Matrices

① $$\begin{vmatrix} A & B \\ C & D \end{vmatrix} = |A|\,|D - CA^{-1}B| = |D|\,|A - BD^{-1}C|$$

② A is of order $m \times n$
B " " " $n \times m$.

$$|I_m - AB| = |I_n - BA|$$

Linear Transformation

$T : V \to W$ a linear transformation

$\Rightarrow$ $\text{Kernel}(T) = \{ v \in V \ni T(v) = 0_W \} \subseteq V$

$\text{Image}(T) = \{ T(v) \in W \ni v \in V \} \subseteq W$

$\Rightarrow$ $\text{Ker } A = N(A)$ (i.e. $A\underset{\sim}{x} = \underset{\sim}{0}$)

$\text{Im } A = \mathcal{C}(A)$ (i.e. $A\underset{\sim}{x} \in \mathcal{C}(A)$) $\subseteq \mathbb{R}^m$

where $A_{m \times n} \ni T = A\underset{\sim}{x} : \mathbb{R}^n \to \mathbb{R}^m$

$\Rightarrow$ $\text{Ker}(T) = \{0_V\} \Rightarrow T$ is one-one (injective)

$\text{Im}(T) = W \Rightarrow T$ is surjective (onto)

V and W are finite dimensional and of equal dimension (i.e. $\dim V = \dim W$)

Then followings are equivalent –

(i) T is one-one.

(ii) T is onto.

(iii) $\text{rank}(T) = \dim V$

$T : V \to W$

$\beta = \{v_1, v_2, \dots, v_n\}$: basis of V

$R(T) = \text{span}(T(\beta))$.

$T : V \to W$ where V is of finite dimension

$\dim N(T) + \dim R(T) = \dim V$

i.e. $\text{nullity}(T) + \text{rank}(T) = \dim V$

$\text{rank}(T) = \text{rank}$ of matrix of T

$T: V \to W$

β = ordered basis of W

α = " " " V

$$[T(v)]_\beta = [T]_\alpha^\beta \, [v]_\alpha$$

Eigen Vector & Eigen Values

To an eigen vector ~~B~~ of A there corresponds an unique eigen value of A

Each eigen value of a real orthogonal matrix has unit modulus.