Regression

(*) We want to predict x_1 on the basis of the information provided by other variables $x_2, x_3, \ldots, x_p$.

The MSE $f(\underset{\sim}{x}_{(2)})$ in predicting x_1 i.e.

$E(e^2) = E(x_1 - f(\underset{\sim}{x}_{(2)}))^2$ is minimised

when $f(\underset{\sim}{x}_{(2)}) = M(\underset{\sim}{x}_{(2)}) = E(x_1 | \underset{\sim}{x}_{(2)})$ with prob. 1.

(*) $corr(x_1, M(\underset{\sim}{x}_{(2)})) = \rho(x_1, M(\underset{\sim}{x}_{(2)})) \geq 0$

and $|\rho(x_1, f(\underset{\sim}{x}_{(2)}))| \leq \rho(x_1, M(\underset{\sim}{x}_{(2)}))$.

equality holds iff $f(\underset{\sim}{x}_{(2)}) = M(\underset{\sim}{x}_{(2)})$.

(*)

Least Square Linear Regression

Suppose the model is,

$x_1 = \alpha + \beta_2 x_2 + \cdots + \beta_p x_p + e$

Then the L.S. estimates of α, β_i's are

(a) $\underset{\sim}{\hat{\beta}} = \Sigma_2^{-1} \underset{\sim}{\sigma}_{(1)}$

and $\hat{\alpha} = \mu_1 - \underset{\sim}{\hat{\beta}}^T \underset{\sim}{\mu}_2$

where, $\Sigma_2 = \begin{pmatrix} \sigma_{22} & \cdots & \sigma_{2p} \\ \vdots & & \vdots \\ \sigma_{p2} & \cdots & \sigma_{pp} \end{pmatrix}$, $\underset{\sim}{\sigma}_{(1)} = \begin{pmatrix} \sigma_{21} \\ \sigma_{31} \\ \vdots \\ \sigma_{p1} \end{pmatrix}$

$\underset{\sim}{\mu}_2 = \begin{pmatrix} \mu_2 \\ \mu_3 \\ \vdots \\ \mu_p \end{pmatrix}$

$\left[x_{1.23\cdots p} + e_{1.23\cdots p} = x_1 \right]$

(b) $E(e_{1.23\cdots p}) = 0$

$cov(e_{1.23\cdots p}, x_j) = 0, \quad j = 2(1)p$

$cov(e_{1.23\cdots p}, x_{1.23\cdots p}) = 0$

$V(e_{1.23\cdots p}) = \sigma_{11} - \underset{\sim}{\sigma}_{(1)}^T \Sigma_2^{-1} \underset{\sim}{\sigma}_{(1)}$

$= \dfrac{|\Sigma|}{|\Sigma_2|}, \quad \Sigma = \begin{bmatrix} \sigma_{11} & \underset{\sim}{\sigma}_{(1)}^T \\ \underset{\sim}{\sigma}_{(1)} & \Sigma_2 \end{bmatrix}$

(*) The correlation coefficient between x_1 and the LS multiple linear regression $x_{1\cdot 23\cdots p}$ is the max^m correlation between x_1 and any linear function of $\underset{\sim}{x}_{(2)}$.

(*) $0 \leq \rho(x_1, x_{1\cdot 23\cdots p}) \leq 1$

As it's a measure of degree of linear relationship of x_1 ~~and~~ with $\underset{\sim}{x}_{(2)}$ and there is no ~~rat~~ direction in this relationship.

(*) $$\rho(x_1, x_{1\cdot 23\cdots p}) = \max \rho(x_1, l_0 + \underset{\sim}{l}^T \underset{\sim}{x}_{(2)})$$

$$= \sqrt{\frac{\hat{\underset{\sim}{\beta}}^T \Sigma_2 \hat{\underset{\sim}{\beta}}}{\sigma_{11}}} = \sqrt{\frac{var(x_{1\cdot 23\cdots p})}{var(x_1)}}$$

$$\rho^2_{1\cdot 23\cdots p} = 1 - \frac{\sigma^2_{1\cdot 23\cdots p}}{\sigma_{11}} = \left(\sqrt{1 - \frac{|\Sigma|}{\sigma_{11}|\Sigma_2|}}\right)^2$$

[Due to L.S. principle]

(*) $$\rho^2_{1\cdot 23\cdots p} \geq \rho^2_{1\cdot 34\cdots p}$$

(*) $$\sigma^2_{1\cdot 23\cdots p} = \sigma_1^2(1-\rho^2_{1\cdot 23\cdots p})$$

(*) Partial Correlation Coefficient

$$\rho_{12\cdot 34\cdots p} = \frac{cov(e_{1\cdot 34\cdots p}, e_{2\cdot 34\cdots p})}{\sqrt{V(e_{1\cdot 34\cdots p})V(e_{2\cdot 34\cdots p})}}$$

$$cov(e_{1\cdot 34\cdots p}, e_{2\cdot 34\cdots p}) = \frac{(-1)^{1+2}\Sigma_{12}}{|\Sigma_3|}$$

Cov (e)

Rank Correlation

\# $R = 1 - \frac{6 \Sigma d_i^2}{n(n^2-1)}$ where n is the number of individuals who are ranked, d_i is the difference between ranks given by two judges to ith individual

\# $-1 \le R \le 1$

\# For perfect agreement $R = 1$ [when $\Sigma d_i^2 = 0$]

" " dis- " $R = -1$ [when $x_i + x_i' = n+1 \ \forall i$]

\# $\sum_{i=1}^{n} d_i^2 \le \frac{n(n^2-1)}{3}$