ASSIGNMENT- OPTIMIZATION

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Problem:

Find the shortest distance of the point (0,c) from the parabola $y = x^2$ where $0 \leq c \leq 5$.

Solution:

Optimization

Any conic of the form

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0$$

can be written as

$$\mathbf{x}^\top \mathbf{A}\mathbf{x} = 0$$

$$\text{where } \mathbf{A} = \begin{pmatrix} \mathbf{V} & \mathbf{u} \\ \mathbf{u}^\top & f \end{pmatrix}, \mathbf{x} = \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

The distance from point $\mathbf{p} = \begin{pmatrix} 0 \\ c \end{pmatrix}$ to the point 'x' on parabola is $\|\mathbf{x} - \mathbf{p}\|^2$

$$\implies \mathbf{x}^\top \mathbf{x} - 2\mathbf{p}^\top \mathbf{x} + \|\mathbf{p}\|^2$$

The above equation can be written as

$$\mathbf{x}^\top \mathbf{C}\mathbf{x}$$

where $\mathbf{C} = \begin{pmatrix} \mathbf{I} & -\mathbf{p} \\ -\mathbf{p}^\top & \|\mathbf{p}\|^2 \end{pmatrix}, \mathbf{x} = \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$

The shortest distance is given by,

$$\min \mathbf{x}^\top \mathbf{C}\mathbf{x}$$

such that,

$$\mathbf{x}^\top \mathbf{A}\mathbf{x} = 0$$

Using SDR(Semi Definite Relaxation), it can be rewritten as

$$\min Tr\left(\mathbf{C}\mathbf{X}\right)$$

Suc that,

$$Tr\left(\mathbf{A}\mathbf{X}\right) = 0,$$
$$\mathbf{X} \geq 0$$

Here , $\mathbf{X}$ is a 3×3 matrix of variables where

$$\mathbf{X} = \mathbf{x}\mathbf{x}^\top$$