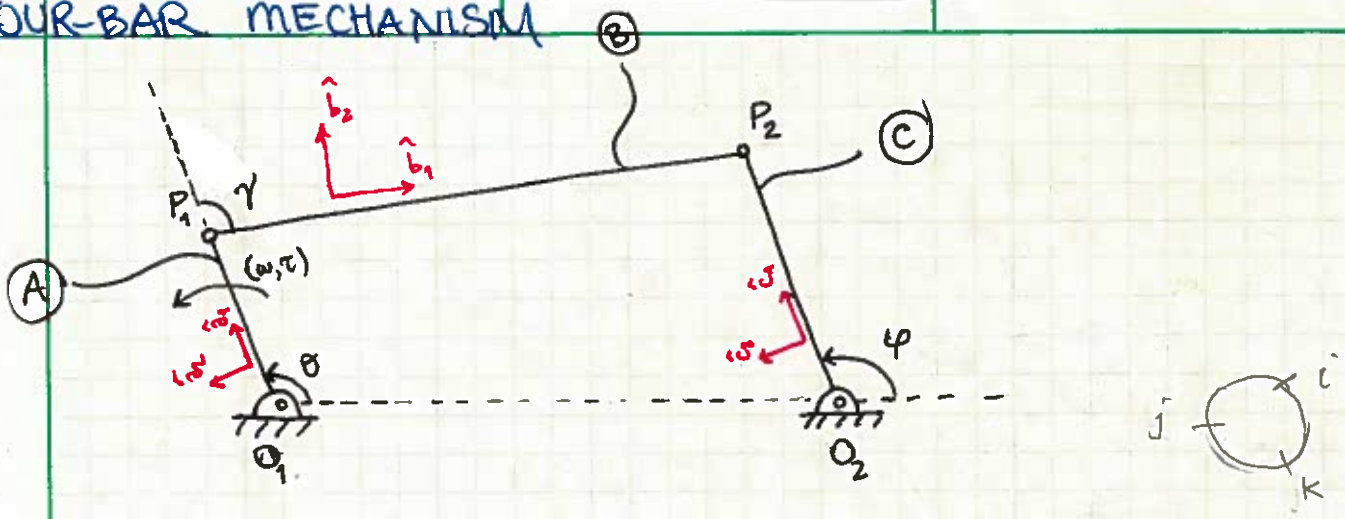
FOUR-BAR MECHANISM



Position Level

$$P_1 + P_2 + P_3 + P_0 = 0$$

$$\Rightarrow l_1 \hat{a}_1 + l_2 \hat{b}_1 - l_3 \hat{c}_1 - l_0 \hat{e}_1 = 0$$

$$\Rightarrow l_1 \begin{pmatrix} c_\theta \\ s_\theta \end{pmatrix} + l_2 \begin{pmatrix} c_{\theta+\gamma} \\ s_{\theta+\gamma} \end{pmatrix} - l_3 \begin{pmatrix} c_\varphi \\ s_\varphi \end{pmatrix} - \begin{pmatrix} l_0 \\ 0 \end{pmatrix} = 0$$

Forward

Known: θ
Unk.: γ, φ

Inverse

Known: φ
Unknown: γ, θ

Velocity level

$$\frac{{}^E d}{dt}\left(P_1 + P_2 + P_3 + P_0\right) = {}^E\omega^A \times P_1 + {}^E\omega^B \times P_2 + {}^E\omega^C \times P_3 = 0$$

$$\Rightarrow \dot{\theta}\,\hat{a}_3 \times l_1 \hat{a}_1 + (\dot{\theta}+\dot{\gamma})\,\hat{b}_3 \times l_2 \hat{b}_1 + \dot{\varphi}\,\hat{c}_3 \times (-l_3 \hat{c}_1) = 0$$

$$\Rightarrow l_1 \dot{\theta}\,\hat{e}_3 \times (c_\theta \hat{e}_1 + s_\theta \hat{e}_2) + l_2(\dot{\theta}+\dot{\gamma})\,\hat{e}_3 \times (c_{\theta+\gamma}\hat{e}_1 + s_{\theta+\gamma}\hat{e}_2) - l_3 \dot{\varphi}\,\hat{e}_3 \times (c_\varphi \hat{e}_1 + s_\varphi \hat{e}_2) = 0$$

$$\Rightarrow l_1 \dot{\theta} \begin{pmatrix} -s_\theta \\ c_\theta \end{pmatrix} + l_2(\dot{\theta}+\dot{\gamma}) \begin{pmatrix} -s_{\theta+\gamma} \\ c_{\theta+\gamma} \end{pmatrix} - l_3 \dot{\varphi} \begin{pmatrix} -s_\varphi \\ c_\varphi \end{pmatrix} = 0$$

Forward

Known: $\dot{\theta}$
Unk.: $\dot{\gamma}, \dot{\varphi}$

Inverse

Known: $\dot{\varphi}$
Unk.: $\dot{\gamma}, \dot{\theta}$

Acceleration level

$$\frac{{}^E d}{dt}\left({}^E\omega^A \times P_1 + {}^E\omega^B \times P_2 + {}^E\omega^C \times P_3\right) = 0$$

$$\Rightarrow {}^E\alpha^A \times P_1 + {}^E\omega^A \times ({}^E\omega^A \times P_1) + {}^E\alpha^B \times P_2 + {}^E\omega^B \times ({}^E\omega^B \times P_2) + {}^E\alpha^C \times P_3 + {}^E\omega^C \times ({}^E\omega^C \times P_3) = 0$$

$$l_1 \ddot{\theta} \begin{pmatrix} -s_\theta \\ c_\theta \end{pmatrix} - l_1 \dot{\theta}^2 \begin{pmatrix} c_\theta \\ s_\theta \end{pmatrix} + l_2(\ddot{\theta}+\ddot{\gamma}) \begin{pmatrix} -s_{\theta+\gamma} \\ c_{\theta+\gamma} \end{pmatrix} - l_2(\dot{\theta}+\dot{\gamma})^2 \begin{pmatrix} c_{\theta+\gamma} \\ s_{\theta+\gamma} \end{pmatrix} - l_3 \ddot{\varphi} \begin{pmatrix} -s_\varphi \\ c_\varphi \end{pmatrix} + l_3 \dot{\varphi}^2 \begin{pmatrix} c_\varphi \\ s_\varphi \end{pmatrix} = 0$$

	Known	Unknown
Forward	$\ddot{\theta}$	$\ddot{\gamma}, \ddot{\varphi}$
Inverse	$\ddot{\varphi}$	$\ddot{\gamma}, \ddot{\theta}$