CS & IT ENGINEERING

Theory of Computation

Finite Automata

Lecture No. 12

By- DEVA Sir

TOPICS TO BE COVERED

01 DFA

02 NFA without ε moves

03 NFA with ε moves

04

05

DFA

① Every DFA is NFA

② $\delta : Q \times \Sigma \to Q$

③ $w \longrightarrow \boxed{\text{DFA}}$

$w \in L$ exactly 1 path

$w \notin L$ exactly 1 path

④ For every regular, Unique min DFA exist

NFA

① NFA may or may not DFA

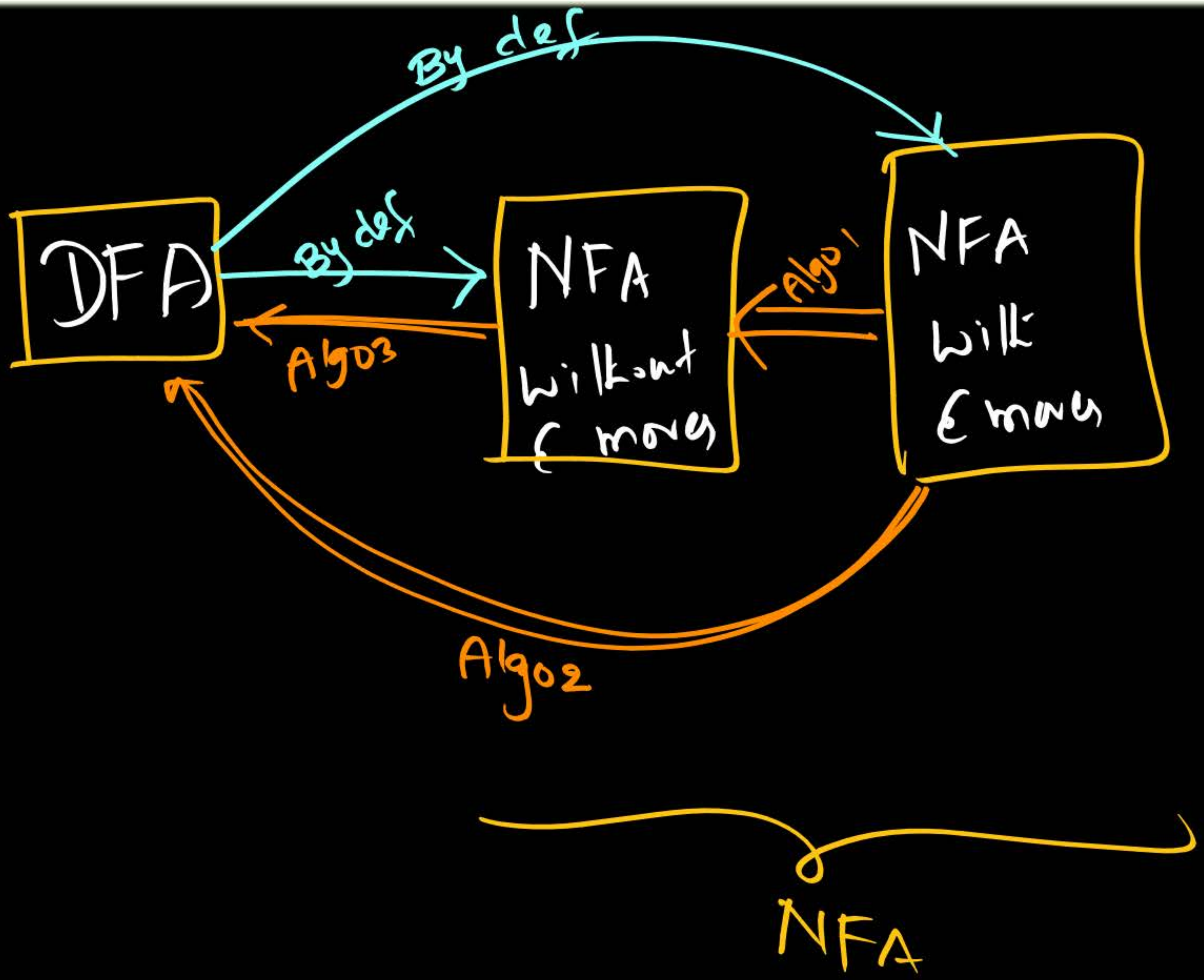
② $\delta : Q \times \Sigma_{\varepsilon} \to 2^{Q}$

③ $w \longrightarrow \boxed{\text{NFA}}$

$w \in L$ atleast 1 path

$w \notin L$ 0 or more paths

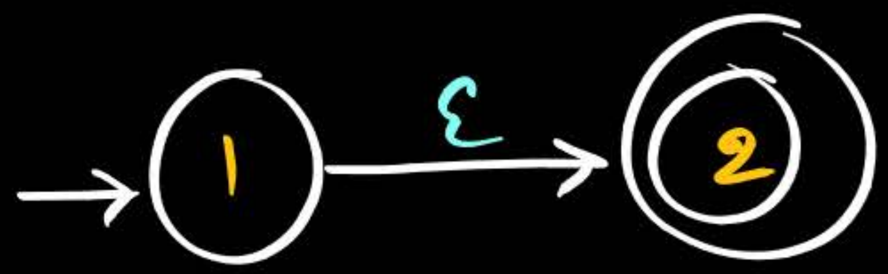
④ For every regular, one or more min NFAs exist


DFA
By def
By def
NFA without ε moves
NFA with ε moves
Algo1
Algo3
Algo2
NFA

$$DFA \cong NFA$$

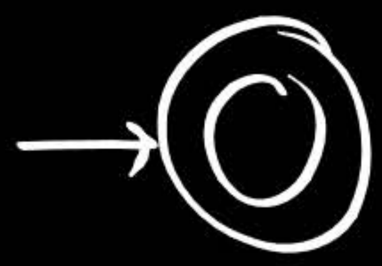
NFA with ε moves:

1



$L = \{\varepsilon\}$

2



$L = \{\varepsilon\}$

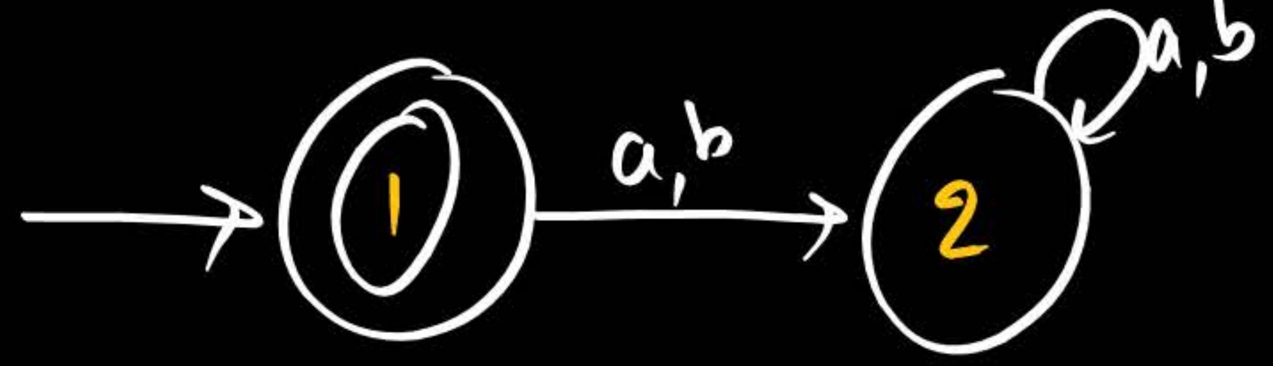
3

$L = \{\varepsilon\}$

ε :

1

$1 \xrightarrow{\varepsilon} 2$

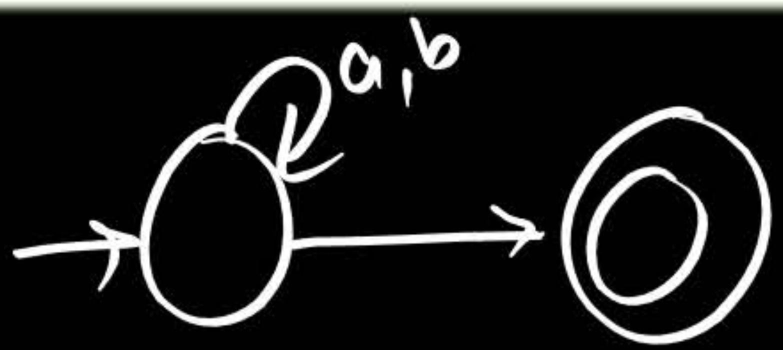


DFA
NFA without ε moves
NFA with ε moves

$$\delta : Q \times \Sigma \cup \{\varepsilon\} \longrightarrow 2^Q$$

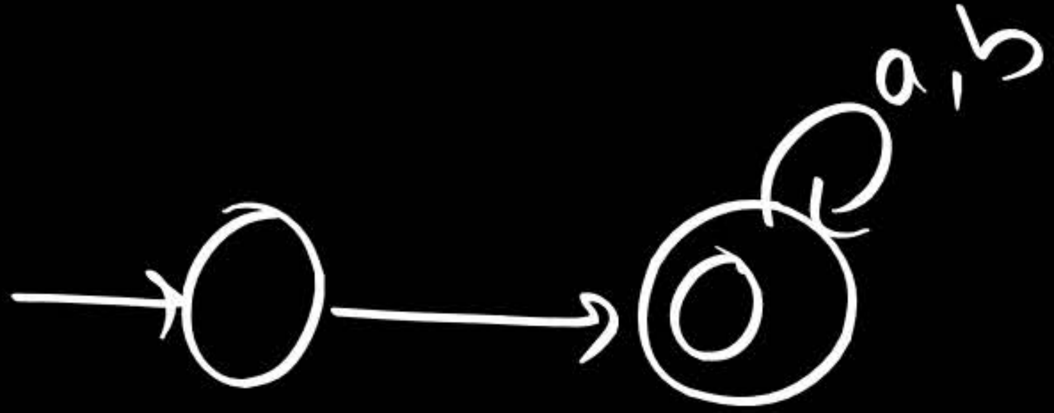
$\delta(1,a) = \{2\}$	$\delta(2,a) = \{2\}$
$\delta(1,b) = \{2\}$	$\delta(2,b) = \{2\}$
$\delta(1,\varepsilon) = \{\ \}$	$\delta(2,\varepsilon) = \{\ \}$

(4)



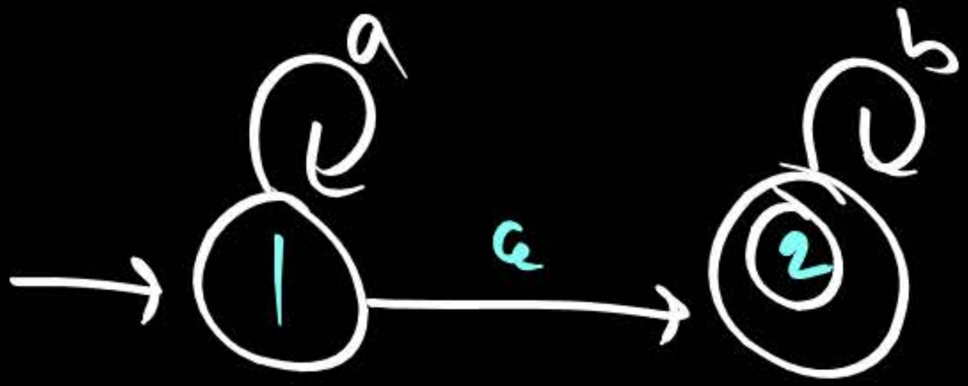
$L = (a+b)^*$

(5)

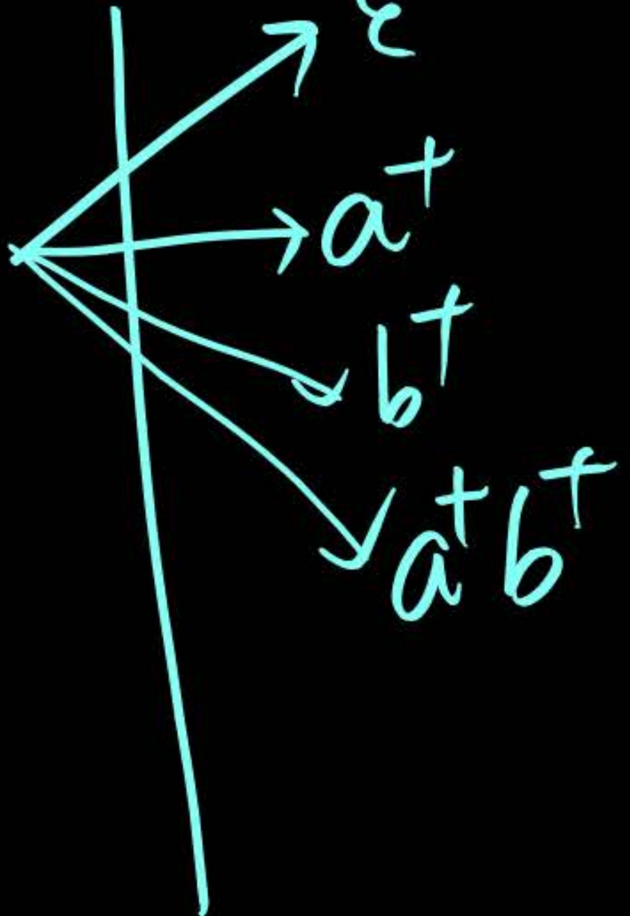


$L = (a+b)^*$

(6)



$L = a^* b^*$



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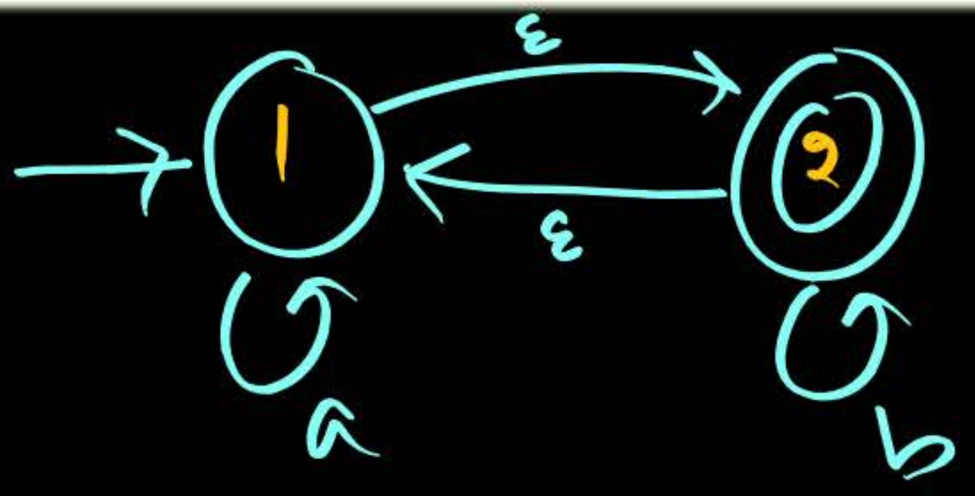
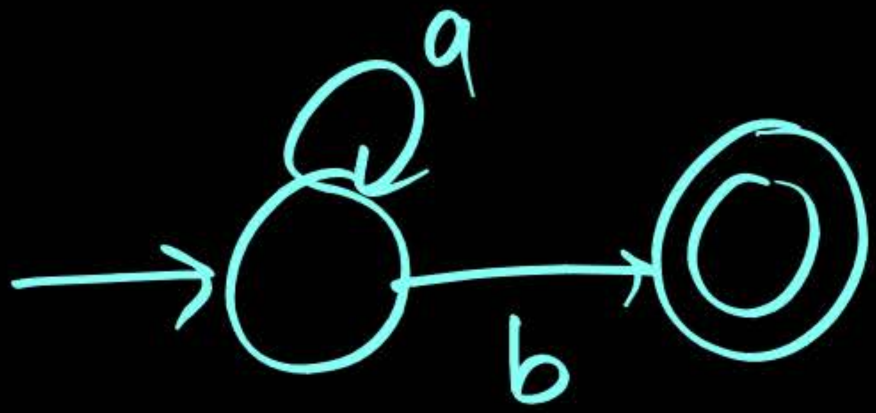


$$1 \xrightarrow{\varepsilon} 2 \xrightarrow{b} 2 \xrightarrow{\varepsilon} 1 \xrightarrow{a} 1 \xrightarrow{\varepsilon} \underbrace{2}_{\text{Final}}$$

$$\varepsilon\, b\, \varepsilon\, a\, \varepsilon = ba$$

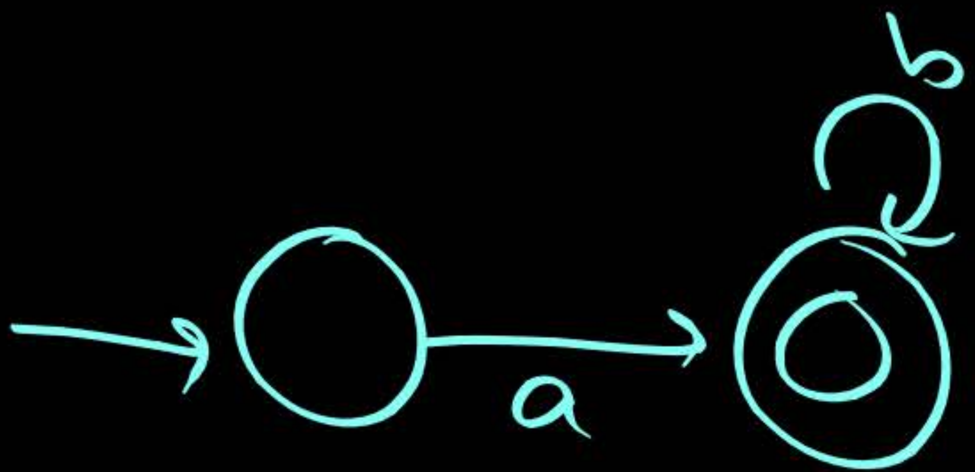
$$L = (a+b)^*$$
$$= (a^* b^*)^*$$
$$= (b^* a^*)^*$$

8



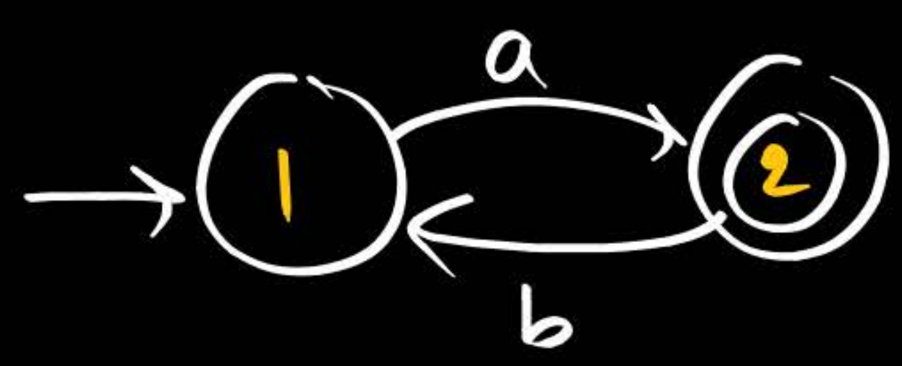
$L = a^* b$

9



$L = a b^*$

(10)

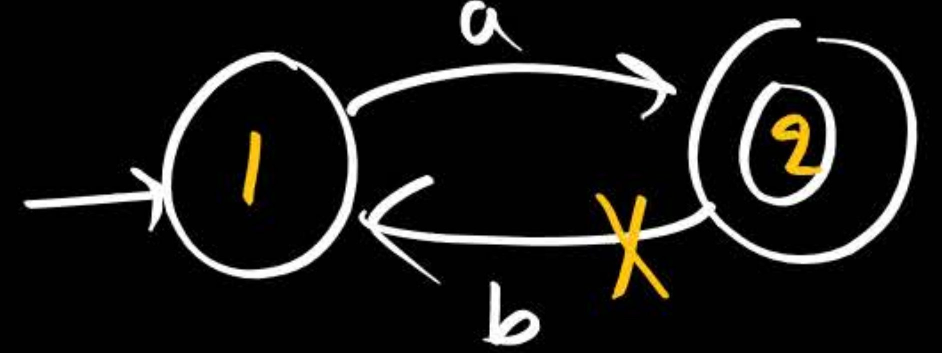


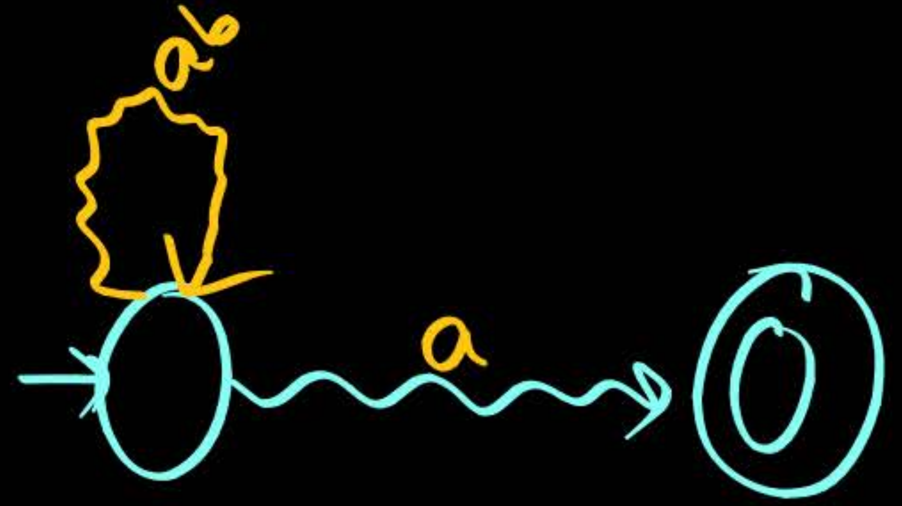
$$L = \{a, aba, ababa, abababa, \ldots\}$$

$$= a(ba)^*$$

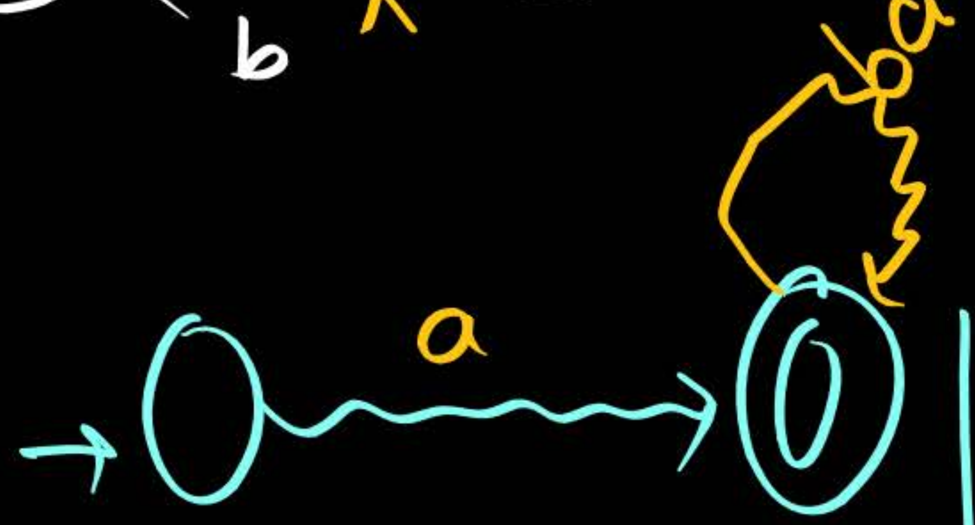
$$= (ab)^*a$$

Important

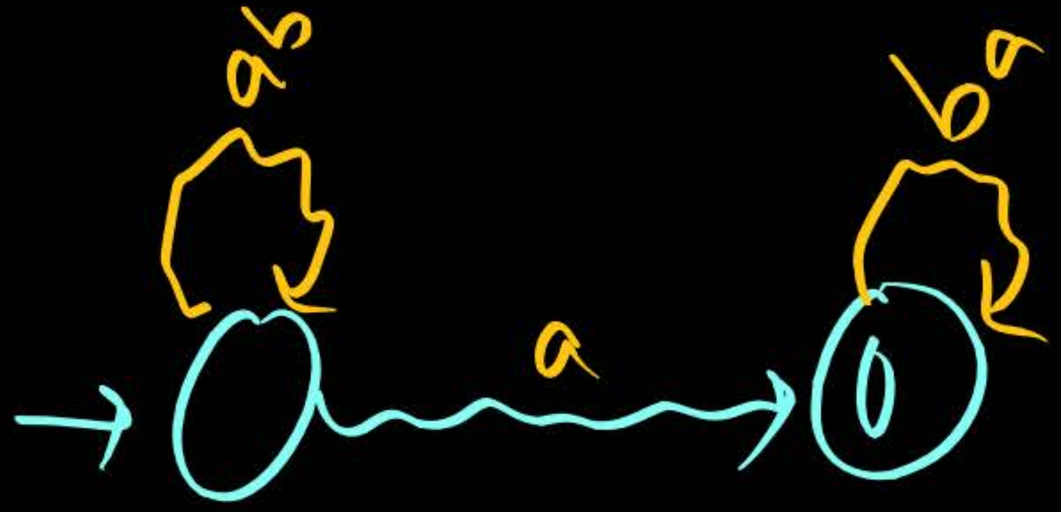




$(ab)^*a$

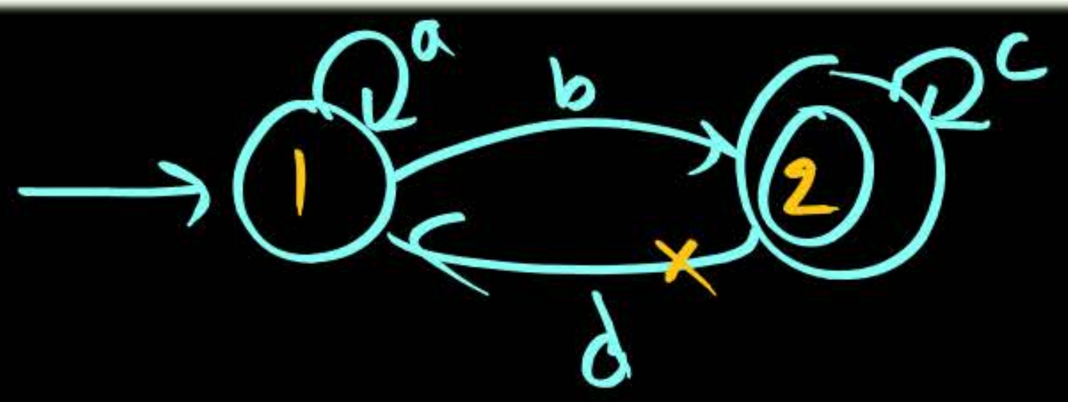


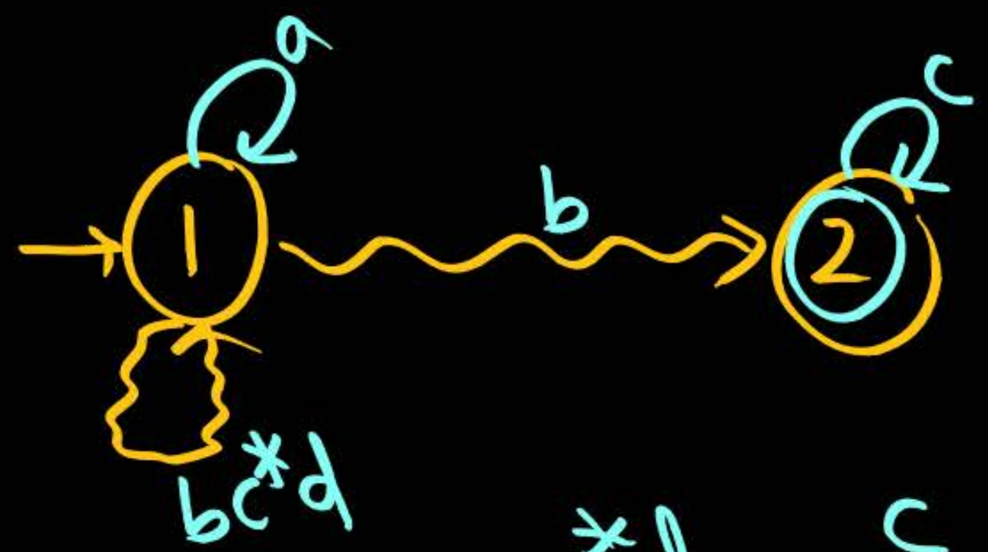
$a(ba)^*$

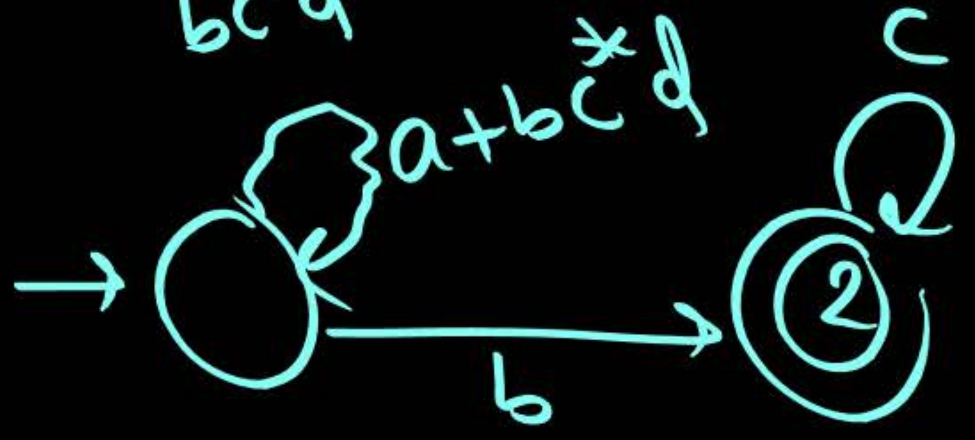


$(ab)^*a(ba)^*$

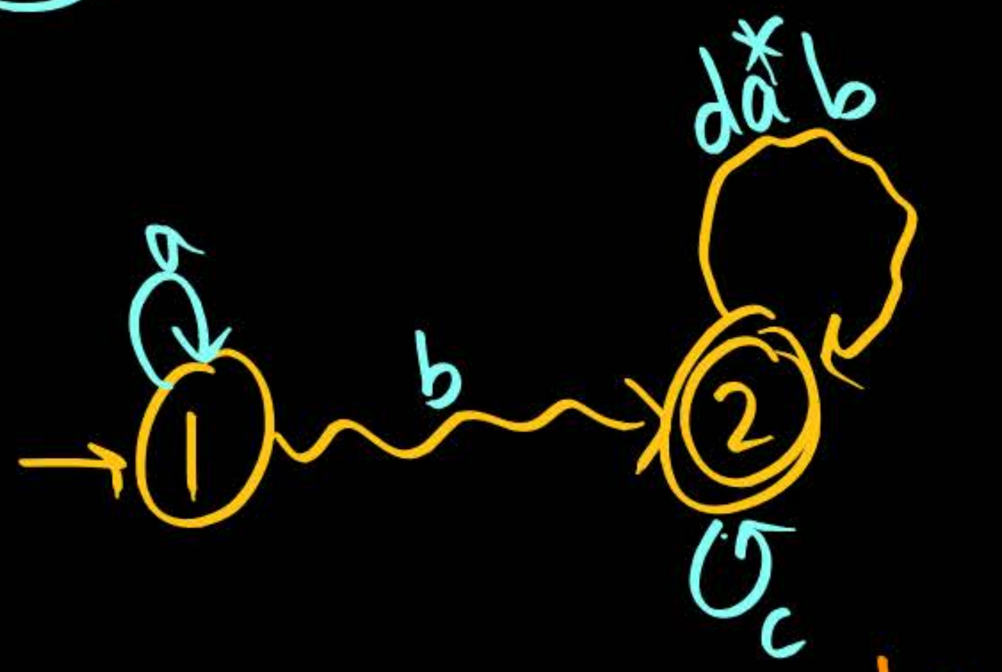
11



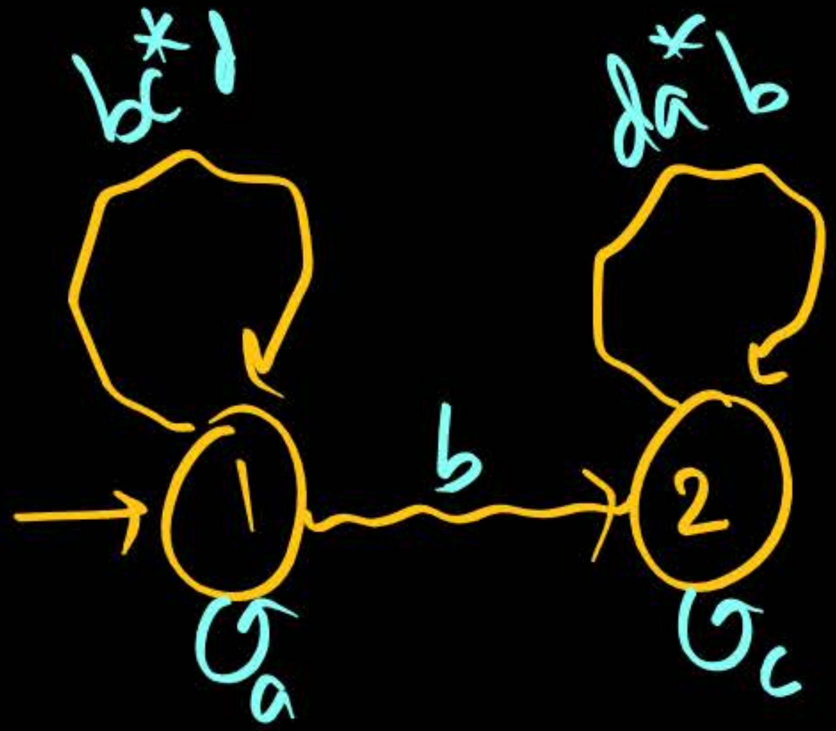




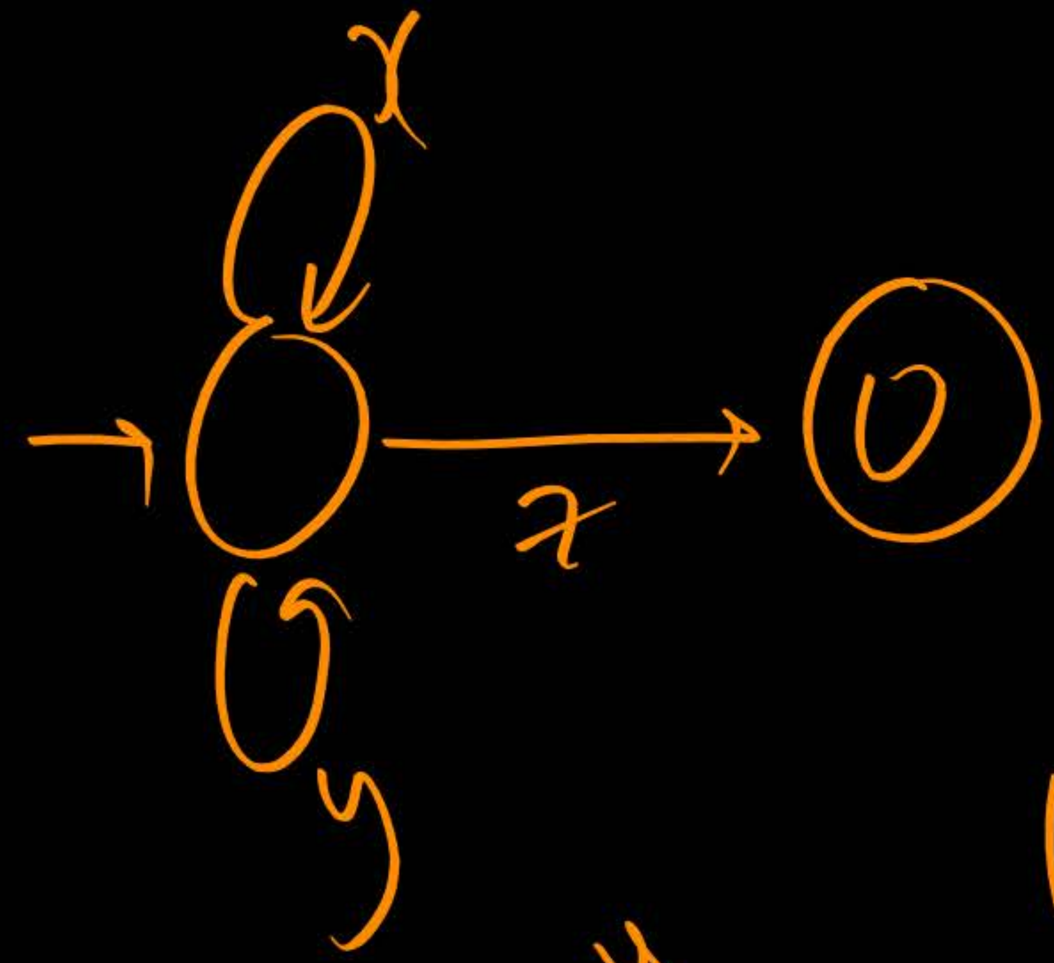
$$=(a+bc^*d)^* b c^*$$



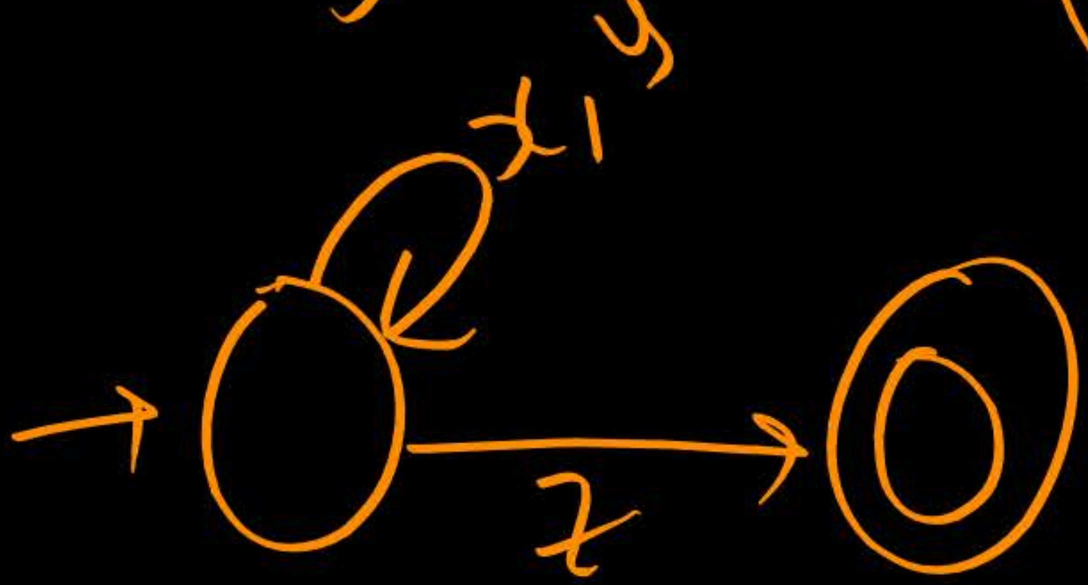
$$=a^* b (da^*b+c)^*$$

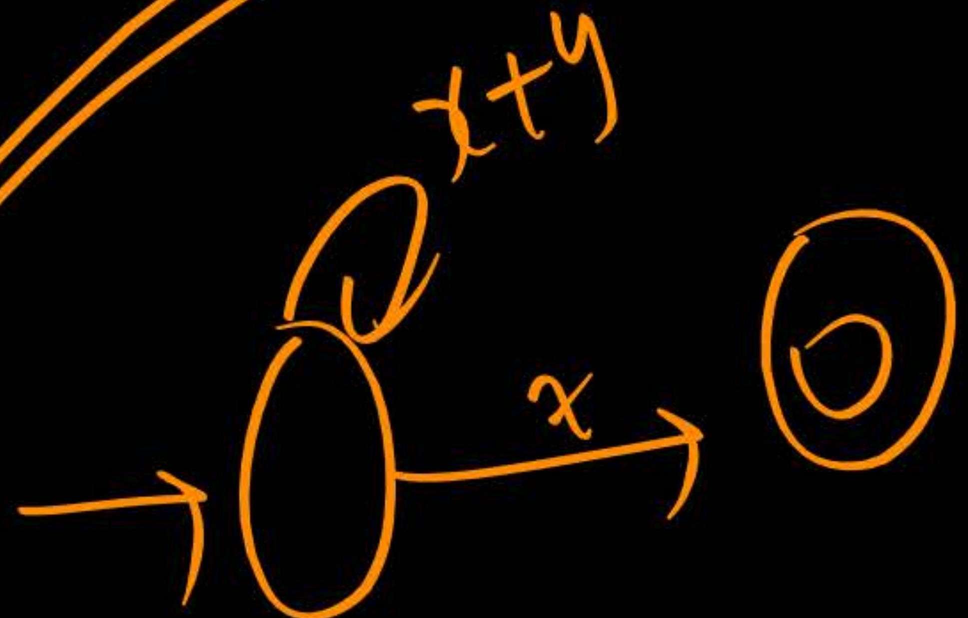


$$=(a+bc^*d)^* b (da^*b+c)^*$$



$(x+y)^* z$





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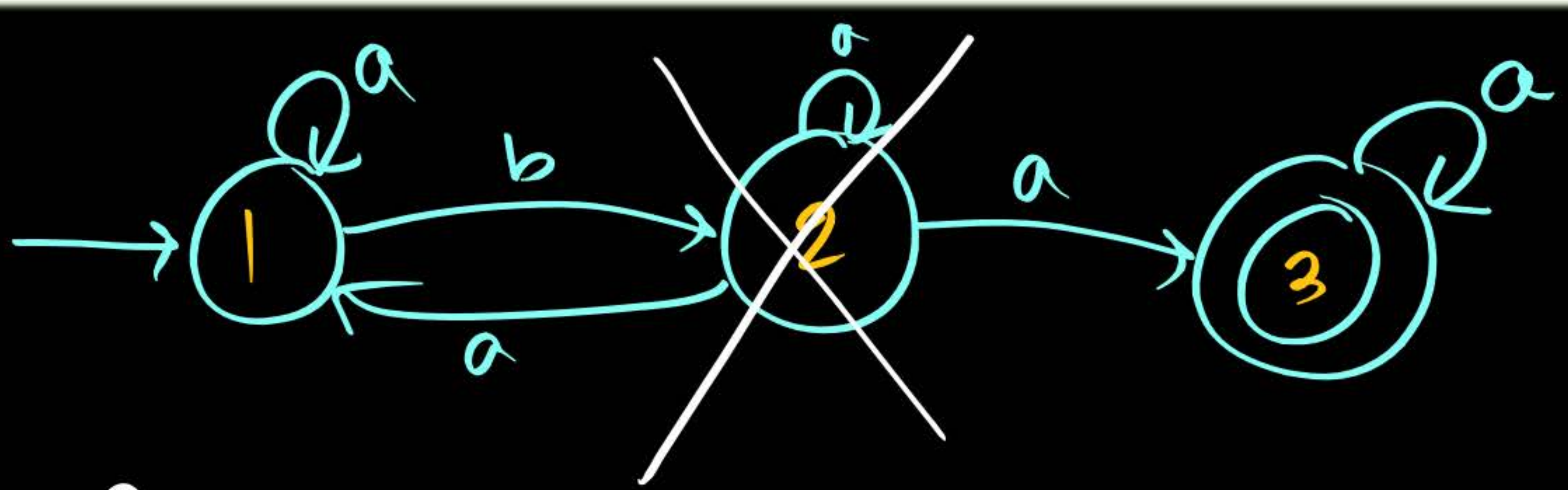
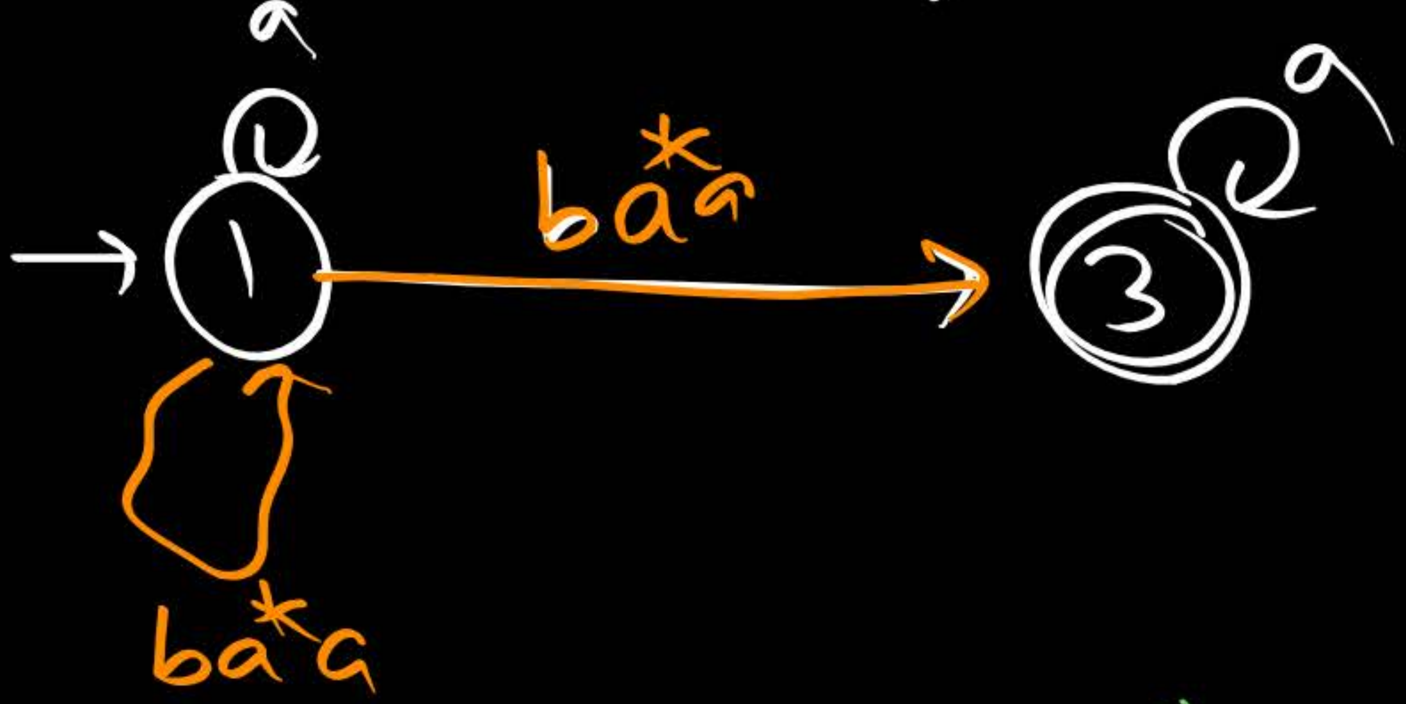
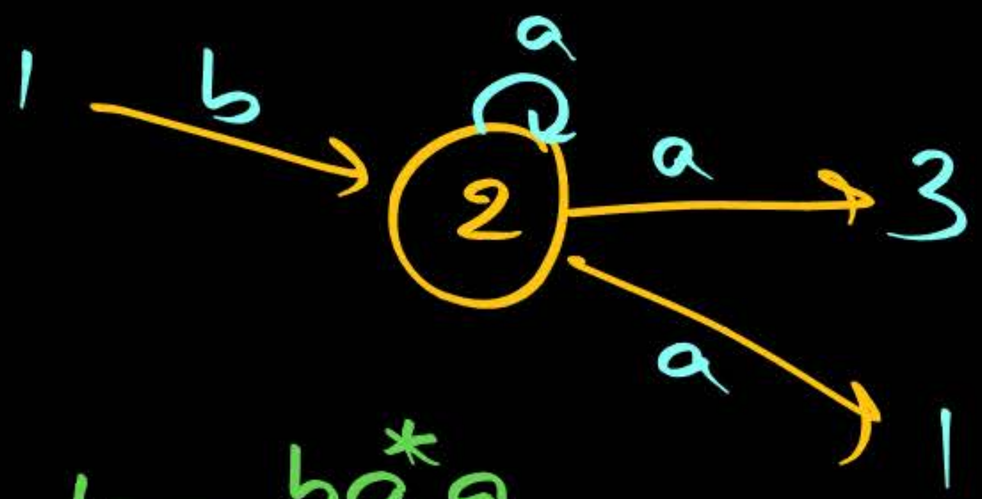




Delete 2

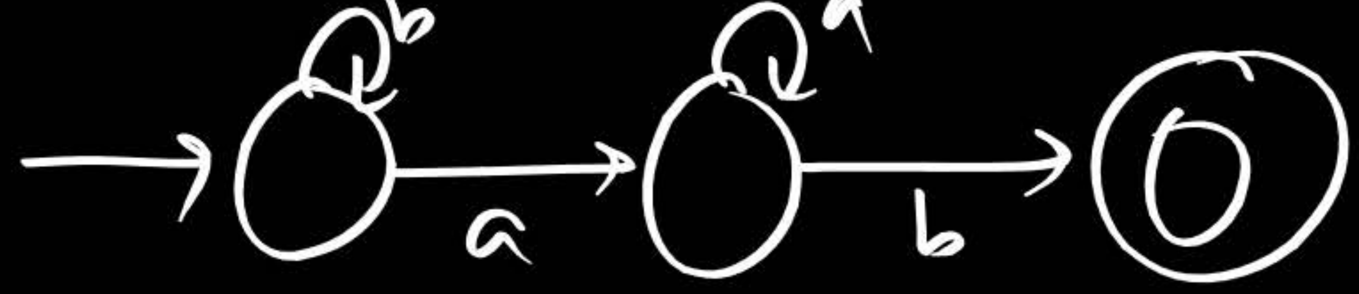


$$1 \xrightarrow{ba^*a} 3$$

$$1 \xrightarrow{ba^*a} 1$$

$$=(a+ba^*a)^*\, ba^*a\, a^*$$

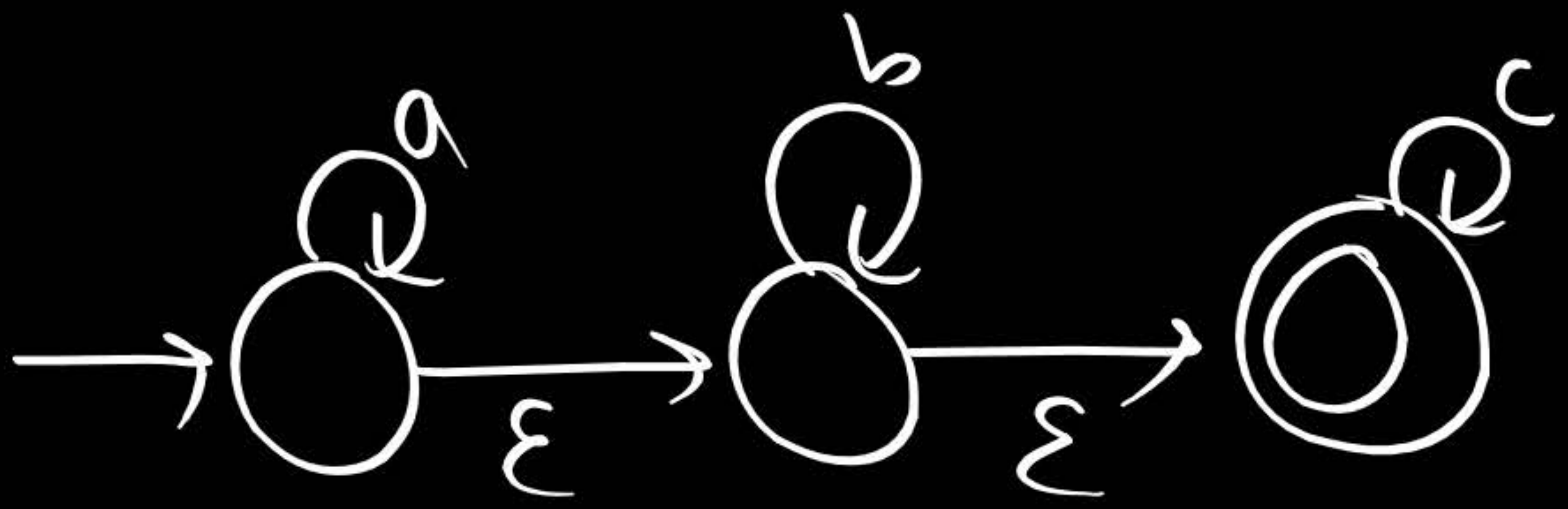
(13)



$L = b^* a a^* b$

$= b^* a^+ b$

(14)



$L = a^* b^* c^*$

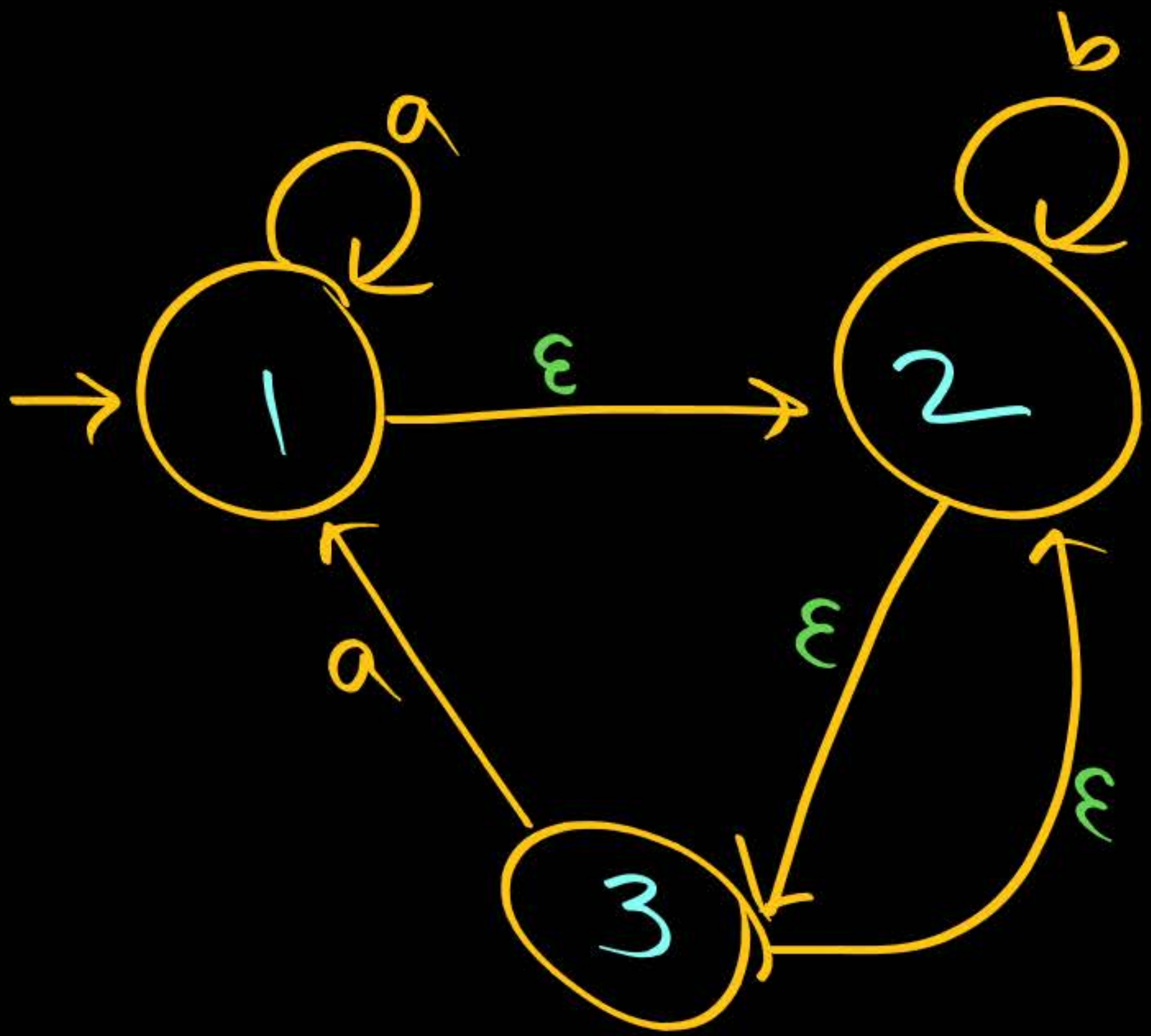
NFA with ε moves :

ε - closure (state) :

$1 \xrightarrow{\varepsilon} 1$

$1 \xrightarrow{\varepsilon} 2$

$1 \xrightarrow{\varepsilon} 3$



ε-closure(1) = {1, 2, 3}

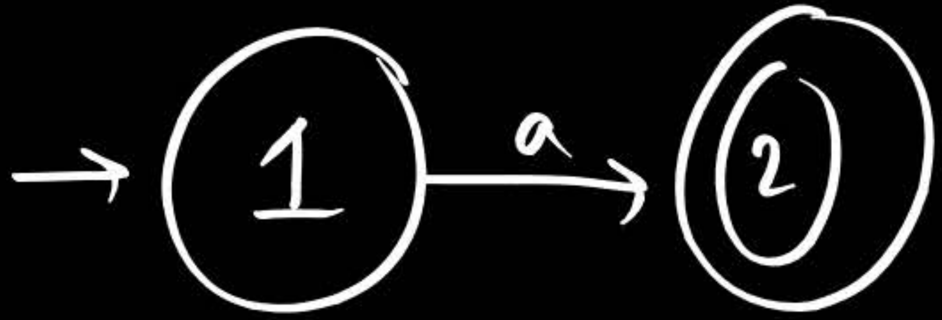
ε-closure(2) = {2, 3}

ε-closure(3) = {3, 2}

d

$$\varepsilon\text{-closure}(q) = \hat{\delta}(q, \varepsilon)$$

$$= \text{Set of all states reachable from state } q \underbrace{\text{without reading any string}}_{\varepsilon}$$



Transition

Symbol or ε

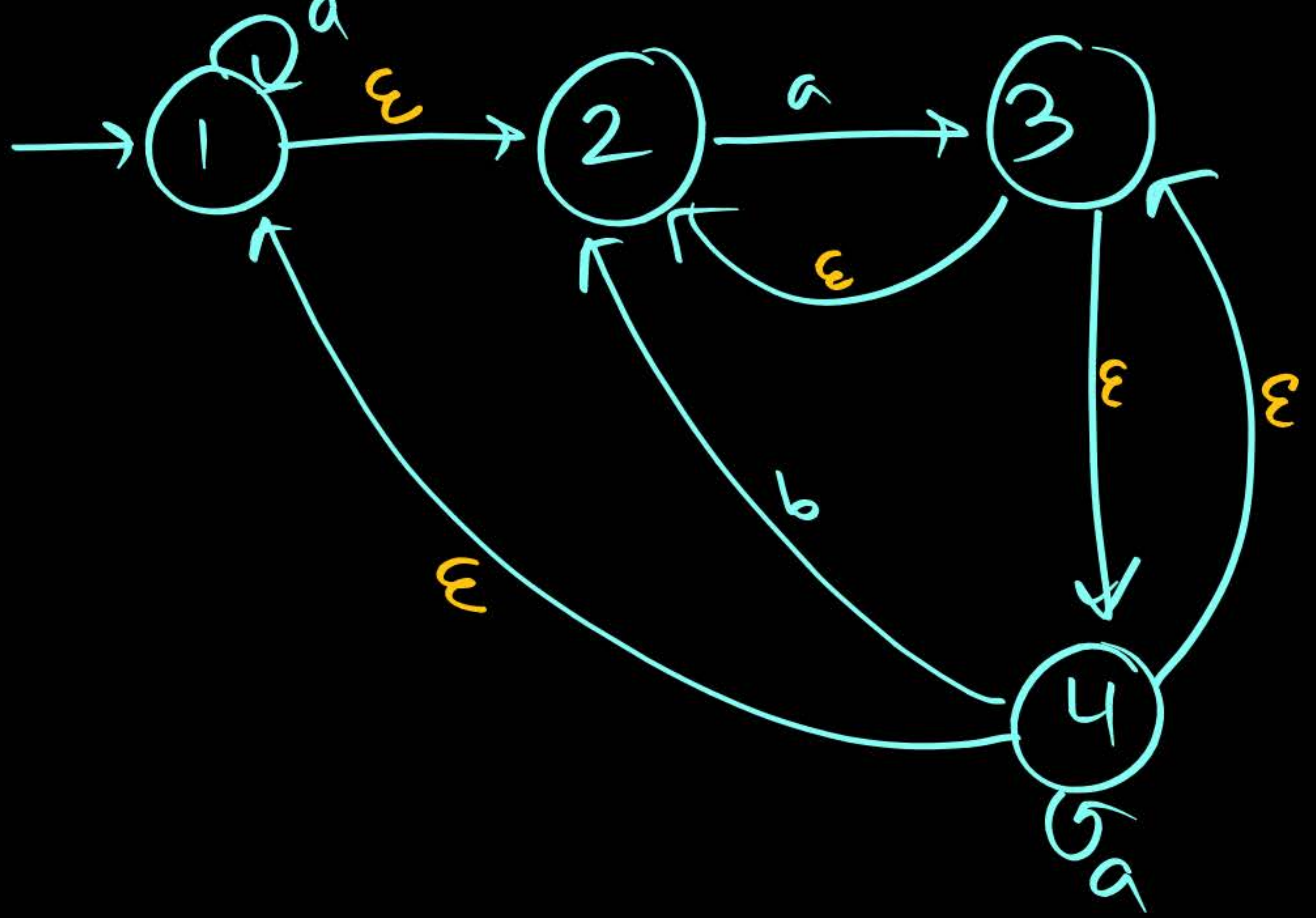
$\delta(1, \boxed{\varepsilon}) = \{\ \}$ From 1, there is no transition on ε

$\hat{\delta}(1, \boxed{\varepsilon}) = \{1\}$ From 1, there is path on ε

Extended Transition

string

path : Zero or more sequence of transition

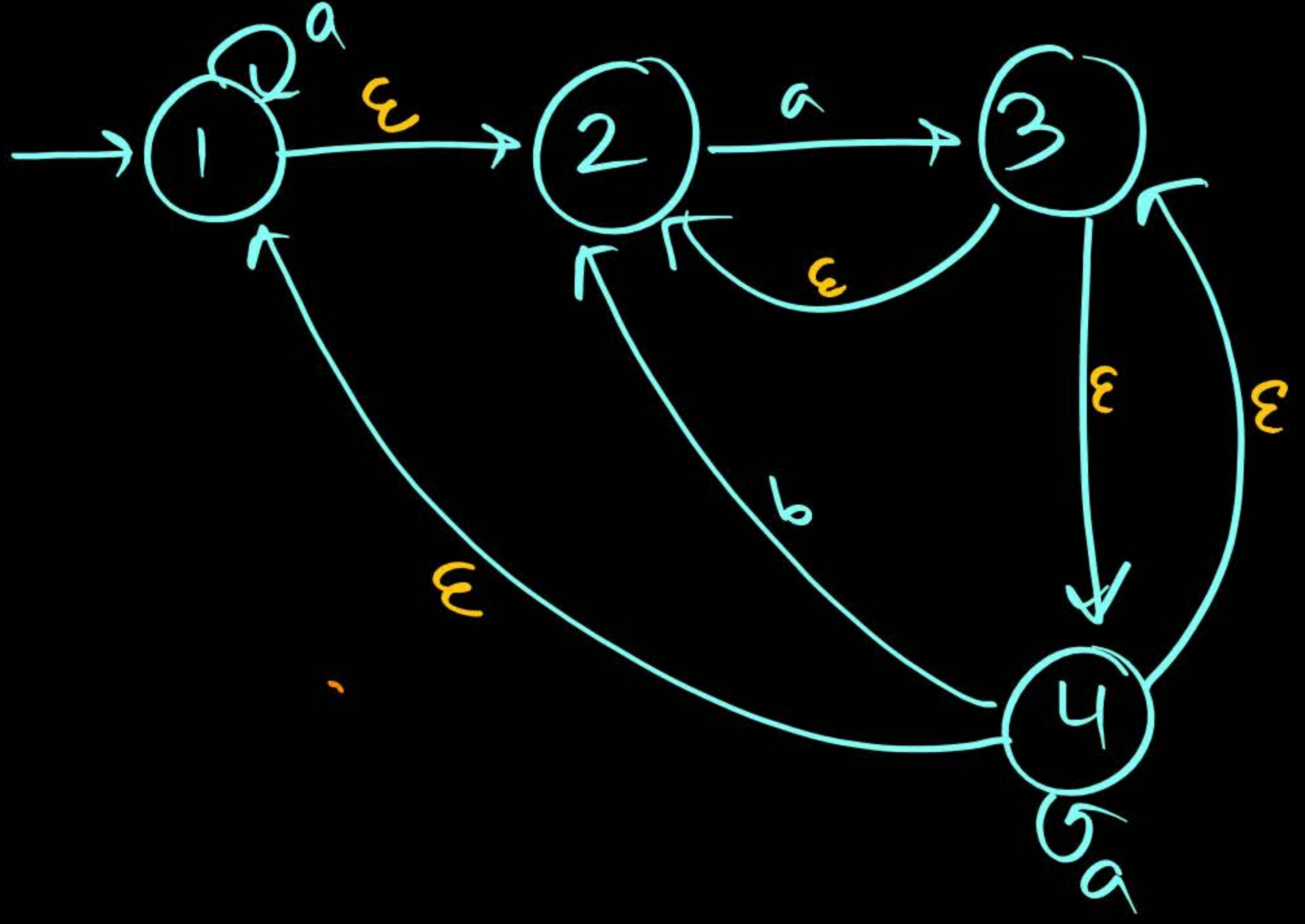


$\varepsilon\text{-closure}(1) = \{1, 2\}$

$\varepsilon\text{-closure}(2) = \{2\}$

$\varepsilon\text{-closure}(3) = \{3, 1, 2, 4\}$

$\varepsilon\text{-closure}(4) = \{4, 1, 2, 3\}$

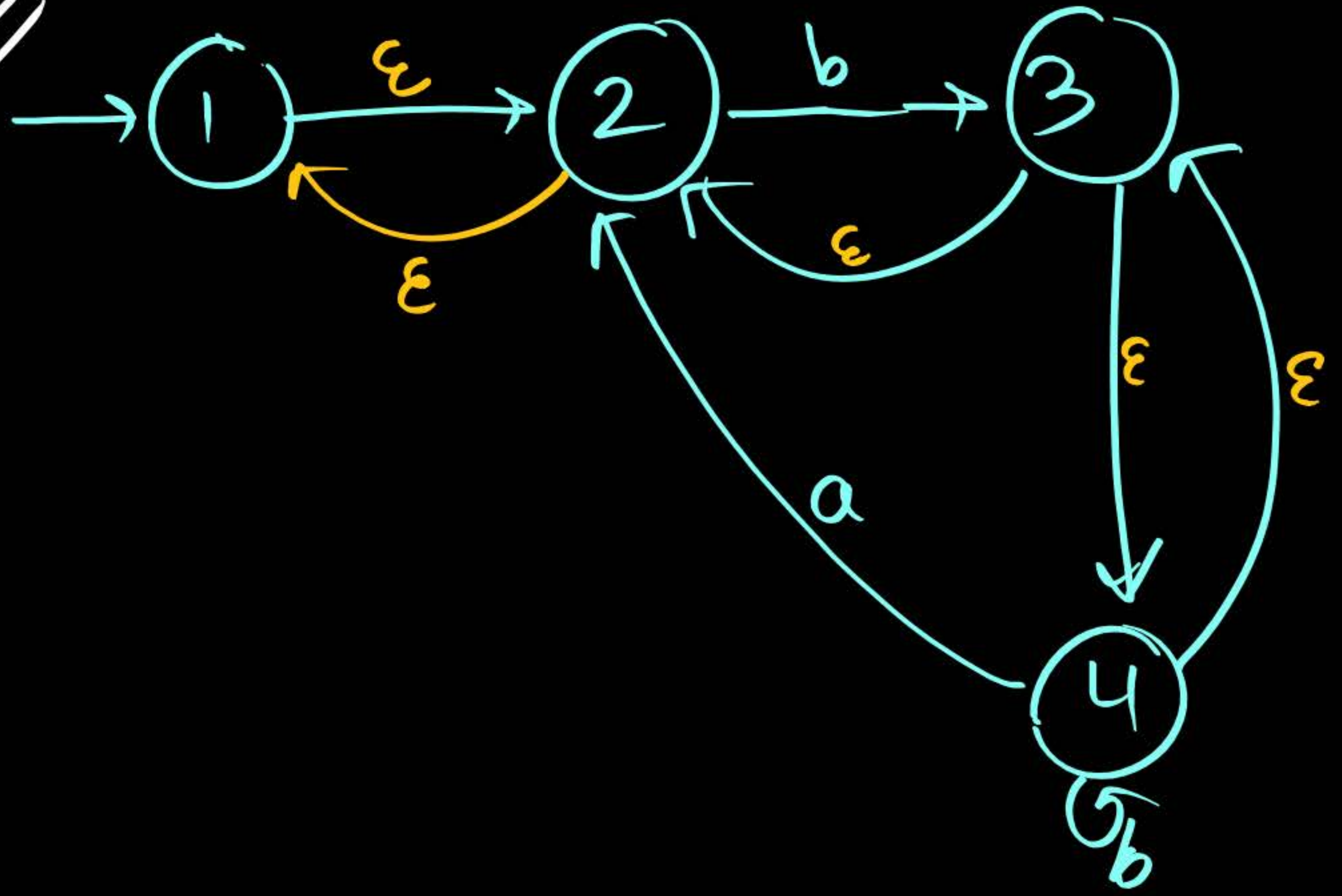


$$\hat{\delta}(3, ab) = ?$$
$$= \{2\}$$

$$\hat{\delta}(4, aa) = ?$$
$$= \{4, 3, 2, 1\}$$

$$4 \xRightarrow{aa} 4$$
$$4 \xRightarrow{aa} 1$$
$$4 \xRightarrow{aa} 2$$
$$4 \xRightarrow{aa} 3$$

H.W.



$$\hat{\delta}(1, aab) = ?$$

$$\hat{\delta}(2, bb) = ?$$

$$\hat{\delta}(3, bab) = ?$$

$$\hat{\delta}(4, ab) = ?$$

H.W.

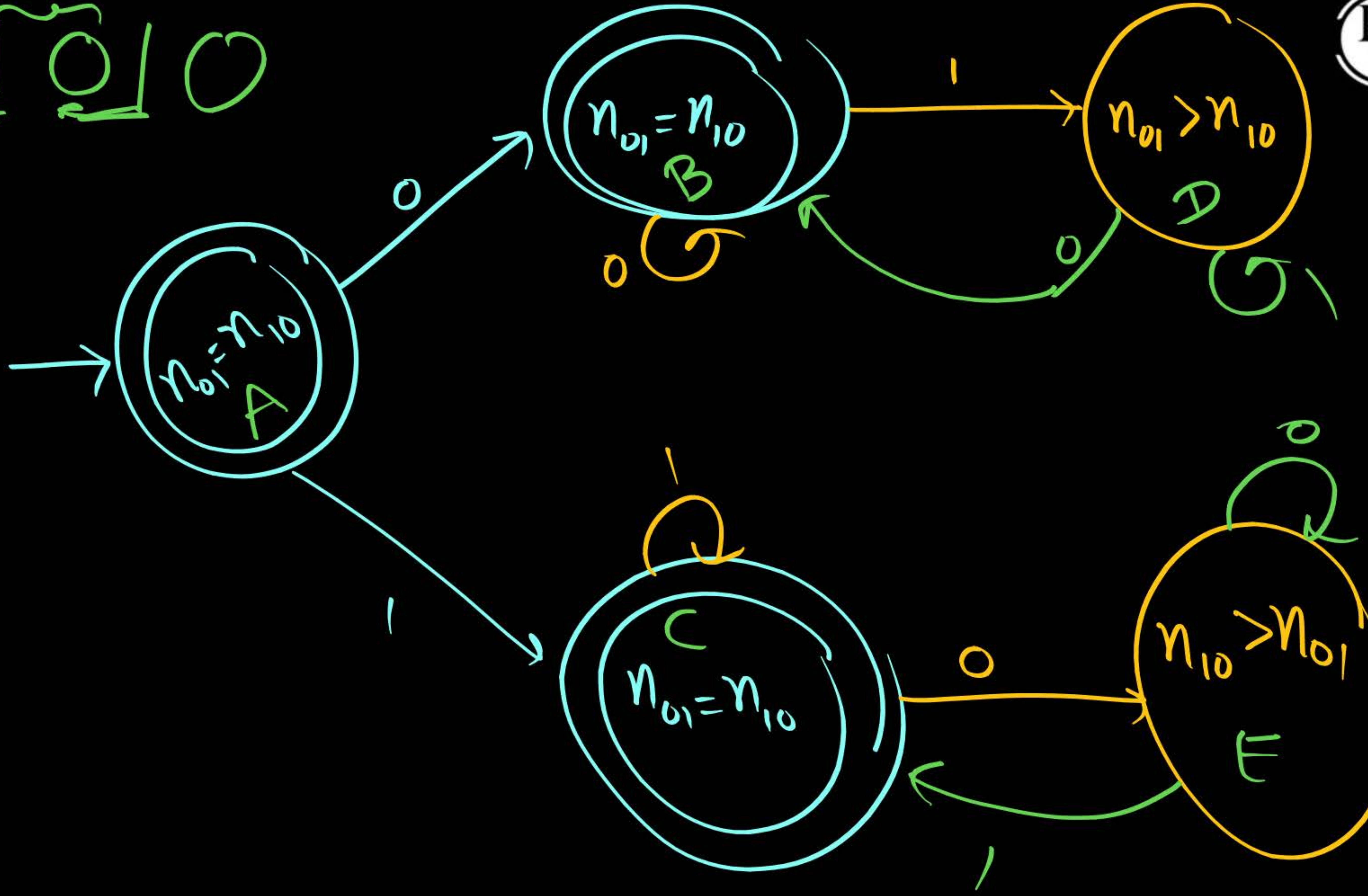
Model-13.

Problem 2.

$$L = \{ w \mid w \in \{0,1\}^*, \#_{01}(w) = \#_{10}(w) \}$$

$$L = \{ \varepsilon, 0, 1, 00, \cancel{\boxed{01}}, \cancel{\boxed{10}}, 11, 000, \cancel{\boxed{001}}, 010, \cancel{\boxed{011}}, \cancel{\boxed{100}}, 101, \cancel{110}, 111, \ldots \}$$

010010



$w = 1\overset{\frown}{0}1\overset{\frown}{0}1\overset{\frown}{0}1\overset{\frown}{0}1$

valid string

$\#_{10}(w) = 4$

$\#_{01}(w) = 4$

In the world

~~If~~ $\#_{01}(w) = 2$

then $\#_{10}(w) = 1$, or 2, or 3

0101

01010

101010

$w \in \{0,1\}^*$

If no. of 0's of $w = K$

then no. of 10's of $w = ?$

$= K$, or $K+1$, or $K-1$

$$\begin{tikzcd} & \Sigma^* \arrow[dl] \arrow[d] \arrow[dr, bend left] & \\ \#_{01} = \#_{10} & \#_{10} = \#_{01} + 1 & \#_{01} = \#_{10} + 1 \end{tikzcd}$$

$$\left| \#_{01}(\omega) - \#_{10}(\omega) \right| = 0 \text{ or } 1$$

$\#_{01}(w) = \#_{10}(w)$ ✓

Non-reg

$\#_{0}(w) = \#_{1}(w)$ ✗

$\#_{00}(w) = \#_{11}(w)$ ✗

Reg

$\#_{001}(w) = \#_{100}(w)$ ✓

Summary

→ NFA with ϵ-moves


THANK YOU
GW
SOLDIERS !