Ig: @studyvalee___

ÍNDICE

Hojas

Ig: @studyvalee

Ponderación

• Asistencia	15
• Práctica	25
• Examen	60
total	100

VARIABLES ALEATORIAS BIVARIANTES BIDIMENSIONALES

ALEATORIAS → Probabilidad

Experimento

Aleatorios No Deterministico - Podemos saber → resultados posibles
Probabilisticos

→ Espacio muestral < Ω, S

tipos de espacio muestral

• Finito Numerable monedo hasta que salga cara
•
•
•

Experimento

sin reemplazo · No orden → Teoria combinatoria combinaciones
· Si orden → permutación

Seleccionar 3 fichas de una urna que contiene 3 fichas rojas 4 fichas verdes y 5 fichas amarillas

3 Rojas
4 Verdes
5 Amarillas

3FR
4FV
3FA
12 total

$\boxed{?}\boxed{?}\boxed{?}$

$\Omega = \{$ RRR, VVV, AAA, RVV, RRV, RAA, RRA, VVA, VAA, RA, V $\}$

Rango ó Recorrido

X: Nro fichas rojas : $R_x = \{0,1,2,3\}$
Y: Nro fichas verdes : $R_y = \{0,1,2,3\}$

* 2do paso función de probabilidad

(X, Y): V.A Discretas Bivariantes (Se puede contar)

restricciones
• $0 \leq x+y \leq 3$

Probabilidades

• $P(X=0, Y=0) = \dfrac{\binom{5}{3}}{\binom{12}{3}} = \dfrac{10}{220}$ (fichas)

$\boxed{A}\boxed{A}\boxed{A}$

Casos favorables #A
Casos posibles #Ω

• $P(X=0, Y=1) = \dfrac{\binom{\cancel{4}}{1}^{4}\binom{\cancel{5}}{2}^{10}}{\binom{12}{3}} = \dfrac{40}{220}$

$\boxed{A}\boxed{A}\boxed{V}$

• $P(X=0, Y=2) = \dfrac{\binom{\cancel{4}}{2}^{6}\binom{\cancel{5}}{1}^{5}}{\binom{12}{3}} = \dfrac{30}{220}$

$\boxed{A}\boxed{V}\boxed{V}$

$C_r^n = {}_nC_r = \binom{n}{r}$

$= \dfrac{n!}{(n-r)! \cdot r!}$; $n! = (n)(n-1)(n-2)\cdots 3\cdot 2\cdot 1$

$0! = 1$

Funcion de cuantia

✓ Ecuacion

$$P(X=x, Y=y) \; \frac{\binom{3}{x}\binom{4}{y}\binom{5}{3-x-y}}{\binom{12}{3}} ;$$

Funcion de cuantia → Para discreta

Función de probabilidad

- $0 \leq x+y \leq 3$

$x = \{0,1,2,3\}$

$y = \{0,1,2,3\}$

✓ Tabla

x \ y	0	1	2	3
0	$\frac{10}{220}$	$\frac{40}{220}$	$\frac{30}{220}$	$\frac{4}{220}$
1	$\frac{30}{220}$	$\frac{60}{220}$	$\frac{18}{220}$	–
2	$\frac{15}{220}$		–	–
3		–	–	–

- $P(x=0\ y=) =$

$x=1\ y=0$ $\quad \frac{\binom{3}{1}\binom{4}{0}\binom{5}{2}}{} = \frac{30}{220}$ (3, 1, 10)

$x=1\ y=1$ $\quad \binom{3}{1}\binom{4}{1}\binom{5}{1}$

Propiedades

i) $P(x,y) \geq 0$; $\forall (x,y) \in$ ✓

ii) $\sum_y \sum_x P(x,y) = 1$ ✓

iii) $P((x,y) \subset R_{xy}) = \sum\limits_{(x,y)\subset R_{xy}} \sum P(x,y)$

Hallar

a) Prob. exactamente 1 verde

b) " almenos 2 rojas

c) " 2 amarillas 1 verde

d) " a lo mas 2 verdes

x \ y	0	1	2	3
0	$\frac{10}{220}$	$\frac{40}{220}$	$\frac{30}{220}$	$\frac{4}{220}$
1	$\frac{30}{220}$	$\frac{60}{220}$	$\frac{18}{220}$	–
2	$\frac{15}{220}$	$\frac{12}{220}$	–	–
3	$\frac{1}{220}$	–	–	–

Sol a):

$P(Y=1 \;\; {}^{x=0,1,2}_{\text{or } = 2,1}) = \frac{112}{220}$

Sol b)

$P(x \geq 2, y = ?) = \frac{28}{220}$

Sol c)

$P(x=0, y=1) = \frac{40}{220}$

Sol d)

$P(x : y \leq 2) = 1 - P(A^c)$

$1 - \frac{4}{220} = \frac{216}{220}$

Ig: estudyvalee

Si A es un evento $P(A) = 1 - P(A^c)$; $P(A^c)$ = complemento

Distribuciones Marginales

$$P(X = x) = P_x(x) = \sum_{Ry} P(x, y) =$$

$$\frac{\binom{3}{x}\binom{9}{x}}{\binom{12}{3}}$$

x \ y	0	1	2	3	P(x)
0	$\frac{10}{220}$	$\frac{40}{220}$	$\frac{30}{220}$	$\frac{4}{220}$	$\frac{84}{220}$
1	$\frac{30}{220}$	$\frac{60}{220}$	$\frac{18}{220}$	–	$\frac{108}{220}$
2	$\frac{15}{220}$	$\frac{12}{220}$	–	–	$\frac{27}{220}$
3	$\frac{1}{220}$	–	–	–	$\frac{1}{220}$
P(y) marginal y	$\frac{56}{220}$	$\frac{112}{220}$	$\frac{48}{220}$	$\frac{4}{220}$	1

$$P(Y = y) = P_y(y) = \sum_{Rx} P(x,y) =$$

$$\frac{\binom{4}{y}\binom{8}{3-y}}{\binom{12}{3}}$$

x	P(x)	$x \cdot P(x)$	$x^2 P(x)$
0	$\frac{84}{220}$	$0 \cdot \frac{84}{220}$	$0^2 \cdot \frac{84}{220}$
1	$\frac{108}{220}$	$1 \cdot \frac{108}{220}$	$1^2 \cdot \frac{108}{220}$
2	$\frac{27}{220}$	$2 \cdot \frac{27}{220}$	$2^2 \cdot \frac{27}{220}$
3	$\frac{1}{220}$	$3 \cdot \frac{1}{220}$	$3^2 \frac{1}{220}$

y	P(y)	y P(y)	$(y - \mu_y)^2 \cdot P(y)$
0	$\frac{56}{220}$	$0 \cdot \frac{56}{220}$	$(0 - \bullet)^2 \cdot \frac{56}{220}$
1	$\frac{112}{220}$	$1 \cdot \frac{112}{220}$	$(1 - \bullet)^2 \cdot \frac{112}{220}$
2	$\frac{48}{220}$	$2 \cdot \frac{48}{220}$	$(2 - \bullet)^2 \cdot \frac{48}{220}$
3	$\frac{4}{220}$	$3 \cdot \frac{4}{220}$	$(3 - \bullet)^2 \cdot \frac{4}{220}$

Esperanza

$$\mu_x = E(x) = \sum_{Rx}^{n} (x \cdot P(x))$$

$$\mu_x = E(y) = \sum_{Ry} (y P(y)) = \bullet$$

Varianza

$$\sigma_x^2 = V[x] = E[(x - \mu_x)^2] = \begin{cases} \sum_{Ry} (x - \mu_x)^2 P(x) \\ E[x^2] - (\mu_x)^2 \end{cases}$$

Desv. Estandar

$$\sigma_x = \sqrt{V[x]}$$

Covarianza

Relacion entre las 2 variables

$$Cov(X,Y) = S_{xy} = \begin{cases} >0 \;\; ¡\exists! & \text{relacion directa [positiva]} : X\uparrow Y\uparrow \\ & : X\downarrow Y\downarrow \\ =0 & \\ <0 & \text{inversa [negativa]} : X\uparrow Y\downarrow \\ & : X\downarrow Y\uparrow \end{cases}$$

$$S_{xy} = E[(X-\mu_x)(Y-\mu_y)] \begin{cases} \sum_y \sum_x (x-\mu_x)(y-\mu_y) \cdot P(x,y) \\ E[X \cdot Y] - (\mu_x \mu_y) \end{cases}$$

$$E[X \cdot Y] = \sum_x \sum_y xy \cdot p(x,y) = \frac{120}{220} \qquad E\left[\frac{X^3}{Y}\right] = \sum_y \sum_x \frac{x^3}{y} \cdot P(x,y)$$

x \ y	0	1	2	3	P(x)
0	$\frac{10}{220}$	$\frac{40}{220}$	$\frac{30}{220}$	$\frac{4}{220}$	$\frac{84}{220}$
1	$\frac{30}{220}$	$\frac{60}{220}$	$\frac{18}{220}$	–	$\frac{108}{220}$
2	$\frac{15}{220}$	$\frac{12}{220}$	–	–	$\frac{27}{220}$
3	$\frac{1}{220}$	–	–	–	$\frac{1}{220}$
P(y) marginal y	$\frac{56}{220}$	$\frac{112}{220}$	$\frac{48}{220}$	$\frac{4}{220}$	1

x·y P(x y)

	0	1	2	3	$\sum_x$ x y P(xy)
0	–	–	–	–	0
1	–	$\frac{60}{220}$	$\frac{36}{220}$	–	$\frac{96}{220}$
2	–	$\frac{24}{220}$	–	–	$\frac{24}{220}$
3	–	–	–	–	
$\sum_y$ x y P(xy)					$\frac{120}{220}$

$$\sum_y \sum_x x \cdot y \, P(x,y)$$

Coeficiente de correlacion Pearson

$$\rho_{xy} = \frac{Cov(X,Y)}{\sigma_x \cdot \sigma_y}$$

INDEPENDENCIA

Sea (x,y) V.A conjuntas (bivariantes) con funcion de cuantia $P(x,y) = P(x=x, Y=y) = P_{xy}(x,y)$ donde X e Y son discretas, entonces x,y son independientes si solo si

$$\boxed{P(x=x, y=y) = P_x(X=x) \; P(y=y)}$$

$x \backslash y$	0	1	2	3	(P_x)
0	$\frac{10}{220}$	$\frac{40}{220}$	$\frac{30}{220}$	$\frac{4}{220}$	$\frac{84}{220}$
1	$\frac{30}{220}$	$\frac{60}{220}$	$\frac{18}{220}$		$\frac{108}{220}$
2	$\frac{15}{220}$	$\frac{12}{220}$			$\frac{27}{220}$
3	$\frac{1}{220}$				$\frac{1}{220}$
$P(Y)$	$\frac{56}{220}$	$\frac{112}{220}$	$\frac{48}{220}$	$\frac{4}{220}$	

$P_y(Y) = P(Y=y)$

$$\sum_y \sum_x P(x,y) = \sum_x \sum_y P(x,y) = \sum_{Rx} P(Y) = \sum_{Ry} P(x) = 1$$

* $P(x,y)$ Si es una funcion de probabilidad

★ basta que en 1 casilla no se cumpla

$$P(x=1, y=2) = P(x=1)\cdot P(Y=2)$$

$$\frac{18}{220} \neq \frac{108}{220}\cdot\frac{48}{220}$$

∴ Entonces x, y no son independientes, son dependientes

V.A bivariantes continua

Sea (x,y) variables Aleatorias continuas con funcion probabilidad (densidad)

•
$$f(x,y) = \begin{cases} k\cdot x^2\cdot y & : 0 \leq x^2 \leq y \leq 1 \\ 0 & e o c \end{cases}$$

a) $k = ?$

recurro a PROPIEDADES

i) $f(x,y) \geq 0 \quad \forall (x,y) \in R_{xy}$ ✓

ii) $\int_{R_{xy}}\int f(x,y)\, dx\, dy = 1$

iii) Sea $A \subset R_{xy}$ entonces

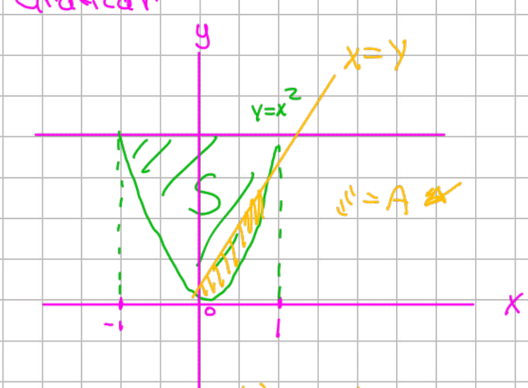
$$\iint_{R_A} f(x,y)\, dx\, dy = P(A)$$

$$\sum_Y \sum_X \underbrace{P(x+y)}_{1} = 1$$

$P(x,y) = c(x+y)$

∴ Si no se cumple No es funcion de probabilidad

Graficar



b) cuanto vale esa area

$$1 = \int_{R_{xy}}\int f(x,y)\, dx\, dy = \int_{-1}^{1}\int_{x^2}^{1} kx^2 y\, dy\, dx$$

$$1 = k\int_{-1}^{1} x^2 \int_{x^2}^{1} y\, dy\, dx = k\int_{-1}^{1} x^2 \left[\frac{y^2}{2}\right]_{x^2}^{1} dx$$

$$1 = k\int_{-1}^{1} x^2\left[\frac{1}{2} - \frac{x^4}{2}\right] dx = \frac{k}{2}\int (x^2 - x^6)\, dx$$

$$= \frac{k}{2}\left[\frac{x^3}{3} - \frac{x^7}{7}\right]_{-1}^{1} = \frac{k}{2}\left[\frac{1}{3} - \frac{1}{7} + \frac{1}{3} - \frac{1}{7}\right]$$

$$= \frac{k}{2}\left[\frac{2}{3} - \frac{2}{7}\right] = k\left[\frac{1}{3} - \frac{1}{7}\right] = k\left[\frac{7-3}{21}\right] = \frac{4k}{21}$$

$1 = \frac{4k}{21}$ $\boxed{K = \frac{21}{4}}$

$f(x,y) \begin{cases} 0 \le \frac{21}{4} x^2 \cdot y \le 1 \\ 0, \quad e\,o\,l \end{cases}$

b) $P(x \geq y) = ?$

$x = y$

$$P(x \ge y) = \int_0^1 \int_{x^2}^{x} \frac{21}{4} x^2 y \; dy\, dx$$

$$= \frac{21}{4} \int_0^1 x^2 \int_{x^2}^{x} y \, dy\, dx$$

$$= \frac{21}{4} \int_0^1 x^2 \int x^2 \left[\frac{y^2}{2}\right]_{x^2}^{x}$$

$$= \frac{21}{4} \int_0^1 x^2 \left[\frac{x^2}{2} - \frac{x^4}{2}\right] dx$$

$$= \frac{21}{4} \int_0^1 \left[\frac{x^5}{10} - \frac{x^7}{14}\right]_0^1$$

$$= \frac{21}{4}\left[\frac{1}{10} - \frac{1}{14}\right]$$

$$\frac{21}{4}\left[\frac{14-10}{140}\right]$$

$$= \frac{21}{\cancel{4}}\left[\frac{\cancel{4}}{140}\right]$$

$$= \frac{21}{140}$$

c) $P(0.5 \le Y \le 0.75)$ otro camino es encontrar la distribucion marginal de Y o funcion prob de Y

$\boxed{f_y(y) = \int_{R_x} f(x,y)\, dx}$ ↳ de la otra var.

$\boxed{f_x(x) = \int_{R_y} f(x,y)\, dy}$

marginal de y

$$f_y(y) = f_y(Y) = \int_{-\sqrt{y}}^{\sqrt{y}} \frac{21}{4} x^2 y \; dx$$

$$\frac{21}{4} y \left[\frac{x^3}{3}\right]_{-\sqrt{y}}^{\sqrt{y}}$$

$$\frac{21}{4} y \left[\frac{y^{3/2}}{3} + \frac{y^{3/2}}{3}\right]$$

$$\boxed{f_y(Y) = \begin{cases} \frac{7}{2} y^{\frac{5}{2}} ; & 0 \le y \le 1 \\ 0 ; & \text{en otro caso} \end{cases}}$$

$\rightarrow F_y(Y) = P(Y \le y) = \int_{-\infty}^{y} f(t)\,dt$

func. dist. acumulativa

★ Hay 2 caminos

$$P(0.5 \le y \le 0.75) = \int_{0.5}^{0.75} \frac{7}{2} y^{5/2}\, dy$$

$$= \frac{7}{2}\left[\frac{y^{5/2+1}}{\frac{5}{2}+1}\right]_{0.5}^{0.75}$$

$$= \frac{\cancel{7}}{2}\left[\frac{y^{7/2}}{\frac{\cancel{7}}{2}}\right]_{0.5}^{0.75} = 0.75^{7/2} - (0.5)^{7/2}$$

Ig: @studyvalee

Valor esperado de X (Media)

$$\boxed{\mu_x = E[X] = \int_{R_X} x \, f(x)\, dx}$$

• marginal de x

\# Poner ec de marginal de x

• $f_X(x) = \int_{x^2}^{1} \frac{21}{4} x^2 y \, dy$

$\frac{21}{4} x^2 \left[\frac{y^2}{2}\right]_{x^2}^{1}$

$\frac{21}{4} x^2 \left[\frac{1}{2} - \frac{x^4}{2}\right]$

$\frac{21}{8} x^2 \left(1 - x^4\right)$

$$f_X(x) \begin{cases} \frac{21}{8} x^2 (1-x^4) & -1 \le x \le 1 \end{cases}$$

• $\mu_x = E[X] = \int_{-1}^{1} x \, \frac{21}{8} x^2 (1-x^4)\, dx$

$= \frac{21}{8} \left[\frac{x^4}{4} - \frac{x^8}{8}\right]_{-1}^{1}$

$\frac{21}{8}\left[\cancel{\frac{1}{4}} - \cancel{\frac{1}{8}} - \cancel{\frac{1}{4}} + \cancel{\frac{1}{8}}\right]$

0

Varianza

$$\sigma_x^2 = V[X] = E[(X-\mu_x)^2] = \begin{cases} \int_{R_X} (x-\mu_x)^2 f(x)\, dx \\ E[X^2] - (\mu_x)^2 \end{cases}$$

• $E[X^2] = \int_{R_X} x^2 f(x)\, dx \quad \{ \quad E[X^2] = \int y^2 f(y)\, dy$

$E[X^2] = \int_{-1}^{1} x^2 \, \frac{21}{8} x^2 (1-x^4)\, dx$

$= \frac{21}{8} \left[\frac{x^5}{5} - \frac{x^9}{9}\right]_{-1}^{1}$

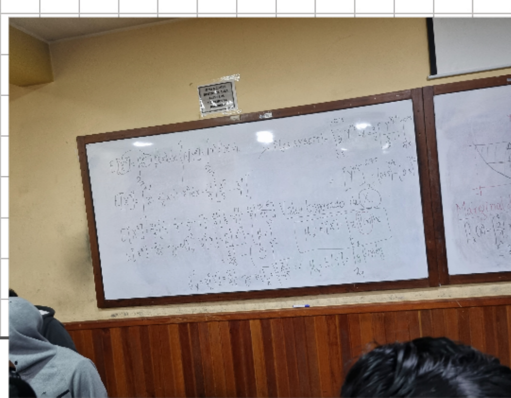
$= \frac{21}{8} \left[\frac{1}{5} - \frac{1}{9} + \frac{1}{5} - \frac{1}{9}\right]$

$\frac{21}{\cancel{8}_4}\left[\frac{2}{5} - \frac{\cancel{2}}{9}\right]$

$\frac{21}{4}\left[\frac{9-5}{45}\right]$

$E[X^2] = \frac{21}{45}$

$\sigma_x^2 = V[X] = E[X^2] - (\mu_x)^2$



Independencia

Sea (X,Y) dos v.a.c. con función de densidad (masa) f(x,y), entonces X e Y son independientes si y solo si

$$\boxed{f(x,y) = f(x)\, f(y)}$$

*

$$\cancel{\frac{21}{4}}x^2 y = \cancel{\frac{21}{8}}\,\cancel{x^2}\,(1-x^4)\cdot\frac{7}{2}\,y^{5/2}$$

$$y \neq \frac{1}{2}(1-x^4)\cdot\frac{7}{2}\,y^{5/2}$$

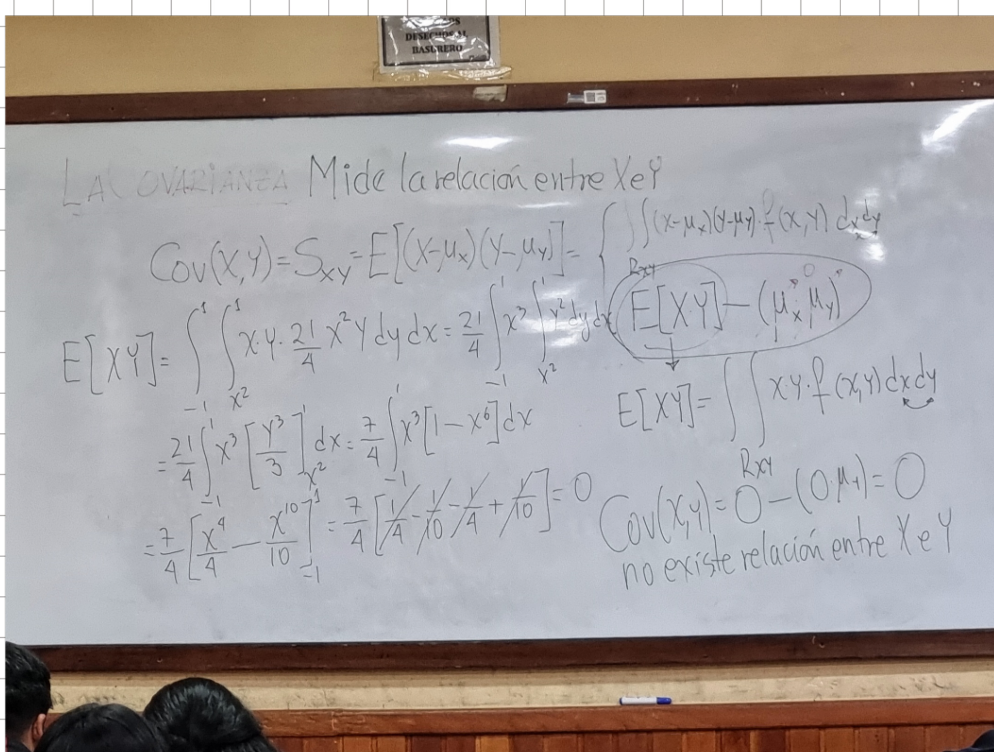
∴ X e Y no son independientes

La covarianza

Mide la relación entre X y Y

$$Cov(X,Y) = S_{xy} = E[(X-\mu_x)(Y-\mu_y)] = \begin{cases} \iint_{R_{xy}} (x-\mu_x)(y-\mu_y)\, f(x,y)\,dx\,dy \\ E[X\cdot Y] - (\mu_x \mu_y) \end{cases}$$

$$E[X\cdot Y] = \iint_{R_{xy}} x\cdot y\cdot f(x,y)\,dx\,dy$$



Coeficiente de correlación de Pearson

$$\rho_{xy} = \rho = \mathrm{Corr}(x,y) = \frac{\mathrm{Cov}(x,y)}{\sigma_x \cdot \sigma_y}$$

$$\frac{\cancel{\mathrm{Cov}(x,y)}^{\,0}}{\sqrt{V[x]\cdot V[y]}}$$

$$\left[-1 \leq \rho_{xy} \leq 1\right]$$

$\rho_{xy} = 0$ ∴ No existe correlacion lineal entre x y Y

iii) $V[aX \pm bY] = a^2 V[X] \pm b^2 V[Y] \pm 2ab\,\mathrm{Cov}(x,y)$
$a, b \in \mathbb{R}$

iv) $\mathrm{Cov}(a_1 \pm b_1 X, a_2 \pm b_2 Y) = b_1 \cdot b_2\, \mathrm{Cov}(x,y)$
$a_1 a_2 b_1 b_2 \in \mathbb{R}$

v) $\rho = \mathrm{Corr}(a_1 \pm b_1 X, a_2 \pm b_2 Y) = \rho_{xy}$

- $E[a] = a$ • $V[a] = 0$ $V[aX] = a^2 V[X]$
- $E[a \pm bX] = a \pm b\, E[X]$
- $V[a \pm bX] = b^2 V[X]$
  $E[X \pm Y] = E[X] \pm E[Y]$

Propiedades

i) $V[X \pm Y] = V[X] \pm V[Y] \pm 2\,\mathrm{cov}(X,Y)$

Ej

$\sigma_x = 5$ $\sigma_y = 10$ $\rho_{xy} = 0.85$

σ_x^2 σ_y^2

$25 + 100 + 2\ \boxed{?}$

Hallamos Cov

$\rho_{xy} = \frac{\mathrm{Cov}(x,y)}{\sigma_x\ \sigma_y} \Rightarrow \mathrm{Cov}(xy) = \rho_{xy} \cdot \sigma_x \sigma_y$

$0.85 \cdot 5 \cdot 10$

ii) Si X e Y son independientes entonces;

$\cancel{\Leftarrow}\!\Rightarrow \mathrm{Cov}(X,y) = 0$

∴ q la cov sea 0 no implica q x,y son independientes

lg: estudyvalee

DISTRIBUCIONES

muestrales

Summer

Distribuciones

M • U • E • S • T • R • A • L • E • S

X Variable aleatoria
N tamaño de la poblacion [Finita o infinita]

$\left.\begin{matrix}\mu \\ \sigma^2 \\ \pi : p \\ \theta\end{matrix}\right\}$ Parametro cualquier medida resumen q viene con todos los tatos de la poblacion

muestra

$\hat{\mu} = \bar{X}$ promedio de la muestra
$\hat{\sigma}^2 = s^2$
$\hat{\pi} = \hat{p} = \frac{a}{n}$

Estadistica estadigrafo

$$\boxed{\hat{\theta} = h(X_1, X_2, \ldots, X_n)}$$

Muestra aleatoria

Sea X una V.A. con funcion de probabilidad $f(x)$ y media μ, varianza σ^2. Si $X_1, X_2, \ldots, X_n$ elementos seleccionados al azar de X son una muestra aleatoria (m.a) Si cumple prop.

Propiedades

i) $X_1, X_2, \ldots, X_n$ son independientes

ii) $X_1, X_2, \ldots, X_n$ estan identicamente distribuidos

iii) son ii independientes e identicamente dist.

Entonces

i) $\mu_{X_i} = E[X_i] = \mu = E[X]$ Promedio

ii) $\sigma^2_{X_i} = V[X_i] = \sigma^2 = V[X]$ Varianza

iii) La función de probabilidad conjunta

$$f(x_1, x_2, \dots, x_n) = f(x_1) \cdot f(x_2) \cdots f(x_n) = \prod_{i=1}^{n} f(x_i)$$

X e Y v.a con f.p.c.
$f(x,y) = \begin{cases} e^{-(x+y)} \\ 0 \text{ eoc} \end{cases}$
Anterior cap

- Ejemplo

Peso de los est

Sea X una v.a. con distribución normal de media 65 k y varianza 4 k^2

Se toma una muestra aleatoria (m.a) $X_1, X_2, \dots, X_{20}$ de tamaño 20 est.

Calcular: Probabilidad de:

a) $P(X_3 - X_5 + X_9 + X_{15} - X_{19} + X_{20} \leq 125)$

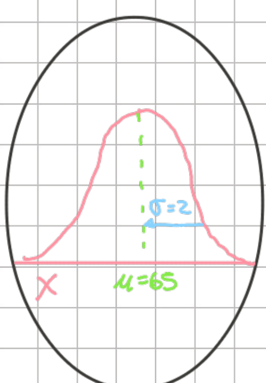
b) $P(X_{10} \geq 60)$

c) Los pesos de los est no superen la capacidad del ascensor de 1350 k

d) Hallar la func. de prob conjunta

- Sol a

X: Pesos de est (kilos)



Normal

$$f(x) = \frac{e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}{\sqrt{2\pi r^2}} \quad ; x \in \mathbb{R}$$

binomial

$X \sim b(n, p)$

$P(x) = \binom{n}{x} p^x \underbrace{(1-p)}_{q}^{n-x}$

$n = 20 \{X_1, X_2, \dots, X_{20}\}$

$X_1 \sim N(\mu_1 = \mu = 65, \sigma_1^2 = \sigma^2 = 4)$

$X_2 \sim N(\mu_2 = \mu = 65, \sigma_2^2 = \sigma^2 = 4)$

⋮

$X_{20} \sim N(\mu_{20} = \mu = 65, \sigma_{20}^2 = \sigma^2 = 4)$

- $S = \overset{N}{X_3} - \overset{N}{X_5} + \overset{N}{X_9} + \overset{N}{X_{15}} - \overset{N}{X_{19}} + \overset{N}{X_{20}}$

1. ¿Qué dist tiene la prob.

$$S \sim N(\mu_s = 130, \sigma_s^2 = 24 \quad)$$

2. Hallar media y varianza

- $\mu_s = E[S] = E[X_3 - X_5 + X_9 + X_{15} - X_{19} + X_{20}]$
$= E[X_3] - E[X_5] + E[X_9] + E[X_{15}] - E[X_{19}] + E[X_{20}]$

Ig: estudyvalee_

$$= \cancel{\mu} - \cancel{\mu} + \cancel{\mu} + \mu - \cancel{\mu} + \mu$$

$$\mu_S = E[S] = 2\mu = 2 \cdot 65 = 130$$

- $$\sigma_S^2 = V[S] = V[X_3 + X_5 + X_9 + X_{15} - X_{19} + X_{20}]$$

* Nunca es −, siempre es +

$$= V[X_3] + V[X_5] + V[X_9] + V[X_{15}] + V[X_{19}] + V[20]$$

$$\sigma^2 + \sigma^2 + \sigma^2 + \sigma^2 + \sigma^2 + \sigma^2$$

$$= 6\sigma^2 = 6 \cdot 4 = 24$$

3. Ahora si hallamos la prob (a))

$$P(X_3 - X_5 + X_9 + X_{15} - X_{19} + X_{20} \leq 125) = P(S \leq 125)$$

$$S \sim N(\mu_S = 130, \sigma^2 = 24)$$

4. Estandarizamos normalizamos

Siempre

$$Z = \frac{S - \mu_S}{\sigma_S} \sim N(0,1)$$

5. eso permite usar la "tabla Z" / Hallar resultado

En la tabla...

$$= P\left(Z \leq \frac{125 - 130}{\sqrt{24}}\right)$$

z	...	02
:		
:		
-1.0		

$$= P(Z \leq -1{,}02)$$

$$= \underbrace{GF\ \phi_Z N}_{\text{tabla Z}}(-1{,}02) =$$

6. Celebrar! ☺

Ig: estudyvalee

- Sol b)

$$X_{10} \sim N(\mu_{10} = \mu = 65,\ \sigma_{10}^2 = \sigma^2 = 4)$$

Estandarizamos

$$Z = \frac{X_{10} - \mu_{10}}{\sigma_{10}} \sim N(0,1)$$

tabla Z

$$P(X_{10} \geq 60) = P\left(Z \geq \frac{60-65}{2}\right)$$

$$= P(Z \geq -2{,}50) = 1 - N_Z(-2{,}50)$$

$$= 1 - \bigcirc$$

$$=$$

En la tabla

Z	0.00
⋮	⋮
-2,5	◯

- Sol c)

$$X_1 + X_2 + X_3 + \dots + X_{20} = T$$

$$P(T \leq 1350)$$

Hallar Dis

$$T \sim N(\mu_T = 1300,\ \sigma^2 = 80)$$

Hallar media y var

tienen a ser ind. si no viene con cov

- $\mu_T = E[T] = E\left[\sum_{i=1}^{20} X_i\right] = \sum_{i=1}^{20} E[X_i]$

$\mu_T = \sum_{i=1}^{20} \mu = 20\mu = 20 \cdot 65 = 1300$

- $\sigma^2 = V[T] = V\left[\sum_{i=1}^{20} X_i\right] = \sum_{i=1}^{20} V[\cancel{X_i}] = \sum_{i=1}^{20} \sigma^2 =$

$V[X]$

$= 20\sigma^2 = 20 \cdot 4 = 80$

Son casi iguales
+ vas bien

Esta... tran... -

$$P(T \leq 1350) = P\left(Z \leq \frac{1350-1300}{\sqrt{80}}\right)$$

$$= P(Z \leq 5.59)$$

$$= 1$$

Si es + y supera 3 o 4 = 1
Si es - = 0

• Sol d)

$$f(X_1, X_2, X_3 \cdots X_{20}) = ?$$

$$f(X_1, X_2, X_3 \cdots X_{20}) = f(x_1) \cdot f(x_2) \cdot f(x_3) \cdots f(x_{20})$$

$$f(X_1, X_2 \cdots X_{20}) = \frac{e^{-\frac{1}{2}\left(\frac{X_1-65}{2}\right)^2}}{\sqrt{8\pi}} \cdot \frac{e^{-\frac{1}{2}\left(\frac{X_2-65}{2}\right)^2}}{\sqrt{8\pi}} \cdots \frac{e^{-\frac{1}{2}\left(\frac{X_{20}-65}{2}\right)^2}}{\sqrt{8\pi}}$$

Simplificamos...

$$\frac{e^{-\frac{1}{2}\sum_{i=1}^{20}\left(\frac{X_i-65}{2}\right)^2}}{(8\pi)^{10}}$$

Distribucion de la media muestral

$$\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n}$$

Ej

X: Edad de los hijos

$X_1 = 5$
$X_2 = 6$
$X_3 = 7$
$X_4 = 7$
$X_5 = 8$

$N = 5$

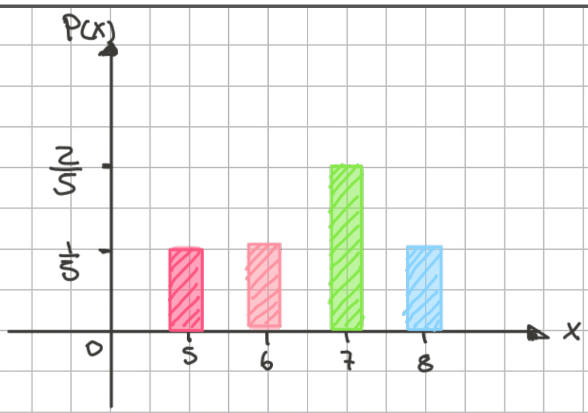
• $\mu = \frac{5+6+7+7+8}{5} = \frac{\sum_{i=1}^{N} X_i}{N}$

$\mu = 6.6$

• $\sigma^2 = \frac{\sum_{i=1}^{N}(X_i-\mu)^2}{N} = \frac{(5-6.6)^2+(6-6.6)^2+(7-6.6)^2+(7-6.6)^2+(8-6.6)^2}{5}$

$\sigma^2 = 1.04$

Ig: @studyvalee



Tomamos muestra

$N=5 \longrightarrow n=2$ S/ reemplazo

(X_1, X_2)

$m = \binom{N}{n} = \binom{5}{2}$ 10 maneras de tomar muestras

tabla funcion de probabilidades

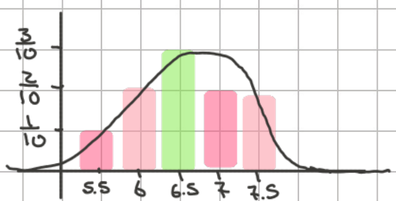
= Promedios

5.5	6	6	6.5
(5,6)	(5,6)	(5,7)	(5,8)
6.5	6.5	7	7
(6,7)	(6,7)	(6,8)	(7,7)
7.5	7.5		
(7,8)	(7,8)		

$\bar{X}$	$P(\bar{X})$	$x(Px)$	$x^2(Px)$
5.5	$\frac{1}{10}$		
6	$\frac{2}{10}$		
6.5	$\frac{3}{10}$		
7	$\frac{2}{10}$		
7.5	$\frac{2}{10}$		
	1		

Podemos graficar

media y var.



Ig: @studyvalee

GRACIAS POR DESCARGAR

Sígueme en mis redes sociales:

Tiktok: @studyvalee

Instagram: @studyvalee__