

# CS & IT ENGINEERING

Theory of Computation  
Finite Automata



Lecture No. 8



By- DEVA Sir



- 01 DFA-Model III (No.of Symbols)
- 02 DFA-Model IV (Length & Remainder)
- 03 DFA-Model V (Symbols & Remainder)
- 04 DFA-Model VI (Sequence)
- 05 DFA-Model VII (Sequence & Number of Symbols)
- 06 DFA-Model VIII (Multiple Conditions on Symbols)



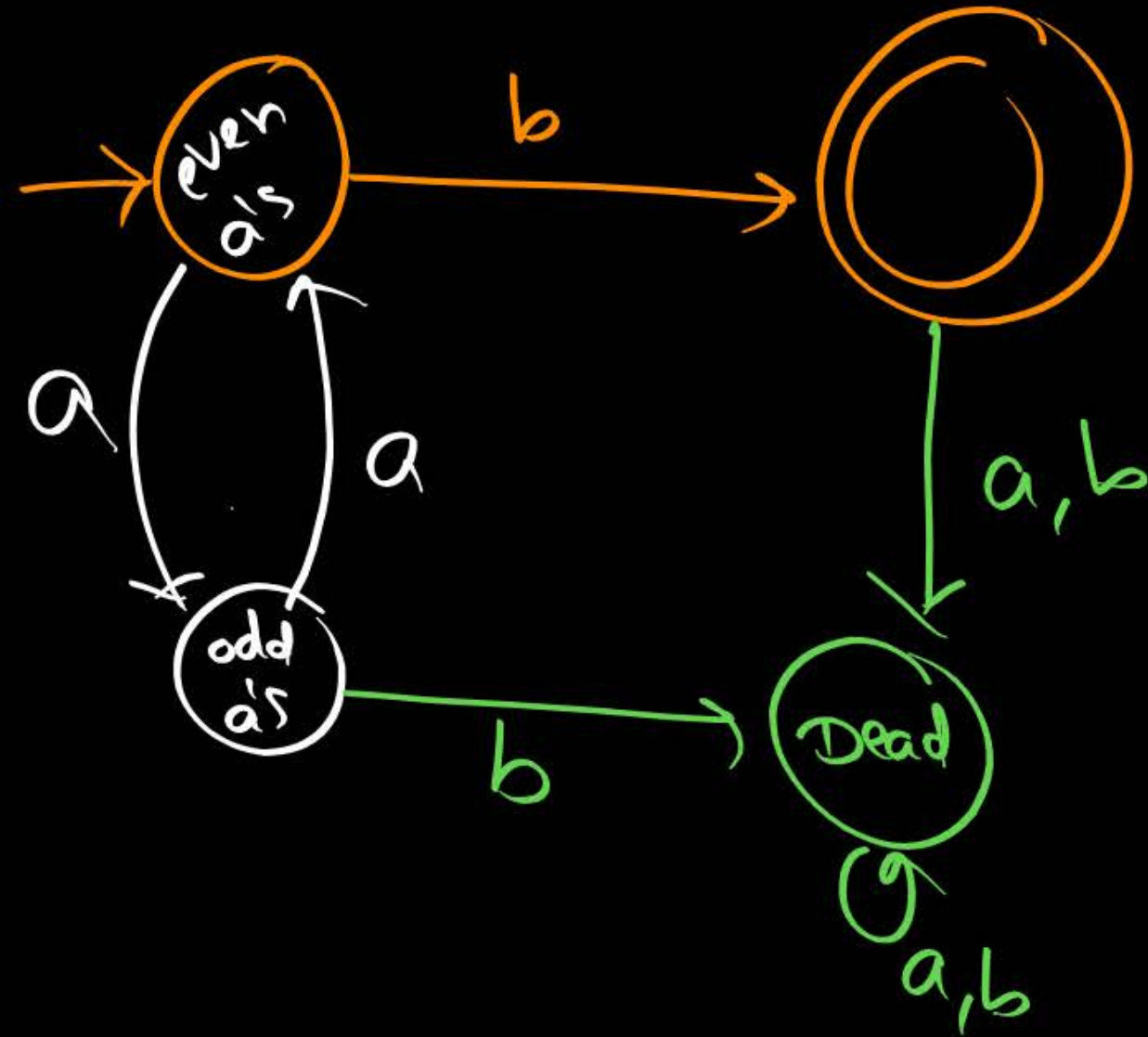
# Model-7: [Sequence, No. of Symbols]



- |   |   |
|---|---|
| ① $\{a^m b^n \mid m=1, n=1\}$ ✓           | ⑥ $\{a^m b^n \mid m=\text{even}, n=1\}$ ✓           |
| ② $\{a^m b^n \mid m=1, n \geq 1\}$ ✓      | ⑦ $\{a^m b^n \mid m=\text{odd}, n=1\}$ ✓            |
| ③ $\{a^m b^n \mid m \geq 1, n \geq 1\}$ ✓ | ⑧ $\{a^m b^n \mid m=1, n=\text{even}\}$ ✓           |
| ④ $\{a^m b^n \mid m \leq 1, n=1\}$ ✓      | ⑨ $\{a^m b^n \mid m=\text{even}, n=\text{even}\}$ ✓ |
| ⑤ $\{a^m b^n \mid m \leq 1, n \leq 1\}$ ✓ | ⑩ $\{a^m b^n \mid m=\text{even}, n=\text{odd}\}$ ✓  |
- H.W.

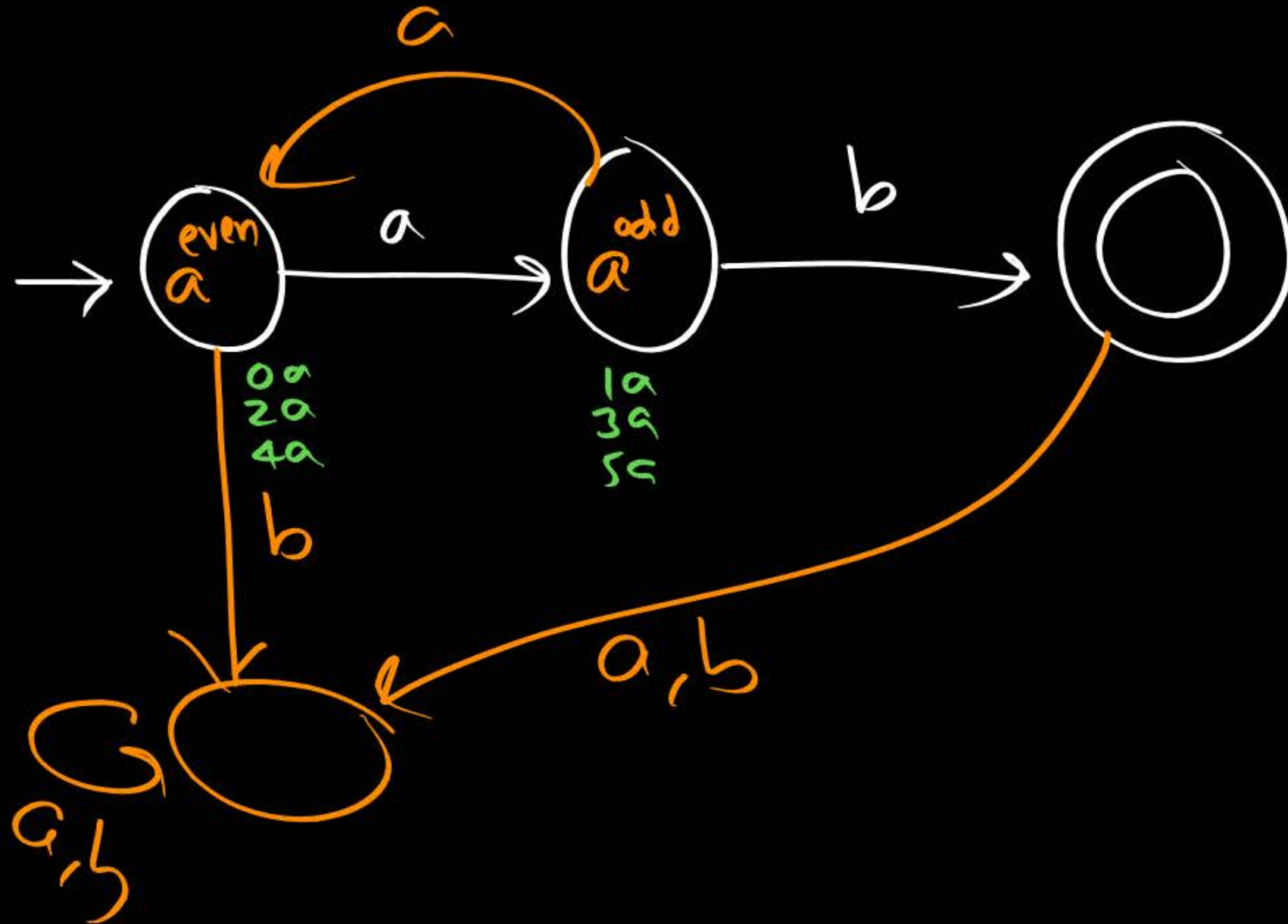
⑥

$$a^{\text{even}} b \Rightarrow (aa)^* b \Rightarrow \{b, a^2 b, a^4 b, a^6 b, \dots\}$$



7

$$a^{odd} b \Rightarrow \underbrace{a(aa)^*}_{odd} \underbrace{b}_1 \Rightarrow \{ab, a^3b, a^5b, a^7b, \dots\}$$

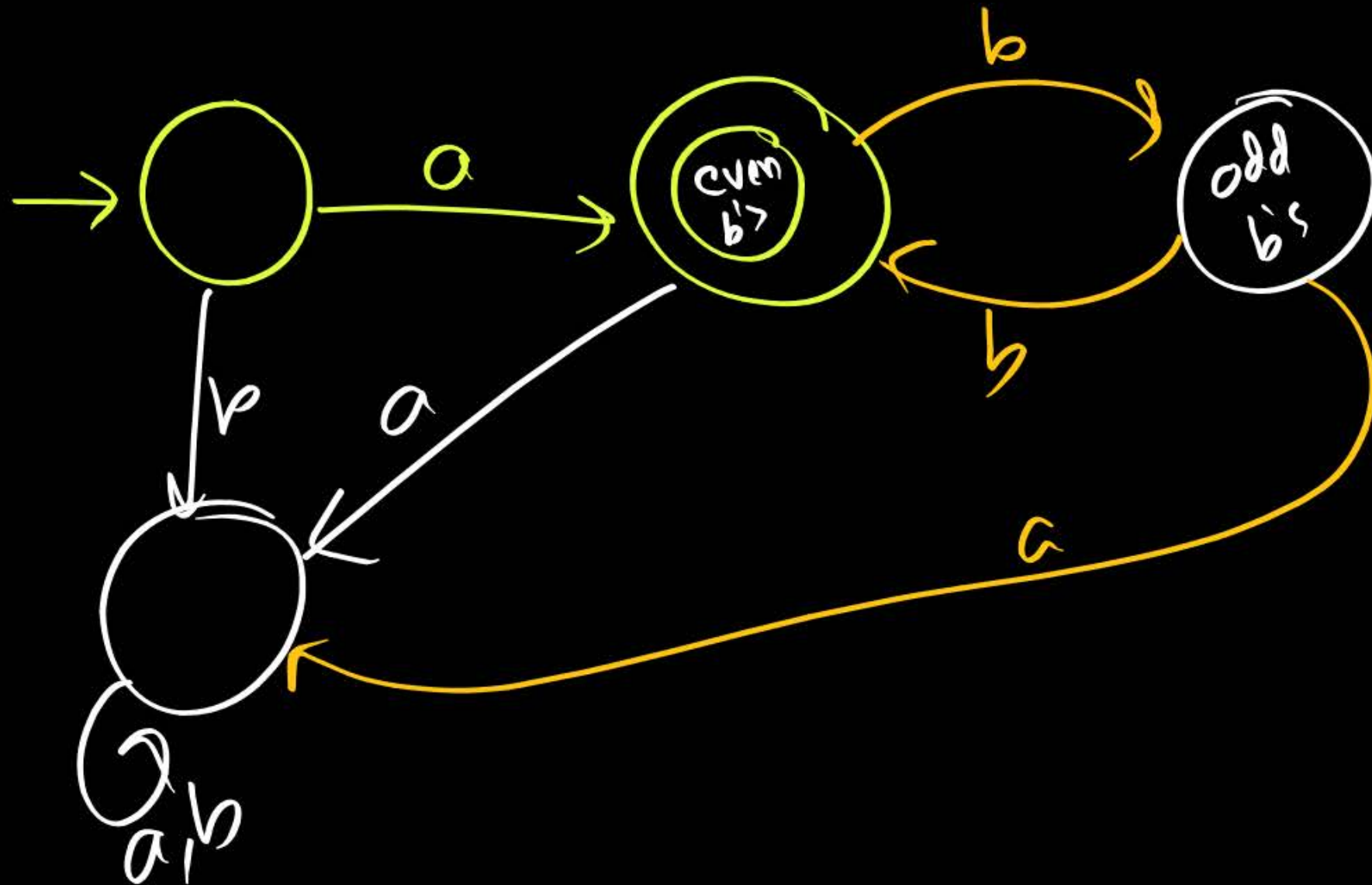
$$\text{Min} = ab$$




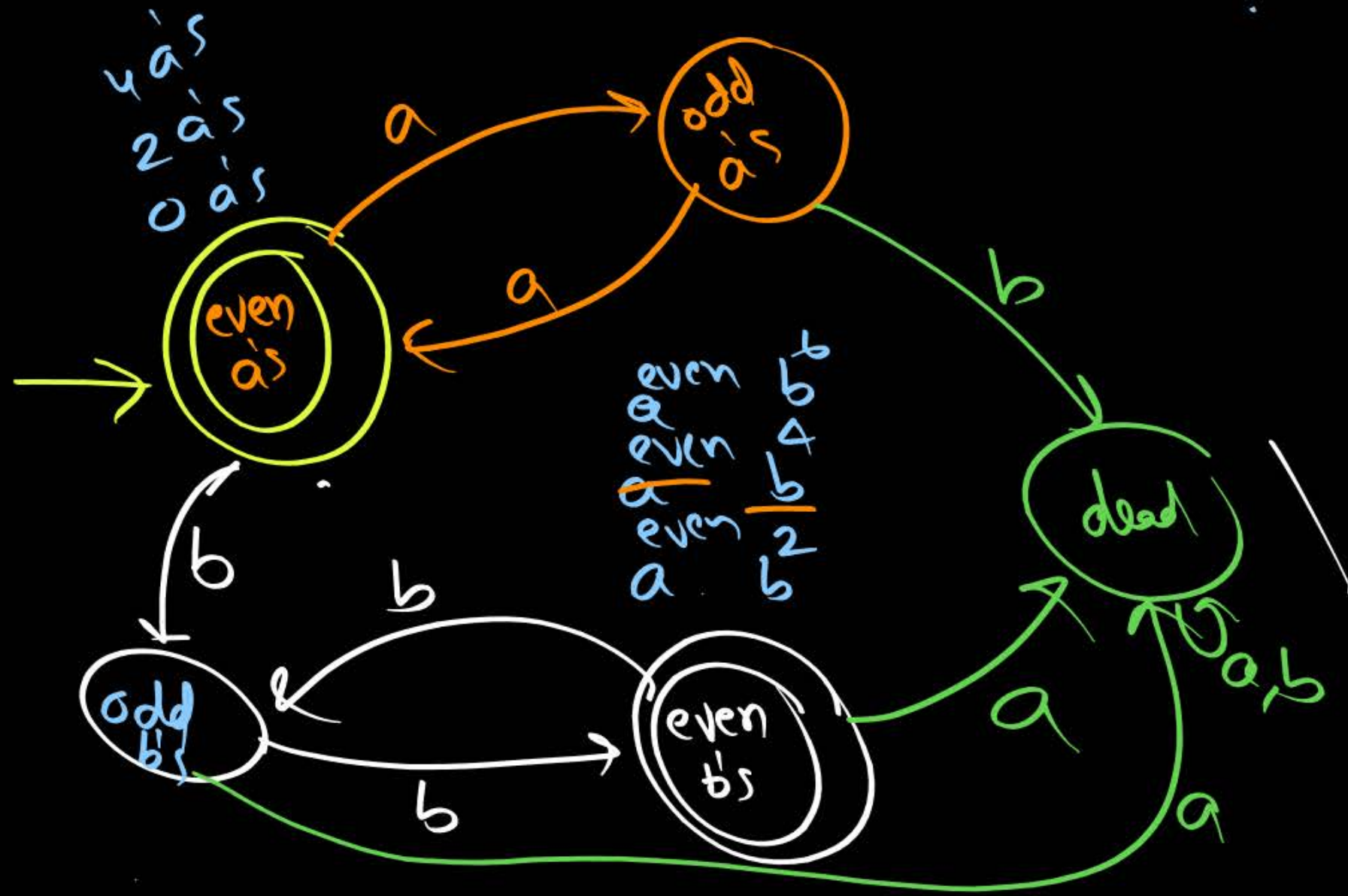
⑧  $a^1 b^{\text{even}} = a (bb)^* \Rightarrow \{\underline{a}, ab^2, ab^4, \dots\}$



Min = a

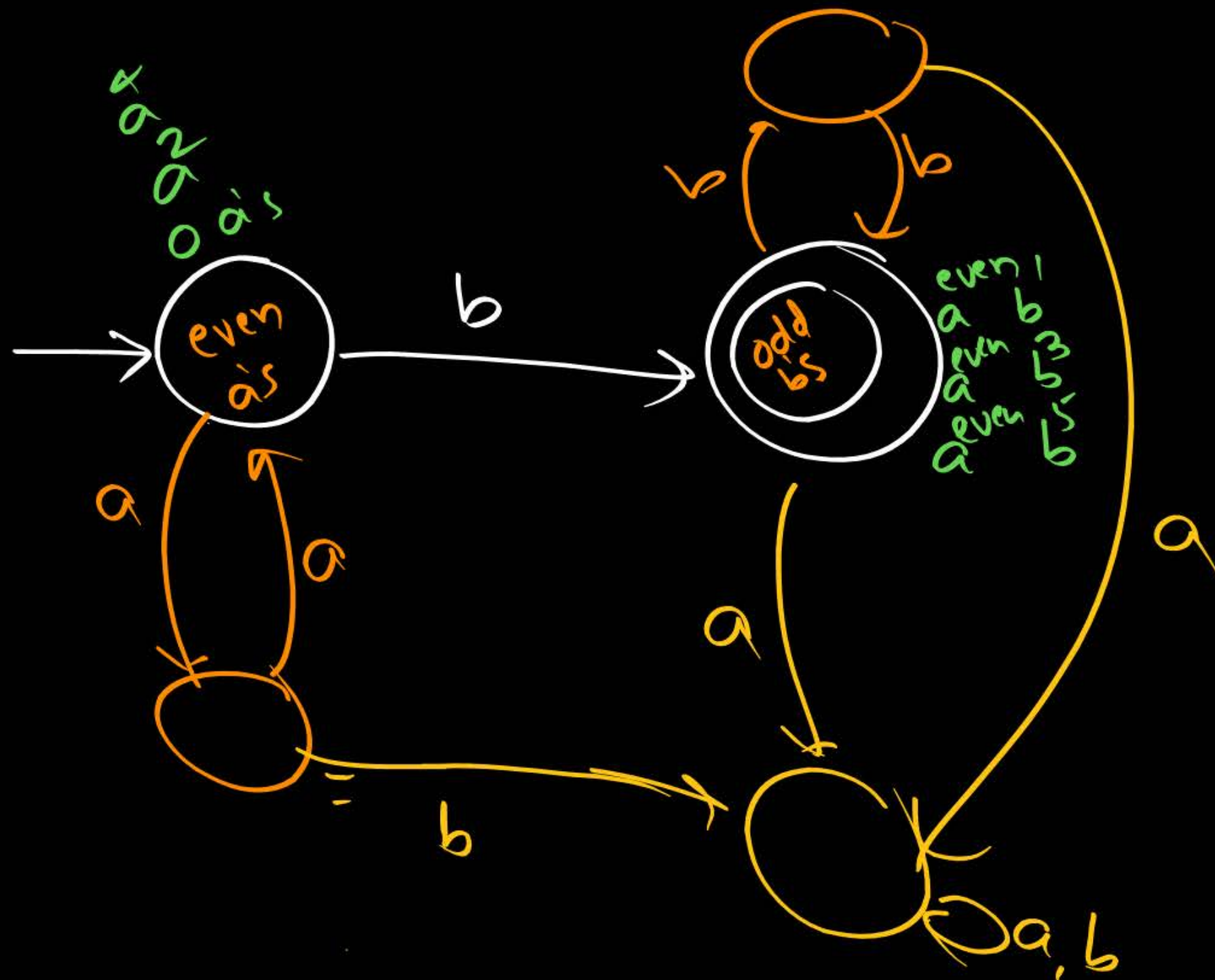


(9)  $\xrightarrow{\begin{matrix} \text{even} & \text{even} \\ a & b \end{matrix}}$   $= \underline{(aa)^*} \underline{(bb)^*}$   
 $= \{ \underline{\epsilon}, a^2, b^2, a^4, b^4, a^2 b^2, a^4 b^4, \dots \}$



⑩

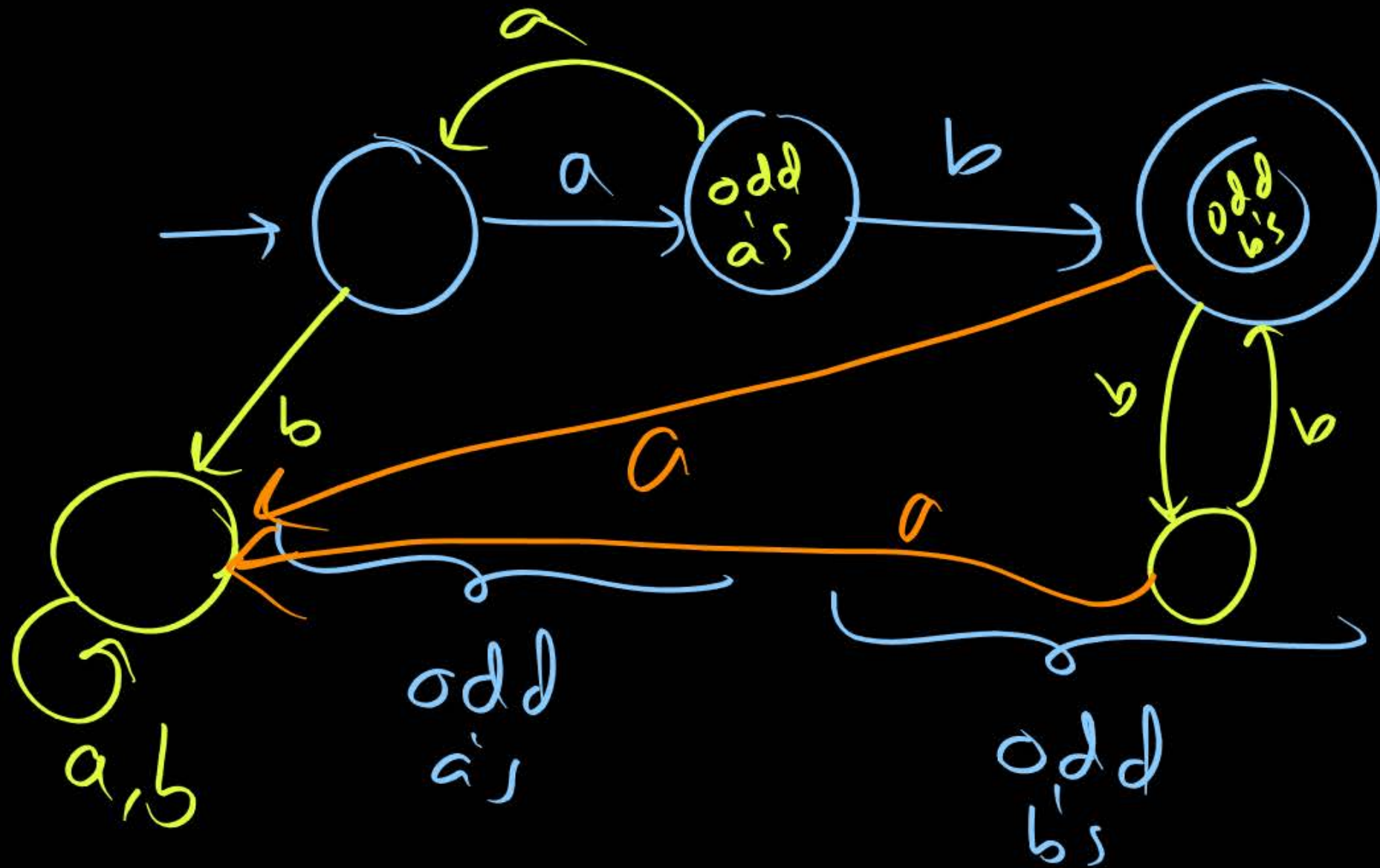
$$a^{\text{even}} b^{\text{odd}} = \underbrace{(aa)^*}_{\text{even } a} \underbrace{b(bb)^*}_{\text{odd } b}$$



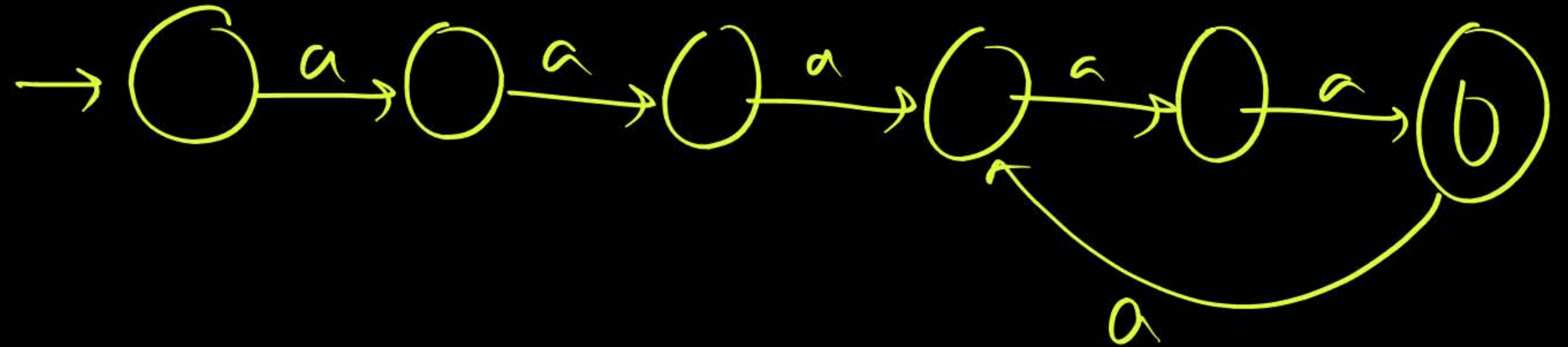
Min = 6



①  $\underline{a}^{odd} b^{odd} = a(aa)^* b(bb)^*$   
 $= \{ \underline{ab}, a^3b, ab^3, a^5b, ab^5, a^7b, ab^7, \dots \}$



$$\{a^{3n+5} \mid n \geq 0\} = \{ \underset{\checkmark}{a^5}, a^8, a^{11}, a^{14}, \dots \}$$





# Model - VIII [Multiple condition, No. of symbols]



$$\textcircled{1} \{ \omega \mid \omega \in \{a, b\}^*, n_a(\omega) = 1, n_b(\omega) = 2 \}$$

$$\textcircled{2} \{ \omega \mid \text{ " }, n_a(\omega) \geq 1, n_b(\omega) \geq 2 \}$$

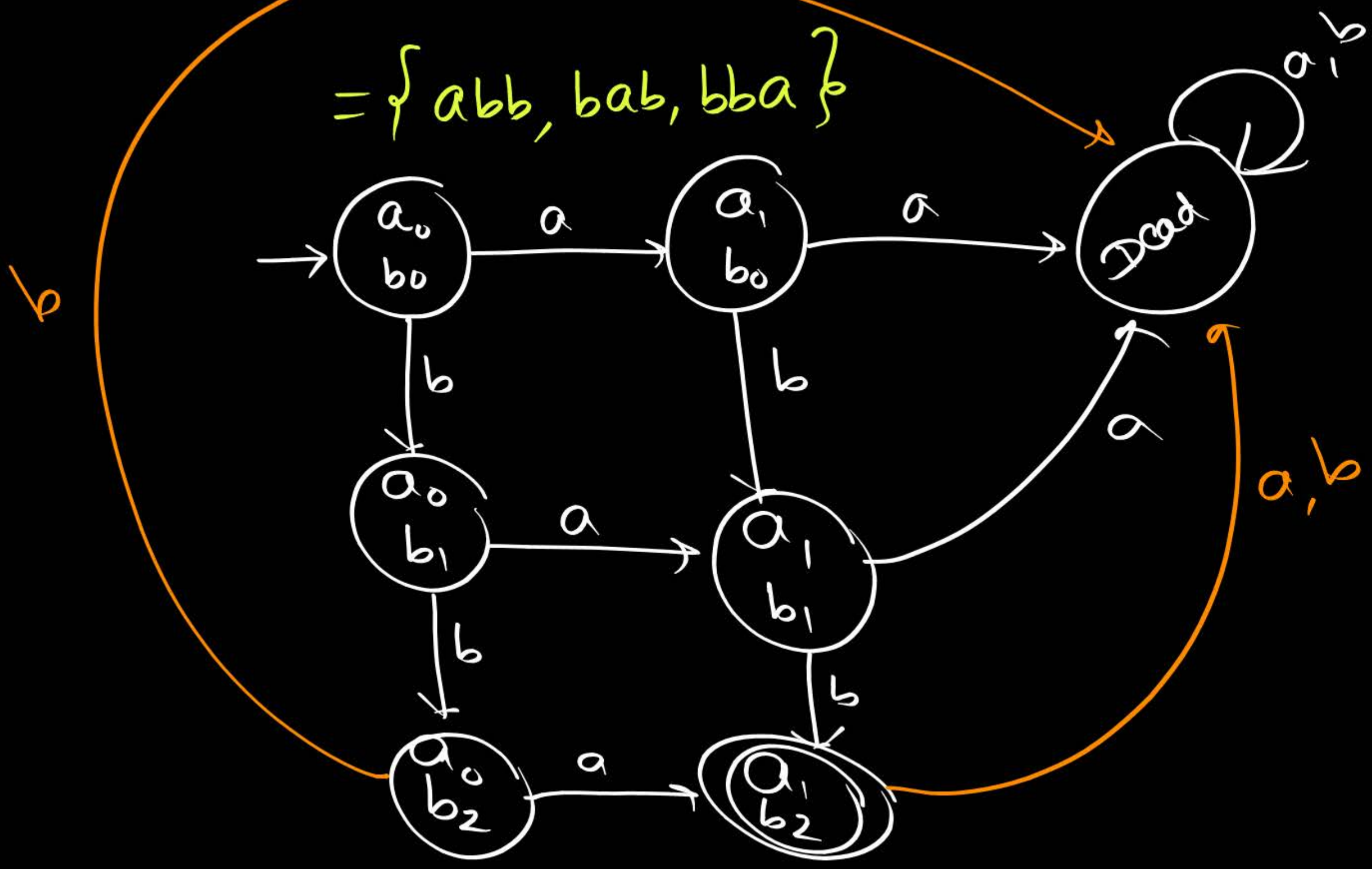
$$\textcircled{3} \{ \omega \mid \text{ " }, n_a(\omega) \leq 1, n_b(\omega) = 2 \}$$

$$\textcircled{4} \{ \omega \mid \text{ " }, n_a(\omega) \leq 1, n_b(\omega) \leq 2 \}$$

$$\textcircled{5} \{ \omega \mid \text{ " }, n_a(\omega) \geq 1, n_b(\omega) \leq 2 \}$$

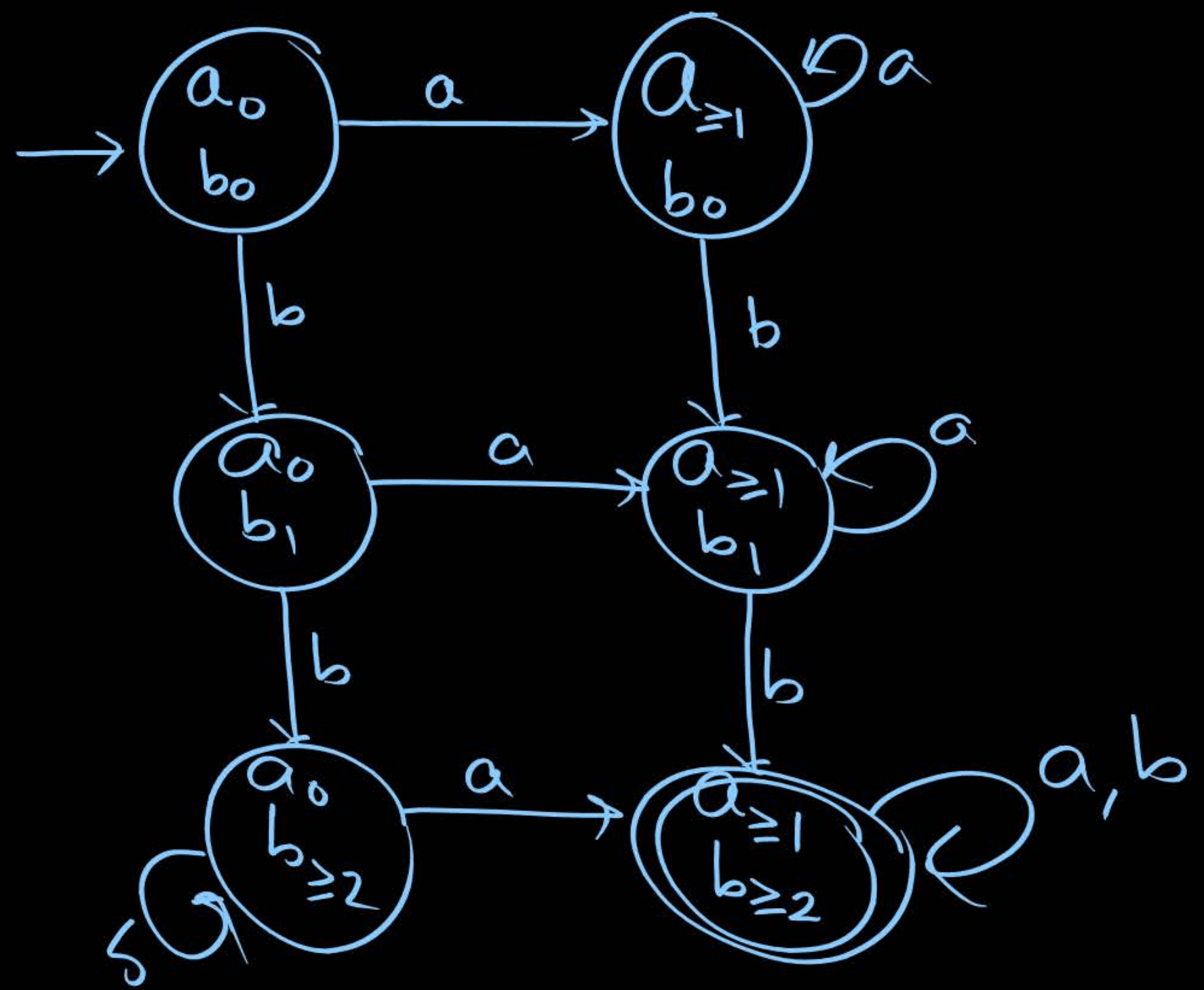
①  $\{w | w \in \{a,b\}^*, n_a(w)=1, n_b(w)=2\}$

$= \{abb, bab, bba\}$





②  $\{w \mid w \in \{a,b\}^*, \underbrace{n_a(w) \geq 1}_{\text{min } 1a}, \underbrace{n_b(w) \geq 2}_{\text{min } 2b}\}$



$a_1 \Rightarrow a_{\geq 1}$

$b_2 \Rightarrow b_{\geq 2}$

③

$\{w \mid w \in \{a, b\}^*,$

$n_a(w) \leq 1,$

Max 1a

$a_0, a_1,$

$a_{\geq 1}$

$n_b(w) \geq 2\}$

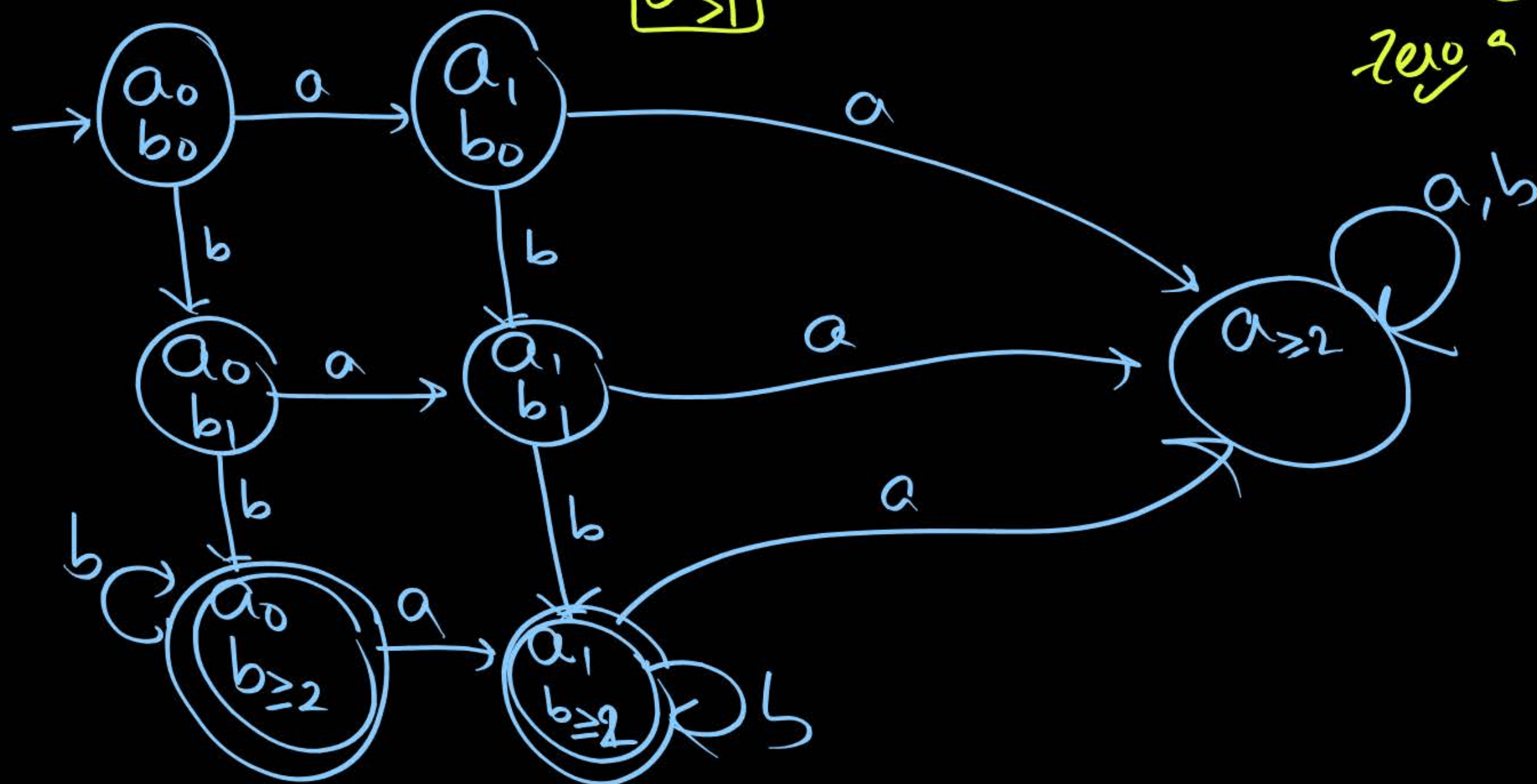
Min 2 b's

$b_0, b_1, b_{\geq 2}$

at most 1a

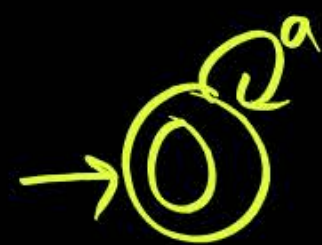
zero  $a$

one  $a$





# Model-IX [Languages over 1 symbol]

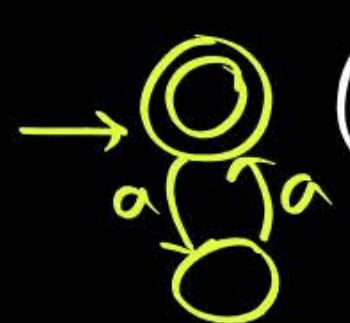


①

$$a^* = \{\epsilon, a, a^2, \dots\}$$

⑤

$$\{a^{3n+1} \mid n \geq 0\} \Rightarrow 3 \text{ states}$$

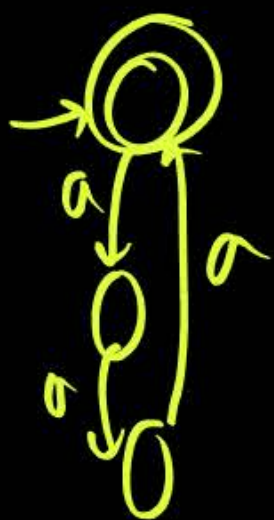


②

$$(aa)^* = \{\epsilon, a^2, a^4, a^6, \dots\}$$

⑥

$$\{a^{100n+123} \mid n \geq 0\} \Rightarrow 124 \text{ states}$$



③

$$\{a^{3n} \mid n \geq 0\} = \{\epsilon, a^3, a^6, a^9, \dots\}$$

⑦

$$\{a^{\text{prime}}\}^*$$

④

$$(aa+aaaa)^* = \{ \}$$

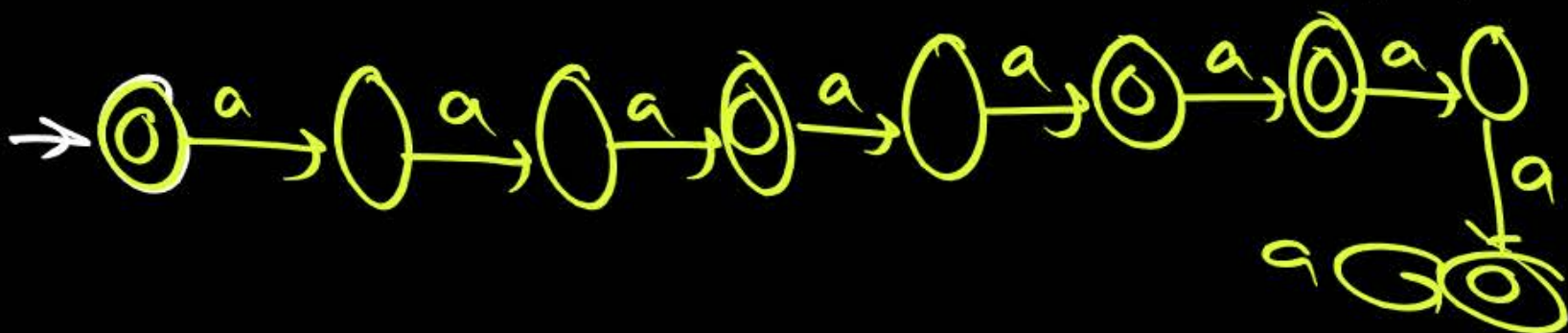
⑧

$$(aaa+aaaaaa)^* = \{\epsilon, a^3, a^5, a^6, \boxed{a^8}, a^9, a^{10}, \dots\}$$

⑤

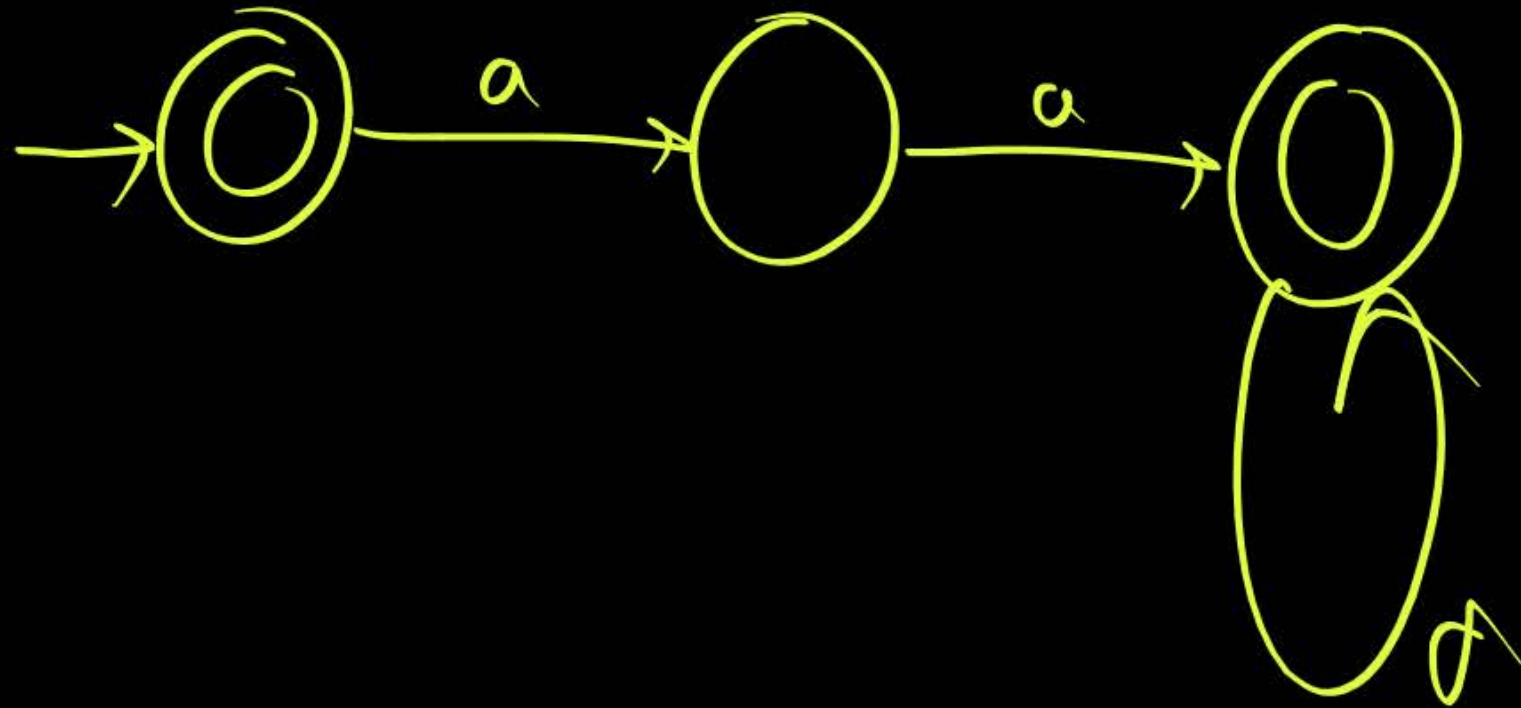
$$(a^3+a^4)^*$$

GATE



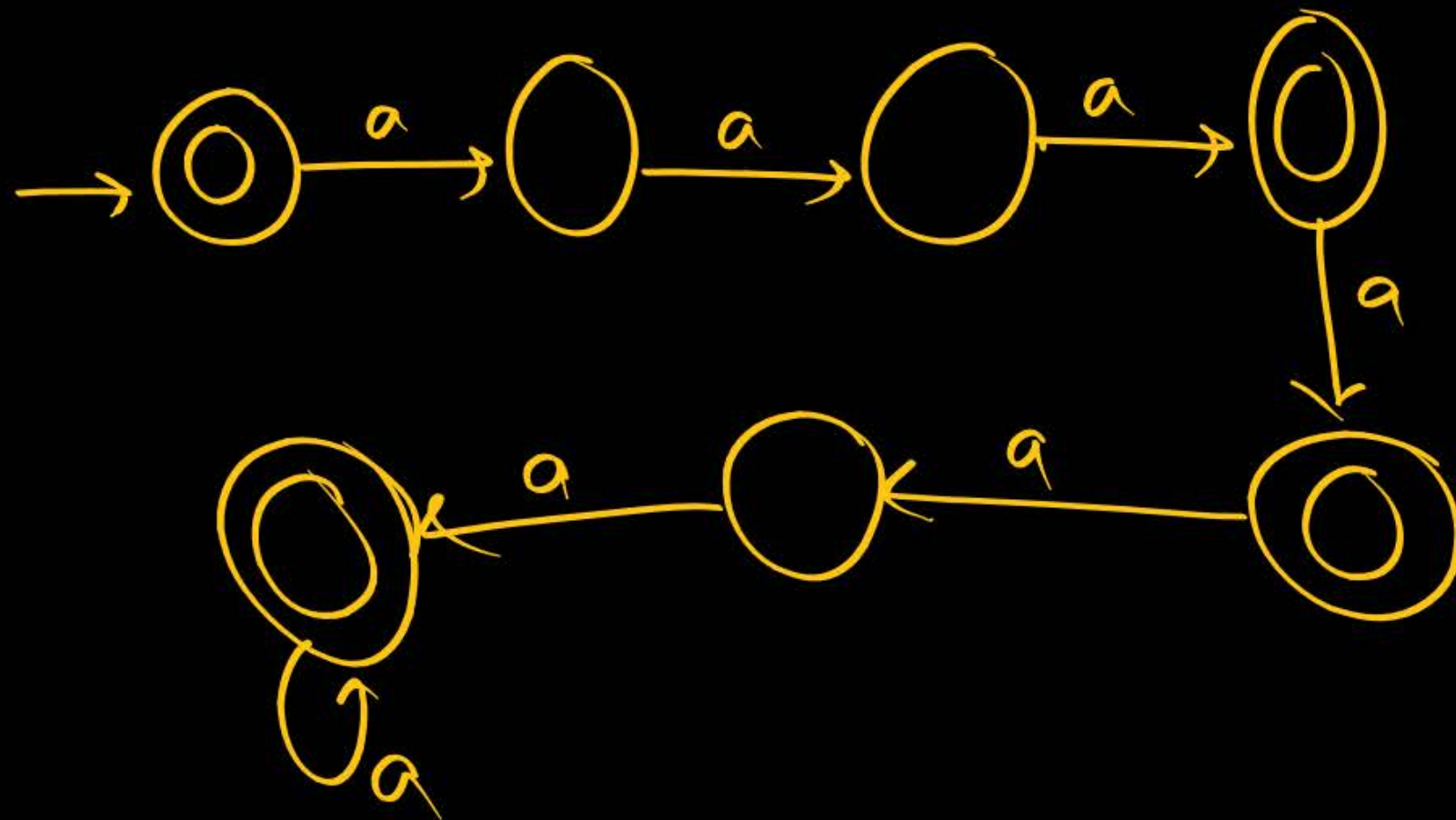
$$(4) \quad (a^2 + a^3)^* = (aa + aaa)^* \Rightarrow \{ \epsilon, \boxed{a^2}, a^3, a^4, a^5, \dots \}$$

forms A.P.





$$(5) (a^3 + a^4)^* = \{ \epsilon, a^3, a^4, \underbrace{a^6}_{a^3}, \underbrace{a^7}_{a^4}, \underbrace{a^8}_{a^5}, \underbrace{a^9}_{a^6}, \underbrace{a^{10}}_{a^7}, \dots \}$$



$a^1 \times$   
 $a^2 \times$   
 $a^5 \times$

$$\textcircled{7} \quad \{a^{\text{prime}}\}^* = \{a^2, a^3, a^5, a^7, a^{11}, a^{13}, \dots\}^*$$

$$\textcircled{4} = \{\epsilon, \boxed{a^2}, a^3, a^4, a^5, a^6, a^7, a^8, \dots\}$$

$$= a^* - \{a\}$$

$$= \epsilon + a a a^*$$

$$= \overline{\{a\}} = (a^2 + a^3)^*$$



Forms A.P.

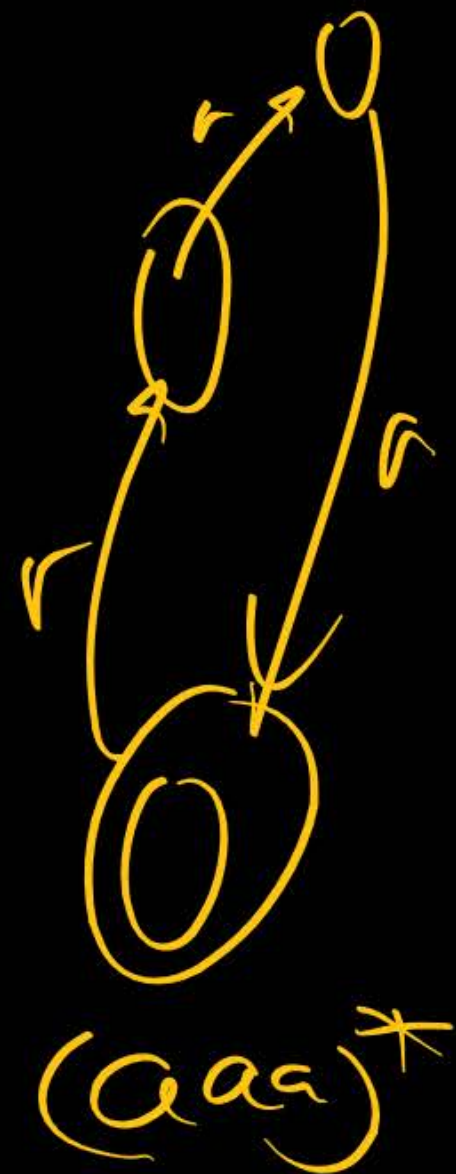
Not Forms A.P.



$a^*$



$(aa)^*$



$(aas)^*$



2, 4, 6, 8, ...

9, 11, 13, 15, ...  
+2 +2 +2

0, 1, 2, 5, 8, 11, ...  
Start



$2n \checkmark$

$3n+100 \checkmark$

$5n+1234 \checkmark$

$\cancel{n^2}$

$\Rightarrow 1^2, 2^2, 3^2, 4^2, 5^2, \dots$

$1, 4, 9, 16, 25, \dots$

$\cancel{n!}$

$\cancel{\text{prime}_2 = 2, 3, 5, 7, 11, \dots}$

$\cancel{2^n}$

$n^4 \cancel{\quad}$



$$a^{2n} \rightarrow a^0 a^2 a^4 \dots$$

$$a^{3n+100}$$

$$a^{5n+1000}$$

forms F.P.

↓  
regular  
DFA  
Regexp

$$a^{n!} \rightarrow a^1 a^2 a^6 a^{24} \dots$$

Not forms F.P.

Not reg  
No DFA  
No Reg Exp

Model- $\Sigma$  : [Start / end / contain]  $\Sigma = \{a, b\}$



①  $L = a(a+b)^*$

⑦  $\Sigma^*ab$

②  $L = b(a+b)^*$

⑧  $\Sigma^*aaa$

③  $L = aaa(a+b)^*$

⑨  $(aa+bb)\Sigma^*$

④  $L = \Sigma^*a\Sigma^*$

⑩  $\Sigma^*(aa+bb)\Sigma^*$

⑤  $L = \Sigma^*aaa\Sigma^*$

⑪  $\Sigma^*(aa+bb)$

⑥  $L = \Sigma^*a$

H.W.

## Summary



→ Model - 7 ✓  
8 ✓  
9 ✓  
10 →



