Branch : CSE/IT **Batch : Hinglish**

Discrete Mathematics II

Set Theory

DPP-01

[NAT]

1. Which of the following statements are true?

I. $\phi \in \phi$

II. $\phi \subset \phi$

III. $\phi \subseteq \phi$

IV. $\phi \in \{\phi\}$

V. $\phi \subset \{\phi\}$

VI. $\phi \subseteq \{\phi\}$

[NAT]

2. If a set A has 63 proper subsets, then what is the cardinality of A?

[MCQ]

3. If a set A has 64 subsets of odd cardinality, then what is |A|?

(a) 6 (b) 63

(c) 7 (d) 128

[NAT]

4. How many subset of {1, 2, 3, …, 11} contain at-least one even integer?

[NAT]

5. Let A = {1, 2, 3, 4, 5, 7, 8, 10, 11, 14, 17, 18}
How many subsets of A contain six elements?

[NAT]

6. Let A = {2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15}
How many six-elements subsets of A contain four even integers and two odd integers?

[NAT]

7. Let A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}
How many subsets of A contain only odd integers?

Answer Key

1. **(4)**
2. **(6)**
3. **(c)**
4. **(1984)**
5. **(924)**
6. **(225)**
7. **(63)**

Hints and Solutions

1. (4)

I. $\phi \in \phi$ is false. The empty set has no members.

II. $\phi \subset \phi$ is false. The empty set is not a proper subset of itself.

III. $\phi \subseteq \phi$ is true. The empty set is a subset of every set

$\therefore$ subset of itself

IV. $\phi \in \{\phi\}$ is true. ϕ is a member here.

V. $\phi \subset \{\phi\}$ is true. The empty set is a proper subset of itself.

VI. $\phi \subseteq \{\phi\}$ is true. The empty set is a subset of every set.

2. (6)

If a set has n elements then the number of subsets will be 2^n and the number of proper subsets will be $2^n - 1$.

A has 63 proper subsets, so $2^n - 1 = 63$

$2^n = 63 + 1$

$2^n = 64$

$2^n = 2^6$

$\therefore$ n = 6

The cardinality of A is 6

3. (c)

The number of subsets for {1, 2, 3, … n} with odd cardinality is 2^{n-1}.

Number of subsets with cardinality i = ${}^nC_i$

So, the number of subsets with odd cardinality

$\Sigma i = 1, 3 \ldots n - 1\ {}^nC_i = 2^{n-1}$.

Now, given

$2^{n-1} = 64$

$2^{n-1} = 2^6$

$n - 1 = 6$

[Bases are same, so equating power]

n = 7

4. (1984)

2^{11} subset for {1, 2, 3, ….11}

2^6 subset for {1, 3, 5, 7, 9, 11} contains none of the even integers {2, 4, 6, 8, 10}.

Hence, there are $2^{11} - 2^6 = 1984$ subsets that contain at least one even integer.

5. (924)

If we choose 6 elements from a set of 12 elements where order does not matter. Then we can do it in ${}^{12}C_6$ ways.

For example consider a set = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12} out of this set we have to choose a subset of 6 elements. This can be done in following ways:

{1, 2, 3, 4, 5, 6}, {2, 3, 4, 5, 6, 7},{3, 4, 5, 6, 7, 8}, {4, 7, 8, 3, 11, 12}, {5, 7, 9, 10, 11, 12}….. and so on.

That means the arrangement or order of elements does not matter, therefore we can do it using combinations.

$${}^{12}C_6 = \frac{\cancel{12} \times 11 \times \cancel{10}^2 \times \cancel{9}^3 \times \cancel{8}^2 \times 7 \times \cancel{6!}}{\cancel{6!} \times \cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1}$$

$= 11 \times 12 \times 7$

$= 77 \times 12$

$= 924$

6. (225)

Out of 6 element subsets, we can choose 4 even integers in ${}^6C_4$ ways.

Similarly to find 2 odd integers out of 6 element subset, can be done in ${}^6C_2$ ways.

$$\text{Therefore } {}^6C_4 * {}^6C_2 * = \frac{\cancel{6}^3 \times 5 \times \cancel{4!}}{\cancel{2} \times \cancel{4!}} \times \frac{\cancel{6}^3 \times 5 \times \cancel{4!}}{\cancel{4!} \times \cancel{2}}$$

$= 15 * 15 = 225$

7. (63)

In the given set, there are 6 odd integers, we have two choices for each odd integer to be included or not included, therefore total possibilities = $2^6 - 1$ = 63.