CS & IT ENGINEERING

Theory of Computation

Finite Automata

Lecture No. 21

By- DEVA Sir

TOPICS TO BE COVERED

01 Closure properties

02 For finite languages

03 For Infinite languages

*** 04 For regular languages

05 For non regular languages

What is closure property?

$(D, \circ)$ is "closed"

D → Domain

o → operation

iff

$\underbrace{\forall x_1, x_2}_{\text{every 2 elements in D}} \in D \ni x_1 \circ x_2 \in D$

$(D, \circ)$ is "not closed"

iff

$\underbrace{\exists x_1, x_2}_{\text{Some 2 elements in D}} \in D \ni x_1 \circ x_2 \notin D$

In Mathematics:

Domain = Set of Natural numbers

operation

$+$

$\cdot 1 \quad \cdot 2 \quad \cdot 3 \quad \cdot 4$

$\cdot 5 \quad \cdot 6 \quad \cdot 7 \quad \cdot 8$

$\cdot 9 \quad \cdot 10 \quad \ldots$

Is $(N, +)$ closed?

$\Downarrow$

$$\boxed{\forall x_1, x_2 \in N \ni x_1 + x_2 \in N}$$

$(N, +)$ is closed

Set of Natural numbers is closed under $+$.

N is closed under $+$

$+$ is closed for natural numbers

$(\{a \mid a \text{ is natural number}\}, +)$ is closed

$(\{1, 2, 3, 4, 5, \ldots\}, +)$ is closed

For Finite languages, which of the following are closed?

I) Union $\Rightarrow$ closed

II) Intersection $\Rightarrow$

$Finite_1 \cup Finite_2 \Rightarrow Finite$

$Finite_1 \cap Finite_2 \Rightarrow Finite$

Domain:
Set of finite languages

In maths;

$(N, +)$ ✓

$(N, \times)$ ✓

$(Z, +)$ ✓

$(Z, \times)$ ✓

$(Z, -)$ ✓

In Mathematics:

operation

$-$

$\boxed{(N, -) \text{ is not closed}}$

Example:

$$\left.\begin{matrix} 2 \in N \\ 5 \in N \end{matrix}\right\} \quad \underbrace{2-5}_{-3} \notin N$$

$$\exists\, x_1, x_2 \in N \ni x_1 - x_2 \notin N$$

Domain = Set of Natural numbers

$\cdot 1 \quad \cdot 2 \quad \cdot 3 \quad \cdot 4$

$\cdot 5 \quad \cdot 6 \quad \cdot 7 \quad \cdot 8$

$\cdot 9 \quad \cdot 10 \quad \dots$

$(\mathbb{N}, -) \Rightarrow$ Not closed

$(\mathbb{N}, /)$

$(\mathbb{Z}, /)$

Types of Domains

- Set of finite languages
- " " Infinite "
- " " Regular "
- " " Not regular "
- " " languages

Types of operations :

- Union
- Intersection
- complement
- Difference
- Kleene star
- ⋮

	Union	Intersection	Complement
For finite languages	closed $F_1 \cup F_2 \Rightarrow$ Finite	closed $F_1 \cap F_2 \Rightarrow$ Finite	*** Not closed $\overline{F_i} \Rightarrow$ Infinite
For Infinite languages	closed $Inf_1 \cup Inf_2 \Rightarrow Inf$	Not closed $I_1 \cap I_2 \Rightarrow$ either Inf or Fin	Not closed $\overline{I} \Rightarrow$ either finite or Inf

closed $\Rightarrow$ proof by Algo

Not closed $\Rightarrow$ proof by Example.

$\mathrm{Inf}_1 \cap \mathrm{Inf}_2 \Rightarrow$ Either finite or Inf (Need not be Infinite)

Example 1: $\underset{\mathrm{Inf}}{a^*} \cap \underset{\mathrm{Inf}}{a^*} \Rightarrow \underset{\mathrm{Inf}}{a^*}$

Example 2: $\underset{\mathrm{Inf}}{a^*} \cap \underset{\mathrm{Inf}}{b^*} \Rightarrow \{\varepsilon\}$ Finite lang

$\{\varepsilon, \ldots\}$ $\{\varepsilon, \ldots\}$

Example 1:

$L = (a+b)^+$ Inf

$\Downarrow$

$\bar{L} = \{\varepsilon\}$

Fin

Example 2:

$L = a(a+b)^*$ Inf

$\Downarrow$

$\bar{L} = b(a+b)^* + \varepsilon$

Inf

over $\Sigma = \{a, b\}$

- $(a+b)^* \to \phi$
- $(a+b)^+ \to \{\varepsilon\}$
- $a^* \to \Sigma^* - a^*$
- $b^* \to \Sigma^* - b^*$
- $a(a+b)^* \to bX + \varepsilon$
- $b(a+b)^* \to aX + \varepsilon$

Infinite no. of Infinite languages

***

I) Complement of Infinite language is either finite or Infinite

II) Complement of Finite language is Always Infinite

III) Complement for Infinite languages is Not Closed

IV) Complement for Finite languages is Not closed

Complement

Is there any $\overline{F_i} \notin D$

$F_1 \to I \quad F_2 \to I \quad F_3 \to I \quad F_4$

$F_5 \quad F_6 \quad \ldots$

$\ldots$

What is complement of L ?

$$\boxed{\overline{L} = \Sigma^* - L}$$

$$\overline{Fin} = \underbrace{\Sigma^*}_{Inf} - Fin$$

$$= \text{Infinite lang}$$

I) $L = \phi \Rightarrow \overline{L} = \Sigma^*$

$\Sigma = \{a\}$

II) $L = \{\varepsilon\} \Rightarrow \overline{L} = \Sigma^+$

III) $L = \{a\} \Rightarrow \overline{L} = \Sigma^* - \{a\} = \{\varepsilon, a^2, a^3, \ldots\}$

$= \{a^n \mid n \neq 1\}$

$= \varepsilon + aaa^*$

IV) $L = \{\varepsilon, aa\} \Rightarrow \overline{L} = \{a, a^3, a^4 \ldots\}$

	Operation	finite languages	Infinite languages
1	Union	✓	✓
2	Intersection	✓	✗
3	Complement	✗	✗
4	Difference	✓	✗
5	Concatenation	✓	✓
6	Reversal	✓	✓
7	Kleene star	✗	✓
8	Kleene plus	✗	✓
9	Subset	✓	✗
10	Symmetric Difference	✓	✗
11	$Deva(L) = \bar{L} \cup L^{Rev}$	✗	✓

	Operation	finite languages	Infinite languages
1	Union	$F_1 \cup F_2 \Rightarrow$ Fin	$I_1 \cup I_2 \Rightarrow$ Inf
2	Intersection		
3	Complement		
4	Difference	$F_1 - F_2 \Rightarrow$ Fin	$I_1 - I_2 \Rightarrow$ either Inf or Fin
5	Concatenation		
6	Reversal		
7	Kleene star	$(F)^* \Rightarrow$ either finite or Inf	$(I)^* \Rightarrow$ always Inf
8	Kleene plus		
9	Subset		
10	Symmetric Difference		
11	$Deva(L) = \overline{L} \cup L^{Rev}$		

Symmetric Difference:

I) $L_1 \oplus L_2 =$

$(L_1 \cup L_2) - (L_1 \cap L_2)$

OR

II) $L_1 \oplus L_2 = (L_1 - L_2) \cup (L_2 - L_1)$

What is operation?

How to Apply on given Domain?

Closure properties for Regular languages

Assume $\Sigma = \{a, b\}$

Domain

Set of all regular languages

- ϕ
- $\{\varepsilon\}$
- $\{a^n b^n \mid n < 10\}$
- a^*
- a^+
- b^*
- $(aa)^*$
- $a(aa)^*$
- $(a+b)^*$
- $(a+b)^+$
- a X
- X a
- X a X
- $\{a^m b^n \mid m \geq 0, n \geq 0\}$

Infinite no. of regular languages

Closure properties for Regular languages:

① Union
② Intersection
③ Complement
④ Difference
⑤ Concatenation
⑥ Reversal
⑦ Kleene star
⑧ Kleene plus
⑨ Symmetric Difference

⑩ $\boxed{\text{Subset}}$
⑪ Prefix
⑫ Suffix
⑬ Substring
⑭ Substitution(L) = f(L)
⑮ Homomorphism(L) = h(L)
⑯ ε-free Homomorphism
⑰ Inverse Homomorphism
⑱ Quotient

⑲ $\frac{1}{2}(L)$ = Half(L)
⑳ Second Half(L)
㉑ one-third (L) = $\frac{1}{3}(L)$
㉒ middle $\frac{1}{3}(L)$
㉓ Last $\frac{1}{3}(L)$

㉔ Finite ∪
㉕ Finite ∩
㉖ Finite −
㉗ Finite ·
㉘ Finite ⊆
㉙ Finite f

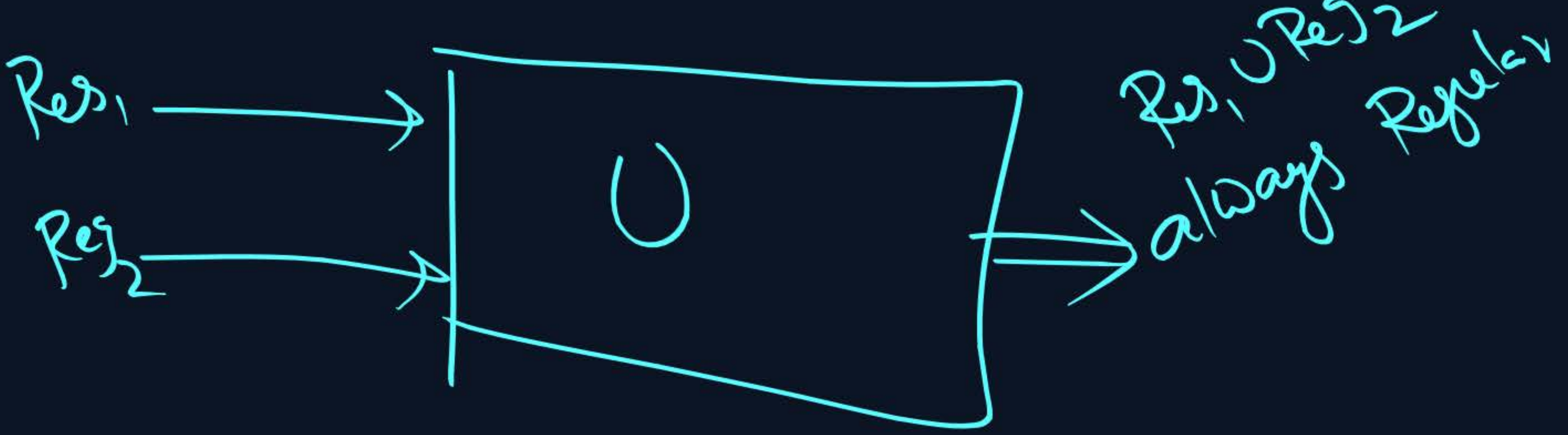
㉚ Inf ∪
㉛ Inf ∩
㉜ Inf −
㉝ Inf ·
㉞ Inf ⊆
㉟ Inf f

$$R^{X}_{SI_{All}}$$

① Union for Regular Languages

→ closed

$Reg_1 \cup Reg_2 \implies$ Always Regular



proof:

Reg Lang ∪ Reg Lang ⇒ Reg Lang

$\Downarrow$ $\Downarrow$

Algorithm: $RegExp_1$ $RegExp_2$

$\Downarrow$ Add + in between

$\underbrace{\boxed{RegExp_1 + RegExp}}_{\text{New RegExp}}$

I) $L_1 = \phi,\ L_2 = \Sigma^* \Rightarrow L_1 \cup L_2 = \Sigma^* = L_2$

II) $L_1 = a^*,\ L_2 = (aa)^* \Rightarrow L_1 \cup L_2 = a^* = L_1$

III) $L_1 = \text{Any},\ L_2 = \phi \Rightarrow L_1 \cup L_2 = L_1$

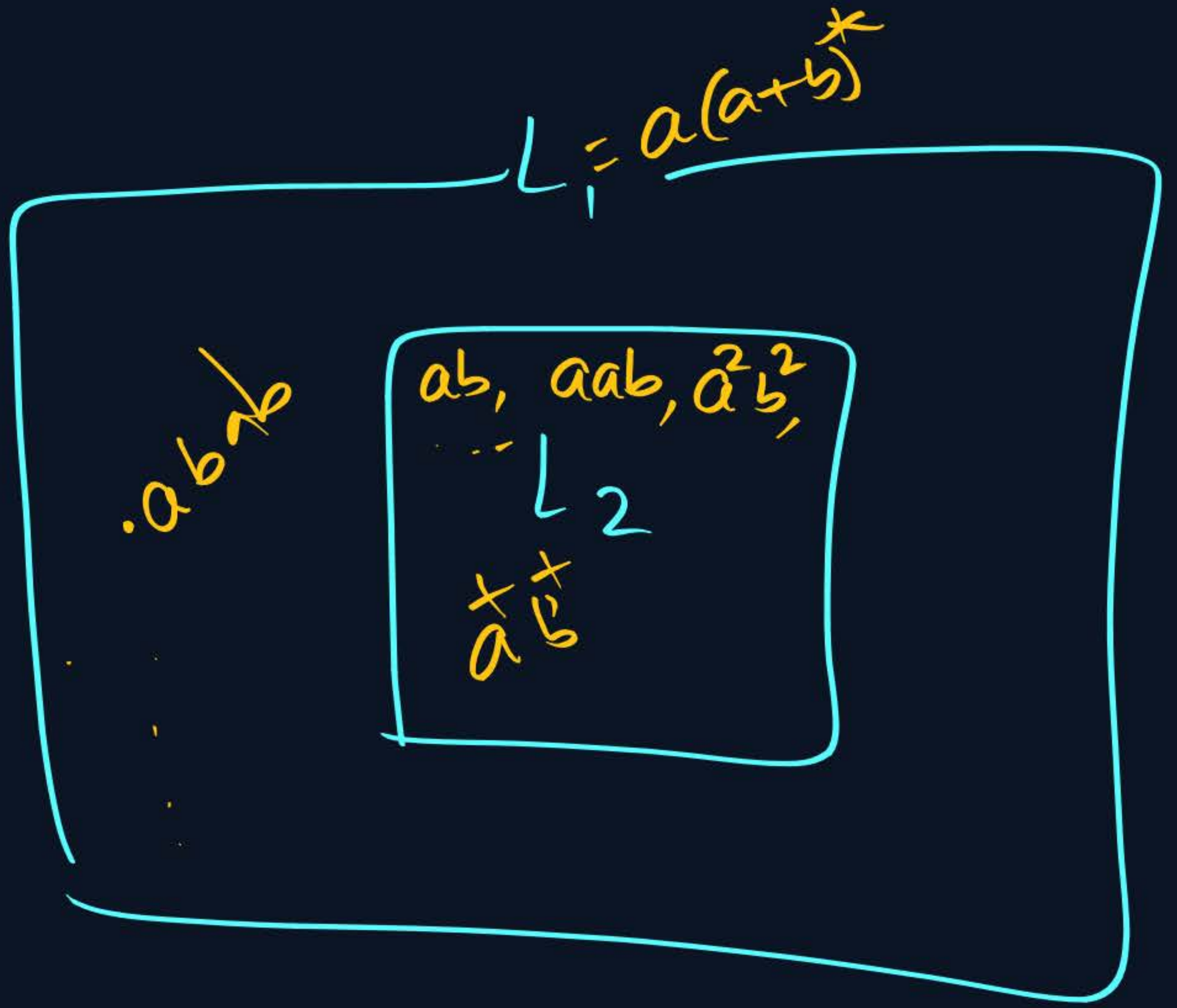
IV) $L_1 = \Sigma^*,\ L_2 = \text{Any} \Rightarrow L_1 \cup L_2 = \Sigma^* = L_1$

V) $L_1 = a^* b^*,\ L_2 = a^* \Rightarrow L_1 \cup L_2 = L_1$

VI) $L_1 = a(a+b)^*,\ L_2 = a^+ b^+ \Rightarrow L_1 \cup L_2 = L_1$

$\boxed{L_1 \supset L_2}$

$\boxed{L_1 \supseteq L_2}$


$L_1 = a(a+b)^*$
$abab$
$ab,\ aab,\ a^2b^2, \ldots$
L_2
a^+b^+

H.W

I) Reg Lang$_1$ $\cup$ Reg Lang$_2$ $\Rightarrow$ Regular

II) Reg Lang $\cup$ Non Reg lang $\Rightarrow$ either Reg or Not reg

III) Non Reg lang$_1$ $\cup$ Non reg lang$_2$ $\Rightarrow$ either Reg or Not reg

A) Regular

B) Not Regular

C) Either Reg or Not reg

D) None

Summary

→ Basics of closure properties


THANK YOU
GW
SOLDIERS !