Study about the conductivity equation

A R Bathri Narayanan P0211501
Ranya Sharma P02115

We have the conductivity equation derived in class

$$\sigma = \sigma_0 e^{\frac{-\Delta}{kT}}$$

Taking a derivative of that with respect to temperature

$$\frac{d\sigma}{dT} = \sigma_0 e^{\frac{-\Delta}{kT}} \frac{\Delta}{kT^2}$$

Means the function will be at it's extrema at T $\to$ 0 or T $\to$ ∞ (T$\neq$0 as it will violate the third law of thermodynamics) Taking the second derivative to check for inflection points

$$\frac{d^2\sigma}{dT^2} = \sigma_0 \frac{\Delta^2}{k^2 T^4} e^{\frac{-\Delta}{kT}} - 2\sigma_0 e^{\frac{-\Delta}{kT}} \frac{\Delta}{kT^3}$$
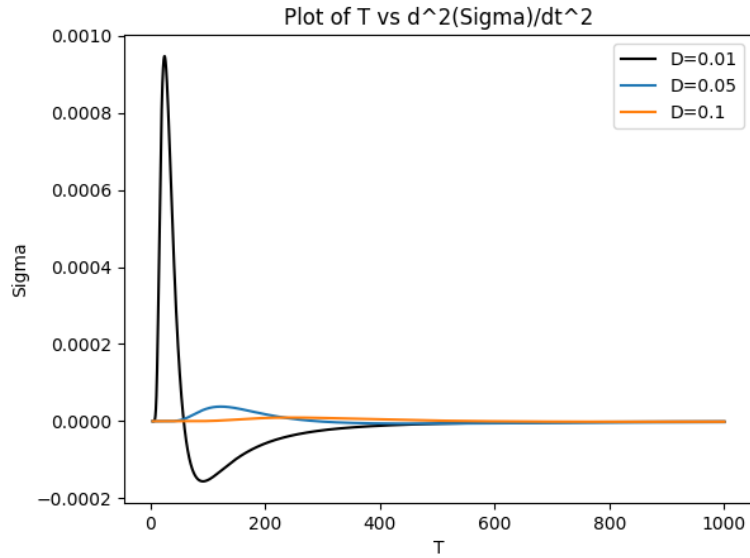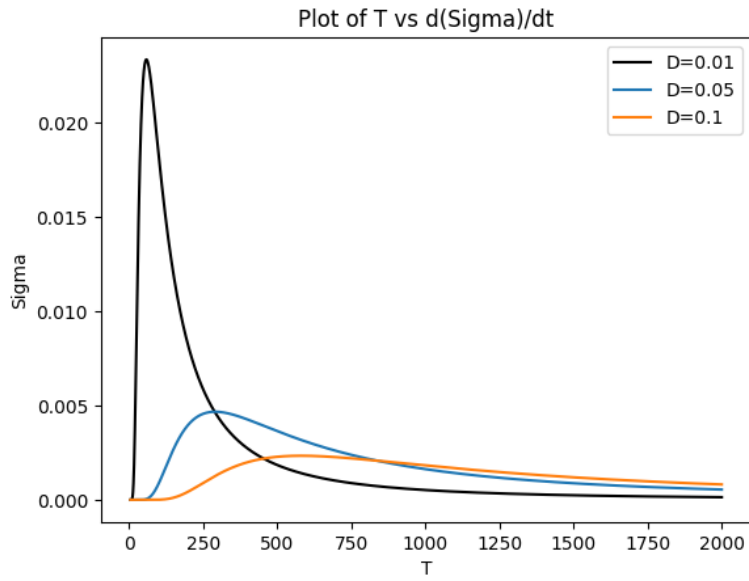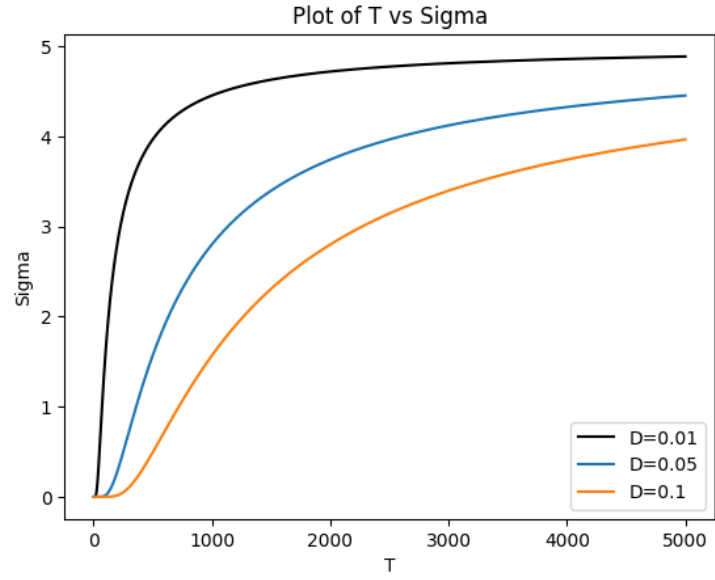
Finding the inflection points

$$\sigma_0 \frac{\Delta^2}{k^2 T^4} e^{\frac{-\Delta}{kT}} = 2\sigma_0 e^{\frac{-\Delta}{kT}} \frac{\Delta}{kT^3}$$

$$\frac{\Delta}{2kT} = 1 \text{ or } T = \frac{\Delta}{2k}$$

But as the function is a monotonically increasing function, this should increase, stop and still increase but at a slower rate. But if we try to find the order of inflection point, taking Δ to be around 0.05-0.5 eV, meaning T should be around $(0.29 - 2.9) \times 10^3 K$.
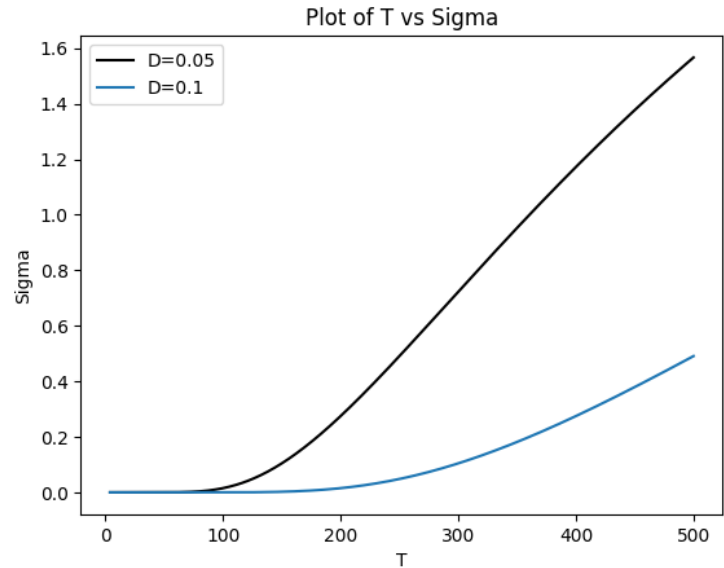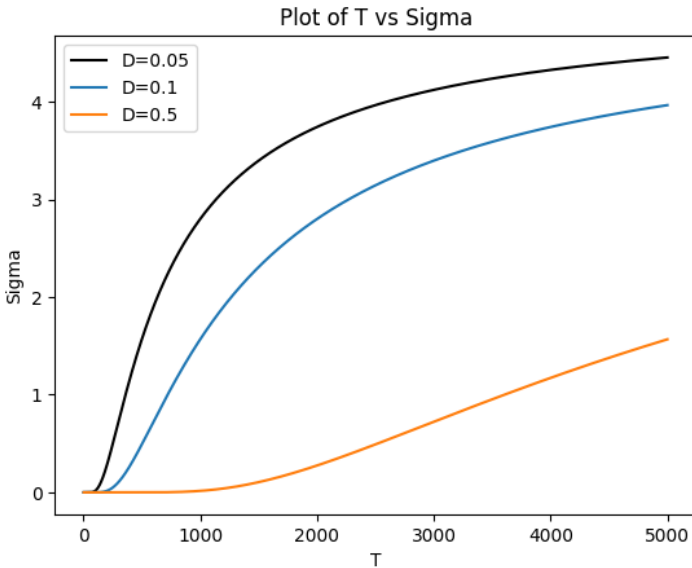
Graphs for reference (Taking σ_0=1 for ease), note that due to resolution, the graph appears to be touching 0, but it starts from 4K, the temperature of liquid Helium.

Now we take Δ's range from 0.01 to 0.1 and try plotting the graphs for σ vs T, $\frac{d\sigma}{dT}$ and $\frac{d^2\sigma}{dT^2}$

Plot of T vs Sigma

Plot of T vs d(Sigma)/dt

Plot of T vs d^2(Sigma)/dt^2

The inflection points are respectively at 58.0 K, 290.1 K and 580.2 K as we increase Δ from 0.01 to 0.1, with 290.1 K at 0.05.

Appendix