CS & IT ENGINEERING

Theory of Computation

Decidability

Lecture No. 1

By- DEVA Sir

SD

01 D, SDUD, NR

SD

02 Problem Vs Language

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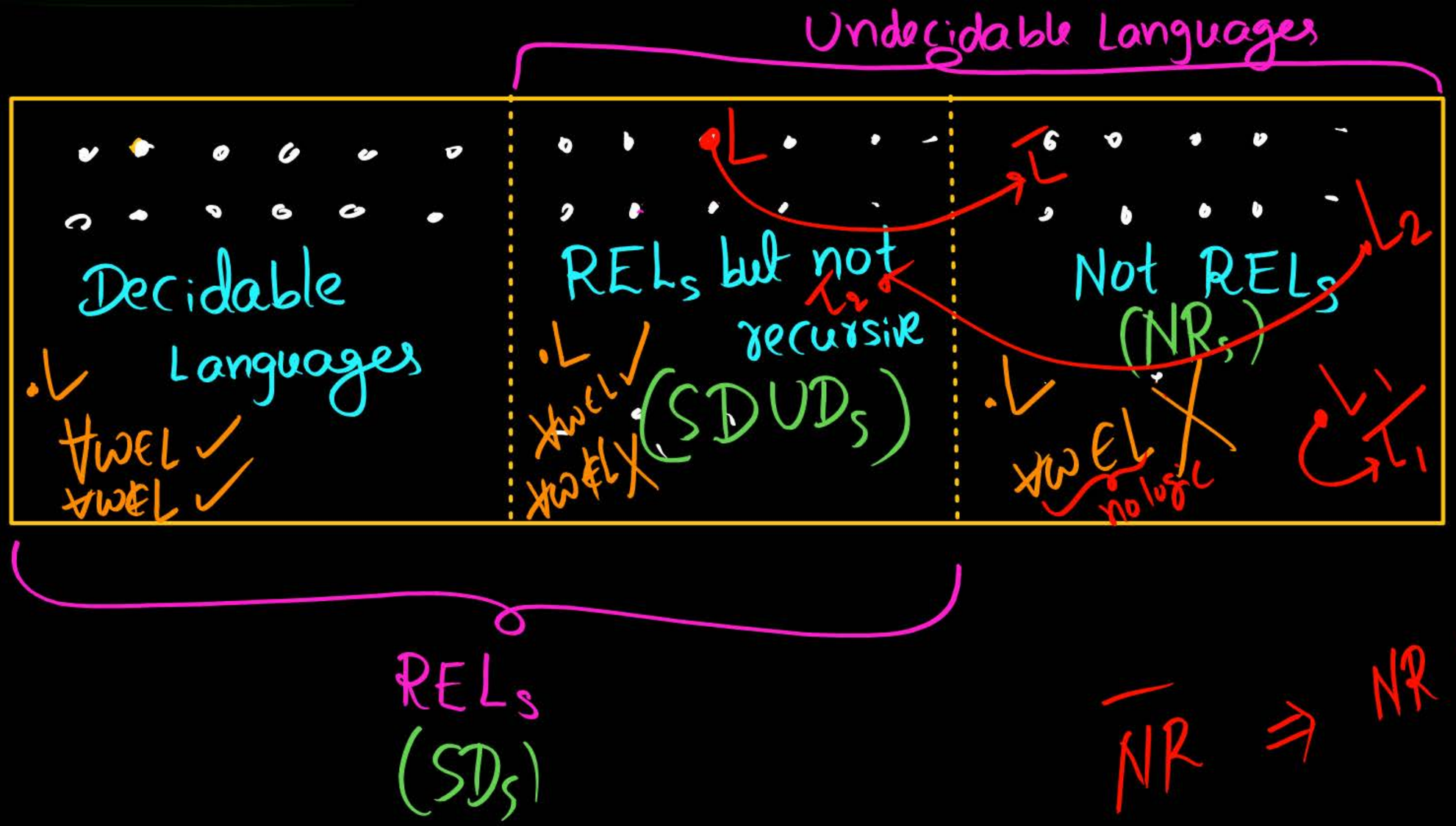
03 Decision properties

04 Languages

05


Undecidable Languages
Decidable Languages
RELs but not recursive
Not RELs (NRs)
L
$\bar{L}$
L_2
L_1
L_1
$\forall w \in L$
$\forall w \notin L$
$\forall w \in L$
no logic
RELs (SDs)
(SDUDs)
$\bar{NR} \Rightarrow$ NR or SDUD

	D	SDUD	NR
Members	Valid → logic exist Invalid → logic exist	Valid → logic exist Invalid → logic not exist	Valid → logic not exist
Machine	HTM exist	TM exist but HTM not exist	TM not exist
Language	Decidable lang (Recursive lang)	SDUD (REL but not rec)	Not REL (Not SD)
Program (Computer)	Halting program exist (Algorithm exist)	program exist but no Algorithm	program not exist

What is Language (Set) ?

- over Σ : **Set of strings over Σ** (Subset of Σ^*)
- Set of meaningful members
  - Set of strings $\Rightarrow \{\varepsilon, a, ab, \ldots\}$
  - Set of Grammars $\Rightarrow \{G_1, G_2, G_3, \ldots\}$
  - Set of Languages $\Rightarrow \{L_1, L_2, L_3, \ldots\}$
  - Set of Automata (machines) $\Rightarrow \{M_1, M_2, M_3, \ldots\}$
  - Set of Objects

$L_1 = a^* = \{\varepsilon, a, aa, \cdots\}$

L_2 = Set of C programs $= \{P_1, P_2, P_3, \cdots\}$

L_3 = Set of Languages over Σ $= \{L_1, L_2, L_3, L_4, \cdots\} = 2^{\Sigma^*}$

$=$ Set of all subsets of $\Sigma^* = P(\Sigma^*)$

L_4 = Set of (all) strings over Σ.

$= \Sigma^*$

L_5 = Set of all regular languages $= \{R_1, R_2, R_3, \cdots\}$

Set of finite languages

Set of infinite "

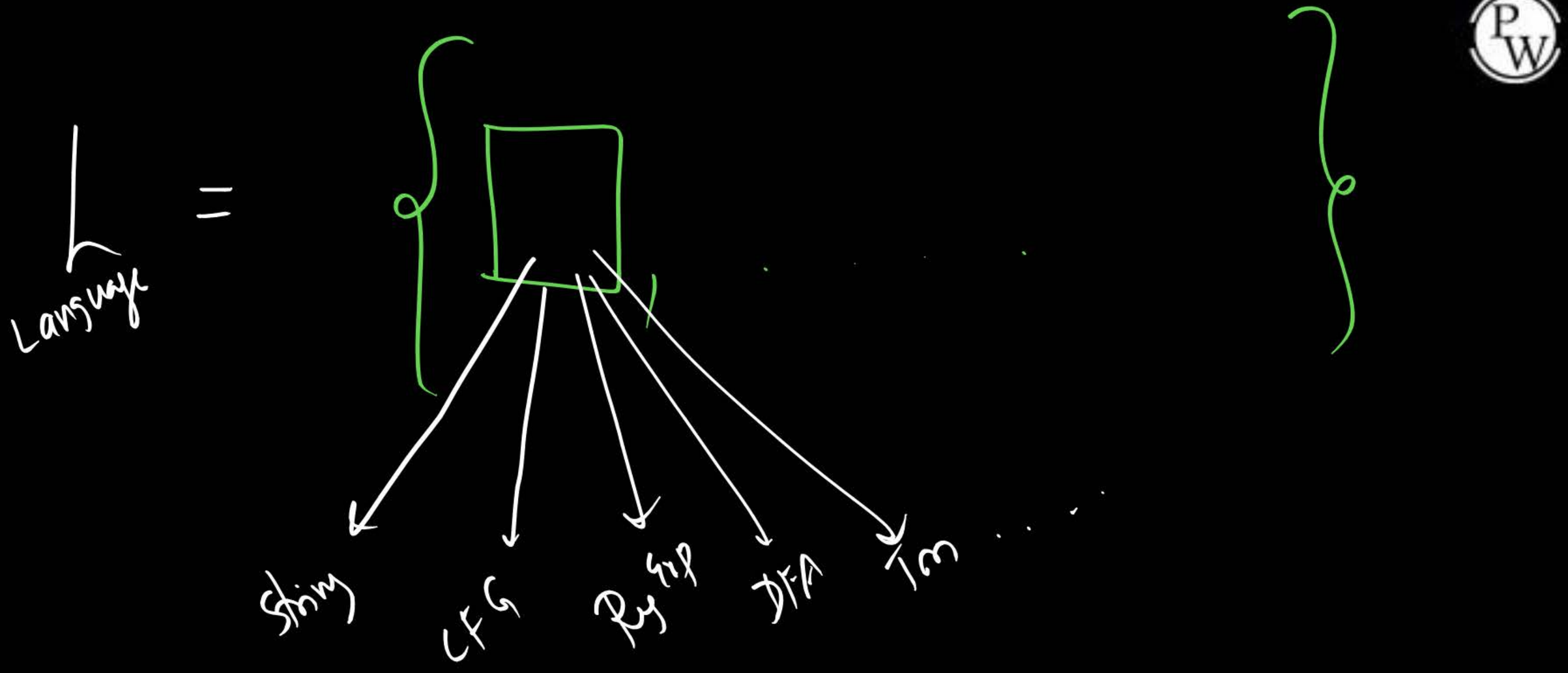
Set of DCFLs

Set of CFLs

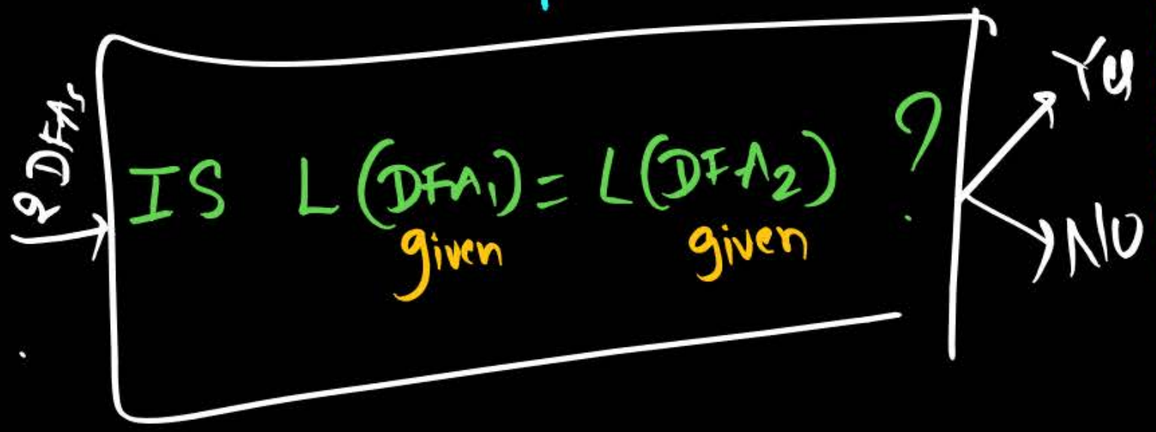
Set of CFGs = {CFG$_1$, CFG$_2$, CFG$_3$, ...}

Set of Regular Exps

Set of Not RELs


L
Language
=
String
CFG
Regular
DFA
TM

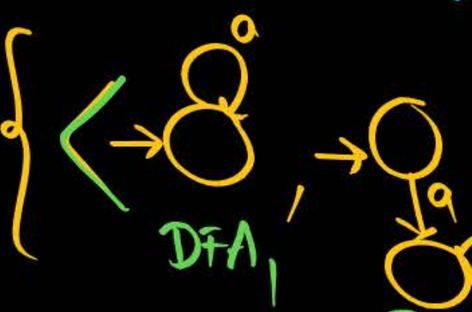
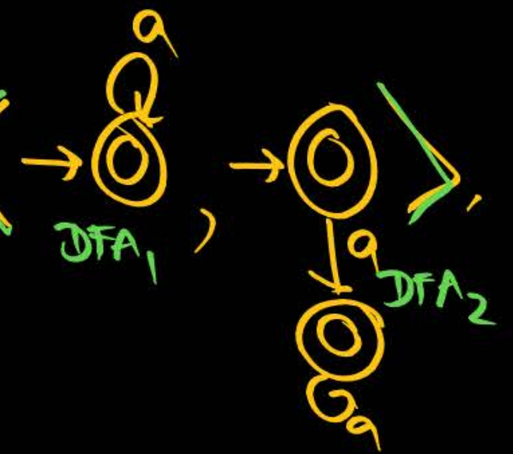
Problem



Language

$$L_1 = \{ \underbrace{\langle DFA_1, DFA_2 \rangle}_{\text{object}} \mid \overbrace{L(DFA_1) = L(DFA_2)}^{DFA_1 \cong DFA_2} \}$$

L_1 = Set of pairs where every pair has 2 equivalent DFAs

$= \{ \langle$  $\rangle, \langle$  $\rangle, \ldots \}$

(Decision property)
Decision Problem

↳ Yes
NO

Language

↳ valid members
Invalid members

Decidable	Yes ⇒ logic exist No ⇒ logic exist } Decidable problem	Decidable Language: Logic exist for both valid & Invalid
SDUD	Yes ⇒ logic exist No ⇒ logic not exist } SDUD problem	SDUD language Logic exist only for valid
NR	Yes ⇒ logic not exist } Not RE problem	Not REL Logic not exist for valid

Decision properties Table [Decision problems]

	Problems	FA / regular lang / regular RG	DPDA / DCFLs	PDA / CFLs / CFGs	HTM / Recursive lang.	TM / RELs / UGs
①	Halting problem	D	D	D	D	UD
②	Membership problem	D	D	D	D	UD
③	Emptiness	D	D	D	UD	UD
④	Finiteness	D	D	D	UD	UD
⑤	Totality	D	D	UD	UD	UD
⑥	Equivalence	D	D	UD	UD	UD
⑦	Disjoint	D	UD	UD	UD	UD
⑧	Set containment	D	UD	UD	UD	UD

I) $\overline{\overline{D}} \Longrightarrow D$

II) $\overline{UD} \Longrightarrow UD$

$\overline{UD} = ?$

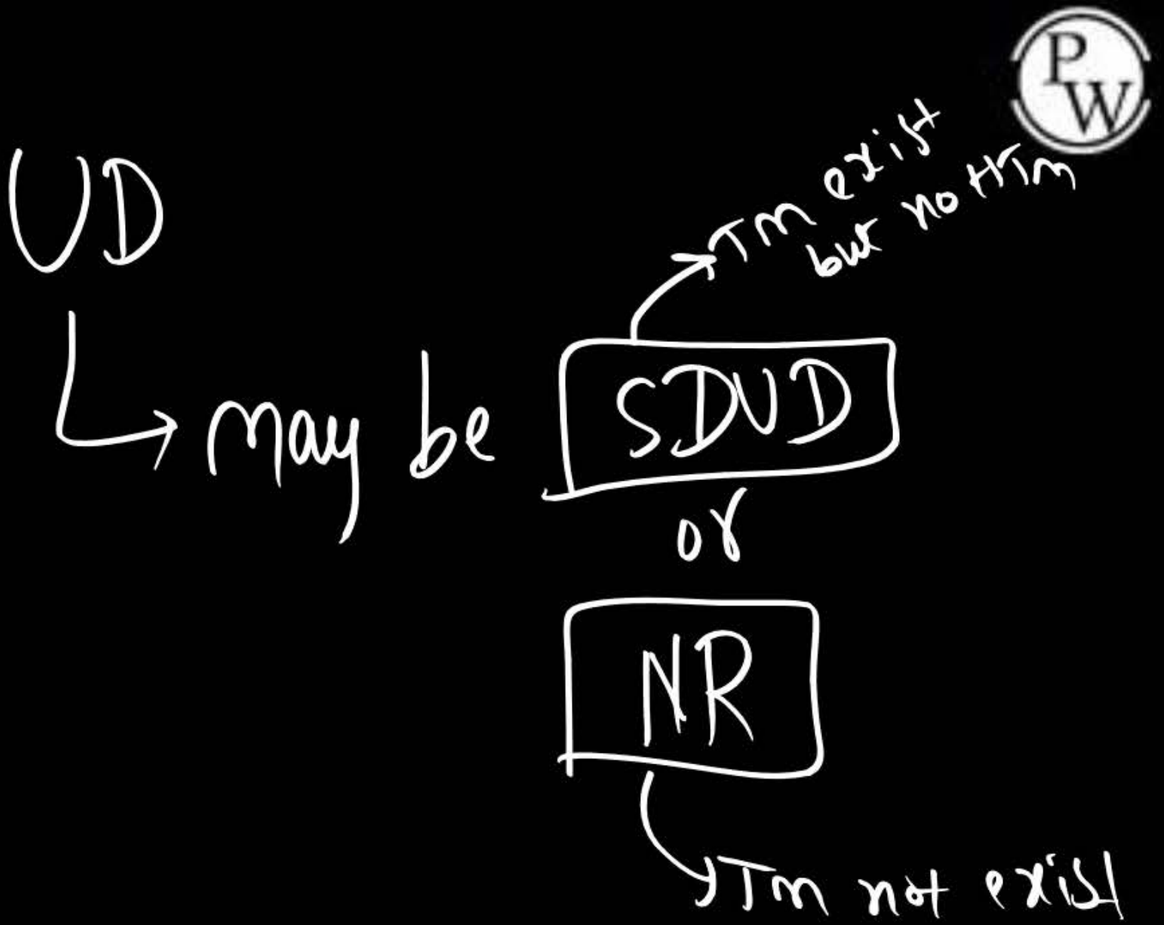
$= \overline{SDUD}$ or $\overline{NR}$

$= NR$ or $(NR$ or $SDUD)$

$= UD$

Decision properties Table [Decision Problems]

	Problems	FA / regular lang / regular RG	DPDA / DCFLs	PDA / CFLs / CFGs	HTM (LBA) / Recursive lang.	TM / RELs / UGs
①	Halting problem	D	D	D	D	SDUD → Which UD? (NR / SDUD)
②	Membership problem	D	D	D	D	?
③	Emptiness	D	D	D	?	?
④	Finiteness	D	D	D	?	?
⑤	Totality	D	D	?	?	?
⑥	Equivalence	D	D	?	?	?
⑦	Disjoint	D	?	?	?	?
⑧	Set containment	D	?	?	?	?


UD
may be
SDUD
TM exist but no HTM
or
NR
TM not exist

(1)$_a$ Halting problem

For FA:
- IS FA halts on string w?
- IS FA halts on ε?
- IS FA halts on a?
- IS FA halt on all?

Question: Is M (given) halts on w (given)?

OR

Statement: M halts on w.

(1)$_b$ Non Halting problem

Is M doesn't halt on ⊔?

OR

"M doesn't halt on ⊔"

(2)a Membership

Is M accepts string w ?

(M: given, string w: given)

OR

Is w accepted by M ?

OR

Is $w \in L(M)$?

OR

M accepts w

(2)b Non membership

Is M doesn't accept w ?

Is w not accepted by M ?

Is $w \notin L(M)$?

M doesn't accept w.

(3)$_a$ Emptiness

given
Is [M] accepts ϕ ?

Is M accepts nothing ?

Is $L(M) = \phi$?

(3)$_b$ Non-emptiness

Is M not accepts ϕ ?

Is M accepts Something ?
atleast one string

Is $L(M) \neq \phi$?

④a Finiteness

Is M accepts finite language?

Is $L(M) =$ finite lang?

④b Non-finiteness (Infiniteness)

Is $L(M) \neq$ finite lang?

Is $L(M) =$ Infinite lang?

(5)a Totality

IS M accepts everything?

IS $L(M) = \Sigma^*$?

(5)b Non totality

IS M not accepts everything?

IS M not accepting atleast one string?

IS $L(M) \neq \Sigma^*$

$\Sigma^* = \phi$

Language

IS M accepts Σ^* ?

Problem

= IS M not accepts Σ^* ?

IS $L(M) \neq \Sigma^*$

(6a) Equivalence

IS $M_1 \cong M_2$?

IS $L(M_1) = L(M_2)$?

(6b) Non equivalence

IS $M_1 \ncong M_2$?

IS $L(M_1) \neq L(M_2)$?

(7)a Disjointness

IS $L(M_1) \cap L(M_2) = \phi$?

(7)b Non-Disjointness

IS $L(M_1) \cap L(M_2) \neq \phi$?

(8)a Set Containment (Subset Checking)

IS $L(M_1) \subseteq L(M_2)$?

IS $L_1 \subseteq L_2$?

L_1 given, L_2 given

(8)b non Set Containment

IS $L_1 \not\subseteq L_2$?

Halting for FA

IS FA halts on ε?
Is FA halts on w?
Is FA halts on ab?

Halting for DPDA

IS DPDA halts on w?

Halting for PDA

IS PDA halts on w?

Halting for HTM

Is HTM halts on w!

By definition all these machines always halts

$\Downarrow$

Decidable

Halting problem for TM

Logic exist

Is TM halts on ε (w) ?

Yes : TM halts on ε

No : TM doesn't halt on ε

Logic not exist

Undecidable

SDUD

Non halting

Is TM doesn't halt on w ?

Yes : ⇨ logic not exist

No :

Undecidable

Not RE

Halt
Logic exist

Doesn't Halt
Never Halt
Inf loop

Logic not exist

Summary

- Decidable & Undecidable problems
- Next: Membership problem

Problems
Languages


THANK YOU
GW
SOLDIERS !