CS & IT ENGINEERING

Discrete Mathematics

Set Theory

DPP 03 Discussion Notes

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01 Question

02 Discussion

Q.1 Let f, g: R → R, where $g(x) = 1 - x + x^2$ and $f(x) = ax + b$. [MSQ]

If $(g \circ f)(x) = 9x^2 - 9x + 3$, determine a, b.

A. $a = -1, b = 3$

B. $a = 3, b = -1$

C. $a = -3, b = 2$

D. $a = 2, b = -3$

$gof = 9x^2 - 9x + 3$.

$gof = 9x^2 - 9x + 3$.

$g(f(x))$

$g(ax+b) =$

$1 - (ax+b) + (ax+b)^2 = 9x^2 - 9x + 3$

$1 - ax - b + a^2x^2 + 2axb + b^2 = 9x^2 - 9x + 3$

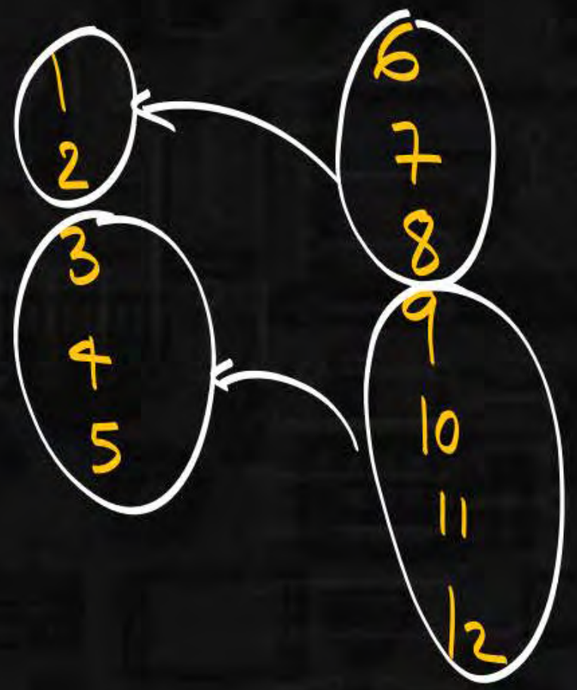
$2axb - ax$

$(2ab - a)x = -9x$

$a^2x^2 = 9x^2$

$a^2 = 9 \quad a = \pm 3$.

Q.2 Let A = {1, 2, 3, 4, 5} and B = {6, 7, 8, 9, 10, 11, 12}. How many functions f: A → B are such that $f^{-1}(\{6, 7, 8\}) = \{1, 2\}$? **[NAT]**



$3^2 \times 4^3 = 576.$

$9 \times 16 \times 4$

64×9

$(60 + 4)\,9$

$540 + 36.$

Q.3 How many function are 1:1 onto both? [NAT]

(a) $f(x) = x + 7$ ✓

(b) $f(x) = 2x - 3$

(c) $f(x) = -x + 5$ ✓

(d) $f(x) = x^2$ ✗

(e) $f(x) = x^2 + x$

(h) $f(x) = x^3$

Q.4

A chemist who has five assistants is engaged in a research project that calls for nine compounds that must be synthesized. In how many ways can the chemist assign these syntheses to the five assistants so that each is working on at least one synthesis?

[MCQ]

A. $5!\, S(9, 5)$ ✓

B. $(4!)\, S(7, 4)$

C. $(3!)\, S(6, 3)$

D. None of the above

onto = _____

$S(9,5) = \frac{\text{onto}}{5!}$

$\text{onto} = 5! \times S(9,5)$

Q.5 Let A = {1, 2, 3, 4} and B = {x, y, z}, now let x is the number of functions from A to B and y be the number of one-to-one functions from A to B. Then the value of x/y is? [MCQ]

A. 81

B. 0

C. undefined ✓

D. 81/4

$A \to B = B^A = 3^4$

$y = 0$

$\frac{x}{y} = \frac{3^4}{0}$

Q.6 If there exists 2187 functions f: A → B and the cardinality of set B is 3. What is the cardinality of set A? **[NAT]**

$f: A \to B$.

$B^A = 2187$

$3^A = 2187$

$A = 7$.

Q.7 If A = {1, 2, 3, 4, 5} and there are 6720 injective functions f: A → B, what is |B| ? [MCQ]

A. 16

B. 17

C. ~~18~~ Ans: 8

D. None of these

1
2
3
4
5

$n = 8$

R.S $P_{L.S} = 6720$

$nP_5 = 6720$

$\frac{n!}{(n-5)!} = 6720$

$n(n-1)(n-2)(n-3)(n-4)\frac{(n-5)!}{(n-5)!} = 6720$

$n(n-1)(n-2)(n-3)(n-4) = 6720$

Q.8 [MCQ]

Let f: $A \to B$, with $A_1, A_2 \subseteq A$. Then choose the correct option from the following regarding the given statements.

S_1: $f(A_1 \cup A_2) = f(A_1) \cup f(A_2)$ ✓

S_2: $f(A_1 \cap A_2) \subseteq f(A_1) \cap f(A_2)$ ✓

S_3: $f(A_1 \cap A_2) = f(A_1) \cap f(A_2)$ when f is one to one

A. Only S_2 and S_3 are true.

B. Only S_1 and S_3 are true.

C. Only S_1 and S_2 are true.

D. All S_1, S_2 and S_3 are true.

THANK YOU GW
SOLDIERS !