Some Important Formulas

(1) Let X be a rv $\ni P(X>0)=1$

then $E(X^r) = \int_0^\infty r x^{r-1}(1-F(x))\,dx$

(2) X and Y are said to be independently distributed if $F_{X,Y}(x,y) = F_X(x)F_Y(y) \ \forall (x,y)$

(3) X and Y are said to be independently distributed iff $P_{X,Y}(x,y) = P_X(x)P_Y(y) \ \forall (x,y)$

iff $f_{X,Y}(x,y) = f_X(x)f_Y(y) \ \forall (x,y)$

(4) $V\left(\sum_1^n l_i x_i\right) = \sum_1^n l_i^2 V(x_i) + \sum_{i \neq j} l_i l_j \operatorname{cov}(x_i, x_j)$

(5) $\operatorname{cov}\left(\sum_1^m a_i x_i, \sum_1^n b_j y_j\right) = \sum_{i=1}^m \sum_{j=1}^n a_i b_j \operatorname{cov}(x_i, y_j)$

(6) For independent rvs X and Y

$E(XY) = E(X)E(Y)$

(7) $E(E(Y|X)) = E(Y)$

$V(Y) = E(V(Y|X)) + V(E(Y|X))$

(8) If the support of joint distn

$\neq$ (the " " X) $\times$ (the support of Y)

then X and Y aren't independently distributed

(9) Chebyshev's inequality

$$P(|X-a| < t) \geq 1 - \frac{E(X-a)^2}{t^2}, \quad t > 0$$

$$P(|X-\mu| < t) \geq 1 - \frac{V(X)}{t^2}, \quad t > 0$$

$$P(|X-\mu| \geq t) \leq \frac{V(X)}{t^2}, \quad t > 0$$

(10) Markov's Inequality

$P(X > 0) = 1$ with $E(X) = \mu < \infty$

$$\Rightarrow P(X \geq t) \leq \frac{E(X)}{t}, \quad t > 0$$

(11) Cantelli's Inequality

X : rv, F : df, $\mu = E(X)$, $\sigma^2 = V(X) < \infty$

Then $$F(x) \leq \frac{\sigma^2}{\sigma^2 + (x-\mu)^2} \quad \forall\ x \leq \mu$$

$$F(x) \geq \frac{(x-\mu)^2}{\sigma^2 + (x-\mu)^2} \quad \forall\ x \geq \mu$$

(12) One-sided Chebyshev's Inequality

X : RV, $\mu = E(X)$, $\sigma^2 = V(X) < \infty$

$$P(X \geq \mu + t\sigma) \leq \frac{1}{1+t^2} \quad \text{[By ''], } t > 0$$

(13) If $P_X(t)$ be the pgf of a rv X then

$P_X(1) = 1$ (obvious)

(14) Suppose X is non-negative and integer-valued and moments of all orders exist.

Then, $$P_X(t) = \sum_{k=0}^{\infty} \left\{ E(X)_k \right\} \frac{(t-1)^k}{k!}, \quad |t| \leq 1$$

(15) For any integer-valued rv x

$$\sum_{n=0}^{\infty} z^n P(x \le n) = (1-z)^{-1} P_x(z)$$

where $P_x(z)$ is the pgf of x.

(16) <u>Continuity Theorem:</u>

If $\{A_n\}$ be an expanding or contracting sequence of events then

$$\lim_{n\to\infty} P(A_n) = P(\lim_{n\to\infty} A_n)$$

(17) ~~V()~~ $V(x) \le E(x^2)$

i.e. $V(x)$ exists if $\mu_2' < \infty$

(18) x be a continuous rv $\ni P(x \ge 0) = 1$ and $E(x)$ exists then (1) $\lim_{x\to\infty} x(1-F(x)) = 0$

(2) $E(x) = \int_0^{\infty} (1-F(x))dx$

(19) For non-negative integer valued rv $x \ni E(x)$ exists. $E(x) = \sum_{i=0}^{\infty} (1-F(i))$

(20) For a continuous rv $\ni E(x)$ exists.

$x(1-F(x)) \to 0$ as $x \to \infty$

$x F(x) \to 0$ as $x \to -\infty$.

$$E(x) = \int_0^{\infty} (1-F(x)-F(-x))\, dx$$

$$V(x) = \int_0^{\infty} 2x(1-F(x)+F(-x))dx - \mu_1^2$$

(21) For integer-valued r.v. $X \geq 0$, $E(X)$ exists.

$$E(X) = \sum_{j=1}^{\infty} (1 - F(j-0) - F(-j))$$

(22) X is said to be symmetrically distributed about a if

$$P(X \leq a-x) = P(X \geq a+x) \quad \forall x$$

or $P(X \geq a-x) = P(X \leq a+x) \quad \forall x$

i.e. $F(a-x) + F(a+x-0) = 1$ or $F(a-x-0) + F(a+x) = 1$

Discrete Case :- iff $P(a-x) = P(a+x) \quad \forall x$

Continuous Case :- iff $f(a-x) = f(a+x) \quad \forall x$

(23) X : continuous r.v. symmetrically distributed about 'a'.

$$Y = \begin{cases} 1 & \text{if } X > a \\ 0 & \text{o.w.} \end{cases}$$

$|X-a|$ and Y are independent of each other.

(24) X : symm. about 'a'. g : odd func^n ;

Then $E(g(X-a)) = 0$ provided expectation exists

(25) Jensen's Inequality

g : convex func^n $\Rightarrow E(g(X)) \geq g(E(X))$

g : concave " $\Rightarrow E(g(X)) \leq g(E(X))$

(26) C.S inequality

$$E(g^2(x))\, E(h^2(x)) \geq E^2(g(x)h(x))$$

holds when $g(x) = \lambda h(x)$ w.p. 1

(27) From the knowledge of marginal distribution determination of joint distribution is not unique.

(28) Legendre's Duplication Formula

$$\Gamma(n)\,\Gamma\left(n+\tfrac{1}{2}\right) = \frac{\Gamma(2n)\cdot\sqrt{\pi}}{2^{2n-1}}$$

(29) Poincaré's Theorem

If A_i's are events in $\mathcal{A}$, then

$$P\left(\bigcup_{i=1}^{n} A_i\right) = S_1 - S_2 + S_3 - \cdots + (-1)^{n-1} S_n$$

$$S_k = \sum_{1 \leq i_1 < i_2 < \cdots < i_k \leq n} P(A_{i_1} A_{i_2} \cdots A_{i_k})$$

(30) Probability of occurrence of exactly m out of n events $A_1, A_2, \ldots, A_n$.

$$\boxed{P_{[m]} = \sum_{r=0}^{n-m} (-1)^r \binom{m+r}{m} S_{m+r}}$$

(31) Probability of occurrence of atleast m out of n events is $P_{\langle m \rangle} = \sum_{r=0}^{n-m} (-1)^r \binom{m+r-1}{m-1} S_{m+r}$

(32) Imp inequalities

$$P\left(\bigcup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} P(A_i) \quad [\text{Boole's}]$$

$$P\left(\bigcap_{i=1}^{n} A_i\right) \geq \sum_{i=1}^{n} P(A_i) - (n-1) \quad [\text{Bonferroni's}]$$