Coordinate Geometry

If the point R divides PQ { $P \equiv (x_1, y_1)$, $Q \equiv (x_2, y_2)$ } internally in the ratio m:n, then

$$R \equiv \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$$

... externally in the ratio m:n, then

$$R \equiv \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n} \right)$$

$A \equiv (x_1, y_1)$, $B \equiv (x_2, y_2)$, $C \equiv (x_3, y_3)$

coordinate of centroid of $\Delta ABC \equiv \left(\frac{\Sigma x_i}{3}, \frac{\Sigma y_i}{3} \right)$

$\overline{BC} = a$, $\overline{CA} = b$, $\overline{AB} = c$

coordinate of incentre $\equiv \left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c} \right)$

Area of $\Delta ABC = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$ sq. unit

A, B and C are collinear if

$$y_1(x_2 - x_3) + y_2(x_3 - x_1) + y_3(x_1 - x_2) = 0$$

centroid $\equiv$ point of concurrence of median of a triangle.

Straight Line

slope of x axis and lines parallel to it is $\tan 0° = 0$

" " y " " " " " " " $\tan\frac{\pi}{2} = \infty$

point of intersection of two intersecting lines

$a_i x + b_i y + c_i = 0 \quad i = 1, 2$

is $\left(\dfrac{b_1 c_2 - c_1 b_2}{a_1 b_2 - a_2 b_1}, \dfrac{a_2 c_1 - a_1 c_2}{a_1 b_2 - a_2 b_1}\right)$

Angle between two st. lines,

$y = m_1 x + c_1$

and $y = m_2 x + c_2$

is, $\theta = \tan^{-1}\left(\pm \dfrac{m_1 - m_2}{1 + m_1 m_2}\right)$

Equn of the st. line $\perp^{r}$ to $ax + by + c = 0$

is $bx - ay + k = 0$ where k is a constant.

The $\perp^{r}$ distance of A from $ax + by + c = 0$ $(\equiv L)$

is $= \dfrac{|a x_1 + b y_1 + c|}{\sqrt{a^2 + b^2}}$

Circle

$(C' \equiv) x^2+y^2+2gx+2fy+c=0$ is a circle with

centre $= (-g, -f)$

radius $= \sqrt{g^2+f^2-c}$

Equ^n of common chord of

$x^2+y^2+2g_ix+2f_iy+c_i=0 \quad i=1,2$

is $\quad 2(g_1-g_2)x + 2(f_1-f_2)y + (c_1-c_2) = 0$

Intercept made by c' on x-axis $= 2\sqrt{g^2-c}$

" " " c' " y-axis $= 2\sqrt{f^2-c}$

Equ^n of any circle through the point of intersection of C_1 and C_2 is,

$C_1 + k C_2 = 0$ where $k \neq -1, 0$ or ∞.

SOURCE: GITHUB.COM/SOURAVSTAT

Parabola

$(P \equiv) y^2 = 4ax$

vertex : $(0,0)$

axis : x-axis

focus = $(a,0)$

length of latus rectum = $4a$

(chord $\perp^r$ to axis passing through focus)

equ^n of directrix, $x = -a \Rightarrow x + a = 0$

$(at^2, 2at)$ is parametric coordinate

$(P' \equiv)$ $(y-\beta)^2 = 4a(x-\alpha)$

vertex = (α, β)

axis = x-axis

focus = $(a+\alpha, \beta)$

directrix, $x + a = \alpha$

If $(at^2, 2at)$ is one end of a focal chord of $y^2 = 4ax$ then the another end is $\left(\frac{a}{t^2}, -\frac{2a}{t}\right)$

If the another end be $(at_1^2, 2at_1)$

then, $t_1 t = -1$

For a double ordinate of length $8a$ of P, the lines joining the vertex to its two ends are at right angles.

$\overline{PS} = l$, $\overline{P'S} = l'$. where PP' is focal chor[d]
then for P, $\frac{1}{l} + \frac{1}{l'} = \frac{1}{a}$

Ellipse

(E ⇒) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $\quad a^2 > b^2$

vertex: $(a,0), (-a,0)$

centre = $(0,0)$

eccentricity $(e) = \sqrt{1 - \frac{b^2}{a^2}} = \frac{\text{distance of any point of ellipse}}{\text{distance of focus from } \dots \text{ line directrix}}$

$0 < e < 1$

focus = $(\pm ae, 0)$

Latus rectum $= \frac{2b^2}{a}$ unit, $x = \pm ae$

directrix ⇒ $x = \pm \frac{a}{e}$

$\dfrac{(x-\alpha)^2}{a^2} + \dfrac{(y-\beta)^2}{b^2} = 1, \quad a^2 > b^2$

vertices $= (\alpha \pm a, \beta)$

centre $= (\alpha, \beta)$

foci $= (\alpha \pm ae, \beta)$

latus rectum, $x = \alpha \pm ae$

directrix, $x = \alpha \pm \frac{a}{e}$

If S and S' be the foci of E, then for any point (P) on it,

$\overline{SP} + \overline{S'P} = 2a$ where $a^2 > b^2$

Hyperbola

$(H \equiv)$ $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$e = \sqrt{1 + \frac{b^2}{a^2}} \quad > 1$

Vertex $= (a, 0), (-a, 0)$

Centre $= (0, 0)$

latus rectum = same as ellipse

directrix = " "

foci = $(\pm ae, 0)$

$(H_1 \equiv)$ $x^2 - y^2 = a^2$

Here $e = \sqrt{2}$