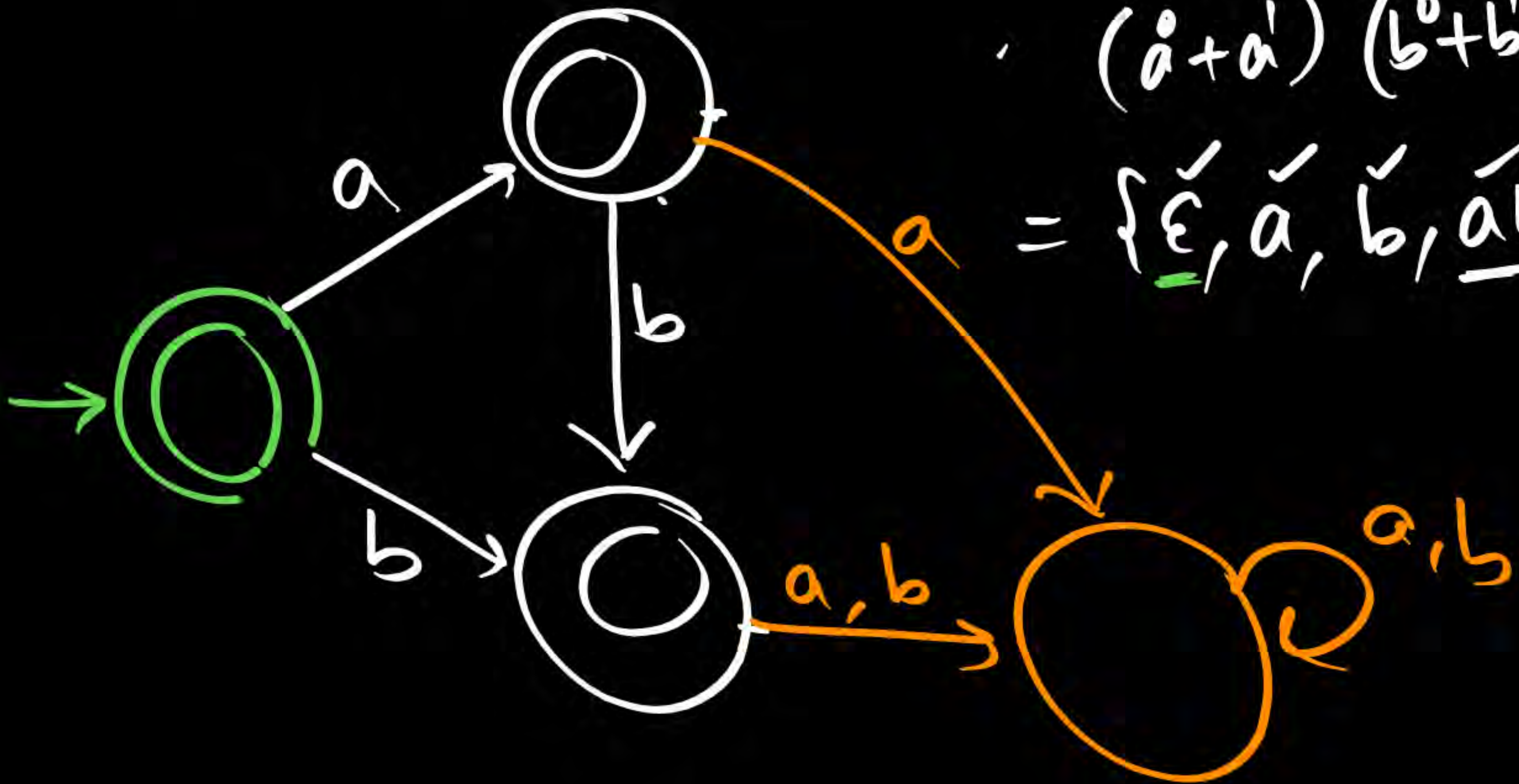
$= 4$ states



⑤ $\{a^m b^n \mid m \le 1, n \le 1\} = \underbrace{(\varepsilon + a)}_{}(\varepsilon + b)$

$(a^0 + a^1)(b^0 + b^1)$

$= \{\varepsilon, a, b, ab\}$



Model-VIII [Multiple conditions, No. of symbols]

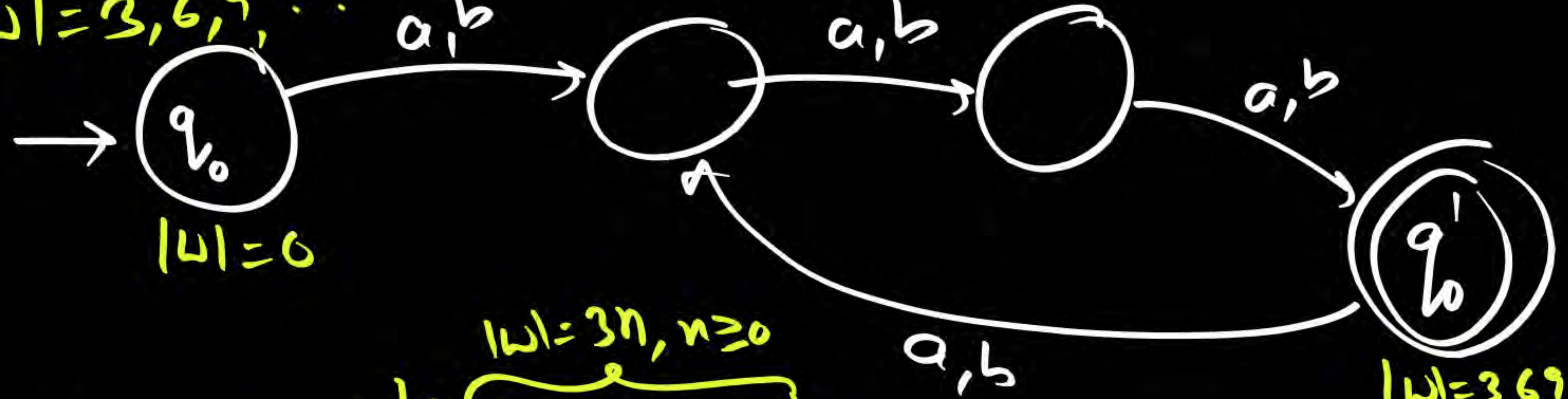
1. $\{w \mid w \in \{a,b\}^*,\ n_a(w) = 1,\ n_b(w) = 2\}$
2. $\{w \mid$ " $,\ n_a(w) \geq 1,\ n_b(w) \geq 2\}$
3. $\{w \mid$ " $,\ n_a(w) \leq 1,\ n_b(w) = 2\}$
4. $\{w \mid$ " $,\ n_a(w) \leq 1,\ n_b(w) \leq 2\}$
5. $\{w \mid$ " $,\ n_a(w) \geq 1,\ n_b(w) \leq 2\}$

I) $\{w \mid w \in \{a,b\}^{\boxed{+}},\ \underbrace{|w| \bmod 3 = 0}\}$

$\Rightarrow |w| = \underline{3n+3}_{\,n \geq 0}$

$= \{aaa, aab, \ldots\}$

$|w| = 3, 6, 9, \ldots$



$|w| = 0$

$|w| = 3, 6, 9, \ldots$

II) $\{w \mid w \in \{a,b\}^{*},\ \overbrace{|w| \% 3 = 0}^{|w| = 3n,\ n \geq 0}\}$

$= \{\varepsilon, aaa, aab, \ldots, aaaaaa, aaaaab, \ldots\}$

$|w| = 0, 3, 6, 9, \ldots$

$|w| = \boxed{3n+5}$, $n \geq 0$ ⟹ min 5 length string

5+1 states

$\boxed{|w| = 3n+6}$, $n \geq 0$ ⟹ min 6 length string

6+1 States

$$L = \underbrace{\Sigma^*}_{\text{Infinite}} \Rightarrow \overline{L} = \underbrace{\phi}_{\text{finite}}$$

$$L = \underbrace{a\Sigma^*}_{\text{Inf}} \Rightarrow \overline{L} = \underbrace{\epsilon + b\Sigma^*}_{\text{Inf}}$$


THANK YOU
GW
SOLDIERS !