Survey of Price Elasticities from Economic Exploration Models of US Oil and Gas Supply

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ABSTRACT: Exploration for oil or gas reserves consists of searching for and finding new reserves. It begins with the study of the geology of an area followed by exploratory or wildcat drilling in promising areas. How much oil or gas is found or the supply of new reserves is a function of exploration, the geology of the area drilled along with a random component measuring the fickleness of mother nature in yielding up her treasures to mere mortals. Knowing how oil and gas prices affect this process (price elasticities) is valuable to those who are involved with this strategic industry. Economic theory suggests that the search for petroleum is affected by prices and can be characterized by functions representing geophysical activities and drilling while the finding of reserves can be characterized by some sort of discovery process. The earliest econometric approach to modeling oil and gas exploration, which yielded price elasticities, was to estimate the search process using a function for wildcat wells drilled (Ww). For example, the discovery process for oil was represented using two equations: a success rate equation % of wells that are commercially successful (W/Ww) and an average oil reserve per successful well equation (O/W). Each of these three dependent variables were dependent on price and other relevant variables. The product of these three variables (Ww*W/Ww*O/W) yields the supply of oil reserves (O). In this paper we survey all U.S. exploration models for oil and gas that include wells drilled, share of successful wells, average reserves per well, or total reserve equations. We focus our survey on price elasticities to capture the effect of market price on the exploration process. Drilling equation results tend to be good with drilling strongly influenced by oil price and we suspect the long run drilling oil price elasticity to be greated than 1. It is much less clear how natural gas prices have affected drilling but their effect seems to be increasing. The most important issue in drilling equations besides including the correct eco-

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nomic variables is more work to clarify what geological variables provide the best forecasts. The econometric estimates for successful wells and average reserves per wells are much poorer and we would recommend more systematic work on discovery models to determine which perform the best. *JEL Category Q41*

I. INTRODUCTION

Oil and gas currently generate about 3–5% of US income. However, U.S. crude oil production fell 16% between 1990 and 1997 while oil reserves fell even faster. At the same time natural gas production rose 8% while gas reserves remained fairly constant.[1] Given the importance of this strategic but declining industry, there have been numerous attempts to econometrically model and quantify how oil and gas prices affect oil and gas exploration. In this paper we briefly discuss the oil and gas exploration process in Section II. The theoretical model for a profit maximizing oil explorer is presented in Section IV and we use this model as a benchmark for our survey of econometric oil and gas exploration papers that follow. In Section III, we present a brief history of the U.S. oil industry to be able to put the models into historical context. In the survey we focus on price elasticities and structure our survey around the earliest econometric model, Fisher (1964), which is discussed in Section V. We include econometric papers that contain equations for wells drilled, success rates for wells, reserves per successful well, or total reserves along with oil and gas price elasticities. We contrast the Fisher approach with other approaches for the oil market in Section VI, summarize exploration models for natural gas in Section VII and present our conclusions and suggestions for further work in Section VIII.

II. OIL AND GAS EXPLORATION PROCESS

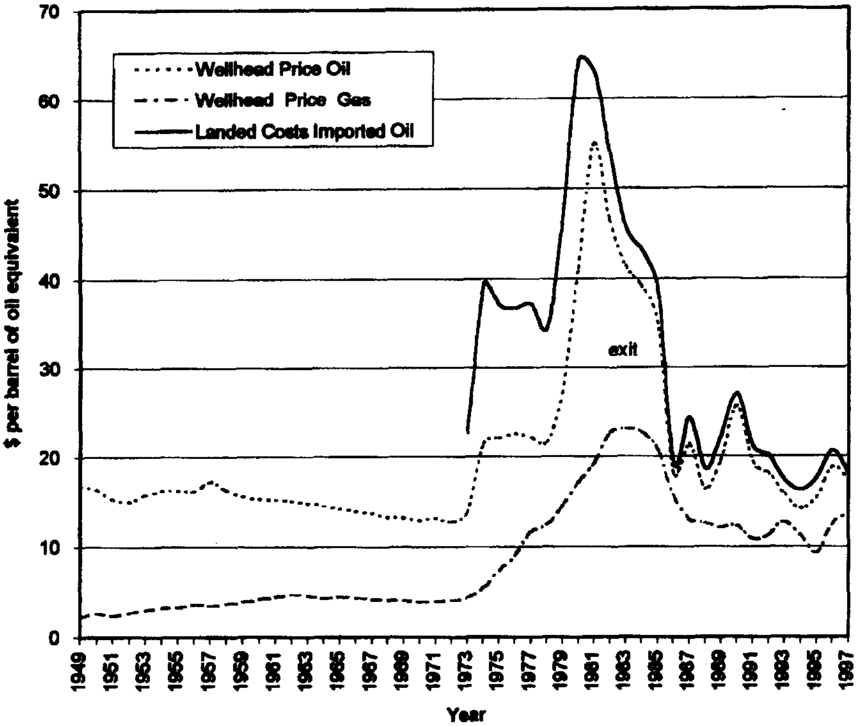
Oil and gas are thought to have been created from decayed organic material that has been under high pressure and temperature for millions of years. Through migration in permeable rock (sandstone, shale, and limestone) it has accumulated under impermeable rock in anticlines, which is an upfold in the earth's strata, under and around saltdomes or in other traps. Since natural gas creation requires more heat and pressure it tends to be found at greater depths than oil, except for methane produced by bacterial action, which is nearer the surface than oil. Exploration is the process of searching for these traps.

In unexplored areas, some surface signs like oil or gas seeps, sedimentary outcrops, or surface deformations may indicate oil and gas. These may be noted from direct observation and from visual, thermal and infrared photography from airplanes and satelites. Anomalies in gravitational and magnetic fields as well as electrical properties, which may also indicate oil and gas, are measured by using gravity trucks, by aerial magnetometer and magnetotelluric surveys.

The last step in the predrilling process is to do seismic surveys. In order to conduct seismic or any onsite exploration activity, leasing arrangements must be made. For privately held land, land men are sent out to make leasing agreements in which typically there are lump sum payments and a royalty agreement in the event of a subsequent discovery. For publically held land either at the state or federal level often the right to explore is gained through a bidding process.

Seismic surveys, which measure how sound waves pass through the earth, provide information on the location and density of subsurface formations. Earlier, sound waves were created by dynamite charges. Now they are more typically created by air guns or steam release in water or by trucks that produce acoustical vibrations on land. The waves are picked up by sensors called hydrophones in water and by geophones on land and the resulting images are sent to computers for processing sometimes via satelites from more remote areas. If the seismic lines are laid in parallel, three dimension (3D) seismic may be conducted. 3D seismic along with improved resolution and interpretation for all of the above exploration tools have been recent technical improvements that have significantly lowered oil and gas exploration costs by improving the probabilities of finding oil and gas. We will refer to all of the above predrilling exploration activities except for leasing as geophysical inputs.

The last step in the exploration process is the drilling of exploration wells (often called wildcat wells), which includes building any necessary access roads and infrastructure, drilling and casing the hole, and all the testing that accompanies this process. These tests



Source: Prices from US EIA/DOE *Annual Energy Review* (1997) deflated by the consumer price index from *Economic Report of the President* (various years) converted to 1997 dollars by the authors.

Figure 1. US Oil and Gas Prices (1997$)

include core samples, cutting samples, drill stem tests for pressure; fluid samples, and wireline logs that test electrical, radioactive, and acoustical properties of the materials surrounding the well bore. We will classify all of these activities under drilling. (Langencamp, 1982; Van Dyck, 1997)

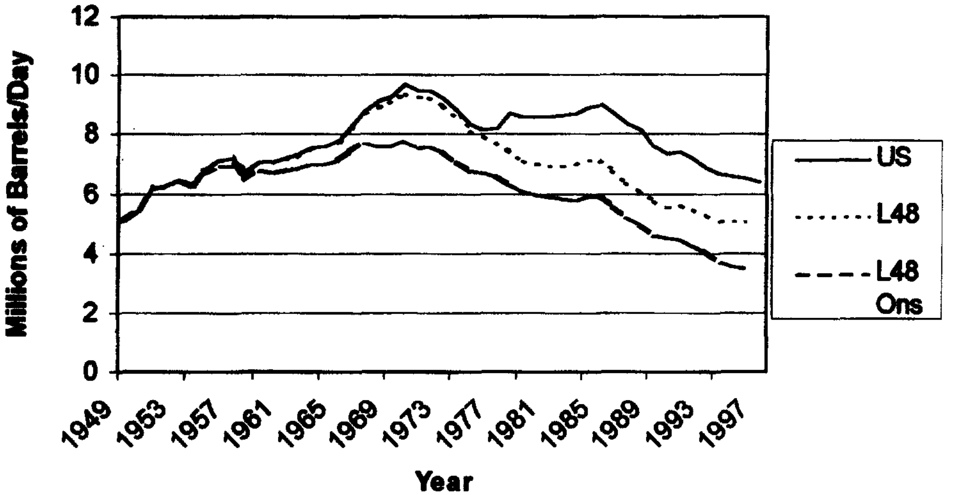
III. HIGHLIGHTS OF U.S. OIL AND GAS SINCE WW II.

All the models that we survey in later sections consider exploration over some time period since WW II. To set the stage for these models and put them in historical context, we give a brief overview of U.S. oil and gas prices, policies, production and reserves. The US emerged from WW II producing well over half of the free world's oil, almost all of which came from lower 48 onshore production. However, this was a position that would erode in the coming decades. By 1948 the U.S. had become a net importer of oil and by 1950 prices in the Middle East were being posted instead of being based on those in the U.S. Through the decade with increasing Middle Eastern and other non-US production, real oil prices in the US began to erode with brief strengthening during the Korean War and the 1957 Suez Crisis. See Figure 1 for the evolution of oil and gas prices in the post war period. (Adelman, 1972; Bourdaire, 1994).

As a result of falling prices the US instituted an oil import quota (voluntary in 1957, mandatory from 1959 to 1973) and OPEC was formed in 1960. During this period state regulatory commissions, the most famous being the Texas Railroad Commission, continued their practice of prorationing (assigning output per well in order to reduce production and support the domestic price of oil) which they had began in the 1930s. (Adelman, 1972). However, neither prorationing nor OPEC were able to stop the downward price trend throughout the 1960s. In the meantime, natural gas prices, which had been under price controls since 1952, were much lower than oil on an energy content basis. Although they had a tendency to rise gently, they still remained well below that of oil.

With the Arab oil embargo of 1973 US wellhead oil prices took a steep jump from 1973 to 1974 and with the Iranian revolution an even steeper jump from 1978 to 1979. In 1974 a two and then a three tier pricing system was instituted with U.S. price controls on old oil but not on new oil or on stripper production. (US EIA/DOE, 1998) The relevant price for wildcatters would then have been the price for new oil which we proxy in the figure by using landed import prices. In 1980 and 1981 a U.S. windfall profit tax was imposed and oil prices were decontrolled.[2] (US EIA/DOE, 1998) The tax varied by tiers and was higher on old oil production than on new or stripper production and was higher in some categories for the majors than the independent companies. (Thompson & Wright, 1985). The relevant price for wildcatters would then not have been wellhead price but wellhead price less the relevant taxes paid.

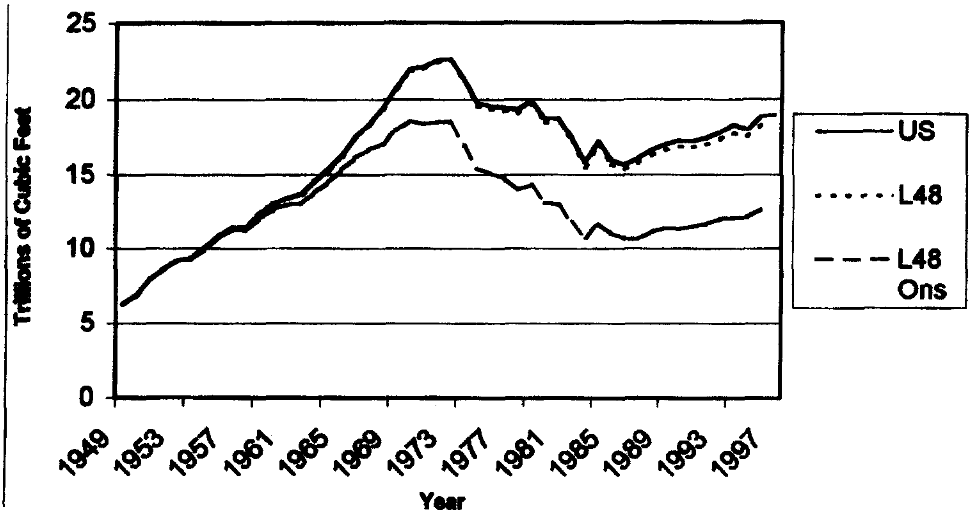
Because oil prices were now being set on world markets, the tax was absorbed by domestic producers and a proxy for the after tax price on new oil, which has not been reported, would be world oil adjusted by the windfall profit tax rate. In Figure 1, we can see the resulting difference between average domestic wellhead world prices and import prices through 1985. Then increased OPEC production caused prices to plummet rendering the windfall profit tax ineffective. The tax was repealed in 1988.



Source: American Petroleum Institute (1998).

Figure 2. Oil Production US, Lower 48, and Lower 48 Onshore

Gas prices remained controlled throughout this period but the Natural Gas Act of 1978 contained provisions to phase out price controls at the wellhead and new gas was totally decontrolled by 1987 (US EIA/DOE, 1989; Some old gas remained under price controls into the 1990s). Average wellhead gas prices tended to follow oil but price controls kept them well below oil and the price changes were more gradual. Again new prices net of tax



Source: American Petroleum Institute (1998).

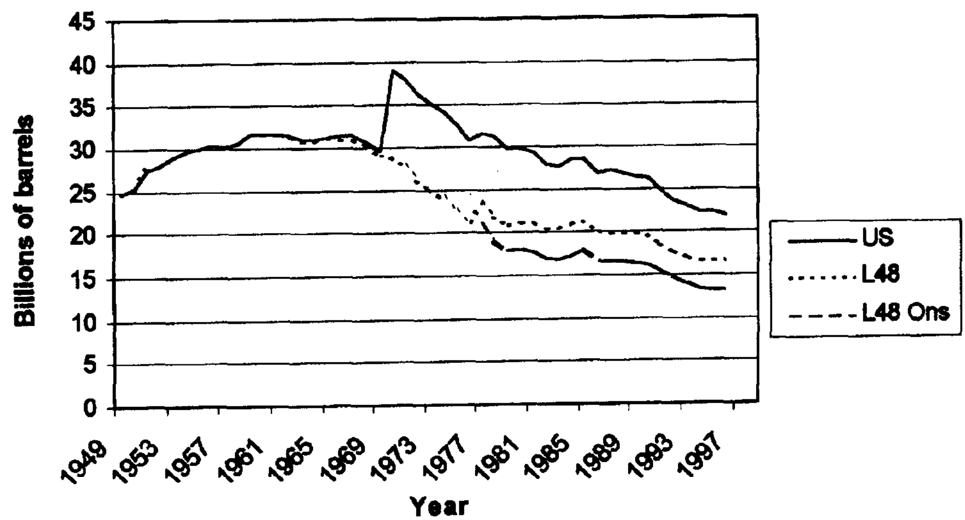
Figure 3. Natural Gas Production US, Lower 48, and Lower 48 Onshore

rather than the average wellhead prices in Figure 1 would be the appropriate price for wildcatters.

By 1987 the majority of gas no longer had price controls and we can see from Figure 1 that US wellhead and import prices track each other reasonable well while the difference in price between oil and gas had narrowed considerably. Despite blips in oil price during the Persian Gulf war in 1990 and during times when markets tighten from strong demand or OPEC discipline, the overall trend for oil since 1987 has been down. After gas price deregulation gas prices did not track oil prices as well as before. The correlation coefficient between oil and gas prices from 1949 to 1986 was 0.90 but from 1987 to 1997 it was only 0.30.

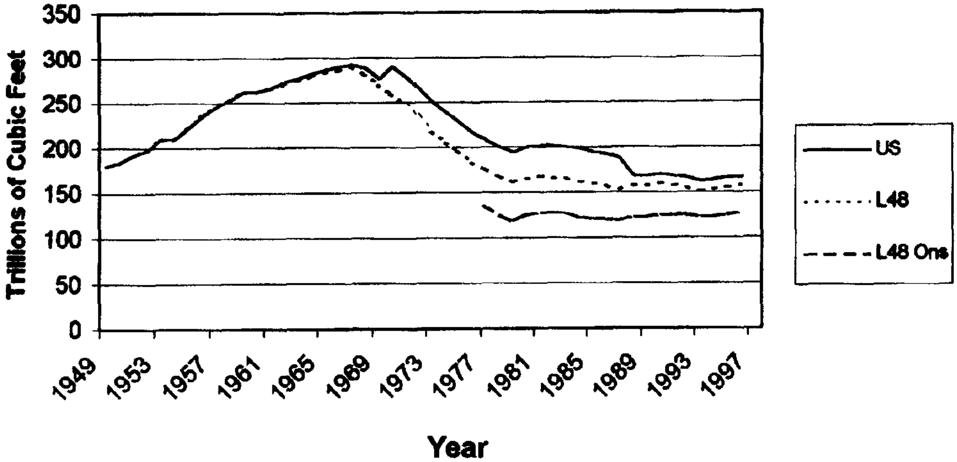
As the markets evolved production and proven reserve patterns changed as well. In 1947 almost all oil and natural gas were produced in the lower 48 states onshore. (See Figures 2 and 3) This lower 48 onshore production trended upwards peaking in 1970 for both oil and for natural gas and then trended down with a slight rebound for oil in the early 1980s and a rebound for gas after 1984 as gas prices were decontrolled. In the meantime offshore oil production trended upwards with a peak in 1971 accounting for almost 18% of total US oil production. It then trended down but rebounded in the early 1980's with higher oil prices and again in the 1990s with more deepwater Gulf production. By 1996 it had almost regained its 1971 high and constituted just over 20% of US production. Offshore gas followed a somewhat different pattern with the upward trend continuing throughout the entire post war period.

The Prudhoe Bay find in 1968 put Alaska in the oil big leagues. Oil production began from Prudhoe Bay in 1970 but did not reach significant proportions for the US until 1977 after the completion of the Alaska pipeline. Production peaked in 1988 when Alaska was producing 25% of US oil production. Its production has fallen faster than the US average



Source: American Petroleum Institute (1998), US EIA/DOE (1996b)

Figure 4. Oil Reserves US, Lower 48 and Lower 48 Onshore



Source: American Petroleum Institute (1998), U.S. EIA/DOE (1996b)

Figure 5. Natural Gas Reserves, US, Lower 48 and Lower 48 Onshore

and its share fell to about 22% by 1996. Alaskan gas production has always been relatively small.

Figure 4 and 5, which show US oil and gas reserves, demonstrate the fruits of exploration and the expansion of the industry before 1970 and the contraction of the industry after 1970. Oil and gas reserves in the lower 48 and oil reserves in Alaska peaked in 1970 and then trended down. The figures also show the relative shift away from the lower 48 towards Alaska and offshore areas. Offshore oil reserves, for which we have found data only since 1977, tended to be more stable with high oil prices keeping reserves up through the 1980s and deepwater Gulf of Mexico causing a rebound in the mid 1990's. By 1996 deepwater reserves were over half of US offshore oil reserves (US EIA/DOE, 1996b). Natural gas reserves offshore have generally trended down since 1977. We can see the increase in Alaskan reserves with Prudhoe Bay and the write down of these reserves in 1988 when lack of market made them un-economic. Although deepwater has also been a hot play for gas with deepwater gas reserves rising to about a quarter of US offshore gas reserves in 1996, these finds have only barely stemmed the reserve decreases in shallower offshore areas.

IV. THEORETICAL MODEL OF OIL AND GAS EXPLORATION

We model the oil and gas explorer or wildcatter as maximizing the expected net present value of exploration. Uhler (1979) puts the inputs into this process, that are described above, under three categories – land, geophysical inputs and drilling. We assume two direct inputs Ws, which represents wildcat wells drilled, and S, which represents seismic and other geophysical inputs. Land use is typically not observed directly and is represented by us through any bonus or upfront costs that permit exploration (B) and a royalty on output which we include below in our lifting taxes. Exploration yields proven reserves of oil (O) and gas (G). Our implicit production function is $F(S, W_w, G, O, N, \varepsilon)$.[3] N represents nature or the geology of the area, such as remaining reserves to be found, and ε represents the random nature of the discover process.[4] The unit cost of drilling and seismic are P_w and P_s, which include both the physical costs and an opportunity cost of capital. Production of oil and gas are functions of the reserves found or $Q_o = f(O)$ and $Q_g = g(G)$. Without enhanced recovery this production path often follows an exponential decline curve with decline rate δ, where production at time i is $Q_o = e^{-\delta i}O$ and $Q_g = e^{-\delta i}G$. (McCray, 1975) The life of the well is n.

The net benefits of finding reserves is the expected (E) discounted present revenues of oil and gas production (where r_i, P_{oi} and P_{gi} are the discount rate, oil price, and gas price at time i) minus lifting costs and taxes for oil and gas production (C_{oi}, C_{gi}, t_{oi}, t_{gi})[5] minus the cost of seismic and drilling and any up front bonus costs or leasing cost (B). The objective is then to maximize[6]

$$\Pi = E\left[\Sigma_{i=1}^{n}\frac{(P_{oi}e^{\delta i}O + P_{gi}e^{-\delta i}G - C_{oi}e^{-\delta i}O - C_{gi}e^{-\delta i}G - t_{oi}e^{-\delta i}O - t_{gi}e^{-\delta i}G)}{\Pi_{j=1}^{i}(1+r_j)}\right.$$
$$\left. - P_wWw - P_sS - B + \lambda F(S, Ww, G, O, N, \varepsilon)\right] \tag{1}$$

with respect to Ww, S, and λ. First order conditions are

$$\Pi_{W_w} = E\left[\Sigma_{i=1}^{n}\frac{e^{-\delta i}}{\Pi_{j=1}^{i}(1+r_j)}\left\{P_{oi}\left(\frac{\partial O}{\partial Ww}\right) + P_{gi}\left(\frac{\partial G}{\partial Ww}\right)\right.\right.$$
$$\left. - C_{oi}\left(\frac{\partial O}{\partial Ww}\right) - C_{gi}\left(\frac{\partial G}{\partial Ww}\right) - t_{oi}\left(\frac{\partial O}{\partial Ww}\right) - t_{gi}\left(\frac{\partial G}{\partial Ww}\right)\right\} \tag{2}$$
$$\left. - P_w + \lambda F_{W_w}\right] = 0$$

$$\Pi_S = E\left[\Sigma_{i=1}^{n}\frac{e^{-\delta i}}{\Pi_{j=1}^{i}(1+r_j)}\left\{P_{oi}\frac{\partial O}{\partial S} + P_{gi}\frac{\partial G}{\partial S}\right.\right.$$
$$\left. - C_{oi}\frac{\partial O}{\partial S} - C_{gi}\frac{\partial G}{\partial S} - t_{oi}\frac{\partial O}{\partial S} - t_{gi}\frac{\partial G}{\partial S}\right\} \tag{3}$$
$$\left. - P_S + \lambda F_S\right] = 0$$

$$\Pi_\lambda = F(S, W_w, G, O, N, \varepsilon) = 0. \tag{4}$$

Solving these three implicit functions for our dependent variables we get a drilling and a seismic equation that are functions of the price of drilling (Pw), the price of seismic (Ps), the decline rate (δ), expected values of all oil and gas prices, costs, taxes, and interest rates over the production horizon and the geology of the area (N). We characterize expected oil and gas prices, costs, taxes and discount rates by the following 7 vectors ($\overline{P}_o, \overline{P}_g, \overline{C}_o, \overline{C}_g, \bar{t}_o, \bar{t}_g, \bar{r}$). (e.g., $\overline{P}_o = E(P_{o1}, P_{o2}, \ldots P_{on})^T$ (T stands for transpose.) The marginal benefits of drilling $\partial O/\partial Ww$, $\partial G/\partial Ww$ are often referred to as drilling discovery rates. These discovery rates along with the marginal products of seismic and other inputs $\partial O/\partial S$, $\partial G/\partial S$ should depend on the parameters of the production function and the geology of the area (N). Solving for Ww and S in terms of all of the exogenous variables should then give us the following functions for estimation:

$$Ww = Ww(\overline{P}_o,\overline{P}_g,\overline{C}_o,\overline{C}_g,\bar{t}_o,\bar{t}_g,\bar{r}, \delta, P_w, P_s, N) \text{ and} \tag{5}$$

$$S = S(\overline{P}_o, \overline{P}_g, \overline{C}_o, \overline{C}_g, \bar{t}_o, \bar{t}_g, \bar{r}, \delta, P_w, P_s, N). \tag{6}$$

Having determined resources devoted to exploration the last step is to change these inputs into new reserves. In the above economic model we would substitute Ww, S and N into the production function to determine O and G discovered as:

$$O = O(Ww, S, N) \tag{7}$$

$$G = G(Ww, S, N) \tag{8}$$

Often these equations are rearranged into some form of discovery rate model by considering reserves per well as follows:

$$O/Ww = O(S, N) \tag{9}$$

$$G/Ww = G(S, N) \tag{10}$$

These functions could be estimated econometrically and total reserves would be measured as drilling rates times discovery rates as follows:

$$Ww^*O/Ww = O \tag{11}$$

$$Ww^*G/Ww = G \tag{12}$$

Alternatively, if we only wanted to forecast reserve additions we could substituted (5) and (6) into (7) and (8) to get reduced form equations for oil and gas discoveries, which could be estimated econometrically.

$$O = Ord(\overline{P}_o,\overline{P}_g,\overline{C}_o,\overline{C}_g,\bar{t}_o, \bar{t}_g, \bar{r}, \delta, P_w, P_s, N) \tag{13}$$

$$G = Grd(\overline{P}_o,\overline{P}_g,\overline{C}_o,\overline{C}_g,\bar{t}_o,\bar{t}_g,\bar{r}, \delta, P_w, P_s, N) \tag{14}$$

Theory suggests we include the above variables in an econometric model. It does not necessarily suggest the functional form our model takes, which should be subject to statistical testing. The error structure needs to be specified and tested as well. Beside the tests for serial correlation and heteroskedasticity, it is likely that errors in these sets of equations will be correlated. If so seemingly unrelated regressions will be more asymtotically efficient if estimating a system in which the exogenous variables in some of the equations vary.

Another econometric issue is the inertia found in many economic systems. In any capital intensive process, such as drilling, changes in prices and costs may cause changes in drilling with a lag. In such a case the short run adjustment is likely to be less than the long run adjustment. This lag is often represented using a partial adjustment model. We can illustrate this model using the following simplified drilling equation, where the change in drilling from $t-1$ to t is some fraction of the difference between desired drilling and drilling last period or

$$Ww_t - Ww_{t-1} = \lambda(Ww_t{}^* - Ww_{t-1})$$

Where desired drilling (Ww^*) is a function of the price of oil or $Ww^* = \alpha Po_t$. Thus long run adjustment to a price change is $dWw^*/dPo = \alpha$. Substituting desired drilling into the adjustment function and solving for current drilling and rearranging yields the estimating equation that contains a lagged endogenous variable or

$$Ww_t = \alpha\lambda Po + (1-\lambda)Ww_{t-1.}$$

Short run adjustment $dWw/dPo = \alpha\lambda$, which would be the coefficient in front of Po in the estimated model, and long run adjustment can be computed from $\alpha\lambda/(1-(1-\lambda)) = \alpha$, which is the coefficient in front of Po divided by one minus the coefficient in front of the lagged endogenous variable. Thus the model in theory can be used to measure both a long and a short run elasticity. In practice, however, the lagged endogenous variable is often collinear with the included variables making it difficult to get stable and significant estimates on coefficients for the exogenous variables allowing us to distinguish between long and short run. At the same time it is often even more collinear with the endogenous variable yielding a good overall fit for the equation.

In addition to the economic and the econometric specification data must be collected. We include Table A1 in Appendix A with some the most common variable sources.

In the following sections we will present econometric estimates of the above oil and gas exploration processes in the US some including one or more lagged endogenous variables. We will see that most papers have some variant of equation (5) but many have not included all of the variables that theory suggests should be included. Unless these excluded variables are constant or are orthogonal to the included variables, the coefficient estimates are biased. Few have attempted to model other inputs into the production process as in (6), although a number have used seismic crews in their estimation of equation (5). Thus investigating geophysical inputs and productivity would be useful but probably quite challenging given the large recent gains in geophysical technology. Researchers have used a variety of techniques for modeling the reserve discovery process (7) – (14). We will see the earliest and one of the more often used econometric approaches in the next section.

V. FISHER TYPE MODELS FOR OIL EXPLORATION

Fisher (1964) modeled the oil and gas exploration process with four equations. The first equation was a wildcat wells drilling equation. He broke the discovery process (9) and (10) each into two parts. The first was an equation for the fraction of wells that are successful W/Ww and the second was the average size of oil or gas discoveries per successful well (O/W, G/W). Henceforth, we will refer to his four equations as drilling, success rate, oil discovery rate and gas discovery rate. Multiplying wildcats by success rate times average size of field discoveries per well for oil or gas gives us the amount of oil reserves (O) and gas reserves (G) found or

$$O = Ww * W/Ww * O/W \tag{15}$$

$$G = Ww * W/Ww * G/W \tag{16}$$

Fisher estimated his equations econometrically on pooled cross section time series data by Petroleum Administration Defense Districts (PADD). In his drilling equation he included oil price, seismic crews, a lagged success rate for oil, a lagged discovery rate, and a lagged success rate for gas. (For a listing of the variables in all equations discussed in this survey, whether coefficients were significant and R^2 see Table A2 in Appendix A) Since he did not explicitly base his model on underlying economic theory, we first discuss the theoretical merits of his model. Using the current price of oil for expected oil prices may be reasonable since Walls (1991) found oil prices to be a random walk. Gas prices are omitted but price controls had kept them low and fairly constant and lagged gas finds might also be a proxy for natural gas prices. Costs are represented by deflating oil price by a wholesale price index in one case and a drilling cost index in the other. As with oil this implicitly assumes that current costs are a good proxy for expected costs. Seismic crews may be a proxy for the price of geophysical inputs but using it does introduce an endogenous variable on the right hand side of the equation with its attendant biases. A seismic cost variable adjusted for productivity would be more desirable. Barring the availability of seismic cost data an instrumental variable approach could have been used for seismic crews to give consistent estimates or a lagged seismic variable has economic appeal and would be appropriate if there is no serial correlation in the model.

Lagged success and discovery rates may be proxies for geology. However, the way they are entered has some complicating implications for long run price elasticities as seen in Appendix B. Since the depletion rate is likely to be fairly constant its omission should not be serious. What is more serious is the omission of lifting taxes, lifting costs, and interest rates. Unless they have all been constant across his sample or they are orthogonal to all of the included variables their omission is biasing his coefficient estimates.[7] At least for interest rates this seems unlikely to be the case. Although the interest rate for discounting may be an expected rate that is fairly constant, the opportunity cost of current drilling depends on current interest rates which vary considerably over the business cycle.

He models well success rate as a function of lagged success rate, lagged discovery rates per successful well for oil and gas and depth, all of which could proxy for the geological variables required in the model. His well success rate equations also contains seismic

Table 1. Summary of Price Elasticities in Oil Exploration Models for the US for Fisher Type Models

Reference	*Prod*	*Sample*	*Wells Drilled*	*Success Rate*	*Discovery Size*	*Total Reserves*	*Comments*
Fisher (1964)	Oil	46-55 PADD 1-5, CT	2.27 *	-0.36	-1.63 *	0.31	Poil/WPI
		LR	1.35	0.14	-5.44	-5.88#	
		46-55 PADD 1-5, CT	2.85 *	-0.39	-2.18 *	0.82	Poil/Drillcost, D, TXShutdown
		LR	1.12	0.184	-5.62	-5.95#	
Erickson (1968)q	Oil	46-59 PADD 1-5, CT	1.54 *	0.27	-0.91	0.89	D*PADD
		LR	1.33	0.27	0.68	0.92	
Erickson & Spann (1971)	Oil	46-59 PADD 1-5, CT	1.48 *	-0.23	-0.42	0.83	D*PADD, Pgas
		LR	1.66	-0.23	-.60	-0.61#	
Pindyck (1974)	Oil	64-69 18 Dis, CT	0.07	1.42 *	Ne		All exploratory wells, Pgas
Eyssel (1978)	Oil	46-59 PADD 1-5, CT	1.20 *	0.01	1.45	2.66	Erickson's model with directionality
		LR	1.20	0.01	1.45		
		46-70 PADD 1-5, CT	1.52 *	-0.03	2.70 *	3.90	Erickson's model with directionality
		LR	1.54	-0.32	2.49	3.71	
		46-70 27 Dist, CT	1.84 *	-0.22	0.37	2.43	D*dist
		LR	1.80	0.22	0.48	2.50	
Kolb (1979)	Oil	46-58 7 Dist, CT	1.62 *	0.90 *	-1.29	1.23	Fisher model
		LR	13.13	49.87	-24.45	90.8#	
	Oil	46-58 6 Dist, CT	2.39 *	0.50	0.04	2.93	Fisher model
		LR	1.69	1.68	1.48	9.81#	
	Oil	46-58 7 Dist, CT	0.91 *	0.12	1.27	3.55	Total elasticity adjusted for directionality, Pgas, Cost, Depletion Effect
AlShami (1995)	Oil	60-90 US TS	0.48 *	0.32 *	0.81 *	1.61	Poil&PGas, Cost & Bond rate, Depletion Effect
		LR	0.42	0.32	-0.58	0.15	

Notes: *indicates a |t statistics| > 1.645. All abbreviations are defined under Table A2 in Appendix A. #includes indirect effects from a gas finding equation.

Table 2. Summary of Own Gas Price Elasticities in Natural Gas Exploration Models for the US

Reference	*Prod*	*Sample*	*Wells Drilled*	*Success Rate*	*Discovery Size*	*Total Reserves*	*Comments*
Erickson & Spann (1971)	Gas	46-59 TS	0.35	0.01	0.33	0.69	D*PADD
Pindyck (1974)	Gas	65-69 CT 18 Dist	-0.03	0.57 *	1.83 *	2.36	D*Dist
MacAvoy & Pindyck (1973)	Gas	65-71 CT 18 Dist			1.46	1.46	Computed from simulations
	GasA	65-71 CT 18 Dist			1.21	1.21	
Pindyck (1978)	Gas	65-72 CT 18 Dist				1.52	Computed from simulations
	GasA	65-72 CT 18 Dist				1.37	
	Gas	65-74 CT 18 Dist				1.17	Computed from simulations
	GasA	65-74 CT 18 Dist				0.48	
Eppel (1975)	Gas	46-68 US TS	0.74 *			2.87 *	Joint production function, Cost,
						3.09	After tax prices
Deagan (1979)	Gas	46-69 8 US Gulf Regions CT				2.10	
Kolb (1979)	Gas	48-70 7 Dist CT	0.84 *	0.33 *	-0.13		poil insignificant, directionality
		54-70 7 Dist CT	-0.10	0.31	-1.23		poil insignificant, directionality
		48-70 7 Dist CT			-0.91		no directionality
					0.67		
Wilkinson (1983)	Gas	58-79 US TS	0.15				
Charlotte (1988)	Gas	Gulf Mexico 56-85 TS			<0.61 sr		
					<0.83 lr		
Iledare (1995)	Gas	WV Wells 77-87 CT	0.52 *		0.56		price = expected profit
			1.06 *		1.10		price = expected wellhead price

Notes: GasA = additions to gas reserves, *indicates coefficients have |t statistics| > 1.645. Abbreviations given under Table A2 in Appendix A.

crews as required in the model along with price. Although our theory above does not explicitly include price in the discovery portion of the model, if the proper geological variable is economic remaining reserves and our geological variables do not capture this price effect, it is appropriate to include price. However, the insignificance of price in this and most of the other success rate equations estimated as seen in Table 1 suggests that it would probably have been statistically more efficient to have excluded oil price from this equation.

His discovery rate for successful wells for oil and for gas are functions of lagged success rates and lagged successful recovery rates for oil and for gas and the price of oil. All of which could again proxy for geological variables. Although economic theory does not tell us how to break discovery rates into success rates and discovery rates per successful well, the product of his success rate and discovery rate per successful well yield the discovery rates as in (9) and (10) which do have the appropriate variables—seismic and geological information- and also an oil price which is quite justifiable on economic grounds. Whether these are the best geological variables remains an empirical question that we will consider as we survey other articles.

The elasticity of total new oil reserves in his model with respect to the price of oil is

$$\partial(\ln(O)/\partial\ln Po = (\ln(Ww)/\partial\ln Po + \partial\ln(W/Ww)/\partial\ln Po + \partial\ln(O/W)/\partial\ln Po. \qquad (17)$$

In this survey we will consider all articles that include any of these elasticities—a drilling elasticity ($\partial\ln(Ww)/\partial\ln Po$), a success rate elasticity ($\partial\ln(W/Ww)/\partial\ln Po$), or a discovery elasticity ($\partial\ln(O/W)/\partial\ln Po$) as well as any estimated or implied total new reserve elasticities ($\partial\ln(O)/\partial\ln Po$). We will also include similar estimates for the gas market. All oil price elasticity estimates from Fisher type models included in this paper are summarized in Table 1 with more complete statistical information and included variables in Table A2 in Appendix A. Own price elasticity estimates exclusively for gas are included in Section VII and Table 2. (|t statistic| > 1.645 is chosen as the significance level for all analyses is this paper and significant coefficients are labeled with * in all the Tables).

Reserves used to create his discovery variable included later reserves credited back to the year of discovery, a practice followed by many later papers. District dummy variables when included tended to pick up the effects of the lagged endogenous variables and so were not retained in the model.

Fisher estimates a demand for seismic crews. He includes no prices but finds that an autoregressive model with various dummy variables fits the data quite well. Putting the equations together as in (1), Fisher obtained a short run reserve price elasticity between 0.31 and 0.82 with his preferred value closer to 0.3. He did not compute long-run price elasticities using the lagged values of the dependent variables which are included in his equations. These lagged values substantially complicate the computation of any long run reserve elasticities as can be seen by the formulas developed in Appendix B. We compute and report next to LR the long run elasticities where they differ from the reported short run values for all models in Table 1. They present a much more confusing picture with long run reserve price elasticities of less than -5. Although lagged values for success rate, discoveries, and gas reserves are added to capture geological factors and, hence, have intuitive appeal, we would caution that they contribute to long run elasticities, which should be computed as part of the check on model performance.

In updates of Fisher's model, Erickson (1968) and Erickson and Spann (1971) used a slightly broader definition of exploration that included new field and new pool wildcats as well as variables to measure the effect of exploration activity of large integrated firms in their drilling and success rate equations along with dummies for Texas shut down days and district dummy variables in all equations. They also exclude taxes and interest rates suggesting that their drilling equation has the same potential bias as that of Fisher. Lagged depth, which was used as a proxy for drilling costs in their drilling equations, had an unexpected statistically significant positive sign. Nor do they include any seismic costs in drilling or a seismic variable in their discovery equations which is again a potential source of bias.

With less lagged endogenous variables their long run results are much easier to compute than for Fisher and they do not vary appreciable from the short run results. It appears that including district dummies but not including so many lagged endogenous variables produces more believable long run results than for the case of Fisher above. It also produces a better fit as measured by $R.^2$ Although district dummies may be easier to use and they may capture geological differences across PADD districts, they do not capture changes over time as would other geological variables.

Erickson found that large integrated firms with better technology tended to have higher success rates. Erickson and Spann (1971) included the price of natural gas in all equations but found little difference. Gas price was insignificant in all three equations suggesting that the low controlled prices for natural gas either did not influence the exploration process or else did not vary enough for econometric work to pick up their effect.

MacAvoy and Pindyck (1973) did not include the success rate equation of Fisher but modeled discoveries per exploratory well as in (9) and (10) on more disaggregated and recent data. One improvement is to explicitly include drilling costs, which they find to be significant. They have some interesting alternative geological variables worth more consideration including a risk index representing the variance in reserve size in their drilling equation. They include cumulative values of drilling in their discovery equations, although quantity of cumulative drilling or cumulative reserves found would likely to have been better. They use total district revenues of oil and gas to represent their price variable. Combining oil and gas prices may be a useful way to get around problems of collinearity between these two variables but it would have been better to have included some weighted average of oil and gas prices or expected revenues per well. They use a weighted average of both costs and prices to represent expectations in their discovery equations. This too is a useful innovation and we encourage more experimenting with expectation models to determine whether current values for prices, costs, and other variables provide the best proxy for their expected value. However, they do not include an interest rate, taxes, or other lifting costs in their model. Nor to they report any price elasticities for oil and the overall fit of their equations is relatively poor.

Pindyck (1974) estimated three equations of the Erickson and Spann model, the drilling, the success rate, and the gas discovery within a larger gas model on 18 US districts also on more recent data. We include his gas equation below with other gas studies. His drilling equation included all exploratory wells instead of just new field wildcats. He found his oil price elasticities to be very different from the earlier Fisher type models and claimed the model was unstable. Although we tend to agree with him for the success rate equation we are not convinced this is the case for the drilling equation given results from all the other studies. He found that the district percent of total new wells which he substituted for exploratory drilling of large firms did most of the explaining in the drilling equation. Since both components of district percent of new wells are influenced by price, they are probably picking up the price effect indirectly. Thus his drilling result may be more a function collinearity than an underlying instability in drilling. His large and significant success rate elasticity is also unusual and may be influenced by his using total exploratory rather than new field wild cats. The American Association of Petroleum Geologists (AAPG) at one time broke exploratory wells into five categories—new field wild cats, new pool wild cats, deeper pool wildcats, shallower pool wildcats, and outposts. Since the last four categories require no new geology and are close to known pools their success rates tends to be higher and Kolb (1978) finds them to be significantly influenced by price.

Eyssel (1978) started with Erickson's model, hence it suffers the same potential biases as discussed earlier. He found the elasticities sensitive to whether PADD1, primarily a gas region, was included in the model. He argued that gas and oil were often not a joint product but rather producers specifically looked for one or the other.[8] Unfortunately data do not exist on drilling intentions, so we only know ex post whether oil or gas or both were found. Both he and Pindyck (1978) attempted to separate out gas's influence by only including success rates for oil wells (Wo/Ww=Wwo) and discoveries per successful oil well (O/Wo=Owo). Success rate price elasticities were insignificant in their different samples, discovery elasticities varied highly from insignificant to 2.7.

Pindyck (1978) is the first to include an interest rate in his drilling equation, which he finds significant. This confirms our earlier thoughts that its earlier omission may be biasing results. His price variable is expected oil and gas revenues for a new well which is an improvement over their earlier total district revenues but unfortunately it is not significant. Thus in some ways his model is getting closer to a correct specification but he still does not include taxes and although he uses success rates and discovery rates to create his expected revenues he includes no other geological variables in his drilling equation. This omission of seismic variables and more disaggregated data, which may not average out some of the randomness, may help explain his poor fit.

Kolb (1979) reestimated Fisher's model but with Texas and Louisiana divided out into separate districts from PADD III to make seven districts. He estimated his equations with and without PADD I, which is a more gas prone district. As expected overall reserve price elasticities suggest that PADD I is a less promising oil area. However, all R^2 are smaller than in the original studies suggesting a poorer fit on more disaggregate data. This suggests more work would be useful to model structural differences across districts. His long run elasticities clearly illustrate the difficulties that can be encountered when numerous lag and cross equation lags are included.

Kolb's updated model included data from 1948 to 1970 with a modification that allowed for directionality in drilling. This slightly more complicated model included an overall drilling and an overall success rate equation but separate discovery rates for successful oil and successful gas wells, the ratio of associated gas to oil, and the ratio of successful gas wells to total gas wells. As in Eyssel and earlier papers, his reserve data includes discoveries credited back to the year of initial discovery. He found deflated oil and gas prices both to be significant in his drilling equation with an oil price elasticity of 0.90 and a gas price elasticity of 0.84. Since their sum is close to earlier estimates of oil price drilling elasticities, earlier estimates, which excluded gas, may be attributing some of the gas effect to oil.

He included drilling costs directly and found that increases in drilling costs decrease drilling but increase the success rate. This suggests that with higher costs, drillers pick their prospects more carefully. He included an estimate of remaining reserves as a geological variable and found it significantly influences his success rate. This and the other models that include directionality are interesting attempts to separate out the effect of gas. Their basic framework can be justified on economic grounds but none, however, contains all of the variables that economic theory suggests should be included. Furthermore, all are now rather dated leaving the way for others to update and improve on this previous work.

The last model we have found that closely follows the Fisher approach is Al Shami (1995), who estimated his equations on aggregate US data for 1960–90. His study, which is on the most aggregate data and only includes time series data, has among the highest R^2 of the studies. Although he omitted lifting costs and lifting taxes and included seismic crews in his drilling equation, thus introducing potential bias, he has a number of interesting features in his model worth considering for future work. His price variable, which was a weighted average of oil and gas prices, performed well in all three equations. This leads us to expect that multicollinearity might be the reason for some previous models finding gas price to be insignificant. He used imports to capture expectations. Higher imports suggest a tightening market and better future prospects for the US industry, which increase drilling. He found a drilling price elasticity of 0.48 in the short run which was quite significant and found no structural break in his sample. Because he found structural stability across time, we suspect that either his price variable or the level of aggregation or both may be the cause of his low drilling price elasticity rather than a decreased elasticity later in the sample. This improvement of results over more aggregated data seemed to occur in a number of studies. We expect that geological differences across districts show up more clearly in disaggregated data unless the appropriate geological variables are included.

Since he omitted seismic activity from his success and discovery rate equations they both suffer from potential bias as well. Unlike other equations, price was significant in both. He included the bond rate in his discovery equation and found it significant. As with price, a case can be made that the bond rate will influence whether reserves are economical or not. His implied reserve elasticity from his three equations is 1.61. However, if you compute the long run effect taking both the direct effect in his drilling equation and the indirect effect through his lagged success rate and lagged drilling you get a long run reserve elasticity of 0.15 which is illogically smaller than the short run effect. Again injudicious use of lagged endogenous variables has gotten us into some trouble and somewhere on the way from short to long run we seem to have mislaid some of our reserves. In functional form testing he found the log linear functional form statistically dominated the linear form.

Variants of the Fisher model have reappeared for over 3 decades. Summarizing these studies, we would offer the following general observations. Their basic structure can be justified on economic grounds. None include all the variables that economic theory suggests should be included and all, thus suffer from potential bias. Most drilling equations suggest that increases in oil price have increased drilling with a price elasticity greater than 0.5 and most have a reasonably good fit as measured by R^2. Since models with numerous cross equation lags often gave implausible results when long run elasticities were calculated, we would caution their use. Rather more work should be done to find geological variables which provide the best explanatory power.

Theory suggests that natural gas prices should be included in the model. However, when they are included separately, they are usually insignificant. When oil and gas prices are included as a weighted average on more recent data, the price elasticity is much more significant but considerably smaller. One suspects that natural gas prices, which had been controlled at low levels prior to the 1980s, probably had a small effect on drilling. After the Natural Gas Policy Act of 1978, which began the decontrol process for natural gas, gas prices should have had a larger effect on drilling. We would guess, however, that the overall effect of natural gas price has been smaller than for oil, since gas over most of the sample has been considered the consolation prize. We would urge more work on correctly specified models to investigate whether gas prices have more influence on drilling now that the prices of natural gas has been decontrolled and whether a combined oil and gas price is a more suitable variable than entering both variables separately. If variables are combined it would be useful to compute the separate oil and gas elasticities that are implied by the model.

Theory suggests that geophysical prices should be included in the drilling equation and geophysical quantities should be included in the discovery process. This has been done in no model. Developing a geophysical price series would be desirable as would including a geophysical equation in the model. Although current crews has been the variable included, it should be endogenous causing bias. At a minimum instrumental variables should be used or a lagged rather than a current seismic variable included. Some work has been done to model expectations. More work would be useful to determine what expectation models provide the best explanatory power for all the variables in oil and gas exploration models.

Although oil prices appear to have affected exploratory drilling, translating this extra drilling into extra oil reserves through success and discovery rates appears to be difficult. Neither empirically nor theoretically is it clear whether oil prices affect either success or discover rates. Most success and discovery rate equations are misspecified and they explain less of the variation (R^2 average 0.70 and 0.69, respectively) than do the drillings equation (R^2 average 90%). Perhaps better measures of geological potential in these equations will help. Further, theory does not indicated how to break discovery rate per well into success rates and discovery rates per successful well. More work could be done to determine whether modeling discover rates per well directly would provide us with a better fit.

VI. OTHER MODELING APPROACHES FOR OIL

A number of studies do not precisely follow the Fisher approach but contain some of the same components—drilling, discovery size, and total reserves equations. We summarize the price elasticities available from these studies in Table 3 with a more complete summary of their equation specifications included in Table A2 in Appendix A.

Many of these studies include drilling equations which suffer from the same sorts of biases as in the Fisher type models. Rice and Smith (1977) use total crude oil revenues as their price variable and omit seismic costs, lifting costs, lifting taxes, and interest rates. Their equation has a rather low explanatory power. Bouhabib (1978), Attanasi (1979), Deacon et al. (1990), Ghouri (1991), and Berman and Tuck (1994) use a better price variable but also suffer from the same omissions. Nevertheless the ones done on time series have a rather good fit and a number have new features worth mentioning. Ghouri (1991) considered exploration at the company level for four US companies—ARCO, Mobil, Phil-

Table 3. Summary of Price Elasticities in Oil Exploration Models for the US Other Models

Reference	Prod	Sample	Wells Drilled SR	Wells Drilled LR	Discovery Size SR	Discovery Size LR	Total Reserves SR	Total Reserves LR	Comments
Erickson & et al (1974)	Oil	50-68 5 PADD CT					0.07 *	0.76	D*PADD, lagged endogenous model stock of reserves, different taxes
Eppel (1975)	Oil	46-68 TS					2.87 *		Joint production function, costs, Expected price
Cox & Wright (1976)	Oil & Gas, NGL	5 L 48 States 59-71 CT					0.03 *		Stock of reserves different taxes
Deagan (1979)	Oil	46-49 8 US Gulf Regions CT					2.10		Joint production function, costs
Cox & Wright (1976)	Oil & Gas, NGL	5 L 48 States 59-71 CT					0.03 *		Stock of reserves different taxes
Deacon et (1991)	Oil	48-81 California TS	0.43 *	1.31					lagged endogenous model
Cleveland & Kaufmann (1991)	Oil	25-88 TS			1.00 *				
Chollett (1988)	Oil	Gulf Mex 56-85 TS			<.50 *	<1.13			reserves per foot, lagged endogenous
Ghouri (1991)	Oil	ARCO US 74-89 TS	1.06 *	1.49					Costs, Almon Lags
	Oil	Mobil 70-88 US TS	0.48 *	1.18					Costs, Almon Lags
	Oil	Phillips 70-88 US TS	0.97 *	1.80					Costs, Almon Lags
	Oil	Shell 70-88 US TS	0.06 *	1.17					Costs, Almon Lags
Walls (1991)	Oil	73-87 TS(m)	0.38 *	0.55			0.42 *	0.63	Price=net present value after tax of oil dynamic optimization model
Porter (1992)	Oil	66-88 TS Low48 Ons	0.46						Price=after tax profit
	Oil	66-88 TS Low48 Ofs	0.26						
	Oil	66-88 TS Low48 AK	0.36						
Walls (1994)	Oil	Gulf OCS 72-88 TS	0.18 *	1.12 *					price=expected after tax profits oil and gas
Berman and Tuck (1994)		US 70-89 TS	1.11 *		1.12 *		2.23		price=after tax profit, reserves per well
Iledare, Pulsipher, & Baumann (1995)	Oil & Gas	Gulf OCS 78-92 CT	0.40 *						Price=before tax cash flow oil and gas, expected tax rate included
			Drill	*RecRT*	*Res/ft*	*Total Reserves*			
Bouhabib (1975)	Oil	66-73 8 L 48 states	-2.25	6.45	-2.33	1.87			D*State
		CT	2.13 *	-1.41	2.90 *	3.62			D*offshore
		3 offshore states	-2.22	19.88 *	-13.91 *	3.75			Offshore states, D*state
		5 onshore states	-2.49	0.76	2.47	0.74			Onshore States, D*state

Notes: *indicates coefficients had a |*t statistics*| > 1.645. Abbreviations are given under Table A2 in Appendix A..

lips, and Shell—using a four period Almon lag on prices. It is desirable to use such a flexible lag to see what the data tell us about expectations. He finds the coefficient on current price is always the largest. However, the rest of the lag pattern is a bit erratic and does not strongly support the lag structure for a geometric lag model, which is used in a number of other studies in this section. Attanasi (1979) also used an Almon model with a ten period lag but on a geological variable representing the value of past reserves discovered and finds the lag tends to follow an inverted V.

Epple (1975) and Deegan (1979) both do complex modeling beginning with a CES joint production function for oil and gas and assuming profit maximization. They derive their total reserve elasticity estimates from a system of estimated equations. However, neither optimizes in a dynamic context and they omit many of the variables of the above models - interest rates, lifting costs and taxes.

Four studies include the net after tax discounted present value of a barrel of reserves in a drilling equation estimated on time series data. Griffin and Moroney (1985) studied the effect of severance tax on drilling data for Texas. Porter (1992) estimated total drilling equations and discovery equations for the lower 48 onshore, offshore, and Alaska for both oil and gas as part of a larger oil model for the US. Walls (1991) estimated an exploratory drilling function and an oil discovery model as part of a larger oil model for the aggregate US to evaluate an oil import tariff. Walls (1994)[9] estimated a total drilling function for total wells and discovery functions for oil, associated gas, and nonassociated gas on the Gulf of Mexico Outer Continental Shelf. A fifth study, Iledare et al. (1995), included a discounted profits and a separate tax variable. They used US company drilling data on the US Outer Continental Shelf to investigate whether there were any differences in drilling or discovery rates for the majors, large independents, and small independents companies.

These studies all have the desirable property of including prices, taxes, lifting costs and the interest rate for discounting, Including them as a combined variable instead of each separately also should help to cope with the collinearity that may exist between these variables. All but one long run drilling profit elasticity are below 0.55, which tends to be lower than earlier drilling price elasticities as we would expect.[10] We would encourage all model builders to include these variables and if they are combined into a profit variable to also compute and report separate elasticities for price and cost to increase the usability of the model to more readers.

Although improvements, these models also have potential for bias. Seismic costs and activity still receive little attention. Walls (1991) assumes a function in which drilling costs increase with drilling rather than using actual costs, presumably because actual costs were not available on a monthly data. She also omits associated gas from her profit variable. Although gas was only 8% of wellhead value of oil well output in 1973 by the end of her sample in 1991 it had risen to 17% suggesting that's its omission may also be a source of bias.[11] Porter (1992) includes lifting costs but assumes they are a constant percent of price and also assumes a drilling cost function that increases with drilling rates instead of actual costs.

These models have some other interesting features and results worth mentioning. Walls (1991) derived her drilling equation for exploratory wells from a dynamic optimization model assuming that firms have rational expectations and maximize the expected discounted after tax net present value of exploration. Porter (1992) also explicitly built his model on an optimization framework by assuming that the discounted finding costs of a barrel of oil net of taxes should be equal to the discounted present value of a barrel of oil net of taxes. He divides total wells drilled into oil and gas wells by allocating dry wells over the two categories in a non specified way. His is the only paper to do so and it would be useful to have more information on this allocation and a comparison of this more disaggregate drilling model with the more traditional approach.

Iledare et al. (1995) also base their model on dynamic optimization. They divide their profit from their tax variable which makes it possible to see their separate effects. A bit more information on how these variables were created would have also been helpful. They found the drilling profit elasticity to be 0.40 and not to vary statistically across their three types of firms—majors, large independents, and small independents. Increasing taxes decreased drilling and the coefficient on cumulative drilling suggested a depletion effect with the majors having a larger response than the independents.

Two studies estimate reserve equations directly, where stock rather than the flow of reserves is considered. Since total reserves (Ro) are equal to total reserves last period minus production this period plus reserve additions ($Ro_t = Ro_{t-1} - Qo_t + O_t$) a reasonable specification would include the variables we would expect to be in a reserve addition equation. Cox and Wright (1976) explicitly derive their equation from a dynamic optimization model for total oil, gas and Natural Gas Liquids (NGL) reserves. Erickson et al. (1974) began with a more informal approach and estimate only oil reserves. Both models suffer from some of the same omissions as above by ignoring lifting costs and seismic activities. Both use cross section time series; both take taxes and interest rates into account in their price variables. Erickson et al. (1974) ignore royalties and severance taxes whereas Cox and Wright (1976) expense royalties for severance tax purposes. Since severance taxes appear to be more often levied on gross revenue, this is a potential for bias. Both studies find total reserves to be responsive to their price variable but the elasticity in the short run is low < 0.1 with an implied new reserve elasticity roughly ten times as large.[12] These new reserve elasticities are not out of line with others that use a profit variable. However, the long run new reserve elasticity in Erickson et al. (1974), that are implied by this adjustment (>7) seems unbelievably large.

Another modification of the Fisher approach is Bouhabib (1979). He argued that reserve additions change with the recovery rate of reserves. In light of this he estimated equations for drilling measured in total feet, finding measured by reserves in place per foot and recovery rate measured as share of reserves in place that are produced. His results on a cross section of eight large oil producing states are very disappointing. Coefficients for price are insignificant in all three of his original equations and for equations estimated on only onshore states. Results are erratic in other stratifications for states with offshore productions. Since this approach explained less than 65% of the recovery rate and reserves per foot it does not seem to provide a superior alternative.

A few studies have modeled finding rates. Chollett (1988) estimated an oil reserve discovery per foot drilled equation using three year averages for the US. He found a depletion effect with discoveries falling as cumulative depth increased, the price coefficient to be significant at the 10% level. However, we do not consider this model to be an improvement over earlier models since the specification has the non intuitive constraint that his own price elasticity gets larger the closer to depletion and he explains only 74% of the variation in discoveries.

Most other studies do not include a price variable in their discovery equations including Wall (1991), (1994), Berman and Tuck (1994), and Iledare (1995). This seems reasonable given its lack of significance in the Fisher models earlier. Porter (1992) and Cleveland and Kaufman (1992) are the two discovery models we find most intriguing. Both are rather simple with relatively good fit. Both are on fairly aggregate data. Porter (1992) regressed cumulative oil discoveries on cumulative wells drilled. His lowest R^2 is 0.97 for Alaskan gas on data from 1966 to 1988. Cleveland and Kaufmann (1991) use an update of the model in Hubbert (1962) and fitted discoveries per foot (YPF) in an exponential model that included price.

$$YPF_t = YPF_0 e^{-\lambda\Sigma Wx} e^{\delta p} e^{\phi Wx}$$

where p is the real well-head price of oil, Wx is the rate of exploratory drilling, and ΣWx is cumulative exploratory drilling. They estimated this equation for various years with their longest sample starting in 1859 and ending in 1988. They get most impressive results for such a long time period. Their price coefficient is highly significant, their other coefficients are significant and plausible, and they explain over 88% of YPF for samples starting after 1925 and 84% for the whole period. Reserves per foot fall as exploratory drilling increases suggesting diminishing marginal product for drilling and reserves per foot fall with cumulative drilling suggesting a depletion affect. Their results are not presented in an elasticity form but we used their estimates to derive a reserve price elasticity of 1.

Both of these models could be the basis for further work. Both include cumulative drilling which may be a proxy for geological variables but neither includes seismic information. Besides developing seismic variables it would be interesting to investigate whether Porter's model fits as well over longer periods and whether a profit variable has any explanatory power in either model. Both models and their offspring could be tried on more disaggregate data as well as checking for stability in the estimates across time.

VII. EXPLORATION MODELS FOR NATURAL GAS

Gas, once the Cinderella of the petroleum world, has been increasing in importance as a clean fuel alternative with abundant reserves.[13] A number of studies have either estimated natural gas price along with their oil exploration model as in Fisher or have completely separate models for natural gas. These own elasticities are reported earlier in Table 2 with more complete model information in Table A2. Gas models tend to suffer the same omissions as in the oil models and equations that deal exclusively with gas usually have a poorer fit. In the Fisher (1964) approach, an equation for gas discoveries per well was included. However, since he did not model gas discoveries to depend on gas prices, no own price elasticity of supply can be determined. But he did find that gas reserves fell as oil prices increased with elasticities varying from -2.01 to -1.55 depending upon how the price was deflated.

Erickson and Spann (1971) applied the Fisher model but included natural gas prices in the gas reserve and all other equations. However, none of these prices were significant. Kolb (1979) also included a discovery equation for gas per successful gas wildcat well. The fit on these equations were even worse than for the oil discovery equations. Neither oil nor gas price was found to significantly influence gas discoveries. All R^2s were less than 0.46. The only significant variables in the equation were the district dummy variables again suggesting that geology rather than price may be a dominating factor with a lot of routine randomness.

Khazzoom (1971) estimated an equation for the volume of natural gas discovered as a function of the prices of oil, gas, natural gas liquids, the effect of price ceilings, and lagged endogenous variables. He reported no elasticities but reported that increases in the price ceiling resulted in a lower price elasticity. When Khazzoom's model was re-estimated by Pindyck (1974), oil and natural gas prices were not found to be significantly correlated with new discoveries. (Kimmel, 1977).

Pindyck (1974) re-estimated the Erickson and Spann model for gas using 18 production districts and total exploratory wells instead of wildcats. The explanatory power of his success rate and gas discovery equation are very low at around 0.25. He found the Erickson-Spann model to be questionable because of their low t-statistics and the inability to replicate their results. MacAvoy and Pindyck (1973) modeled the gas industry's new discoveries in a similar but more detailed approach than in Erickson and Spann. They included estimates of exploratory wells, new discoveries of oil and gas, oil and gas well success ratios, development wells, gas and oil extensions, gas and oil revisions, production from reserves, pipeline mark up, an offshore submodel, demand by sector and region, and depletion variables.

They did not provide us with direct elasticities but we can infer elasticities from their simulations (p. 489) as well as simulations of their model provided from updated versions of their model through 1972 and 1974 in Pindyck (1978, p. 190). These simulations imply the following elasticities with respect to gas price which is set exogenously by regulations. On data prior to 1974 new gas discovery elasticities are near 1.5 with the elasticity of additions somewhat smaller. On data through 1974, we see a substantial decrease in elasticities, the new gas discovery elasticity is 1.17 and the reserve addition elasticity is 0.48. However, since the price of natural gas was not significant in any of the relevant equations for these simulations we expect that none of these elasticities is statistically significant.

Two authors discussed earlier estimated natural gas elasticities using models derived from a joint production function from which we could compute elasticities. Again the basic framework has intuitive appeal but too little statistical information is reported to properly evaluate the studies. Surprisingly Epple (1975) found a significant drilling elasticity with respect to natural gas price of 0.74, whereas oil price was not significant, and his total reserve elasticity for gas was the same as for oil varying from 2.87. Deegan (1979) found a total reserve elasticity of 2.10 when oil and gas price changed by the same percent.

Wilkinson (1983) had a fairly extensive model of the natural gas market. His 17 supply side equations include reserve additions, well costs, wells drilled, abandonments, and reserve discoveries. All equations were estimated as linear equations with no elasticities given so we compute elasticities based on the values for prices and quantities for the simulation in the paper for 1980. His elasticity of wildcat gas wells drilled with respect to wellhead gas price is 0.15, which is very near to his elasticity for total wells drilled.

Chollett (1988) estimated a reserve discovery per foot equation for natural gas using a similar model to his oil equation. He found a depletion effect for gas with discoveries falling with increases in cumulative depth and a highly significant price elasticity, which is less than 0.61 in the short run and less than 0.83 in the long run. As in his oil discoveries equation, his specification has the undesirable property that discovery elasticity goes up as you approach depletion.

Iledare (1995) modeled natural gas reserve additions in W. Virginia using a drilling equation and a total new reserves equation on well data for 1977–1987. His is the only model that includes a profit rather than a price variable. He does, however, also report a price elasticity. Since gas production is over 9 times as large as oil production in W. Virginia, drillers are probably targeting their search for natural gas. His value or profit elasticity was a significant 0.52 which implied an elasticity with respect to wellhead price of 1.02. He found drilling to decrease with increases in cumulative feet drilled and with increases in the reserve over production ratio. His total new reserve additions per foot are not directly a function of profits but the equation includes drilling, cumulative drilling, a time trend, and formation dummies. He found a depletion effect with reserves per foot falling as cumulative drilling increases. At the same time he found reserves per foot increasing over time which might result from improved technology. He found more drilling in Mississippi and Upper Devonian formations but less gas found per foot drilled showing the importance of geological factors. The fit on his equations were unusually good compared to the other studies on natural gas with R^2's over 0.92 for both equations. We attribute at least part of this to the homogeneity of his sample. His discovery model and Porter (1992), which also has good fits on his gas equations, provide promising models for further work on natural gas. One could also experiment to see if Cleveland and Kaufman (1991) provide any long term explanatory power for gas as it did for oil.

From the above studies there is less evidence that drilling responds to natural gas price than to oil. However, the most recent study on well data in a gas prone region suggests that there is significant response with a price elasticity near 1 with a smaller profit drilling elasticity. We attribute the earlier lack of drilling response to low controlled gas prices and would urge more work estimating drilling in the post decontrol era to get a clearer picture of the effect of gas prices.

VIII. CONCLUSIONS

Numerous econometric models have been estimated over the years to help explain the oil and gas exploration process and its response to oil and gas prices. Most models include equations for drilling and some way of measuring discoveries such as well success rates and discoveries per successful exploration effort. Where all three equations are available they can be combined to yield total new reserves. In some cases total reserves or total new reserves are estimated directly or estimated with drilling in a production function context. The best results have been obtained on drilling equations and we would argue that this equation should be retained in the model but with improved specification. Dividing discovery per effort into success rate and discovery rates per successful exploration effort has not been as successful. We would suggest that discovery per effort is probably the better dependent variable with more work along the lines of Cleveland and Kaufman (1991) or Porter (1992).

Since the majority of the studies omit some costs in their drilling equation either lifting costs, taxes or capital costs or both, they are poorly specified and elasticity estimates are potentially biased. Most studies include some proxy for geological information but there has been no systematic study of which geological variables are the best. Little attention has been paid to geophysical inputs.

In all of the above modeling there is overwhelming evidence that drilling responds to oil price and oil profits. Both weakly specified models and strong ones suggest such a relationship and most find the overall fit for drilling equations to be relatively good. The results to date suggest that the drilling price elasticity may be greater than 1 whereas the drilling profit elasticity is likely to be less than 1. Although discounted profit is the desirable variable, it would be helpful for studies to compute their implied price, cost and tax elasticities to increase their usefulness. Further, since associated gas by 1996 accounted for 25% of revenues from oil wells,[14] gas prices should be included in the profit variable with a separate gas price elasticity also computed.

Drilling appears to respond the most to current prices but not all adjustment seems to occur in one year. More work on expectations and the precise shape of the lags would be useful. The explanatory power for the well success rate and discovery per successful well equations were not very good and we would encourage dropping these equations from the econometric model and instead model discoveries per well or per foot. Two particular models on fairly aggregate data seemed to show promise. One regressed cumulative discoveries on cumulative drilling another estimated an exponential model of discoveries per foot on cumulative drilling, current drilling and price. Both these models could be modified to include geophysical activity, oil and gas after tax profits, and they could be checked for stability across time and regions.

Although oil has been the target of more of the modeling efforts, some studies have specifically targeted natural gas. It appears that gas has been the more difficult to model in earlier studies, perhaps because of price controls and the dominance of oil. Gas prices were seldom found to influence the exploration process. However a recent study on a gas prone region found quite acceptable results for a drilling equation with a price elasticity similar to that for oil. Combining oil and gas prices together into a profit variable has also shown promise. More work along these lines for other regions would be encouraged.

Geologists confirm that directionality of drilling exists and directionality was explicitly included in a few models. This was done by considering discoveries per successful oil or gas well instead of discoveries per successful wells and in one case the mix between wells was explicitly modeled. Since these models did not improve on earlier models, and if anything displayed poorer fits, more work could be done to try to incorporate directionality of drilling into the modeling effort but on models that model gas discoveries directly as in Porter (1992) rather than success rates.

In Fisher and some of the later work numerous lagged endogenous variables were included in each equation. Although this has a tendency to improve the fit of the model and probably helped pick up some of the variation across regions, the implied long run elasticities when all these indirect effects are taken into account are sometimes quite doubtful. We would urge caution in their use. If they are used their implications on the long run elasticities should be explicitly taken into account.

Some studies account for regional variation by using dummy variables. However, very little work has been done to more explicitly investigate differences across regions. PADD I, which is more predominantly a gas region, appears to be different and has been left out of some estimates. However, no systematic differences have been detected. Offshore may have a lower drilling elasticity. One study explicitly tried to investigate differences between onshore and offshore but results were too poor in both cases to make any inferences from them. We would urge more work at the regional level to explicitly investigate differences across regions and more work on geological variables available to evaluate their relative performance in forecasting drilling and discovery rates.

APPENDIX A

Table A1. Data Sources for Oil ad Gas Exploration Models

Years	*Time*	*Aggregate*	*Source*	*Notes*
Wellhead Price of Crude Oil:				
1918-1996	annual	U.S.	A*, S	
1947-1996	annual	state	B*	
1859-1993	annual	U.S.	E*	
1974-1993	monthly	U.S.	E*	
1901-1992	annual	state	E*	
current	weekly	U.S.	O*	
Wellhead Price of Natural Gas:				
1947-1996	annual	U.S.	B*, S	
1947-1996	annual	state	B*, S	
1959-1996	annual	U.S.	F*	Real
1949-1996	annual	U.S.	F*	Nominal
1922-1996	annual	U.S.	A*	
1973-1995	annual	U.S.	C*	
1993-1994	monthly	U.S.	C*	
New Oil Reserves:				
1977-1995	annual	U.S.	F*	
1918-1997	annual	U.S.	A*	
periodical	annual	state	A*	
1966-1996	annual	lower 48	A*	
1947-1996	annual	U.S.	B*, T	
1947-1996	annual	state	B*, V, 2	
1990	annual	U.S.	G*	Undiscovered, Onshore, Offshore, Inferred
1990	annual	lower 48	G*	Unproved, Proved
1966-1973	annual	U.S.	H*	
1994	annual	regional	I*	Undiscovered conventional, unconventional, onshore, state offshore
New Natural Gas Reserves:				
1960-1995	annual	U.S.	J*	
1977-1995	annual	U.S.	F*	
1981-1995	annual	U.S.	K*	Natural Gas Liquids
1947-1996	annual	U.S.	B*, K, T, V	
1947-1996	annual	U.S.	B*, S, T, V	Natural Gas Liquids

1947-1997	annual	state	B*	
1966-1996	annual	lower 48	A*	
1994	annual	regional	L*	
1977-1996	annual	state and subdivision	F*	Reported Reserves of Natural Gas, Non-associated and Associated Dissolved Gas
Cost of Production:				
1973-1991	annual	U.S.	B*, 8	Exploration, Development, and Production
1991-1995	annual	U.S.	B*, Y	U.S. Oil Industry Capital Expenditures
1978-1991	annual	U.S.	B*, Y	Oil Finding and Development Costs
current	annual	PADD	G*	Summary of Fixed Costs by PADD
Lifting/Severance Taxes				
various years	annual	state	A*, S	
Mar. 1980 - Dec. 1985	quarterly	U.S.	B*	
Interest Rates:				
1947-1996	annual	U.S.	B*, S	
Drilling Costs:				
1959-1996	annual	U.S.	A*	
1959-1995	annual	U.S.	A*	Offshore
1959-1996	annual	U.S.	B*, Z, 6, 7	Offshore, Onshore
current	annual	U.S.	P*	Estimated Costs of Drilling and Equipping Wells, plus lower 48
Seismic Variables:				
1949-1996	annual	U.S.	F*	
1965-1993	annual	U.S.	F*	Onshore, Offshore
1949-1996	annual	U.S.	A*	Seismic Activity and Rotary Rings Running in the U.S.
1954-1991	annual	U.S.	B*	International Petroleum Geophysical Exploration (crew months), U.S.
FQ 1988 - FQ 1998	quarterly	U.S.	D*	
FQ 1988 - FQ 1999	year-to-date	U.S.	D*	
Total Oil Production:				
1949-1996	annual	U.S.	F*	
1954-1996	annual	U.S.	F*	Crude Oil and Lease Condensate Production
1977-1995	annual	U.S.	F*	Crude Oil Ultimate Recovery
1918-1997	annual	U.S.	A*	U.S. Crude Oil Production During Year
1918-1996	annual	U.S.	A*	United States Domestic Production of Crude Oil
1918-1996	annual	PADD	A*	Crude Oil Production by PAD District
1921-1996	annual	PADD	A*	Daily Average Crude Oil Production Per Producing Oil Well

(continued)

Table A1. (Continued)

Years	*Time*	*Aggregate*	*Source*	*Notes*
Total Oil Production:				
current	annual	U.S.	N*	USA Petroleum Production
1971-1995	annual	U.S.	M*	
1947-1996	annual	U.S.	B*, U, T, S	U.S. total Production of Energy Resources by Major Source, Petroleum
1947-1996	annual	state	B*, 3	U.S. Crude Oil Production per Well Per Day by State
1917-1997	monthly	U.S.	B*	U.S. Crude Oil Production by Month
1859-1946	annual	state	B*	Historical U.S. Crude Oil Production by State
cumulative to 1953, 1954-1996	annual	U.S.	B*, 5	Production and Value of U.S. Crude Oil and Condensate, On vs. Offshore
two previous years	monthly	state	H*	OGJ Production Report
two prev. months for two prev. years	monthly	state	I*	U.S. Oil Production
current week for current and prev. year	weekly	state	J*	
1973-1995	annual	U.S.	C*,	Crude Oil Field Production
1977-1965	annual	U.S.	F*	Natural Gas Fields Cumulative Production
1918-1996	annual	U.S.	A*	U.S. Domestic Production of Natural Gas Plant Liquids
1966-1996	annual	Lower 48	A*	Lower 48 States Natural Gas Production
1946-1996	annual	U.S.	A*	U.S. Natural Gas Liquids Production
1918-1996	annual	PADD	A*	Market Production of Natural Gas, by PAD District
1960-1979	annual	U.S.	K*	Preliminary Net Production during Year of Natural Gas in the U.S.
1979-1995	annual	U.S.	K*	Production of Dry Natural Gas in the U.S.
1971-1994	annual	U.S.	K*	Supply of Gas in the U.S. Marketed Production and Gross Production
current	annual	U.S.	N*	World Petroleum Production (including Natural Gas Liquids)
1947-1996	annual	U.S.	B*, S, T, W	U.S. Total Production of Energy Resources by Major Source, Natural Gas
cumulative to 1953, 1954-1996	annual	U.S.	B*	Production and Value of U.S. NATURAL GAS, ON VS. OFFSHORE
1947-1996	annual	state	B*, S	United States Marketed Production of Natural Gas by State
monthly for two previous years	monthly	state	H*	OGJ Production Report, by State
total YTD for current and prev. years	annual	U.S.	O*	Production (dry gas)
1973-1995	annual	U.S.	C*, X	Natural Gas Plant Liquids Field Production, Dry Gas Production

three most recent years by month	monthly	U.S.	C*, X	Natural Gas Plant Liquids Field Production, Dry Gas Production
Drilling Variables:				
1949-1996	annual	U.S.	F*	Oil and Gas Exploratory and Development Wells
1949-1996	annual	U.S.	F*	Oil and Gas Exploratory Wells
1938-1996	annual	U.S.	A*	Exploratory Wells Drilled in the U.S.
1944-1996	annual	U.S.	A*	Total New Field Wildcats, Number of Productive New Field Wildcats
1947-1997	annual	U.S.	B*, Z	U.S. Exploratory Wells and Footage, New-Field Wildcat Wells
1859-1946	annual	U.S.	B*, 4, W	Historical Number of Wells Drilled in the U.S. by Type
1973-1995, 3 most recent yrs. by mo.	annual	U.S.	C*	Oil and Gas Wells Drilled; Exploratory, Development, Total
FQ 1988 - FQ 1998	FQ, YTD	U.S.	D*	Summary of Estimated Well Completions and Footage Drilled
1988-1998	quarterly	U.S.	D*	U.S. Total: Exploratory and Development Wells, Wells and Footage
1988-1998	YTD	U.S.	D*	Exploratory, Development, New-Field Wildcat Well Completions and Footage Drilled

Notes:
A DeGolyer and Mac Naughton, Twentieth Century Petroleum Statistics
B American Petroleum Institute (API), Basic Petroleum Data Book, Petroleum Industry Statistics
C Energy Information Administration (EIA), Monthly Energy Review
D American Petroleum Institute, Quarterly Well Completion Report
E Oil & Gas Journal, Energy Database, Refining Statistics Sourcebook
F www.eia.doe.gov
G Enery Information Administration, Supplement to the Annual Energy Outlook
H *Oil & Gas Journal*, Data Book
I *World Oil*, "industry at a glance"
J *Oil & Gas Journal*, OGJ Production Report
K Gas Facts
L USGS, *Economics and the 1995 Assessment of United States Oil and Gas Resources*, USGS circular 1145
M International Energy Agency, Energy Balances of OECD Countries
N Financial Times, Energy Yearbooks, Oil & Gas
O *Oil & Gas Journal*, Statistics
P API, Joint Association Survey on Drilling Costs
Q www.census.gov
R various state Dept. of Rev. or Bureau of Taxation internet sites
S U.S. Energy Information Administration
T U.S. Department of Energy
U U.S. Bureau of Mines
V American Gas Association
W U.S. Bureau of Mines
X EIA, Natural Gas Annual
Y The Chase Manhattan Bank
Z American Association of Petroleum Geologists
1 API
2 CPA
3 U.S. Bureau of Mines, Minerals Yearbook and Annual Petroleum Statement
4 *Oil & Gas Journal*
5 U.S. Department of Interior, Bureau of Mines, Mineral Industry Surveys
6 Independent Petroleum Association of America
7 Mid-Continent Oil and Gas Association
8 U.S. Department of Commerce, Bureau of Census
9 Internal Revenue Service, SOI Bulletin
10 Standard & Poor's, Bond Guide

Source: *indicates the reference is a primary sources

Table A2. Additional Information for Oil and Gas Exploration Models

Reference	*Sample*	*Dependent Variable*	*Independent Variables*	*R2*
Fisher (1964)	46-55 PADD 1-5, CT	Ww	Seismic+*, Poil deflated WPI+*, O/Wws(-1)+*, Wws/Ww(-1)-*, G/Wws(-1)-*	0.84
		Wws/Ww	Wws/Ww(-1)+*, O/Wws(-1)-*, G/Wws(-1)+*, Depth(-1)-*, Seismic+*, Poil deflated WPI	0.71
		O/Wws	O/Wws(-1)+*, Wws/Ww(-1)+*, G/Wws(-1)+*, Poil deflated WPI-*	0.85
		G/Wws	O/Wws(-1), Wws/Ww(-1)+*, G/Wws(-1), Poil deflated WPI-*	0.32
		Ww	Seismic+*, Poil deflated by drill cost+*, O/Wws(-1)+*, Wws/Ww(-1)-*,G/Wws(-1)-*, Depth(-1)-*,D*TXshut down, D*nonTxshutdown +*	0.91
		Wws/Ww	Wws/Ww(-1)+*, O/Wws(-1)+*, G/Wws(-1), Depth(-1), Seismic+*,Poil deflated by drill costs	0.72
		O/Wws	O/Wws(-1)+*, Wws/Ww(-1)+*, G/Wws(-1)-*,Poil deflated by drill cost-*	0.86
		G/Wws	O/Wws(-1), Wws/Ww(-1)+*,G/Wws(-1), Poil deflated by drill cost-*	0.23
		Seismic	Seismic(-1)+*,4 D*PADD some*, D*Txshut down on Tx, D*TXshutdown+*	0.97
Erickson (1968)q	46-58 PADD 1-5, CT	Wwp	Deflated Poil+*, Depth(-1)+*, Wwps/Wwp(-1)-*, Wwlarge/Wwus+*, D*PADD some*,D*TXshut down, D*NonTXshutdown+*	0.97
		Wwps/Wwp	Deflated Poil, Wwlarge/Ww+*, D*PADD some*, D*Txshutdown, D*NonTXshut-down	0.71
		O/Wwps	Deflated Poil, Wwps/Wwp(-1)+*, D*PADD some*, D*Txshutdown-*, D*NonTX-shutdown	0.89
Erickson & Spann(1971)	46-59 PADD 1-5, CT	Wwp	Deflated Poil+*, Deflated Pgas, Depth(-1)+*, Wwps/Wwp(-1)-*, Wwlarge/WwUS+*, D*PADD some*, D*TXshut down, D*NonTXshutdown	0.97
		Wwps/Wwp	Deflated Poil, Deflated Pgas, D*PADD, D*TXshut down, D*NonTXshutdown	0.80
		O/Wwps	Deflated Poil, Deflated Pgas, Wwps/Wwp(-1), D*PADD some*, D*TXshut down-*, D*NonTXshutdown	0.89
		G/Wwps	Deflated Poil, Deflated Pgas,Wwps/Wwp(-1), D*PADD some*, D*TXshut down, D*NonTXshutdown	0.60
MacAvoy and Pindyck (1973)	64-71 18 Dist, CT	Wx	3 D*dist some *, Deflated revenue oil and gas+*,drill cost(-1)-*, risk index of variance in district discovery size-*	0.50
		Gna/Wx	3 D*dist some *, 3ma Pgas+*, 3ma drill cost+*, Σ value of exploratory well since 1963*	0.63
		Ga/Wx	3 D*dist some *, 3ma Poil, 3ma drill cost+*, Σ value of exploratory well since 1963*	0.60

Erickson et al. (1974)	50-68 5 PADD CT	Ro	Deflated Poil net of taxes+*, D*Txshutdown, D*NonTxshutdown, Ro(-1)+*, D*PADD	0.99
Pindyck (1974)	64-69 18 Dist, CT	Wx	Deflated Poil, Deflated Pgas, Depth(-1), Wxs/W(-1), Ratio of district wells to total wells drilled+*, 3 D*Dist	0.99
		Wxs/Wx	Deflated Poil+*, Deflated Pgas+*, 3 D*Dist some *	0.27
		G/Wxs	Deflated Pgas+*, Deflated Poil, 3 D*Dist some *, Wxs/Wx(-1)	0.27
Bouhabib (1975)	66-73 8 Lower 48	Wx(ft)	Poil delated WPI, rate of change in RecRt, RecRt(-1), Ro/Qo(-1)+*, depth(-1), 7 D*State some *	0.94
	States	RecRt	Poil deflated WPI, rate of change in RecRt-*, RecRt(-1)*, Ro/Qo(-1)*, Wds+*, 7 D*State some *	0.05
		OIP/Wx(ft)	Poil deflated WPI, rate of change in RecRt, RecRt(-1)+*, ΣWx(ft)-*, depth, 7 D*State some *	0.63
		Wx(ft)	Poil deflated WPI+*, rate of change in RecRt, RecRt(-1)+*, Ro/Qo(-1)+*, depth(-1), 2D*Offshore state	0.83
		RecRt	Poil deflated WPI, rate of change in RecRt, RecRt(-1)+*, Wws/Ww(-1), 2D*Offshore state	0.13
		OIP/Wx(ft)	Poil deflated WPI+*, rate of change in RecRt, RecRt(-1)-*, ΣWx(ft), depth, 2D*Offshore state+*	0.23
	66-73 3 Lower 48	Wx(ft)	Poil deflated WPI, rate of change in RecRt, RecRt(-1)*, Ro/Qo(-1), depth(-1), 2D*Offshore state some *	0.98
	Offshore States	RecRt	Poil deflated WPI+S*, rate of change in RecRt*, RecRt(-1)* Ro/Qo(-1), Wws/Ww, 2D*Offshore state*	0.44
		OIP/Wx(ft)	Poil deflated WPI-*, rate of change in RecRt, RecRt(-1), ΣWx(ft)*, depth, 2D*Offshore	0.18
	66-73 5 Lower 48	Wx(ft)	Poil deflated WPI, rate of change in RecRt, RecRt(-1), Ro/Qo(-1), depth(-1),4 D*Onshore state	0.81
	Onshore States	RecRt	Poil deflated WPI-*, rate of change in RecRt*, RecRt(-1)-*, Wds+*, 4 D*Onshore state some *	0.14
		OIP/Wx(ft)	Poil deflated WPI, rate of change in RecRt, RecRt(-1)+*, ΣWx(ft)-*, depth, 4 D*Onshore some *	0.64
Epple (1975)	46-68 TS	Ww(ft)	Drill costs-*,Expected Poil+*, Expected Pgas+*, Σ Ww(ft)+*	nr
		O	Drill costs-*,Expected Poil+*, Expected Pgas/Poil, Σ Ww(ft)-*	nr
		G	Drilling costs-*,Expected Pgas+*, Expected Poil/Pgsas, Σ Ww(ft)-*	nr
Cox & Wright (1976)	5 States CT	Rogn	After tax price index Poil, Pgas, Pngl+*,output of oil, Ng, and NGL+*, % days proration TX-*, time-*	0.97

(continued)

Table A2. (Continued)

Reference	Sample	Dependent Variable	Independent Variables	R2
Rice & Smith (1977)	46-73 TS	Ww	Price variable is total crude oil revenue for US, Drill costs, Wws(-1)+*	0.88
Eyssel (1978)	46-59 PADD1-5, CT	Ww	Deflated Poil+*, Depth(-1), Wwos/Ww(-1),% days proration TX-*, % days proration non-TX, D*PADD(nr)	0.94
		Wwos/Ww	Deflated Poil, Wwlarge/Wwus, D*PADD(nr), % days proration TX, % days proration non-TX+*	0.92
		O/Wwos	Deflated Poil, Wwos/Ww(-1),D*PADD(nr),% days proration TX, % days proration non-TX	0.84
	46-70 PADD1-5, CT	Ww	Deflated Poil+*, Depth(-1), Wwos/Ww(-1), % days proration TX, % days proration non-TX-*, D*PADD(nr)	0.94
		Wwos/Ww	Deflated Poil, Wwlarge/Wwus+*, D*PADD(nr), % days proration TX, % days proration non-TX+*	0.77
		O/Wwos	Deflated Poil+*, Wwos/Ww(-1)+*, D*PADD(nr), % days proration TX+*,% days proration non-TX	0.76
	46-70 27 Dist, CT	Ww	Deflated Poil+*, Wwlarge/Ww+*,Depth(-1)+*, Wws/Ww(-1)+*, 10 state proration variables some *, D*PADD(nr)	0.84
		Wws/Ww	Deflated Poil,Wwlarge/WwUS, D*PADD(nr),10 proration variables some *	0.46
		O/Wws	Deflated Poil, Wws/Ww(-1)+*, D*PADD(nr),10 proration variables some *	0.86
Pindyck (1978)	64-74 18 Dist, CT	Wx	4 D*dist some *, 3ma expected oil and gas revenues for a new well, S. Louisiana price dummy, well cost -*, bond rate(-1)-*	0.30
		Wxos/Wx	trinomial logit, 18 D*dist some *,	0.75
		Wxgs/Wx	trinomial logit, 18 D*dist some *,	0.78
		O/Wxos	3 D*dist*, Σ Wx-*	0.80
		G/Wxgs	3 D*dist some *, deflated Pgas plus expected change Pgas, deflated Poil plus expected change Pgas-*, ΣWx	0.66
Attanasi (1979)	Denver Basin 49-73	Ww	Expected gross profit not discounted+*,Σ Ww, $value of discoveries(0),and (-1) to (-10)+*	0.92
Deegan (1979)	53-69 CT	Gna/O	Elasticity comes from the estimation of these three nonlinear equations. Ro/Qo, Rg/Qna,Pgas/Poil	0.68
	46-69	O	D*regional land input+*,Pgas(nr),Poil(nr),WPI(nr),	0.78
	8 US Gulf Regions	Gna	D*regional land input+*,Pgas(nr),Poil(nr),WPI(nr),	0.78
Kolb (1979)	46-58 7 Dist, CT	Ww	Wws/Ww(-1), O/Wws(-1), G/Wws(-1)-*, Depth-*, Seismic+*, Poil deflated WPI+*	0.75

		Wws/Ww	O/Wws(-1), Wws/Ww(-1)+*, G/Wws(-1)+*, Depth , Seismic+*, Poil deflated WPI+*	0.65
		O/Wws	O/Wws(-1)+*, Wws/Ww(-1), G/Wws(-1), Poil deflated WPI-*	0.50
		G/Wws	O/Wws(-1), Wws/Ww(-1)+*, G/Wws(-1)+*, Poil deflated WPI, Pgas deflated by WPI	0.30
	46-58 6 Dist, CT	Ww	Wws/Ww(-1), O/Wws(-1), G/Wws(-1)-*, Depth-*, Seismic+*, Poil deflated WPI+*	0.73
		Wws/Ww	O/Wws(-1), Wws/Ww(-1)+*, G/Wws(-1)+*, Depth, Seismic+*, Poil deflated WPI	0.73
		O/Wws	O/Wws(-1)+*, Wws/Ww(-1), G/Wws(-1), Poil deflated WPI	0.33
		G/Wws	O/Wws(-1), Wws/Ww(-1)+*, G/Wws(-1)+*, Poil deflated WPI, Pgas deflated by WPI	0.32
	48-70 7 Dist, CT	Ww	Poil deflated WPI+*, Pgas delated WPI+*, capacity utilization, drill cost-*, 4 ma Wws/Ww-*,Ro plus Rg, 7D*dist*	0.88
		Wws/Ww	Poil deflated WPI, Pgas deflated WPI+*, capacity utilization, drill cost+*, Ro plus Rg+*, 7 D*dist*	0.67
		O/Wwos	Poil deflated WPI, Pgas deflated WPI, capacity utilization, drill cost, 4ma Wws/Ww+*,4ma G/Wwsg,Ro, 7D*dist	0.57
		Gna/Wwgs	Poil deflated WPI, Pgas deflated WPI, capacity, drill cost, 4ma Wws/Ww-*,4ma O/Wwos-*,Rg, 7D*dist	0.31
		Ga/O	Poil deflated WPI-*, Pgas deflated WPI, 7D*dist	0.34
		Wwgs/Wws	Poil deflated WPI-*, Pgas deflated WPI+*, 4ma O/Wwos,4ma Gna/Wsgs,Rg/Ro,7D*dist	0.65
		Depth	Ro,Rg-*,7D*Dist*	0.95
Wilkinson (1983)	58-79 US TS	Wwg	Ww+*, Wwg/Ww(-1)+*, Pgas/(Poil-WPT)+*	0.82
		G/Wwg	G/Wwg(-1)+*,Rg+*	0.93
Griffin & Moroney (1985)	Texas	W	Expected profit variable	
Chollett (1988)	Gulf Mexico 56-85 TS	3ma(O/Wx(km))	ΣWx(km)(-1)-*,ΣO/Rou(-1) times (Poil-WPT)+*, 3ma(O/Wx(km))(-1)+*	0.74
	Offshore	3ma(G/Wx(km))	ΣWx(km)(-1)-*,ΣG/Rgu(-1) times Pgas+*, 3ma(G/Wx(km))(-1)+*	0.61
Deacon et al. (1990)	48-81 California TS	Wx	Poil deflated by WPI*, Wx(-1)*,time*, D*WPT	0.91
Cleveland& Kaufmann (1991)	US 25-88 TS	O/Wx(frt)	ΣWx(ft)-*, deflated Poil+*, Wx(ft)-*	0.89
Ghouri (1991)	ARCO US 74-89 TS	Wx	Deflated Poil+*, Poil(-1), Poil(-2), Poil(-3)+*, Poil(-4), Exploration costs-*	0.90
	Mobil 70-88 US TS	Wx	Deflated Poil+*, Poil(-1)+*, Poil(-2), Poil(-3)+*, Poil(-4)+*, Exploration costs-*	0.94
	Phillips 70-88 US TS	Wx	Deflated Poil+*, Poil(-1)+*, Poil(-2), Poil(-3)+*, Poil(-4)+*, Exploration costs-*	0.96
	Shell 70-88 US TS	Wx	Deflated Poil+*, Poil(-1), Poil(-2)+*, Poil(-3)+*, Poil(-4)+*, Exploration costs-*	0.94

(continued)

Table A2. Additional Information for Oil and Gas Exploration Models

Reference	Sample	*Dependent Variable*	*Independent Variables*	*R2*
Walls (1991)	73-87 TS	Wx	Price variable is present value of after tax cash flow(nr), marginal product of drilling(nr),ΣWx	0.99
		O/Wx	O/Wx(-1)+*	0.45
Porter (1992)	66-88 TS Lower 48	Wo(ft)	marginal after tax oil profit per barrel adjusted for drilling productivity+*	0.80
	Onshore	Wg(ft)	marginal after tax oil profit per barrel adjusted for drilling productivity+*	0.88
	66-88 TS Lower 48	Wo(ft)	marginal after tax oil profit per barrel adjusted for drilling productivity+*	0.81
	Offshore	Wg(ft)	marginal after tax oil profit per barrel adjusted for drilling productivity+*	0.72
	66-88 TS Lower 48	Wo(ft)	marginal after tax oil profit per barrel adjusted for drilling productivity+*	0.76
	AK	Wg(ft)	marginal after tax oil profit per barrel adjusted for drilling productivity+*	0.25
	66-88 TS Lower 48	ΣO	time, ΣWo+*	0.99
	Onshore	ΣG	time, ΣWg+*	0.99
	66-88 TS Lower 48	ΣO	time*, ΣWo+*	0.99
	Offshore	ΣG	time, ΣWg+*	0.99
	66-88 TS Lower 48	ΣO	time, ΣWo+*	0.99
	AK	ΣG	time, ΣWg+*	0.97
Berman & Tuck (1994)	US 70-89 TS	W	Poil*,ΣO/ΣW*,Time*	0.91
		ΣO/W	Poil*, W*, ΣO/ΣW*, Time*	0.62
Walls (1994)	Gulf OCS 72-88 TS	W	Price variable is profits per well+*, OCS tracts leased, D*area wide leasing	0.64
		3maO/3maWs	ΣO-*	0.84
		3maGna/3maWs	ΣGna-*	0.44
		3maGa/3maWs	3maO/3maWs+*	0.38
AlShami (1995)	60-90 US TS	Ww	Weighted Average Poil& Pgas+*, Bond rate, Seismic+*, Drill Cost-*, Ww(-1+)*, Wws/Ww(-1)-*, Crude imports+*	0.97
		Wws/Ww	Weighted Average Poil& Pgas+*, ΣWw-*, Depth(-1)+*	0.87
		O/Wws	Weighted Average Poil& Pgas+*, Bond rate-*, Wds/Wd(-1)*, Σ Ww, Ww(-1)-*, Wws/Ww(-1)-*	0.85
Iledare et al. (1995)	Gulf OCS 78-92 CT	W(ft)	Price variable is expected pre-tax net cash flow from oil and gas+*, taxes-*, 3D* firm size*,3D*firm size times Σ W all firms*	0.93
		O&G/W(ft)	D*3 firm size*, 3D*firm size * Σ W all firms*, time	0.76
Iledare (1995)	WV Wells 77-87 CT	W(ft)	ΣW(ft)-*, price = expected profits+*, Rg/Qg-*,2 D*Geological Formation*	0.92
		G/W(ft)	W(ft),ΣW-*,time+*,2 D*Geological Formation	0.97

Notes: * absolute value of the t statistic is greater than 1.645, the sign before * is the sign of the relevant coefficient. (e.g. Poil+* indicates Poil had a positive coefficient with t > 1.645

Abbreviations

% days proration TX = the % of oil that is prorationed in Texas
% days proration non-TX = the % of oil prorationed outside of Texas
(-1) variable is lagged one period
(ft) measured in feet
(km) measured in kilometes
(nr) indicates a coefficient or statistic has not been reported
3D*firm size = three dummies representing different firm sizes
adj = adjusted r
AK = Alaska
ARCO = ARCO Oil and Gas Company
capacity utilization= the percent of productive capacity being used
CT = cross section time series data
D* = a dummy variable
D*dist = a district dummy variable (e.g. 2 D*dist indicates 2 district dummy variables have been included
D*regional land input = a regional land input dummy variable
D*WPT = dummy for the windfall profit tax
depth = well depth
drill costs = drilling costs
G=gas reserves discovered
Ga = new associated gas reserves found
Gas = natural gas
GasA = associated gas
Gna = new nonassociated gas reserves found
Gulf OCS = Gulf of Mexico outer continental shelf
lower 48 equal the lower 48 states
ma = a moving average (e.g. 3ma = a 3 period moving average of following variable)
Mobil = Mobil Oil Company
NGL = natural gas liquids
O=oil reserves discovered
Offshore States = states that have offshore productions
OIP= oil in place
Onshore States = states with only onshore production
PADD = petroleum administrative defense district
Pgas = the price of natural gas
Phillips = Phillips Petroleum Company
Poil = the price of oil
Qg = production of natural gas
Qo = production of oil
RecRt = primary recovery rate, which is the percent of oil in place that is recovered
Rg = remaining reserves of gas
Rgu = ultimate gas reserves
Ro = remaining reserves of oil
Rogn = remaining reserves of oil gas and natural gas liquids
Rou = ultimate oil reserves
S indicates the following variable has been accumulated
Seismic = seismic crews
Shell = Shell Oil Company
time = a time trend
trinomial logit = estimated using a trinomial logit model
TS = time series data
TX shut down days = the number of days a Texas well was shut down by order of the Texas Railroad Commission
W = total wells drilled
Wds = successful development wells
Wg = total gas wells drilled
Wo = total oil wells drilled
WPI = wholesale price index
WPT = the windfall profit tax on oil
Ws = total successful wells drilled
WV = West Virginia
Ww= new field wildcat wells drilled
Wwg = gas wildcat wells including dry wells
Wwgs = successful gas wildcat wells drilled
Wwlarge = the number of wildcat wells drilled by large firms
Wwos = successful oil wildcat wells drilled
Wwp = new field plus new pool wildcat wells drilled
Wwps = successful new field plus new pool wildcat wells drilled
Wws = successful wild cat wells drilled
Wwus = wildcat wells drilled in the US
Wx = exploratory wells drilled
Wxgs = successful gas exploratory wells drilled
Wxos = successful oil exploratory wells drilled
Wxs = successful exploratory wells drilled

APPENDIX B

The most general form of the Fisher system, which includes all of the models we have encountered, is

$$LnWW = a_{wp}lnP_t + a_{ws}\, lnSW_{-1} + a_{wo}\, lnOW_{-1} + a_{wg}\, lnGW_{-1} + a_{ww}\, lnWW_{-1}$$

$$LnSW = a_{sp}lnP_t + a_{ss}\, lnSW_{-1} + a_{so}\, lnOW_{-1} + a_{sg}\, lnGW_{-1} + a_{sw}\, lnWW_{-1}$$

$$LnOW = a_{op}lnP_t + a_{os}\, lnSW_{-1} + a_{oo}\, lnOW_{-1} + a_{og}\, lnGW_{-1} + a_{ow}\, lnWW_{-1}$$

$$LnGW = a_{gp}lnP_t + a_{gs}\, lnSW_{-1} + a_{go}\, lnOW_{-1} + a_{gg}\, lnGW_{-1} + a_{gw}\, lnWW_{-1}$$

The short run drilling price elasticity is

$$\frac{\partial \ln Ww_1}{\partial \ln P_1} = \alpha_{wp}$$

The Long Run Drilling Price Elasticity (Ww_L) = $\sum_{i=1}^{\infty} \frac{d\ln Ww_i}{d\ln P_1}$
Where:

$$\frac{d\ln Ww_1}{d\ln P_1} = \alpha_{wp}$$

$$\frac{d\ln Ww_2}{d\ln P_1} = \frac{\partial \ln Ww_2 d\ln Sw_1}{\partial \ln Sw_1 d\ln P_1} + \frac{\partial \ln Ww_2 d\ln Ow_1}{\partial \ln Ow_1 d\ln P_1} + \frac{\partial \ln Ww_2 d\ln Gw_2}{\partial \ln Gw_2 d\ln P_1} + \frac{\partial \ln Ww_2 d\ln Ww_2}{\partial \ln Ww_1 d\ln P_1}$$

$$\frac{d\ln Ww_3}{d\ln P_1} = \frac{\partial \ln Ww_3 d\ln Sw_2}{\partial \ln Sw_2 d\ln P_1} + \frac{\partial \ln Ww_3 d\ln Ow_2}{\partial \ln Ow_2 d\ln P_1} + \frac{\partial \ln Ww_3 d\ln Gw_2}{\partial \ln Gw_2 d\ln P_1} + \frac{\partial \ln Ww_3 d\ln Ww_2}{\partial \ln Ww_2 d\ln P_1}$$

Therefore:

$$Ww_L = \alpha_{wp} + \alpha_{ws} \sum_{i=1}^{\infty} \frac{d\ln Sw_j}{d\ln P_1} + \alpha_{wo} \sum_{i=1}^{\infty} \frac{d\ln Ow_j}{d\ln P_1}$$

$$\alpha_{wg} \sum_{i=1}^{\infty} \frac{d\ln Gw_i}{d\ln P_1} + \alpha_{ww} \sum_{i=1}^{\infty} \frac{d\ln Ww_j}{d\ln P_1}$$

Where the expressions in the first three summation signs are the long run elasticity price elasticities for Sw, Ow and Gw, respectively. Call them Sw_L , Ow_L, and Gw_L. And the last is the long run price elasticity for Ww. Then

$$Ww_L = \alpha_{wp} + \alpha_{ws}Sw_L + \alpha_{wo}Ow_L + \alpha_{wg}Gw_L + \alpha_{ww}\, Ww_L \tag{1}$$

By a similar technique you can show that

$$Sw_L = \alpha_{sp} + \alpha_{ss}\, Sw_L + \alpha_{so}\, Ow_L + \alpha_{sg}\, Gw_L + \alpha_{sw}\, Ww_L \tag{2}$$

$$Ow_L = \alpha_{op} + \alpha_{os}\, Sw_L + \alpha_{oo}\, Ow_L + \alpha_{og}Gw_L + \alpha_{ow}\, Ww_L \tag{3}$$

$$Gw_L = \alpha_{gp} + \alpha_{gs}\, Sw_L + \alpha_{go}\, Ow_L + \alpha_{gg}\, Gw_L + \alpha_{gw}\, Ww_L \tag{4}$$

Equations (1)-(4) provide 4 linear equations in 4 unknowns which can be written in matrix notation as:

$$\begin{bmatrix} 1-\alpha_{ww} & -\alpha_{ws} & -\alpha_{wo} & -\alpha_{wg} \\ -\alpha_{sw} & 1-\alpha_{ss} & -\alpha_{so} & -\alpha_{sg} \\ -\alpha_{ow} & -\alpha_{os} & 1-\alpha_{oo} & -\alpha_{og} \\ -\alpha_{gw} & -\alpha_{gs} & -\alpha_{go} & +\ 1-\alpha_{gg} \end{bmatrix} \begin{bmatrix} Ww \\ Sw_L \\ Ow_L \\ Gw_L \end{bmatrix} = \begin{bmatrix} \alpha_{wp} \\ \alpha_{ap} \\ \alpha_{op} \\ \alpha_{gp} \end{bmatrix}$$

With the solution equal to

$$\begin{bmatrix} Ww \\ Sw_L \\ Ow_L \\ Gw_L \end{bmatrix} = \begin{bmatrix} 1-\alpha_{ww} & -\alpha_{ws} & -\alpha_{wo} & -\alpha_{wg} \\ -\alpha_{sw} & 1-\alpha_{ss} & -\alpha_{so} & -\alpha_{sg} \\ -\alpha_{ow} & -\alpha_{os} & 1-\alpha_{oo} & -\alpha_{og} \\ -\alpha_{gw} & -\alpha_{gs} & -\alpha_{go} & 1-\alpha_{gg} \end{bmatrix}^{-1} \begin{bmatrix} \alpha_{wp} \\ \alpha_{sp} \\ \alpha_{op} \\ \alpha_{gp} \end{bmatrix}$$

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NOTES

1. *Oil and Gas Journal*, Worldwide Issue, Dec 31, 1990 and Dec. 29, 1997 and Forecast and Review Issue, January 26, 1998.
2. The depletion allowance was reduced from 27 to 22% in 1970, was eliminated for large produces (those who produced more than 1000 barrels per day) in 1975 and had been further reduced to 15% by 1983. However, as it affected net profits rather than lifting costs per barrel, these changes should not have affected production directly. ((EIA/DOE (1998), Thompson and Wright (1985)).
3. We could model directionality of drilling, which assumes that the driller knows in advance whether oil or gas is more likely to be found and is specifically targeting one or other. This would simply require dividing the Ww variable into a Ww for oil and Ww for gas in the production function and optimizing with respect to each of the drilling variables. Since the results would not be qualitatively different and separate variables are not available for oil and gas wild cat wells drilled, we forego this slight complication.
4. For example Porter (1992) found that discoveries of new fields resembled a normal distribution.
5. For more detail on tax modeling, see Porter (1992; 1995) or Cox and Wright (1976).
6. A more esoteric approach would be to use optimal control to model a time path of drilling explicitly tying cumulative reserves found to N the geological variable. We do not pursue this approach since we did not find any additional insights were gained for modeling a discrete econometric model but only found an added level of difficulty.
7. If price and costs are highly correlated both in the sample and in the forecasting period their omission will not be so serious. For example, suppose the true drilling model is $W = \beta_0 + \beta_1 P - \beta_2 C$ and C is perfectly correlated with P or $C = aP$. Then $W = \beta_0 + \beta_1 P - \beta_2 aP = \beta_0 + (\beta_1 - \beta_2 a)P$. If you estimate the model including only a constant and price your coefficient on P is an estimate for $(\beta_1 - \beta_2 a)$. Your forecast at price P_o would be $\beta_0 + (\beta_1 - \beta_2 a)P_o = \beta_0 + \beta_1 P_o - \beta_2 aP_o$, which would be a good forecast if cost remained correlated in the same way with price as during the sample period. (i.e. $C_o = aP_o$)
8. Such directionality of drilling has been confirmed in private conversations with Professor John Curtis in the Department of Geology and Director of the Potential Gas Committee at the Colorado School of Mines.
9. Elasticities were not reported for either Walls (1991) or Walls (1994) and have been computed from simulation inforormation in the papers. For Walls (1991) the elasticities reported here are based on the simulations of a $5 tariff for 1989 to 1998 using the base oil price for 1988 of $15.40. Since there is some instability in first and second year, the short run estimate is taken as the average elasticity for year 1 and year 2, while the long run elasticity is taken from year 10. No t statistics are available for these elasticities but since most of the coefficients in the estimated model are significant we assume the elasticities are significant as well. For Walls (1994) no profit elasticity was reported but a numerical example implied an oil price drilling elasticity of 0.18.
10. It is easy to show how a drilling profit and price elasticity are related. Let the price elasticity = $\varepsilon_p = (\partial W/\partial P)P/W = (\partial W/\partial \pi)(\partial \pi/\partial P)(P/W)(\pi/\pi)$. Rearranging this last expression we get $\varepsilon_p = (\partial W/\partial \pi)(\pi/W) * (\partial \pi/\partial P)(p/\pi)$ or the drilling price elasticity is equal to the drilling profit elasticity times the elasticity of profits with respect to price of $\varepsilon_p = \varepsilon_\pi * e_{\pi,p}$. Then let profit per barrel $\pi = P - C$. If price changes then $e_{\pi,p} = (\partial \pi \partial P)(P/\pi) = (1)P/(P\text{-}C)$ implying that the price elasticity is the profit elasticity times the price over the profit margin.
11. Computed from information in the US EIA/DOE (1996a).

12. To show how a total reserve and a new reserve elasticity are related let Rot be total reserves and O be new reserves. The total reserve elasticity $(\partial Ro_t/\partial P)(P/Ro_t) = \varepsilon_t$. this elasticity can be rewritten as $(\partial Ro_t/\partial O)(\partial O/\partial P)(R/Ro_t)(O/O)$. Separating out the new reserve elasticity we get $(\partial O/\partial P)(P/O)$ $(\partial Ro_t/\partial O)(O/Ro_t) = \varepsilon_0 \varepsilon_{RO}$ Thus the total reserve elasticity equals the new reserve price elasticity times the elasticity of total reserves with respect to a change in new reserves where ε_{RO} has averaged just over 10 from 1977 to 1995.
13. Gas reserves on an energy content basis have been larger in the US than oil reserves in the post WWII period. (Computed from data in American Petroleum Institute (1998)).
14. Computed from information in US EIA/DOE (1996a).

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