CS & IT ENGINEERING

Theory of Computation

Finite Automata

Lecture No. 4

By- DEVA Sir

TOPICS TO BE COVERED

01 Practice Questions

02 GATE PYQs

03 Model GATE Questions

04 Important Regular Exps

05 Revision

Practice Questions

How to write Regular Expressions?

$$\boxed{\begin{array}{c}\text{Regular Language} \\ \cong \\ \text{Regular Expression}\end{array}}$$

1. $L = \{\ \}$ over $\Sigma = \{a\}$

$$\boxed{R = \phi} = \phi . a = \phi . a^* \ \ldots$$

2. $L = \{\ \}$ over $\Sigma = \{a, b\}$

$$\boxed{R = \phi} = b . \phi . a = \phi . a^* b^* \ \ldots$$

3. $L = \{\ \}$ over $\Sigma = \{a, b, c\}$

$$\boxed{R = \phi} = \phi . (c+a) = \phi . (a+b+c)^* \ \ldots$$

Note: To represent one regular language,
How many regular expressions?

= Infinite

$\Sigma = \{a\}$

$L = \{\ \} \Longrightarrow \left\{ \begin{array}{l} \phi \\ \phi \cdot a \\ \phi \cdot a^* \\ \phi \cdot \text{any} \end{array} \right\} \left\{ \begin{array}{l} a \cdot \phi \\ aa \cdot \phi \\ a^3 \cdot \phi \\ a^* \cdot \phi \cdot a^+ \end{array} \right.$

④ $L = \{\varepsilon\}$ over $\Sigma = \{a\}$

$$\boxed{R = \varepsilon} = \phi^* = \phi^* \cdot a^0$$

⑤ $L = \{\varepsilon\}$ over $\Sigma = \{a, b\}$

$$\boxed{R = \varepsilon} = \phi^0 = b^0 = a^0$$

$$= (a+b)^0 = (ab)^0$$

⑥ L = Set of all strings over $\Sigma = \{a\}$

$$\boxed{R = a^* = \Sigma^*}$$

⑦ L = Set of all strings over $\Sigma = \{a, b\}$

$$\boxed{R = (a+b)^* = \Sigma^*}$$

$$a^* = (a^*)^2 = (a^*)^* = (a^*)^+ = (a^+)^*$$

$$= a^+ + \varepsilon$$

$$= (a+\varepsilon)^+$$

$$= (a+\varepsilon)^*$$

$$= \phi^* \cdot a^*$$

$= a^* +$ any exp over Σ

$$(\underline{a+b})^* = (a+b)^+ + \varepsilon$$

$$= (a+b+\varepsilon)^+$$

$$= (a^* + b^*)^*$$

$$= (a^* b^*)^*$$

$$= (b^* a^*)^*$$

Later

$$\boxed{a^* b^*} \neq \boxed{b^* a^*}$$

$$(a^* b^*)^* = (b^* a^*)^*$$

Difference?

$$a^* b^* = \varepsilon + a + a^2 + \dots + b + b^2 + \dots + ab + aab + abb + \dots$$

$$b^* a^* = \boxed{\varepsilon} + \boxed{a^+} + \boxed{b^+} + \boxed{a^+ b^+}$$

$$b^0 a^0 \qquad b^0 a^+ \qquad b^+ a^0$$

$$(a^* b^*)^* = (a^* b^*)^0 + (a^* b^*)^1 + (a^* b^*)^2 + \dots$$

$\varepsilon \quad b$

$a \quad \varepsilon$

$$= \varepsilon + a^* b^* + a^* b^* a^* b^* + \dots$$

$\varepsilon \quad b^1 \quad a^1 \quad \varepsilon$

(8) $L = \{ w \mid w \in \{a, b\}^*, \ w \text{ starts with } a \}$

$= \{ aw \mid w \in \{a, b\}^* \}$

= Set of all strings starting with a

$a\ \cancel{\ }$

$R = a(a+b)^*$

$= a\Sigma^*$

$= a(a^* b^*)^*$

$L = \{ w \mid \underbrace{w \in \{a,b\}^*}_{w \in \Sigma^*},\ w \text{ starts with } a \}$

$= \{ a, aa, ab, a\boxed{a}\boxed{a}, aab, aba, abb, \ldots \}$

$A = \{ x \mid \underbrace{x \in \mathbb{N}}_{\text{cond 1}}, \underbrace{x > 2}_{\text{cond 2}} \}$

Every x which is natural and greater than 2

$= \{ 3, 4, 5, 6, 7, \ldots \}$

(9) Set of all strings starting with ab over $\Sigma = \{a, b\}$

$$R = ab.(a+b)^*$$

(10) $L = \{ w \mid w \in \{a, b\}^*, \text{ w starts with aaa} \}$

$$R = aaa(a+b)^*$$

(11) $L = \{ w \mid w \in \{a, b\}^*, \text{ w ends with a} \}$

$$R = (a+b)^* a$$

Match Equivalent

12 Group - I

Group - II

1. $a(a+b)^*$ — I. $(ab^*)^+ = \{a, aa, ab, \ldots\}$

2. $b(a+b)^*$ — II. $(b^*a)^+ = (a+b)^*a$

3. $(a+b)^*a$ — III. $(ba^*)^+ = b(a+b)^*$

4. $(a+b)^*b$ — IV. $(a^*b)^+ = (a+b)^*b$

$a^0 = \varepsilon$

$a^* \quad a^1 = a$

$a^2 = aa$

$(ab^*)^+$

$(\underbrace{ab^*}_{b^0})(\underbrace{ab^*}_{b^0})$

$ab^0 \quad ab^0 \rightarrow aa$

(13) $L = \{ w_1 \, a \, w_2 \mid w_1, w_2 \in \{a,b\}^* \}$

= Set of all strings containing 'a' as substring

$= \{a, aa, ab, ba, \ldots \}$

$$= (a+b)^* \, a \, (a+b)^*$$

same

$$= b^* \, \underline{a} \, (a+b)^*$$

(14) $L = \{ w \mid w \in \{a,b\}^*, \underbrace{|w| = 2}_{} \}$

Exactly 2 length strings

$\Sigma = \{a, b\}$
$= a+b$

$R = ab + ba + aa + bb$

$= \underbrace{(a+b)}_{1} \cdot \underbrace{(a+b)}_{1}$

$= (a+b)^2 = \Sigma^2$

$xx = x^2$

$(a+b) \cdot (a+b) = (a+b)^2$

$\Sigma \cdot \Sigma = \Sigma^2$

Exactly k length

$= (a+b)^k$

(15) $\{w \mid w \in \{a,b\}^*,\ \underbrace{|w| \le 2}_{\text{Atmost 2 length strings}}\}$

$$= \varepsilon + (a+b) + (a+b)^2$$

$$R = \varepsilon + a + b + aa + ab + ba + bb$$

$$= \Sigma^0 + \Sigma^1 + \Sigma^2$$

$$= (\varepsilon + a + b)^2$$

$|w| \le K$ constant

$= (a+b+\varepsilon)^K$

Strings	Length
ε	0
$a+b$	1
$a+b+\varepsilon$	0 or 1

≤ 1

atmost 1 length

b	b
b	a
a	b
a	a

$(a+b+\varepsilon)\cdot(a+b+\varepsilon) = (a+b+\varepsilon)^2$

≤ 2 length

$\varepsilon \cdot \varepsilon \rightarrow \varepsilon$

$\varepsilon \cdot a \rightarrow a$

$a \cdot \varepsilon \rightarrow a$

$\varepsilon \cdot b \rightarrow b$

$b \cdot \varepsilon \rightarrow b$

Note:

$$\underbrace{a^*b^*}_{} + \underbrace{b^*a^*}_{} = \underbrace{\varepsilon + a^+ + b^+ + a^+b^+ + b^+a^+}_{}$$

a^*b^*:

ε

a^+

b^+

a^+b^+

b^*a^*:

ε

a^+

b^+

b^+a^+

$$= a^* + b^* + a^+b^+ + b^+a^+$$

$$= a^* + b^+ + a^+b^+ + b^+a^+$$

$$= a^+ + b^* + a^+b^+ + b^+a^+$$

(16) $\{w \mid w \in \{a,b\}^*,\ \underbrace{|w| \geq 2}_{\text{Atleast 2 length}}\}$

$$x^+x^+ = xx^+$$
$$= xxx^*$$
$$= xx^*x$$
$$= x^*xx$$
$$= (x^+)^2$$

$$= \underbrace{\underbrace{(a+b)^2}_{\text{exactly 2 length}}\ \underbrace{(a+b)^*}_{\text{any length}}}_{\text{atleast 2 length}}$$

$$= (a+b)^*(a+b)^2$$
$$= (a+b)(a+b)(a+b)^*$$
$$= (a+b)(a+b)^+$$
$$= (a+b)^+(a+b)$$

(17) $\{w \mid w \in \{a,b\}^*,\ n_a(w) = 2\}$

$= \underbrace{b^* a}_{\text{must}} \underbrace{b^* a}_{\text{must}} b^*$

$= (b^* a)^2 b^*$

$= b^* (a b^*)^2$

$n_a(w)$

$\#_a(w)$

no. of a's(w)

$$b^* \rightarrow \begin{cases} \rightarrow \varepsilon \\ \rightarrow b^+ \end{cases}$$

Zero b's
or
more b's

18) $\{w \mid w \in \{a,b\}^*,\ n_a(w) \leq 2\}$

$= b^* + b^*ab^* + b^*\underline{a}b^*\underline{a}b^*$

$= b^*(\varepsilon+a)\,b^*\,(\varepsilon+a)\,b^*$

19) $\{w \mid w \in \{a,b\}^*,\ n_a(w) \geq 2\}$

$= (a+b)^*\,\underline{a}\,(a+b)^*\,\underline{a}\,(a+b)^*$

$X\,a\,X\,a\,X$

zero a's $\Rightarrow b^*$

one a $\Rightarrow b^*ab^*$

2 a's $\Rightarrow b^*ab^*ab^*$

3 a's $\Rightarrow b^*ab^*ab^*ab^*$

$$= \underbrace{b^*}_{=\text{zero a's}} + \underbrace{b^*ab^*}_{=1a} + \underbrace{b^*ab^*ab^*}_{=2a's}$$

$$= \boxed{b^*(\varepsilon+a)\,b^*\,(\varepsilon+a)\,b^*}$$

$b^*\ \varepsilon\ b^*\ \varepsilon\ b^* \longrightarrow b^*$

$b^*\ \varepsilon\ b^*\ a\ b^* \longrightarrow b^*ab^*$

$b^*\ a\ b^*\ \varepsilon\ b^* \longrightarrow b^*ab^*$

$b^*\ a\ b^*\ a\ b^* \longrightarrow b^*ab^*ab^*$

19

$$= (a+b)^* \boxed{a} (a+b)^* \boxed{a} (a+b)^*$$

$$= b^* \boxed{a} b^* \boxed{a} (a+b)^*$$

$$= (a+b)^* \boxed{a} b^* \boxed{a} b^*$$

$$= b^* \boxed{a} (a+b)^* \boxed{a} b^*$$

Equivalent

Atmost 2 a's

$\Sigma = \{a, b\}$

$\varepsilon + a$

zero a

1 a

almost 1 a

$(\varepsilon + a)\ (\varepsilon + a)$

$\Sigma = \{a, b\}$

$b^*(\varepsilon + a)\ b^*\ (\varepsilon + a)\ b^*$

(20) $\{w \mid w \in \{a,b\}^*, \underbrace{|w| \text{ is even}}\}$

|w| is divisible by 2

|w| is multiple of 2

$(|w| \% 2) = 0$

$|w| = 2n$ $n \geq 0$

$= \left[(a+b)^2\right]^*$

(21) $\{w \mid w \in \{a,b\}^*, \underbrace{|w| \text{ is odd}}_{|w| = 2n+1}\}$

$= \underbrace{\underbrace{\left[(a+b)^2\right]^*}_{\text{even}} \underbrace{(a+b)}_{1}}_{\text{odd}} = (a+b)\left[(a+b)^2\right]^*$

$$\left((a+b)^2\right)^* \rightarrow \begin{cases} (\ \ )^0 = \varepsilon \\ (\ \ )^1 = (a+b)^2 \\ (\ \ )^2 = \left[(a+b)^2\right]^2 = (a+b)^4 \\ (\ \ )^3 = (a+b)^6 \end{cases}$$

Even length strings

$$= \{ \underbrace{\varepsilon}_{0}, \underbrace{aa, ab, ba, bb}_{2}, \underbrace{aaaa, aaab, aaba, \ldots}_{4} \ldots \}$$

(22) $L = \{ \underset{\text{3 len}}{x}\, a\, \underset{\text{any}}{y} \mid x, y \in \{a,b\}^*, |x| = 3 \}$

$= \boxed{4^{\text{th}} \text{ symbol from begin is } a}$

$= \underbrace{(a+b)^3}_{\text{3 length}}\ \underbrace{a}_{4^{\text{th}}\text{ symbol}}\ (a+b)^*$

$= \square\square\square\ \underset{4^{\text{th}}}{a}\ \underbrace{\qquad}_{}\text{any}$

23. $\{ x a y \mid x, y \in \{a,b\}^*, |y| = 2 \}$

x: any y: 2 len

$= (a+b)^* \boxed{a} (a+b)^2$

3rd from end is a

$= (a+b)^* \, a \, (a+b) \, (a+b)$

$\xleftrightarrow{\text{any}}$ a $\square$ $\square$

(24) $\{a^m b^n \mid m, n \geq 0\}$

$$\boxed{R = a^* b^*}$$

(25) $\{a^m b^n \mid m \geq 0, n \geq 1\}$

$$\boxed{R = a^* b^+}$$

(26) $\{a^m b^n \mid m \geq 1, n \geq 1\}$

$$\boxed{R = a^+ b^+}$$

27) $\{a^m b^n \mid m = \text{even}, n \geq 0\} \implies R = (aa)^* b^*$

28) $\{a^m b^n \mid m = \text{odd}, n \geq 1\} \implies R = a(aa)^* b^+$

29) $\{a^m b^n \mid m \geq 2, n = \text{even}\} \implies R = aaa^* (bb)^*$

30) $\{a^m b^n \mid m \geq 2, n \geq 3\} \implies R = aaa^* bbbb^*$

31) $\{a^m b^n \mid m \leq 1, n \geq 2\} \implies R = \underbrace{(\epsilon + a)}_{\text{max } 1a} \underbrace{bbb^*}_{\text{min } 2 \text{ b's}}$

$a^{\leq 2} \Rightarrow \underbrace{\varepsilon + a + a^2}_{\text{atmost } 2a\text{'s}}$

$a^{\text{even}} = (aa)^* = (a^2)^*$

$a^{\text{odd}} = a(aa)^*, = (aa)^* a$

$a^{\geq 2} = aaa^*$

$a^{\leq 1} = a^0 + a^1 = \underbrace{\varepsilon + a}_{\text{atmost } 1a}$

*** Home work

32) $\{w \mid w \in \{a,b\}^*, \ n_a(w) = odd\}$

$= b^*(b^*ab^*ab^*)^*b^*ab^*$

$= b^*(ab^*ab^*)^*ab^*$

$= (b^*ab^*a)^*b^*ab^*$

$= b^*ab^*(b^*ab^*ab^*)^*b^*$

Wrong

$\cancel{(ab^*a)^*a}$

$\cancel{(ab^*a)^*b^*a}$

$\cancel{b^*(aa)^*a}$

*** Homework

33) $\{w \mid w \in \{a,b\}^*,\ n_a(w) = \text{even}\} = \{\varepsilon, b, bb, aa, \dots\}$

$= b^*(b^*\boxed{a}b^*\boxed{a}b^*)^*b^*$

$= b^*(\boxed{a}b^*\boxed{a}b^*)^*$

$= (b^*\boxed{a}b^*\boxed{a})^*b^*$

$= (b^*\boxed{a}b^*\boxed{a}b^*)^* + b^*$

valid:

- ε ✓
- b ✓
- aa ✓
- aba ✓
- baa ✓
- aab ✓
- bbaa ✓
- baba ✓

This expression can't generate all

$(b^*ab^*ab^*)^*$

wrong

$\boxed{b}$ is having even a's

$\boxed{b}$ is valid

b is not possible here

$$L = \boxed{w \in \{a,b\}^*,\ n_a(w) = \text{even}}$$

$$\Downarrow$$

$$\Sigma^* = \{\varepsilon, a, b, aa, ab, ba, bb, aaa, aab, \dots\}$$

$$L = \{\varepsilon, b, aa, bb, aab, \dots\}$$

Zero a's

$(aa)^*$

b's can come anywhere

even no. of a's

$\Downarrow$

zero a's

2 a's

4 a's

#a's = even

ε ✓

a ×

b ✓

bb ✓

aa ✓

aba ✓

aaa ×

✓ valid $\in L$

× Invalid $\notin L$

34) $\{w \mid w \in a^*,\ n_a(w) = \text{odd}\} = \{a^{2n+1} \mid n \geq 0\}$

$$R = a(aa)^*$$

$$= (aa)^* a$$

35) $\{w \mid w \in a^* b^*,\ n_a(w) = \text{odd}\}$

$$= a(aa)^* b^*$$

$b \notin (b^*ab^*ab^*)^*$

baa

must must

$ba\varepsilon a\varepsilon \rightarrow baa$

$$(a+ba^*)^* = (a+b)^*$$

$$\boxed{(a+b)^*}$$

$$(a+ba)^*$$

$$(a+ba^2)^*$$

$a+b$

$\downarrow$

a ✓

b ✓

ab ✗

ba ✗

ab

$\downarrow$

ab ✓

a ✗

b ✗

1

$L = \{w \in \{a, b\}^* : \#_a(w) \leq 3\}$.

A. $b^* (a \cup \varepsilon) b^* (a \cup \varepsilon) b^* (a \cup \varepsilon) b^*$

B. $b^* (a) b^* (a \cup \varepsilon) b^* (a \cup \varepsilon) b^*$

C. $b^* (a) b^* (a) b^* (a \cup \varepsilon) b^*$

D. $b^* (a \cup \varepsilon) b^* (a) b^* (a) b^*$

$\left.\begin{matrix} + \\ \cup \\ | \end{matrix}\right\}$ same

(2)

$L = \{w \in \{a, b\}^* : \#_a(w) >= 3\}$.

A. $b^* (a \cup \varepsilon) b^* (a \cup \varepsilon) b^* (a \cup \varepsilon) b^*$

B. $(a \cup b)^* (a) b^* (a \cup \varepsilon) b^* (a \cup \varepsilon) b^*$

C. $b^* (a) b^* (a) b^* (a \cup \varepsilon) b^*$

D. $(a \cup b)^* a (a \cup b)^* a (a \cup b)^* a (a \cup b)^*$

3

$L1 = a^*b^*$
$L2 = a^+b^+$

Find L2-L1.

A. a^*
B. b^*
C. $a^* + b^*$
D. None

4

$L1 = a^* + b^*$ and $L2 = a^*b^*$.
Which of the following is TRUE?

A. $L1 = L2$
B. $L1 \cup L2 = (a+b)^*$
C. $L1^* = L2^*$
D. None

5

$L1 = a^*+b^*$ and $L2 = a^*b^*$.
Which of the following is TRUE?

A. L1 is subset of L2
B. L2 is subset of L1
C. L1 U L2 = L1
D. None

6

$L1 = a^+b^+$ and $L2 = a^*b^*$.

Which of the following is FALSE?

A. L1 is subset of L2

B. $L1^* = L2^*$

C. L1 U L2 = L2

D. None

7

$L1 = a^+ + b^+$ and $L2 = a^* + b^*$.
Which of the following is TRUE?

A. $L1 = L2$
B. $L1^+ = L2^+$
C. $L1 \cup L2 = L2$
D. None

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$L1 = a^+$ and $L2 = a^*$

Which of the following is TRUE?

A. $L1+ = L2^*$

B. $L1^+ = L2^+$

C. $L1^* = L2^+$

D. None

9

$A = a^*$ and $B = b^*$

$AB = ?$

A. $\{ a^n b^n \mid n >= 0 \}$
B. $\{ a^m b^n \mid m, n >= 0 \}$
C. $(a+b)^*$
D. None

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$A = aa^*$ and $B = bb^*$

$(AUB)^* = ?$

A. $\{ a^n b^n \mid n >= 0 \}$
B. $\{ a^m b^n \mid m, n >= 0 \}$
C. $(a+b)^*$
D. None

Given the language L = {ab, aa, baa},
which of the following strings are not in L*?

1) abaabaaabaa
2) aaaabaaaa
3) baaaaabaaaab

A. 1 only
B. 2 only
C. 3 only
D. None

11

12

The length of the shortest string NOT in the language (over $\Sigma = \{a, b\}$) of the following regular expression is ______________.

$a^*(ba)^*a^*$

A. 1
B. 2
C. 3
D. 4

13

$L = a(a+b)^*$ is equivalent to ________

A. $(ab^*)^+$

B. $(a^+b^*)^+$

C. $a^*(ab^*)^+$

D. All of the above

$L = (a+b)^*b$ is equivalent to ________

(14)

A. $(ab^*)^+$

B. $(a^+b^*)^+$

C. $b^*(ab^*)^* b$

D. None

15

(b + ba) (b + a)* (ab + b)

A. (a+b)*
B. a(a+b)*a
C. b(a+b)*b
D. None

$\{w \in \{a, b\}^* : \#_a(w) \equiv_3 0\}$.

16

A. $(b^*ab^*ab^*a)^*b^*$

B. $(b^*ab^*ab^*a)^*$

C. $(ab^*ab^*a)^*$

D. $(ab^*ab^*a)^*b^*$

17

$(a \cup b)^* (a \cup \varepsilon) b^* =$

A. $(a+b)(a+b)^*$

B. $(a+b)^*$

C. $(aa+b)^*$

D. None

18

$L = \{w \in \{a, b\}^* \mid w \text{ has bba as a substring}\}$
Which of the following describes L ?

A. $(a \cup b)^*\, bba\, (a \cup b)^*$
B. $(a \cup b)^+\, bba\, (a \cup b)^*$
C. $(a \cup b)^+\, bba\, (a \cup b)^+$
D. $(a \cup b)^*\, bba\, (a \cup b)^+$

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$L = \{w \in \{a, b\}^*\}$

1. $(a + b)^*$
2. $(a + b + \text{epsilon})^+$
3. $\text{Epsilon} + (a + b)^+$
4. $(a^*b^*)^*$
5. $(b^*a^*)^*$
6. $(a^+b^+)^*$

How many of above are equivalent to given L ?

A. 4 B. 5 C. 6 D. 3

Which Two of the following four regular expressions are equivalent?

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(i) $(00)^*(\varepsilon + 0)$ (ii) $(00)^*$ (iii) 0^* (iv) $0\,(00)^*$

(a) (i) and (ii) (b) (ii) and (iii)

(c) (i) and (iii) (d) (iii) and (iv)

(GATE - 96)

If the regular set A is represented by $A = (01+1)^*$ and the regular set 'B' is represented by $B = ((01)^* 1^*)^*$, which of the following is true? **(GATE - 98)**

21

(a) $A \subset B$

(b) $B \subset A$

(c) A and B are incomparable

(d) $A = B$

The string 1101 does not belong to the set represented by **(GATE - 98)**

(a) $110^* (0+1)$

(b) $1(0+1)^* 101$

(c) $(10)^* (01)^* (00+11)^*$

(d) $(00+(11)^* 0)^*$

22

Let S and T be languages over $\sum = \{a, b\}$ represented by the regular expressions $(a + b^*)^*$ and $(a + b)^*$, respectively.

Which of the following is true? **(GATE - 2000)**

(a) $S \subset T$

(b) $T \subset S$

(c) $S = T$

(d) $S \cap T = \phi$

23

Consider the set $\sum^*$ of all strings over the alphabet $\sum = \{0, 1\}$. $\sum^*$ with the concatenation operator for strings **(GATE - 03)**

(a) Does not form a group

(b) Forms a non-commutative group

(c) Does not have a right identity element

(d) Forms a group if the empty string is removed from $\sum^*$

24

TOC
&
Discrete
math

The regular expression $0^*(10^*)^*$ denotes the same set as **(GATE - 03)**

(a) $(1^*0)^*1^*$

(b) $0+(0+10)^*$

(c) $(0+1)^*10(0+1)^*$

(d) None of the above

25

Which one of the following languages over the alphabet $\{0, 1\}$ is described by the regular expression $(0+1)^*0(0+1)^*0(0+1)^*$ **(GATE - 09)**

(a) The set of all strings containing the substring 00

(b) The set of all strings containing at most two 0's

(c) The set of all strings containing at least two 0's

(d) The set of all strings that begin and end with either 0 or 1

26

Consider the languages $L_1 = \phi$ and $L_2 = \{a\}$. Which one of the following represents $L_1 L_2^* \cup L_1^*$? **(GATE - 13)**

(a) $\{\varepsilon\}$

(b) ϕ

(c) a^*

(d) $\{\varepsilon, a\}$

27

The length of the shortest string NOT in the language (over $\Sigma = \{a, b\}$) of the following regular expression is ________.

$$a^*b^*(ba)^*a^*$$

(GATE – 14-SET3)

28

Which one of the following regular expressions represents the language: the set of all binary strings having two consecutive 0's and two consecutive 1's? **(GATE – 16 – SET1)**

(a) $(0+1)^* 0011(0+1)^* + (0+1)^* 1100(0+1)^*$

(b) $(0+1)^* (00(0+1)^* 11 + 11(0+1)^* 00)(0+1)^*$

(c) $(0+1)^* 00(0+1)^* + (0+1)^* 11(0+1)^*$

(d) $00(0+1)^* 11 + 11(0+1)^* 00$

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Let $r = 1(1+0)^*$, $s = 11^*0$ and $t = 1^*0$ be three regular expressions. Which one of the following is true? **(GATE - 91)**

(a) $L(s) \subseteq L(r)$ and $L(s) \subseteq L(t)$

(b) $L(r) \subseteq L(s)$ and $L(s) \subseteq L(t)$

(c) $L(s) \subseteq L(t)$ and $L(s) \subseteq L(r)$

(d) $L(t) \subseteq L(s)$ and $L(s) \subseteq L(r)$

30

Which of the following regular expression identities are true?

(GATE - 92)

(a) $r(^*) = r^*$

(b) $(r^*s^*)^* = (r+s)^*$

(c) $(r+s)^* = r^* + s^*$

(d) $r^*s^* = r^* + s^*$

31

Which one of the following regular expressions represents the set of all binary strings with an odd number of $1'$s?

A. $((0+1)^*1(0+1)^*1)^*10^*$

B. $(0^*10^*10^*)^*0^*1$

C. $10^*(0^*10^*10^*)^*$

D. $(0^*10^*10^*)^*10^*$

GATE 2020

32

Which one of the following regular expressions over $\{0, 1\}$ denotes the set of all strings **not** containing 100 as a substring?

(GATE - 97)

(a) $0^*(1^+0)^*$

(b) 0^*1010^*

(c) $0^*1^*01^*$

(d) $0^*(10+1)^*$

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Model GATE Questions

- Identify correct regular expression for given Language.
- Find the expression that can generate given String.
- Find the string that can be generated by given expression.
- Find shortest length string generated by given expression.
- Identify the equivalent expression for given expression.
- Identify equivalent expressions from given expressions.
- Find number of equivalence classes for the language generated by given expression.
- Find the language generated by expression is finite or infinite.

Important Reg Exps

$(a+b)^* = ?$

$a(a+b)^* = ?$

$b(a+b)^* = ?$

$(a+b)^*a = ?$

$(a+b)^*b = ?$

equivalent Answers

$$xxx^* = xx^+$$

$$= (a+b)\underbrace{(a+b)(a+b)^*}$$

$$= (a+b)\,(a+b)^+$$

$$\underbrace{(b^*a)^+} \qquad\qquad \underbrace{(a+b)^*a}$$

$$\downarrow \qquad\qquad\qquad \downarrow$$

$$(b^0a)^1 \longleftrightarrow a$$

$$(b^1a)^1 \longleftrightarrow ba$$

$$(b^0a)^2 \longleftrightarrow aa$$

$$(b^0a)^3 \longleftrightarrow aaa$$

$$(b^0a)^1(b^1a)^1 \longleftrightarrow aba$$

$$\vdots \qquad\qquad\qquad \vdots$$


THANK YOU GW
SOLDIERS !