

CS & IT ENGINEERING

Theory of Computation

Push Down Automata



Lecture No. 03



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TOPICS TO BE COVERED

01

CFG ✓

02

PDA

03

04

05

Every Regular Language is CFL

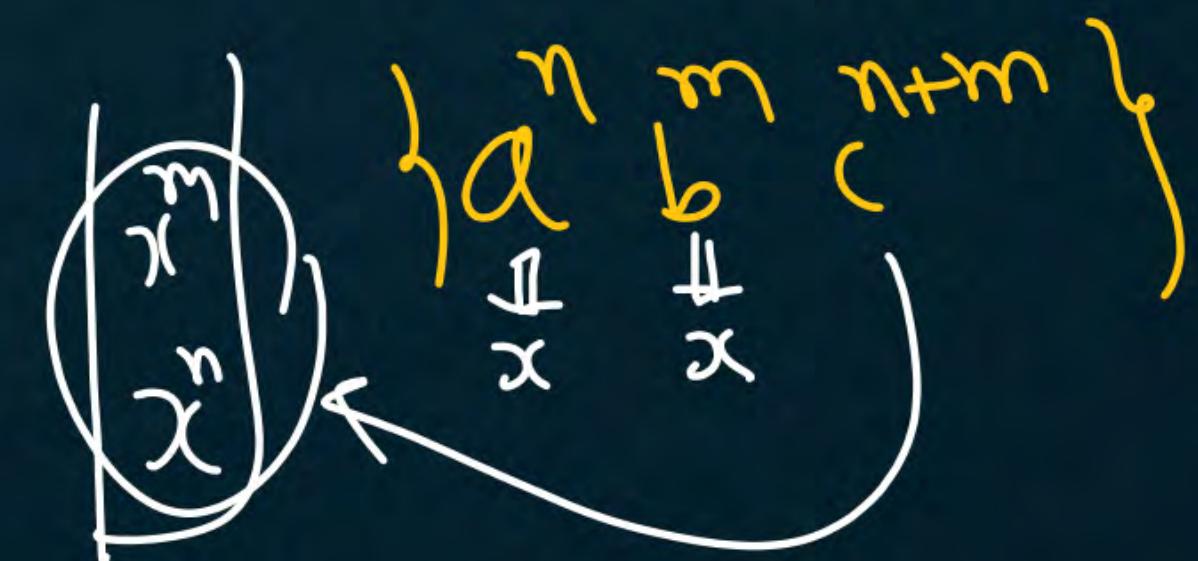
Some non regular languages are CFLs.

$$\{a^n b^n\} \xrightarrow{\text{non reg}} \text{CFL}$$

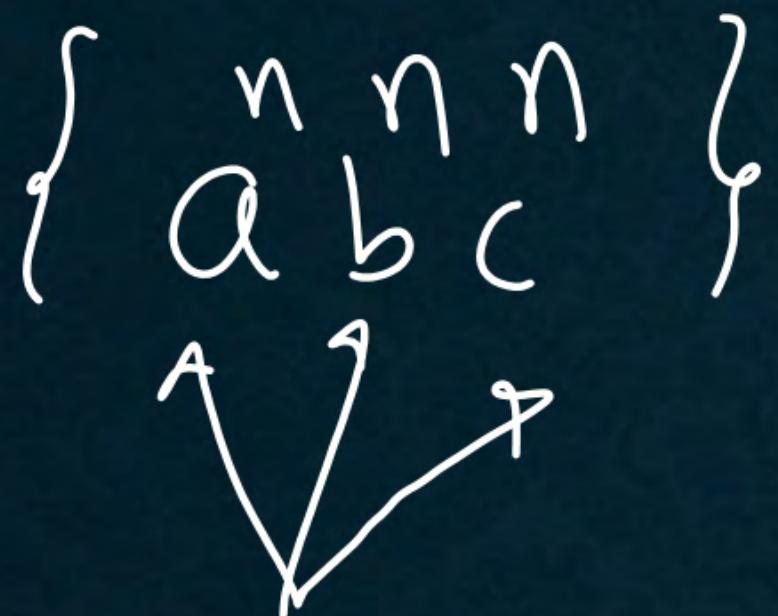
$$\{a^n b^{2n}\}$$

$$\{a^{2n} b^n\}$$

$$\begin{aligned} & \{ww^R \mid w \in \{a,b\}^*\} \\ & \{w\#w^R \mid " \} \end{aligned}$$



not CFLs

$$\{ a^{\frac{n}{2}} b^{\frac{n}{2}} c^{\frac{n}{2}} \}$$


prime
 $a^{\frac{n}{2}}$
 $a^{\frac{n}{2}+1}$
 $a^{\frac{n}{2}-1}$

Context Free Language

\equiv

PDA

\equiv

NPDA
(non-det PDA)

DPDA
 \equiv
DCFL

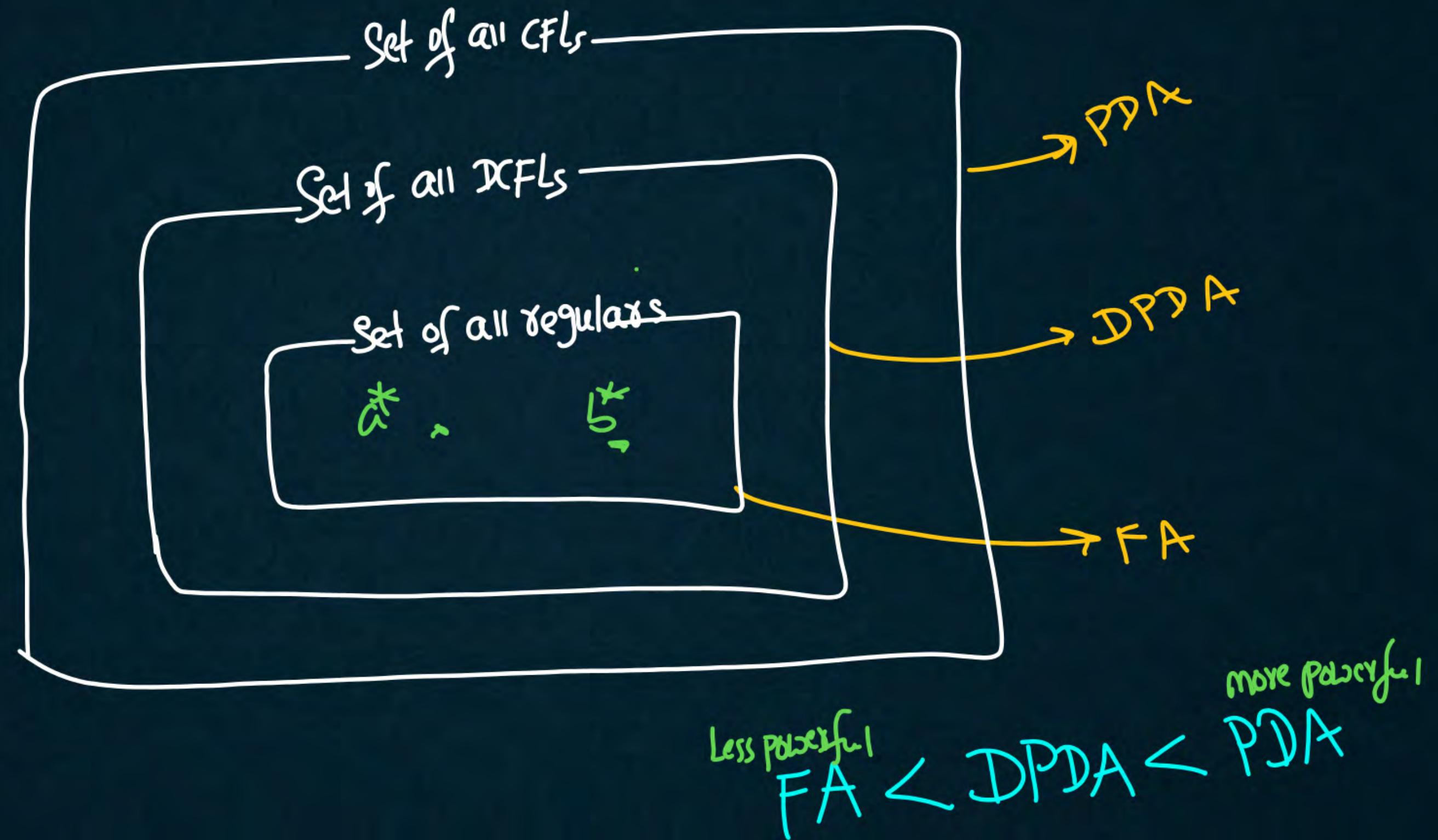
PDA (NPDA)
 \geq
CFL

Every DCFL is CFL

CFL need not be DCFL

Every DPDA is PDA

PDA need not be DPDA



Which of the following statement is TRUE ?

~~FALSE~~ I) Regular Language is Subset of CFL

~~TRUE~~ II) Set of all regulars is Subset of set of all CFLs

I) Every Regular is DCFL
is CFL

II) Every DCFL is CFL

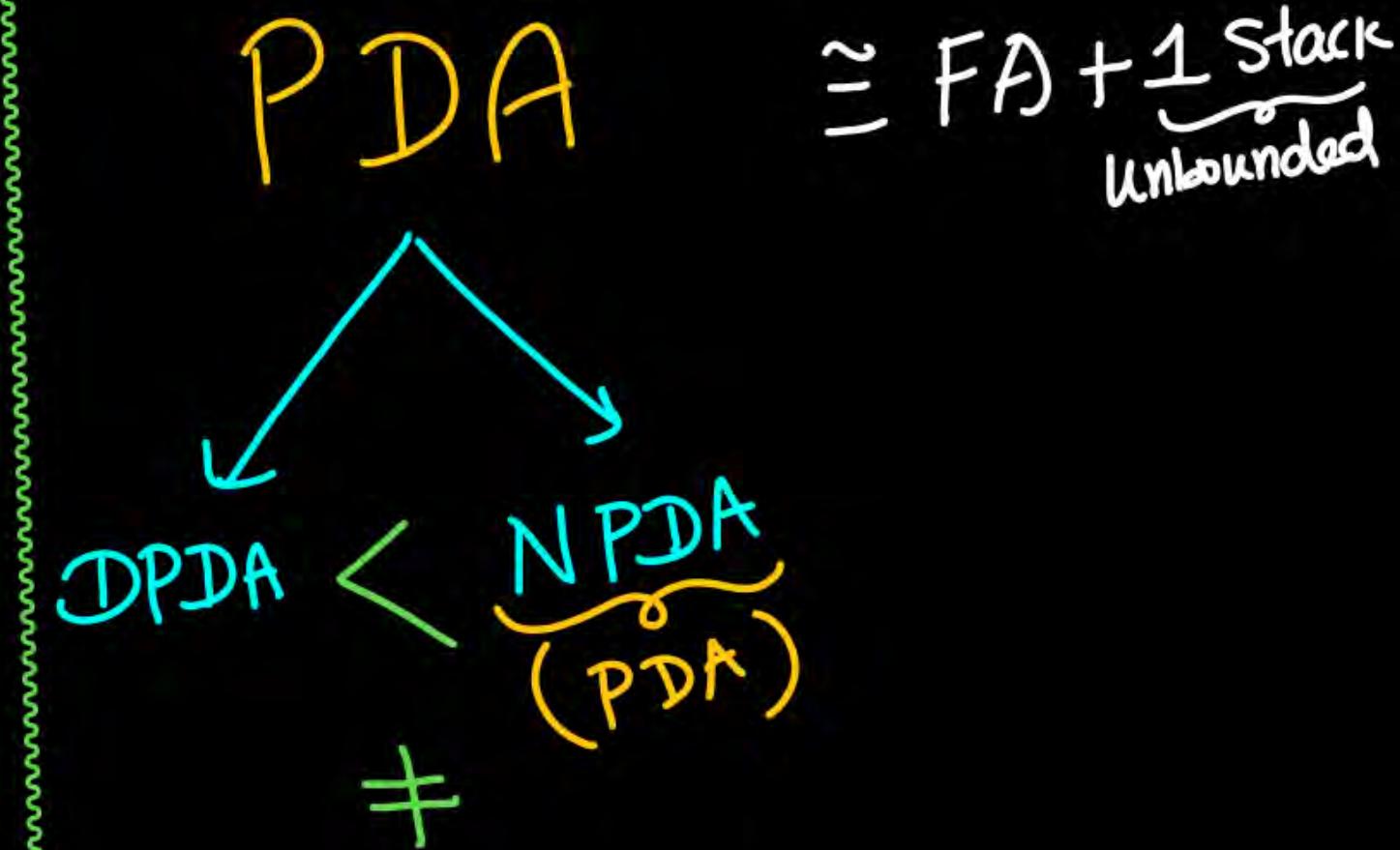
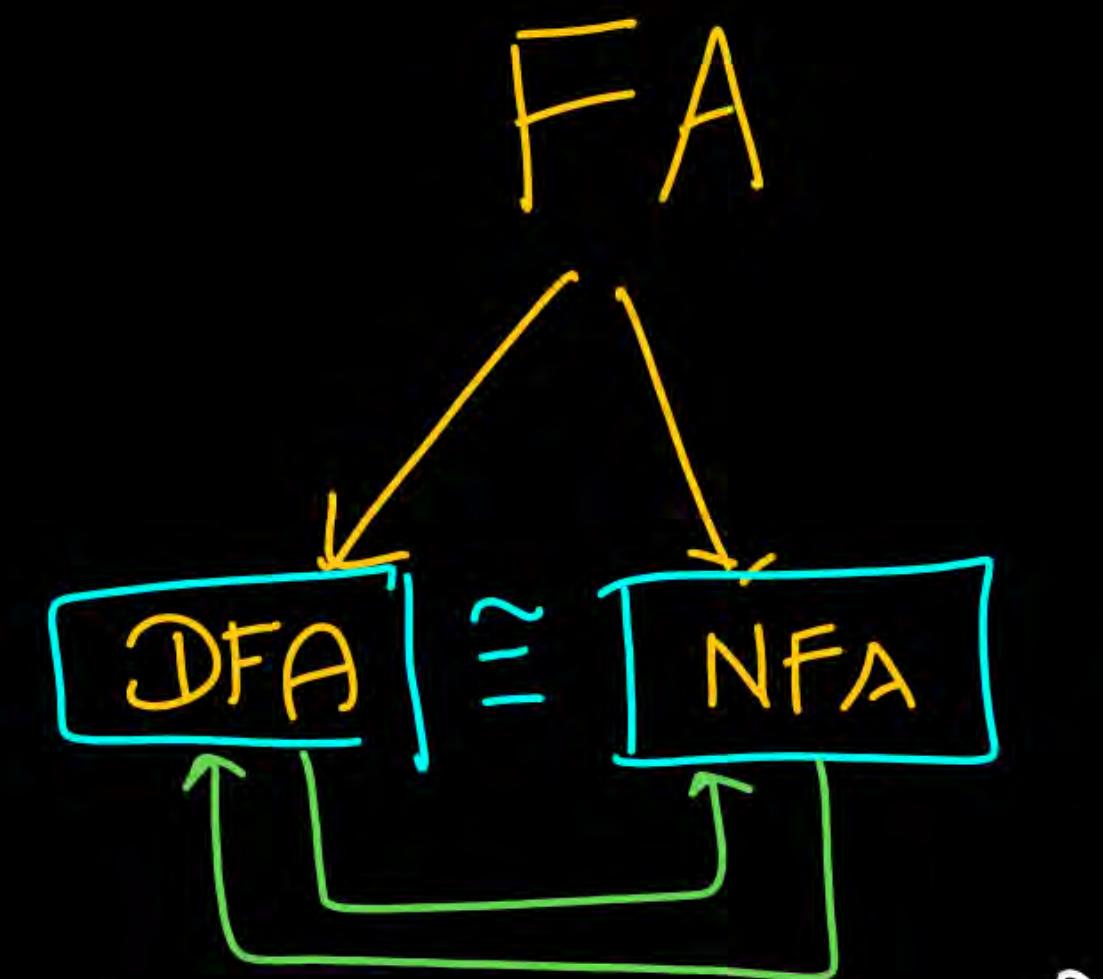
III) Regular need not be subset of DCFL

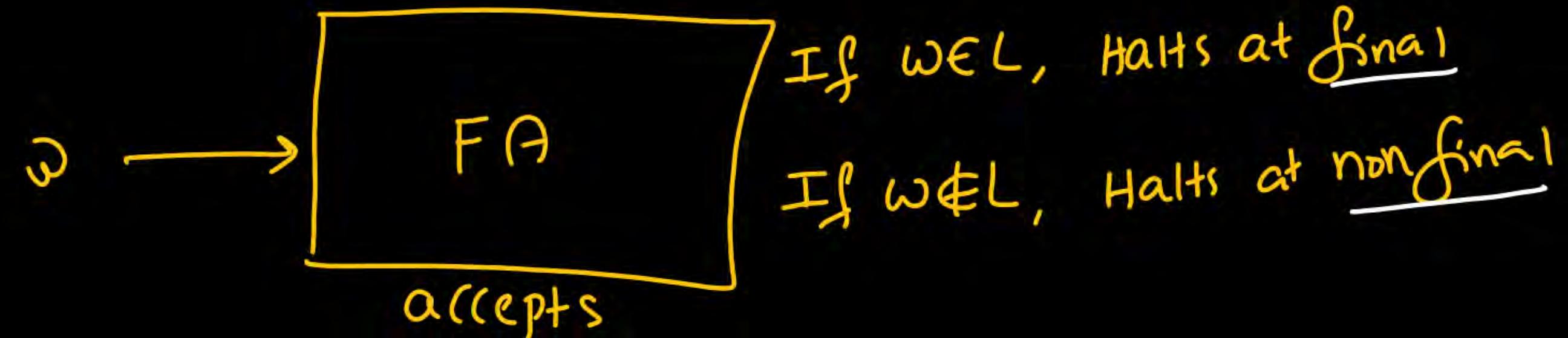
IV) Set of all Regulars \subset Set of all DCFLs

V) Set of all Regulars \subset Set of all CFLs

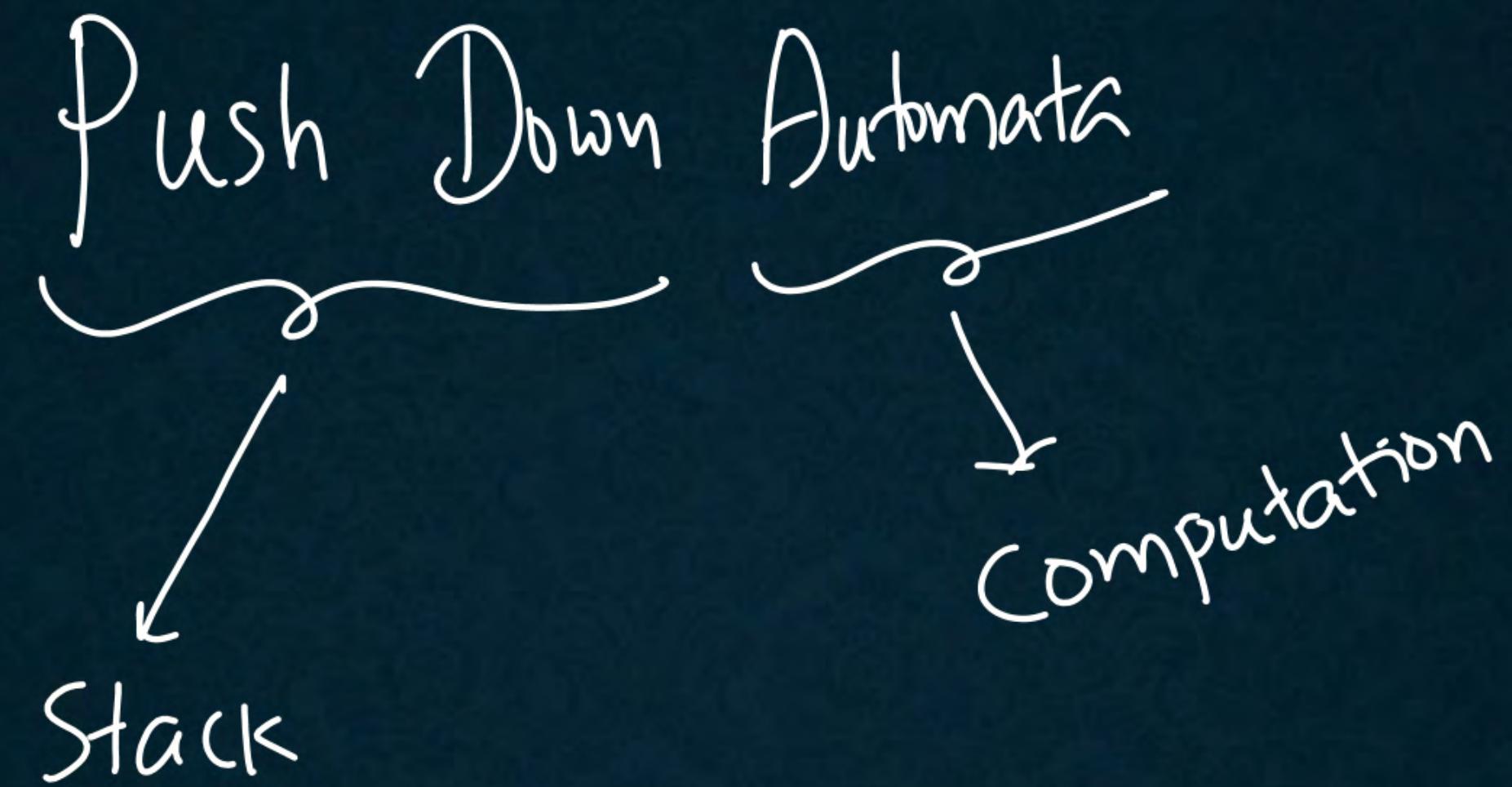
VI) Set of all DCFLs \subset Set of all CFLs

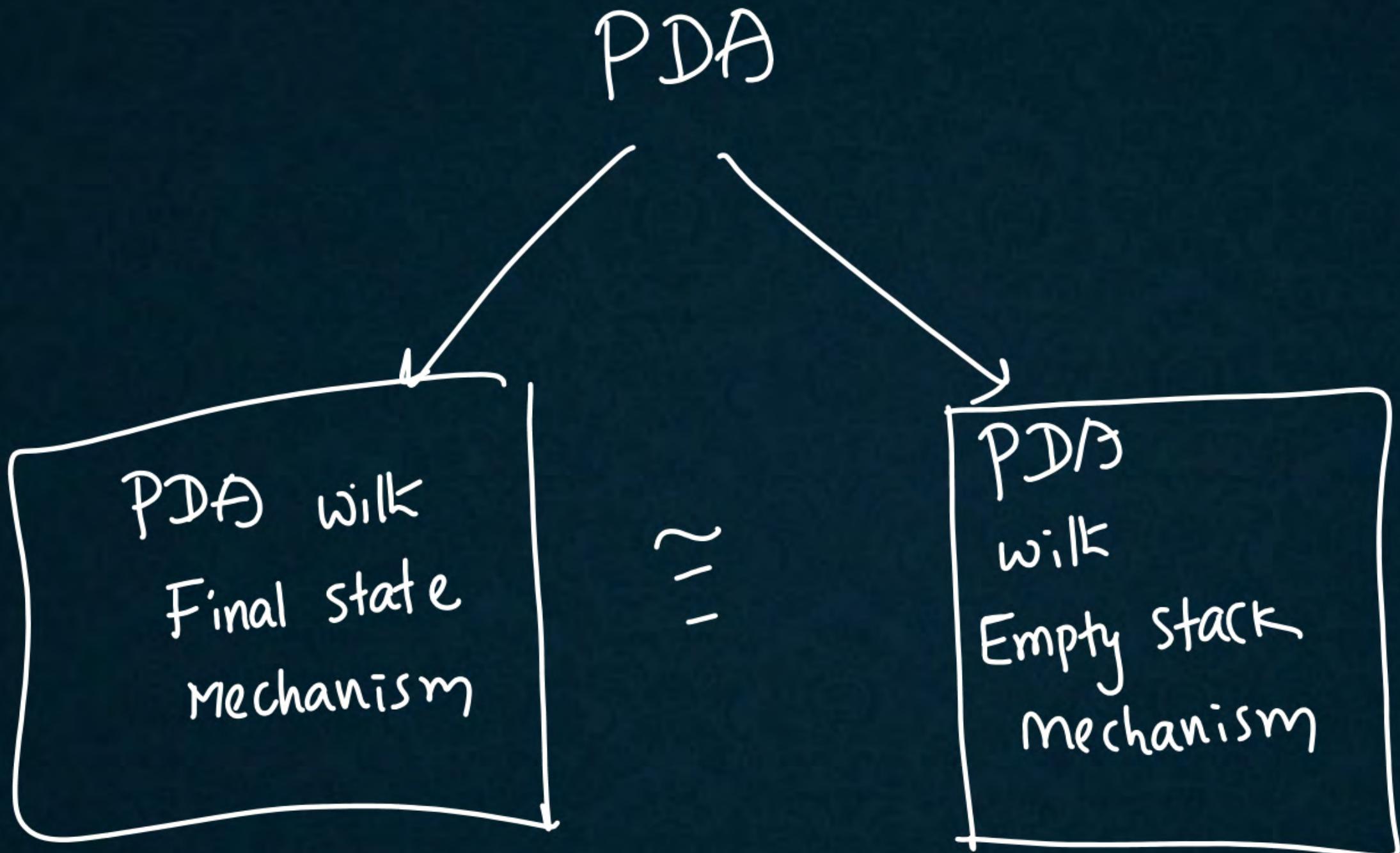
Push Down Automata :





Final state mechanism
(Universal)





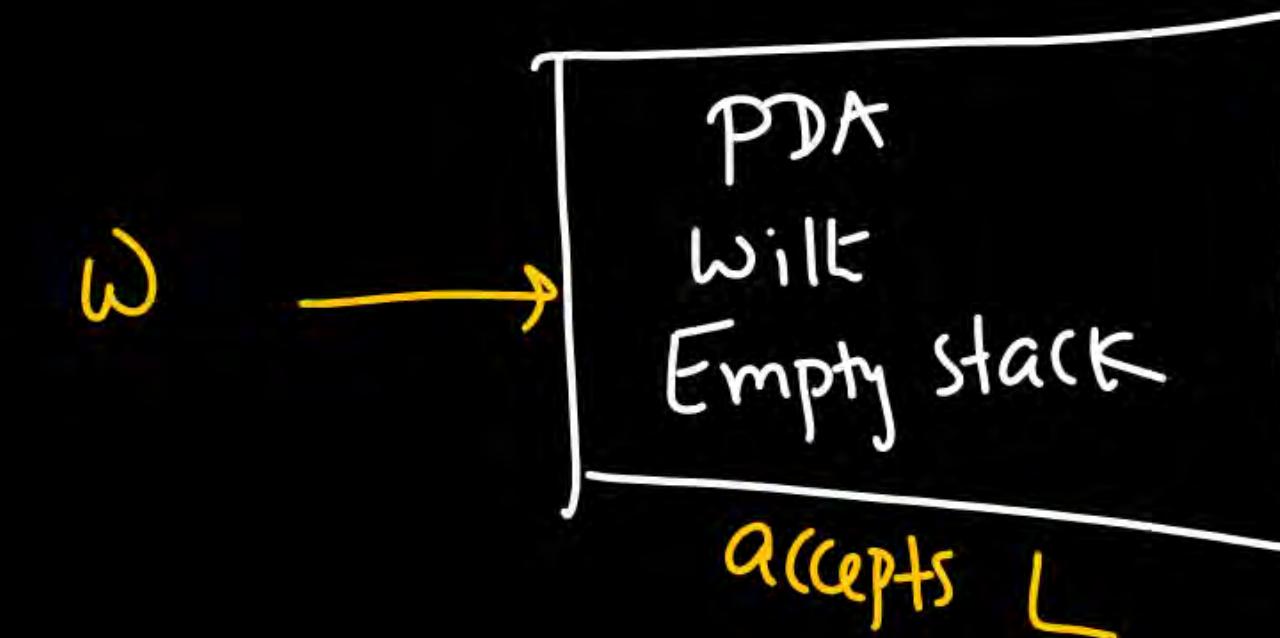
DPDA
(uses Final State mechanism only)

PDA (NPDA)

- By default non-det m/c
- It accepts CFL



If weL, atleast one palt exist
that halts at finalstate



If weL, atleast one palt exist
that makes empty stack.

$L_1 = L_2 = L_3 = L_4 = L_5$

$L_1 = \text{Set of all languages represented by PDA}$

$L_2 = \text{" " " " .. NPDA}$

$L_3 = \text{" " " " .. PDA with final state}$

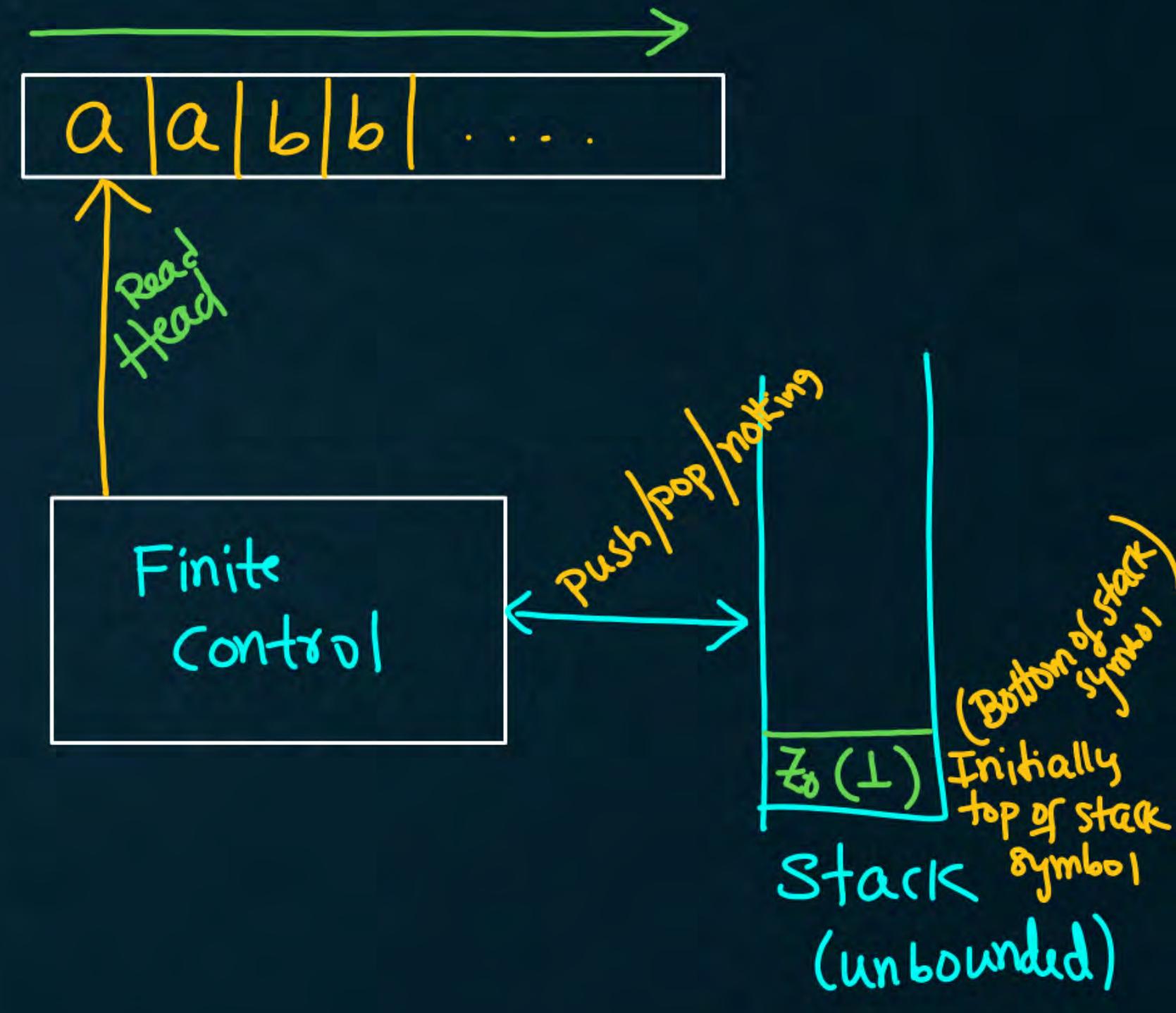
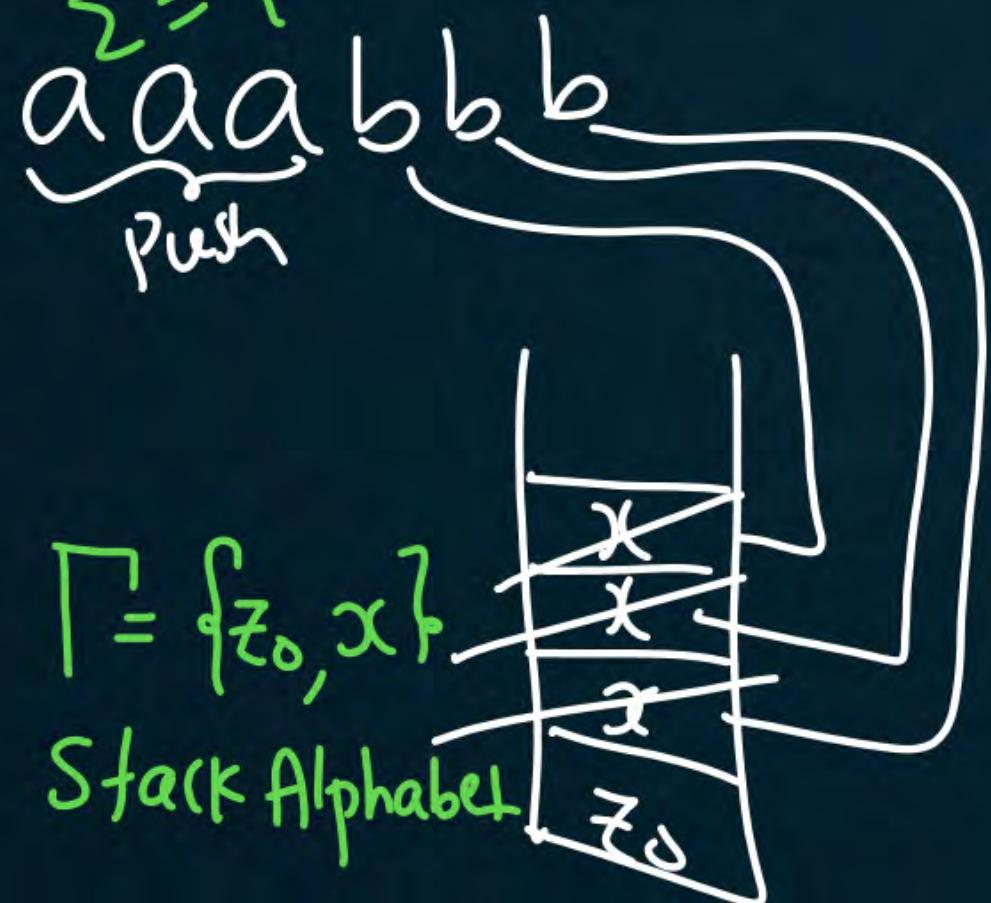
$L_4 = \text{" " " " .. PDA with empty stack}$

$L_5 = \text{Set of all CFLs}$

Push Down Automata

$a^n b^n$

Input Alphabet
 $\Sigma = \{a, b\}$



$$FA = (Q, \Sigma, \delta, q_0, F)$$

$$\delta_{NFA} : Q \times \Sigma \rightarrow 2^Q$$

$$\delta_{DFA} : Q \times \Sigma \rightarrow Q$$

$$PDA = (Q, \Sigma, \delta, q_0, F, Z_0, \Gamma)$$

$$\delta_{PDA} : Q \times \Sigma \times \Gamma^* \rightarrow Q \times \Gamma^* (\perp)$$

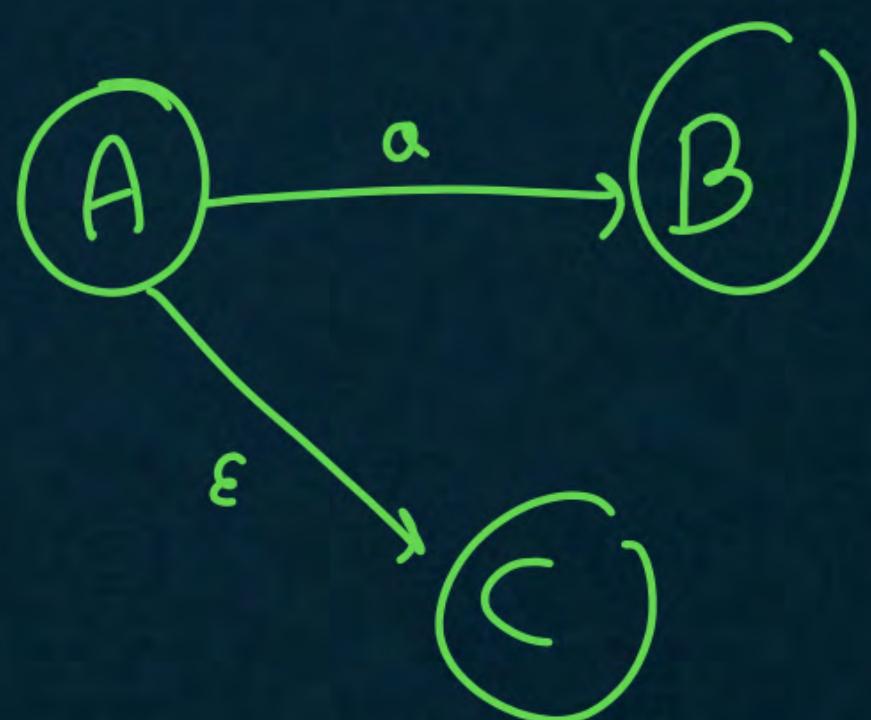
$$\delta_{DPDA} : Q \times \Sigma \times \Gamma \rightarrow Q \times \Gamma^*$$

PDA \cong FA + one stack

Stack Alphabet
(Set of stack symbols)

Initial top of stack symbol
(Bottom of stack symbol)

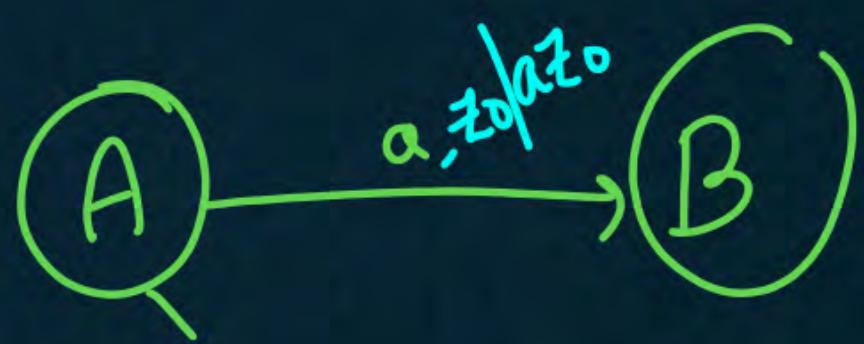
$$\alpha \times \sum_{\xi} \rightarrow \text{?}$$



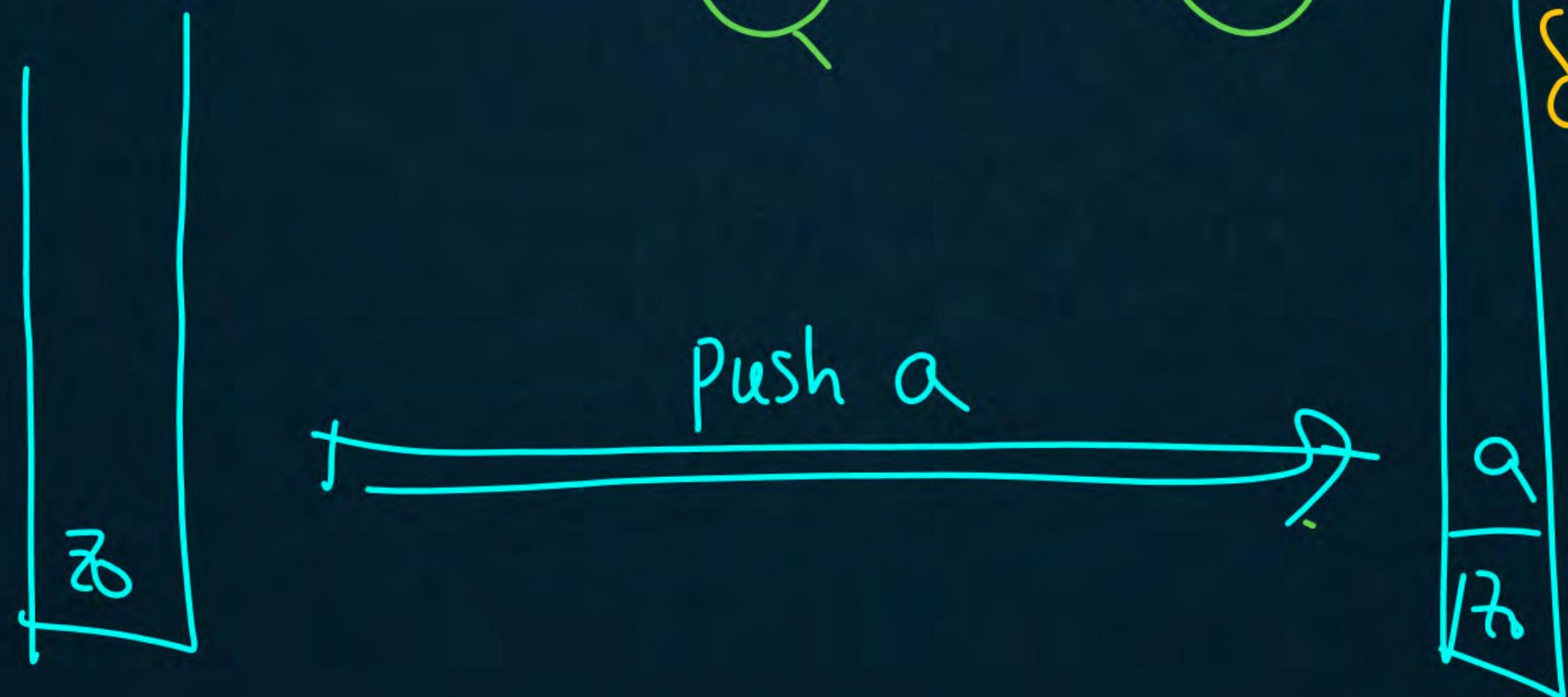
$$\delta(A, a) = B$$
$$\delta(B, ε) = C$$

$$Q \times \sum_{\varepsilon} \times \cap_{\varepsilon} \rightarrow Q$$

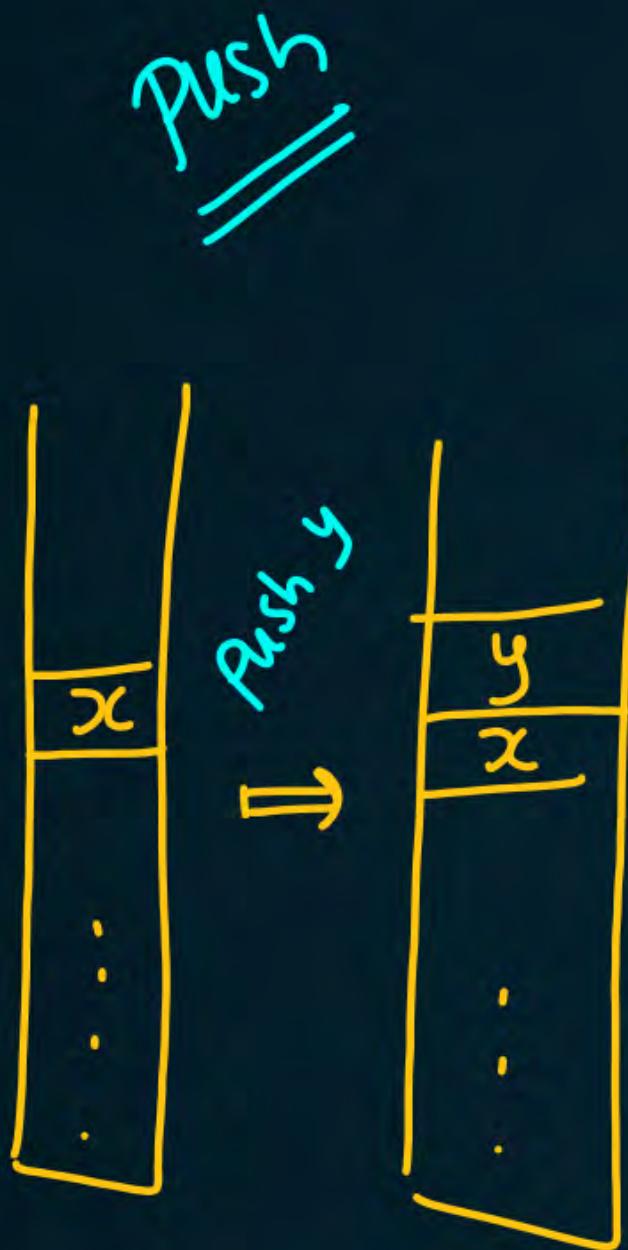
$$Q \times \cap^*$$



$$\delta(A, a, z_0) = (B, a^2)$$



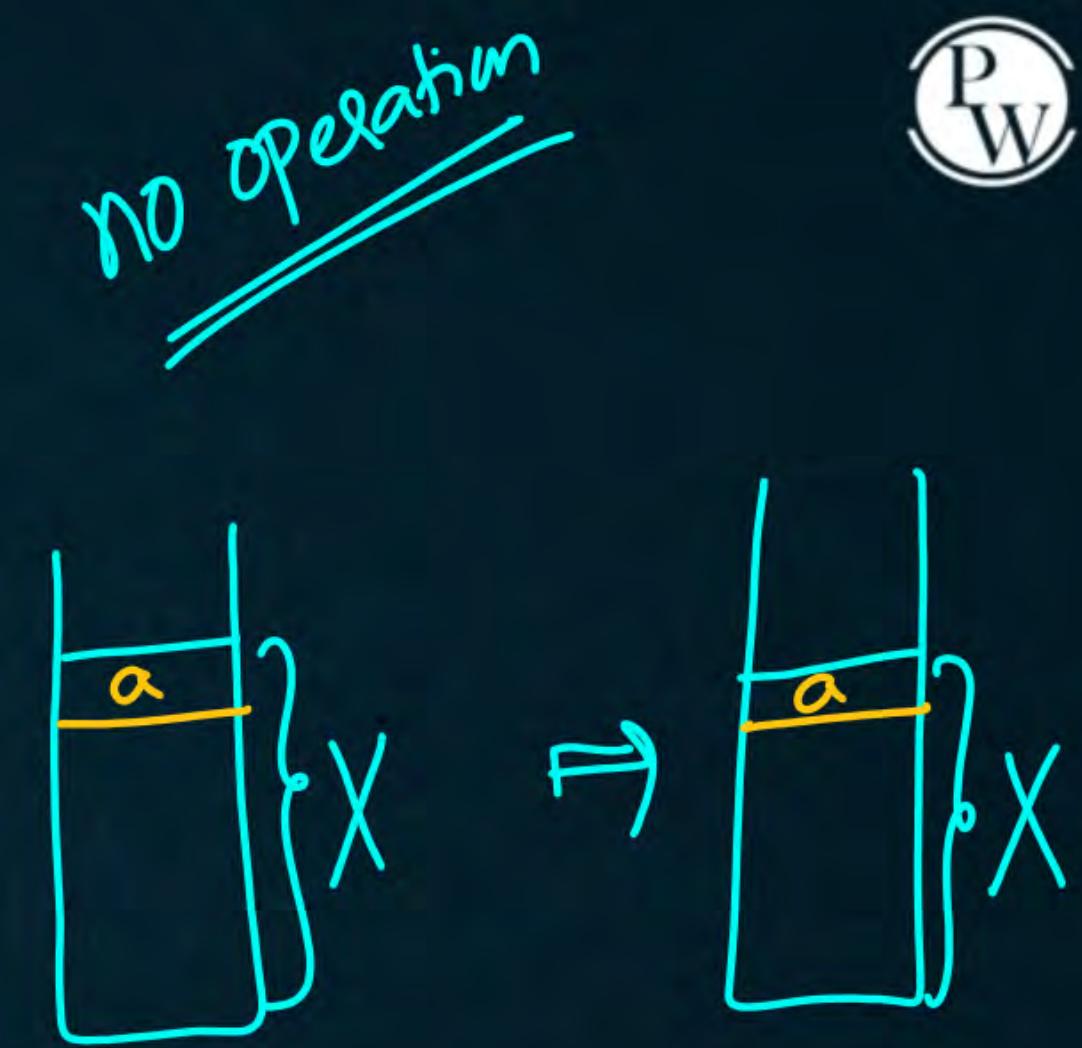
$$\epsilon \cap^*$$



$x \rightarrow yx$
 x/yx



x/ϵ



a/a
 \times/\times
 ϵ/ϵ

$$\Sigma_\epsilon \Rightarrow \Sigma \cup \{\epsilon\}$$

Σ^* = set of strings over Σ

$$\Gamma_\epsilon \Rightarrow \Gamma \cup \{\epsilon\}$$

Γ^* = set of strings over Γ

$$\Gamma = \{x, z_0\}$$

$$\Gamma^* = \{\epsilon, x, z_0, xx, xz_0, \dots\}$$

- ① $a/a \Rightarrow$ no operation performed on stack
- ② $a/\epsilon \Rightarrow$ pop a
- ③ $\epsilon/a \Rightarrow$ push a
- ④ $\epsilon/\epsilon \Rightarrow$
- ⑤ $a/a/a \Rightarrow$ push a
- ⑥ $a/a/\epsilon \Rightarrow$ pop 2 a's
- ⑦ $a/b \Rightarrow$ not valid

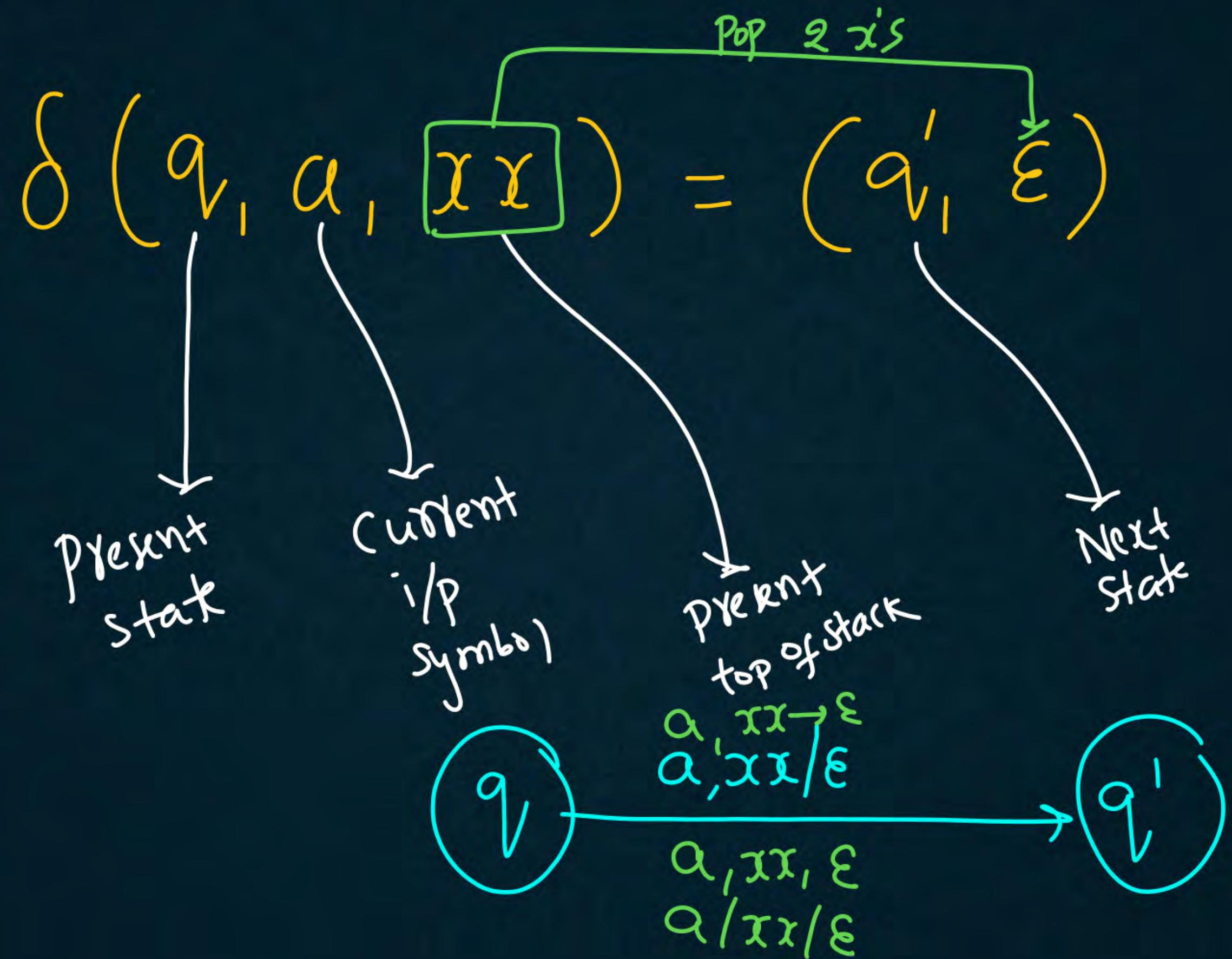
$$\delta(q, a, \overbrace{xx}^{c \in \cap}) = (q', \epsilon)$$

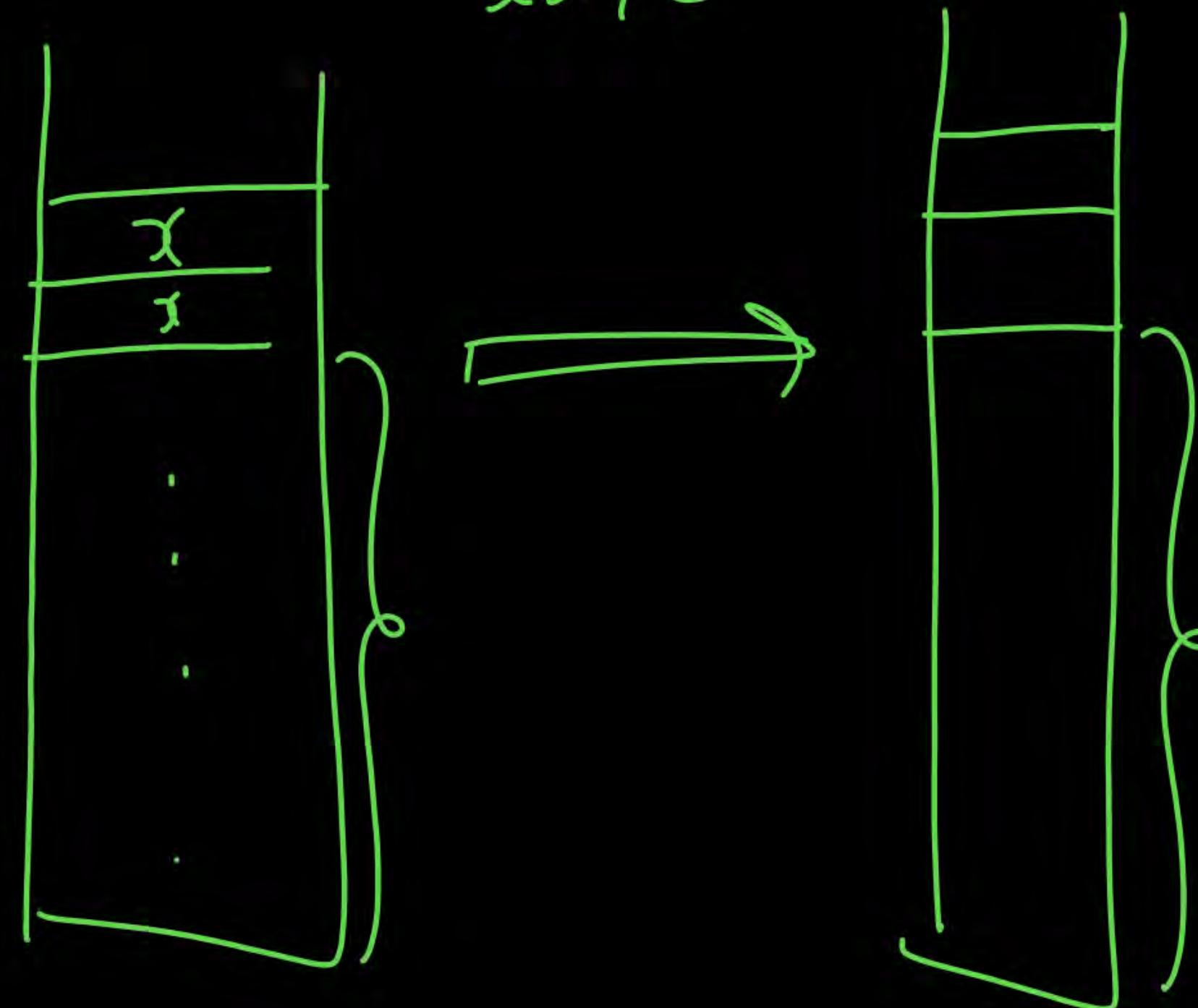
DPDA: $Q \times \Sigma \times \Gamma \rightarrow$

PDA: $Q \times \Sigma_\epsilon \times \Gamma^* \rightarrow$

Valid in PDA

not valid in DPDA



$\alpha\alpha/\epsilon$ 

xx / ε

xx → ε

xx, ε

xx : ε

Find transition valid or not

If valid, is it applicable in DPDA or not?
also find meaning.

$$\delta(q_1, \epsilon, \epsilon) = - - - -$$

Reading ϵ on tape

Reading ϵ on stack

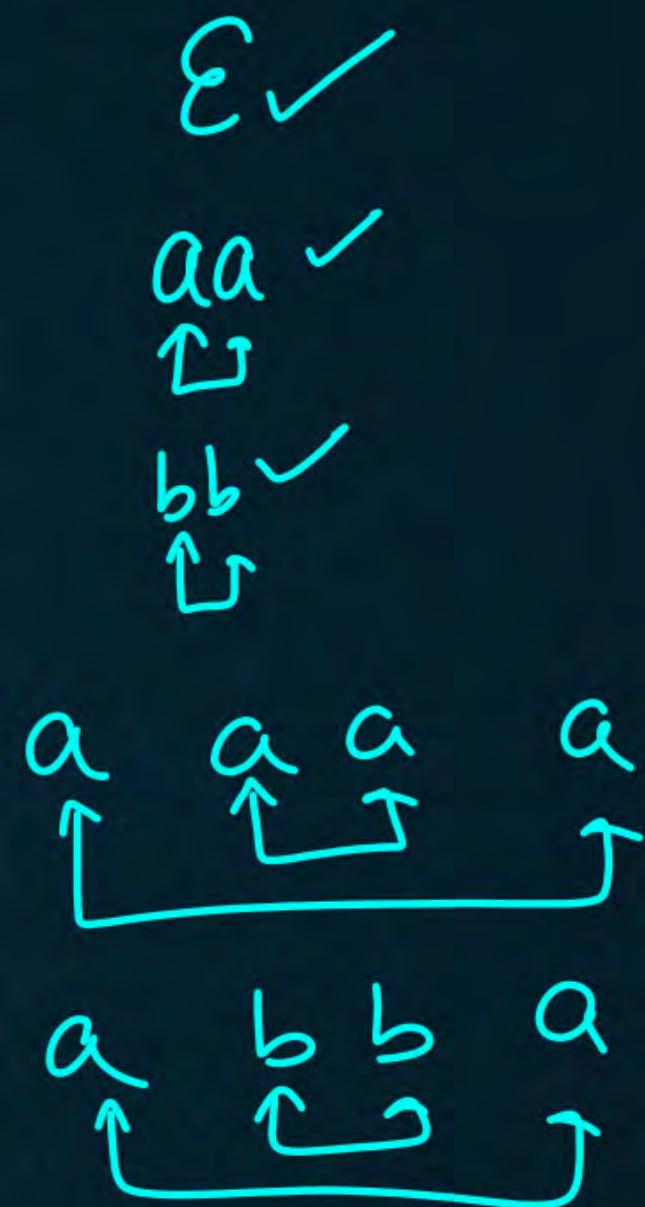
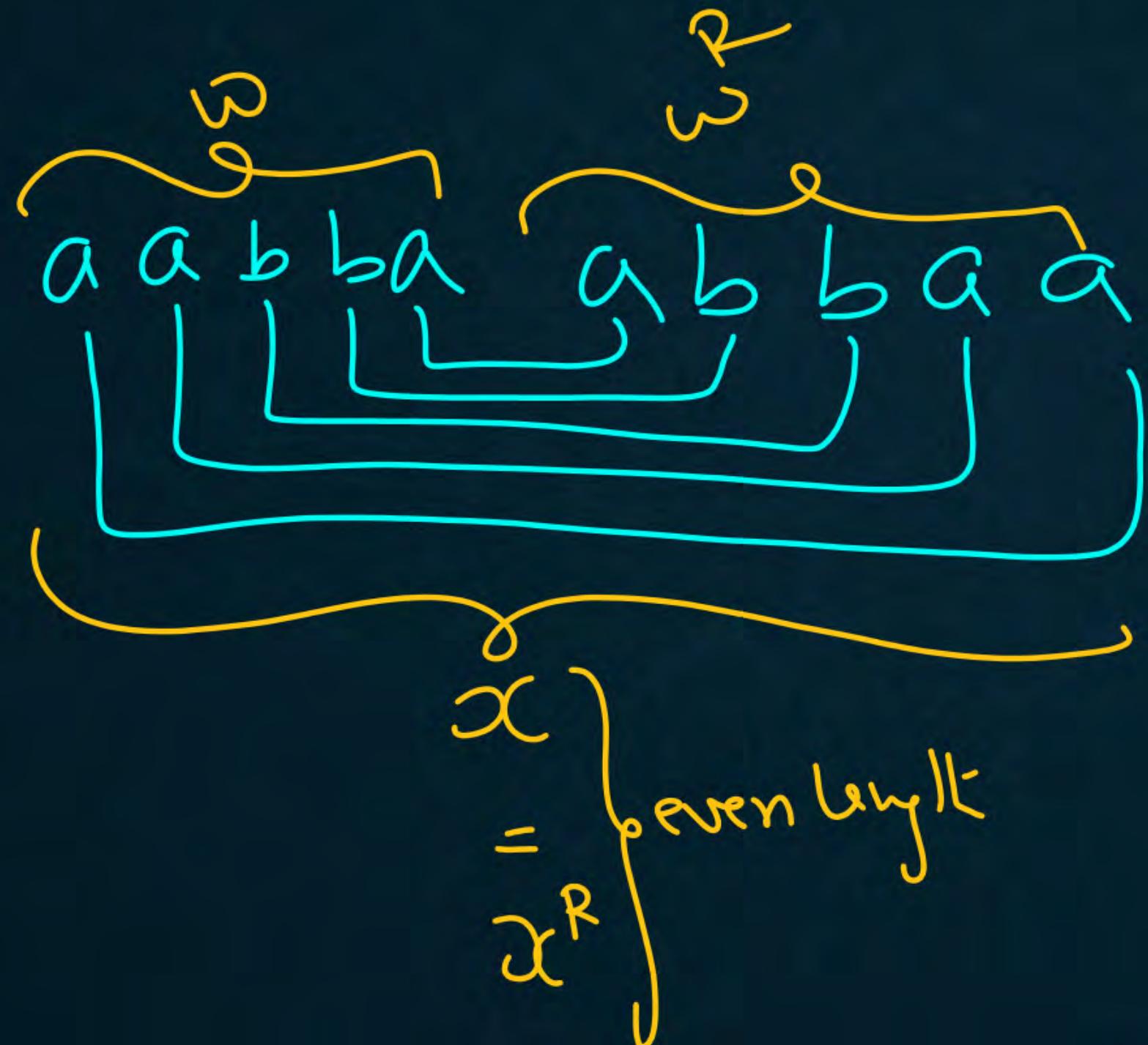
(Not looking at stack)

(not reading any i/p on tape)

$\textcircled{1} \quad \delta(q, \epsilon, \epsilon) = (q', a)$	PDA ✓ DPDA ✗	Without reading i/p, without looking at tos Push 'a'
$\textcircled{2} \quad \delta(q, \epsilon, a) = (q', a)$	PDA ✓ DPDA ✗	Without reading i/p, when tos = a, No operation performed on stack
$\textcircled{3} \quad \delta(q, a, \epsilon) = (q', a)$	PDA ✓ DPDA ✗	When i/p = a, without looking at stack Push 'a'
$\textcircled{4} \quad \delta(q, a, b) = (q', ab)$	PDA ✓ DPDA ✓	When i/p = a, tos = b, push a
$\textcircled{5} \quad \delta(q, a, b) = (q', aaaab)$	PDA ✓ DPDA ✓	When i/p = a, tos = b - push 3 a's

$$S \rightarrow aSa/bSb \propto$$

$\overbrace{[w \propto w^r]}$

$S \rightarrow aSa | bSb | \epsilon$ 

$$S \rightarrow aSa \mid bSb \mid a \mid b$$

$$= \{ w(a+b)w^R \mid w \in \{a,b\}^* \}$$

$$= \{ x \mid x \in \{a,b\}^*, x = x^R, |x| = \text{odd} \}$$

a b a
b a b
b b b
.

= Set of all odd length palindromes

New concept (41) $S \rightarrow aSa \mid bSb \mid \epsilon \Rightarrow L = \{ww^R \mid w \in \{a,b\}^*\}$
 = Set of all even length palindromes
 $= \{x \mid x \in \{a,b\}^*, x = x^R, |x| = \text{even}\}$

(42) $S \rightarrow aSa \mid bSb \mid a \mid b \Rightarrow L = \text{Set of all odd length palindromes}$

(43) $S \rightarrow aSa \mid bSb \mid a \mid b \mid \epsilon \Rightarrow L = \text{Set of all palindromes}$
 $= \{x \mid x = x^R, x \in \{a,b\}^*\}$

(44) $S \rightarrow aSa \mid bSb \mid \#$

$$L = \{ww^R \mid w \in \{a,b\}^*\}$$

$\boxed{\begin{array}{l} = \{x \mid x = x^R, x \in \{a,b\}^*\} \\ = \{wxw^R \mid w \in \{a,b\}^*, \\ x \in \{a,b,\epsilon\}\} \end{array}}$

P
W

$$L = \{ w \# w^R \mid w \in \{a, b\}^* \}$$

$$= \{ \#, a\#a, b\#b, ab\#ba, aa\#aa, ba\#ab, \dots \}$$

$$= \{ w C w^R \mid w \in \{a, b\}^* \}$$

$$= \{ w \$ w^R \mid w \in \{a, b\}^* \}$$

45

$$L = \left\{ \begin{matrix} x & y & z \\ a & c & b \end{matrix} \mid x = y + z \right\}$$

$$\left\{ \begin{matrix} K+n & n & K \\ a & c & b \end{matrix} \right\}$$

$$\left\{ \omega \mid \omega \in a^* c^* b^*, \quad N_a(\omega) = N_b(\omega) + N_c(\omega) \right\}$$

P
W

④5) $S \rightarrow aSb | A$

$$A \rightarrow aAc | \epsilon$$

$$A = a^n c^n$$

$$L = a^k \underbrace{a^n c^n}_A b = \{a^{k+n} c^n b\}$$

④6) $S \rightarrow AB$

$$A \rightarrow aAb | \epsilon$$

$$B \rightarrow cBd | \epsilon$$

$$\begin{aligned} B &= c^n d^n \\ A &= a^k b^k \end{aligned}$$

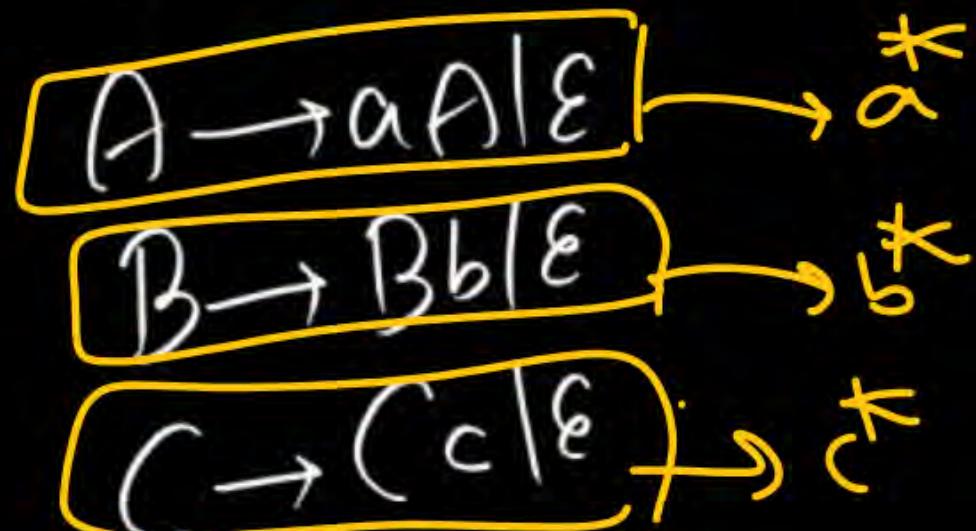
$$L = \{a^k b^k c^n d^n\}$$

④7) $S \rightarrow aSb | A$

$$A \rightarrow cAb | \epsilon$$

$$\left. \begin{aligned} A &= c^n b^n \\ L &= a^k c^n b^n \underbrace{c^n b^n}_{n+k} \\ &= \{a^k c^n b^{n+k}\} \end{aligned} \right\}$$

④8) $S \rightarrow ABC$



$$L = a^* b^* c^*$$

④9

$$S \rightarrow SS | (S) | \epsilon$$

$L = \text{Set of all balanced parentheses}$

P
W

⑤0

$$E \rightarrow E + E \mid E * E \mid (E) \mid a$$

$L = \text{Set of all arithmetic expressions}$
 $\text{using } +, *$

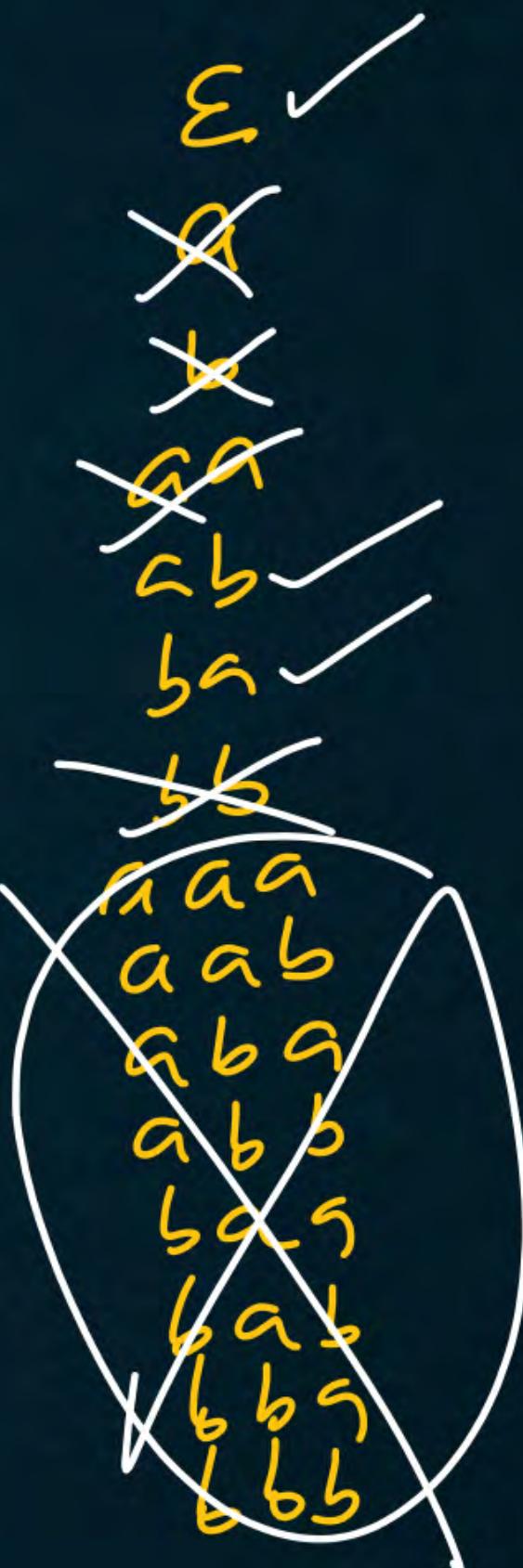
Functional(f)
(meaning)

(51)

 $S \rightarrow aSb \mid bSa \mid SS \mid \epsilon$

$L = \{ w \mid w \in \{a, b\}^*, n_a(w) = n_b(w) \}$

aabb
abab
baba
baab
⋮



52

$$S \rightarrow aSb \mid bSa \mid \underbrace{SS}_{\sim} \mid aS \mid \varepsilon$$

$$L = \{ \omega \mid \omega \in \{a, b\}^*, n_a(\omega) \geq n_b(\omega) \}$$

53

$$S \rightarrow aSb \mid bSa \mid SS \mid \underbrace{bS}_{\sim} \mid \varepsilon$$

$$L = \{ \omega \mid \omega \in \{a, b\}^*, n_a(\omega) \leq n_b(\omega) \}$$

(S4)

$$S \rightarrow aSb \mid bSa \mid SS \mid aS \mid a$$

$$\boxed{n_a(\omega) > n_b(\omega)}$$

(S5)

$$S \rightarrow SaSbS \mid SbSaS \mid \epsilon$$

ε ✓

ab ✓

ba ✓

aabb ✓

bbaa ✓

qbab ✓

baba' ✓
 abba' ✓
 baab' ✓

$$L = \{ \omega \mid \omega \in \{a, b\}^*, n_a(\omega) = n_b(\omega) \}$$

H.W. 
① $L = \{ xy \mid |x|=|y|, x \neq y, x, y \in \{a, b\}^*\}$

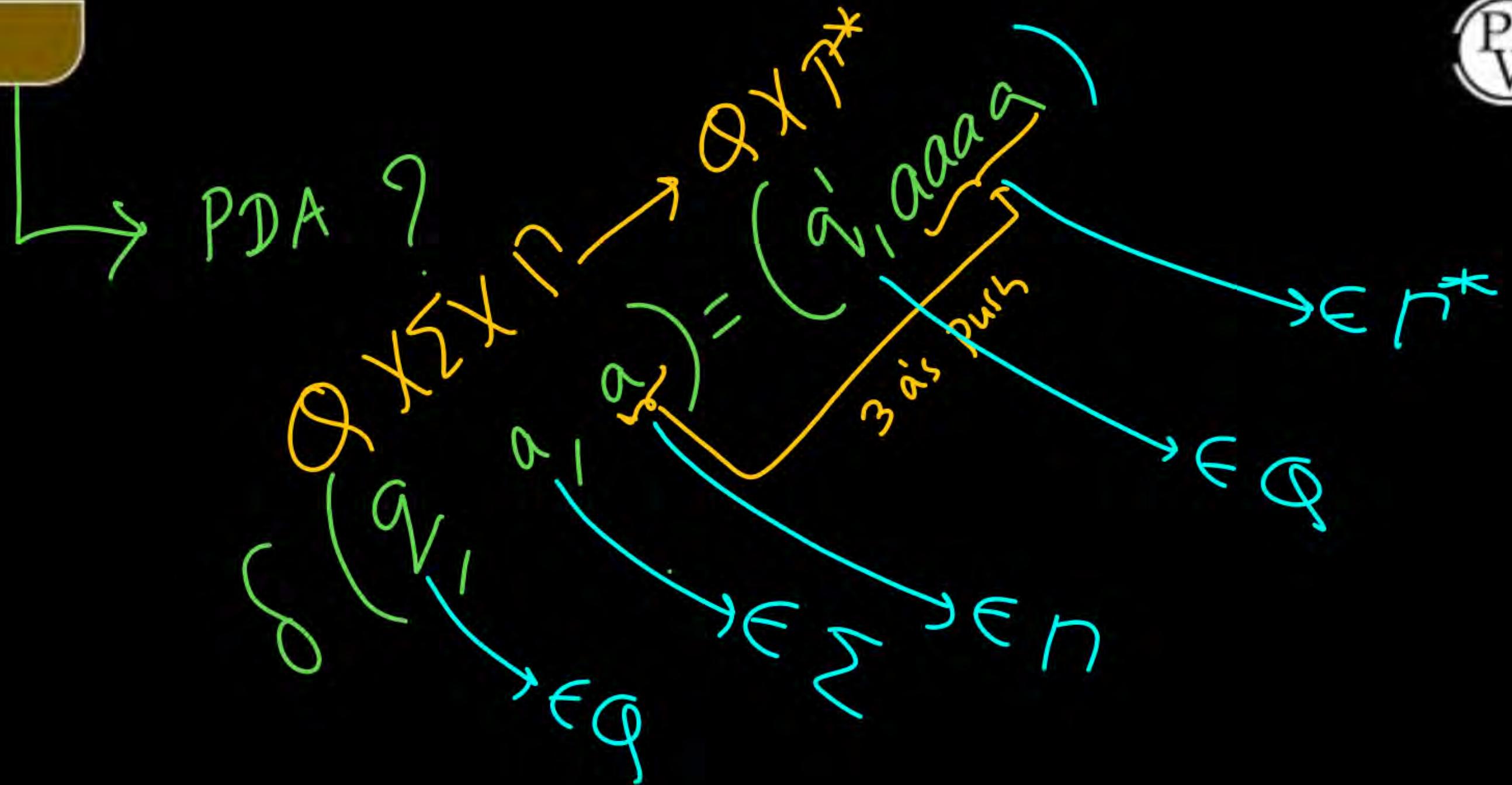
write CFG

② $L = \{ x \mid x \in \{a, b\}^*, x \neq x^R \}$

write CFG

Summary

P
W



next step in DPAK

$$\delta(q, a, aa) = (q', \epsilon)$$

$$Q \times \Sigma \times \Gamma \longrightarrow$$

$$\Gamma = \{a, b\}$$

$$aa \notin \Gamma$$

$$aa \in \Gamma^*$$

