Search on Superconducting Qubits

International Young Quantum Meet 2024

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Introduction to the talk

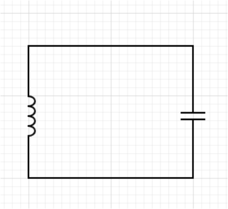
Qubits can be realised in a lot of ways, examples include

1. Ion trap quantum computing
2. NMR quantum computing
3. Optical photon quantum computing to name a few.

My talk will be focused more on Superconducting qubits and applying it to model a theoretical problem.

How do you build Superconducting Qubits

By using so called "Artificial atoms". You cool a LC circuit to the material's critical temperature, into it's superconducting phase.

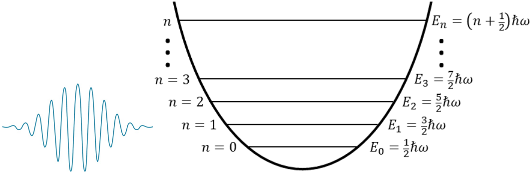


Where, —⌒⌒⌒— means Inductor and —||— means Capacitor. The Hamiltonian of this circuit is given by

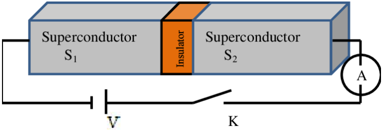
$$H = \frac{\hat{Q}}{2C} + \frac{\hat{\Phi}}{2L} = \hbar\omega(aa^{\dagger} + \frac{1}{2})$$

Issues with simple superconducting qubits

Energy levels of the circuit mimic a linear Harmonic oscillator



This can't be an ideal two qubit system as all energy levels are equally spaced. But how do we modify the energy levels?
Enter **Josephson Junctions** !!



How can you modify Superconducting Qubits (Josephson Junctions and dc-SQUIDs)

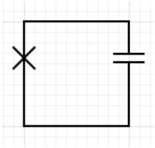
Josephson Junctions are represented by  . I am not focusing more on the derivation of V, I and L of Josephson Junctions, I just mention it here. Interested can refer to Feynman lectures volume 3 last chapter.

$$V_J = \frac{\Phi_0}{2\pi}\frac{d\varphi_J}{dt}$$

$$I_J = I_c sin\varphi_J$$

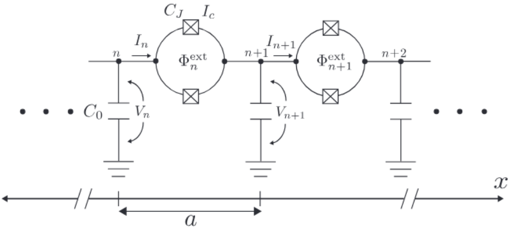
$$L_J = \frac{\phi_J}{I_c sin(2\pi\phi_J/\phi_0)} = \frac{\phi_0\varphi_J}{2\pi I_c sin\varphi_J}$$

The circuit can be drawn like



Analog gravity using superconducting qubits

We construct an infinite circuit like



Now, we apply Kirchoff law and Faraday's law as

$$I_n - I_{n+1} = \frac{dQ_{n+1}}{dt}$$

$$V_n - V_{n+1} = \frac{\Phi_0}{2\pi}\frac{d\varphi_{Jn}}{dt}$$

Obtaining the equation of motion

Plugging in the formulas obtained for Josephson junctions, we can obtain the current entering the nth unit cell as

$$I_n = 2C_J\frac{\Phi_0}{2\pi}\frac{d^2\varphi_{Jn}}{dt^2} + 2I_c cos\left(\frac{\pi\Phi_n^{ext}}{\Phi_0}\right)sin\varphi_{Jn}$$

Applying φ_{Jn}=φ_n-φ_{n+1} and plugging the above in the V_n equation, we obtain

$$(C_0+4C_J)\left(\frac{\Phi_0}{2\pi}\right)^2\frac{d^2\varphi_n}{dt^2} - 2C_J\left(\frac{\Phi_0}{2\pi}\right)^2\left(\frac{d^2\varphi_{n-1}}{dt^2}+\frac{d^2\varphi_{n+1}}{dt^2}\right)$$

$$= -2E_J cos\left(\frac{\pi\Phi_n^{ext}}{\Phi_0}\right)sin(\varphi_n-\varphi_{n+1}) + 2E_J cos\left(\frac{\pi\Phi_{n-1}^{ext}}{\Phi_0}\right)sin(\varphi_{n-1}-\varphi_n)$$

Obtaining the EM phase speed

We can guess the Lagrangian, and eventually the Hamiltonian, which is nothing but

$$H = \int_{-\infty}^{\infty} dx \left[\left(\frac{2\pi}{\Phi_0} \right)^2 \frac{p^2}{2\mathcal{C}} + E_J acos \left(\frac{\pi \Phi^{ext}}{\Phi_0} \right) \left(\frac{\partial \varphi}{\partial x} \right)^2 \right]$$

Of course we make a few approximations like the continuum approximation, neglecting the Josephson junction capacitance term. We get the following wave equation

$$\frac{\partial^2 \varphi}{\partial t^2} = \frac{\partial}{\partial x} \left(c^2 \frac{\partial \varphi}{\partial x} \right), c = \frac{1}{\sqrt{\mathcal{L}\mathcal{C}}}$$

Where,

$$\mathcal{C} = C_0/a, \mathcal{L} = \frac{\Phi_0}{4\pi I_c a} sec \left(\frac{\pi \Phi^{ext}}{\Phi_0} \right)$$

Getting the covariant wave equation

Now, we transform as x'=x-ut, t'=t, the wave equation becomes

$$\left(-\frac{\partial^2}{\partial t^2}+2u\frac{\partial^2}{\partial x\partial t}+\frac{\partial}{\partial x}(c^2-u^2)\frac{\partial}{\partial x}\right)\varphi=0=\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\varphi)$$

Where,

$$g^{\mu\nu}=\frac{1}{c}\begin{pmatrix}-1 & u\\ u & c^2-u^2\end{pmatrix}$$

We finally take the Hawking temperature

$$T_H=\frac{\hbar}{2\pi k_B}\left|\frac{\partial c}{\partial x}\right|_{x_h}$$

And plug in the respective values to get an estimate (for $\left|\frac{\partial c}{\partial x}\right|_{x_h}$ $\approx$0.01c_0/a, C_0=1μ F, I_c=5μA, T_h= 70 mK)

Conclusions

So in this talk, we have discussed about

1. A brief introduction to Superconducting qubits
2. Construction of superconducting qubits
3. A brief application of superconducting qubits in Analogue Gravity and Hawking radiation.

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Thank You !!