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Nonlinear inverse demand curves in electricity market modeling

Yi Wan, Tom Kober, Martin Densing *

Energy Economics Group, Paul Scherrer Institute, Forschungsstrasse 111, 5232 Villigen PSI, Switzerland

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ABSTRACT

In large-scale energy market models, the price–demand relationship is usually represented by a linear function. In this paper, nonlinear demand functions are fitted to electricity market bid data; in particular, exponential and polynomial (cubic) functions are estimated from EPEX day-ahead data (i.e. Central Western European market area). The corresponding game-theoretic, large-scale electricity models were successfully solved using the Extended Mathematical Programming framework after a suitable adaptation for conjectural variations. Additionally, sufficient conditions for the existence of equilibrium solutions are tested. Numerical results show that nonlinear demand curves lead to an improved modeling especially in high price (peak) load periods and to lower levels of implied market power, which can be considered to be more realistic for markets that have strong transparency measures.

1. Introduction

In fundamental electricity market models, the inverse demand curve, that is, the functional price–demand relationship in the markets, is usually assumed to be linear. This is in contrast to the supply curve, which is often modeled more detailed using estimates of marginal costs and capacities of supply technologies. One reason for the assumption of linearity is that such market equilibrium models can be numerically solved relatively easily, and more analytical properties can be deduced. Another reason is that empirical market data on the inverse demand function were formerly not available. However, an oversimplified representation of demand curves may raise several issues. For example, a linear demand function means that the marginal volume (i.e. the cleared amount of demand) in response to a marginal price is constant regardless whether market prices are high or low. In power markets, a change in demand is usually sensitive to a price change when the price is low (which means high elasticity) while the demand is less elastic when the price is high because a certain minimal amount of electricity is usually always asked for at the market as a necessary good. Thus, especially in periods of high (peak) prices, a linear demand curve cannot represent market information accurately.

This issue is even more pronounced if linear demand curves are used in Nash–Cournot market models, which are themselves prone to unrealistic high prices and low volumes because the rivals' supply functions are unaffected by a player's strategic quantity reduction; a detailed case study exhibiting the high prices is for example in Koschker and Möst (2016); for a general introduction in Nash–Cournot models, see e.g. Vives (1999). Nash–Cournot models can be extended by conjectural variation (CV), where players can vary their degree how their production amounts are affected by the decision of other players (e.g. see the review on electricity market modeling trends of Ventosa et al. (2005)). CV allows different strategic behavior to be modeled between (pure) Nash–Cournot and perfect competition by introducing a scaling parameter to reduce the price response on demand changes of a player. A non-vanishing CV parameter indicates deviation from marginal-cost based pricing (perfect competition). Models which allow for CV yield in most cases results more inline with empirical data (de Haro et al., 2007), which can be also theoretically justified under certain conditions; for example, constant CVs can be shown to be consistent in linear models if the number of players is fixed and marginal costs are constant (Perry, 1982). Electricity market models with CV are for example used in García-Alcalde et al. (2002), Song et al. (2003), de Haro et al. (2007), Linares et al. (2008), Orgaz et al. (2017, 2019), Orea and Steinbuks (2018).

The aforementioned model approaches assume generally a linear price–demand relationship, whereas the shape of the demand curve has a large impact on results of such Nash–Cournot-type of models (Kahn, 1998). In fact, unrealistic losses of volume when prices rise can also be prevented by more accurate, nonlinear demand functions due to their high sensitivities of prices to demands in high price regions. Nonlinear demand curves are rarely implemented in electricity market optimization modeling due to computational challenges (Pintér, 2018).

* Corresponding author.
E-mail address: martin.densing@psi.ch (M. Densing).

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Further, finding a global optimal solution is more difficult if the nonlinearity leads to non-convex problems because of the likely existence of several local optima or saddle points (Szu and Hartley, 1987). A case study using a nonlinear demand curve with CV is given in Lagarto et al. (2014), where a sigmoid function is used to approximate the demand curve in the case of the Iberian day-ahead electricity market. Based on long-term empirical regression, Abada et al. (2013) consider an S-shape curve (hyperbolic tangent function) to model gas demand by capturing fuel substitution and historic prices. Outside the field of energy markets, an example of using a nonlinear demand function is Caliskan-Demirag et al. (2010) in the case of logistics, illustrating that exponential demand functions can coordinate more model parameters than linear functions if the retailer is the only fairness-concerned party. The scarcity of research work on nonlinear demand curve estimation and application is a further motivation for this paper.

Nowadays, more powerful computational tools for market equilibrium models allow for tractable, nonlinear demand curves, and, due to the data transparency measures imposed by market regulators (e.g. in the case of Europe EU, 2011), detailed historical market data of (inverse) demand curves are now available for relevant electricity markets. This also motivates us to investigate the impacts of nonlinear demand curves on electricity prices and market volumes in liquid, short-term electricity markets, where non-linear demand curves can be estimated empirically.

In this paper, we apply statistical nonlinear curve fitting (exponential and polynomial (cubic) functions) to estimate nonlinear demand functions and investigate their impacts on electricity markets with a market equilibrium model. The new data-based estimation of the curves covers several regions (five countries in Central Western Europe) in our case. In particular, we are able to combine nonlinear demand functions together with CV into electricity market modeling in a robust numerical framework, and we can determine the interrelation between the CV parameter and the use of different demand curve assumptions. For the estimation method of the CV parameter, we use a similar method as in de Haro et al. (2007), which is based on differences between observed prices and the modeled marginal costs. By our results, given a level of CV parameter, which is a measure for market distortion, we achieve a higher Lerner index, which is a common empirical measure for market power (Lerner, 1995) and relates in electricity markets to price-peaks. In our model, we can explain such peaks with less market distortion. Hence, our work contributes also to an improved attribution of observed price mark-ups, which can be due partially to market power; for the empirical estimation of such mark-ups, see for example Weigt and Hirschhausen (2008), Koschker and Möst (2016), Pape et al. (2016), Orgaz et al. (2019); a review of estimation methods is by Pham (2019).

However, we focus on curve representations by differentiable functions; indeed, non-smooth (piecewise constant) functions would be exactly factual, but severe computational issues may arise (Hobbs and Pang, 2007).

This paper is organized as follows. Nonlinear inverse demand curves in different formats are introduced in Section 2. In Section 3, we describe a game-theoretic market model which integrates nonlinear inverse demand curves, and we investigate the basics of the optimality conditions of the equilibrium problem with nonlinear curves. In Section 4, we discuss several aspects of the obtained numerical results: market price and volume changes, market power impacts, and historical prices comparisons. We conclude in Section 5.

2. Inverse demand curves

Demand curves represent the assumed relationship between market prices and demands. A point of the curve indicates how many goods are requested by buyers for a given price. Obviously, traded volumes and prices are negatively correlated: If prices increase, buyers reduce consumption, and if prices drop, more goods are bought. Fig. 1 is

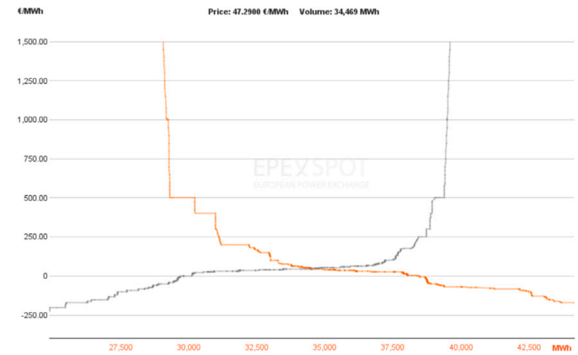


Fig. 1. Aggregate curve of the German day-ahead market (purchasing bids in red color; selling bids in black color), Hour 10, 01-03-2019 (EPEX, 2019); clearing price: 47.29 EUR/MWh, clearing volume: 34469 MWh. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

an example from the German day-ahead electricity market. It shows the stepwise decreasing demand curve comprising the bids of different (undisclosed) market participants.

Considering the issues for the solvability of a numerical model raised by the non-differentiability of the curve, we think about smooth approximations of price–demand curves. Because of their simplicity, linear functions are most commonly used in price–demand modeling, and we will use linear functions as a benchmark of the model results of nonlinear demand curves. The linear (inverse) demand function is

$$p(d) = \alpha - \beta d, \tag{1}$$

where p is the market price given as a function of demand d, and the (sign-reversed) slope is β. We have $\alpha > 0$ and $\beta > 0$ under the usual assumption that for any inverse demand function it holds that $p(0) > 0$ and $p(d)$ is monotonously strictly decreasing in d.

By using a sigmoid function for the demand-price relationship, Lagarto et al. (2014) achieved a satisfactory fit to observed demand curves in the case of the Iberian day-ahead electricity market. However, we found by computational experiments that sigmoids cannot be estimated satisfactorily in the case of the day-ahead market data from EPEX SPOT (EPEX, 2019). In this electricity market, the maximal and minimal allowed clearing prices are set in accordance with the Capacity Allocation and Congestion Management (CACM) Regulation (EU, 2015); the maximal cap is 3000 EUR/MWh and represents the assumed value of lost load (VOLL). Indeed, a sigmoid function is suitable to represent the VOLL price step, which shelves the prices for sufficiently low demands. However, the EPEX SPOT market was never cleared at the VOLL (up to year 2019) such that this ability of sigmoids is irrelevant for cleared price/volumes, and the numerical fitting of sigmoids leads to poor results at historical price/demand points because the fitting aims also to approximate the shelving price at VOLL.

An alternative are exponential functions, which neglect per definition such (unrealistic) price shelves. Using exponential functions, the rate of price change is proportional to the price. In other words, the price changes more slowly for low prices, which can be considered as a plausible assumption for electricity markets. We can write an exponential inverse demand function as

$$p(d) = \alpha + \beta e^{-\gamma d}, \tag{2}$$

where α, β, and γ are (real-valued) parameters. If we assume that $p(d)$ is strictly monotonously decreasing, and that $p(d)$ is convex, we have $\beta > 0$ and $\gamma > 0$.

Based on the form of empirical demand data (cf. Fig. 1), we consider also polynomial functions as a further option. To avoid over-fitting, the polynomial inverse demand function is chosen to be cubic:

$$p(d) = a_0 + a_1 d + a_2 d^2 + a_3 d^3, \tag{3}$$

with parameters a_0, a_1, a_2, and a_3; we have $a_3 < 0$ under the assumption of a strictly decreasing function $p(d)$.

3. Implementation in the example of the BEM model

The nonlinear inverse demand curves are implemented in the Cross-Border Electricity Market (BEM) model, which is developed at the Paul Scherrer Institute, and exhibits in its base version linear inverse demand functions (Panos et al., 2017; Panos and Densing, 2019). Using this large-scale electricity market model, we investigate the performance of nonlinear inverse demand curves with respect to linear curves, and analyze their impacts on modeled market behavior. The substitution of linear demand curves with nonlinear ones is computational challenging because – as we will see – the profit maximization problems become non-convex for some parameter ranges. Related, to restate the large-scale BEM model for different nonlinear inverse demand curves, we use Extended Mathematical Programming (EMP), which is a flexible computational reformulation tool (Ferris et al., 2009).

The BEM model is a technically detailed electricity market model designed for real-world data application in Switzerland and its neighboring countries, Austria (AT), France (FR), Germany (DE), and Italy (IT), to capture market behavior of power producers on the Central Western European electricity market by assuming imperfect competition through the Nash–Cournot mechanism, which is extended in BEM by the CV mechanism (Panos et al., 2017; Panos and Densing, 2019). The BEM model is implemented in GAMS (Brooke et al., 2013). In the model, the year is divided into four seasons, and each season is represented by a typical daily profile of 24 h. Hence, the time slices of the model comprise $4 \times 24 = 96$ hours.

The BEM model includes currently five strategically acting (production) players, which are modeled as the production mix of countries (AT, CH, DE, FR, and IT). Each country (node) represents a corresponding demand load. As an additional price-taking player, the Transmission System Operator (TSO) clears cost-optimally production and demand by importing/exporting between the nodes.[1] In each load period, the production players compete and choose production amounts (using their different production and storage capacities) to maximize their profits. The market demand of electricity is represented by the price–demand function. The market equilibrium is met subject to additional side-constraints, for example on available capacity, limits on transmission and storage, and other operational constraints.

To represent the demand side of the electricity market of Switzerland and its surrounding countries, market data of the modeled countries' hourly day-ahead electricity prices and purchase volumes is an input to the BEM model. An exception is the Italian market, as hourly market purchase data was not suitably available. For Italy, we use the day-ahead volume purchase data of Germany to estimate the Italian market proportionally to market size (assuming similar characteristics of short-term markets). The input on the supply side includes the main characteristics of each technology, which comprise for example the availability, capacity, efficiency, and costs . The technologies are detailed as aggregated technology types: lignite plants, hard coal plants, oil fueled, gas fueled (divided by steam, gas turbine, and combined cycle), nuclear, biomass (which includes waste), hydro run-of-river, hydro pumped-storage, wind power, and solar PV. Run-of-river has seasonal, and solar and wind power have daily and seasonally varying availabilities. The pump-storage hydro plant-type is modeled as a daily and also seasonal storage technology; the storage level is modeled explicitly. The optimization chooses when and how much water to store or to produce electricity over days and the seasons of the year. Capacity constraints of turbines and pumps and seasonal inflow are considered. More detailed model assumptions and descriptions can be found in Panos et al. (2017) and Panos and Densing (2019), where the mathematical description is given as supplementary material.

The main output of the model comprises the electricity production of different power plant types, the import/export between demand nodes, and the prices and volumes of the cleared market. Note that while stored hydro power is modeled with zero operational marginal costs and it has a relatively high share in supply in the considered countries over the year, the cleared market prices are usually non-zero because in load periods of low demand and of high renewables supply (by wind, solar) water is usually not dispatched but rather stored for later use.

3.1. Market equilibrium with nonlinear inverse demand curves

The objective for each production player is to maximize the operational profit.[2] The profit is the sum of market revenues minus production costs over all time slices (load periods). Basically – by simplifying to a single production technology – the objective function for a player i is:

$$\max_{q_{nil}} \sum_{n=1}^{5} \sum_{l=1}^{96} t_l (p_{nl} - c_{nil}) q_{nil} \qquad (i \neq \text{TSO}) \tag{4}$$

$$\max_{a_{nil}} \sum_{n=1}^{5} \sum_{l=1}^{96} p_{nl} a_{nil} \qquad (i = \text{TSO}) \tag{5}$$

$$\text{s.t.} \begin{cases} d_{nl} = \sum_{i=1}^{5} (q_{nil} + a_{nil}), \\ \text{technical (linear) constraints,} \end{cases} \tag{6}$$

where t_l is the time length of load period l and p_{nl} is the market price in node n and load period l. Production player i decides to produce q_{nil} under marginal costs c_{nil}. The variables a_{nil} of net import amounts into node n are associated to the TSO (the rationale of the TSO's optimization problem is further detailed in Hobbs and Pang (2007), using a similar linearized DC transmission between the players).

In the case of linear inverse demand functions, the well-developed theory of linear complementarity problems (LCP) can be used to show existence and uniqueness properties of Nash–Cournot equilibria having several production players interconnected with transmission constraints (Metzler et al., 2003). In our case with non-linearities, we limit the discussion to the existence of an equilibrium.

For the existence, we use the representation of Nash–Cournot-type games by variational inequalities, which holds under the following conditions: (i) the feasible set of each player is closed, bounded, and convex, (ii) the objective value of the players is concave in the player's variables (fixing the variables of the other players) and is continuously differentiable in all variables (Scutari et al., 2010, (18), p. 41). The conditions are sufficient for the existence of an equilibrium solution, and ensures that the set of solutions is bounded and closed. Because the set of possible production amounts of each production player is given by linear inequalities and is bounded, condition (i) holds; the condition holds also for the linear transmission constraints of the TSO player. Eventually, we have to check concavity (condition (ii)). The TSO player has a linear objective function, such that it remains to check condition (ii) for the production players.

Let f_i be the objective function of production player i given by (4). The first derivative with respect to player's variable q_{nil} in node n and load period l is

$$\frac{\partial f_i}{\partial q_{nil}} = p_{nl}(d_{nl}) + \theta \frac{\partial p_{nl}}{\partial d_{nl}} q_{nil} - c_{nil}, \tag{7}$$

[1] The TSO is modeled as a (passive) price-taking player because EMP requires all variables of the model (incl. import/export amounts) to be assigned to a player.

[2] The version of the BEM model used in this paper has capacity expansion disabled.

where $\theta \in [0,1]$ is the CV parameter (Corts, 1999; de Haro et al., 2007) (cf. Sections 1 and 3.2). Using (6), we can replace the demand by production, such that (7) can be written as

$$\frac{\partial f_i}{\partial q_{nil}} = p_{nl}(q_{nil}) + \theta \frac{\partial p_{nl}}{\partial q_{nil}} q_{nil} - c_{nil}. \tag{8}$$

The second derivative is

$$\frac{\partial^2 f_i}{\partial q_{nil}^2} = \frac{\partial p_{nl}}{\partial q_{nil}} + \theta \Big(\frac{\partial^2 p_{nl}}{\partial q_{nil}^2} q_{nil} + \frac{\partial p_{nl}}{\partial q_{nil}} \Big), \tag{9}$$

where we use the assumption that the marginal cost c_{nil} is constant in quantity. In the case of the exponential inverse demand curve (2), the derivatives (7) and (9) are

$$\frac{\partial f_i}{\partial q_{nil}} = \alpha + \beta e^{-\gamma q_{nil}} - \theta \beta \gamma e^{-\gamma q_{nil}} q_{nil} - c_{nil}, \tag{10}$$

$$\frac{\partial^2 f_i}{\partial q_{nil}^2} = -\beta \gamma e^{-\gamma q_{nil}} + \theta \beta \gamma^2 e^{-\gamma q_{nil}} q_{nil} - \theta \beta \gamma e^{-\gamma q_{nil}}. \tag{11}$$

Concavity means $\partial^2 f_i / \partial q_{nil}^2 \le 0$, which implies $\theta(\gamma q_{nil} - 1) \le 0$ under our assumptions $\beta, \gamma > 0$. In case $\theta = 0$, that is, without CV, this inequality is fulfilled, else the inequality is

$$q_{nil} \le \frac{1}{\gamma}\Big(1 + \frac{1}{\theta}\Big),$$

which may not be fulfilled in all time periods; for example, in our numerical experiments, it is satisfied in 79% of all load periods (with q_{nil} set to the historical load per node n and period l averaged over years 2015 and 2016, and the corresponding θ is calibrated to these years by the model) .

In the case of the polynomial (cubic) inverse demand curve (3), the derivatives (7) and (9) are

$$\frac{\partial f_i}{\partial q_{nil}} = a_0 + a_1 q_{nil} + a_2 q_{nil}^2 + a_3 q_{nil}^3 + \theta \big(a_1 + 2a_2 q_{nil} + 3a_3 q_{nil}^2 \big) q_{nil} - c_{nil}, \tag{12}$$

$$\frac{\partial^2 f_i}{\partial q_{nil}^2} = (1+\theta) a_1 + 2(1+2\theta) a_2 q_{nil} + 3(1+3\theta) a_3 q_{nil}^2. \tag{13}$$

Concavity means $\partial^2 f_i / \partial q_{nil}^2 \le 0$, which implies (by solving $\partial^2 f_i / \partial q_{nil}^2 = 0$) that production is outside the following interval:

$$q_{nil} \notin \frac{1}{3a_3(1+3\theta)} [-a_2(1+2\theta) + a, -a_2(1+2\theta) - a], \tag{14}$$

with abbreviation

$$a := \sqrt{a_2^2(1+2\theta)^2 - 3a_1 a_2 (1+\theta)(1+3\theta)}.$$

This is fulfilled in 92% of the load periods of the estimated parameters a_1, a_2, a_3, and θ for the years 2015 and 2016. We note that with polynomial inverse demand functions, the obtained numerical equilibrium solutions were usually simpler to be found by the solver algorithm (faster and less sensitive to the choice of initial point) than with exponential inverse demand functions, which may be due to the fewer load periods having non-linearities.

3.2. Implementation in EMP

The most common approach to numerically solve a game-theoretic Nash–Cournot equilibrium model is to derive for each player the Karush–Kuhn–Tucker (KKT) optimality conditions. The KKT conditions are a system of equations and complementarity relations over all players, that is, the system forms a mixed-complementarity problem (MCP) (Kuhn and Tucker, 1951). The MCP is then input to a numerical solver, which is in our case PATH (Ferris and Munson, 2000). Stating the KKT conditions for large-scale optimization problems is error prone and time consuming, especially for experimental research where small changes in problem structures are frequent. Extended Mathematical Programming (EMP) is a reformulation framework that allows multiple format models' automatic generation and reformulation to improve solving speed and robustness, such that users can focus on problem description (Ferris et al., 2009). It is available in the form of the (meta-)solver JAMS in the modeling framework GAMS.[3] We use EMP to automatically reformulate the optimization problems of the players into an MCP, which is helpful because we consider several model variants with different inverse demand functions.

The advanced features of EMP are required in the modeling: shared constraints and shared variables. A constraint is shared if it appears in several players' problems or involves multiple players. For example, the constraint that total net imported electricity over all nodes is zero involves all players, hence it is a shared constraint. Similarly, a variable is shared if it is owned by several players and is equal across these players. Shared variables in the BEM model are the net imports a_{nil}, prices p_{nl}, and demands d_{nl}. Sharing in EMP allows players to have common as well as unique variables and constraints, which is for example required in the case of transmission between players.[4] On the other hand, EMP requires that all variables and constraints are associated to at least one player. Hence, the variables of net imports are owned by the TSO, which is modeled as a (auxiliary) player, too.

Moreover, because the BEM model employs CVs, the automatic derivation mechanism of EMP for the KKT conditions requires the following reformulation of the objective function. Given a node n and a load period l, the derivative of the objective function (4) of a production player i is (with indices n, l dropped for simplicity)

$$\frac{\partial}{\partial q_i} \big(p(d) q_i - c_i q_i \big) = p(d) + \frac{\partial p}{\partial q_i} q_i - c_i. \tag{15}$$

If we assume perfect competition, then $\frac{\partial p}{\partial q_i} = 0$, that is, an unilateral (strategic) decision of a player i to reduce (or increase) production amounts cannot influence market prices; in other words, the player has no market power. In a (pure) Nash–Cournot model, the derivative (price-response) is non-zero because we assume an explicit dependence via (6):

$$\frac{\partial p(d)}{\partial q_i} = \frac{\mathrm{d}p(d)}{\mathrm{d}d} \frac{\partial d}{\partial q_i} = \frac{\mathrm{d}p(d)}{\mathrm{d}d}. \tag{16}$$

For example, in the case of a linear inverse demand function (1), $\frac{\mathrm{d}p(d)}{\mathrm{d}d} = -\beta$. The price-response in a model with CV is assumed to be between perfect competition and a pure Nash–Cournot game by reducing the Nash–Cournot price-response by a factor $\theta \in (0,1)$:

$$\text{Cournot:} \quad \frac{\mathrm{d}p(d)}{\mathrm{d}d} \quad \rightarrow \quad \text{CV:} \quad \theta \frac{\mathrm{d}p(d)}{\mathrm{d}d}. \tag{17}$$

In other words, the explicit derivative of the Nash–Cournot game is adjusted by the CV parameter θ to better represent limited market power (cf. (9) and Section 1). The CV mechanism can be implemented into EMP by reformulating (15) with help of a new price variable $\tilde{p}$, which does not depend on d explicitly, and a new variable $\tilde{q}^{(i)}$ of production amount, which does not explicitly belong to player i but only indirectly via an additional equation $\tilde{q}^{(i)} = q_i$. Then, the derivative

$$\frac{\partial}{\partial q_i} \big(\tilde{p} q_i + \theta p(d) \tilde{q}^{(i)} - c_i q_i \big) = \tilde{p} + \theta \frac{\partial p}{\partial q_i} \tilde{q}^{(i)} - c_i. \tag{18}$$

is a reformulation of (15) under the ad-hoc substitution (17). Hence, the objective function of a production player i is reformulated as

$$\max_{q_{nil}} \sum_{n=1}^{5} \sum_{l=1}^{96} t_l \big(\tilde{p}_{nl} q_{nil} + \theta p_{nl}(d_{nl}) \tilde{q}_{nl}^{(i)} - c_{nil} q_{nil} \big), \tag{19}$$

which yields the correct expression by the automatic derivation of EMP.

[3] https://www.gams.com/latest/docs/S_JAMS.html.

[4] Shared constraints and variables are activated by the keywords `ShareEqu` and `implicit` in the EMP control file `jams.opt`.

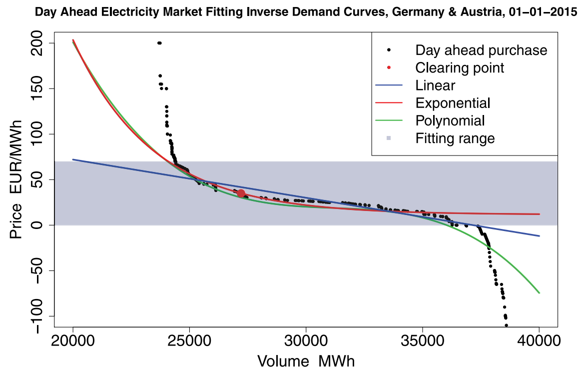


Fig. 2. Fitting of inverse demand functions of the German & Austria day-ahead market, hour 01, 01-01-2015.

Table 1
Average MSE of fitted nonlinear inverse demand curve for hourly day ahead market data of Germany & Austria, 2015[a] (standardized market data).

Curve	Average MSE (standardized market data)
Linear	0.1237
Exponential	0.0557
Polynomial	0.0395

[a]Germany and Austria was a common bidding zone before 1 October 2018.

4. Numerical analysis and model results

In this section, we present estimation results of the different inverse demand functions based on EPEX purchase bidding data, and subsequently discuss the corresponding results of the BEM electricity market model.

4.1. Inverse demand curves fitting

The inverse demand curves of EPEX bidding data are fitted to linear (see (1)), exponential (2), and polynomial (3) functions using least-square estimation. An example of the fitted functions is shown for the German & Austria day-ahead market in Fig. 2. For the fit, the raw data of day-ahead market prices and volumes were (separately) normalized to increase fitting accuracy. Then, a floating interval was set centered on the clearing price p and volume v at a specific hour of historical data as $[(1-f)p,(1+f)p]$ and $[(1-f)v,(1+f)v]$, which represents the relevant range of price/volume determination based on historical market clearing data. By numerical experiments, a suitable interval is found to be $f=0.9$ for data ranges in year 2015.

Table 1 shows the mean squared error (MSE) in the example of the German & Austria day-ahead market, where the MSE is averaged over the hourly estimation of year 2015. In our experiments, the best fit is the polynomial inverse demand function (also for the other market areas).

4.2. Elasticity analysis based on fitted nonlinear inverse demand curves

Given the fit of the empirical inverse demand curve to linear, exponential and polynomial functions, we can consider the corresponding price elasticity of hourly electricity demand. The price elasticity is defined as

$$\epsilon(p_{nl}) = \frac{\partial d_{nl}/d_{nl}}{\partial p_{nl}/p_{nl}}, \tag{20}$$

where p_{nl} is the price in market (node) n and hour (load period) l, and d_{nl} is the corresponding demand. In the case of the linear inverse demand function (1), the elasticity (20) is

$$\epsilon(p_{nl}) = \frac{p_{nl}}{p_{nl}-\alpha}. \tag{21}$$

In the case of an exponential inverse demand function (2), the elasticity becomes

$$\epsilon(p_{nl}) = \frac{p_{nl}}{p_{nl}-\alpha}\frac{1}{\ln\left(\frac{p_{nl}-\alpha}{\beta}\right)}, \tag{22}$$

and for a polynomial inverse demand function (3), the elasticity becomes

$$\epsilon(p_{nl}) = \frac{p_{nl}}{a_1 d(p_{nl}) + 2a_2\left(d(p_{nl})\right)^2 + 3a_3\left(d(p_{nl})\right)^3}, \tag{23}$$

where $d(p)$ is the inverse of the function $p(d)$; we can assume that the inverse exists because an inverse demand function $p(d)$ can be assumed to be monotonously strictly decreasing. In all three cases we have $\lim_{p\searrow 0}\epsilon(p)=0$ if we assume a monotonously strictly decreasing function $p(d)$ (and additionally for the polynomial case $a_0 > 0$). Moreover, under these assumptions, the elasticity attains arbitrarily large negative values in the limit $\lim_{p\nearrow\bar{p}}\epsilon(p)=-\infty$, where $\bar{p} := \alpha$ in the linear case, $\bar{p} := \alpha+\beta$ in the exponential case, and $\bar{p} := a_0$ in the polynomial case. Despite that these limits are equal, intermediate values can differ as follows.

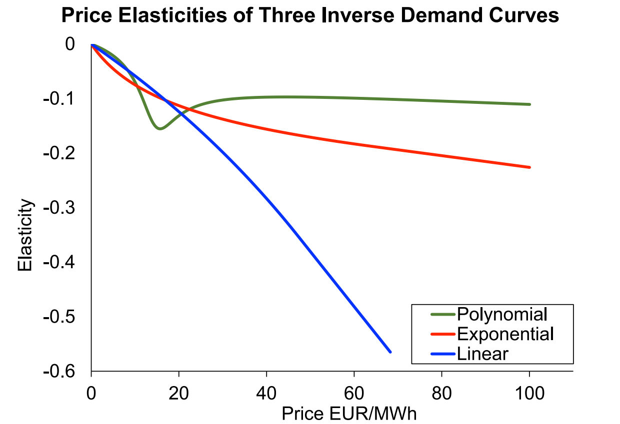


Fig. 3. Elasticity as a function of price for the three inverse demand curves. Function parameters for Hour 10, 01-01-2015, Germany & Austria market.

Based on the fitted parameters, the elasticities are calculated numerically for each hour from year 2006 to year 2015. As an example, Fig. 3 depicts the elasticities of the three different inverse demand functions for hour 10, 01-01-2015, in the Germany & Austria day-ahead market. The figure shows that the elasticities of the linear inverse demand curve decrease monotonically in price, and the slope becomes steeper with increasing market prices. The elasticity of the exponential inverse demand function is also monotonically decreasing, but with flatter slope for increasing prices. At very high (peak) prices, the exponential inverse demand function is approaching a relatively constant elasticity. Similarly, the case of a polynomial function exhibits also a flattening under sufficiently high electricity prices. A constant elasticity means relative price changes are proportional to relative demand changes irrespective of the price level. Starting already at moderate price levels (i.e. > 20 EUR/MWh), the nonlinear inverse demand functions' elasticities becomes smaller (in absolute values) than for the linear function; hence, the non-linear functions explain better the stiffness of the most necessary electricity demand, that is, of those high price regions where demand can decrease only slightly. Accordingly, an equilibrium market model with nonlinear inverse demand curves will exhibit usually higher price variation results compared with linear ones (see Section 4.3).

Next, we analyze the empirical elasticities of the Germany & Austria day-ahead market over time from year 2006 to 2015. The hourly

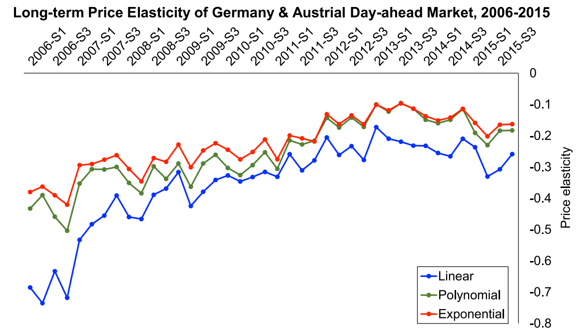


Fig. 4. Average elasticities of three approaches to inverse demand curves, 2006–2015, Germany & Austria day-ahead market.

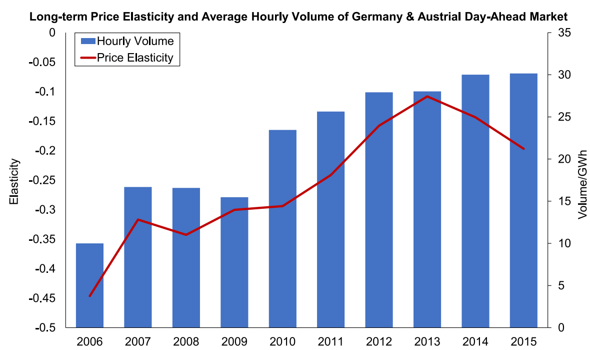


Fig. 5. Hourly elasticities of inverse demand curves, averaged over a year and over the three approaches (linear, exponential, polynomial); market volumes, 2006–2015, Germany & Austria day-ahead market.

elasticities $\epsilon(p)$ given by the three different inverse demand functions ((21), (22), and (23)) are calculated at the hourly clearing price p. Fig. 4 shows the elasticities averaged for each season over historical years. The three different approaches for inverse demand function lead to similar trends of elasticity over the years. Fig. 5 shows for the Germany & Austria day-ahead market the day-averaged elasticity (additionally averaged over the three approaches) and the market volume over the years from 2006 to 2015. In the years 2008–2009, the global financial crises affected also Germany and lead to a reduced electricity consumption because of diminished industrial and commercial demand and corresponding lower trading volume. Accordingly, the absolute value of elasticity decreases in 2008 and 2009. After the beginning of year 2010, German transmission system operators were required to trade energy produced under the Renewable Energy Act (REA) on spot markets (Cludius et al., 2014), which expands the trading volume significantly. After 2013, the absolute value of elasticity increases while German electricity prices decrease and volumes slightly increase. One of the reason could be that the players in the now matured market can rely on more sophisticated forecasts, which helps to flatten the inverse demand curve, and hence the market becomes more informed and competitive.

Our approach is based on market data of bids. By contrast, Knaut and Paulus (2016) quantify the price elasticity by a two-stage regression approach using wind generation to estimate electricity prices and then a regression on electricity demand. Our approach of considering the elasticity of the demand-bids (and not the (not directly observable) end-consumer price elasticity) leads to relatively higher elasticities.

4.3. Impact on market modeling results

In the following, we use the BEM model to investigate price reactions under different market situations and impacts on market power estimation.

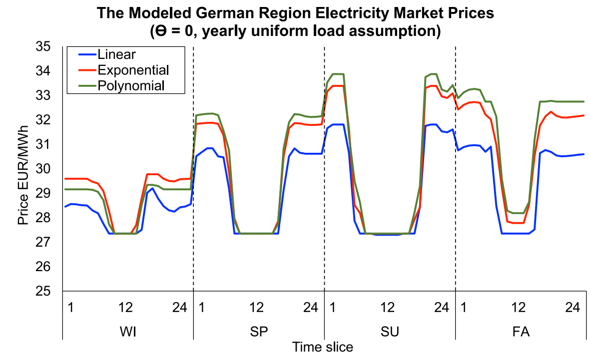


Fig. 6. Electricity day-ahead market prices of the modeled German region for the three approaches of inverse demand curves in the case of $\theta = 0$. The load reference point for each approach is fixed over the load periods. Horizontal axis: hours of the typical day in winter, spring, summer and autumn.

4.3.1. Market behavior change/response

We analyze the influence of nonlinearity of inverse demand curves in the case of high and of low renewable supply while keeping the demand unchanged, which means that the inverse demand curve is the same for all load periods (as mentioned, the BEM model has four typical days with hourly load profiles for each season). Fig. 6 shows the modeled electricity market prices of Germany for the three approaches of inverse demand curves in the case of $\theta = 0$ (and with the same demand curve for the four typical days). Note that while the exclusion of side-effects by neglecting demand variations and by setting $\theta = 0$ allows a cleaner comparison of the different curves, the modeled prices (based only on marginal costs and in winter influenced by larger wind availability) may not approximate well real-world market prices; modeled prices with varying CV parameters and demands are shown later in Fig. 9.

In the periods of high renewable supply (e.g. from hour 11 to hour 13), solar PV generation is high, which means an increase of supply in the market. Consequently, electricity prices decrease, and market volumes increase. The cleared price/volume points move to the flatter part of inverse demand curves, resulting in similar market prices for linear and nonlinear inverse demand curves. By contrast, in periods of low renewable supply (e.g. from hour 1 to hour 5), the clearing prices of nonlinear inverse demand curves are relatively higher than of linear ones, while differences in clearing volume are relatively small (see Fig. 7). Indeed, in such load periods, the nonlinearity of inverse demand curves leads to relatively small volume reductions, which can qualitatively be considered as a more realistic description of the markets.

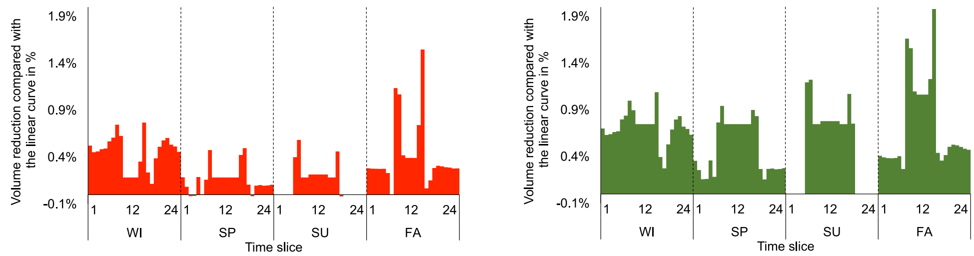
In summary, nonlinear inverse demand functions have a large impact on modeled market prices in periods of low supply, but small impacts in periods of abundant supply compared with linear curves.

4.3.2. Estimation of market power

In an imperfect market, observed prices are usually above marginal costs of production, caused by market power or other market distortions. The Lerner index is a basic metric for the market power evaluation of players:

$$L = \frac{p - m}{p}, \tag{24}$$

where p is the market price and m is the marginal production cost (Lerner, 1995). When market power is high (the market is less competitive), the Lerner index is high. In the BEM model, we consider each country as a player; consequently, we note that the Lerner index has the corresponding granularity. Table 2 shows the Lerner index of the modeled German electricity market area averaged over the load periods in 2015. In these model runs, we preconditioned the market



(a) Exponential curve

(b) Polynomial curve

Fig. 7. Volume reduction of electricity day-ahead market of the modeled German region for the exponential and polynomial inverse demand curves with respect to linear curves. The load reference point for each approach is fixed over the load periods. Horizontal axis: hours of the typical day in winter, spring, summer and autumn.

Table 2
Lerner indexes of modeled German day-ahead market prices for the three different approaches of inverse demand functions; CV parameter is fixed: $\theta = 0.02$.

Curve	Lerner index
Linear	0.057
Exponential	0.074
Polynomial	0.106

Table 3
The standard deviations over day hours of model price results of three demand curves, averaged over days of year; Historical: 2015 data.

Curve	AT	CH	DE	FR	IT
Linear	3.428	3.821	4.852	5.023	5.240
Exponential	9.608	5.3890	5.378	12.038	8.376
Polynomial	9.706	5.661	7.605	9.585	6.219
Historical	7.845	8.714	8.811	8.214	7.002

power of the model, hence, the CV level θ is fixed. The table shows that the Lerner index is highest in the case of the polynomial inverse demand curve. In other words, given a small level θ, nonlinear inverse demand curves provide market results with less competition, that is, more oligopolistic market structure is explained for a given level of assumed market distortion. In European electricity markets, the strong transparency measures of the market regulators do not allow very high levels of market power, such that the θ value, which represents market power in the modeling, should realistically be chosen relatively small. Therefore, using nonlinear inverse demand curves, we are able to model price peaks under relatively moderate assumption of market distortions.

Alternatively, the CV parameter θ can also be estimated by the model itself. We estimate θ based on historical data of market prices and volumes of the year 2015, and calculate the inverse demand function. Using (15), θ is estimated as the scaled difference between realized prices and marginal cost of production:

$$\theta_{nil} = \frac{\bar{c}_{nil} - p_{nl}}{q_{nil} \frac{\partial p_{nl}}{\partial d_{nl}}},$$

where $\bar{c}_{nil}$ is the highest marginal cost over all technologies that are producing in demand node n, in load period l, and for player i. Fig. 8 shows the distribution of the conjectural variation parameter θ of the involved players (i.e. the production mix of the regions) for the three approaches of inverse demand functions.

According to the figure, for all players, θ is lower in case of nonlinear inverse demand functions (especially of the polynomial function) than for the linear function. Hence, we yield again the result that nonlinear inverse demand functions require relatively weak assumptions on market distortion (at least in the case of the considered Central Western European market region and the specific modeling, which excludes some other possible factors (e.g. start-up costs, must-run conditions and specific trading behavior)).

4.3.3. Calibrated model results

Based on the estimation of the CV parameter θ for different inverse demand functions, we used the BEM electricity market model to calculate θ results for the year 2015 (similar to the implicit estimation method in de Haro et al. (2007)). Fig. 9 compares model results for the different approaches of inverse demand curves with historical market prices. Model results with nonlinear inverse demand curves are closer to historical results than with linear demand curves. Especially the polynomial inverse demand functions provide more realistic price results with lower CV parameters than exponential functions. The standard deviations over day hours of the model results are compared in Table 3 with respect to the three demand curves. The comparison shows that the market model with nonlinear demand curves yields more price volatility, which is closer to the historical data.

Given also the previous analyses, polynomial (cubic) inverse demand functions seem to be a considerable improvement over linear functions for electricity market modeling. On the other hand, the used MCP-solver needs more computation time than for linear curves.[5]

5. Conclusion

This paper presents a general data-based approach to empirically estimate nonlinear inverse demand curves using exponential and polynomial (cubic) functions (tested for Central Western European market areas). The curves are applied in a technology-detailed Nash–Cournot-type electricity market model with market power adjustments by CV and with transmission lines between players. For the model formulation, we use the EMP framework and adapt it for CV. To our knowledge, it is the first time that EMP is applied with joint constrains and CV, which helps to expand the applicability of EMP to large-scale energy market modeling.

The numerical results show that nonlinear inverse demand curves can improve model performance in load periods of high (peak) prices. Nonlinear curves are especially relevant in the cases of fluctuant supply (e.g. intermittent renewables) and of low supply (high demand) scenarios. In particular, polynomial inverse demand functions provide a superior fit to the underlying market data and yield more realistic model results compared with linear and exponential functions in our considered cases.

From a modeling point of view, this work provides a means to better represent demand in equilibrium market models. For electricity market modeling, accurate representations of demand curves are increasingly relevant as the supply curve becomes more fluctuating due to the increasing generation from inflexible resources, for example from

[5] The average solving time is 22 s for linear curves, 54 s for exponential curves and 43 s for polynomial curves with calibrated θ and variable start points 0 (origin) on an Intel Xeon 2.4 GHz processor with 16 GB of RAM.

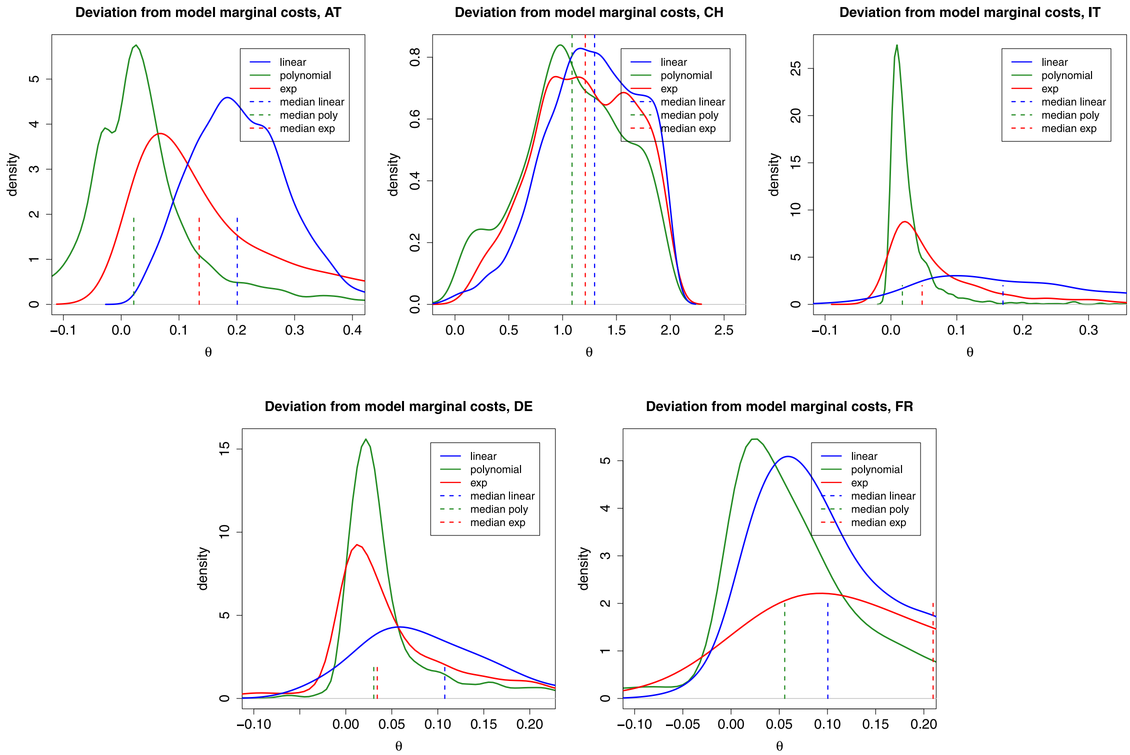


Fig. 8. Frequency distribution of estimated CV parameters for the different countries: Austria (AT), Switzerland (CH), Italy (IT), Germany (DE), France (FR).

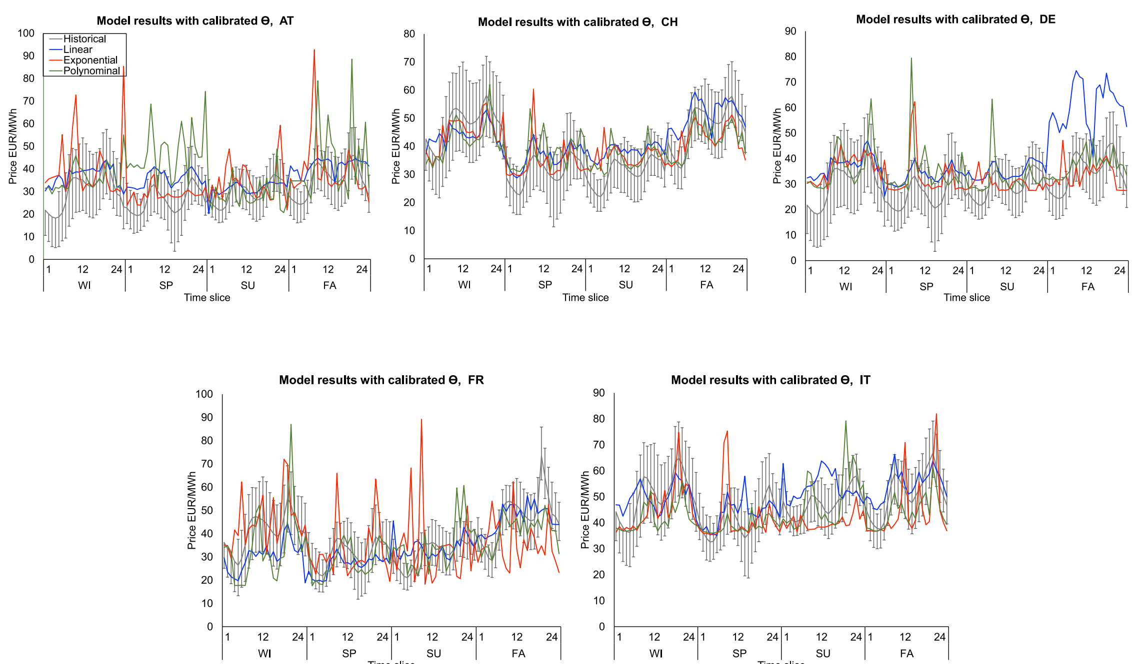


Fig. 9. BEM model price results for year 2015 in the five market regions. Horizontal axis: typical daily load profiles per season (96 time slices). Vertical axis: prices (EUR/MWh). Historical mean and standard deviation per load period.

solar and wind power. Additionally, the development of power-to-gas technologies, the increased deployment of electricity storage, and the advent of prosumage (production, consumption, storage) players in the market increase the complexity of the demand side, which requires proper demand curve representation. While the paper discusses electricity market modeling, the proposed approach can in principle also be used in other energy markets, as long the commodity is homogeneous, for example in oil or natural gas markets, and provided empirical market data is available. Generally, in markets where prices elasticities vary with trading volumes, nonlinear demand curves – properly estimated – can provide a tool to better understand current and future expected market outcomes.

However, the proposed method comes with more involved analytical and computational issues than (conventional) linear functions. The fitted nonlinear inverse demand curves violate in some load periods the convexity assumptions in the considered markets, such that a theoretical proven solvability of such models remains an open question, whereas we were nevertheless able to obtain numerical results in all cases.

There are several further extensions possible. Other forms of non-linear inverse demand curves could be applied, for example splines, which can be fitted to different segments of empirical bid curves. So far, we have considered market modeling results for today's market conditions; modeling of future scenarios, including capacity expansion, is also possible in principle, but the choice of a suitable value for the CV parameter is an open issue (that is, the market power and the corresponding price peaks in the future).

CRediT authorship contribution statement

Yi Wan: Conceptualization, Methodology, Writing – original draft, Software, Data curation, Validation, Formal analysis. **Tom Kober:** Writing – review & editing, Validation. **Martin Densing:** Conceptualization, Methodology, Writing – original draft, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References

Abada, I., Briat, V., Massol, O., 2013. Construction of a fuel demand function portraying interfuel substitution, A system dynamics approach. Energy 49, 240–251. http://dx.doi.org/10.1016/j.energy.2012.10.063.

Brooke, A., Kendrick, D., Meeraus, A., 2013. GAMS: A User's Guide: Release 24.2.1. Scientific Press, Washington DC, USA.

Caliskan-Demirag, O., Chen, Y., Li, J., 2010. Channel coordination under fairness concerns and nonlinear demand. European J. Oper. Res. 207, 1321–1326. http://dx.doi.org/10.1016/j.ejor.2010.07.017.

Cludius, J., Hermann, H., Matthes, F., Graichen, V., 2014. The merit order effect of wind and photovoltaic electricity generation in Germany 2008–2016: Estimation and distributional implications. Energy Econ. 44, 302–313. http://dx.doi.org/10.1016/j.eneco.2014.04.020.

Corts, K., 1999. Conduct parameters and the measurement of market power. J. Econometrics 88, 227–250. http://dx.doi.org/10.1016/S0304-4076(98)00028-1.

EPEX, 2019. Market data day-ahead auction. https://www.epexspot.com/en/market-data. Accessed March 2, 2019.

EU, 2011. Regulation (EU) no 1227/2011 of the European parliament and of the council of 25 october 2011 on wholesale energy market integrity and transparency. Official Journal of the European Union 1227, 1–16, http://data.europa.eu/eli/reg/2011/1227/oj.

EU, 2015. Commission regulation (EU) 2015/1222 of 24 july 2015 establishing a guideline on capacity allocation and congestion management. Official J. Eur. Union 1222, 24–72, http://data.europa.eu/eli/reg/2015/1222/oj.

Ferris, M., Dirkse, S., Jagla, J., Meeraus, A., 2009. An extended mathematical programming framework. Comput. Chem. Eng. 33, 1973–1982. http://dx.doi.org/10.1016/j.compchemeng.2009.06.013.

Ferris, M., Munson, T., 2000. Complementarity problems in GAMS and the PATH solver. J. Econom. Dynam. Control 24, 165–188. http://dx.doi.org/10.1016/S0165-1889(98)00092-X.

García-Alcalde, A., Ventosa, M., Rivier, M., Ramos, A., Relano, G., 2002. Fitting electricity market models: A conjectural variations approach. In: Proc. 14th Power Systems Computation Conference (PSCC), 2002, Vol. 12. https://pascua.iit.comillas.edu/aramos/papers/PSCCSeville_s12p03.pdf.

de Haro, S., Martin, P., Ardiz, J., Caro, J., 2007. Estimating conjectural variations for electricity market models. European J. Oper. Res. 181, 1322–1338. http://dx.doi.org/10.1016/j.ejor.2005.12.039.

Hobbs, B., Pang, J., 2007. Nash-cournot equilibria in electric power markets with piecewise linear demand functions and joint constraints. Oper. Res. 55, 113–127. http://dx.doi.org/10.1287/opre.1060.0342.

Kahn, E., 1998. Numerical techniques for analyzing market power in electricity. Electr. J. 11, 34–43. http://dx.doi.org/10.1016/S1040-6190(98)00057-8.

Knaut, A., Paulus, S., 2016. Hourly Price Elasticity Pattern of Electricity Demand in the German Day-Ahead Market. Technical Report 16/07, EWI, Cologne, Germany, https://www.ewi.uni-koeln.de/cms/wp-content/uploads/2016/08/EWI_WP_16_07_Hourly_Price_Elasticity.pdf.

Koschker, S., Möst, D., 2016. Perfect competition vs. strategic behaviour models to derive electricity prices and the influence of renewables on market power. OR Spectrum 38, 661–686. http://dx.doi.org/10.1007/s00291-015-0415-x.

Kuhn, H., Tucker, A., 1951. Nonlinear programming. In: Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability, Vol. 1950. University of California Press, Berkeley and Los Angeles, pp. 481–492.

Lagarto, J., Sousa, J., Martins, A., Ferrão, P., 2014. Market power analysis in the iberian electricity market using a conjectural variations model. Energy 76, 292–305. http://dx.doi.org/10.1016/j.energy.2014.08.014.

Lerner, A., 1995. The concept of monopoly and the measurement of monopoly power. In: Estrin, S., Marin, A. (Eds.), Essential Readings in Economics. Palgrave, Springer, London, pp. 55–76. http://dx.doi.org/10.1007/978-1-349-24002-9_4.

Linares, P., Santos, F., Ventosa, M., Lapiedra, L., 2008. Incorporating oligopoly, CO_2 emissions trading and green certificates into a power generation expansion model. Automatica 44, 1608–1620. http://dx.doi.org/10.1016/j.automatica.2008.03.006.

Metzler, C., Hobbs, B., Pang, J., 2003. Nash-cournot equilibria in power markets on a linearized dc network with arbitrage: Formulations and properties. Netw. Spat. Econ. 3, 123–150. http://dx.doi.org/10.1023/A:1023907818360.

Orea, L., Steinbuks, J., 2018. Estimating market power in homogenous product markets using a composed error model: application to the California electricity market. Econ. Inq. 56, 1296–1321. http://dx.doi.org/10.1111/ecin.12539.

Orgaz, A., Bello, A., Reneses, J., 2017. Multi-area electricity market equilibrium model and its application to the European case. In: 2017 14th International Conference on the European Energy Market (EEM). IEEE, pp. 1–6. http://dx.doi.org/10.1109/EEM.2017.7981924.

Orgaz, A., Bello, A., Reneses, J., 2019. A new model to simulate local market power in a multi-area electricity market: Application to the European case. Energies 12 (2068), http://dx.doi.org/10.3390/en12112068.

Panos, E., Densing, M., 2019. The future developments of the electricity prices in view of the implementation of the Paris agreements: Will the current trends prevail, or a reversal is ahead? Energy Econ. 84, 1–17. http://dx.doi.org/10.1016/j.eneco.2019.104476.

Panos, E., Densing, M., Schmedders, K., 2017. Oligopolistic Capacity Expansion with Subsequent Market Bidding under Transmission Constraints (OCESM). Technical Report, Paul Scherrer Institute & Swiss Federal Office of Energy (SFOE, Bern, switzerland, http://dx.doi.org/10.5167/uzh-151127.

Pape, C., Hagemann, S., Weber, C., 2016. Are fundamentals enough? explaining price variations in the German day-ahead and intraday power market. Energy Econ. 54, 376–387. http://dx.doi.org/10.1016/j.eneco.2015.12.013.

Perry, M., 1982. Oligopoly and consistent conjectural variations. Bell J. Econ. 13, 197–205. http://dx.doi.org/10.2307/3003440.

Pham, T., 2019. Market power issues in liberalized wholesale electricity markets: A review of the literature with a look into the future. Revue D'économie Politique 129, 325–354. http://dx.doi.org/10.3917/redp.293.0325.

Pintér, J., 2018. How difficult is nonlinear optimization? A practical solver tuning approach, with illustrative results. Ann. Oper. Res. 265, 119–141. http://dx.doi.org/10.1007/s10479-017-2518-z.

Scutari, G., Palomar, D., Facchinei, F., Pang, J., 2010. Convex optimization, game theory, and variational inequality theory. IEEE Signal Process. Mag. 27, 35–49. http://dx.doi.org/10.1109/MSP.2010.936021.

Song, Y., Ni, Y., Wen, F., Hou, Z., Wu, F., 2003. Conjectural variation based bidding strategy in spot markets: fundamentals and comparison with classical game theoretical bidding strategies. Electr. Power Syst. Res. 67, 45–51. http://dx.doi.org/10.1016/S0378-7796(03)00042-7.

Szu, H., Hartley, R., 1987. Nonconvex optimization by fast simulated annealing. Proc. IEEE 75, 1538–1540. http://dx.doi.org/10.1109/PROC.1987.13916.

Ventosa, M., Baíllo, A., Ramos, A., Rivier, M., 2005. Electricity market modeling trends. Energy Policy 33, 897–913. http://dx.doi.org/10.1016/j.enpol.2003.10.013.

Vives, X., 1999. Oligopoly Pricing: Old Ideas and New Tools. MIT Press, Cambridge, Mass.

Weigt, H., Hirschhausen, C., 2008. Price formation and market power in the german wholesale electricity market in 2006. Energy Policy 36, 4227–4234. http://dx.doi.org/10.1016/j.enpol.2008.07.020.