CS & IT ENGINEERING

DISCRETE MATHS
SET THEORY

Lecture No. 09

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TOPICS

01 equivalance relation

02 Partial order relation

3 Poset

Reflexive:

$\{(1\,1)(2\,2)(3\,3)\ldots\ldots\}$

Symmetric:

$\{$

→ Demands flipping.

→ allows same element.

Antisymmetric:

$$\forall a \forall b \left[(a,b) \in R \wedge (b,a) \in R \rightarrow a=b \right]$$

$R_1 = \{\ \}$ ✓ $\underbrace{\underbrace{\underbrace{(a,b)\in R}_{F} \wedge \underbrace{(b,a)\in R}_{F}}_{F} \rightarrow a=b}_{\text{True}}$

if $(a,b) \in R \wedge (b,a) \in R$ then $a=b$.

$R_2 = \{ (1,1) \}$ ✓

$(a,b) \in R \wedge (b,a) \in R \rightarrow a = b$

$\underline{(1,1) \in R} \wedge \underline{(1,1) \in R} \rightarrow 1 = 1.$

$T \wedge T \rightarrow T.$

$a=1 \quad b=1$

Anti:

- → allows same element. ✓
- → does not allows flipping.

$R_3 = \{ (1,2), (2,1) \}$

not Anti

$(a,b) \in R \wedge (b,a) \in R \rightarrow a = b$

$\underline{(1,2) \in R} \wedge \underline{(2,1) \in R} \rightarrow 1 = 2.$

$\underbrace{T. \wedge T.}_{} \rightarrow F.$

$\underbrace{T \rightarrow F.}_{\text{False.}}$

$R_2 = \{ \}$ ✓

$R_3 = \{ (1\,2) \}$ ✓

$(a,b) \in R \wedge (b,a) \in R \rightarrow a = b$

$\underline{(1,2) \in R_3} \wedge \underline{(2,1) \in R_3} \rightarrow$

$\underline{T \wedge F} \longrightarrow$

$\underline{F \longrightarrow}$

$T.$

$R_4 = \{ (2\,1) \}$ ✓

$R_5 = \{ (1\,1)(1\,2) \}$ ✓

$R_6 = \{ (1\,1)(1\,2)(2\,2)(2\,3) \}$ ✓

$R_7 = \{ (1\,1)(2\,2)(3\,3)\boxed{(1\,2)(2\,1)} \}$ X.

12 21

23 32

13 31.

Symmetric

allows same element

$\cap$

Demands flipping.

Anti

allows same element

no flipping.

$R_5 = \{ \}$ Sy $\cap$ Anti

$R_2 = \{(11)(22)(33)\}$ ✓

$R_3 = \{(11)\}$ ✓

$R_4 = \{(22)\}$ ✓

present

$\boxed{(12)\ (21)}$ → absent

present

$\{(12)\}$ ✓

$\{(21)\}$ ✓

$\{\ \}$ ✓

$$\begin{bmatrix} (11) & (12) & (13) \\ (21) & (22) & (23) \\ (31) & (32) & (33) \end{bmatrix} \rightarrow 2^n$$

Total = n^2

Diago = n.

non diago = $n^2 - n$.

boxes = $\frac{n^2-n}{2}$.

Dia. boxes

↓

→ $2^n \cdot 3^{\frac{n^2-n}{2}}$

present
absent

$\{(1,1)\ (2,2)\}$

box possibilities $\to 3$

$\overset{3}{(12)\ (21)}$ $\qquad \boxed{} \qquad \boxed{}$

$3 \times 3 \times 3 \times \ldots$

$\{(12)\}$ ✓

$\{(21)\}$ ✓

$\{\ \}$ ✓

$$3^{\frac{n^2-n}{2}}$$

$A \to$ nonempty set.

$R_1 = \{(1\,2)\}$ $R_2 = \{(2\,1)\}$.

$R_1 \to$ Anti

$R_2 \to$ Anti

$R_1 \cup R_2 =$ not Anti

$R_1 \cap R_2 =$ Anti

$R_1 \cup R_2 = \{(1\,2)(2\,1)\}$ ✗

$R_1 \cap R_2 = \{\ \}$ ✓

$R_1 = \{(1\,1)\}$ Anti ✓

$R_2 = \{(2\,2)\}$ Anti ✓

$R_1 \cup R_2 = \{(1\,1)(2\,2)\}$ ✓

$R_1 \cap R_2 = \{\ \}$ ✓

$R_1 = \{(1\,2)\}$ ✓

$R_2 = \{(2\,1)\}$ ✓

$R_1 \cup R_2 = \{(1\,2)(2\,1)\}$ X.

$R_1 \cap R_2 = \{\ \}$ ✓

Asymmetric :

$\forall a \forall b ((a,b) \in R \rightarrow (b,a) \notin R)$

Asy :
→ does not allows same element.

$R_1 = \{ \}$ ✓ $\underbrace{\underline{(a,b) \in R} \rightarrow (b,a) \notin R}_{\text{True}}$.

F →

$R_2 = \{ (1,1) \}$

$(a,b) \in R \rightarrow (b,a) \notin R$.

$(1,1) \in R \rightarrow (1,1) \notin R$.

T → F.

$a=1$
$b=1$.

$R_1 = \{ (1 2) \}$

$(a,b) \in R \rightarrow (b,a) \notin R.$

$\underbrace{(1,2) \in R}_{T} \rightarrow \underbrace{(2,1) \notin R}_{T}.$

$T \rightarrow T$

(1,2) present $\rightarrow$ (2,1) absent

$R_2 = \{ (1 2) (2 1) \}$

$(a,b) \in R \rightarrow (b,a) \notin R.$

$\underbrace{(1,2) \in R}_{T.} \rightarrow \underbrace{(2,1) \notin R}_{F.}$

→ does not allows flipping.

→ does not allows same element.

$$\begin{bmatrix} (11) & (12) & (13) \\ (21) & (22) & (23) \\ (31) & (32) & (33) \end{bmatrix} \to \frac{1\ \text{choice}}{\text{absent}}$$

$$\boxed{(12)\ (21)}$$

P P absent

1 box → 3 choices.

$$\to 1^n \cdot 3^{\frac{n^2-n}{2}}$$

Symmetric	Antisymmetric	Asymmetric.
$(a,b) \in R \to (b,a) \in R$.	$(a,b) \in R \wedge (b,a) \in R \to a=b$	$(a,b) \in R \to (b,a) \notin R$
→ allows same.	→ allows same	→ no same element.
→ allows flipping.	→ no flipping.	→ no flipping.
$2^n \cdot 2^{\frac{n^2-n}{2}}$	$2^n \cdot 3^{\frac{n^2-n}{2}}$	$1^n \cdot 3^{\frac{n^2-n}{2}}$.

2^n.

$3^{n^2-n/2}$

(12) $R_1 \to$ Asy $\quad R_1 \cup R_2 \to \times$ not Asymmetric.

(21) $R_2 \to$ Asy $\quad R_1 \cap R_2 \to \checkmark$ Asymmetric. ✓

IRREFLEXIVE:

$\forall a \in A \ (a,a) \notin R.$

hates same element

$R_1 = \{(1\,1)\}$ not IRR.
not Reflexive

$R_2 = \{(1\,1)(2\,2)(3\,3)\}$ IRR ✗
RFL ✓

IRREFLEXIVE
$\neq$ not Reflexive.

$R_3 = \{\ \}$ IRR ✓

$R_4 = \{(1\,2)(2\,1)(2\,3)\}$ ✓

$R_5 = \{(1\,2)(2\,1)(2\,2)\}$ ✗.

$$\begin{bmatrix} (11) & (12) & (13) \\ (21) & (22) & (23) \\ (31) & (32) & (33) \end{bmatrix} \rightarrow \text{1 choice absent}$$

Total $= n^2$.

diagonal $= n$.

non diagonal $= n^2 - n$.

Total IRR $= 2^{n^2 - n}$

(11) (22) (33) (12) (21) (13) (31) (23) (32)

absent

$\downarrow$ 2. $\downarrow$ 2

2^{n^2}

$\{\ \}$

$\boxed{A}$ (12)

$\boxed{A}$ (23)

IRR 2^{n^2-n}

$\{(12)(11)\}$

Reflexive ×

IRR ×

$\boxed{11\ 22\ 33}$ (12)

$\boxed{11\ 22\ 33}$ (23)

Refl 2^{n^2-n}

Transitive Relation.

$$\forall a \forall b \forall c \left\{ (a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R. \right\}$$

$R_1 = \{\ \}$

$$(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R.$$

$$\underbrace{\underbrace{\underbrace{F \wedge F}_{F} \rightarrow}}_{\text{True}}$$

$R_2 = \{ (1\,1) \}$ ✓

$(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$

$(1,1) \in R \wedge (1,1) \in R \rightarrow (1,1) \in R$

$a=1$, $b=1$ $b=1$, $c=1$ $a=1$, $c=1$

M2: $a=1$, $b=1$

$(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$

$(1,1) \in R \wedge \boxed{(1,\ )}$

F

F $\longrightarrow$

True.

$R_3 = \{(1\,3)\,(3\,2)\,(1\,2)\}$ ✓

$(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R.$

$(1,3) \in R \wedge (3,2) \rightarrow (1,2) \in R.$

$a = 1$ $\quad$ $b = 3$

$b = 3$ $\quad$ $c = 2.$

$\forall a \forall b \forall c.$

$R_4 = \{(1,2)\,(2\,1)\,(1\,1)\}$ ✗

$(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R.$

$(1,2) \in R \wedge (2,1) \in R \rightarrow (1,1) \in R.$

$(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R.$

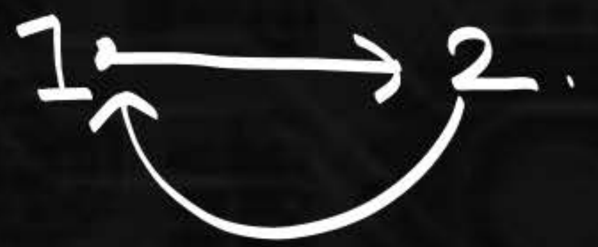
$\underbrace{(2,1) \in R}_{T.} \wedge \underbrace{(1,2) \in R}_{T.} \rightarrow \underbrace{(2,2) \in R}_{F.}$

$a = 2$

$b = 1.$

$R_1 = \{ (1\,2)\,(2\,1) \}$

Tree :



$$\left\{ \begin{array}{l} 1 \to \{ (1,2)\,(1,1) \\ 2 \to (2,1)\,(2,2) \end{array} \right\}$$

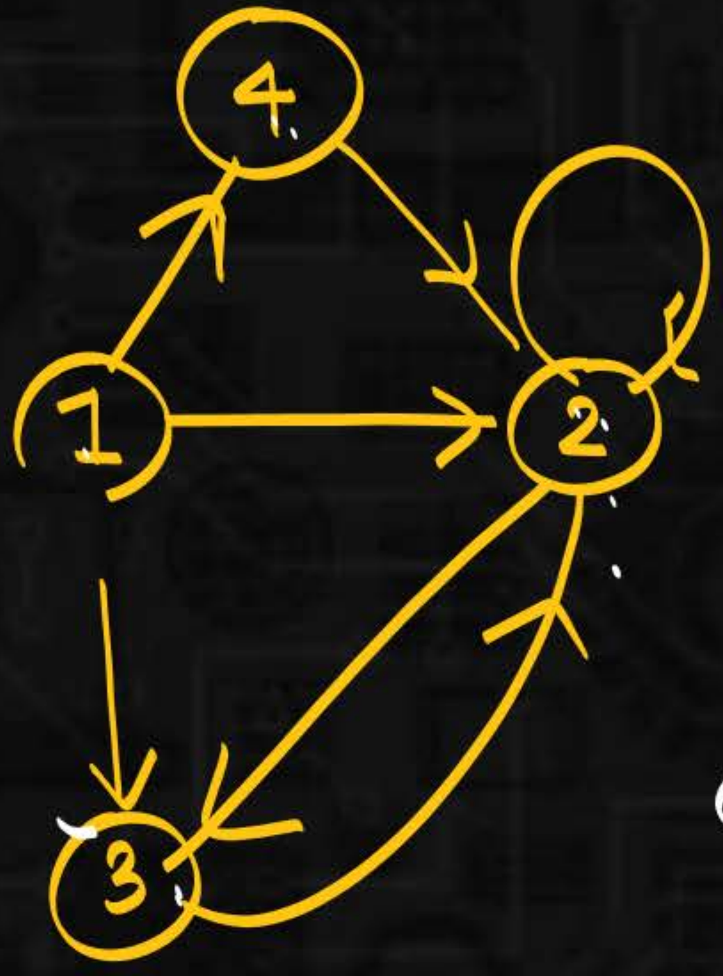
$R_1 = \{ (1,2)\ (2,3)$ — $(3,3)$

$(1,3)\ (3,2)$

$(2,2)\ (1,4)$

$(4,2) \}$

not transitive



R''

$1 \to \{ (1,2)\ (1,3)\ (1,4)$

$2 \to (2,2)\ (2,3)$

$3 \to (3,2)\ (3,3)$ new

$4 \to (4,2)\ (4,3)$ new $\}$


THANK YOU GW
SOLDIERS !