CS & IT ENGINEERING

Theory of Computation

Finite Automata

Lecture No. 18

By- DEVA Sir

TOPICS TO BE COVERED

01 Regular Grammar

02 Pumping Lemma

03

04

05

Home Work :

2nd is 'a'

(34) $S \to Sa \mid Sb \mid A$

$A \to aa \mid ba$

$L = S(a+b)^* = A.(a+b)^* = (aa+ba)(a+b)^*$

$= (a+b)a(a+b)^*$

(36) $S \to aS \mid bS \mid cS \mid \varepsilon$

$L = (a+b+c)^* \varepsilon$

$= (a+b+c)^*$

2nd sym from end is 'a'

(35) $S \to Aa \mid Ab$

$A \to Ba$

$B \to Ba \mid Bb \mid \varepsilon$

$B = (a+b)^*$

$A = (a+b)^* a$

$S = L = A(a+b)$

$= (a+b)^* a (a+b)$

(37) $S \to Aa \mid Ab \mid \varepsilon$

$A \to a \mid b \mid \varepsilon$

$A = a+b+\varepsilon$

$S = L = (a+b+\varepsilon)a + (a+b+\varepsilon)b + \varepsilon$

$= \{\varepsilon, a, b, aa, ab, ba, bb\}$

$L = \{w \mid w \in \{a,b\}^*, |w| \le 2\}$

$= (\varepsilon + a + b)^2$

(38) $S \rightarrow Sa \mid Sb \mid A$

$A \rightarrow Bab$

$B \rightarrow Ba \mid Bb \mid \epsilon$

$B = (a+b)^*$

$A = (a+b)^* ab$

$L = S.(a+b)^*$

$= A.(a+b)^* = (a+b)^* ab (a+b)^*$

(40) $S \rightarrow aA \mid bA$

$A \rightarrow aB \mid bB$

$B \rightarrow aB \mid bB \mid \epsilon$

$\boxed{\text{Atleast 2 length strings}}$

$B = (a+b)^*$

$A = (a+b).B$

$= (a+b)(a+b)^*$

$S = (a+b).A$

$= (a+b)^2 (a+b)^*$

(39) $S \rightarrow aS \mid bS \mid A$

$A \rightarrow abB$

$B \rightarrow aB \mid bB \mid \epsilon$

$B = (a+b)^*$

$A = ab(a+b)^*$

$L = (a+b)^* A$

$= (a+b)^* ab (a+b)^*$

(41) $S \rightarrow Aa \mid Ab$

$A \rightarrow Ba \mid Bb$

$B \rightarrow Ba \mid Bb \mid \epsilon$

$B = (a+b)^*$

$A = B(a+b)$

$= (a+b)^*(a+b)$

$S = A(a+b)$

$= (a+b)^*(a+b)^2$

$= (a+b)^2(a+b)^*$

(42) $S \to Sa \mid Sb \mid \varepsilon \mid A$

$A \to Aa \mid Ab \mid \varepsilon$

$$A = (a+b)^*$$

$$S = S.(a+b)^*$$

$$= (\varepsilon + A)(a+b)^*$$

$$= (\varepsilon + (a+b)^*)(a+b)^*$$

$$= (a+b)^*.(a+b)^*$$

$$= (a+b)^*$$

i) $(a+b)^*.(a+b)^*$

ii) $(a+b)^+$

iii) $(a+b)^*$

iv) $(a+b)^* + (a+b)^*$

***** 43**

$S \to Sa \mid Ab \mid c$

$A \to Sb \mid d$

$A = Sb + d \longrightarrow (1)$

$S \to Sa \mid Ab \mid c$

$S \to Sa \mid Sbb \mid db \mid c$

$L = S(a+bb)^*$

$= (db+c)(a+bb)^*$

*** (44) $S \rightarrow Saa \mid Ab$

$A \rightarrow Sb \mid Ac \mid e$

Step 1: $A \rightarrow Ac \mid \underbrace{Sb \mid e}$

$\Downarrow$

$A = (Sb + e)c^* = Sbc^* + ec^*$

Step 2: $S \rightarrow Saa \mid Ab$

$\Downarrow$

$S \rightarrow Saa \mid Sbc^*b \mid \underbrace{ec^*}b$

$L = ec^*b(aa + bc^*b)^*$ ✓

(45) $S \to Sa \mid Saa \mid \varepsilon$

$L = a^* = (a+aa)^*$

(46) $S \to Sa \mid Sb \mid a \mid b$

$L = (a+b)^+$

(47) $S \to aS \mid bS \mid \underbrace{a \mid b \mid \varepsilon}$

$L = (a+b)^* . S$

$= (a+b)^* . (a+b+\varepsilon)$

$= (a+b)^*$

(48) $S \to aS \mid bS \mid abS \mid \varepsilon$

$L = (a+b+\underline{\underline{ab}})^*$

$= (a+b)^*$

*** (49)

$S \to Sa \mid Ab \mid a$

useless

$\boxed{A \to Ab}$ useless

$\boxed{B \to Ba \mid Bb \mid \epsilon}$ unreachable from S

$\Downarrow$

$S \to Sa \mid a$

$$\boxed{L = S = a^+}$$

i) $(a+b)^*$

ii) a^*

iii) a^+

iv) None

50

$S \rightarrow Sa \mid Sb$

$L = \phi$

$= \{ \}$

Homework :

① $L = a^*$ I) LLG II) RLG	④ $(a+b)^+$	⑦ $(a+b)^*a$
② $L = a^+$ I) LLG II) RLG	⑤ ab^*	⑧ $a(a+b)^*$
③ $(a+b)^*$	⑥ a^*b	⑨ $(a+b)^*a(a+b)^*$

Pumping Lemma for Regular Languages:

- It is a Lemma
  (proof)
- It satisfies regular languages
- It can also be used to prove nonregulars using - Contradiction.

Note: This pumping lemma can't be used to check unknown language is regular or not.

Regular Language → P.L. for Regulars → It proves Regular language using systematic proof

Non Regular Language → P.L. for regulars Using contradiction → It proves non regular using systematic proof

(P.L. for non regulars)

P. L. for Regular Language: (L)

Step 1 : choose any constant $\boxed{P}$ ($P \geq$ no. of states in min DFA for L)

Step 2 : Select any string $w \in L$
Such that $|w| \geq P$

Step 3 : Divide the string w into 3 parts.
$w = xyz$; $\underbrace{|y| \geq 1}_{y \neq \epsilon}$, $|xy| \leq P$

Step 4 : $\forall_{i \geq 0}\ xy^iz \in L$ iff L is Regular

P

$w = xyz$

$x \quad y \quad z$

$y \neq \varepsilon$

$xy \leq P$

$\forall_{i \geq 0} \; xy^i z \in L$ iff L is reg

Choose p

String w

Divide

Regular

Constant P $\quad$ $P \geq$ no. of states in min DFA (ignore dead state)

$\Downarrow$

string w $\quad$ $|w| \geq P$

$\Downarrow$

3 parts xyz $\quad$ $xyz = w,\ |y| \neq 0,\ |xy| \leq P$

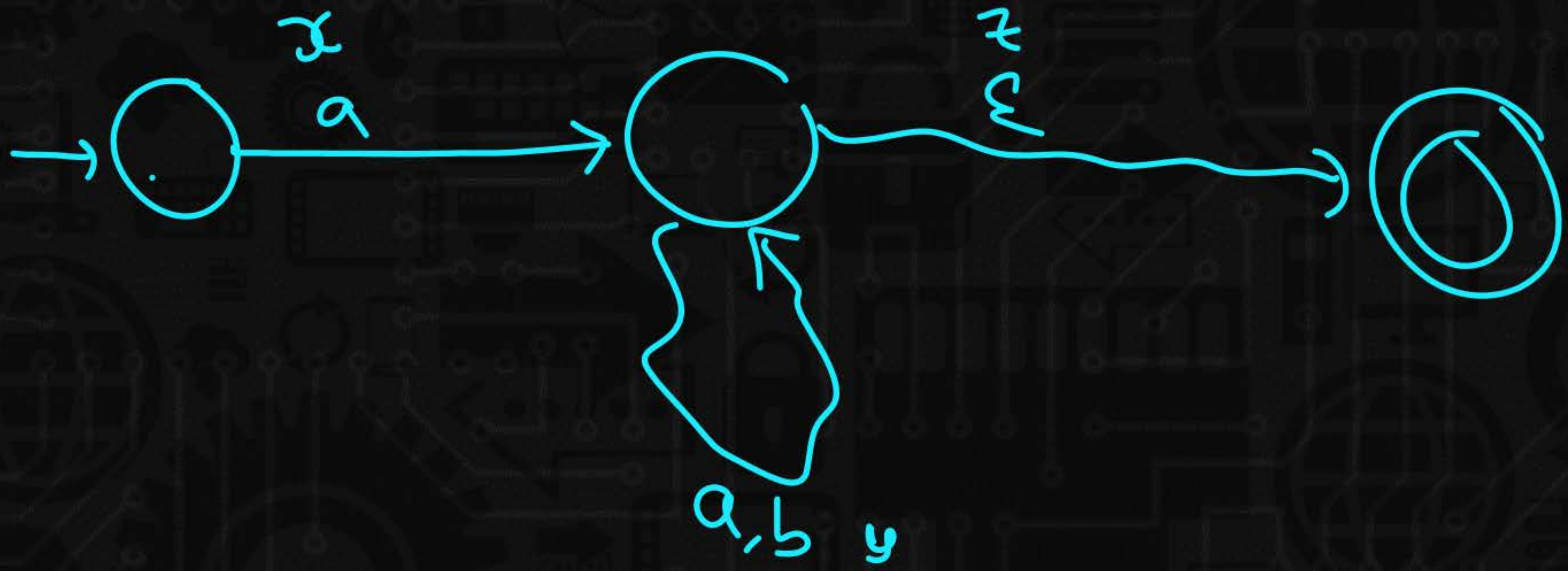
$\Downarrow$

Repeat y $\quad$ xy^iz $\quad$ $\forall i \geq 0\ xy^iz \in L$

Language is Regular
and
Infinite

$a(a+b)^*$

$\forall i \geq 0 \quad x\,\boxed{y}\,z \in L$



$L = a(a+b)^*$

P.L. proof

i) choose constant

$\boxed{P \geq \text{no. of states in min DFA}}$

$P = 3$

ii) $w = aab$

many choices to divide w into 3 parts

$P = 4$

$w = abaab$

$a \quad ba \quad ab$

$P = 5$

$w = aaaaaaab$

$|w| \geq 5$

$|w| = 8$ ✓

$a \quad aa \quad aaaab$

$y \neq \varepsilon$

$|xy| \leq P$

$P = 6$

$P \geq 3$

or

$P \geq 2$

(ignore dead)

. . . .

$P = 3$

$w = aab$
$|w| \geq p$

$w = aaba$
$|w| \geq p$

$w = aaab$
$|w| \geq p$

. . . .

Infinite options to choose w based on p.
$|w| \geq p$

Step1: $P = 3$

$\Downarrow$

Step2: $W = aab = xyz$

$a(ab)^i\varepsilon$

$i=0 \Rightarrow a \in L$

$i=1 \Rightarrow aab \in L$

$L = a(a+b)^*$

$xyz = w$

Step 3:

$x = a$
$y = a$
$z = b$

Step4: $\boxed{\forall i \geq 0 \; xy^iz \in L}$

$\forall i \geq 0 \; a(a)^ib \in L$

$x = \varepsilon$
$y = a$
$z = ab$

$\forall i \geq 0, \; \varepsilon.(a)^i ab.$
$\in L$

$x = a$
$y = ab$
$z = \varepsilon$

$a(ab)^i\varepsilon$
$\in L$

$x = \varepsilon$
$y = aab$
$z = \varepsilon$

$\varepsilon.(aab)^i\varepsilon$

$\forall i \geq 0$

This is not proper choice to divide w into 3 parts

$\forall i \geq 0 \quad a(a)^i b \in L$

$L = a(a+b)^*$

$i=0 \Rightarrow ab \in L$

$i=1 \Rightarrow aab \in L$

$i=2 \Rightarrow a a^2 b \in L$

$\vdots$

Q1) P.L. satisfies Regular Language

Q2) P.L. proves Non regular language using Contradiction

Q3) P.L. uses Pegim hole principle

Q4) For $L = (a+b)^* aaa (a+b)^*$, which of the following can't be pumping constant ?

~~A.~~ 1
~~B.~~ 2 } wrong
C. 5
D. 9

#states in min DFA = 4

$P \geq 4$

$L = (a+b)^* aaa$

Step1:

$P = 4$

$\Downarrow$ $\underline{|w| \geq 4}$ every string will have proof

$w = abaaa$

a (b) aaa

$\forall i \geq 0 \; xy^iz \in L$

Step1:

$P = 6$

$w = abbaaa$

a (b) baaa

$\boxed{\forall i \; xy^iz \in L}$

Step1:

$P = 7$

Step 1:

$P = 10$

How to prove Non-regular as Non regular ?

↳ P.L with Contradiction

L is Nonreg → Assume L is Reg → step1 → step2 → step3 → step4 → our Assumption is wrong (contradiction) → L is Non-reg

$\exists i,\ xy^iz \notin L$

P.L for Regular

P.L. for Non reg using Contradiction

Pumping Lemma proof for Non regular using P.L for reg with contradiction

Step 1: Assume L is Regular

Step 2: Choose P

Step 3: Select $w \in L$, $|w| \geq P$

Step 4: $w = xyz$, $|y| \neq \varepsilon$, $|xy| \leq P$

Step 5: $\forall i$ $xy^iz \in L$ iff L is Reg

But it fails, some i, $xy^iz \notin L$

Step 6: Contradiction. So, L is Non reg

A Assumption

C

S

D

R

C Contradiction

$L = a^n b^n$ is Non reg

Step 1: Assume L is Regular

Step 2: $P = 2n$

Step 3: $W = \underbrace{a^n b^n}_{\text{I will choose this}}$, $a^{n+1} b^{n+1}$, $a^{n+1000} b^{n+1000}$, $a^{2n} b^{2n}$, ...

Step 4: $\underbrace{a^{n-1} \; a}\; b^n$ $\quad |y| \neq 0$, $|xy| \leq P$

Step 5: $\forall i \; a^{n-1}(a)^i b^n \in L$ iff L is Reg

Step 6: Our assumption is wrong, some i, $xy^iz \notin L$

$$a^n b^n$$

$$a^{n-1} \quad a \quad b^n$$

$i=0 \Rightarrow a^{n-1}(a)^0 b^n$

$= a^{n-1} b^n \notin L$

$$a^n b^n$$

$$a^{n-1} \quad ab \quad b^{n-1}$$

$i=0 \Rightarrow a^{n-1}(ab)^0 b^{n-1}$

$a^{n-1} b^{n-1} \in L$

$i=1 \Rightarrow a^{n-1}(ab)^1 b^{n-1}$

$= a^n b^n \in L$

$i=2 \Rightarrow a^{n-1}\,\underline{ab}\,\underline{ab}\, b^{n-1} \notin L$

$$a^n b^n$$

$$a^{n-2} \quad aa \quad b^n$$

$i=0$ ✓

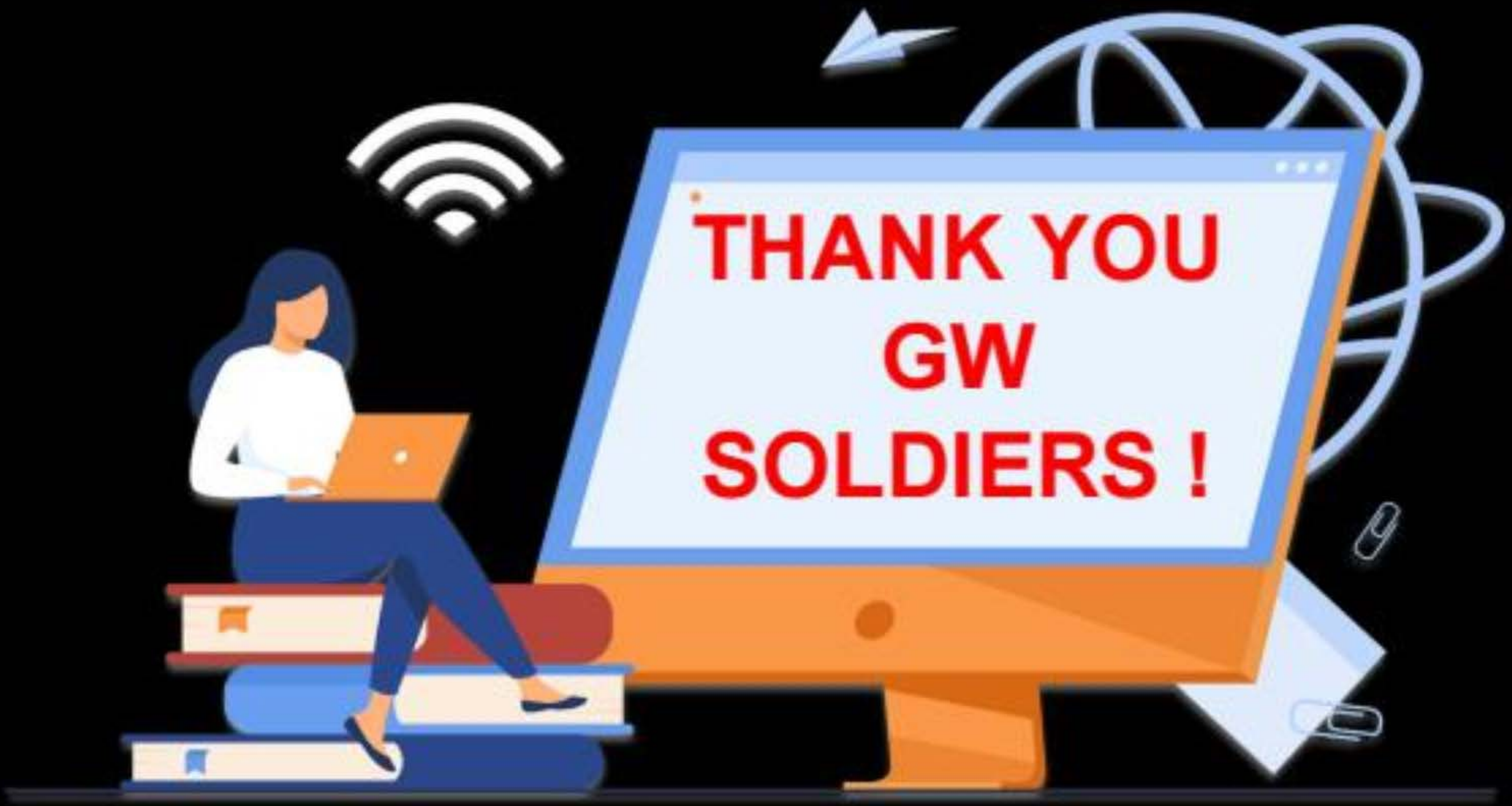
i exist

. . . . .

Summary

→ P.L. for Reg

→ P.L using contradiction for Non reg


THANK YOU
GW
SOLDIERS !