

# Search on Superconducting Qubits

## International Young Quantum Meet 2024

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Qubits can be realised in a lot of ways, examples include

1. Ion trap quantum computing
2. NMR quantum computing
3. Optical photon quantum computing to name a few.

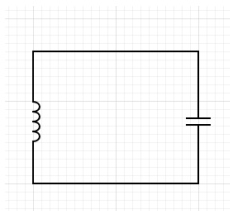
My talk will be focused more on Superconducting qubits and applying it to model a theoretical problem.



# How do you build Superconducting Qubits

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By using so called "Artificial atoms". You cool a LC circuit to the material's critical temperature, into it's superconducting phase.

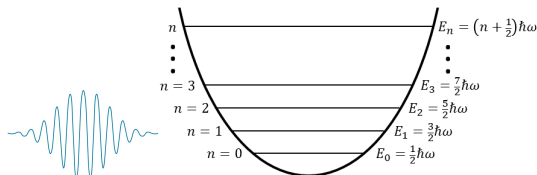


Where,  means Inductor and  means Capacitor.  
The Hamiltonian of this circuit is given by

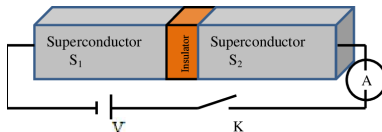
$$H = \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}}{2L} = \hbar\omega\left(aa^\dagger + \frac{1}{2}\right)$$

# Issues with simple superconducting qubits


Energy levels of the circuit mimic a linear Harmonic oscillator



This can't be an ideal two qubit system as all energy levels are equally spaced. But how do we modify the energy levels?  
Enter **Josephson Junctions** !!



# How can you modify Superconducting Qubits (Josephson Junctions and dc-SQUIDs)

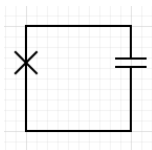
Josephson Junctions are represented by . I am not focusing more on the derivation of  $V$ ,  $I$  and  $L$  of Josephson Junctions, I just mention it here. Interested can refer to Feynman lectures volume 3 last chapter.

$$V_J = \frac{\Phi_0}{2\pi} \frac{d\varphi_J}{dt}$$

$$I_J = I_c \sin \varphi_J$$

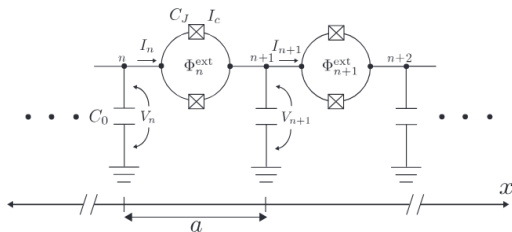
$$L_J = \frac{\phi_J}{I_c \sin(2\pi \phi_J / \phi_0)} = \frac{\phi_0 \varphi_J}{2\pi I_c \sin \varphi_J}$$

The circuit can be drawn like



# Analog gravity using superconducting qubits

We construct an infinite circuit like



Now, we apply Kirchoff law and Faraday's law as

$$I_n - I_{n+1} = \frac{dQ_{n+1}}{dt}$$

$$V_n - V_{n+1} = \frac{\Phi_0}{2\pi} \frac{d\varphi_{Jn}}{dt}$$

# Obtaining the equation of motion

Plugging in the formulas obtained for Josephson junctions, we can obtain the current entering the nth unit cell as

$$I_n = 2C_J \frac{\Phi_0}{2\pi} \frac{d^2 \varphi_{Jn}}{dt^2} + 2I_c \cos\left(\frac{\pi \Phi_n^{\text{ext}}}{\Phi_0}\right) \sin \varphi_{Jn}$$

Applying  $\varphi_{Jn} = \varphi_n - \varphi_{n+1}$  and plugging the above in the  $V_n$  equation, we obtain

$$\begin{aligned} & (C_0 + 4C_J) \left(\frac{\Phi_0}{2\pi}\right)^2 \frac{d^2 \varphi_n}{dt^2} - 2C_J \left(\frac{\Phi_0}{2\pi}\right)^2 \left(\frac{d^2 \varphi_{n-1}}{dt^2} + \frac{d^2 \varphi_{n+1}}{dt^2}\right) \\ &= -2E_J \cos\left(\frac{\pi \Phi_n^{\text{ext}}}{\Phi_0}\right) \sin(\varphi_n - \varphi_{n+1}) + 2E_J \cos\left(\frac{\pi \Phi_{n-1}^{\text{ext}}}{\Phi_0}\right) \sin(\varphi_{n-1} - \varphi_n) \end{aligned}$$

# Obtaining the EM phase speed

We can guess the Lagrangian, and eventually the Hamiltonian, which is nothing but

$$H = \int_{-\infty}^{\infty} dx \left[ \left( \frac{2\pi}{\Phi_0} \right)^2 \frac{p^2}{2\mathcal{C}} + E_J a \cos \left( \frac{\pi \Phi^{\text{ext}}}{\Phi_0} \right) \left( \frac{\partial \varphi}{\partial x} \right)^2 \right]$$

Of course we make a few approximations like the continuum approximation, neglecting the Josephson junction capacitance term. We get the following wave equation

$$\frac{\partial^2 \varphi}{\partial t^2} = \frac{\partial}{\partial x} \left( c^2 \frac{\partial \varphi}{\partial x} \right), c = \frac{1}{\sqrt{\mathcal{L}\mathcal{C}}}$$

Where,

$$\mathcal{C} = C_0/a, \mathcal{L} = \frac{\Phi_0}{4\pi I_c a} \sec \left( \frac{\pi \Phi^{\text{ext}}}{\Phi_0} \right)$$



# Getting the covariant wave equation

Now, we transform as  $x'=x-ut$ ,  $t'=t$ , the wave equation becomes

$$\left(-\frac{\partial^2}{\partial t^2} + 2u \frac{\partial^2}{\partial x \partial t} + \frac{\partial}{\partial x}(c^2 - u^2) \frac{\partial}{\partial x}\right) \varphi = 0 = \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \varphi)$$

Where,

$$g^{\mu\nu} = \frac{1}{c} \begin{pmatrix} -1 & u \\ u & c^2 - u^2 \end{pmatrix}$$

We finally take the Hawking temperature

$$T_H = \frac{\hbar}{2\pi k_B} \left| \frac{\partial c}{\partial x} \right|_{x_h}$$

And plug in the respective values to get an estimate (for

$$\left| \frac{\partial c}{\partial x} \right|_{x_h} \approx 0.01 c_0/a, C_0=1\mu\text{ F}, I_c=5\mu\text{A}, T_h=70\text{ mK})$$

So in this talk, we have discussed about

1. A brief introduction to Superconducting qubits
2. Construction of superconducting qubits
3. A brief application of superconducting qubits in Analogue Gravity and Hawking radiation.

In the future I have planned for

1. Trying to model the Unruh effect by following the footsteps of Blencowe
2. Trying to study on Current mirror qubits as introduced by Kitaev.

I would like to thank the following people for guiding me in the right direction

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4. Prof. Ujjwal Sen, Professor-H, Harish-Chandra Research institute

# Thank You !!

Any doubts :

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