The Gaussian Beam

Presentation for
Femtosecond and Attosecond Pulses (P-704)

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Prelude

A paraxial wave is a plane wave travelling along the z direction (e^{-ikz}) with wavenumber $\frac{2\pi}{\lambda}$ for wavelength λ, modulated by a complex envelope A($\mathbf{r}$), being a slowly varying function of position. The complex amplitude is

$$U(\mathbf{r}) = A(\mathbf{r})e^{-ikz}$$

The envelope is taken to be approximately constant within a neighborhood of size λ, so that the wave locally maintains its plane- wave nature but exhibits wavefront normals that are paraxial rays.
In order that the complex amplitude U($\mathbf{r}$) satisfy the Helmholtz equation, $\nabla^2 U + k^2 U = 0$. The paraxial envelope should ba a solution of the Paraxial Helmholtz equation

$$\nabla_T^2 A - j2k\frac{\partial A}{\partial z} = 0, \nabla_T^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

The Gaussian Solution

We insert a Gaussian solution of the form

$$A(\mathbf{r}) = \frac{A'}{q(z)} exp\big(-ik\frac{\rho^2}{2q(z)}\big), \rho^2 = x^2 + y^2, q(z) = z - \eta$$

The η can be real or complex. If we put $\eta = -iz_0$, we get

$$A(\mathbf{r}) = \frac{A'}{q(z)} exp\big(-ik\frac{\rho^2}{2q(z)}\big), q(z) = z - iz_0$$

q(z) is the q-parameter of the beam, and z_0 is called the Rayleigh range of the beam. Substituting

$$\frac{1}{q(z)} = \frac{1}{R(z)} - i\frac{\lambda}{\pi W^2(z)}$$

The Gaussian Solution

Where,

$$R(z) = z\left[1 + (\frac{z}{z_0})^2\right], W(z) = W_0\sqrt{1 + (\frac{z}{z_0})^2}, W_0 = \sqrt{\frac{\lambda z_0}{\pi}}$$

We get the equation for complex amplitude,

$$\boxed{U(\mathbf{r}) = A_0 \frac{W_0}{W(z)} exp\left(-\frac{\rho^2}{W^2(z)}\right) exp\left(-jkz - jk\frac{\rho^2}{2R(z)} + j\zeta(z)\right)}$$

Where we define,

$$\zeta(z) = tan^{-1}\frac{z}{z_0}, A_0 = \frac{A'}{iz_0}$$

The equation above depends on two independent parameters, A_0 and z_0, which depends on the boundary conditions. All the others are dependent on z_0 and λ

Properties of the Gaussian Beam

The properties we are going to look at are

1. Intensity
2. Power
3. Beam Width
4. Beam Divergence
5. Depth of Focus
6. Phase
7. Wavefronts

And we describe how to characterise them

Intensity

Intensity

Power

Power

Beam Width

Beam Divergence

Depth Focus

Phase

Wavefronts

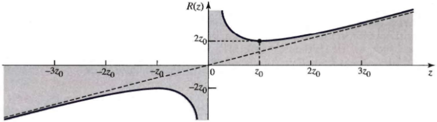


Figure 3.1-6 The radius of curvature $R(z)$ of the wavefronts of a Gaussian beam as a function of position along the beam axis. The dashed line is the radius of curvature of a spherical wave.

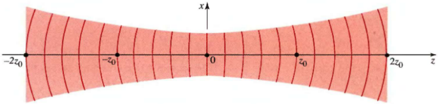
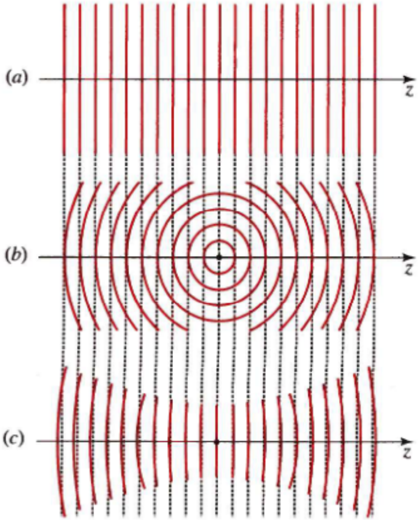


Figure 3.1-7 Wavefronts of a Gaussian beam.

Wavefronts

Wavefronts



Characterisation of Gaussian Beam

To Summarize

Beam Quality

Beam Quality

Take-Home Messages

Thank You!!