Theoretical studies of Ej/Ec ratio and applying it to solve a theoretical physics problem

A R Bathri Narayanan; Carin Samer

UM DAE CEBS, India; Alexandria, Egypt

How can you modify Superconducting Qubits (Josephson Junctions and dc-SQUIDs)

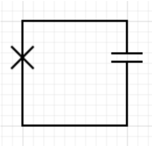
Josephson Junctions are represented by . This has the properties

$$V_J = \frac{\Phi_0}{2\pi}\frac{d\varphi_J}{dt}$$

$$I_J = I_c sin\varphi_J$$

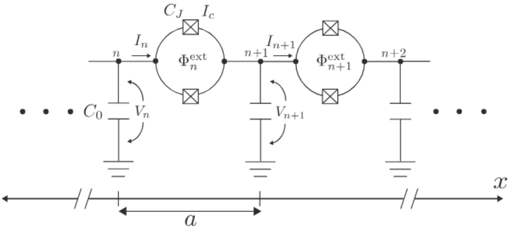
$$L_J = \frac{\phi_J}{I_c sin(2\pi\phi_J/\phi_0)} = \frac{\phi_0\varphi_J}{2\pi I_c sin\varphi_J}$$

The circuit can be drawn like



Analog gravity using superconducting qubits

We construct an infinite circuit like



Now, we apply Kirchoff law and Faraday's law as

$$I_n - I_{n+1} = \frac{dQ_{n+1}}{dt}$$

$$V_n - V_{n+1} = \frac{\Phi_0}{2\pi}\frac{d\varphi_{Jn}}{dt}$$

Obtaining the equation of motion

Plugging in the formulas obtained for Josephson junctions, we can obtain the current entering the nth unit cell as

$$I_n = 2C_J \frac{\Phi_0}{2\pi} \frac{d^2\varphi_{Jn}}{dt^2} + 2I_c cos\left(\frac{\pi\Phi_n^{ext}}{\Phi_0}\right) sin\varphi_{Jn}$$

Applying φ_{Jn}=φ_n-φ_{n+1} and plugging the above in the V_n equation, we obtain

$$(C_0 + 4C_J)\left(\frac{\Phi_0}{2\pi}\right)^2 \frac{d^2\varphi_n}{dt^2} - 2C_J\left(\frac{\Phi_0}{2\pi}\right)^2 \left(\frac{d^2\varphi_{n-1}}{dt^2} + \frac{d^2\varphi_{n+1}}{dt^2}\right)$$

$$= -2E_J cos\left(\frac{\pi\Phi_n^{ext}}{\Phi_0}\right) sin(\varphi_n - \varphi_{n+1}) + 2E_J cos\left(\frac{\pi\Phi_{n-1}^{ext}}{\Phi_0}\right) sin(\varphi_{n-1} - \varphi_n)$$

Obtaining the EM phase speed

We can guess the Lagrangian, and eventually the Hamiltonian, which is nothing but

$$H = \int_{-\infty}^{\infty} dx \left[\left(\frac{2\pi}{\Phi_0} \right)^2 \frac{p^2}{2\mathcal{C}} + E_J a cos \left(\frac{\pi \Phi^{ext}}{\Phi_0} \right) \left(\frac{\partial \varphi}{\partial x} \right)^2 \right]$$

Of course we make a few approximations like the continuum approximation, neglecting the Josephson junction capacitance term. We get the following wave equation

$$\frac{\partial^2 \varphi}{\partial t^2} = \frac{\partial}{\partial x} \left(c^2 \frac{\partial \varphi}{\partial x} \right), c = \frac{1}{\sqrt{\mathcal{LC}}}$$

Where,

$$\mathcal{C} = C_0/a, \mathcal{L} = \frac{\Phi_0}{4\pi I_c a} sec \left(\frac{\pi \Phi^{ext}}{\Phi_0} \right)$$

Getting the covariant wave equation

Now, we transform as x'=x-ut, t'=t, the wave equation becomes

$$\left(-\frac{\partial^2}{\partial t^2} + 2u\frac{\partial^2}{\partial x \partial t} + \frac{\partial}{\partial x}(c^2 - u^2)\frac{\partial}{\partial x} \right)\varphi = 0 = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\varphi)$$

Where,

$$g^{\mu\nu} = \frac{1}{c}\begin{pmatrix} -1 & u \\ u & c^2 - u^2 \end{pmatrix}$$

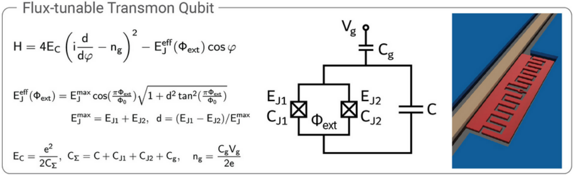
We finally take the Hawking temperature

$$T_H = \frac{\hbar}{2\pi k_B}\left|\frac{\partial c}{\partial x}\right|_{x_h}$$

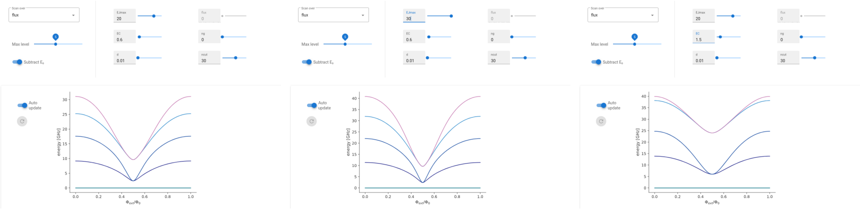
And plug in the respective values to get an estimate (for $\left|\frac{\partial c}{\partial x}\right|_{x_h} \approx 0.01 c_0/\mathrm{a}$, $C_0 = 1\mu$ F, $I_c = 5\mu$A, $T_h =$ 70 mK)

The Ej/Ec ratio

Usually in transmon qubits, the Ej, that is Josephson energy of the circuit, is greater than Ec, the capacitive energy of the circuit. To put it mathematically, Ej/Ec »1.
This gives an advantage of less interference of charge noise, and a disadvantage of anharmonicity. But this can be overcome by tuning the microwave pulse appropriately.
We try varying the Ej/Ec ration of a single tunable transmon qubit, which forms a basic building block of our system.



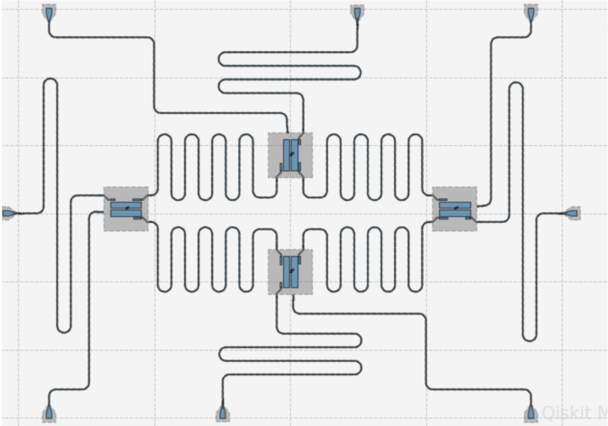
Taken from scqubits



The flux graph taken for various ratios of Ej/Ec

The bigger aim

We have constructed a circular Blencowian qubit. And we would like to analyse on this more. This resembles more like a current mirror qubit, which is even more insensitive to errors as found out by Koch.



References

Miles Blencowe Paper on Analogue Gravity Sc Qubits, Qiskit Metal Documentation