Complex Number

① For $z = a + ib$.

$|z| = \sqrt{a^2+b^2} \geq 0$

$|z|^2 = z\bar{z}$

amplitude = argument $= \theta = \tan^{-1}\left(\frac{b}{a}\right)$

For $-\pi < \theta \leq \pi$, θ is called Principal value.

$-|z| \leq a \leq |z|$, $-|z| \leq b \leq |z|$

② $z_j = a_j + ib_j$; $j = 1, 2$

$|z_1 z_2| = |z_1||z_2|$

$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$

③ $|z_1 + z_2| = |z_1 - z_2| \iff Arg(z_1) - Arg(z_2) = \frac{\pi}{2}$

$|z_1 + z_2| = |z_1| + |z_2| \iff Arg(z_1) = Arg(z_2)$

$|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 \iff \frac{z_1}{z_2}$ is purely imaginary

~~For $|z_1|, |z_2| \leq 1$~~ &.

$|z_1| \leq |z_2| \leq 1 \Rightarrow |z_1 + z_2| \leq |z_1| + |z_2|$

④ $Arg\left(\prod z_i\right) = \sum Arg(z_i)$

$Arg\left(\frac{z_1}{z_2}\right) = Arg(z_1) - Arg(z_2)$

$Arg(\bar{z}) = -Arg(z)$

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(5) $\sqrt{a+ib} = \pm\left[\sqrt{\frac{|z|+a}{2}} \pm i\sqrt{\frac{|z|-a}{2}}\right]$. accor

as $b>0$, $b<0$.

(6) $x^3=1 \Rightarrow x=1, \omega, \omega^2$

$\omega = \frac{-1-i\sqrt{3}}{2}$

$\omega^2 = \frac{-1+i\sqrt{3}}{2}$

$(1+x+x^2) = (x-\omega)(x-\omega^2)$

$\omega^3 = 1$

$\overline{\omega} = \omega^2$

(7) $z = a+ib = r(\cos\theta + i\sin\theta) = re^{i\theta}$

where $r = |z|$, $\theta = \tan^{-1}\left(\frac{b}{a}\right)$

$\ln z = \ln r + i\theta$

(8) $z_1 = i^i$

$\Rightarrow \ln z_1 = e^{-\pi/2}$

(9) $z = \cos\theta + i\sin\theta = e^{i\theta}$

$\prod_{r=1}^{n} (\cos r\theta + i\sin r\theta) = \prod_{r=1}^{n} e^{i\theta r}$

$= \cos(\Sigma r)\theta + i\sin(\Sigma r)\theta$

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(10) $\sum_{k=1}^{n}\left[\sin\frac{2k\pi}{n+1} - i\cos\frac{2k\pi}{n}\right.$

(10) $(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$

(11) For $z^n = 1$, the roots of the equ^n are

$z = \cos\left(\frac{2r\pi}{n}\right) + i\sin\left(\frac{2r\pi}{n}\right)$ $\quad r = 0, 1, 2, \ldots, (n-1)$

$= e^{i\frac{2r\pi}{n}}$

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