Some important transformations

- $X_1, X_2 \overset{iid}{\sim} C(0,1) \Rightarrow X_1 + X_2 \sim C(0,2)$
- $x_1, x_2, \ldots, x_n$: r.s. from $C(\mu, 1)$

  Then $\Sigma x_i \sim C(n\mu, n)$ and $\bar{x} \sim C(\mu, 1)$
- AM $(A) = \bar{x}$ and HM $(H) = \dfrac{n}{\sum^{n} \frac{1}{x_i}}$ has the same distn if $x_1, x_2, \ldots, x_n \overset{iid}{\sim} C(0,1)$

- $X \sim \gamma(m)$, $Y \sim \gamma(n)$, $Z \sim \gamma(p)$ independently

  Then
  $$X+Y+Z \sim \gamma(m+n+p)$$
  $$\frac{Y}{Y+Z} \sim \beta_1(n, p)$$
  $$\frac{X}{X+Y+Z} \sim \beta_1(m, n+p)$$
  $$\frac{X+Y}{X+Y+Z} \sim \beta_1(m+n, p)$$

- $X \sim \gamma(n)$, $Y \sim \gamma(n+\frac{1}{2})$ independent

  Then $2\sqrt{XY} \sim \gamma(2n)$

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- $X_1 \sim \beta_1(n_1, n_2)$, $X_2 \sim \beta_1(n_1+\frac{1}{2}, n_2)$ independent $\Rightarrow \sqrt{X_1 X_2} \sim \beta_1(2n_1, 2n_2)$

- $X \sim \beta_1(a, b)$, $Y \sim \beta_1(c, d)$ independent and $a = c+d$ then $XY \sim \beta_1(c, b+d)$

- $X \sim R(0,1)$, $Y \sim R(0,1)$ independent

  $U = \sqrt{-2\ln X}\cos 2\pi Y$, $V = \sqrt{-2\ln X}\sin 2\pi Y$

  Then $U, V \overset{iid}{\sim} N(0,1)$

- $X, Y \overset{iid}{\sim} N(0,1)$

  Then $\frac{XY}{\sqrt{X^2+Y^2}}, \frac{X^2-Y^2}{2\sqrt{X^2+Y^2}} \overset{iid}{\sim} N(0, \frac{1}{4})$

- Suppose $X_i \overset{iid}{\sim} N(0,1)$, $i = 1(1)4$.

  Then $M_{X_1 X_2}(t) = \frac{1}{\sqrt{1-t^2}}$, $|t| < 1$

  and $M_{X_1X_2 - X_3X_4}(t) = \frac{1}{1-t^2}$, $|t| < 1$

  i.e. $X_1X_2 - X_3X_4 \sim$ Standard Laplace Distn

- ~~$X \sim C(0,1) \Rightarrow \frac{2X}{1+X^2} \sim C(0,1)$ [Using 3rd theorem result]~~

- $X \sim U(0,1)$, $V = \min(X, 1-X)$, $U = \max(X, 1-X)$

  $\Rightarrow \frac{U}{V} \sim$ Beta(2, 1) of 2nd kind.

$Y \sim F_{n_1, n_2}$

Then as $n_2 \to \infty$ ~~then~~ $Z = n_1 Y \sim \chi^2_{n_1}$

$X \sim \beta_1(\theta, 1)$ i.e. $f_X(x) = \frac{x^{\theta-1}}{B(\theta,1)} I_x(0,1), \theta>0$

Then $-2\theta \ln X \sim \chi^2_2$

$X \sim R(0,1) \Rightarrow -2\ln X \sim \chi^2_2$

Order Statistics

\# The df of the rth order statistic $X_{(r)}$ is,

$$F_{X_{(r)}}(x) = \sum_{j=r}^{n} \binom{n}{j} (F(x))^j (1-F(x))^{n-j}$$

$$\left(\because F_r(x) = P(\text{atleast } r \text{ of the } X_i\text{'s are} \le x) = P(Y \ge r) = \sum_{j=r}^{n} P(Y=j) = \sum_{j=r}^{n} \binom{n}{j} \{F(x)\}^j \{1-F(x)\}^{n-j} \right)$$

So, $F_r(x) = I_{F(x)}(r, n-r+1)$

Discrete Case

$$P_r(x) = I_{F(x)}(r, n-r+1) - I_{F(x-0)}(r, n-r+1)$$

Continuous Case

$$f_r(x) = \frac{n!}{(r-1)!\,(n-r)!} (F(x))^{r-1} (1-F(x))^{n-r} f(x)$$

\# $X_1, X_2, \ldots, X_n$: R.S. from $R(0,1)$

$X_{(r)} \sim B_1(r, n-r+1)$

$$E(X_{(r)}) = \frac{r}{n+1}$$

\# $F(X_1), F(X_2), \ldots, F(X_n)$: iid $R(0,1)$

$$\Rightarrow E(F(X_{(r)})) = \frac{r}{n+1}$$

Joint distn of $X_{(r)}, X_{(s)}$

<u>Continuous Case</u>

$$f_{r,s}(x,y) = \frac{n!}{(r-1)!\,(s-r-1)!\,(n-s)!}\left(F(x)\right)^{r-1}\left(F(y)-F(x)\right)^{s-r-1}\left(1-F(y)\right)^{n-s} f(x)f(y)$$

$x < y$

$r < s$

<u>Discrete Case</u>

$$p_{r,s}(x,y) = F_{r,s}(x,y) - F_{r,s}(x-0,y) - F_{r,s}(x,y-0) + F_{r,s}(x-0,y-0)$$

$R(0,1) \rightarrow$ R.S. : $x_1, x_2, \ldots, x_n$

$$E(R) = \frac{n-1}{n+1} \quad \text{where } R = x_{(n)} - x_{(1)} \sim \beta_1(n-1, 2)$$

$Y \sim F_{n_1, n_2}$

Then as $n_2 \to \infty$ then $Z = n_1 Y \sim \chi^2_{n_1}$

$X \sim \beta_1(\theta, 1)$ i.e. $f_X(x) = \frac{x^{\theta-1}}{B(\theta,1)} I_x(0,1)$, $\theta > 0$

Then $-2\theta \ln X \sim \chi^2_2$

$X \sim R(0,1) \Rightarrow -2\ln X \sim \chi^2_2$

If $f_X(x) = \frac{\theta}{x^{\theta+1}} I\, x>0$ then $\cancel{\ln X \sim \text{Exp}(\text{mean}=\frac{1}{\theta})}$ $2\theta \ln x \sim \chi^2_2$

If $X_i \sim \text{Bin}(n_i, p)$ $i=1,2$ then

$X_1 \mid X_1 + X_2 = s \sim$ ~~Bin~~ Hypergeometric $(N = n_1 + n_2, n = s, p = \frac{n_1}{N})$

For $X_i \overset{iid}{\sim} \text{Exp}(\text{mean}=1)$ $i=1,2$ then $X_1 - X_2 \sim DE(0,1)$

$X_1, X_2, X_3 \overset{iid}{\sim} N(0,1)$ then define

$$Y_1 = \frac{X_1 + X_2 + X_3}{\sqrt{3}}, \quad Y_2 = \frac{X_1 - X_2}{\sqrt{2}}, \quad Y_3 = \frac{X_1 + X_2 - 2X_3}{\sqrt{6}}$$

where $Y_i \overset{iid}{\sim} N(0,1)$ $i = 1, 2, 3$.