CS & IT ENGINEERING

Theory of Computation

Finite Automata

Lecture No. 23

By- DEVA Sir

TOPICS TO BE COVERED

01 Closure properties for Regulars

02

03

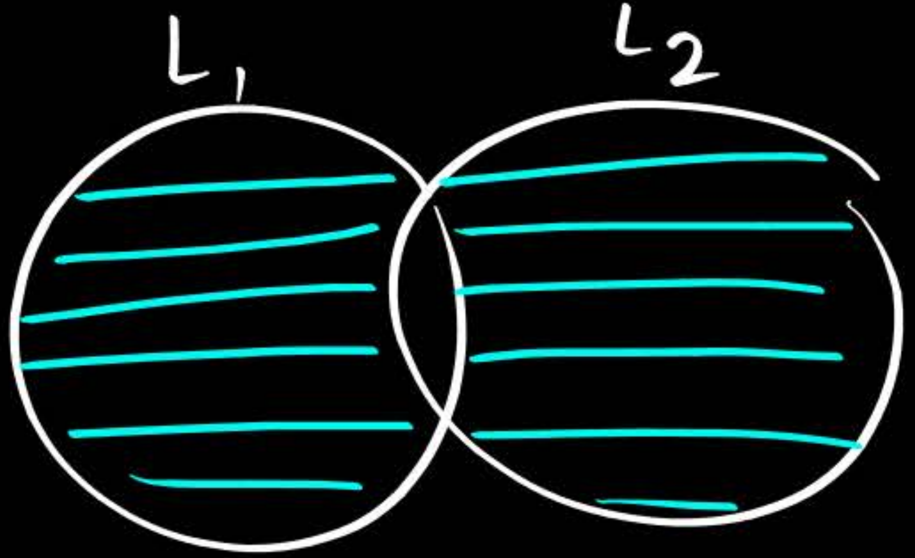
04

05

(9) Symmetric Difference for Regulars

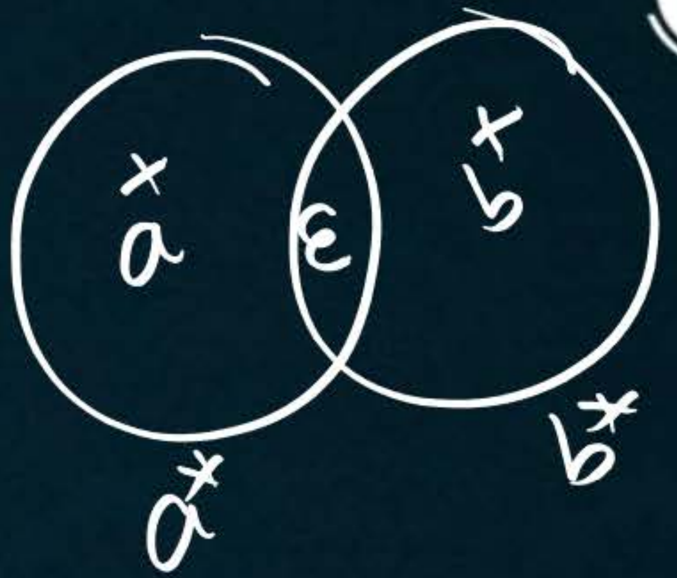
↳ closed

$$L_1 \oplus L_2 = L_1 \Delta L_2 = (L_1 - L_2) \cup (L_2 - L_1)$$



$$= (L_1 \cup L_2) - (L_1 \cap L_2)$$

$$\left.\begin{array}{l} 1)\quad L_1 = a^* \\ \quad\;\; L_2 = b^* \end{array}\right\} \Rightarrow L_1 \Delta L_2 = a^+ + b^+$$



$\Sigma = \{a, b\}$

$$\left.\begin{array}{l} 2)\quad L_1 = a\Sigma^* \\ \quad\;\; L_2 = b\Sigma^* \end{array}\right\} \Rightarrow L_1 \Delta L_2 = a\Sigma^* + b\Sigma^*$$

$$\boxed{\begin{array}{l} \text{If } L_1 \cap L_2 = \phi \\ \text{then } L_1 \Delta L_2 = L_1 \cup L_2 \end{array}}$$

*** (10) Subset for Regulars

→ Not closed

Subset ($\subseteq$)

Every Subset of a Regular language is ______ Either Reg or Not reg

$L = a^*$

ϕ, $\{\epsilon\}$, a^*, a^+ — Reg

$a^{n!}$ — Not reg

a^{prime}

$L = (a+b)^*$

Subset of L

ϕ — reg

a^* — reg

b^* — reg

$a^n b^n$ — not reg

a^{prime} — not reg

In Maths:

$A = \{1, 2, 3\}$

Subsets of A

$S_1 = \{\}$

$S_2 = \{1\}$

$S_3 = \{2\}$

$S_4 = \{3\}$

$S_5 = \{1, 2\}$

$S_6 = \{1, 3\}$

$S_7 = \{2, 3\}$

$S_8 = \{1, 2, 3\}$

No. of Subsets of A

$= 2^{|A|}$

$= 2^3 = 8$

***

I) Every Subset of Finite language is Finite language (Regular)

II) Every Subset of Infinite Language is either Infinite or not Infinite

III) Every Subset of Regular language is either Regular or not reg

IV) Every Subset of Not regular language is either Regular or not reg

For language:

$\underset{\text{(Init)}}{\text{Prefix}}(L) = \{u \mid w \in L, w = uv\} = \{u \mid uv \in L\}$

$\text{Suffix}(L) = \{v \mid uv \in L\}$

$\underset{\text{(Subword)}}{\text{Substring}}(L) = \{y \mid xyz \in L\}$

For string:

$\text{prefix}(w) = \{u \mid uv = w\}$ $\text{pref}(ab) = \{\varepsilon, a, ab\}$

$\text{Suffix}(w) = \{v \mid uv = w\}$ $\text{suffix}(ab) = \{\varepsilon, b, ab\}$

$\text{Substring}(w) = \{y \mid xyz = w\}$ $\text{substring}(ab) = \{\varepsilon, a, b, ab\}$

(11) prefix(Reg lang) is Reg

$L = \{abc\}$

$\Downarrow$

$\Downarrow$ From initial to final state path, make every state is final

$\Downarrow$

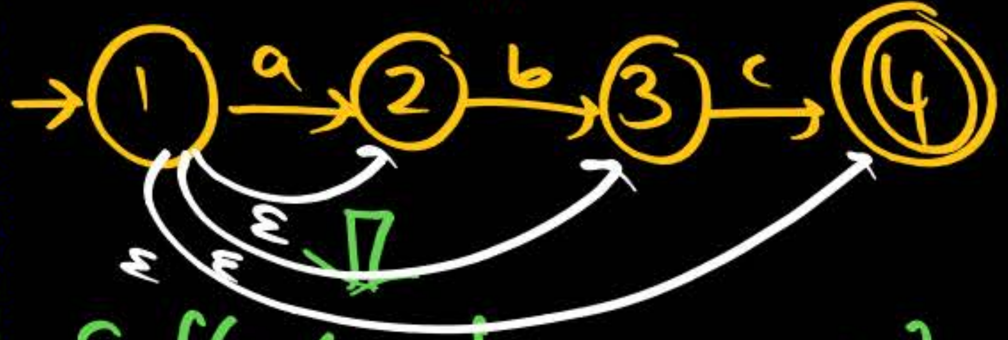
$Prefix(L) = \{\varepsilon, a, ab, abc\}$

(12) Suffix(Reg) is Reg

$L = \{abc\}$

$\Downarrow$

$\Downarrow$ From initial to final state path, add ε moves from initial to every state



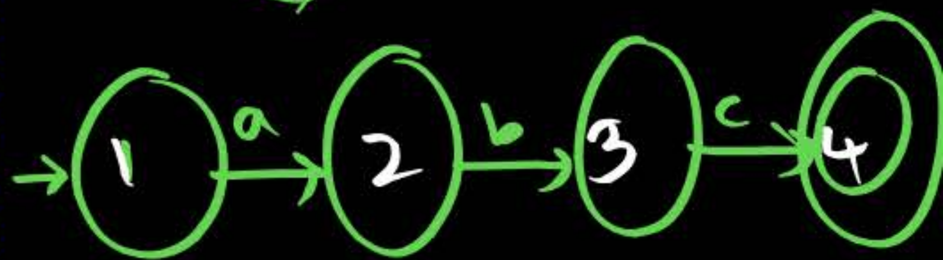
$\Downarrow$

$Suffix(L) = \{\varepsilon, c, bc, abc\}$

(13) Substring(Reg) is Reg

$L = \{abc\}$

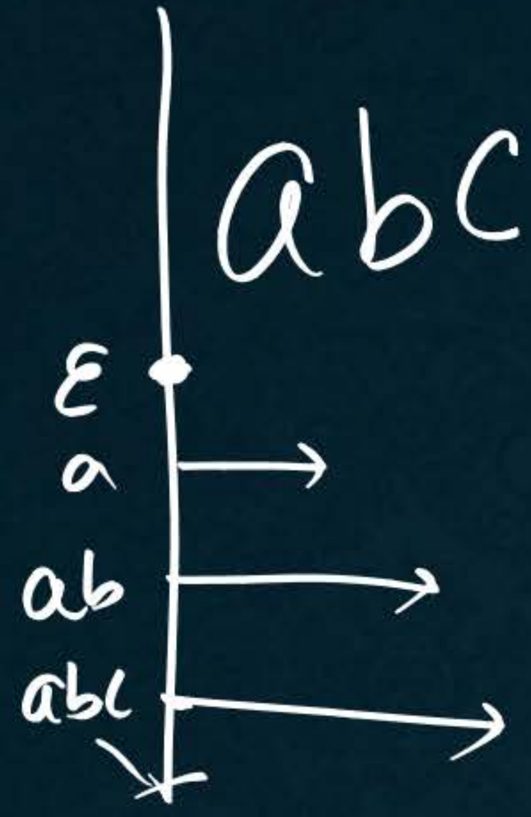
$\Downarrow$



$\Downarrow$ combine both prefix Algo & Suffix Algo

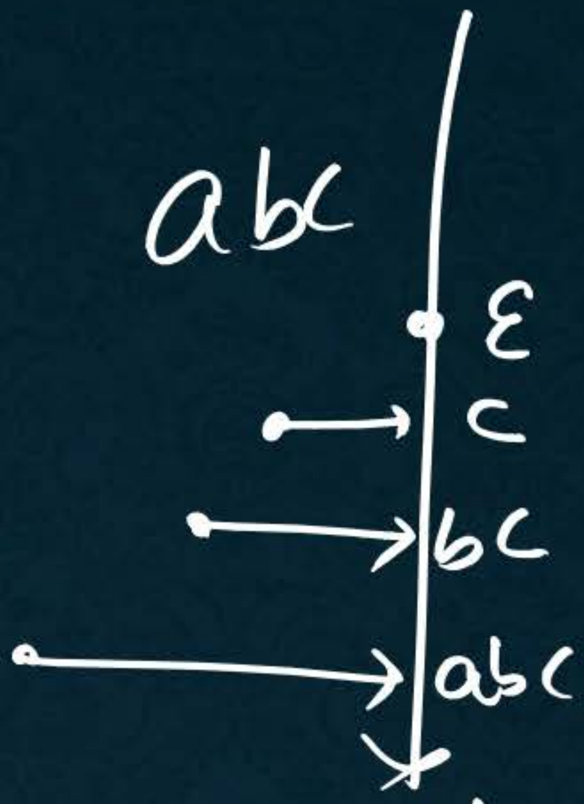
$\Downarrow$

$Substring(L) = \{\varepsilon, a, b, c, ab, bc, abc\}$



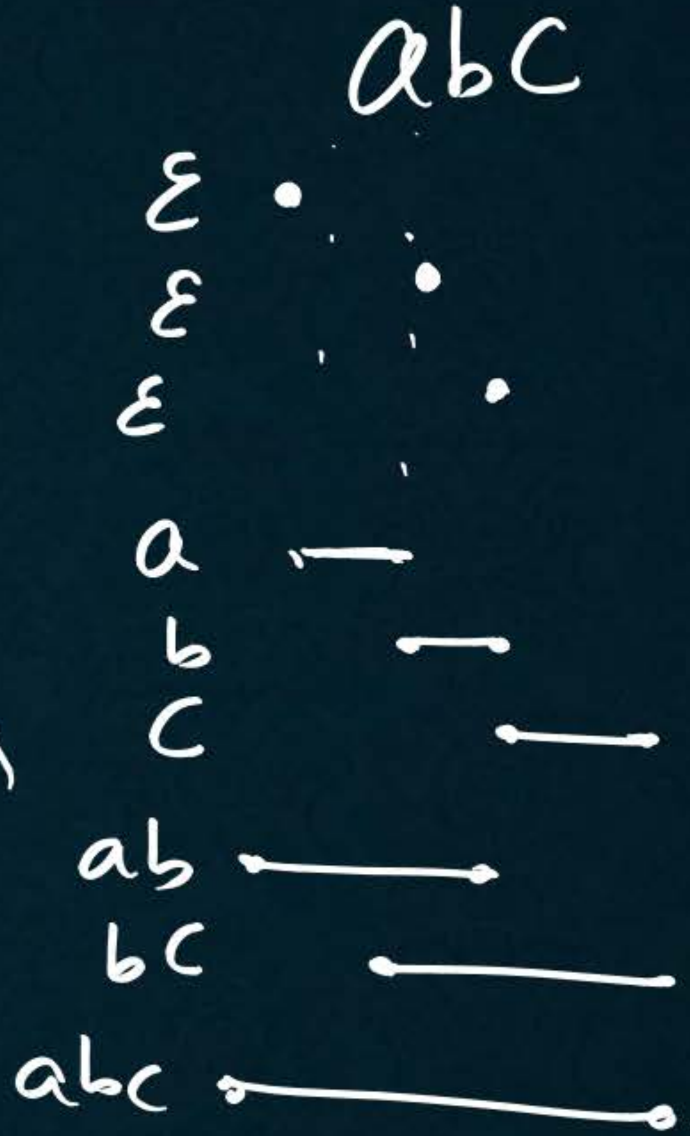
Begin is fixed

Prefixes



end is fixed

Suffixes



Start anywhere & end anywhere

H.W. :

① $L = \{ab\} \Rightarrow$ $\boxed{\begin{array}{l} \text{prefix}(L) = ? \\ \text{Suffix}(L) = ? \\ \text{Substring}(L) = ? \end{array}}$

② $L = a^* \Rightarrow$ "

③ $L = a^+ \Rightarrow$ "

④ $L = a\Sigma^* \Rightarrow$ "

⑤ $L = \Sigma^* b \Rightarrow$ "

⑥ $L = \Sigma^* a \Sigma^* \Rightarrow$ "

$\Sigma = \{a, b\}$

$$a\Sigma^* = a(a+b)^*$$

$$= \{a, aa, ab, aaa, aab, aba, abb, \dots\}$$

Prefix(L)

$$\{\boxed{\varepsilon}, a, aa, ab, aaa, \dots\}$$

$$= \varepsilon + a\Sigma^*$$

Suffix(L)

$$= \Sigma^*$$

Substring(L)

$$= \Sigma^*$$

(14) Substitution for Regular languages

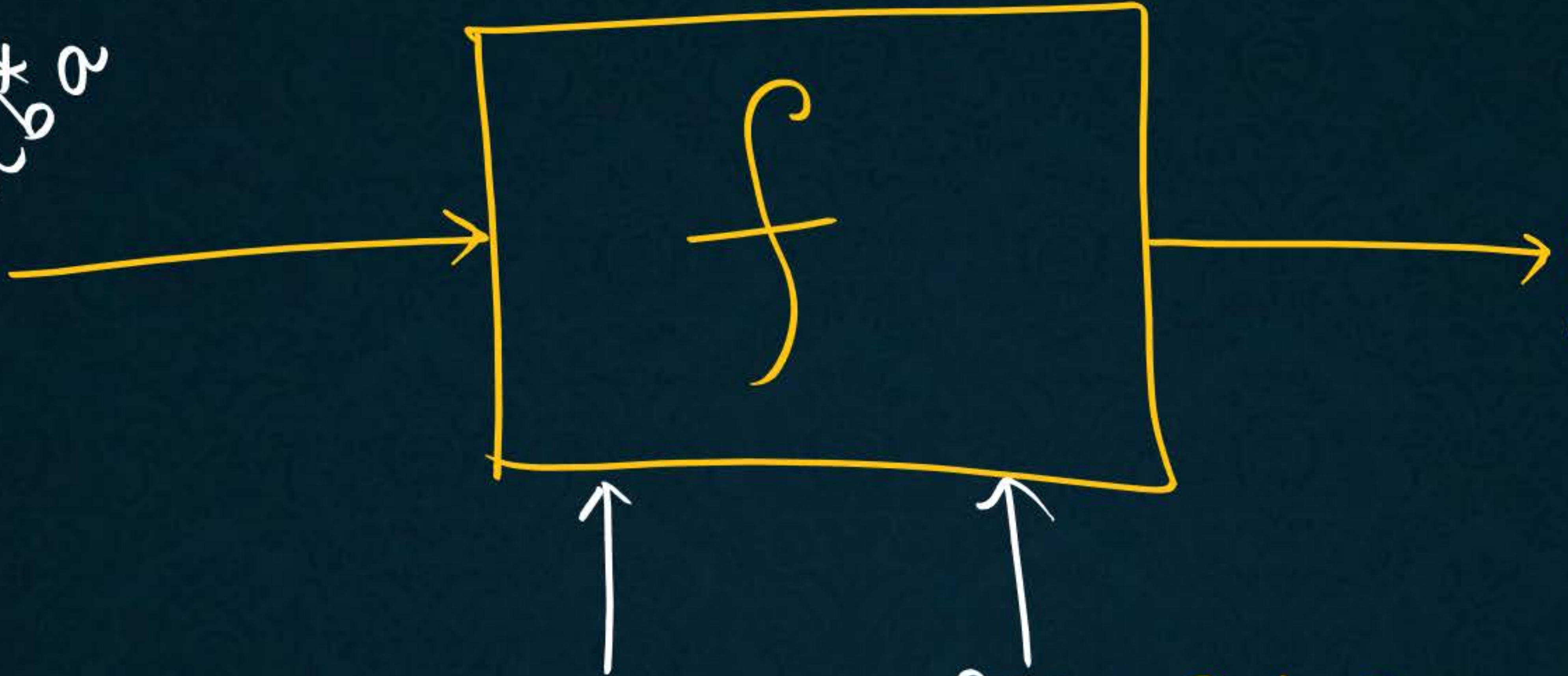
↳ closed

Substitution of a Regular language is ____ Regular Language

(Regular Substitution)

$\Sigma = \{a, b\}$

$\underset{(Reg)}{L} = ab^*a$



$f(L)$ is Regular

$= 1^*(01(11)^*)^*1^*$

$f(a) = $ Reg Lang $= 1^*.$

$f(b) = $ Reg Lang $01(11)^*$

$\Delta = \{0, 1\}$

$$L = \{ ab, \quad abba \}$$

$$f(L) = \{ 1^*01(11)^*, \quad 1^*(01(11)^*)^2 1^* \}$$

$$f(a) = 1^*$$

$$f(b) = 01(11)^*$$

Symbol

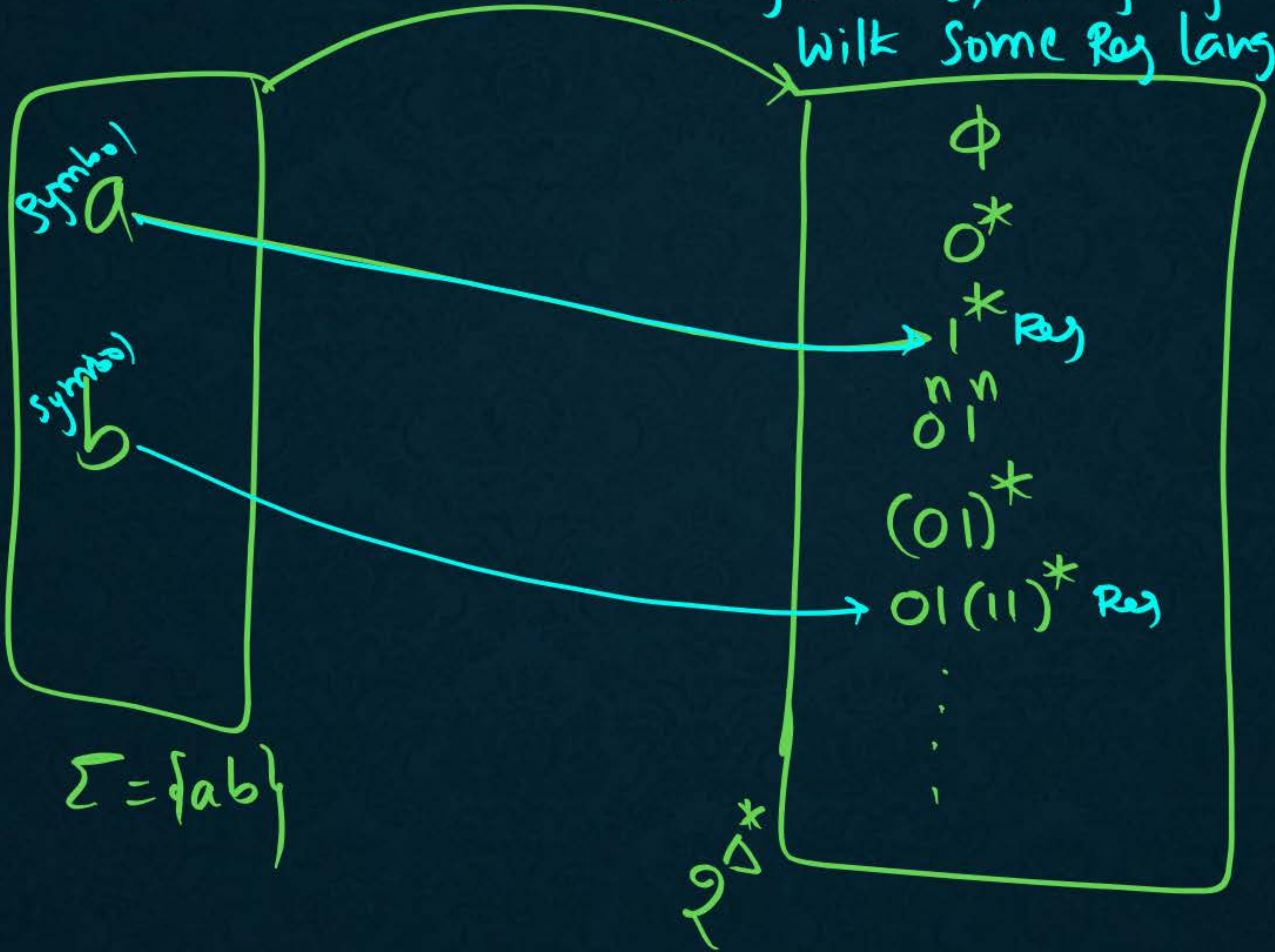
Reg lang

$$f : \text{Set of symbols} \longrightarrow \text{Set of languages}$$

$$f : \Sigma \longrightarrow 2^{\Delta^*}$$

f : It is a mapping

In given Reg, every symbol over Σ is Substituted with Some Reg lang over Δ



$\Delta = \{0,1\}$

$\Sigma = \{0, 1\}$

$\Sigma^* = \{\varepsilon, 0, 1, 00, 01, 10, 11, \ldots\} = (0+1)^*$

2^{Σ^*} = Powerset(Σ^*) = Set of all subsets of Σ^*

= Set of languages = Set of all languages over Σ

$= \{L_1, L_2, L_3, \ldots\ldots\}$

1) $L = (ab)^* \text{ and } f(a) = \{\varepsilon\}, f(b) = 0^*$

$$f(L) = (f(a) \cdot f(b))^*$$

$$= (\varepsilon \cdot 0^*)^*$$

$$= 0^*$$

2) $L = a.(a+b)^*$, $f(a) = 0^*$, $f(b) = 1^*$

$$\underbrace{}$$

$\Downarrow$

$$f(L) = f\left(a\,(a+b)^*\right)$$

$$= f(a).\left(f(a)+f(b)\right)^*$$

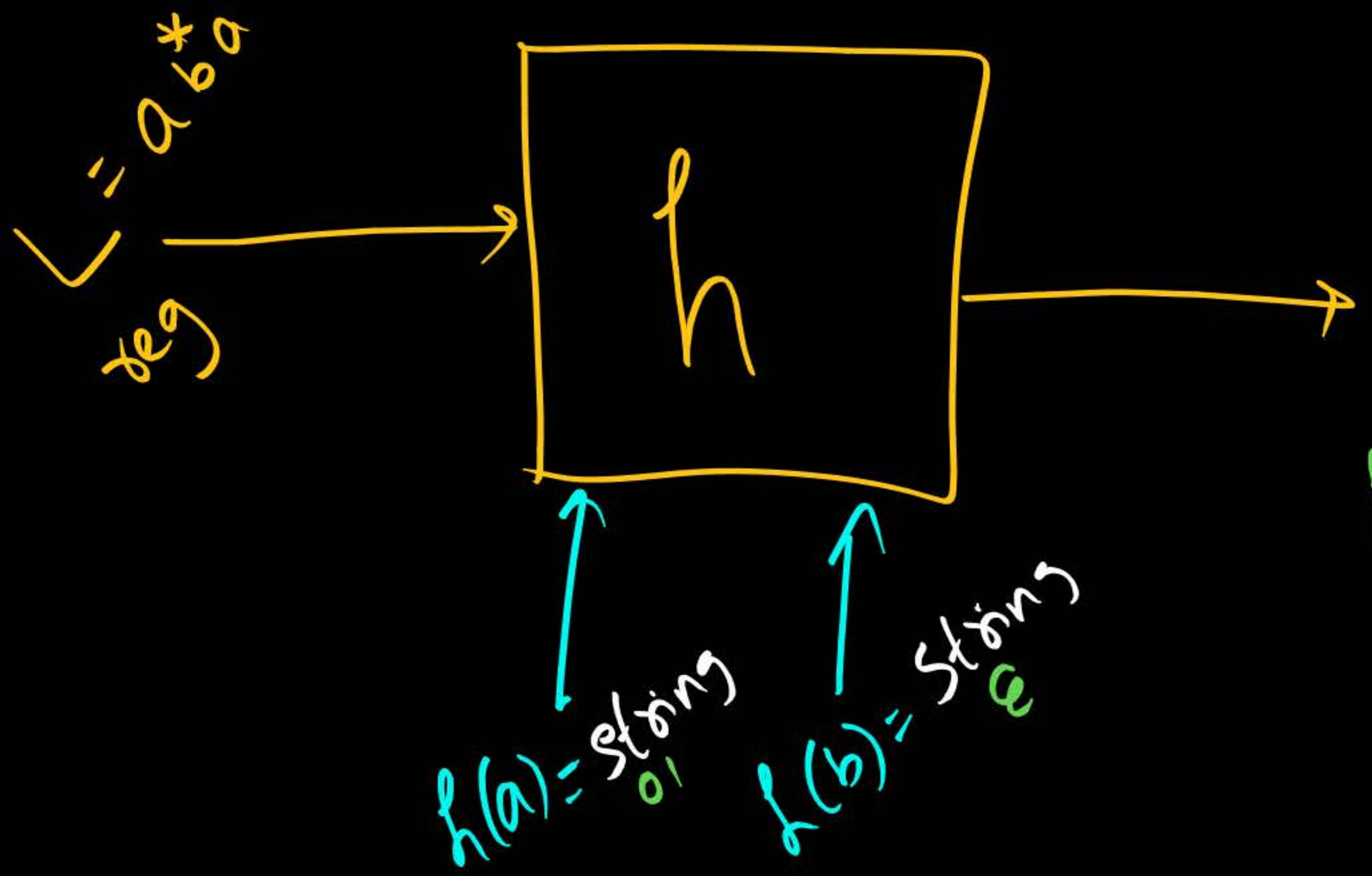
$$= 0^*.\left(0^*+1^*\right)^*$$

$$= (0+1)^*$$

$\left.\begin{matrix}\Sigma \\ \Delta\end{matrix}\right\}$ may or may not be same

(15) Homomorphism

[string substitution]



$h(L)$ is Reg

$h(L) = h(a) \cdot (h(b))^* \, h(a)$

$= 01 \cdot \epsilon^* \cdot 01$

$= 0101$

$h(L)$: In given reg L over Σ, every symbol is substituted with some $\boxed{\text{string}}$ over Δ.

$$h : \underset{\text{set of symbols}}{\Sigma} \longrightarrow \underset{\text{set of strings}}{\Delta^*}$$

(16) ε-free Homomorphism:

$$\Sigma \longrightarrow \Delta^+$$

non empty string (other than ε)

** (17) Inverse Homomorphism for Regular languages

→ closed

given $L = \{1, 111\}$ Reg → h^{-1} → $h^{-1}(L)$ is Reg

$h^{-1}(1) = a$

$h^{-1}(111) = \{aaa, ab, ba\}$

$h^{-1}(L) = \{a, aaa, ab, ba\}$

$h(a) = 1$ $h(b) = 11$

First find h^{-1}

$$\boxed{\begin{aligned} h^{-1}(1) &= a \\ h^{-1}(11) &= b \end{aligned}}$$

$$
h^{-1}(111) \begin{cases} \nearrow \; h^{-1}(1) \cdot h^{-1}(1) \cdot h^{-1}(1) \Rightarrow aaa \\ \rightarrow \; h^{-1}(1) \cdot h^{-1}(11) \Rightarrow ab \\ \searrow \; h^{-1}(11) \, h^{-1}(1) \Rightarrow ba \end{cases}
$$

$$h^{-1}(111) = \{aaa, ab, ba\}$$

1st	2nd	3rd
Everything is Imp.	Difficult $\Rightarrow$ Imp Try to make it easy	After confidence $\Downarrow$ performance ✓

Knowledge $\propto$ confidence $\propto$ performance

$$\propto \frac{1}{\text{errors}}$$

$$K \propto C \propto P \propto \frac{1}{E} \propto \text{Best Rank}$$

18 Quotient (/)

$$L_1/L_2 = \{u \mid uv \in L_1,\ v \in L_2\}$$

1) $abc/\varepsilon = abc$

2) $abc/a = \times$

3) $abc/b = \times$

4) $abc/c = ab$

5) $abc/ab = \times$

6) $abc/bc = a$

7) $abc/abc = \varepsilon$

8) $\varepsilon/abc = \times$

9) $\varepsilon/\varepsilon = \varepsilon$

10) $L/\varepsilon = L$

11) $a/a = \varepsilon$

$uv/v = u$

$42/2 = 4$

1) $\left.\begin{array}{l} L_1 = a^* \\ L_2 = b^* \end{array}\right\} \Rightarrow$

i) $L_1/L_2 = a^*/b^* = \underbrace{\{a^*/\varepsilon,\ a^*/b,\ a^*/bb,\ \ldots\}}_{\times} = a^*$

ii) $L_2/L_1 = b^*/a^* = b^*$

$$a^*/b^* = \{\varepsilon, a, aa, aaa, \ldots\} / \{\varepsilon, b, bb, bbb, \ldots\}$$

$$= \{a^*/\varepsilon,\ a^*/b,\ a^*/bb,\ \ldots\}$$

$$\varepsilon/\varepsilon,\ a/\varepsilon,\ a^2/\varepsilon,\ \ldots$$

$$a^*$$

$$\varepsilon/b,\ a/b,\ aa/b,\ \ldots$$

$$\times,\ \times,\ \times$$

$$\varepsilon/bb,\ a/bb,\ a^2/bb,\ \ldots$$

$$\times\ \ \times\ \ \times$$

2) $L_1 = \{\varepsilon, a, bb\}$

$L_2 = \{b, ab, aaa\}$

i) $L_1/L_2 = \{\varepsilon, a, bb\} / \{b, ab, aaa\}$

$= \{\varepsilon/b, \varepsilon/ab, \varepsilon/aaa, a/b, a/ab, a/aaa, bb/b, bb/ab, bb/aaa\}$

$= \{b\}$

ii) $L_2/L_1 = \{b/\varepsilon, b/a, b/bb, ab/\varepsilon, ab/a, ab/bb, aaa/\varepsilon, aaa/a, aaa/bb\}$

$= \{b, ab, aaa, aa\}$

$$x = 3/3x = 3/x$$

3) $a^*/a = \{\varepsilon, a, aa, \dots\}/a$

$= \{\varepsilon/a, a/a, aa/a, \dots\} = a^*$

$\quad\;\; \times \quad\; \varepsilon \quad\; a$

4) $a/a^* = \{a/\varepsilon, a/a, \underbrace{a/aa, \dots}_{\times}\}$

$\quad\;\; a \quad\; \varepsilon$

$= \{\varepsilon, a\}$

$a^*a = a^+$

$a^* + a = a^*$

$a^*/a = a^*$

$$a^*/a = \{\varepsilon, a, aa, aaa, \ldots\}/a$$

$$= \{\varepsilon/a,\ a/a,\ aa/a,\ aaa/a,\ \ldots\}$$

$$\times \qquad \Downarrow \qquad \Downarrow \qquad \Downarrow$$

$$\qquad \varepsilon \qquad a \qquad aa$$

u v

✓

H.W.:

$$\left.\begin{array}{l} L_1 = a^* b \\ L_2 = a b^* \end{array}\right\} \Rightarrow \begin{array}{l} \text{i) } L_1 / L_2 = \\ \text{ii) } L_2 / L_1 = \end{array}$$

$L = \{\varepsilon, a, ab, abc, abcd, abcda, abbbcd\}$

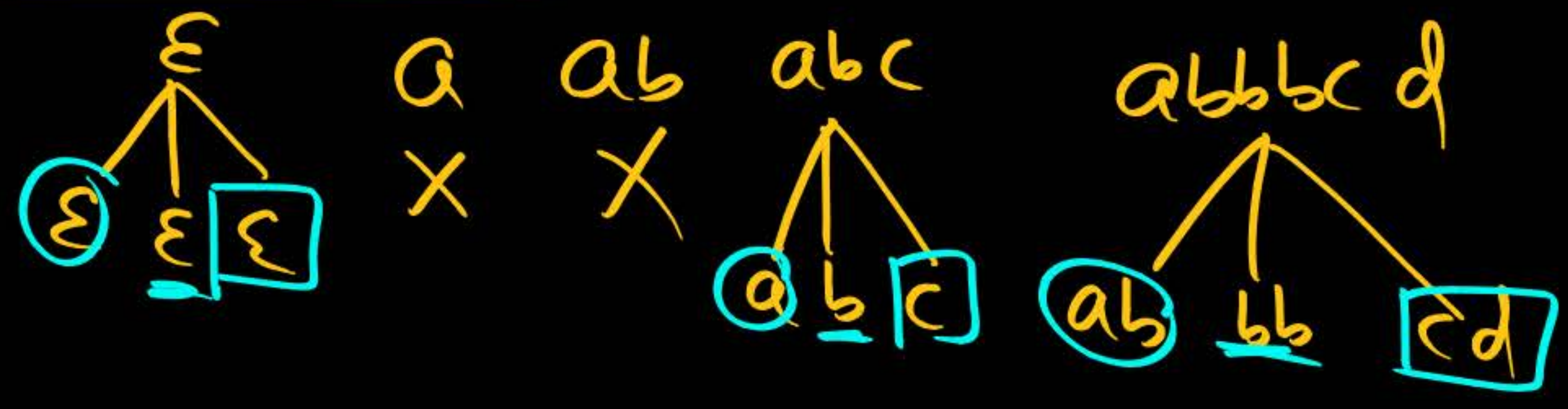
(19) $\frac{1}{2}(L) = \{\varepsilon, a, ab, abb\}$

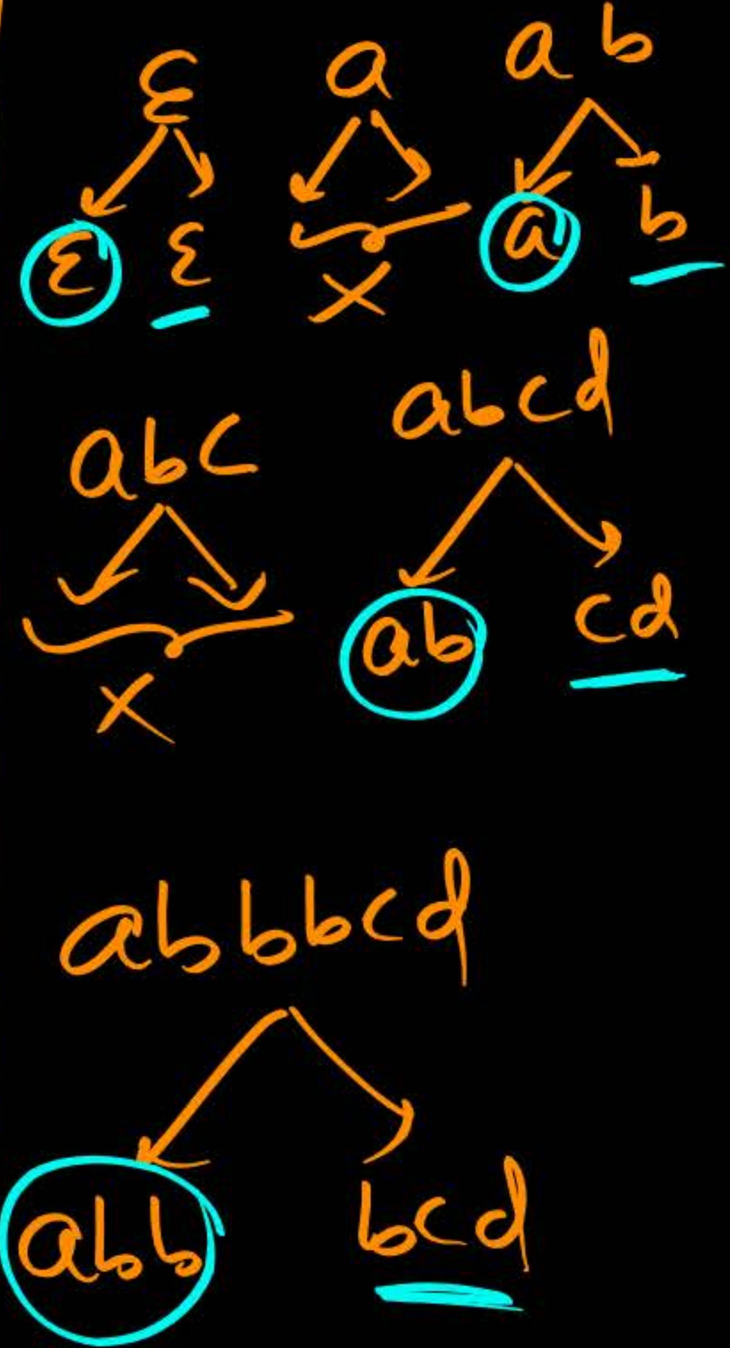
(20) second $\frac{1}{2}(L) = \{\varepsilon, b, cd, bcd\}$

(21) $\frac{1}{3}(L) = \{\varepsilon, a, ab\}$

(22) Middle $\frac{1}{3}(L) = \{\varepsilon, b, bb\}$

(23) Last $\frac{1}{3}(L) = \{\varepsilon, c, cd\}$





(19) $\frac{1}{2}(L) = \{ x \mid xy \in L, |x| = |y| \}$

(20) second $\frac{1}{2}(L) = \{ y \mid$ " " $\}$

(21) $\frac{1}{3}(L) = \{ x \mid xyz \in L, |x| = |y| = |z| \}$

(22) Middle $\frac{1}{3}(L) = \{ y \mid$ " " $\}$

(23) Last $\frac{1}{3}(L) = \{ z \mid$ " " $\}$

(24) Finite union is closed for regular languages

$L_i \to$ reg

$$L_1 \cup L_2 \cup L_3 \cup L_4 \cdots \cup L_{k} \Rightarrow \text{Regular}$$

constant

(25) Finite Intersection is closed for regulars

$$L_1 \cap L_2 \cap L_3 \cap L_4 \cap \cdots \cap L_{k} \Rightarrow \text{Regular}$$

constant
finite

(26) $L_1 - L_2 - L_3 - \cdots - L_k \Rightarrow$ Regular

(27) $L_1 \cdot L_2 \cdot L_3 \cdot \cdots L_k \Rightarrow$ Regular

(28) Finite subset of Regular Set is ~~finite set~~ (Regular)

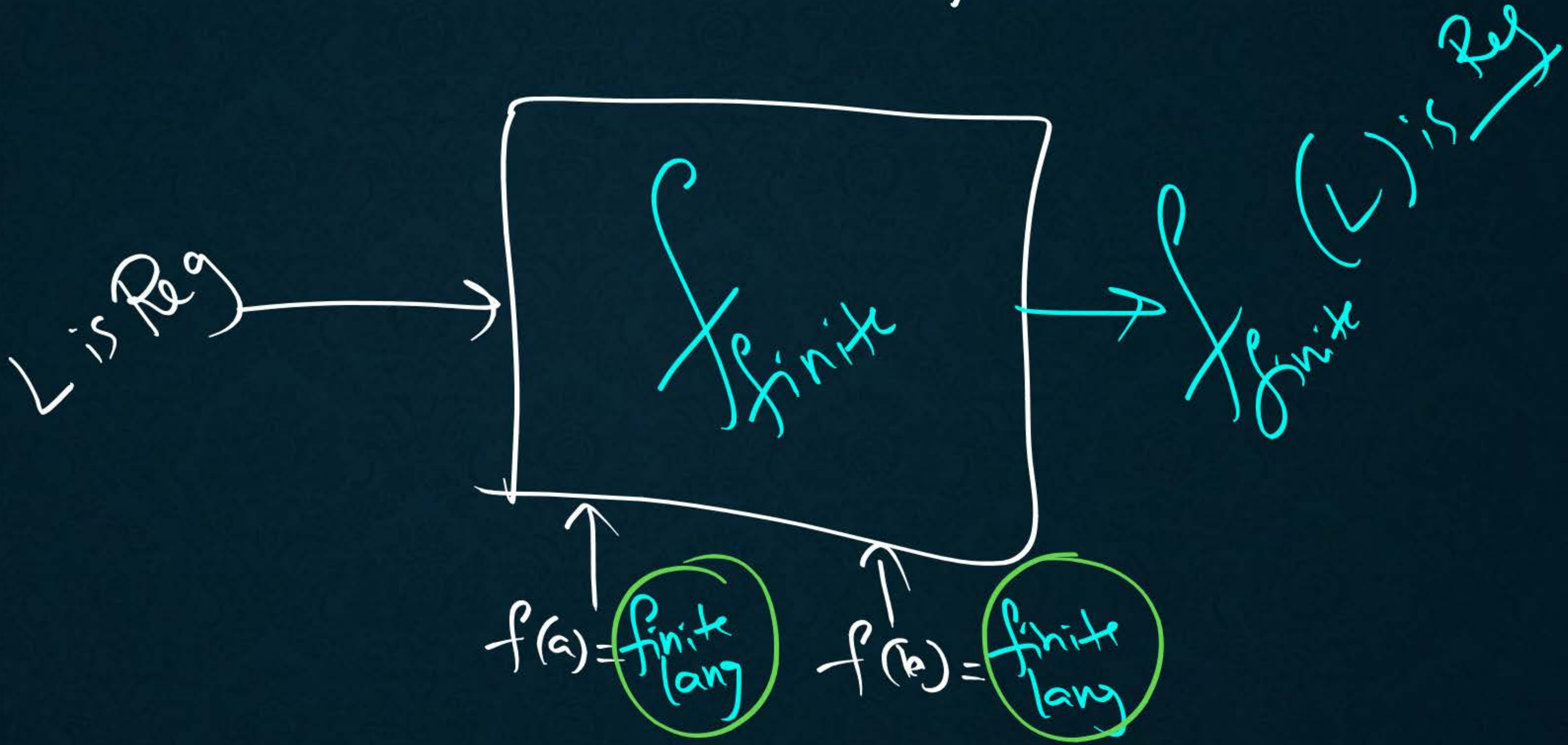
Note: Finite Subset of any language is ~~finite~~

$$L = (a+b)^*$$

$\Downarrow$ Finite Subsets

result must be finite set

(29) Finite Substution (Regular) $\Rightarrow$ Regular



Inf... not closed

(30) $R_1 \cup R_2 \cup R_3 \cup \ldots$ $\Rightarrow$ either Reg or Not reg

(31) $R_1 \cap R_2 \cap R_3 \cap \ldots$ $\Rightarrow$ "

(32) $R_1 - R_2 - R_3 - \ldots$ $\Rightarrow$ "

(33) $R_1 \cdot R_2 \cdot R_3 \cdot \ldots$ $\Rightarrow$ "

(34) Infinite subset of Regular language $\Rightarrow$ "

(35) Infinite Substitution of Regular $\Rightarrow$ "

Summary

→ closure prop ✓

Next : Revision of Regulars

FA

Reg ExP

Reg Grammar


THANK YOU
GW
SOLDIERS !