CS & IT ENGINEERING

Planarity Part -2 ,3

Lecture No.13

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TOPICS TO BE COVERED

01 Inequalities thms in planarity part 1

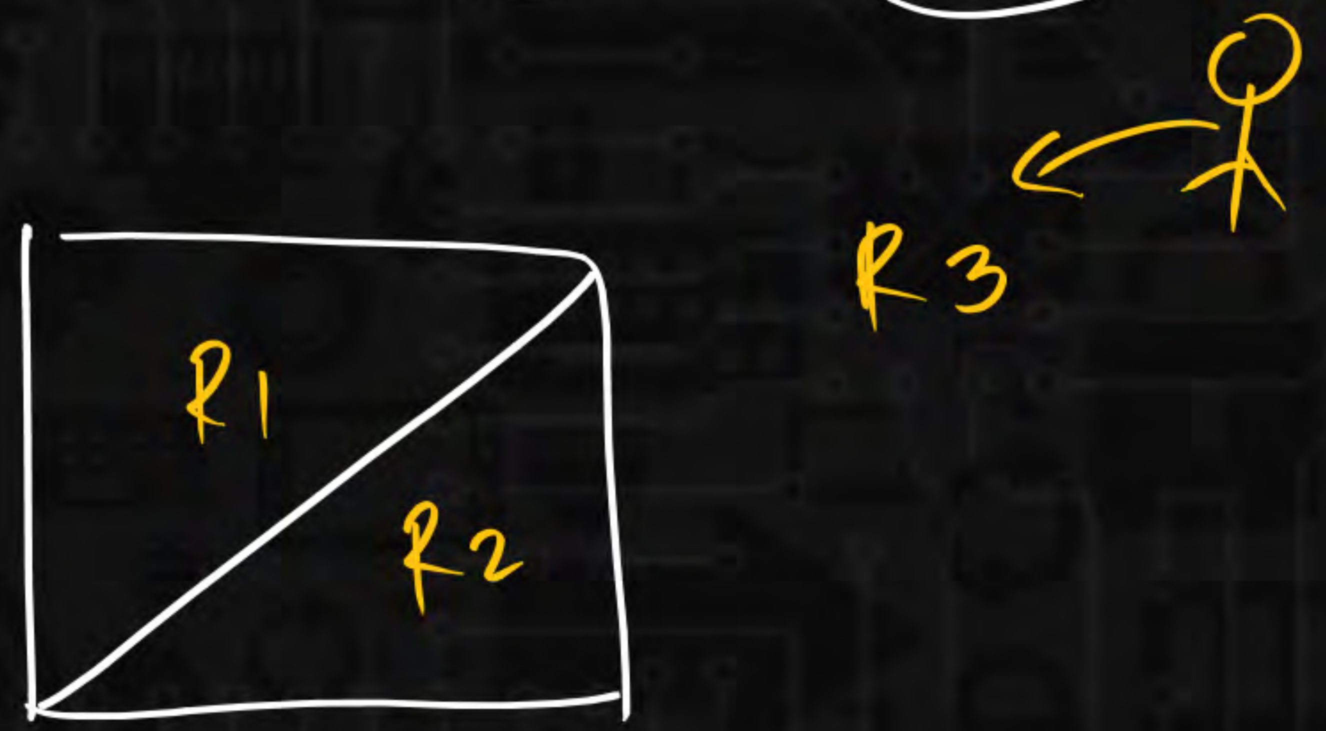
~~02 inequalities thms in planarity part 2~~,3

03 Sub Graphs

04 Graph Operations part 1

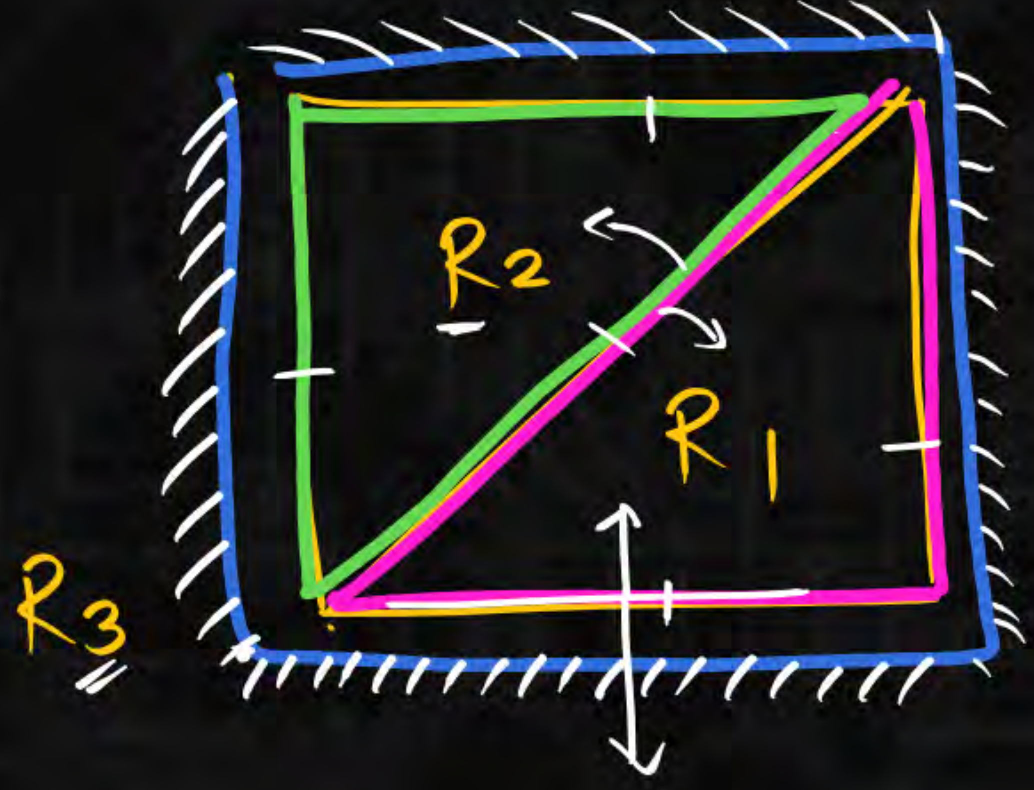
05 Graph Operation Part 2

Planar Graph → Draw → Region/faces



Degree of Regions ($d(R_i)$)

no. of edges present in Region.



$d(R_1) = 3$

$d(R_2) = 3$

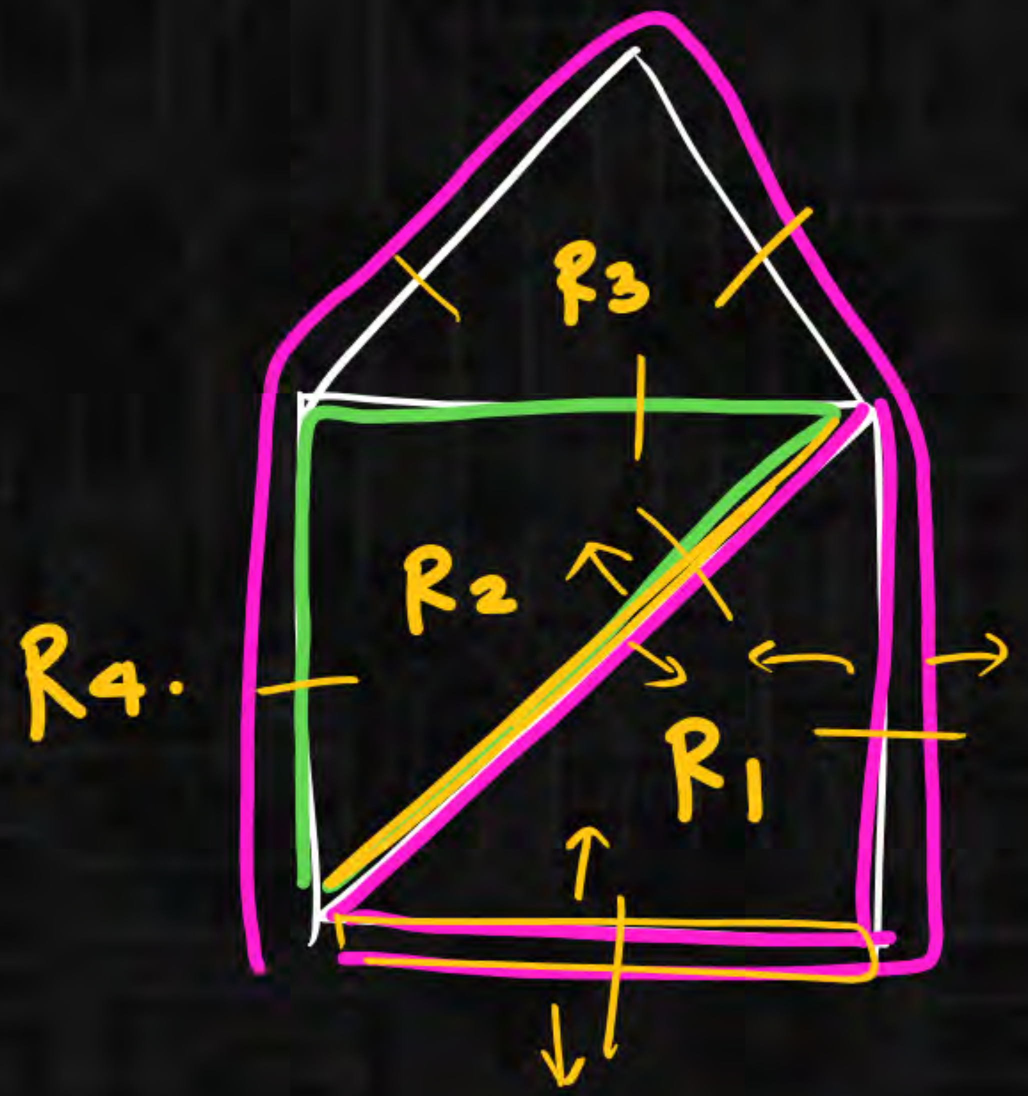
$d(R_3) = 4$

$d(R_1) + d(R_2) + d(R_3)$

$= 3 + 3 + 4 = 10 = 2 \times 5$

no. of edges

$= 2 \times e$



$d(R_1) = 3$

$d(R_2) = 3$

$d(R_3) = 3$

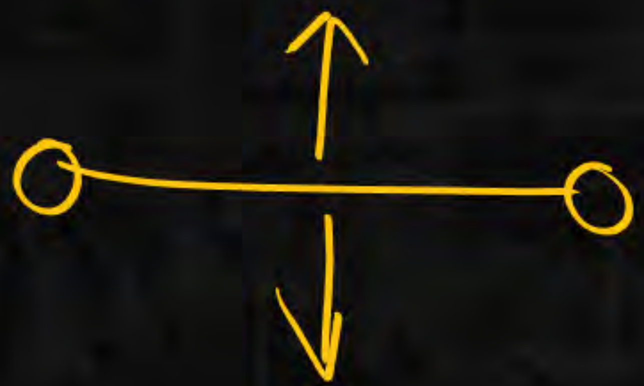
$d(R_4) = 5$

$$d(R_1) + d(R_2) + d(R_3) + d(R_4)$$

$$= 3 + 3 + 3 + 5$$

$$= 14 = 2 \times 7$$

no. of edges



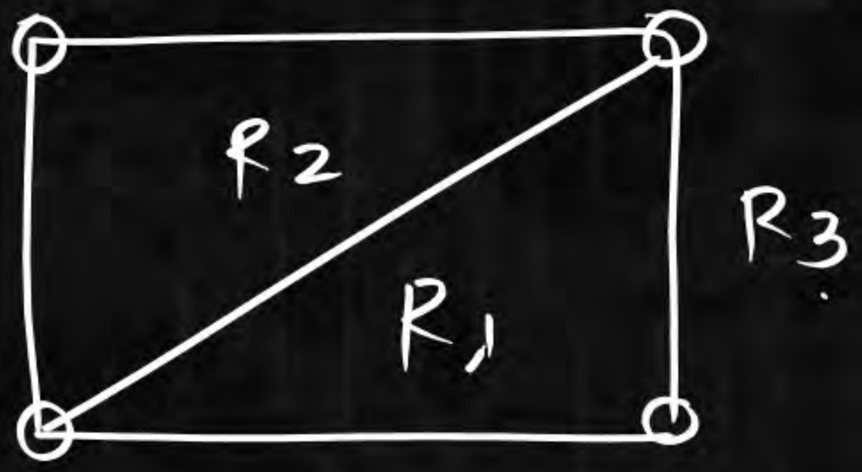
$$\boxed{\sum d(R_i) = 2e}$$

Thm: Sum of degrees of all the regions is equals to twice the no. of edges.

$e = 1$.

$d(R_1) = 2$.

Total no. of Regions = 1.



$\deg(R_1) \geq 3 = 3$

$\deg(R_2) \geq 3$

$\deg(R_3) \geq 3$

$\sum d(R_i) = 2e$

$(n \geq 3)$

$\deg(R_1) + \deg(R_2) + \deg(R_3) \geq 3+3+3 = 9$

$\geq 3 \cdot 3 \rightarrow$ no. of faces

$\boxed{2e \geq 3f}$

$\boxed{\sum d(R_i) \geq 3f}$

$$2e \geq 3f$$

$$2e \geq 3(2+e-n)$$

$$2e \geq 6+3e-3n.$$

$$3n-6 \geq 3e-2e.$$

$$3n-6 \geq e$$

$$\boxed{e \leq 3n-6.}$$

Euler's:

$$n-e+f=2.$$

$$f=2+e-n.$$

Thm:

if Graph is planar then $e \leq 3n-6$ $(n \geq 3)$

K_5

$n = 5$

$e = 10$

$e \leq 3n-6$

$10 \leq 3(5) - 6$

$10 \leq 15 - 6$

$10 \leq 9$ (false)

{vice versa is not true

$K_{3,3}$

$n = 6$

$e = 9$

non planar

$e \leq 3n - 6$

$9 \leq 3(6) - 6$

$9 \leq 18 - 6$

$9 \leq 12$ (True)

{Planar $\rightarrow e \leq 3n - 6$

{$e \leq 3n - 6 \not\Rightarrow$ Planar

eg: $K_{3,3}$

$$\boxed{\delta(G) \le \frac{2e}{n}} \le \Delta(G) \le n-1.$$

Planar $\rightarrow e \le 3n-6.$

$$\delta(G) \le \frac{2e}{n}$$

$$\delta(G) \le \frac{2(3n-6)}{n}$$

$$\delta(G) \le \frac{6n-12}{n}$$

$$\delta(G) \le \frac{6\cancel{n}}{\cancel{n}} - \frac{12}{n}$$

$$\delta(G) \le 6 - \frac{12}{n}$$

$$\boxed{\delta(G) \le 5}$$

$\underline{\text{Thm}}$. if G is Planar Graph then $\delta(G) \le 5$. ✓

$$\delta(G) \le \frac{2e}{n} \qquad e \le 3n-6.$$

$$\delta(G) \le \frac{2(3n-6)}{n} \qquad \delta(G) \le 6 - \frac{12}{n}$$

$$\delta(G) \le \frac{6n-12}{n} \qquad \delta(G) \le 5$$

should not contains triangle.

$\deg(R_1) \geq 4$

$\deg(R_2) \geq 4$

$\deg(R_3) \geq 4$

$\geq 4 + 4 + 4$

≥ 12

$\geq 4 \cdot 3$

$$\underline{\deg(R_1) + \deg(R_2) + \deg(R_3)} \geq 4 \cdot f \geq 4 \cdot \times 3$$

$$2e \geq 4f$$

$2e \geq 3f$

R_2

$\deg(R_1) \geq 4$

$\deg(R_2) \geq 4$

$\deg(R_1) + \deg(R_2) \geq 4 + 4 \geq 4(2) \geq 4f$

$\boxed{2e \geq 4f} \rightarrow$ does not contains triangle

$$2e \geq 4f$$

$$n - e + f = 2$$

$$f = 2 + e - n$$

$$\cancel{2}e \geq \cancel{4}^{\,2}(2 + e - n)$$

$$e \geq 2(2 + e - n)$$

$$e \geq 4 + 2e - 2n$$

$$2n - 4 \geq e$$

$$\boxed{e \leq 2n - 4} \rightarrow \left\{ \text{G does not contains triangle.} \right.$$

Thm: if G is Planar then $e \leq 3n-6$ $(n \geq 3)$

Thm: if G is planar & does not contain triangle $e \leq 2n-4$ $(n \geq 4)$

Subgraph: (C)

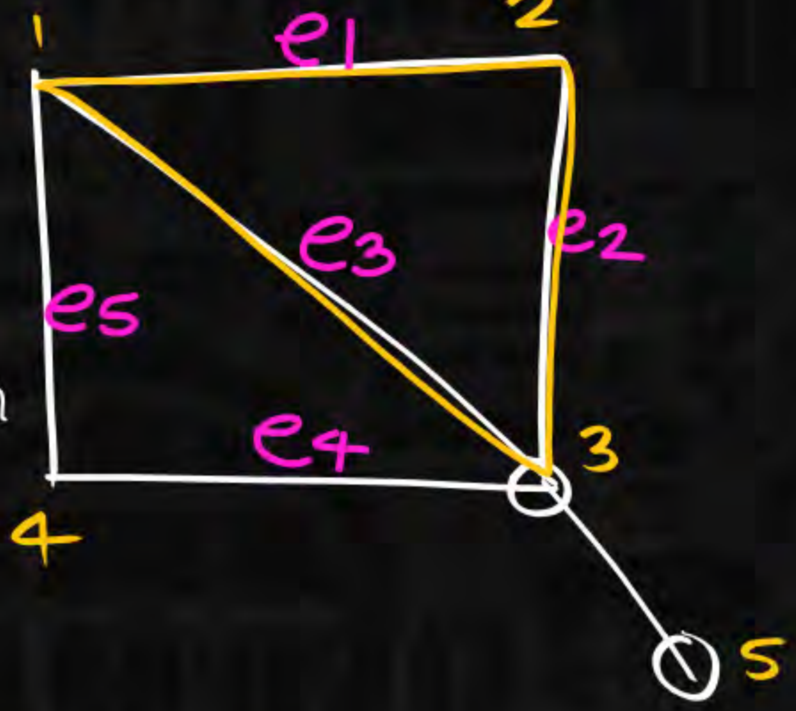
$G_1 = (V_1, E_1)$

$G_2 = (V_2, E_2)$

G_1 is subgraph of G_2.

$\boxed{G_1 \subseteq G_2}$

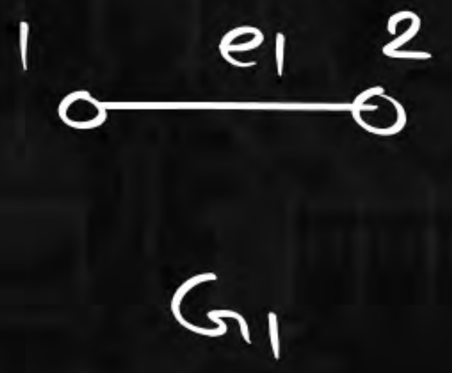
is subgraph



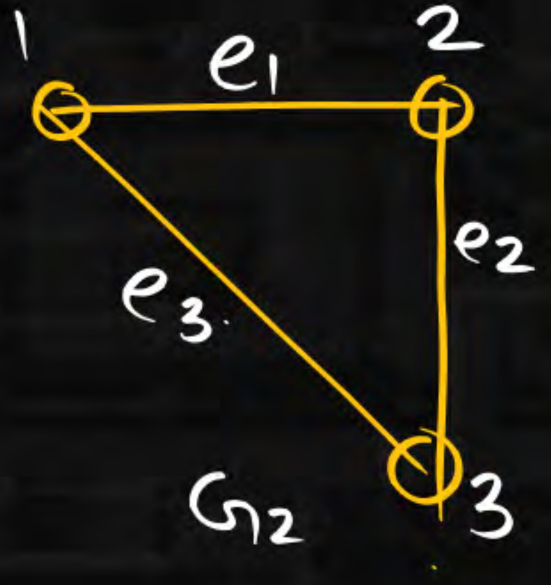
$G_1 = (V1, E1)$ $\qquad$ $G_2 = (V2, E2)$

$\emptyset \neq V1 \subseteq V2$

$E1 \subseteq E2$.

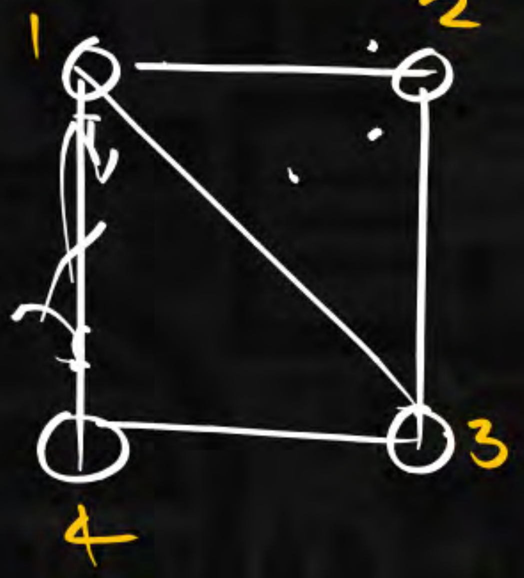


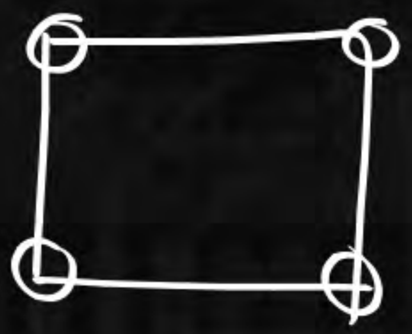
$\subset$



G_1 is subgraph of G_2.

$\subset$





$\subset$

1. Every Graph is subgraph of itself ($G \subset G$)

2. Single vertex of Graph is subgraph of a Graph.

3. Single edge is also subgraph of a Graph.

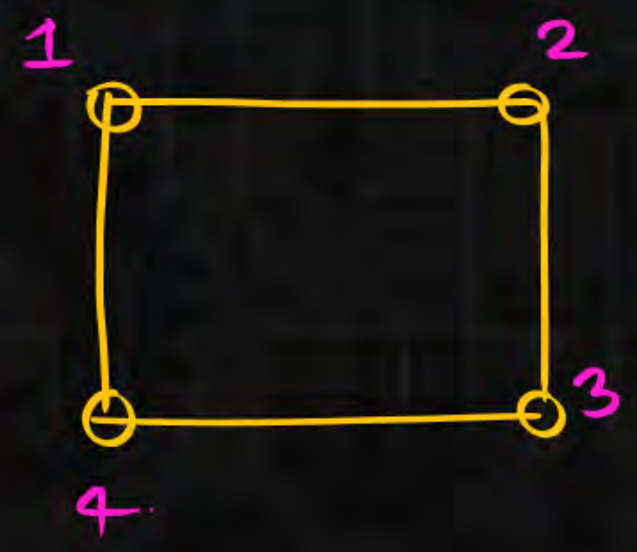
4. $\underline{a} \subset \underline{b} \subset G$

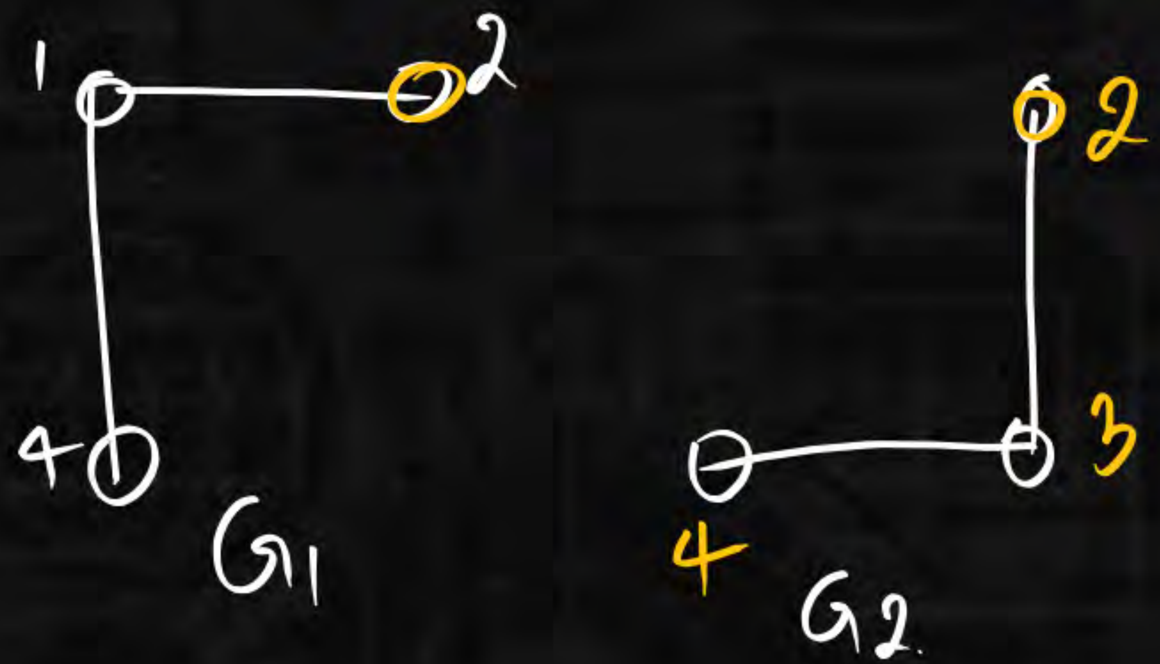
$\underline{a} \subset G$

subgraph of a subgraph of a Graph is subgraph of a Graph.

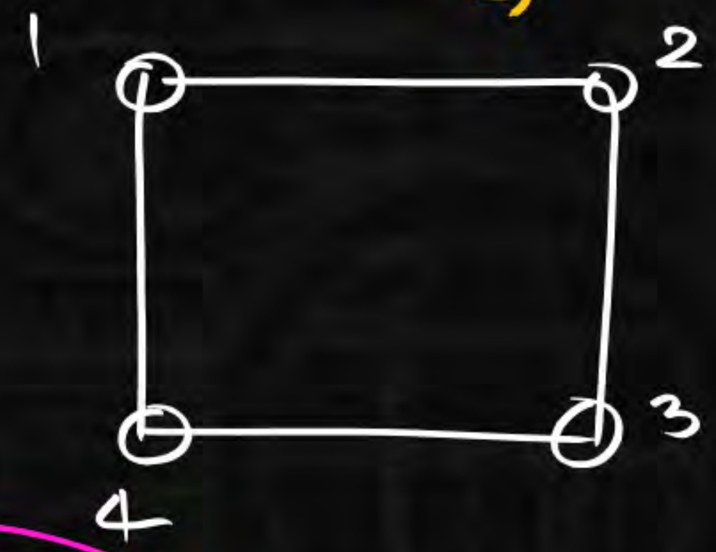
edge disjoint subgraph:

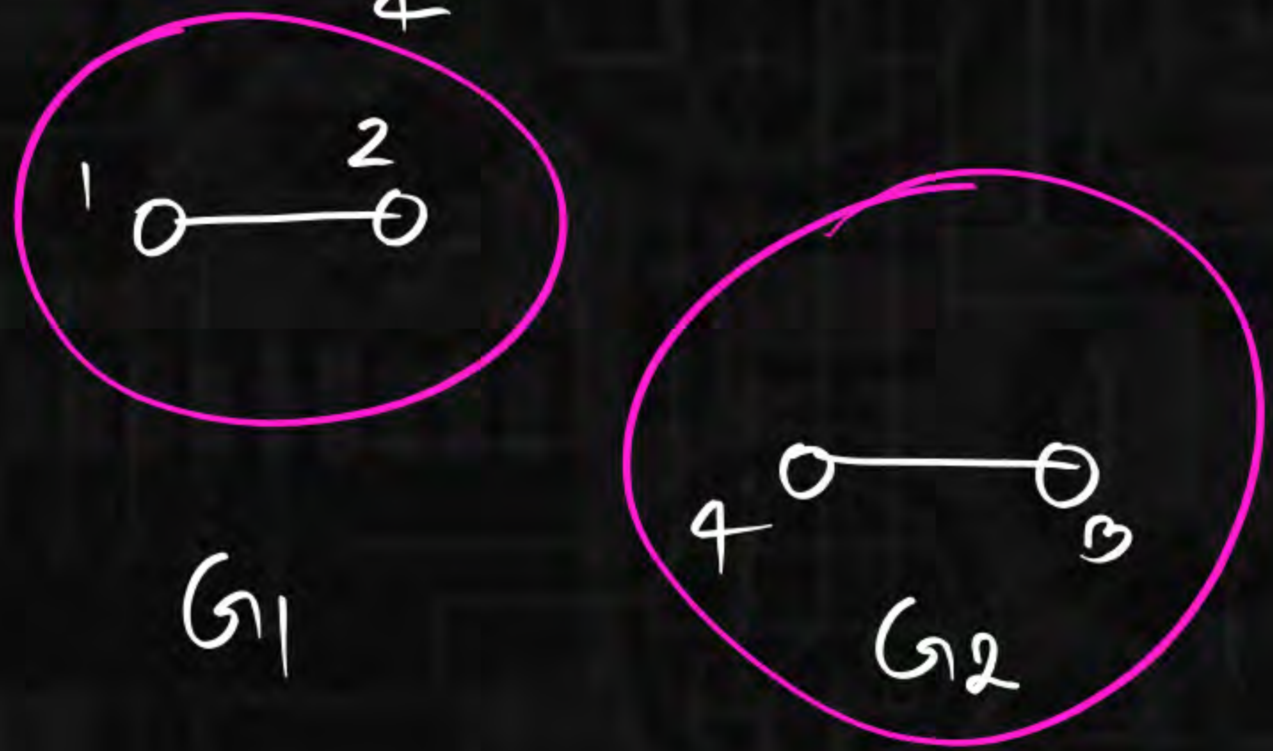
no common edges:





vertex disjoint subgraph.





1. edge disjoint Graphs.

we <u>may</u> get common vertex but no common edge.

2. vertex disjoin graphs.

no common vertices.

no common edges.

Graph operation:

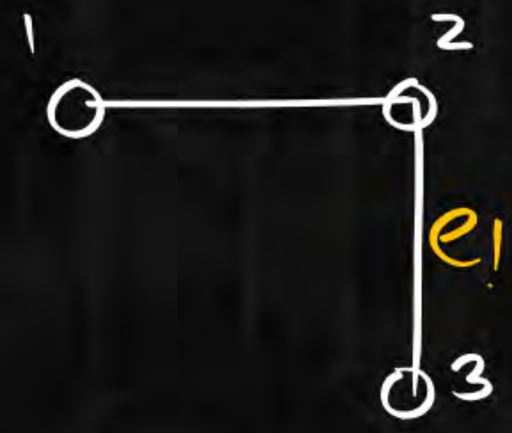
1. Union ($\cup$)

2. Intersection ($\cap$)

3. Ring sum ($\oplus$)

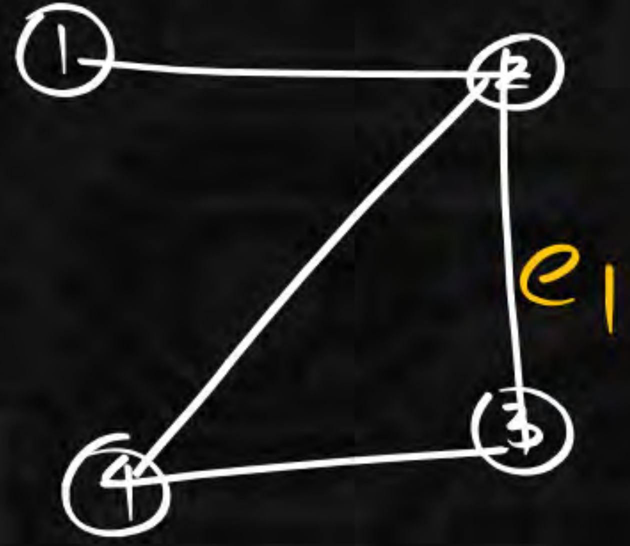
$G_1 = (V_1, E_1)$ $\quad$ $G_2 = (V_2, E_2)$

$$\begin{cases} G_3 = (V_3, E_3) \\ G_3 = G_1 \cup G_2 \\ V_3 = V_1 \cup V_2 \\ E_3 = E_1 \cup E_2 \end{cases}$$



G_1

G_2

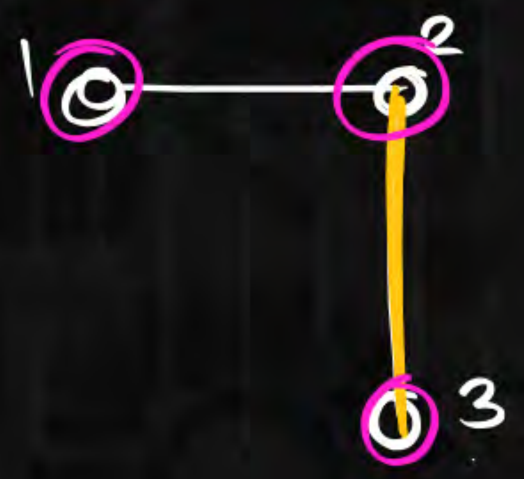


$G_1 = (V_1, E_1)$ $\quad$ $G_2 = (V_2, E_2)$ $\quad$ $G_3 = (V_3, E_3)$

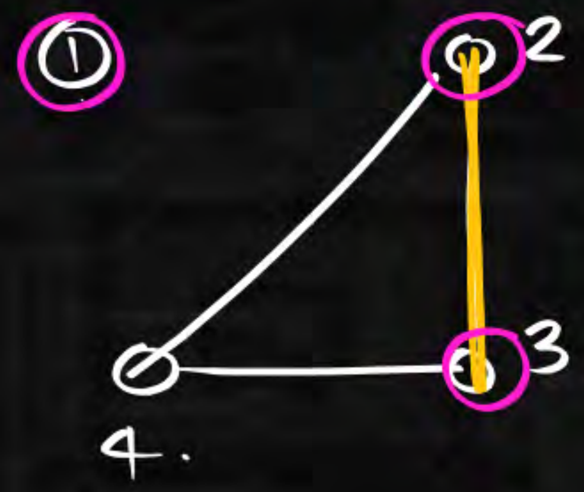
$V_1 = \{1, 2, 3\} \cup V_2 = \{1, 2, 3, 4\}$ $\quad$ $V_3 = V_1 \cup V_2 = \{1\ 2\ 3\ 4\}$

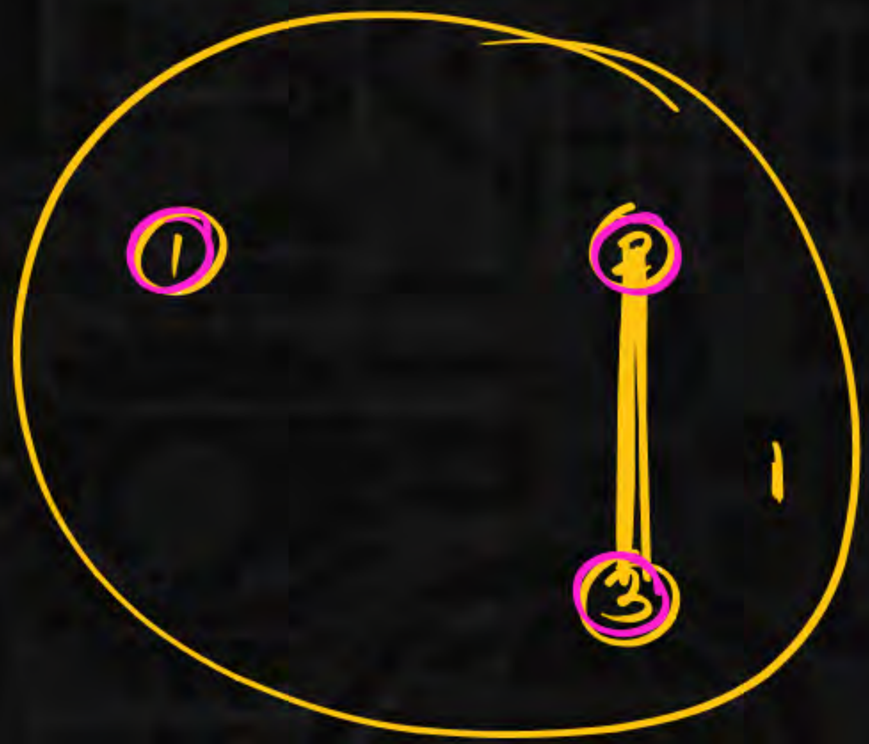
$E_1 = \{ \ldots e_1 \}$ $\quad$ $E_2 = \{ \cdots e_1 \cdots \}$ $\quad$ $E_3 = E_1 \cup E_2 = \{ e_1 \cup e_1 \}$

e_1



$\cap$





G_4

G_1

G_2

$G_1 = (V_1, E_1)$

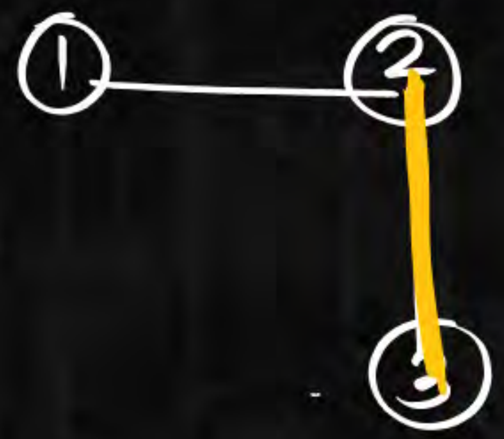
$G_2 = (V_2, E_2)$

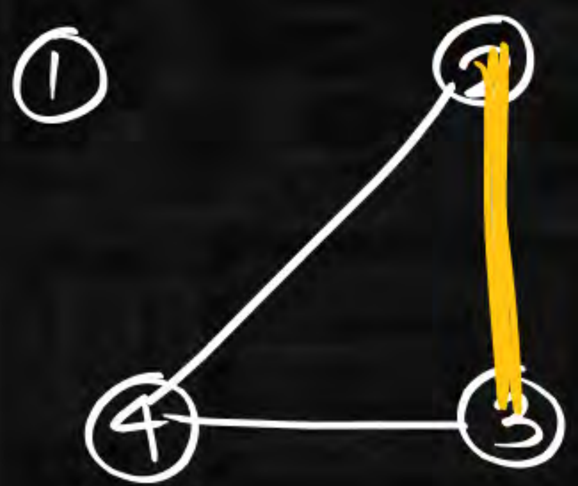
$G_4 = (V_4, E_4)$

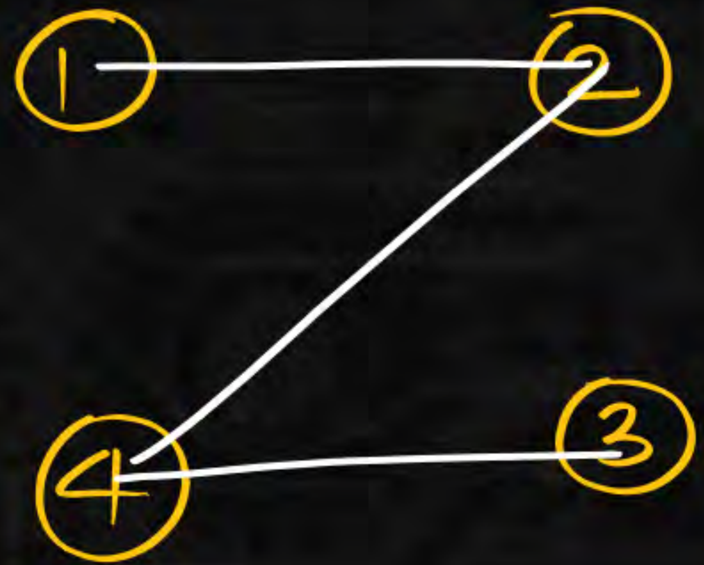
$\boxed{G_4 = G_1 \cap G_2}$

$V_4 = V_1 \cap V_2$

$E_4 = E_1 \cap E_2$







$G_1 = (V_1, E_1)$

$G_2 = (V_2, E_2)$

$G_5 = G_1 \oplus G_2$

$V_5 = V_1 \cup V_2$

$E_5 = (E_1 \cup E_2) - (E_1 \cap E_2)$

edges should be in G_1, G_2 but not in both

$$G \cup G = G$$

$$G \cap G = G$$

$$G \oplus G = \text{null graph}$$

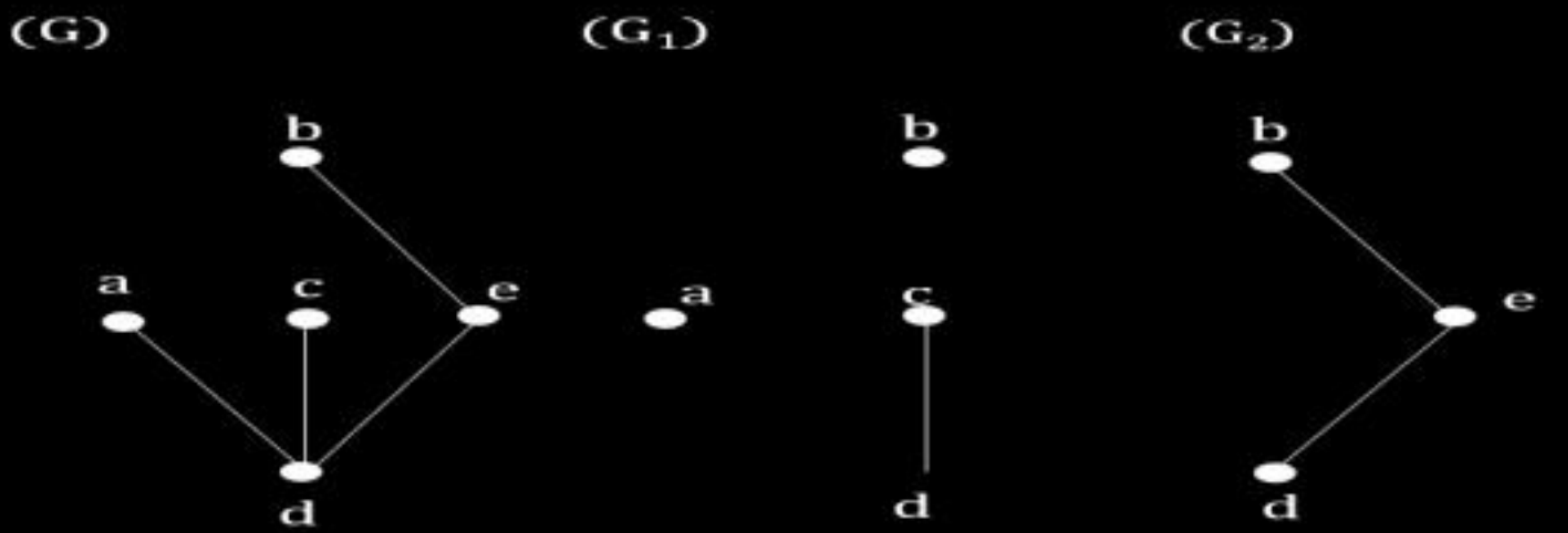
$$G_1 \cup G_2 = G_2 \cup G_1.$$

$$G_1 \cap G_2 = G_2 \cap G_1.$$

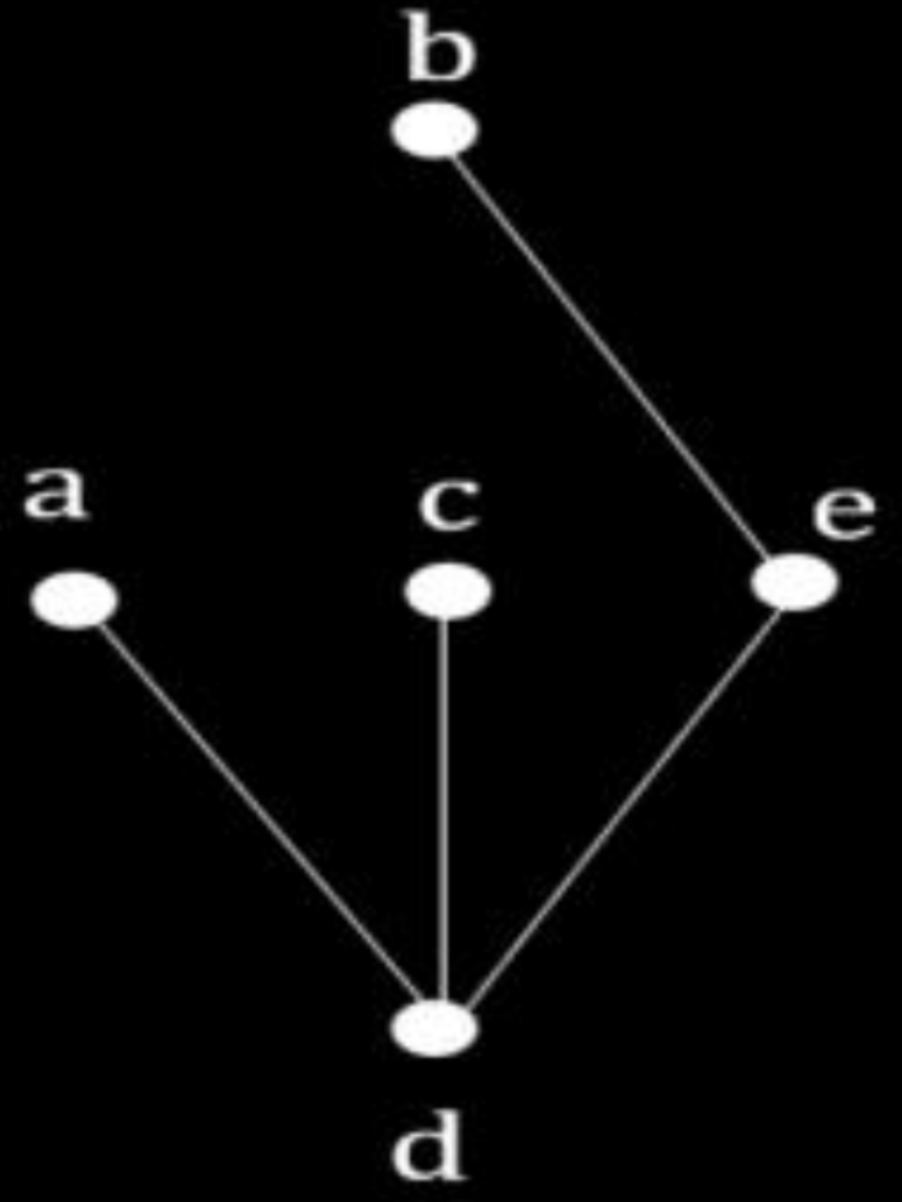
$$G_1 \oplus G_2 = G_2 \oplus G_1$$

A graph g is said to be a subgraph of a graph G if all the vertices and all the edges of g are in G, and each edge of g has the same end vertices in g as in G.

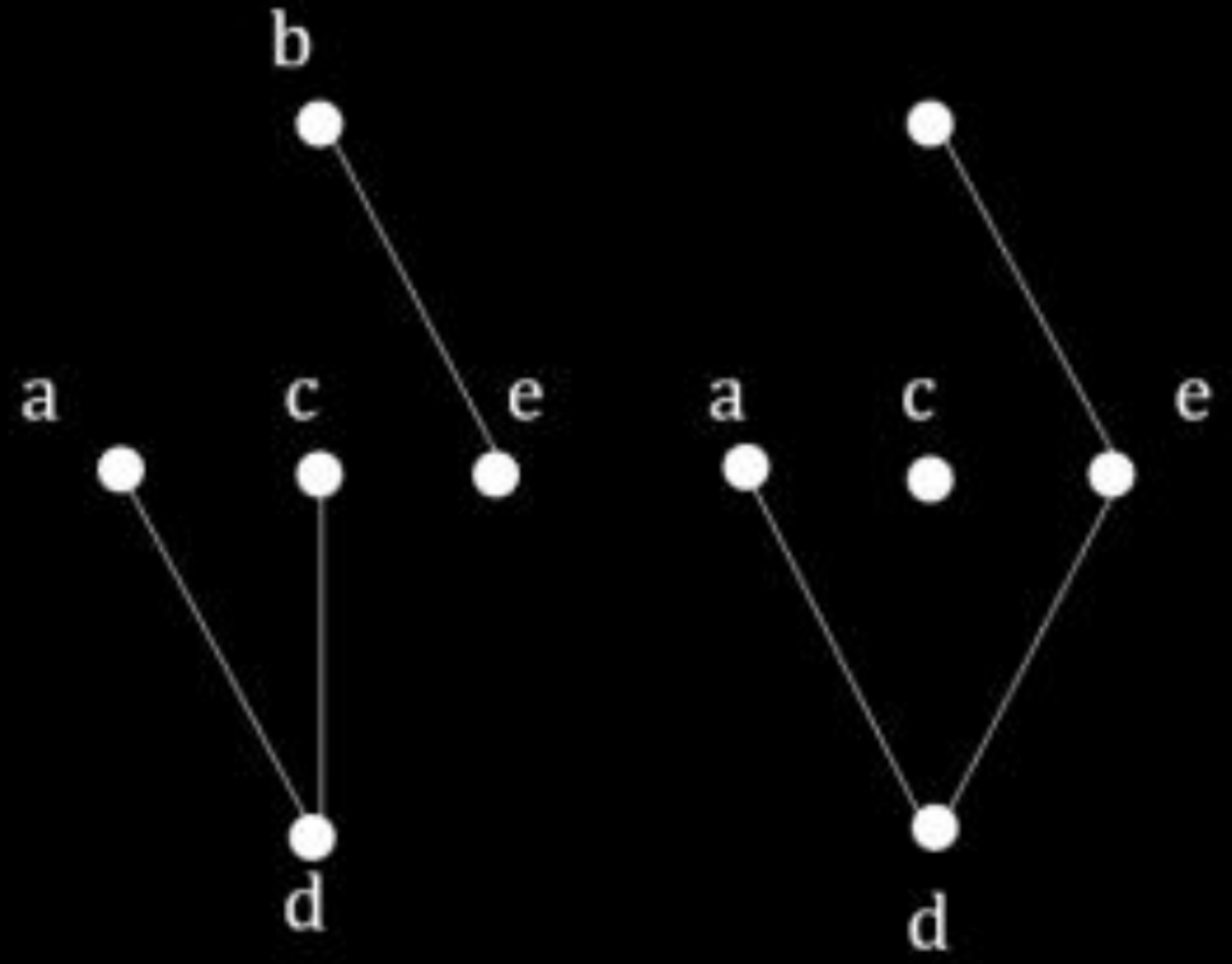
(Subgraph) Let G and H be graphs. We call G a subgraph of H provided $V(G) \subseteq V(H)$ and $E(G) \subseteq E(H)$



Given a (directed or undirected) graph $G = (V, E)$, let $G_1 = (V_1, E_1)$ be a subgraph of G. If $V_1 = V$, then G_1 is called a spanning subgraph of G.



(G3) (G_4)



The following observations can be made immediately:

1. Every graph is its own subgraph.
2. A subgraph of a subgraph of G is a subgraph of G.
3. A single vertex in a graph G is a subgraph of G.
4. A single edge in G, together with its end vertices, is also a subgraph of G.

It is obvious from their definitions that the three operations just mentioned are commutative. That is,

$G_1 \cup G_2 = G_2 \cup G_1$,

$G_1 \cap G_2 = G_2 \cap G_1$

$G_1 \oplus G_2 = G_2 \oplus G_1$.

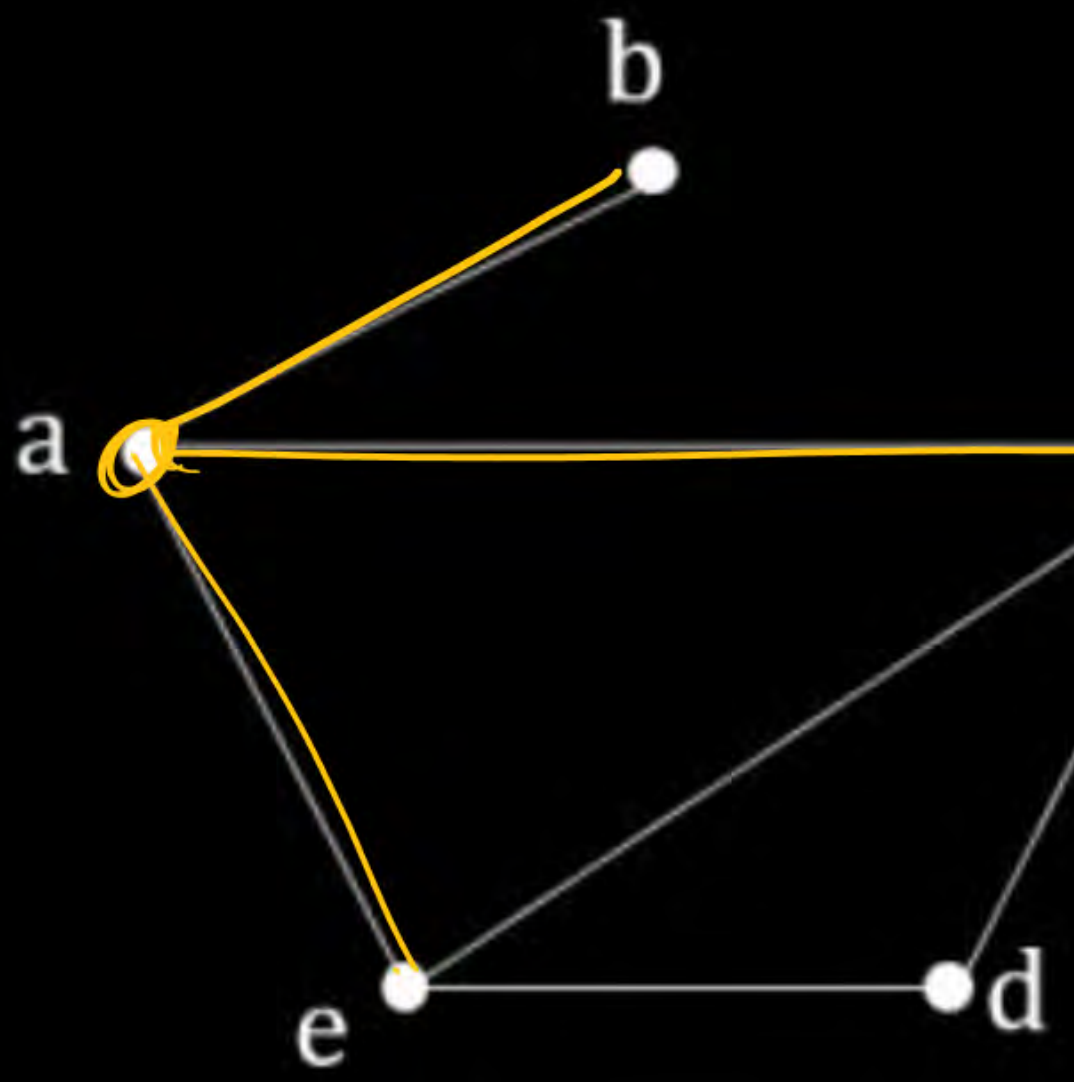
If G_1 and G_2 are edge disjoint, then $G_1 \cap G_2$ is a null graph,

$G_1 \oplus G_2 = G_1 \cup G_2$.

If G_1 and G_2 are vertex disjoint, then $G_1 \cap G_2$ is empty.

For any graph G,

$G \cup G = G \cap G = G$ and $G \oplus G =$ a null graph

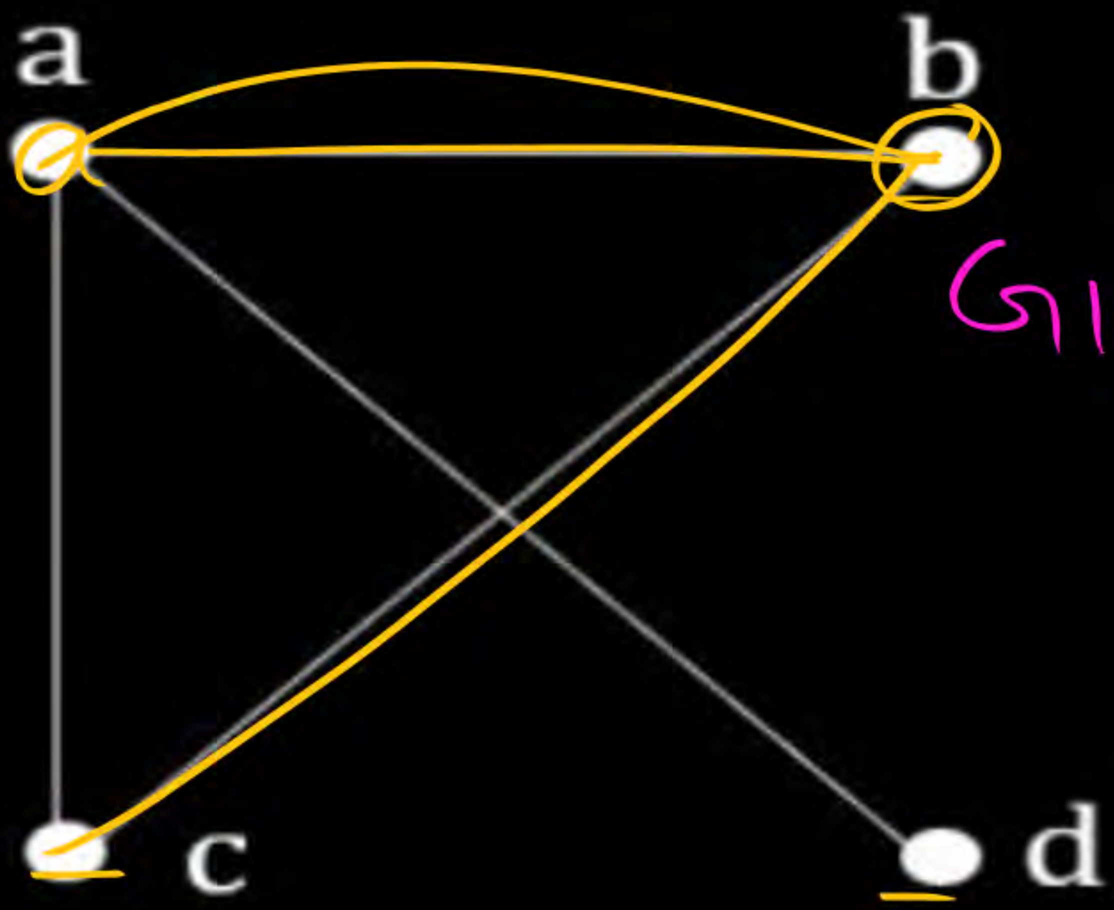


Simple Graph

An adjacent List for a Simple Graph

Vertex	Adjacent Vertices
a	b, c, e
b	a
c	a, d, e
d	c, e
e	a, c, d

We order the vertices as a, b, c, d.
The matrix representing this graph is



$G_1 = (V_1, E_1)$

$$\begin{array}{c|c} & \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{array}{c} \rightarrow 3 \\ \rightarrow 2 \\ \rightarrow 2 \\ \rightarrow 1 \end{array} \end{array}$$

$$\begin{array}{c|c} & \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ & & & \\ & & & \end{bmatrix} \end{array}$$


THANK YOU GW
SOLDIERS !