CS & IT ENGINEERING

Theory of Computation

Push Down Automata

Lecture No. 5

By- DEVA Sir

01 DPDA

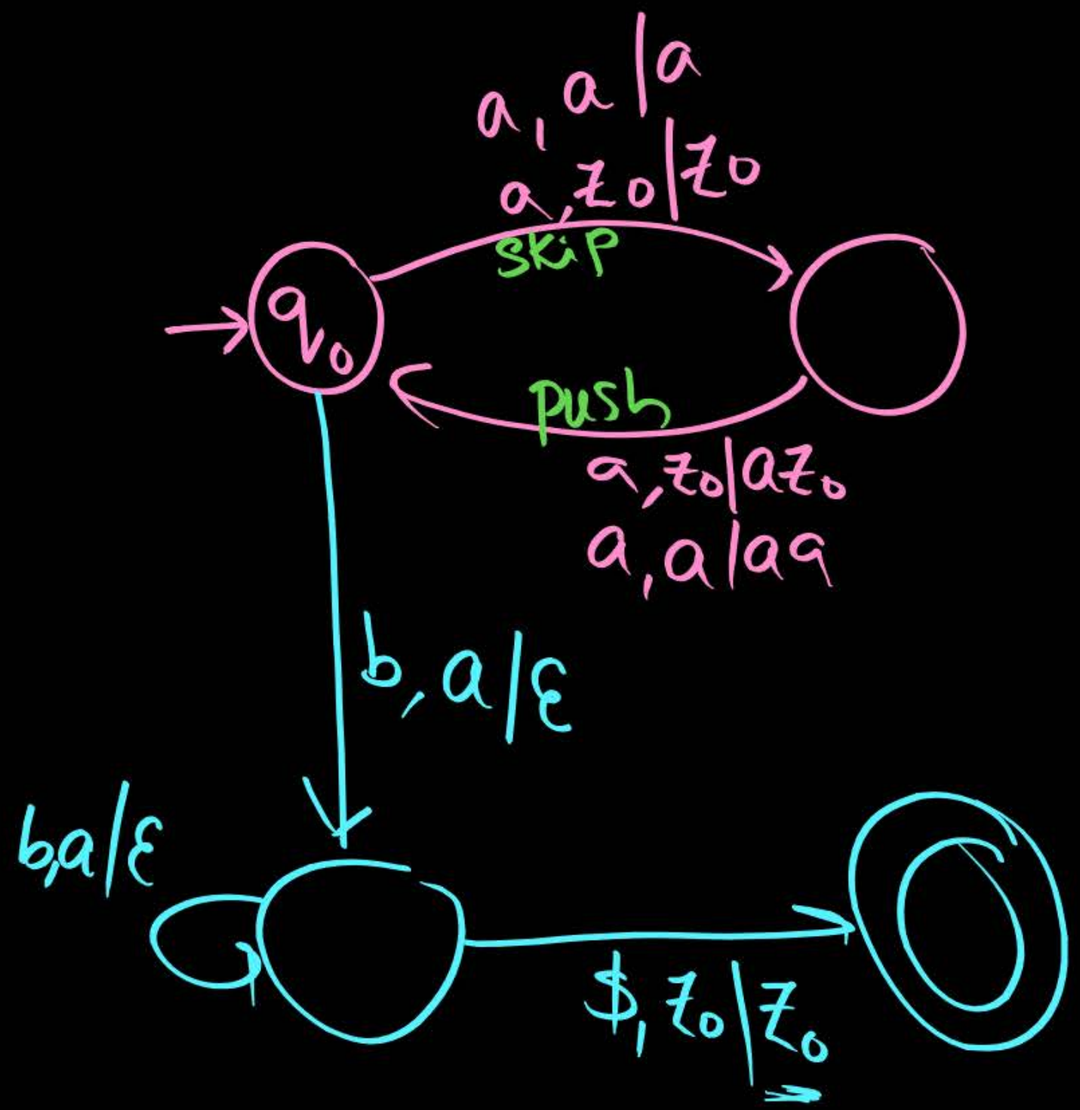
02 PDA

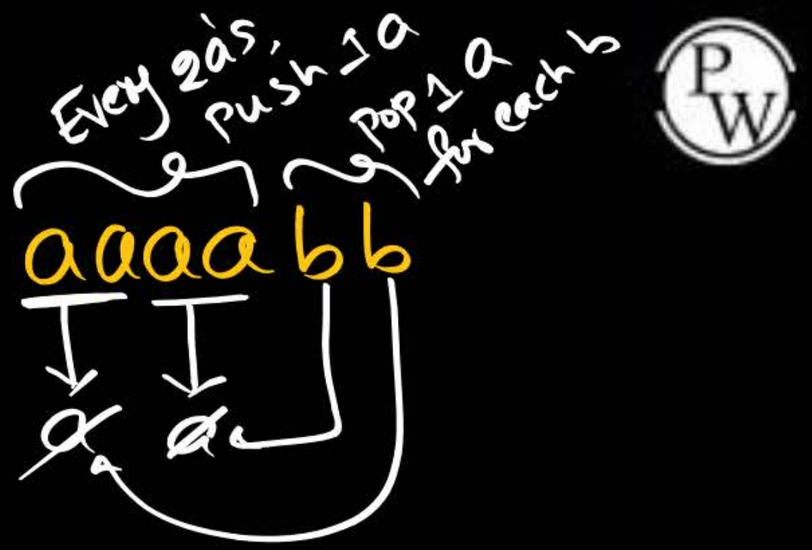
03

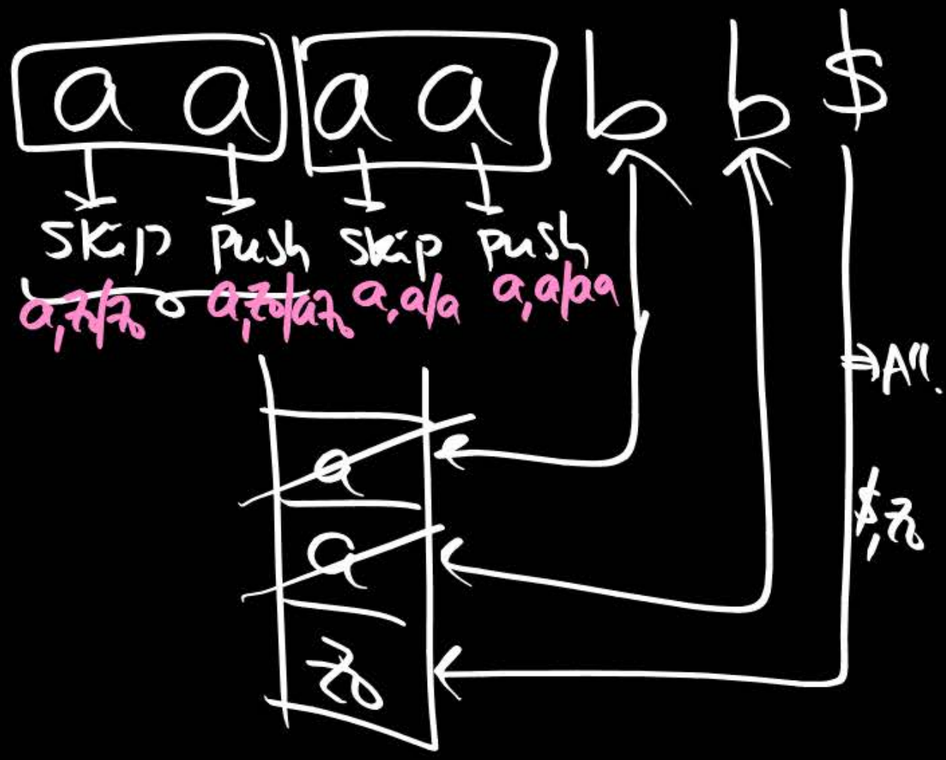
04

05

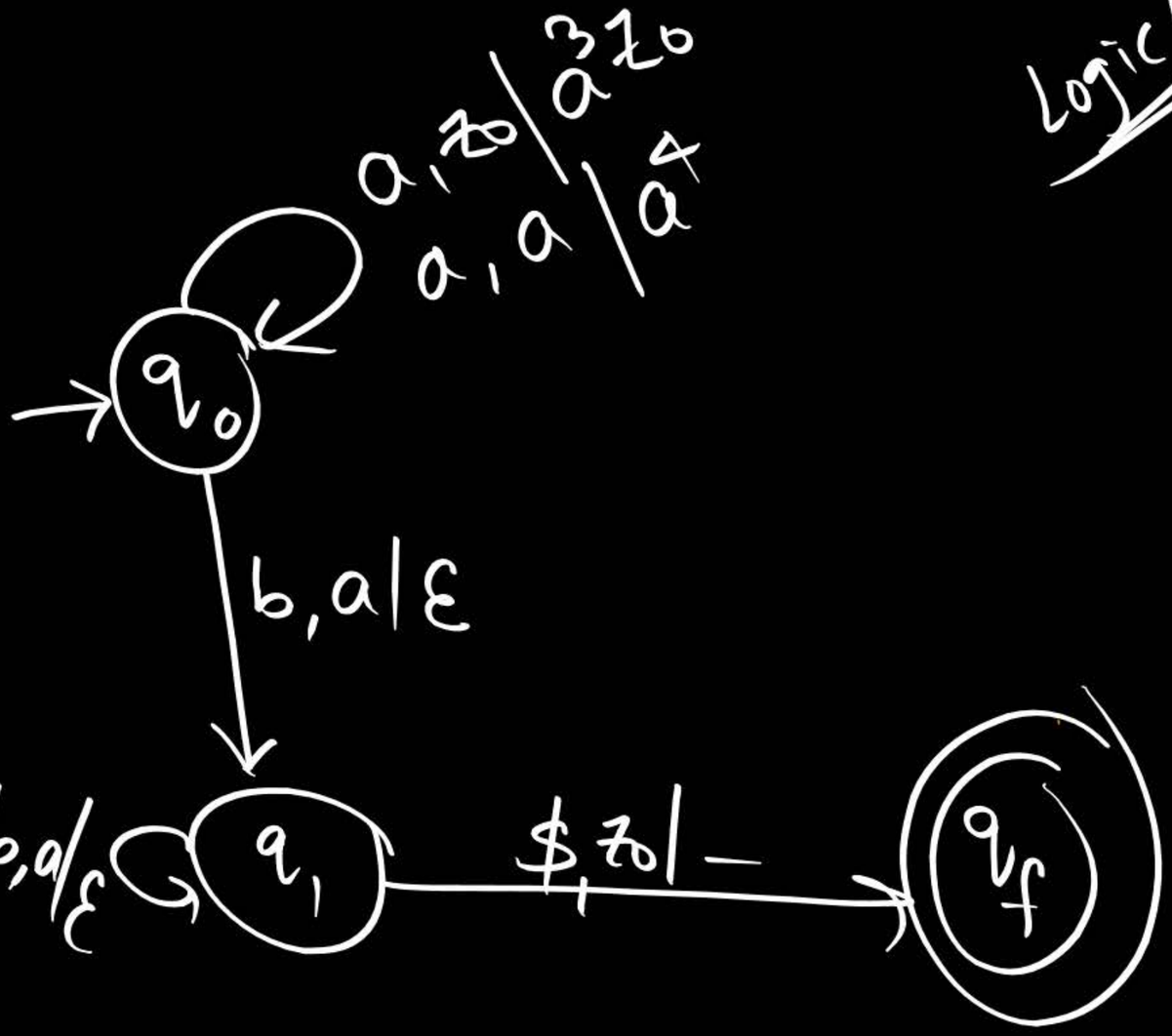
(14) $\{a^{2n}b^{n} \mid n \geq 1\} = \{a^{m}b^{n} \mid m = 2n,\ m,n \geq 1\}$







(15) $\{a^n b^{3n} \mid n \geq 1\}$



Logic 1: Push 3a's for each a

Every b, pop 1a

aabbbbbb $

a, z0 / aaa z0

a, a / aaaa

Accept

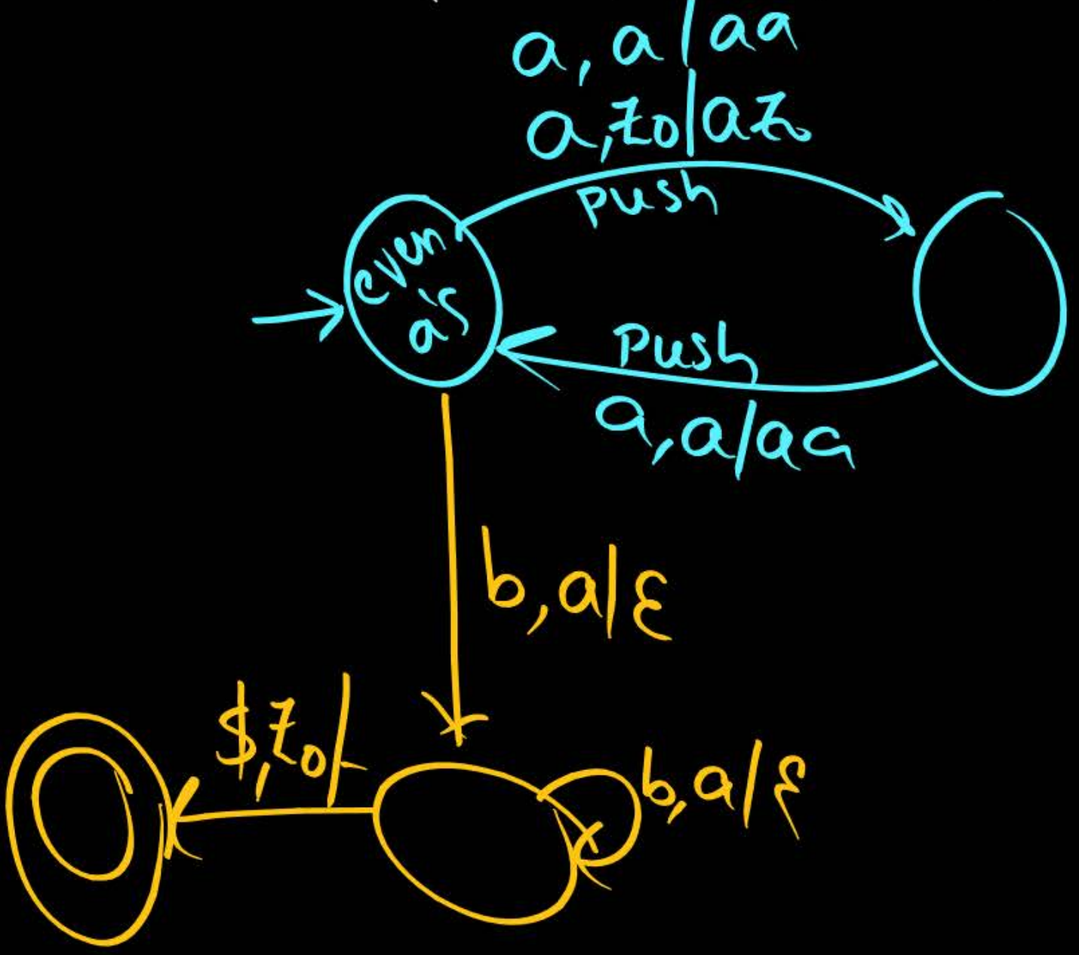
Logic 2: every 3b's, pop 1a

aabbbbbb

Push 1a for each a

(16) $\{a^{2n}b^{2n} \mid n \geq 1\} = \{a^m b^n \mid m,n \geq 1, \underbrace{m=n=\text{even}}\}$

$= \{a^2b^2, a^4b^4, a^6b^6, a^8b^8, \ldots\}$



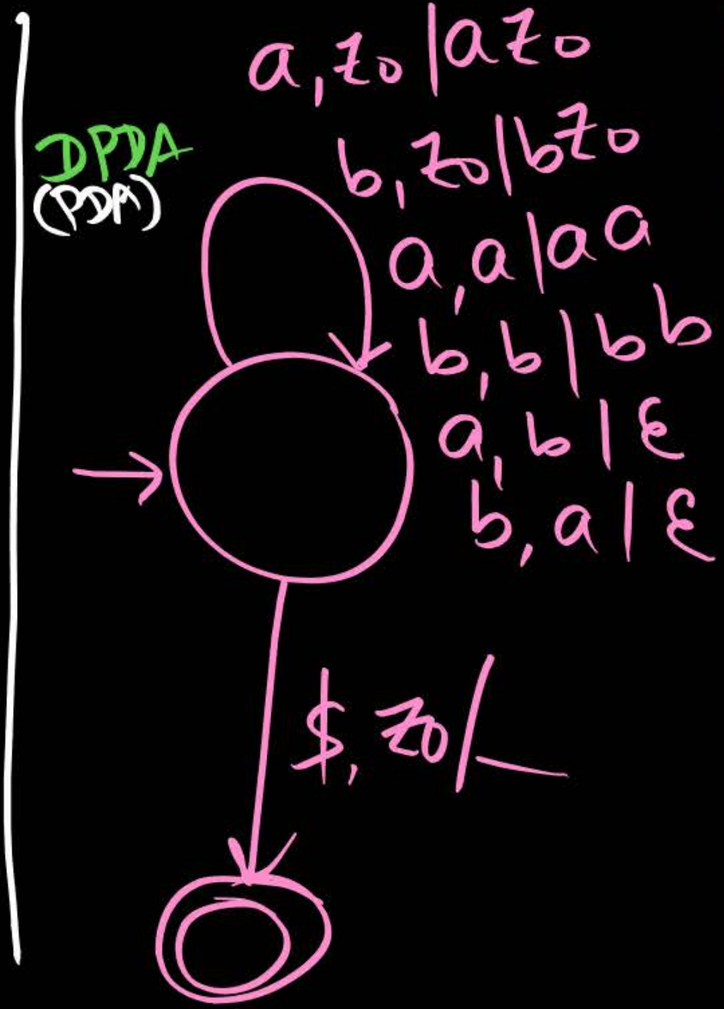
aaaabbbb$

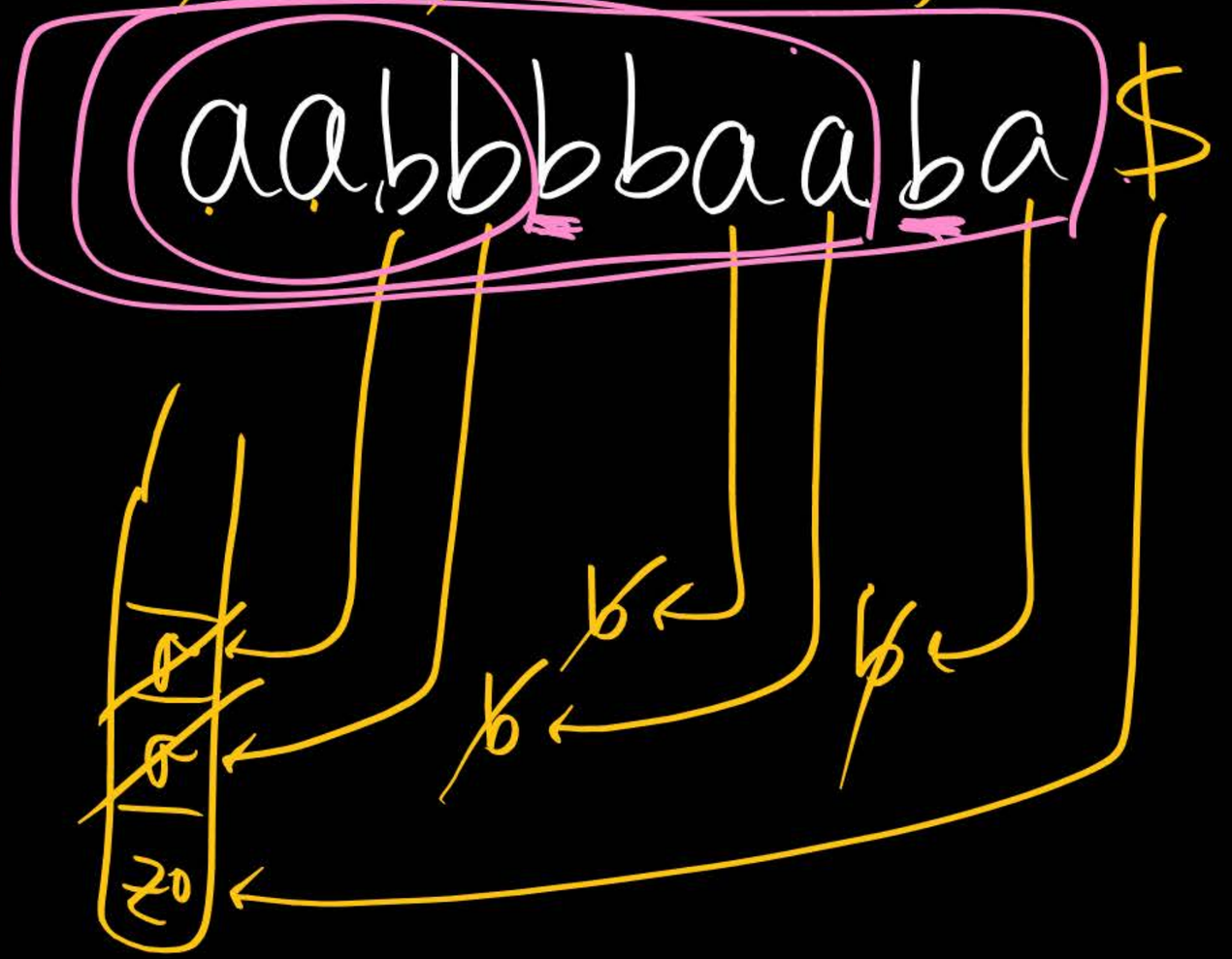
Push 1a for each a & guarantee even a's

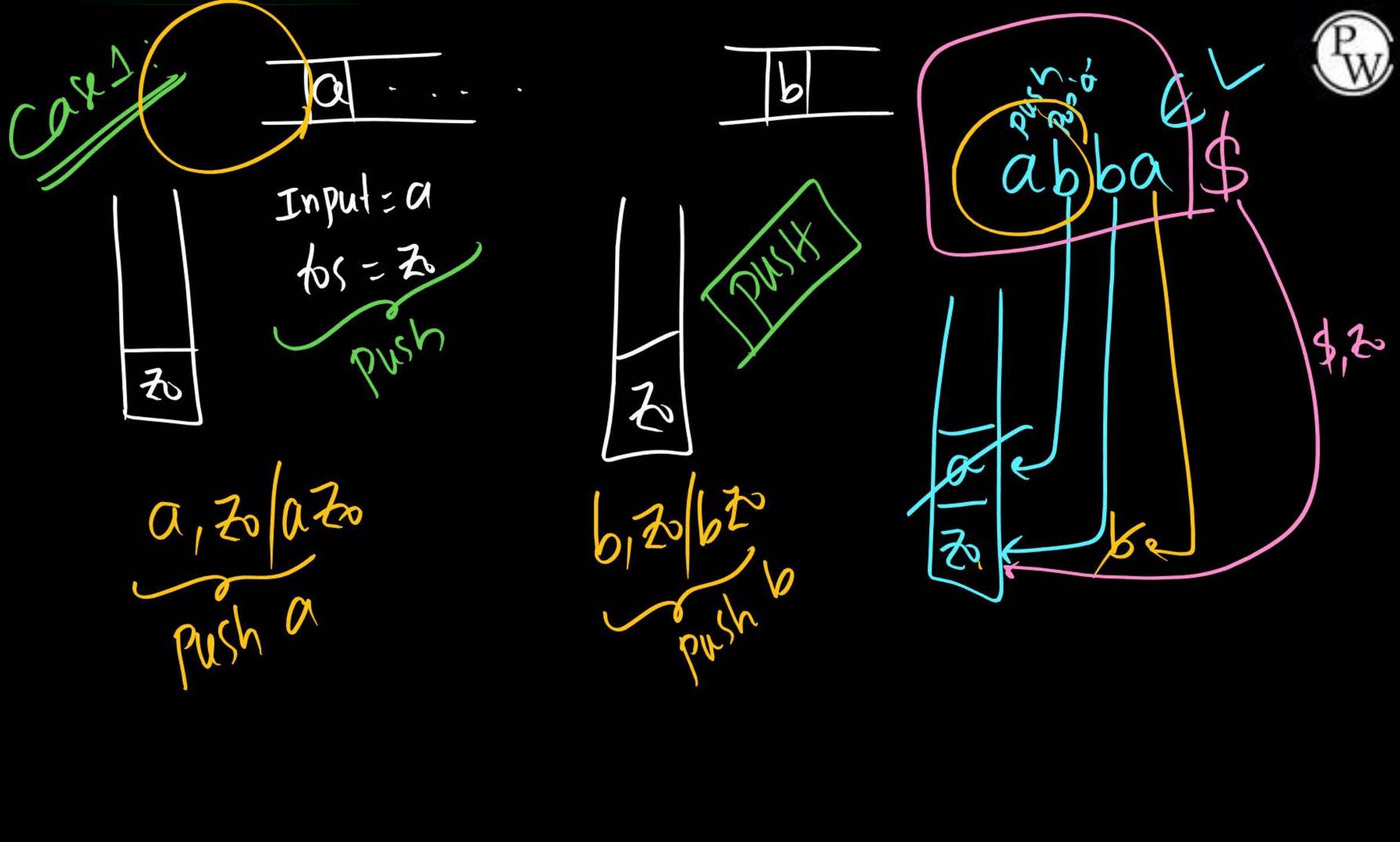
Every b, pop 1a

(17) $\{ w \mid w \in \{a,b\}^*, n_a(w) = n_b(w) \}$

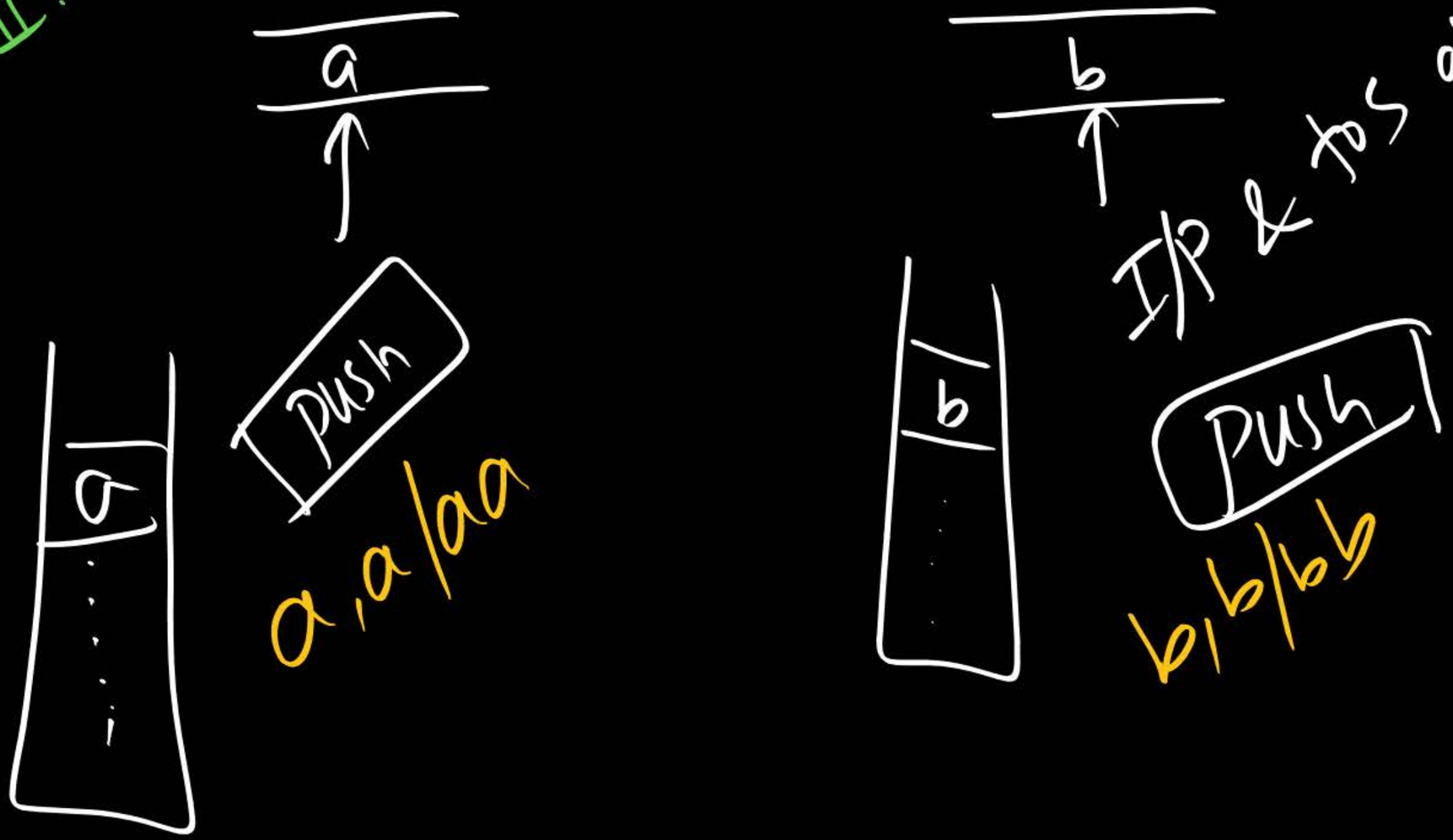
$= \{ \varepsilon, ab, ba, aabb, abab, baab, baba, bbaa, \ldots \}$



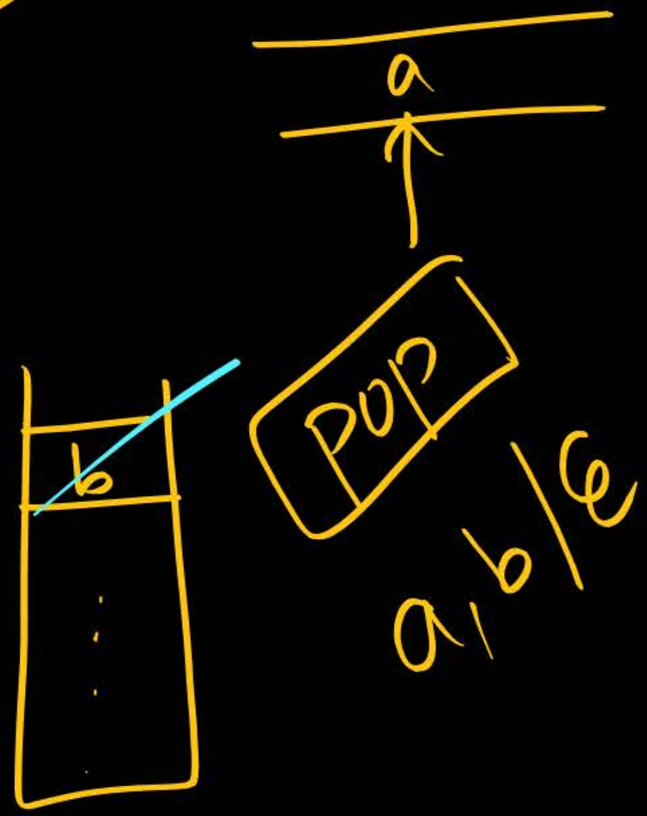



Case 1:
a
b
Input = a
tos = Z0
Push
Z0
PUSH
Z0
a, Z0/aZ0
Push a
b, Z0/bZ0
push b
abba
$
$, Z0
a
b
Z0

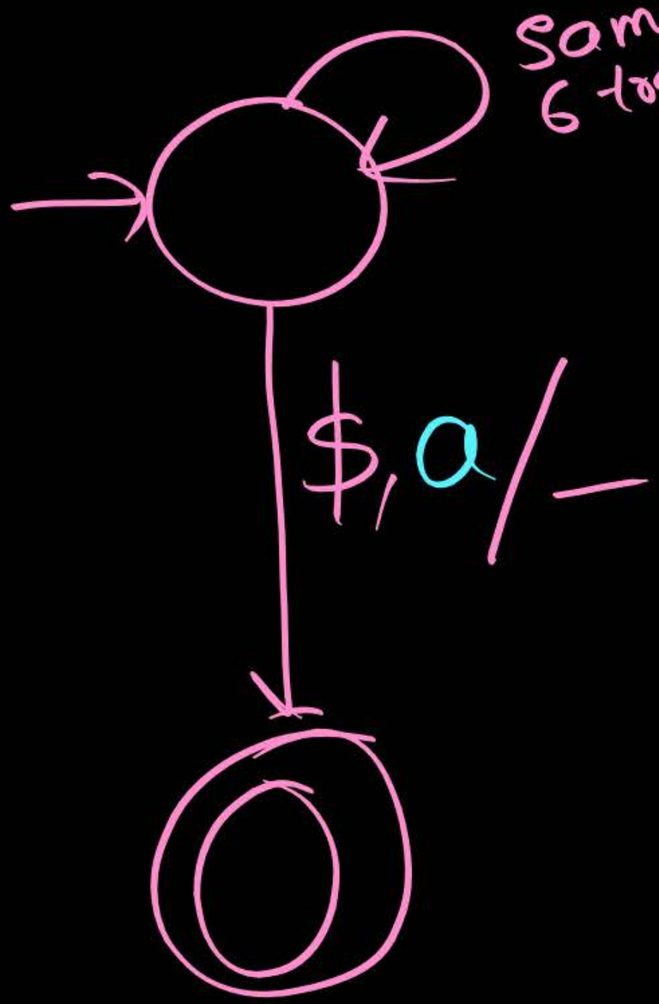
Case II:



Case III

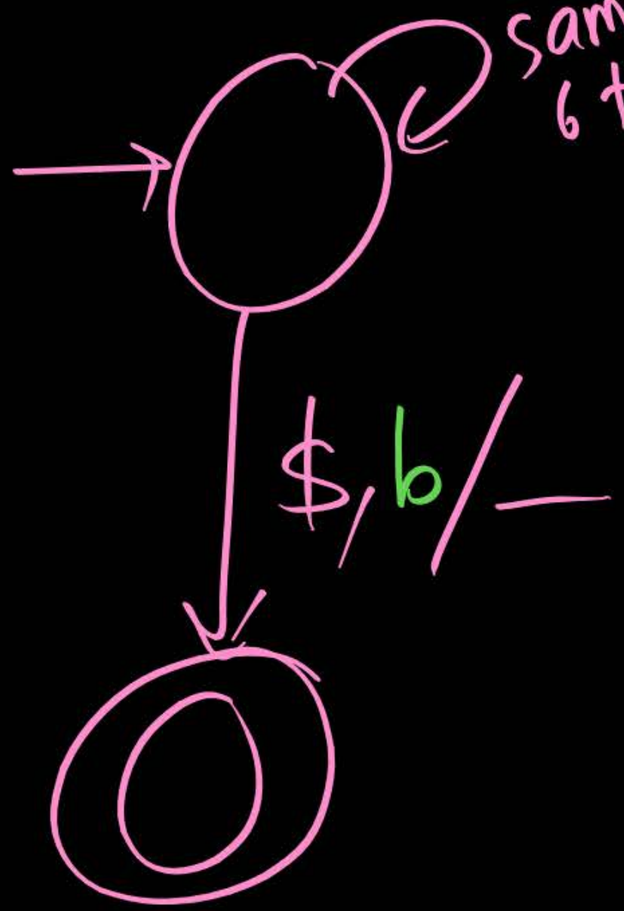


(18) $\{w \mid w \in \{a,b\}^*, \underbrace{n_a(w)}_{\text{more}} > n_b(w)\}$



At the end of i/p string, stack has a's

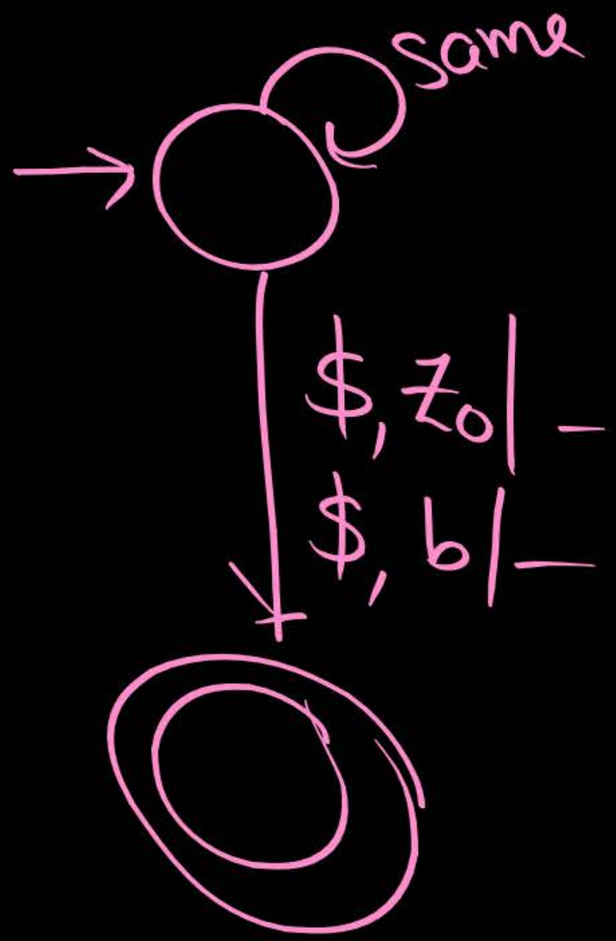
(19) $\{w \mid w \in \{a,b\}^*, n_a(w) < \underbrace{n_b(w)}_{\text{more}}\}$



At end, stack left with b's

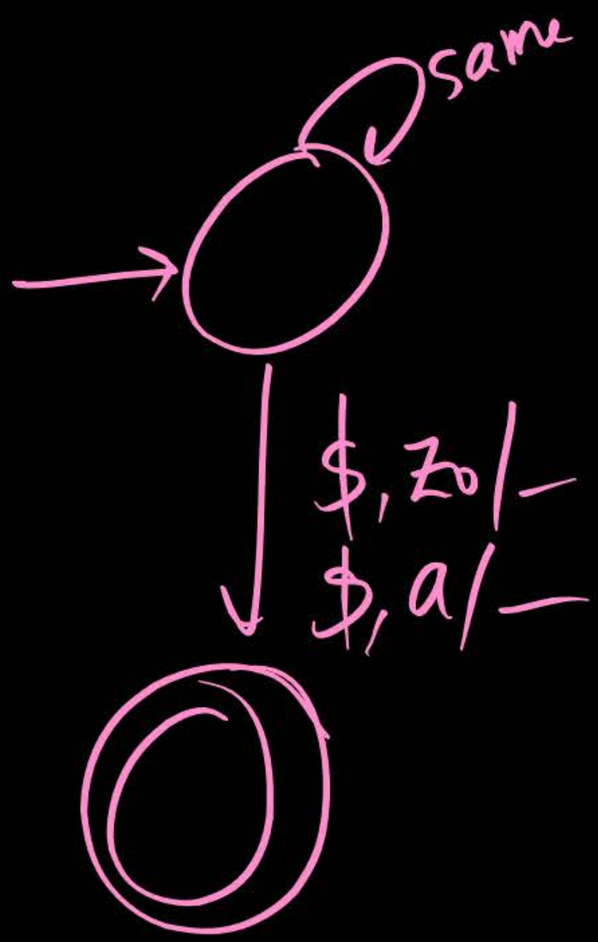
(20) $\{w \mid w \in \{a,b\}^*, n_a(w) \leq n_b(w)\}$

equal or more b's



(21) $\{w \mid w \in \{a,b\}^*, n_a(w) \geq n_b(w)\}$

equal or more a's



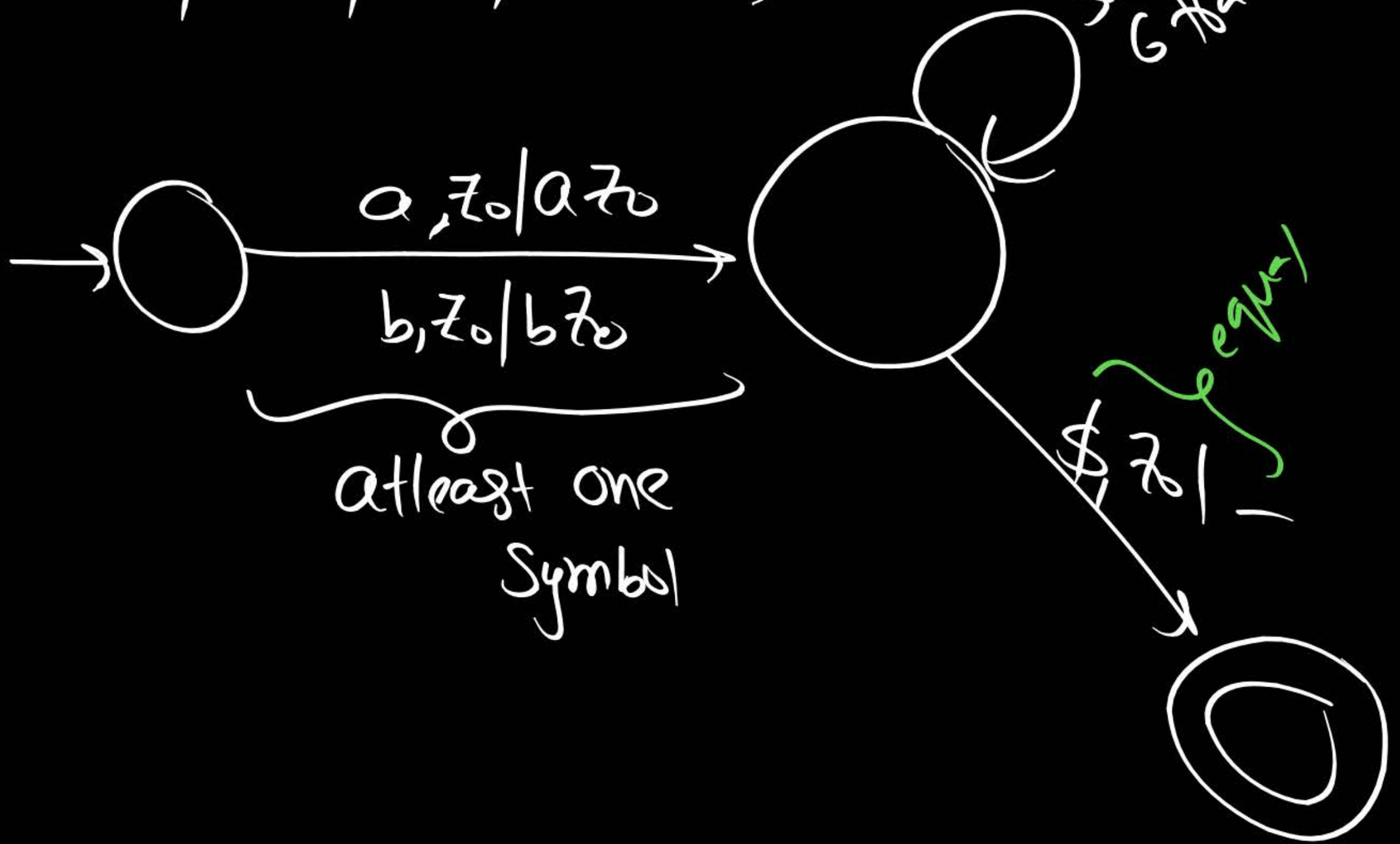
(22) $\{w \mid w \in \{a,b\}^*, n_a(w) \neq n_b(w)\}$

a's more OR b's more

Note:

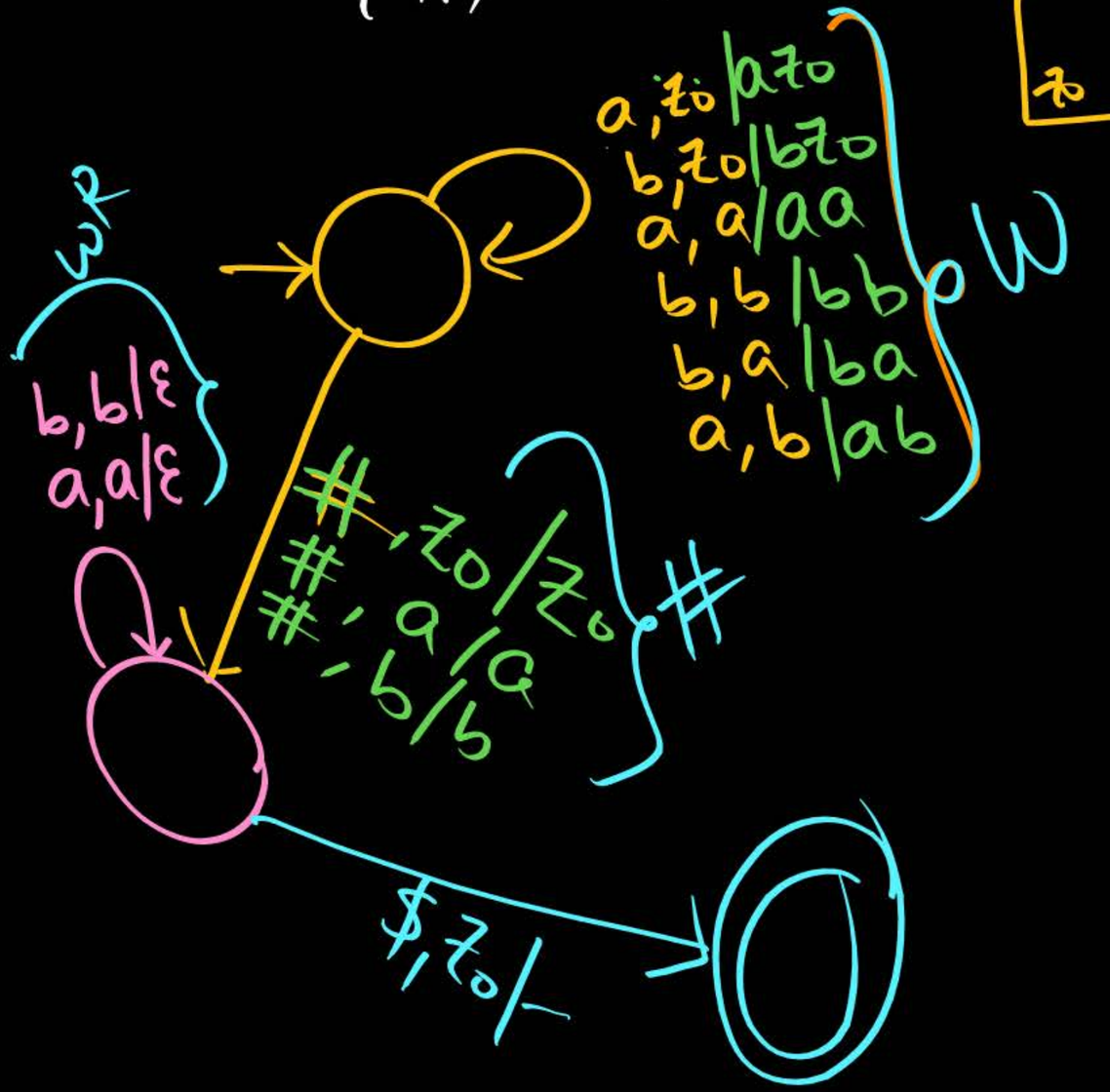
$$L = \{ w \mid w \in \{a,b\}^+, \underbrace{n_a(w) = n_b(w)}_{\text{equal}} \}$$

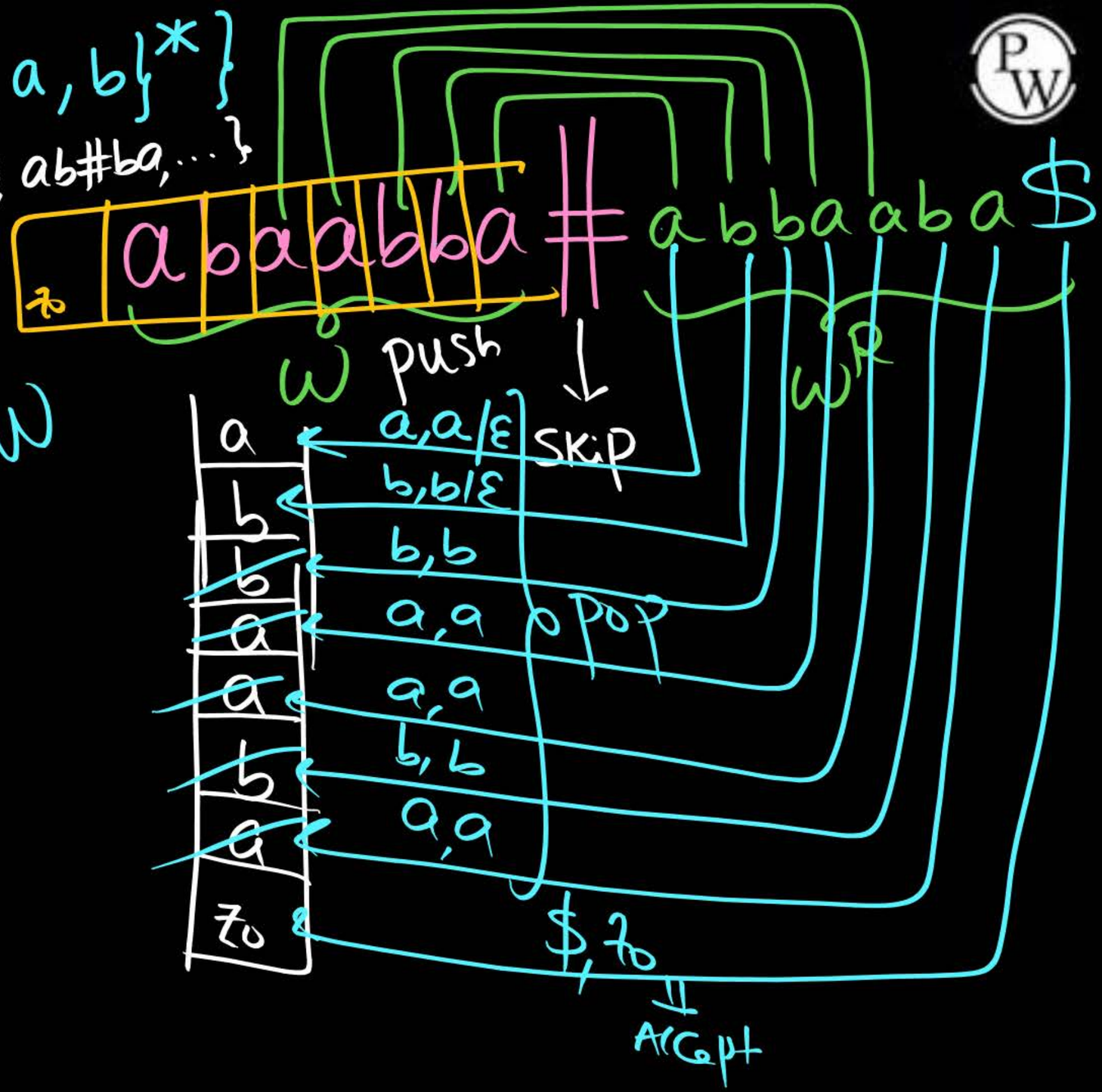
$$= \{ ab, ba, \ldots \}$$



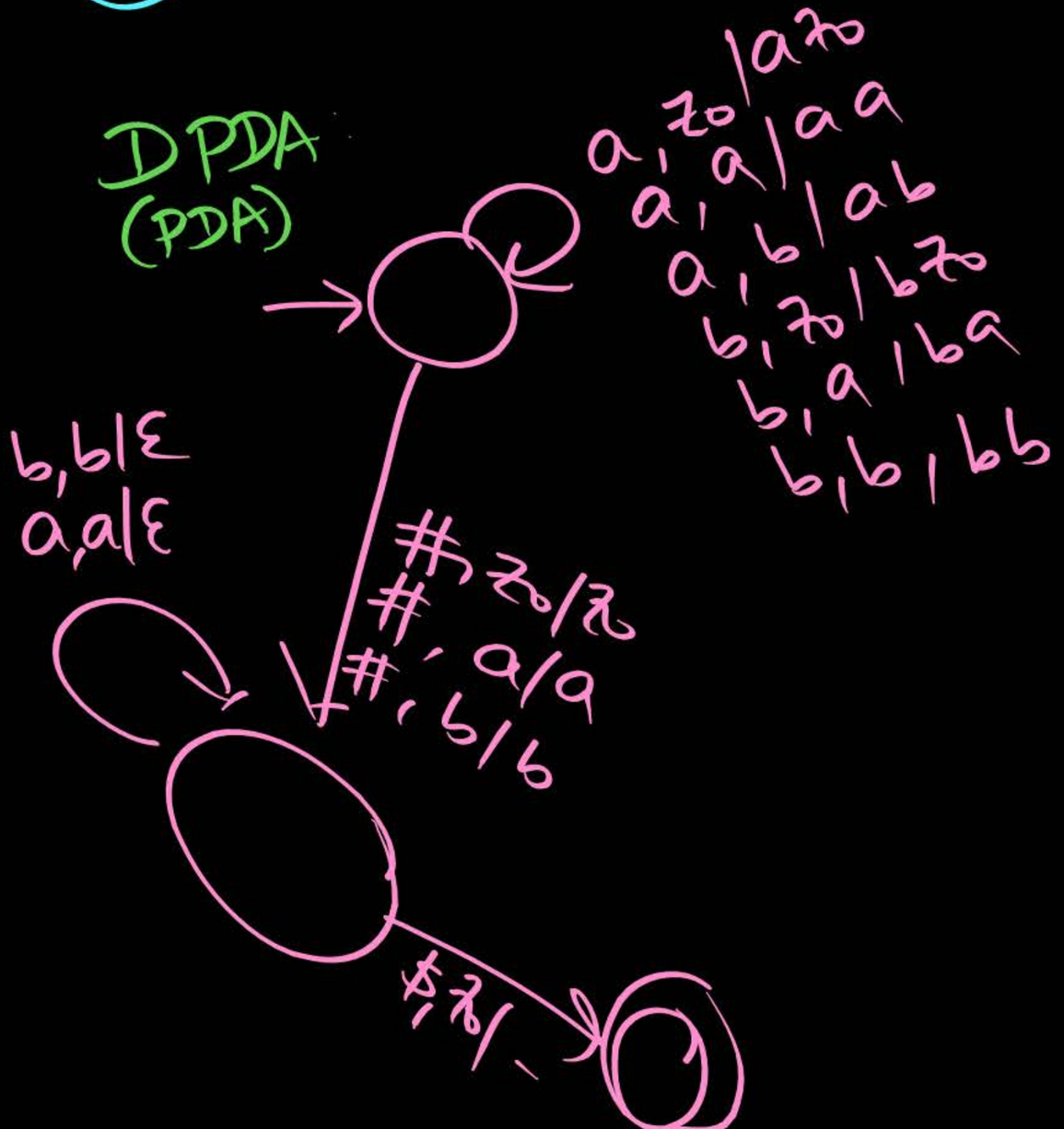
**(23) $L = \{ w \# w^R \mid w \in \{a,b\}^* \}$

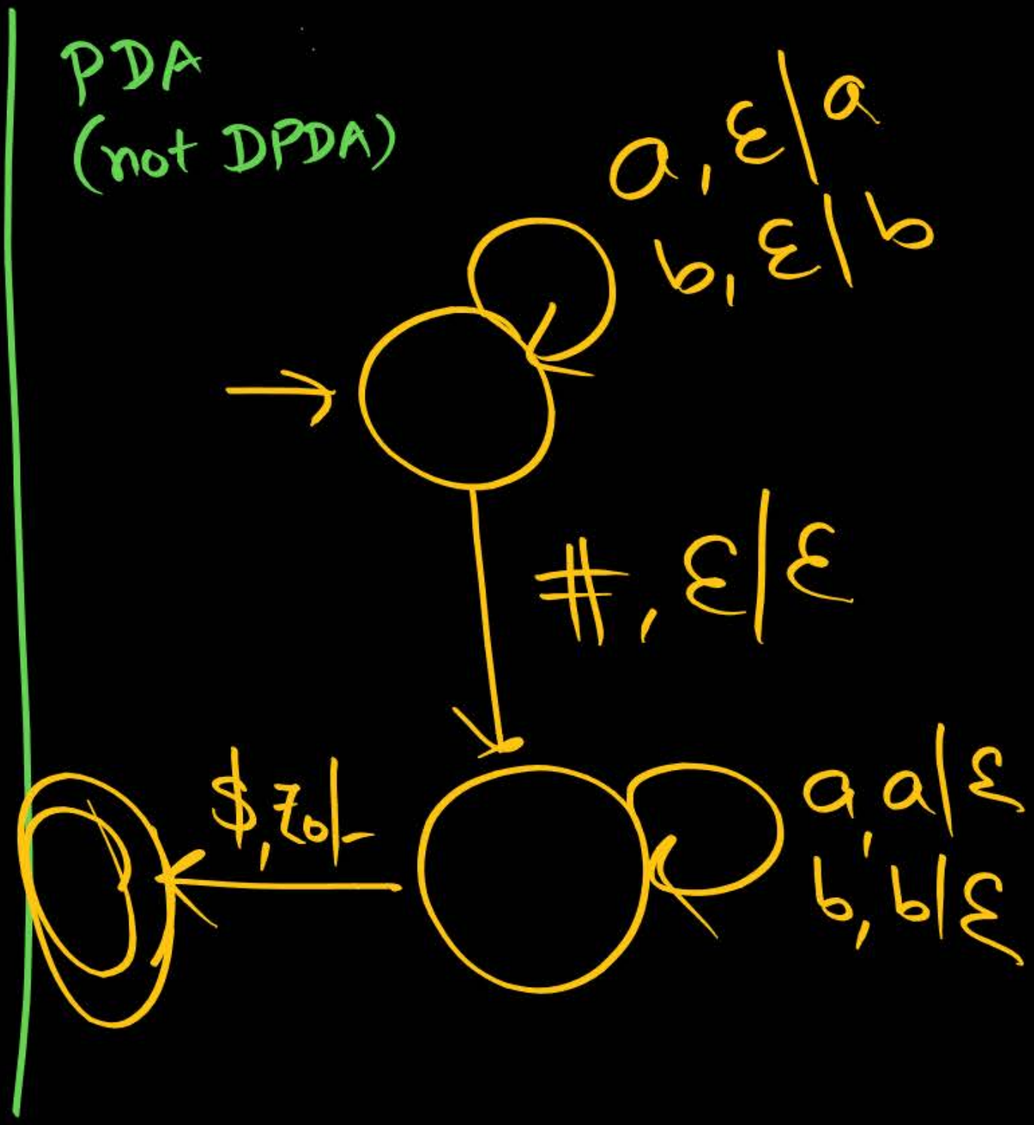
$= \{ \#, a\#a, b\#b, aa\#aa, ab\#ba, \ldots \}$





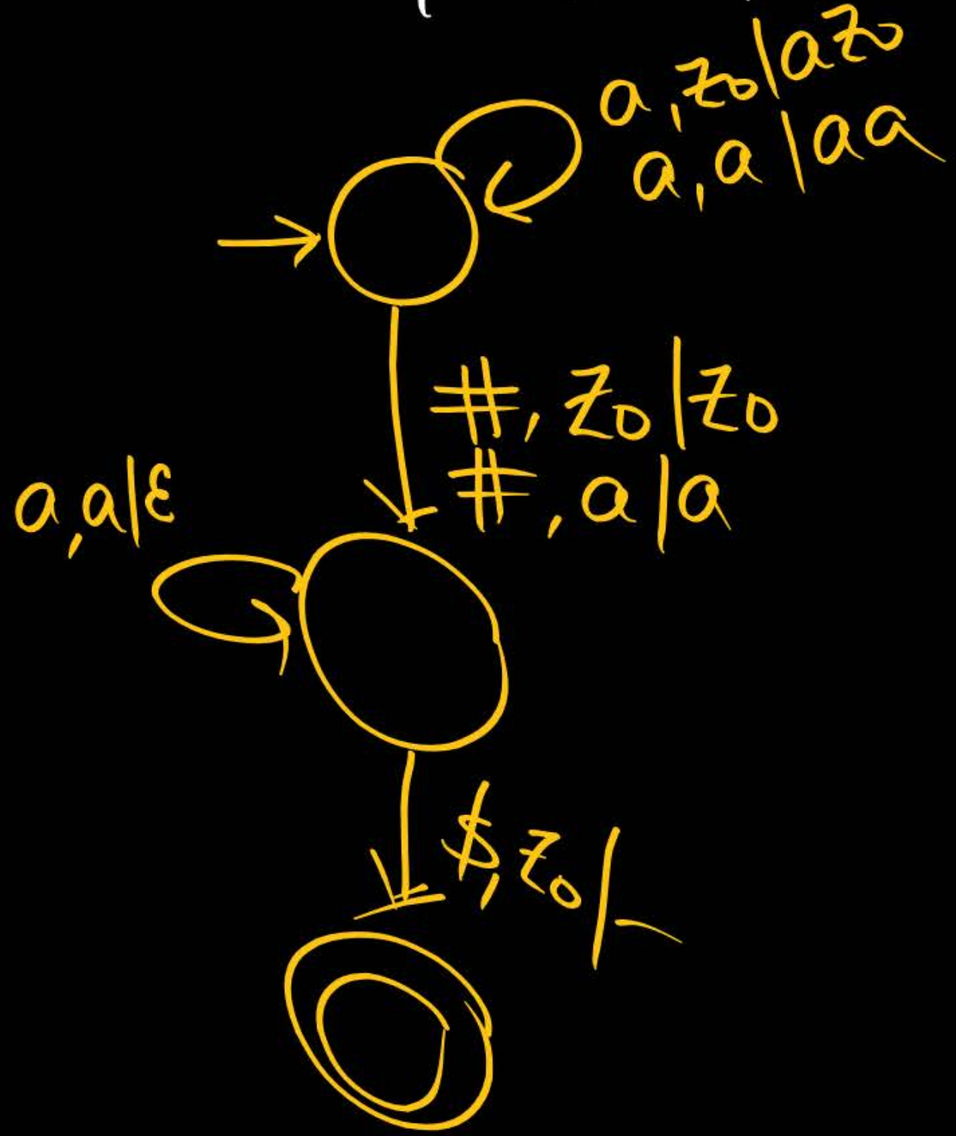
(23) $\{w\#w^R \mid w \in \{a,b\}^*\}$

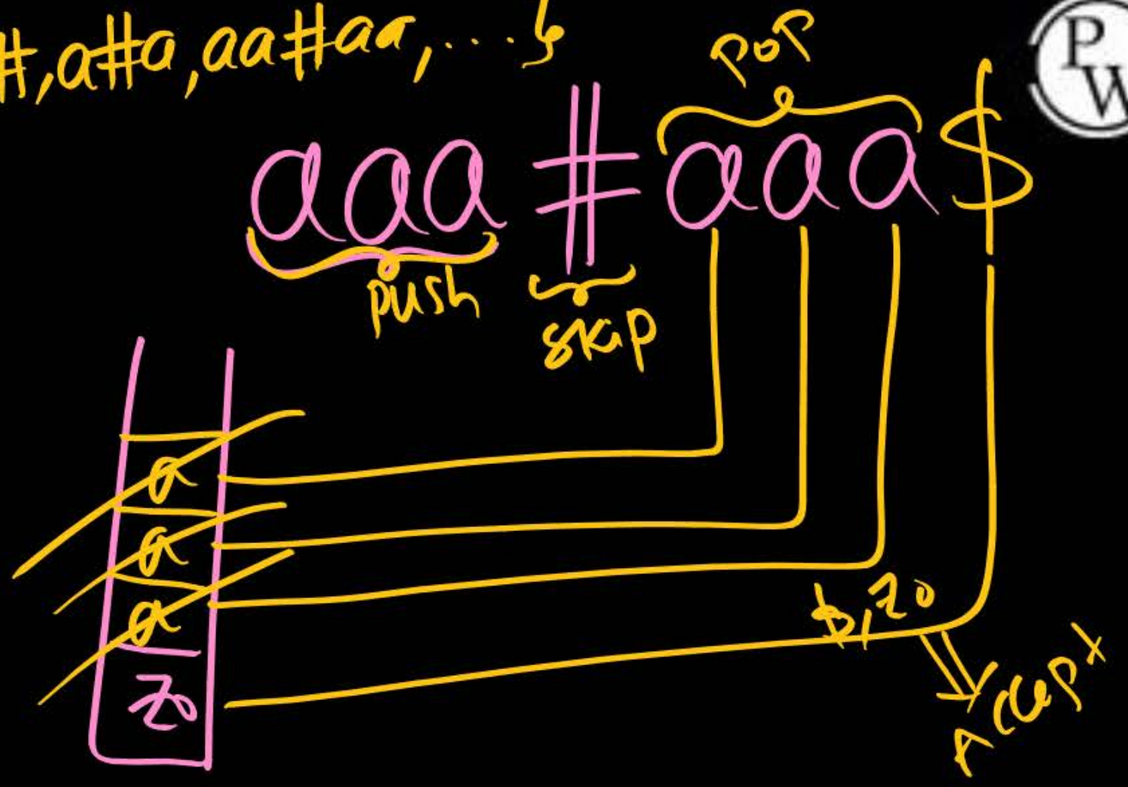




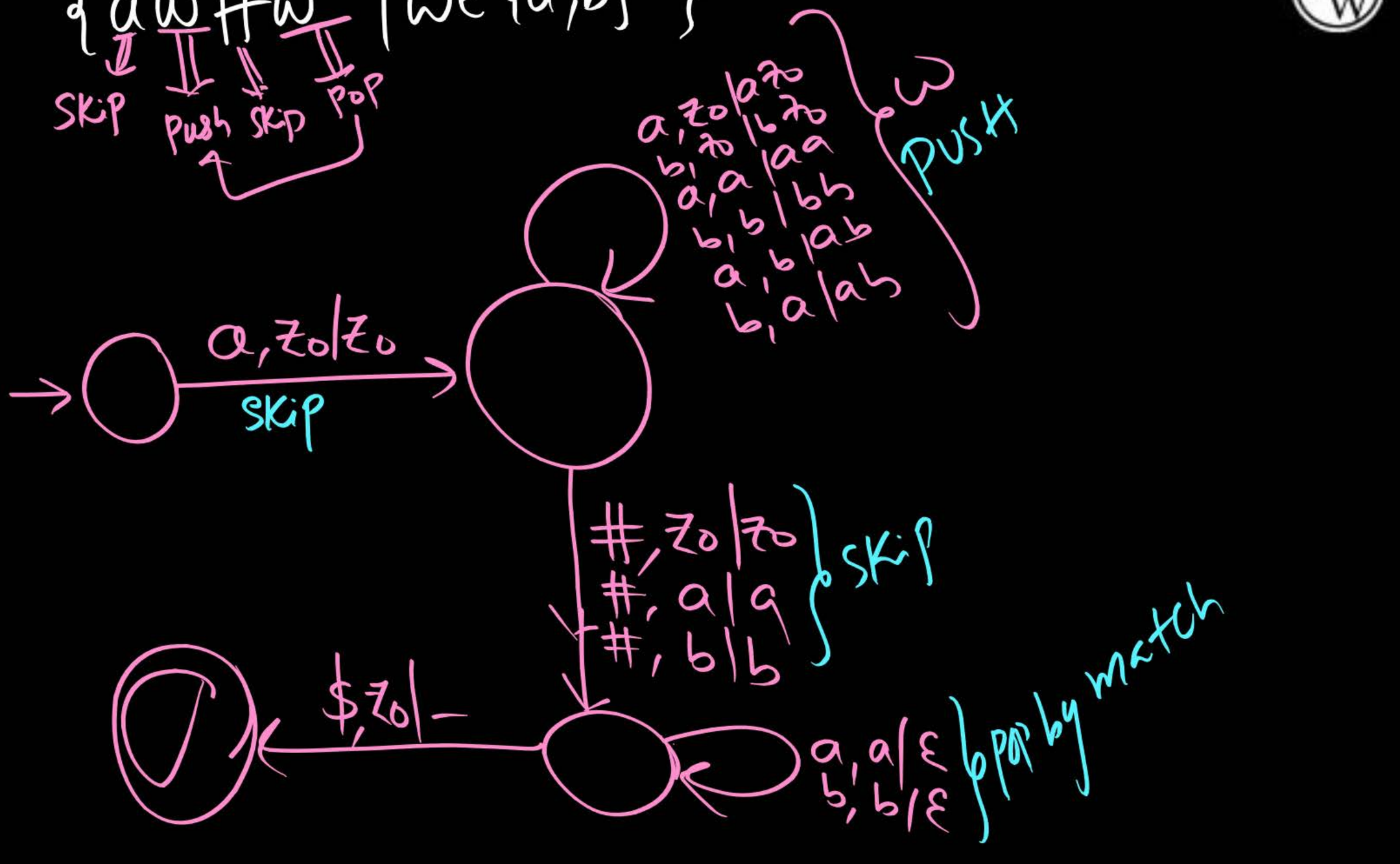
(24) $L = \{a^n \# a^n \mid n \geq 0\} = \{\#, a\#a, aa\#aa, \ldots\}$

$= \{w \# w^R \mid w \in a^*\}$





(25) $\{aw\#w^R \mid w \in \{a,b\}^*\}$



(1) ⟶ (25) Languages

A) DCFLs

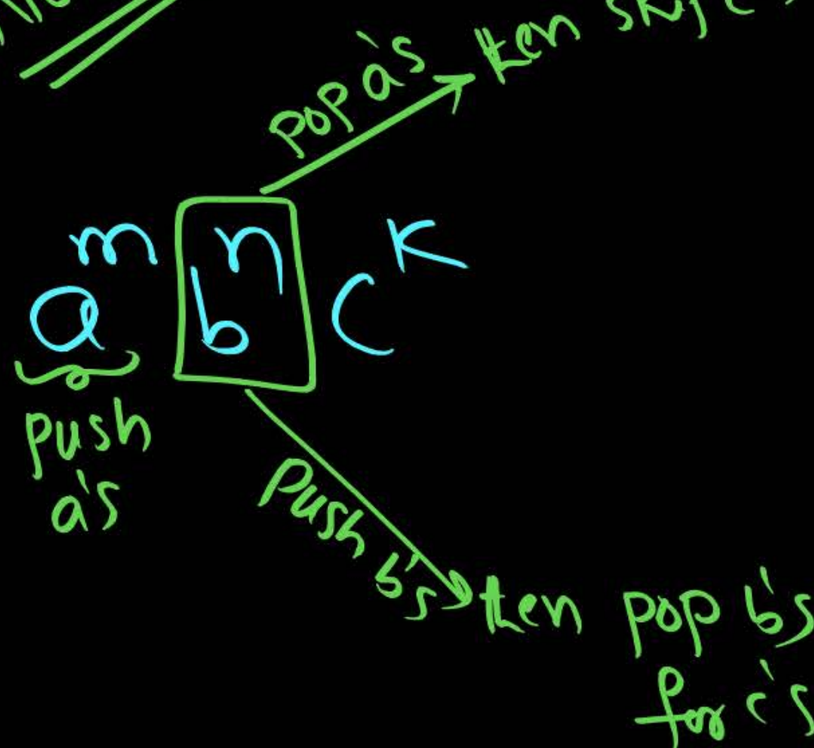
B) CFLs

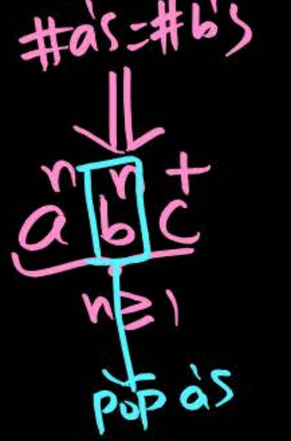
C) Some are not DCFLs

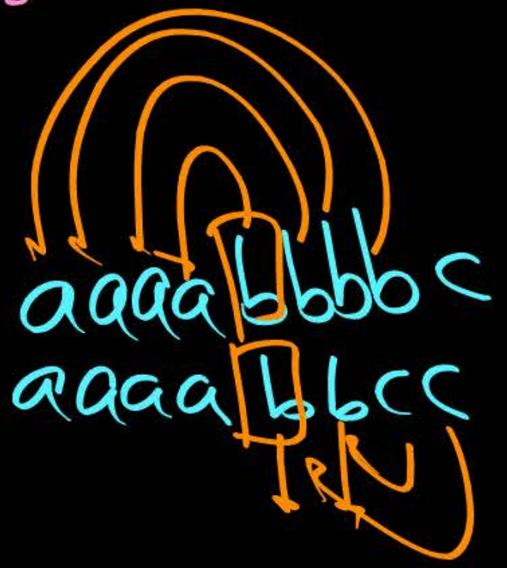
D) Some are not CFLs

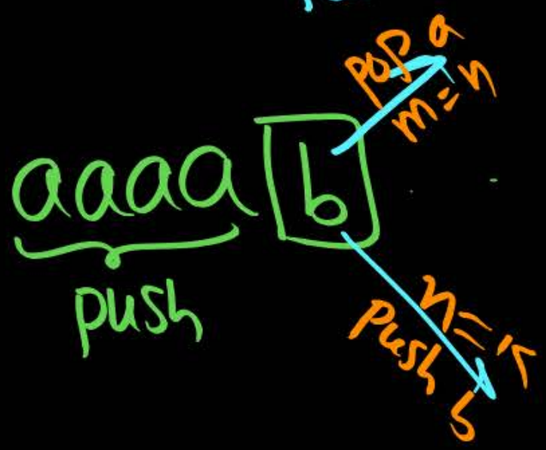
(26) No DPDA $\{a^m b^n c^k \mid m,n,k \geq 1,\ m=n \text{ OR } n=k\}$

CFL but not DCFL



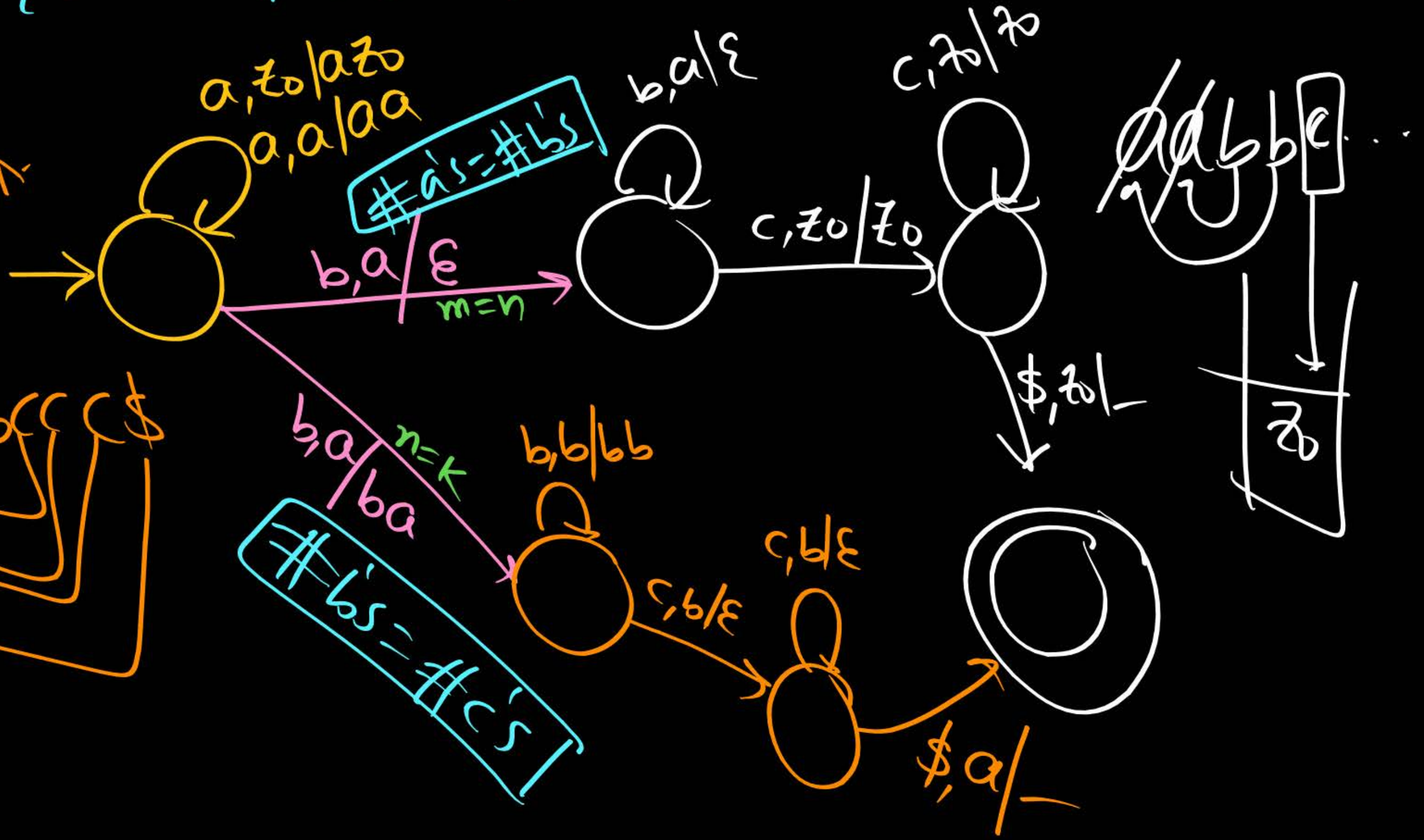




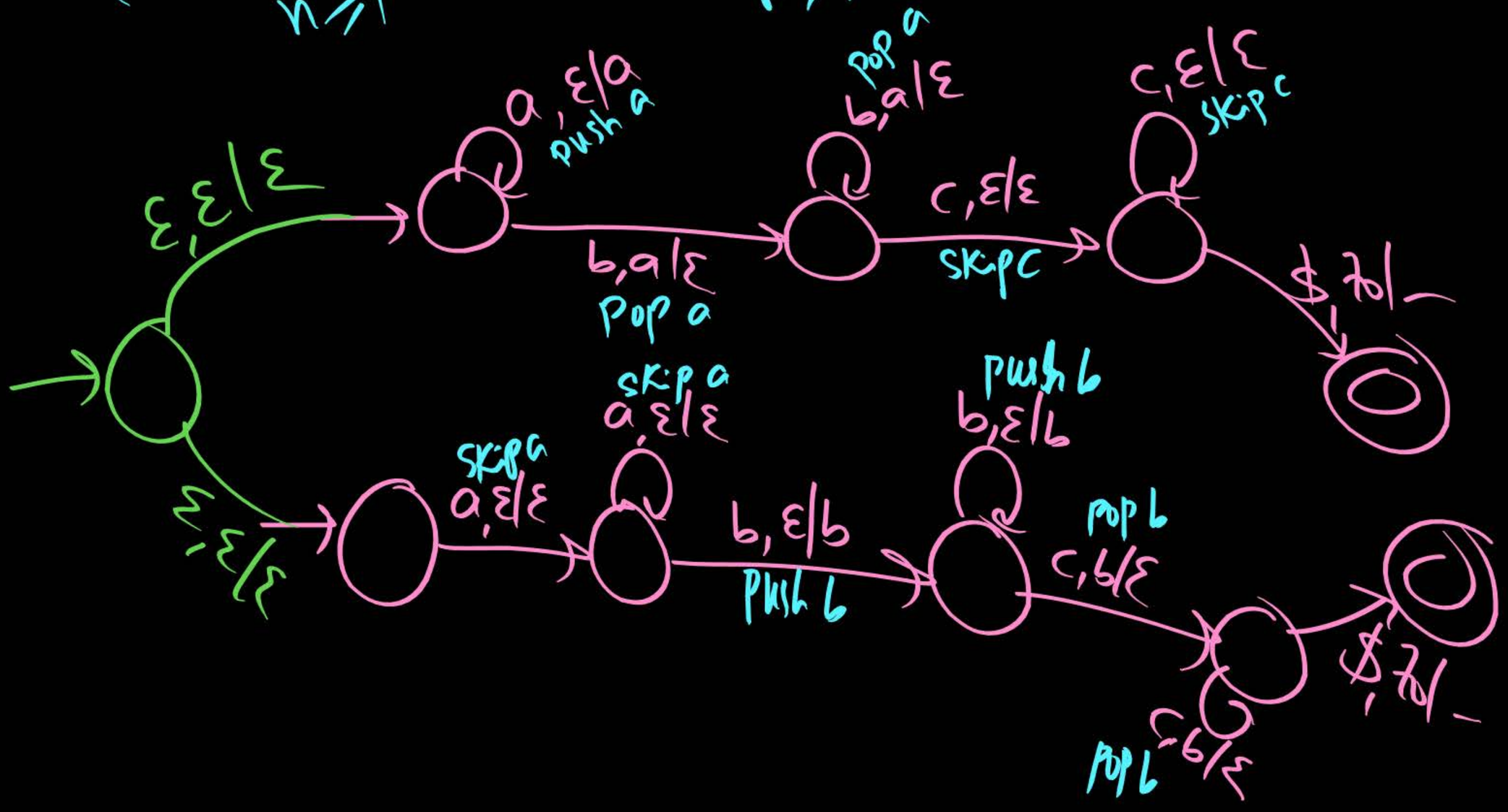


(26) $\{a^m b^n c^k \mid m,n,k \geq 1,\ m=n \text{ OR } n=k\}$

It is PDA
It is not DPDA.



$$\{a^n b^n c^+ \mid n \geq 1\} \cup \{a^+ b^n c^n \mid n \geq 1\}$$



CFLs but not DCFL

(27) $\{a^m b^n c^k \mid m,n,k \geq 1,$ $m=n$ OR $m=k\}$

a's = b's

$a^n b^n c^+$, $n \geq 1$

a's = c's

$a^n b^+ c^n$, $n \geq 1$

Pop a's

Skip b's

without reading c's Pop b's then Every c, Pop a

(28) $\{a^m b^n c^k \mid m,n,k \geq 1,$ $m=k$ OR $n=k\}$

$\#a$'s = $\#c$'s

$\#b$'s = $\#c$'s

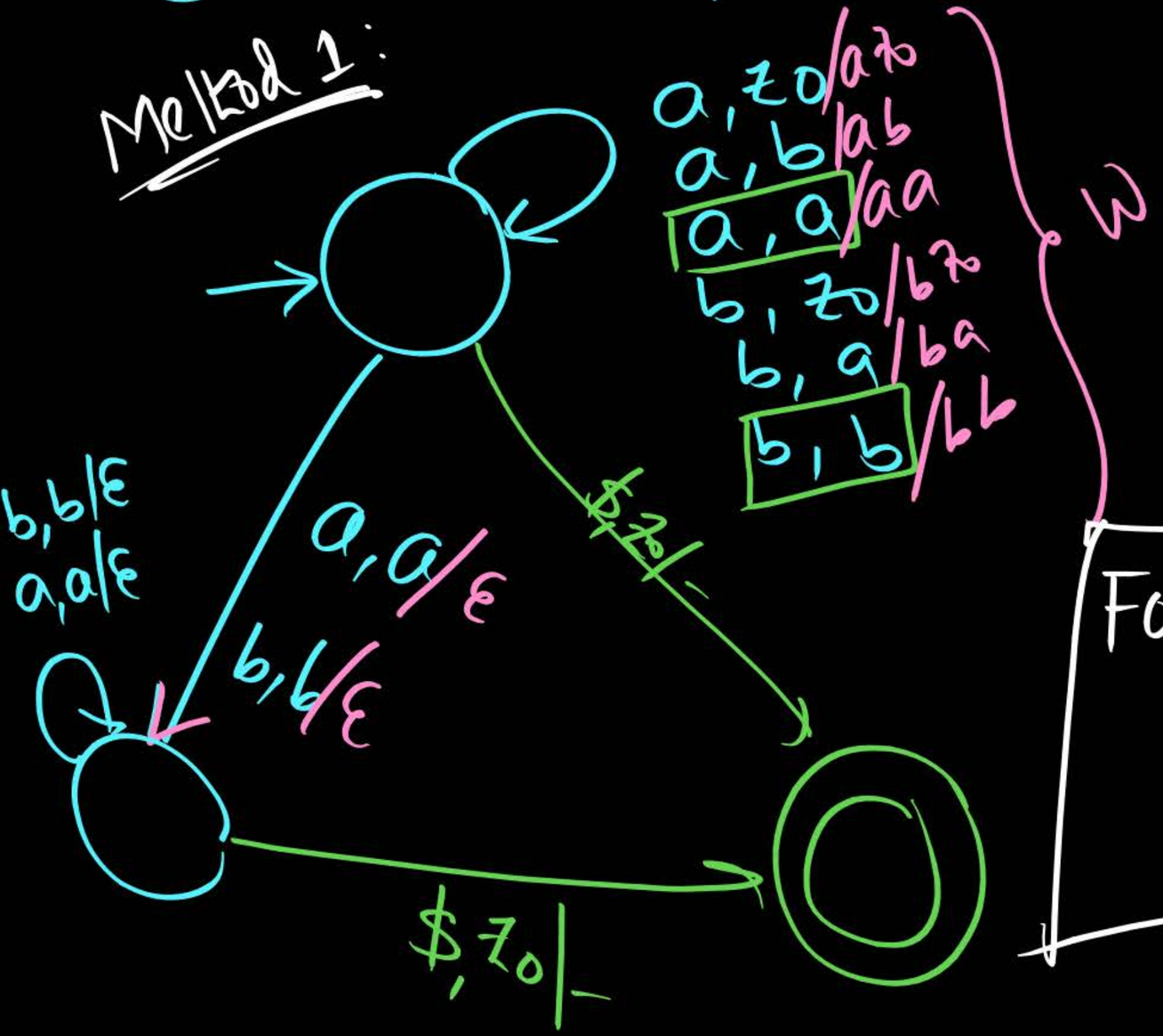
Push a's

Push b's

Pop b's for c's

*** 29 $\{ww^R \mid w \in \{a,b\}^*\} = \{\varepsilon, aa, bb, aaaa, abba, \ldots\}$

Method 1:



When will w^R begin?

↳ Don't know

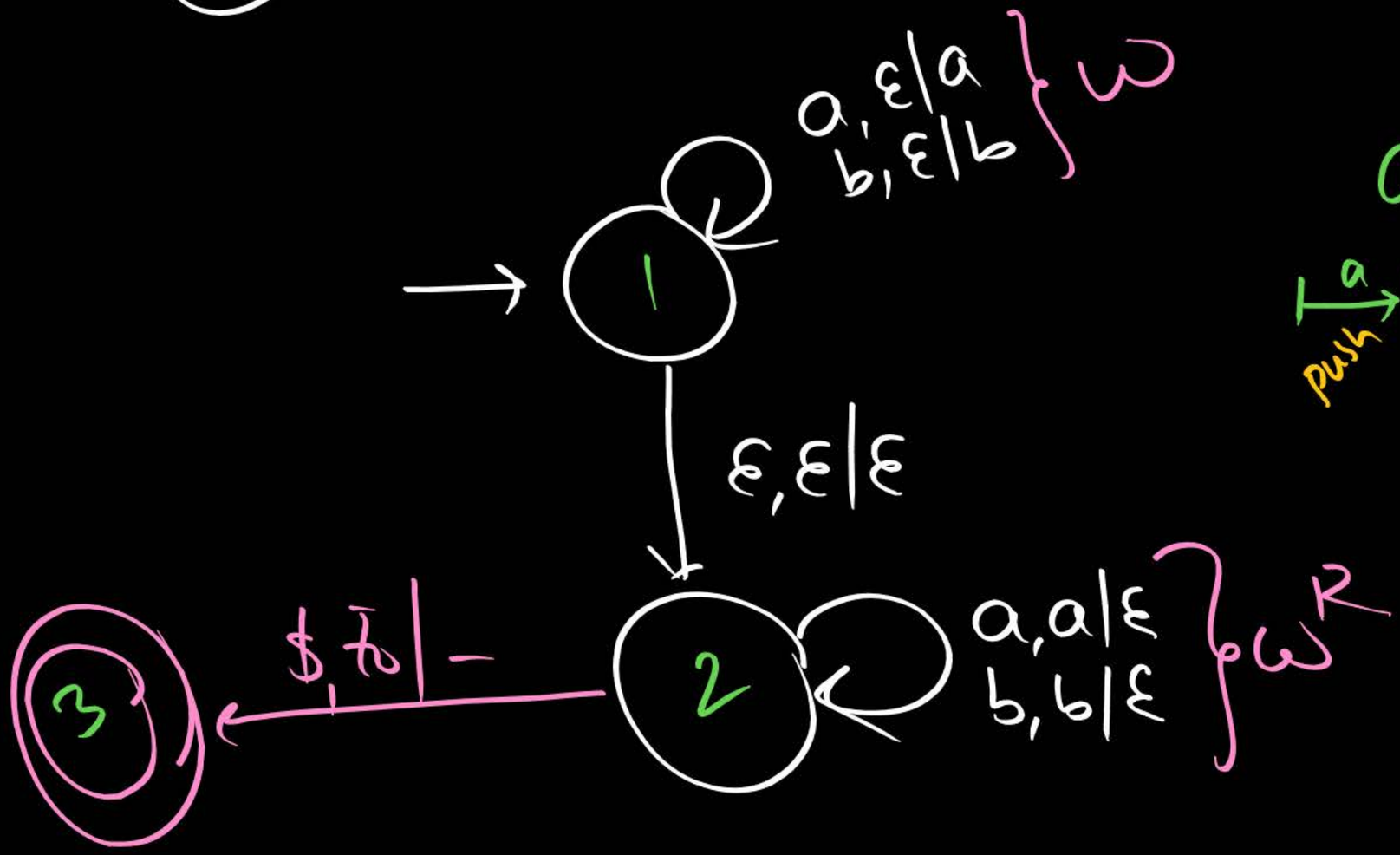
If w^R begins, what logic?

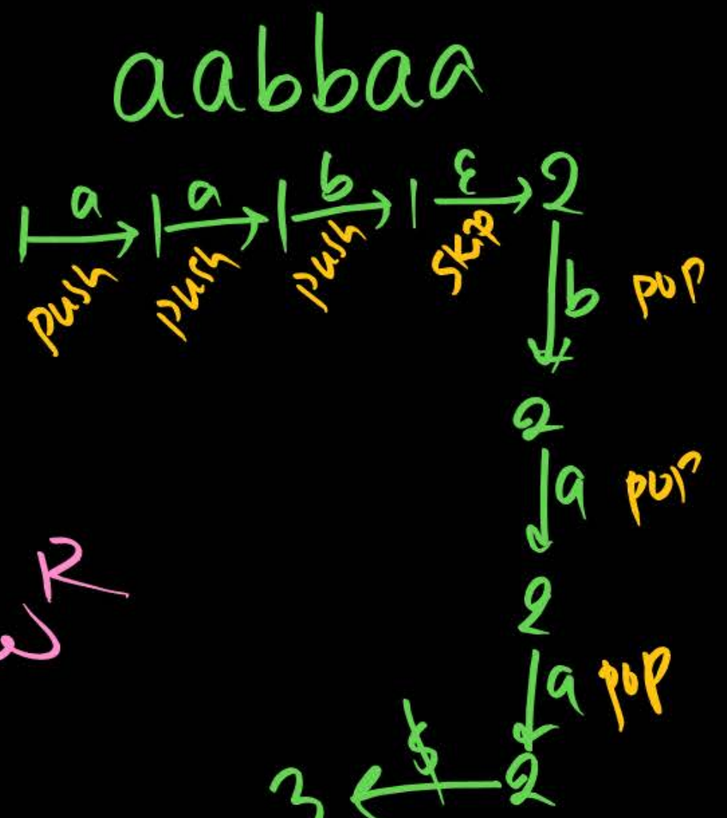
present i/p = tos

previous i/p

For valid string, atleast 1 path that halts at final

(29) $\{ww^R \mid w \in \{a,b\}^*\}$ ⇒ CFL but not DCFL





CFLs

(30) $\{ w w^R x \mid w, x \in \{a,b\}^+ \}$ ⇒ CFL but not DCFL

Not Reg
Not DCFL

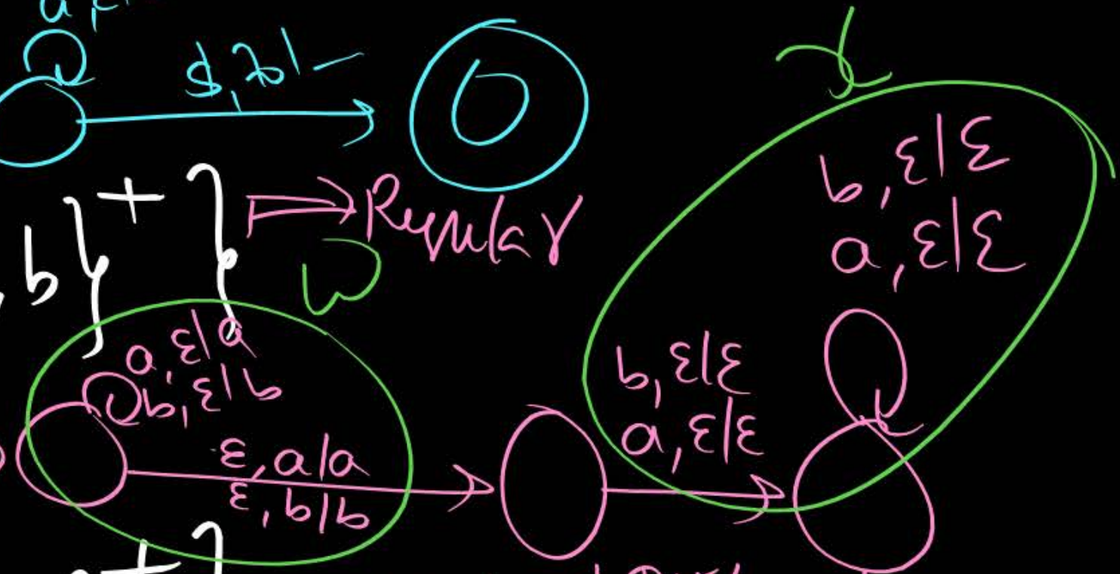
Reg, DCFL

(31) $\{ w x w^R \mid w, x \in \{a,b\}^+ \}$ ⇒ Regular

FA exist

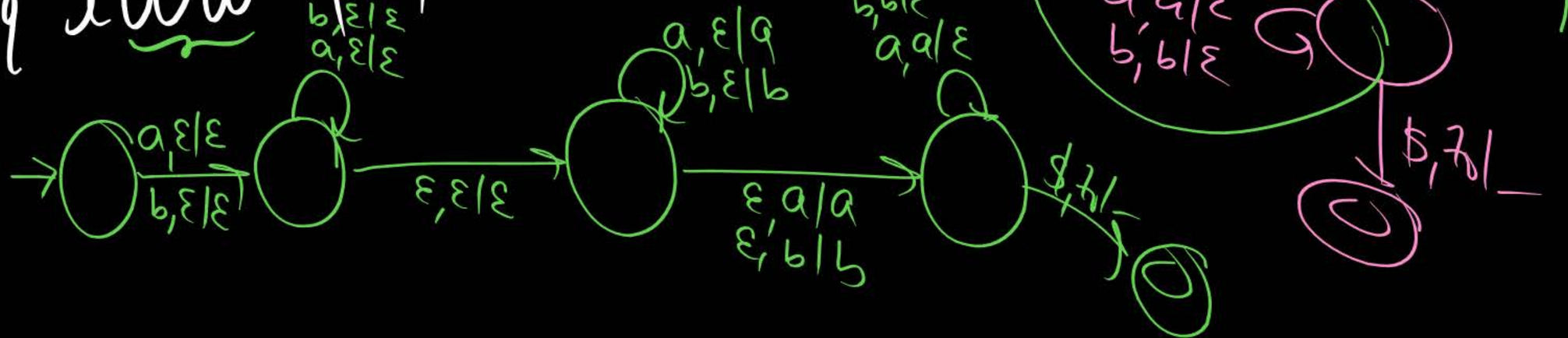
$axa + bxb$

NFA
DFA
DPDA
PDA



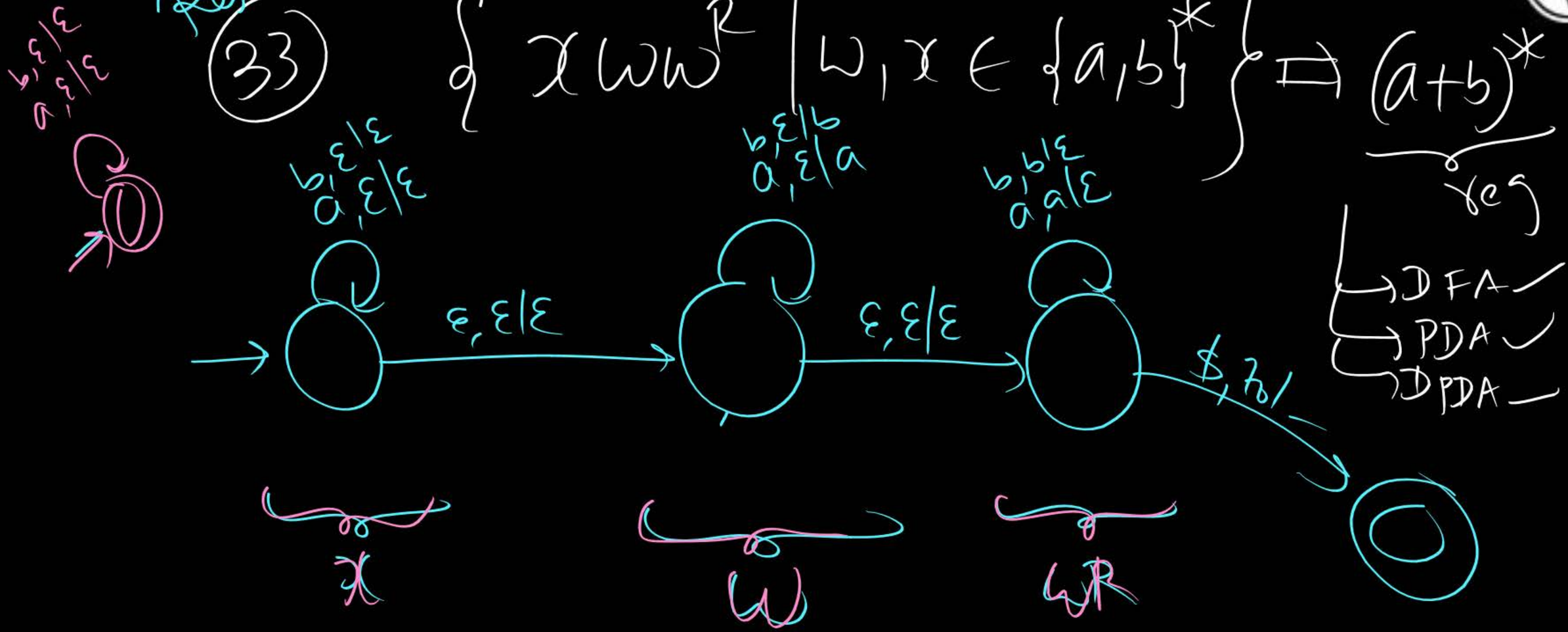
Not DCFL
Not Reg

(32) $\{ x w w^R \mid w, x \in \{a,b\}^+ \}$ ⇒ CFL but not DCFL

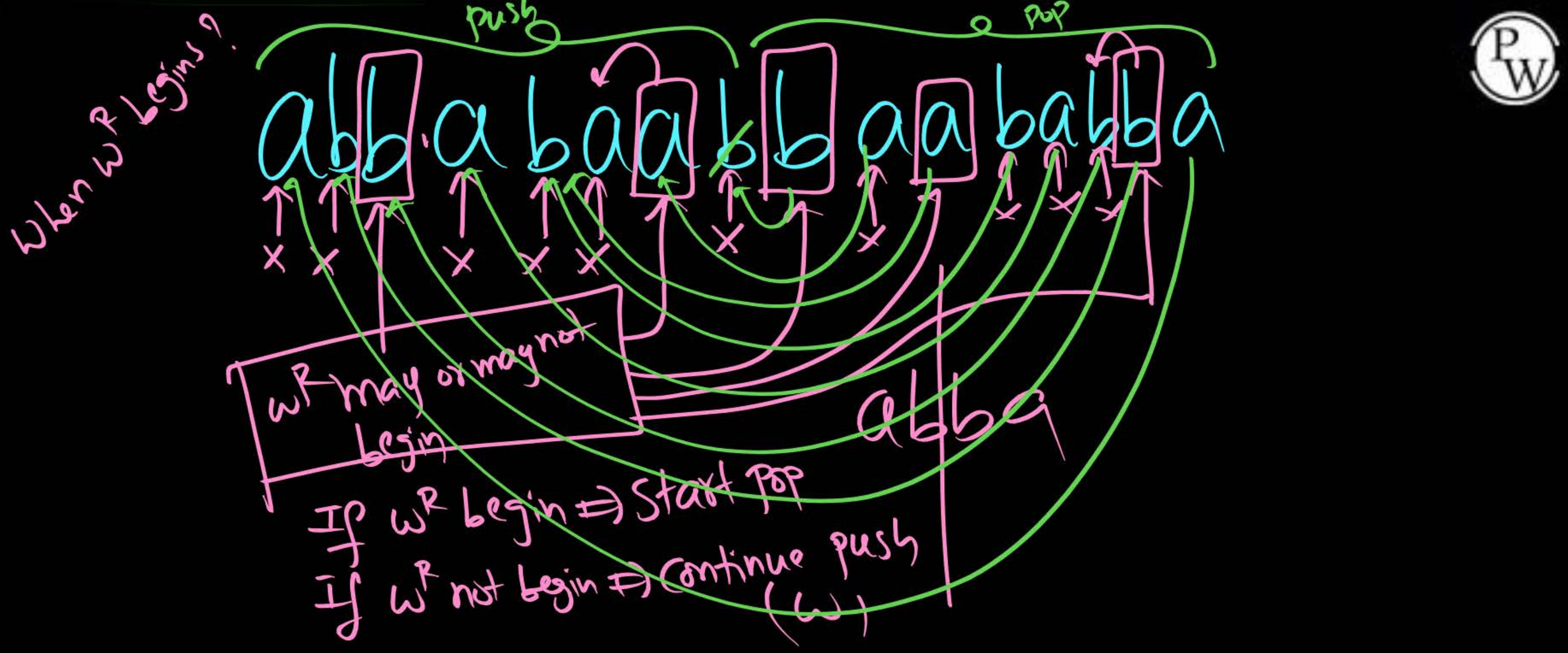


Regular

(33) $\{ x w w^R \mid w, x \in \{a,b\}^* \} \Rightarrow \underbrace{(a+b)^*}_{\text{Reg}}$



- DFA
- PDA
- DPDA


When w^R begins?
push
pop
abbaabbaabba
w^R may or may not begin
abba
If w^R begin ⇒ Start pop
If w^R not begin ⇒ Continue push (w)

H.W

(34) $\{a^n b^n c^+ \mid n \geq 1\}$

(35) $\{a^n b^+ c^n \mid n \geq 1\}$

(36) $\{a^+ b^n c^n \mid n \geq 1\}$

(37) $\{a^m b^n c^{m+n} \mid m,n \geq 1\}$ — Push, Push

(38) $\{a^m b^{m+n} c^n \mid m,n \geq 1\}$

(39) $\{a^{m+n} b^m c^n \mid m,n \geq 1\}$ — Push

DCFLs

(40) $\{a^n b^n c^m d^m \mid m,n \geq 1\}$ — Push, Push

Summary

↳ DPDA Vs PDA

CFLs & DCFLs

$a^n b^n c^m d^m$

DPDA exist ?


THANK YOU
GW
SOLDIERS !