Confidence Intervals for The Parameters Of Univariate Normal, Two Independent Normal and One Parameter Exponential Distn

Univariate Normal

For mean μ

(i) with σ known

$$P\left[\bar{x} - \frac{\sigma}{\sqrt{n}} \tau_{\frac{\alpha}{2}} \leq \mu \leq \bar{x} + \frac{\sigma}{\sqrt{n}} \tau_{\frac{\alpha}{2}}\right] = 1-\alpha$$

So, $\left[\bar{x} - \frac{\sigma}{\sqrt{n}}, \bar{x} + \frac{\sigma}{\sqrt{n}}\right]$ is $100(1-\alpha)\%$ confidence interval for μ.

Interval length $= \frac{2\sigma}{\sqrt{n}} \tau_{\frac{\alpha}{2}}$

(ii) with σ unknown

$$P\left[\bar{x} - \frac{s'}{\sqrt{n}} t_{\frac{\alpha}{2}; n-1} \leq \mu \leq \bar{x} + \frac{s'}{\sqrt{n}} t_{\frac{\alpha}{2}; n-1}\right] = 1-\alpha$$

where $s'^2 = \frac{1}{n-1} \Sigma (x_i - \bar{x})^2$

Interval length $= \frac{2s'}{\sqrt{n}} t_{\frac{\alpha}{2}; n-1}$

τ_α and $t_{\alpha;n}$ are upper $(100\times\alpha)\%$ point of $N(0,1)$ and t_n distn respectively.

For σ^2

(i) with μ known

$$P\left[\frac{\Sigma(x_i-\mu)^2}{\chi^2_{\alpha/2;n}} < \sigma^2 < \frac{\Sigma(x_i-\mu)^2}{\chi^2_{1-\frac{\alpha}{2};n}}\right] = 1-\alpha$$

$$\text{Interval length} = \frac{\Sigma(x_i-\mu)^2}{\chi^2_{1-\frac{\alpha}{2};n}} - \frac{\Sigma(x_i-\mu)^2}{\chi^2_{\frac{\alpha}{2};n}}$$

" " for σ : $\sqrt{\frac{\Sigma(x_i-\mu)^2}{\chi^2_{1-\frac{\alpha}{2};n}}} - \sqrt{\frac{\Sigma(x_i-\mu)^2}{\chi^2_{\alpha/2;n}}}$

(ii) with μ unknown

$$P\left[\frac{\Sigma(x_i-\bar{x})^2}{\chi^2_{\alpha/2;n-1}} < \sigma^2 < \frac{\Sigma(x_i-\bar{x})^2}{\chi^2_{1-\frac{\alpha}{2};n-1}}\right] = 1-\alpha$$

Two Independent Normal

For $l_1\mu_1 + l_2\mu_2$

(i) with σ_1 and σ_2 known

$$P\left[l_1\bar{x}_1 + l_2\bar{x}_2 - \tau_{\frac{\alpha}{2}}\sqrt{\frac{l_1^2\sigma_1^2}{n_1}+\frac{l_2^2\sigma_2^2}{n_2}} < l_1\mu_1 + l_2\mu_2 < l_1\bar{x}_1 + l_2\bar{x}_2 + \tau_{\frac{\alpha}{2}}\sqrt{\frac{l_1^2\sigma_1^2}{n_1}+\frac{l_2^2\sigma_2^2}{n_2}}\right] = 1-\alpha$$

$$\text{Interval length} = 2\tau_{\frac{\alpha}{2}}\sqrt{\frac{l_1^2\sigma_1^2}{n_1}+\frac{l_2^2\sigma_2^2}{n_2}}$$

(ii) with σ_1 and σ_2 unknown

$$P\left[l_1\bar{x}_1 + l_2\bar{x}_2 - \tau_{\frac{\alpha}{2};n_1+n_2-2}\sqrt{s^2\left(\frac{l_1^2}{n_1}+\frac{l_2^2}{n_2}\right)} < l_1\mu_1 + l_2\mu_2 < l_1\bar{x}_1 + l_2\bar{x}_2 + \tau_{\frac{\alpha}{2};n_1+n_2-2}\sqrt{s^2\left(\frac{l_1^2}{n_1}+\frac{l_2^2}{n_2}\right)}\right] = 1-\alpha$$

$$\text{Interval length} = 2s\tau_{\frac{\alpha}{2};n_1+n_2-2}\sqrt{\frac{l_1^2}{n_1}+\frac{l_2^2}{n_2}}$$

where $s = \frac{(n_1-1)s_1'^2 + (n_2-1)s_2'^2}{n_1+n_2-2}$

For $\theta = \frac{\sigma_1}{\sigma_2}$

① with μ_1 and μ_2 known

$$P\left[F_{1-\frac{\alpha}{2};n_1,n_2} \le \frac{\sum(x_{1i}-\mu_1)^2/n_1}{\sum(x_{2i}-\mu_2)^2/n_2}\,\frac{1}{\theta^2} \le F_{\frac{\alpha}{2};n_1,n_2}\right] = 1-\alpha$$

$$\Rightarrow P\left[\frac{1}{\sqrt{F_{\frac{\alpha}{2};n_1,n_2}}}\sqrt{\frac{\sum(x_{1i}-\mu_1)^2/n_1}{\sum(x_{2i}-\mu_2)^2/n_2}} \le \theta \le \frac{1}{\sqrt{F_{1-\frac{\alpha}{2};n_1,n_2}}}\sqrt{\frac{\sum(x_{1i}-\mu_1)^2/n_1}{\sum(x_{2i}-\mu_2)^2/n_2}}\right]$$

② with μ_1 and μ_2 unknown

Same as above just put $n_i - 1$ in place of n_i, $i = 1, 2$ and $\bar{x}_i$ in place of μ_i

If p value is less than the level of significance α then we reject null hypothesis.