Important Formulae

(1) $(1-z)^{-r} = \sum_{x=0}^{\infty} \binom{x+r-1}{x} z^{x}$ for $|z|<1$

$\cancel{(2)\ \ln(1+x) = -1 + x - x^2 + x^3 - \cdots}$

$\cancel{(3)\ \ln(1-x) = -(1 + x + x^2 + x^3 + \cdots)}$

(2) $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$

(3) $\ln(1-x) = -\left(x + \frac{x^2}{2} + \frac{x^3}{3} + \cdots\right)$

(4) $$\int\int\cdots\int_{x_1^2+x_2^2+\cdots+x_n^2<a^2} dx_1\,dx_2\cdots dx_n = \frac{(a\sqrt{\pi})^n}{\Gamma\left(\frac{n}{2}+1\right)}$$

(5) $$\int_{\underline{x}'A\underline{x}<k} d\underline{x} = \frac{1}{\sqrt{|A|}}\,\frac{(\sqrt{\pi k})^n}{\Gamma\left(\frac{n}{2}+1\right)}$$

(6) $\boxed{\text{Leibnitz Integral Rule}}$

$$\frac{d}{dx}\int_{a(x)}^{b(x)} f(x,t)\,dt = f(x,b(x))\frac{d}{dx}b(x) - f(x,a(x))\frac{d}{dx}a(x) + \int_{a(x)}^{b(x)} \frac{\partial}{\partial x} f(x,t)\,dt$$

(7) $$\int_0^{\pi/2} \sin^p\theta \cos^q\theta \, d\theta = \frac{1}{2} B\left(\frac{p+1}{2}, \frac{q+1}{2}\right)$$ where $p>-1$, $q>-1$

(8) $$\int_0^{\infty} e^{-x^2} = \frac{\sqrt{\pi}}{2}$$

Calculus

① $\lim_{x\to 0} \frac{a^x - 1}{x} = \ln a \quad (a>0)$

② $\lim_{n\to\infty} \left(1+\frac{\mu}{n}\right)^n = e^{\mu}$ or $\lim_{x\to 0} (1+\mu x)^{\frac{1}{x}}$

③ $\lim_{x\to 0} \frac{\log_e(1+x)}{x} = 1$

④ $\lim_{x\to a} \{f(x)\}^{g(x)} = e^{\lim_{x\to a} \{f(x)-1\} g(x)}$

given $f(x) \to 1$, $g(x) \to \infty$ as $x \to a$

⑤ $\int \frac{dx}{x^2+a^2} = \frac{1}{a} \tan^{-1}\frac{x}{a} + c$

⑥ $\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left|\frac{x-a}{x+a}\right| + c$

⑦ $\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left|\frac{a+x}{a-x}\right| + c$

⑧ $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left| x + \sqrt{x^2 \pm a^2} \right| + c$

⑨ $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$

⑩ $\int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2+b^2} (a \sin bx - b \cos bx) + c$

⑪ $\int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2+b^2} (a \cos bx + b \sin bx) + c$

Rectification

\# $f(x)$ is $\ni$ $f'(x)$ is continuous.
Then the arc lengths of the curve $y=f(x)$ from $A(a, f(a))$ to $B(b, f(b))$ is

$$= \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

For $x=f(y)$ from the point $y=c$ to $y=d$ is

$$= \int_c^d \sqrt{1+\left(\frac{dx}{dy}\right)^2} dy$$ where $f'(y)$ is continuous.

\# For $x=f(t)$, $y=\phi(t)$, the length of the ~~curve~~ arc AB is $\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\, dt$

where $t=t_1$ at A and $t=t_2$ at B.

\# For the polar equ^n $r=f(\theta)$, the length of the arc AB of the curve is

$$\int_{\theta_1}^{\theta_2} \sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

where θ_1 and θ_2 are vectorial angles for A and B.

Quadrature

For $r = f(\theta)$

$\frac{1}{2}\int_{\alpha}^{\beta} r^2 d\theta$ = the area bounded by the curve $r = f(\theta)$, the radii vectors $\theta = \alpha$, $\theta = \beta$.

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Volumes and Surfaces of Revolution

For the continuous function $y = f(x)$, $a \le x \le b$, the region enclosed by the ordinates $x = a$, $x = b$ and the x-axis be rotated about x-axis, then the volume of revolution $= \pi \int_a^b y^2 dx = \pi \int_a^b \{f(x)\}^2 dx$

Similarly wrt y-axis we will have the volume of revolution $= \pi \int_c^d x^2 dy = \pi \int_c^d \{f(y)\}^2 dy$

For the above enclosed area the surface of revolution

$$= 2\pi \int_a^b y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= 2\pi \int_a^b f(x) \sqrt{1 + (f'(x))^2}\, dx$$

Let AB be the curve where the vectorial angles of A and B are respectively θ_1 and θ_2. If AB is revolved about the polar axis OX then (i) Volume of Revolution $= \pi \int_{\theta_1}^{\theta_2} r^2 \sin^2\theta \, d(r\cos\theta)$

(ii) surface of " $= 2\pi \int_{\theta_1}^{\theta_2} r \sin\theta \sqrt{(dr)^2 + r^2 (d\theta)^2}$