CS & IT ENGINEERING

Theory of Computation

Finite Automata

Lecture No. 19

By- DEVA Sir

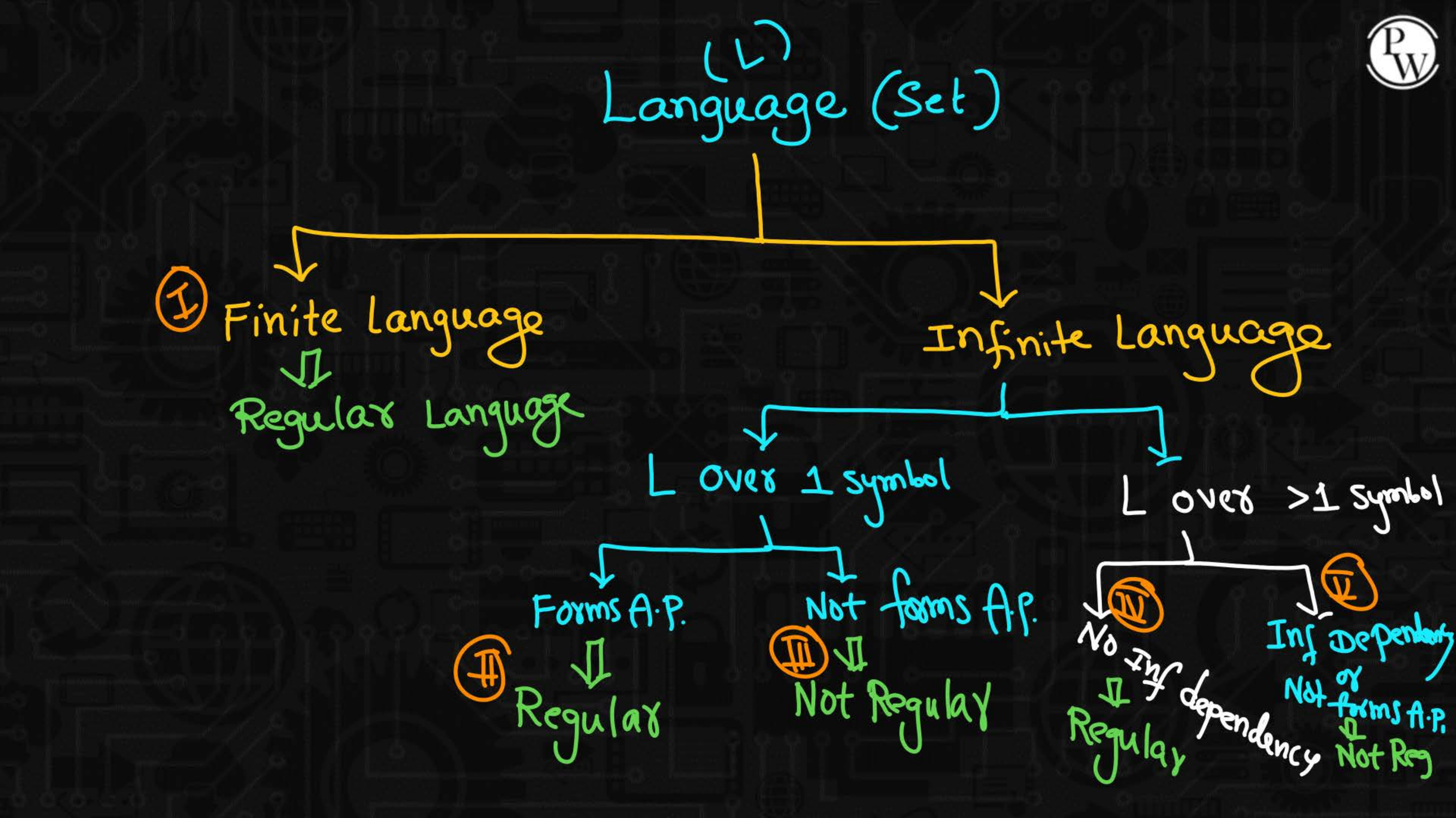
01 Regular

02 Non Regular

03 Finite

04 Infinite

05


(L)
Language (Set)
I
Finite Language
⇓
Regular Language
Infinite Language
L over 1 symbol
Forms A.P.
II
⇓
Regular
Not forms A.P.
III
⇓
Not Regular
L over >1 Symbol
IV
No Inf dependency
⇓
Regular
V
Inf Dependency
or
Not forms A.P.
⇓
Not Reg

Language (Set) (L)

(I) Finite Language

$L_1 = \phi$

$L_2 = \{\varepsilon\}$

$L_3 = \{a^n b^n \mid n < 10\}$

$L_4 = \{w \mid w \in (a+b)^*, |w| \le 100\}$

Infinite Language

L over 1 symbol

Forms A.P.

$L_1 = a^*$

$L_2 = a^{2n}$

$L_3 = a^{3n+100}$

Not forms A.P.

$L_1 = a^{\text{Prime}}$

$L_2 = a^{n!}$

L over >1 Symbol

(II) No inf dep.

$L_1 = a^* b^*$

$L_2 = \{a^n b^n c^* \mid n < 100\}$

(III) Inf Dependency Not or forms A.P.

$L_1 = a^n b^n$

Every Finite language is Regular. [TRUE]

Every Infinite language is Regular. [FALSE]

↳ Some Infinite languages are Regular

Some " " " not regular.

a^* — Reg

a^{prime} — Not Reg

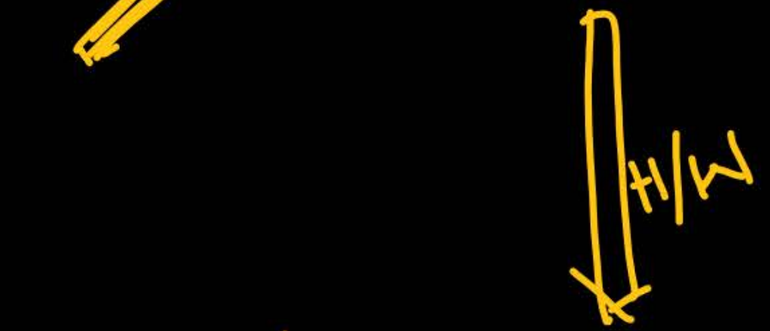
Focus ?

~~I)~~ Will you decide just by looking ?

~~II)~~ " " " by understanding strings of language?

EBIT

Start



Confidence ⇒ End

Eye

⇩

Brain

⇩

Heart

L is Regular

iff

L has FA

iff

L has Reg Exp

iff

L has RG

L is Not Reg

iff

No FA for L

iff

No Reg Exp for L

iff

No RG for L

Identify Regulars and Non regulars.

① $\{a^m b^n\} = \{a^m b^n \mid m, n \geq 0\} = a^* b^*$ ⇒ Regular

$= \{a^+, a^2b, a^3b, \cdots, a^3b^2, a^4b^2, \cdots\}$

② $\{a^m b^n \mid m > n\}$ ⇒ Not regular language

③ $\{a^m b^n \mid m < n\}$

④ $\{a^m b^n \mid m = \text{even}, n = \text{even}\} = (aa)^*(bb)^*$

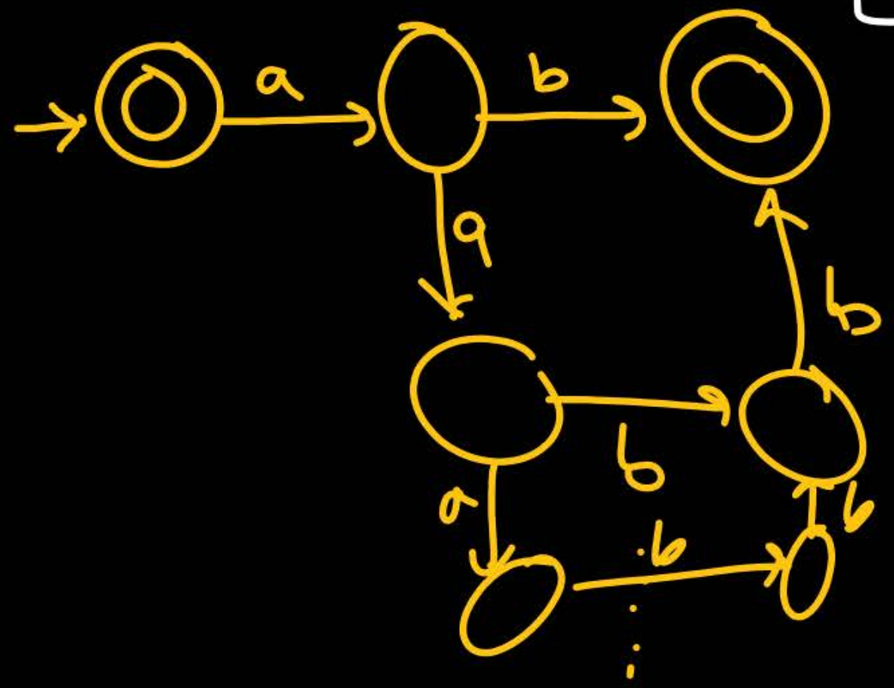
⑤ $\{a^m b^n \mid \underbrace{m = n}_{\text{equal}} = \text{even}\} = \{a^{2n} b^{2n}\}$ ⇒ Not regular

(6) $\{a^n b^n \mid n \geq 0\} = \{\varepsilon, ab, aabb, aaabbb, \ldots\}$

→ Not regular

aaaaa bbbbb

equal



no FA

Infinite no. of state required

$$\{a^n b^n\} = \{\omega \mid \omega \in a^* b^*,\ n_a(\omega) = n_b(\omega)\}$$

$$= \{\varepsilon, ab, a^2b^2, a^3b^3, \cdots\}$$

(7) $\{a^n b^{n+1}\}$ ⇒ Not regular

(8) $\{a^{n+2} b^{n+100}\}$

(9) $\{a^m b^n \mid m=n \text{ OR } m \neq n\} = \{a^m b^n\} = a^* b^*$ ⇒ Regular

(10) $\{a^m b^n \mid m<n<100\}$ ⇒ Finite language ⇒ Regular

(11) $\{a^m b^n \mid m>n>100\}$ ⇒ Not regular

$a^m b^n$

ω

Rel b/w m & n

OR

Rel b/w a's & b's

(12) $\{a^m b^n \mid \gcd(m,n)=1\} = \{a^2b^3, a^2b^5, a^2b^7, \ldots\}$ → Not reg

(13) $\{a^m b^n \mid \text{lcm}(m,n)=1\} = \{ab\}$ ⇒ Finite Set ⇒ Regular

(14) $\{a^m b^n \mid m+n=1\} = \{b, a\}$

(15) $\{a^m b^n \mid m \times n = 1\} = \{ab\} = (13)$

(16) $\{a^m b^n \mid m=n=1\} = (15) = (13)$

(17) $\{a^m b^n \mid m=2,\ n=\text{odd}\} = aa\,b(bb)^* \Rightarrow$ Regular

(18) $\{a^m b^n \mid \text{if } (\underbrace{m=\text{even}}_{P}) \text{ then } (\underbrace{n=\text{odd}}_{Q})\}$

$= \{a^m b^n \mid \underbrace{m=\text{odd}}_{m\neq\text{even}} \text{ OR } n=\text{odd}\} = \underbrace{a(aa)^*}_{a's=\text{odd}} b^* + a^* \underbrace{b(bb)^*}_{b's=\text{odd}}$

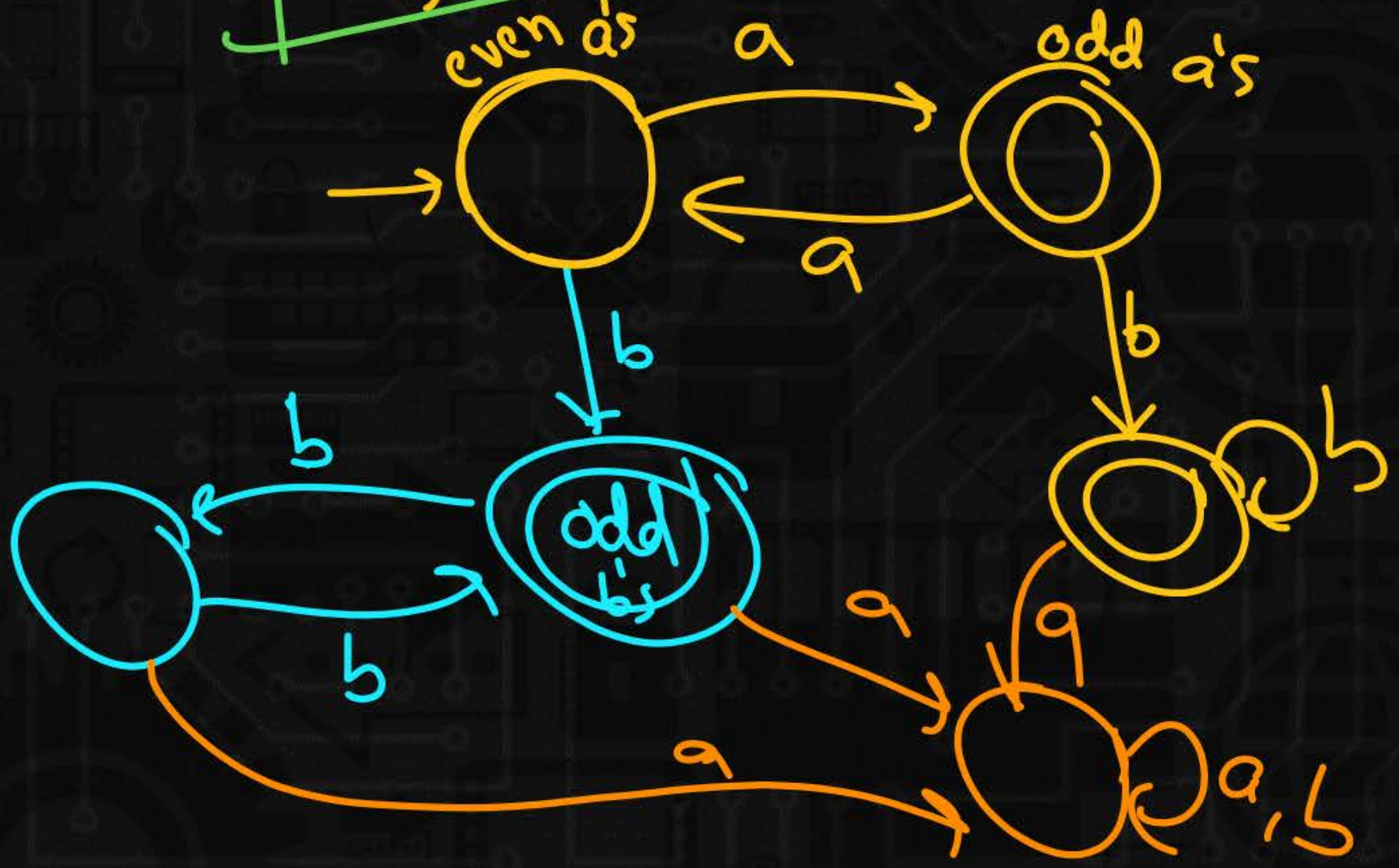
(19) $\{a^m b^n \mid \underbrace{m<100}_{m\leq 99},\ \underbrace{n\geq 100}_{n\geq 101}\} = (\varepsilon + a + a^2 + \cdots + a^{99})\, b^{101}\, b^* \Rightarrow$ Regular

(20) $\{a^n b^{2n}\} \Rightarrow$ Not regular

$a^m b^n$

If #a's = even ⇨ #b's = odd

If #a's ≠ even ⇨ #b's = any



If P then Q

$P \Rightarrow Q$

$\sim P \vee Q$

Either P False OR Q True

P	Q	$P \Rightarrow Q$
T	T	T
F	T	
F	F	

T $\Rightarrow$ F $\Rightarrow$ F

If a's = even $\Rightarrow$ b's = odd

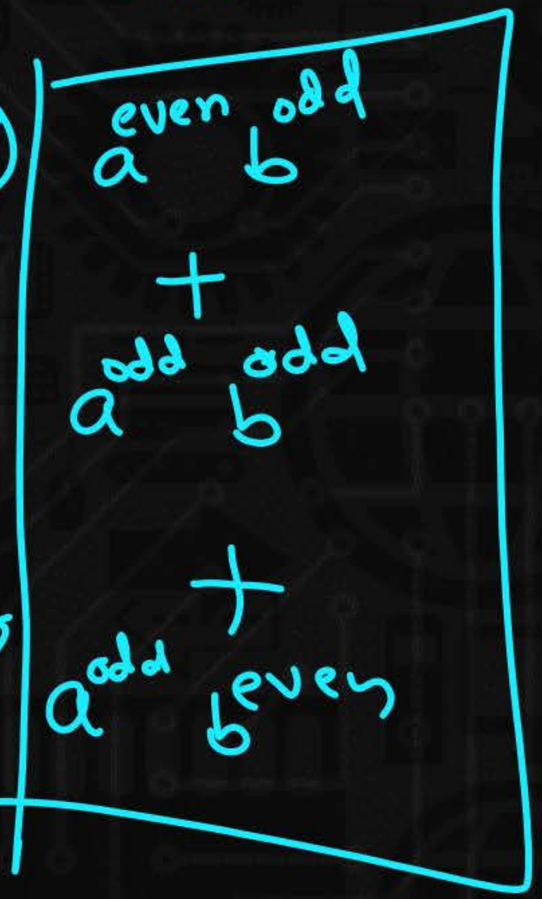
P: T → Q: T
P: F → Q: T/F

I) $\sim p \vee q$

a's ≠ even OR b's = odd

a's = odd

II)



T T

F T

F F

(21) $\{a^n b^n c^n\}$

(22) $\{a^n b^{2n} c^{3n}\}$

(23) $\{a^n b^{n+1} c^{n+2}\}$

(24) $\{a^n b^n c^*\}$

(25) $\{a^n b^* c^n\}$

(26) $\{a^n b^{n+5} c^{2n}\}$

$\Rightarrow$ Not regular

(27) $\{a^n \mid n \geq 0\} = a^*$

(28) $\{a^{2n}\} = (aa)^*$

(29) $\{a^{3n+100}\}$

$\Rightarrow$ Reg

(30) $\{a^{\text{Prime}}\}$ $\Rightarrow$ Not regular

(31) $\{a^{n!}\}$

(32) $\{a^{2^n}\}$

(33) $\{a^{n^2}\}$

(34) $\{a^{100^n}\}$

(35) $\{a^{n^{45}}\}$

$\Rightarrow$ Not reg

(36) $\{a^{100n}\}$ $\Rightarrow$ Regular

(37) $\{a^{2024n+1}\}$

(38) $\{a^{100n}\} \cup \{b^{1000k}\}$

n
$2n$
$3n+100$
$100n$
} A.P.

$n!$
prime
2^n
n^2
n^{100}
} not forms A.P.

a^{2^n}

$n=0 \Rightarrow a^{2^0} = a^1$

$n=1 \Rightarrow a^{2^1} = a^2$

$n=2 \Rightarrow a^{2^2} = a^4$

$n=3 \Rightarrow a^{2^3} = a^8$

a^{2n}

$n=0 \Rightarrow a^0 = \varepsilon$

$n=1 \Rightarrow a^2$

$n=2 \Rightarrow a^4$

$n=3 \Rightarrow a^6$

Reg (39) $\{a^{2n}\}^* = (aa)^*$ ⇒ Reg

Reg (40) $\{a^{\text{prime}}\}^* = \{a^n \mid n \neq 1\}$

Not Reg (41) $\{a^{n^n}\} = \{a^1, a^4, a^{27}, \ldots\}$

Reg (42) $\{a^{m^n}\} = \{\varepsilon, a, a^2, \ldots\} = a^*$

$n=1$

$m=1, n=1$

$m=2, n=1$

(43) $\{a^{2^n}\}^* = a^*$

(44) $\{a^{n^2}\}^* = a^*$

(45) $\{a^{n^n}\}^* = a^*$

(46) $\{a^{m^n}\}^* = a^*$

Reg equal

$$\{a^{2^n}\}^* = \{a^{2^n} \mid n \geq 0\}^*$$

$n=0 \Rightarrow \{a^{2^0}\}^* = a^*$

$$= \{a^{2^n}\}^0 \cup \{a^{2^n}\}^1 \cup \{a^{2^n}\}^2 \cup \cdots$$

$\{a^{2^{n_1}}\} \cdot \{a^{2^{n_2}}\}$

$a \cdot aa$

$$= \{\varepsilon, a, a^2, a^3, \ldots\}$$

Not Reg

(47) $\{a^{*}\ b^{\text{prime}}\}$

Reg

(48) $\{a^{2n}\ b^{3m}\} = (aa)^{*}(bbb)^{*}$

(49) $\{a^{2n}\ b^{m^2}\}$

(50) $\{a^{n!}\ b^{\text{prime}}\}$

(51) $\{a^{n^2}\ b^{2^m}\}$

$\Rightarrow$ Not Reg

(52) $\{a^{n}\ b^{n^2}\}$

(53) $\{a^{n^2}\ b^{2^n}\}$

(54) $\{a^{n}\ b^{n!}\}$

(55) $\{a^{n}\ b^{\text{prime}}\ c^{2^k}\ d^{j!}\}$

$a^{2n}\ b^{m^2} = a^{\text{even}}\ b^{m^2}$

Not forms A.P.

no logic for reg

over >1 Symbol

→ Forms A.P. or not

→ Dep exist or not

$\{b^{m^2}\} \Rightarrow$ Not reg

$\{a^{m^2}\}$

ε.ε a.a aa.aa aaa.aaa

(56) $\{ww \mid w \in a^*\} = \{\epsilon, a^2, a^4, a^6, \ldots\} = (aa)^* = a^{2n} \Rightarrow$ Reg

(57) $\{w\#w \mid w \in a^*\} = \{\#, a\#a, a^2\#a^2, \ldots\} = \{a^n \# a^n\} \Rightarrow$ Not reg

(58) $\{ww^R \mid w \in a^*\} = (56) \Rightarrow$ Reg

(59) $\{w\#w^R \mid w \in a^*\} = (57) \Rightarrow$ Not Reg

(60) $\{ww \mid w \in \{a,b\}^*\}$

(61) $\{w\#w \mid$ " $\}$

(62) $\{ww^R \mid$ " $\}$

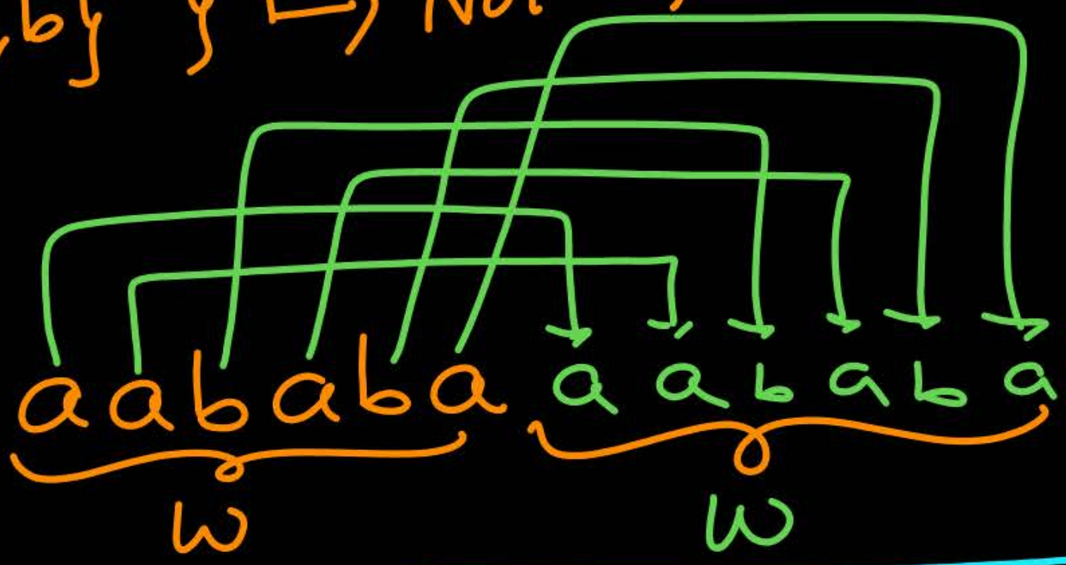
(63) $\{w\#w^R \mid$ " $\}$

$\Rightarrow$ Not regular

(64) $\{w_1w_2 \mid w_1, w_2 \in a^*\}$

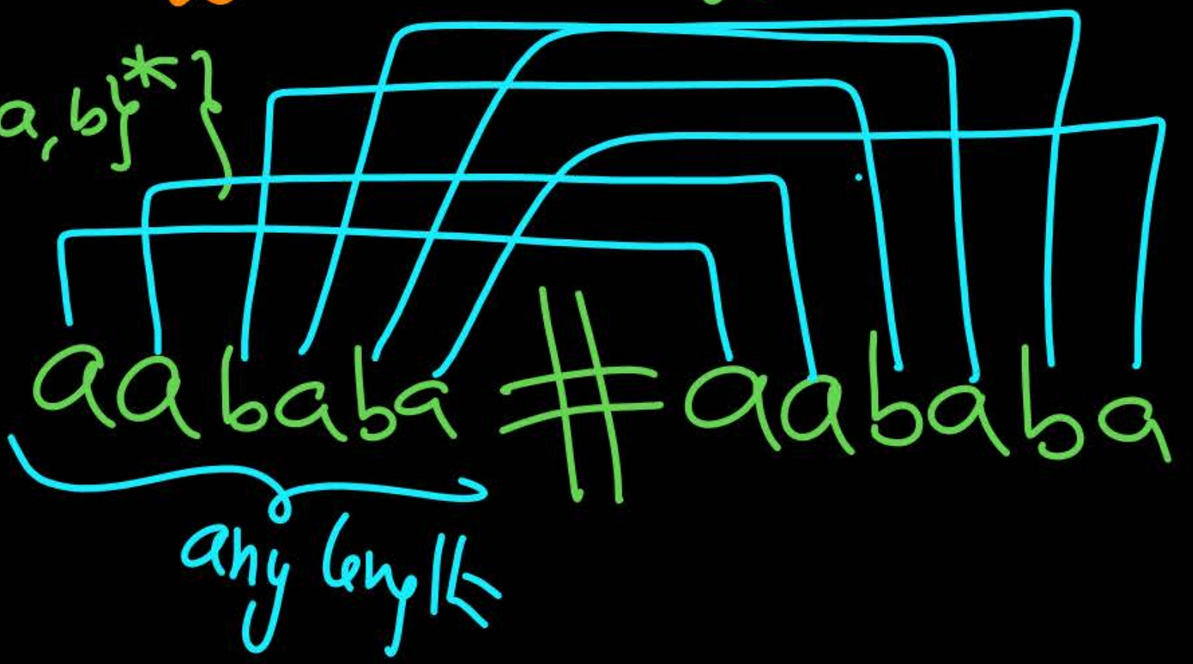
$= a^* \Rightarrow$ Reg

(60) $\{ww \mid w \in \{a,b\}^*\}$ ⇒ Not regular



(61) $\{w\#w \mid w \in \{a,b\}^*\}$

↓ Not reg



(62) $\{ww^R \mid w \in \{a,b\}^*\}$

↓

Not reg

$= \{\varepsilon, aa, bb, aaaa, abba, baab, bbbb, \ldots\}$

(63) $\{w\#w^R \mid w \in \{a,b\}^*\}$ ⇒ Not reg

$\{w\#w \mid w \in \{a,b\}^*\}$

$\Sigma = \{a, b, \#\}$

$\{\omega x \omega \mid \omega, x \in \{a,b\}^*\}$

$\Sigma = \{a, b\}$

Home work :

(65) $\{ww^Rx \mid w,x \in \{a,b\}^*\}$

(66) $\{wxw \mid$ " $\}$

(67) $\{xww \mid$ " $\}$

(68) $\{ww^Rx \mid$ " $\}$

(69) $\{wxw^R \mid$ " $\}$

(70) $\{xww^R \mid$ " $\}$

(71) $\{ww^Rx \mid w,x \in \{a,b\}^+\}$

(72) $\{wxw \mid$ " $\}$

(73) $\{xww \mid$ " $\}$

(74) $\{ww^Rx \mid$ " $\}$

(75) $\{wxw^R \mid$ " $\}$

(76) $\{xww^R \mid$ " $\}$

Summary

→ Reg & Non Reg

Sunday 10 AM


THANK YOU
GW
SOLDIERS !