# CS & IT ENGINEERING

## Theory of Computation

## Finite Automata

**Lecture No.** 17

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01 Regular Grammar

02 Pumping Lemma

03

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Grammar(G) = (V, T, P, S)

- V → Set of variables (non-terminals)
- T → Set of terminals
- P → set of rules
- S → Start symbol

- It is set of rules (productions)
- It generates Set of strings (language)

$S \rightarrow AaB \,(1) \mid a \,(2)$ OR

$A \rightarrow bc$ (3)

$B \rightarrow Sa$ (4)

$V = \{S, A, B\}$

$T = \{a, b, c\}$

$P = \{S \rightarrow AaB,\ S \rightarrow a,\ A \rightarrow bc,\ B \rightarrow Sa\}$

$S = S$

# Regular Grammar (RG) :

↳ It is Left Linear Grammar (LLG) OR Right Linear Grammar (RLG)

## LLG

$$V \longrightarrow VT^* \mid T^*$$

Example :

$$S \rightarrow Sab \mid \varepsilon \mid ab$$

## RLG

$$V \rightarrow T^*V \mid T^*$$

Example :

$$S \rightarrow \varepsilon \mid a \mid ab \mid aabS$$

# Identify RG

① $S \rightarrow \varepsilon$

- LLG ✓
- RLG ✓
- RG ✓

② $S \rightarrow a \mid bc \mid \varepsilon$

- LLG ✓
- RLG ✓
- RG ✓

③ $S \rightarrow A$

$A \rightarrow a \mid S \mid \varepsilon$

- LLG ✓
- RLG ✓
- RG ✓

④

$S \rightarrow Sa$

$S \rightarrow bS$

$S \rightarrow a$

Neither LLG nor RLG

↳ not LLG

↳ not RLG

↳ not RG

⑤ $S \rightarrow Sa \mid b$

- LLG ✓
- RLGX
- RG ✓

⑥ $S \rightarrow abS \mid a$

- LLGX
- RLG ✓
- RG ✓

(7) $S \longrightarrow aSb \mid ab \mid \varepsilon$

→ not RG

(8) $S \longrightarrow AaB \mid \varepsilon$

→ not RG

(9) $S \longrightarrow ABa \mid \varepsilon$

# Find Regular Language from the following RGs.

① $S \rightarrow \varepsilon$ $\quad L = \{\varepsilon\}$

② $S \rightarrow a \mid bb \mid \varepsilon$ $\quad L = \{\varepsilon, a, bb\}$
$= \varepsilon + a + bb$

③ $S \rightarrow \varepsilon \mid a \mid aa \mid aaa$ $\quad L = \varepsilon + a + aa + aaa$
$= \{a^n \mid n \leq 3\}$

④ $S \rightarrow Aa$ $\quad L = \{\} = \phi$

(5) $S \rightarrow \underbrace{Aa}_{\text{useless}} \mid b$

$L = \{b\}$

(6) $S \rightarrow Aa$

$A \rightarrow \varepsilon$

$L = \{a\}$

(7) $S \rightarrow Aa$

$A \rightarrow a \mid b \mid \varepsilon$

$L = (\varepsilon + a + b).a$

$= a + aa + ba$

(8) $S \rightarrow Aa \mid Bb$

$A \rightarrow c$

$B \rightarrow d$

$L = \{ca, db\}$

$\varepsilon a = a$

$a\varepsilon = a$

$\underbrace{\varepsilon\varepsilon\varepsilon\varepsilon = \varepsilon}$

$S \rightarrow Aa$

$A \rightarrow \varepsilon$

$S$

$\Downarrow$

$Aa$

$\Downarrow$

$\varepsilon a = a$

⑨ $S \rightarrow Aa \mid Bb$

$A \rightarrow \varepsilon \mid a$

$B \rightarrow \varepsilon \mid b$

$L = \{aa, a, bb, b\}$

$A = a + \varepsilon$

$B = b + \varepsilon$

$\Rightarrow S = Aa + Bb$

$= (a+\varepsilon)a + (b+\varepsilon)b$

(10) $S \rightarrow Sa \mid b$

b ✓

ba ✓

baa ✓

baaa ✓

baaaa ✓

⋮

$L = ba^*$

$L = b^*a$

$X \rightarrow X\alpha \mid \beta$

$L = \beta\alpha^*$

(11) $S \rightarrow Sa \mid \varepsilon$ $\quad L = a^*$

(12) $S \rightarrow Saa \mid \varepsilon$ $\quad L = (aa)^* = \{a^{2n} \mid n \geq 0\} = \{a^n \mid n = \text{even}\}$

(13) $S \rightarrow Sab \mid \varepsilon$ $\quad L = (ab)^*$

(14) $S \rightarrow Sab \mid c$ $\quad L = c(ab)^*$

15 $S \rightarrow aS \mid b$ $L = a^*b$

16 $S \rightarrow aS \mid a$ $L = a^*a = aa^* = a^+$

17 $S \rightarrow aaS \mid \varepsilon$ $L = (aa)^*$

18 $S \rightarrow aaS \mid a$ $L = (aa)^*a = a(aa)^*$

19 $S \rightarrow abS \mid \varepsilon$ $L = (ab)^*$

*** 20 $S \rightarrow Sa$

$L = \phi$

S
⇓
Sa
⇓
Saa
⇓
⋮
never ending

(21) $S \rightarrow Sa \mid b \mid c$

$= Sa^*$

$L = (b+c)a^* = ba^* + ca^*$

(22) $S \to abS \mid c \mid de \mid fgh$

$L = (ab)^* (c+de+fgh)$

(23) $S \to Sa \mid b \mid c$

$L = (b+c)a^*$

(24) $S \to Sa \mid \varepsilon \mid b$

$L = (\varepsilon+b)a^*$

*** (25) $S \rightarrow Sa \mid Sb \mid c$

$L = c(a+b)^*$

c ✓, ca ✓, cb ✓, caa, cab, cba, cbb, ...

| c | ca | cb | caa | cab | cba |
|---|---|---|---|---|---|
| S | S | S | S | S | S |
| ⇓ | ⇓ | ⇓ | ⇓ | ⇓ | ⇓ |
| c | Sa | Sb | Sa | Sb | Sa |
| | ⇓ | ⇓ | ⇓ | ⇓ | ⇓ |
| | ca | cb | Saa | Sab | Sba |
| | | | ⇓ | ⇓ | ⇓ |
| | | | caa | cab | cba |

$L = c(a+b)^*$

$= c(a^*b^*)^*$

$= c(b^*a^*)^*$

$$X \rightarrow X\alpha_1 \mid X\alpha_2 \mid X\alpha_3 \mid \beta_1 \mid \beta_2$$

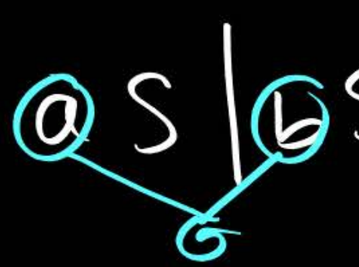
$$X \rightarrow X(\alpha_1+\alpha_2+\alpha_3)^*$$

$$L = (\beta_1+\beta_2)(\alpha_1+\alpha_2+\alpha_3)^*$$

(26) $S \rightarrow Sa \mid Sb \mid \varepsilon$

$L = \varepsilon (a+b)^* = (a+b)^*$

(27) $S \rightarrow aS \mid bS \mid \varepsilon$

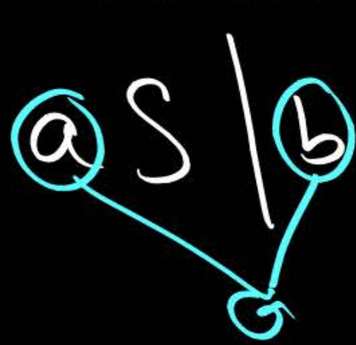


$L = (a+b)^* \varepsilon = (a+b)^*$

(28) $S \rightarrow Sa \mid Sb \mid a$

$L = a(a+b)^*$

(29) $S \rightarrow aS \mid bS \mid b$



$L = (a+b)^* b$

(30) $S \rightarrow Aa$

$A \rightarrow Aa \mid Ab \mid \varepsilon$

$A = (a+b)^*$

$L = S = Aa$

$= (a+b)^* a$

a) $a(a+b)^*$

b) $(a+b)^* a$

c) $(ab^*)^+$

d) $(b^* a)^+$

same

$$(b^{*}a)^{+}$$

a b a

$$(\quad)^{2} = \underset{\varepsilon a}{(b^{0}a)^{1}}\underset{ba}{(b^{1}a)} = \underline{\underline{aba}}$$

(31) $S \rightarrow aA$

$A \rightarrow aA \mid bA \mid \varepsilon$

$L = a(a+b)^*$

(32) $S \rightarrow Sa \mid Sb \mid a$

$L = a(a+b)^*$

(33) $S \rightarrow aS \mid bS \mid a$

$L = (a+b)^* a$

Home Work :

(34) $S \rightarrow Sa \mid Sb \mid A$

$A \rightarrow aa \mid ba$

(35) $S \rightarrow Aa \mid Ab$

$A \rightarrow Ba$

$B \rightarrow Ba \mid Bb \mid \varepsilon$

(36) $S \rightarrow aS \mid bS \mid cS \mid \varepsilon$

(37) $S \rightarrow Aa \mid Ab \mid \varepsilon$

$A \rightarrow a \mid b \mid \varepsilon$

(38)

$S \rightarrow Sa \mid Sb \mid A$

$A \rightarrow Bab$

$B \rightarrow Ba \mid Bb \mid \varepsilon$

(39)

$S \rightarrow aS \mid bS \mid A$

$A \rightarrow abB$

$B \rightarrow aB \mid bB \mid \varepsilon$

(40)

$S \rightarrow aA \mid bA$

$A \rightarrow aB \mid bB$

$B \rightarrow aB \mid bB \mid \varepsilon$

(41)

$S \rightarrow Aa \mid Ab$

$A \rightarrow Ba \mid Bb$

$B \rightarrow Ba \mid Bb \mid \varepsilon$

## Summary

- Regular Grammars ✓

Next: pumping lemma.


THANK YOU
GW
SOLDIERS !