

# MESMER

Master Equation Solver for Multi-Energy well Reactions

## User's Manual

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## 1. Introduction

The modelling of unimolecular systems has application in a variety of environmental and industrial contexts. In recent years a great deal of progress has been made in the understanding and modelling of unimolecular systems over a range of temperatures and pressures. The quantities of particular interest are the rate coefficients, time dependent species profiles, product yields, and branching ratios of the system being investigated. Each of these quantities typically shows a complex dependence on pressure and temperature. The modelling of industrial or environmental processes often involves conditions that are difficult to access experimentally, so it is important to be able to generate experimentally validated rate coefficient models that may be extrapolated to the sorts of conditions of interest in larger scale simulations. For example, typical experimental kinetics measurements of reactions important in the atmosphere are performed at very low pressures; however, in the atmosphere, these reactions occur at high pressures – so the low pressure results must be extrapolated to the higher pressures characteristic of the lower atmosphere. The use of stochastic techniques for describing the evolution of unimolecular systems – in particular the master equation (ME) – is a common means of linking laboratory studies and larger scale modelling. MESMER uses matrix techniques to formulate and solve the energy grained master equation (EGME) for unimolecular systems composed of an arbitrary number of wells, transition states, sinks, and reactants.

A unimolecular system is characterised by one or more potential wells (local minima) on the potential energy surface (henceforth, PES) which describes the energy of the atoms as a function of position. Each well represents a (meta-) stable species that can, in principle, be isolated. Wells are connected by transition states (TS) and a species in one well may be converted to another by passing through the TS that connects the wells. In many systems the TS can be associated with a saddle point on the system PES and so there is an energy barrier to inter-conversion of species. Thus, to convert from one species to another, the reactant must be activated – i.e., energy must be supplied to overcome the barrier separating the two wells. Typically, energy is supplied through collisions with bath gas molecules - the reactant will undergo a number of collisions with bath gas molecules some of which will be activating (net increase in reactant energy) and some of which will be deactivating (net decrease in reactant

energy). Since collision events and the amount of energy transferred are random quantities, the energy transfer process can be regarded as a random walk, and treated using techniques from stochastic process theory.

MESMER has been designed to improve upon and generalize previous developments in older master equation codes. MESMER has been written to offer a flexible approach to ME treatments of more complex systems. We have also attempted to incorporate various facilities that make it easy to apply the ME to gas kinetics. Some of the design goals which we had in mind while writing MESMER include:

1. Use standard, off-the-shelf technologies, so that the code may be readily maintained and extended. For example, we developed MESMER using the Microsoft's VS2005 integrated development environment, we use XML data representation for the input stream, we use Firefox as a PES viewer to aid in the construction of input files, and current developments are underway to increase compatibility between MESMER and such open source projects as OpenBabel.
2. Use open source C++ to write well-structured, object-oriented, cross-platform code that may be easily maintained and built upon by future developers. Where possible, we have designed the code so that future developers may be able to add increasing functionality via the use of plug-in classes. In addition, the code is commented, with references that indicate the methodologies used.

## **2. Getting Started**

### **2.1. Citation**

If you published results using MESMER, we would appreciate it if you would cite us. We are currently preparing a paper in which we detail some of the more interesting things we have implemented in MESMER, and we will ask you to cite that paper, but for the moment, please cite as follows:

Robertson, S. H., Glowacki, D. R., Liang, C.-H.; Morley, C., Pilling, M. J., MESMER (Master Equation Solver for Multi-Energy Well Reactions), 2008; an object oriented C++ program for carrying out ME calculations and eigenvalue-eigenvector analysis on arbitrary multiple well systems. <http://sourceforge.net/projects/mesmer>.

### **2.2. Accessing MESMER**

MESMER is hosted by SourceForge website and can be accessed either by using the search facility provided or following the link:

<http://sourceforge.net/projects/mesmer/>

There are a number of tracker facilities that allow one to enter bugs and features requests and we strongly encourage this. If you wish to receive an email notification of items being added to these trackers, subscribe to the mesmer-notify mailing list which can also be found on the MESMER SourceForge project site.

### **2.3. Compilation and Execution**

MESMER has been designed to be cross platform, and we have compiled it under Windows, LINUX/UNIX, and Mac operating systems, on both 32 and 64-bit architectures. The installation details describe how to get started using MESMER on several of these platforms. In general, we do our development and debugging on Windows. For large production runs, where we may want to run several hundreds or even thousands of ME



calculations to explore the sensitivity of the results over the model parameter space, we tend to use LINUX.

## **2.3.1. Windows**

### **2.3.1.1. Installing the Binary on Windows**

Download the Windows installer and execute it. It will ask you to accept the LGPL licence, and to input the folder where you want it installed. It sets the PATH, MESMER\_DIR and MESMER\_AUTHOR environment variables. An item is added to StartMenu/Programs with links to the documentation, but actually running MESMER would normally be done from a DOS command window. To remove MESMER from your computer, use Add and Remove Programs or click the StartMenu/Programs item Uninstall.

### **2.3.1.2. Compiling it yourself on Windows**

If you *want* to build MESMER yourself, if an executable release is not available so that you *have* to build it yourself, or if you want to develop the code yourself, then we recommend the use of Visual C++. We wrote MESMER using Microsoft's Visual C++ 2005/08 integrated development environment. Building and developing MESMER can be done using either the free Visual C++ 2008 Express Edition or the full version of Visual Studio (which isn't free). .

To build MESMER using VS2008, you could download the MESMER tar.gz distribution as described in section 2.3.2, but we would recommend using SVN (possibly with Tortoise SVN) when you will have the opportunity to use either the most recent development code by checking out

<https://mesmer.svn.sourceforge.net/svnroot/mesmer/trunk>

or using a released version like

<https://mesmer.svn.sourceforge.net/svnroot/mesmer/tags/mesmer-0.1>.

To build it, go to the \Windows VC9 folder, and use Visual C++ to open the file MESMER.sln. Clicking on the VS2008 command Build the Solution will build the binary executable in \Windows VC9\Mesmer folder. (Depending on the MESMER release that you download, you may see a \Windows VC8 folder. This folder contains the VS2005 project and solution files for MESMER.)

### 2.3.1.3. Running on Windows

Open a DOS command window, most conveniently in the folder containing the data file. If the Windows installer has been used, or if the folder containing `mesmer.exe` has been manually added to the PATH environment variable, MESMER may be called as follows:

```
mesmer filename.xml
```

where `filename.xml` refers to the input XML file described below. See section 2.3.4 for a more complete description of the options and syntax of the command line.

### 2.3.2. Linux/UNIX/Mac

The first step to using MESMER is downloading it from the SourceForge project website. The downloaded release is distributed using `tar.gz` compression, which retains the directory structure. To uncompress the files in Windows, you can use free software like WinRAR. Under Linux/UNIX/Mac, you type the following command:

```
tar xvfz filename.tar.gz
```

where `filename` is the name of the particular MESMER release that you have downloaded.

Linux/UNIX compilation involves three easy steps: (1) compile the TinyXML libraries, (2) compile the QD libraries, and finally (3) compile the main executable. These steps are described sequentially below.

#### 2.3.2.1. Compiling TinyXML

To compile TinyXML, which is what MESMER uses for input/output, the library has to be created by typing the following command under the `/tinyxml` folder:

```
make -f MakeLib
```

#### 2.3.2.2. Compiling QD for higher precision arithmetic

MESMER uses numerical matrix techniques to formulate and solve the ME. Because of this, MESMER is not immune to numerical precision problems. In the Energy Grained Master Equation, the origin of these effects and when they occur is reasonably well understood, although solutions to these problems are less well understood. In general, numerical problems arise for very deep wells, very low temperatures, and very low pressures. MESMER includes a few different ways of dealing with numerical precision problems when they arise. The Reservoir state and contracted basis set approaches are elegant ways of

manipulating the mathematical formulation of the ME to delay the onset of numerical problems; however, we have also written MESMER to incorporate a brute force technique for doing the same – carrying out arithmetic using significantly increased precision available in the so-called QD libraries written by Yozo Hida. To accommodate the increased precision libraries, MESMER may be built with different versions of QD. For the compilation of QD package please refer to <http://crd.lbl.gov/~dhbailey/mpdist/>. The QD installation steps are described in `INSTALL` file of the `/qd` folder.

Briefly, QD installation should require no more than the following three commands executed within the directory `Mesmer/qd/`:

```
chmod +x configure
./configure
make
```

When QD executes `configure`, it requires that certain environment variables are defined, and in our experience, the most common difficulties to installing QD concern the fact that these environment variables are not defined on a particular system. Executing

```
./configure --help
```

will show the list of important environmental variables required by QD. For example, QD may not recognize the appropriate C++ and Fortran compilers, and this is mostly like because the environment variables `FC` and `CXX` have no value. The user may check the value of the `CXX` and `FC` environment variables on their system by executing

```
echo $CXX; echo $FC
```

If the above commands return no value, and the respective C++ and Fortran compilers available on the system are `g++` and `gfortran`, the environment variables may be set as follows:

```
CXX=g++; export CXX; FC=gfortran; export FC
```

In some cases where the user has no system administrator's privilege to install the library, they will need to ask the system administrator for help installing the QD package.

### **2.3.2.3. Compiling and Running the Main Executable**

Following successful compilation of both the TinyXML and QD libraries, the main MESMER executable may be compiled. If the QD libraries have been built using files other

than `qd_real.h` and `dd_real.h`, then the header file `MesmerPrecision.h` must be altered so that it refers to the correct filenames. For the standard QD install that we expect most users will utilize, changing `MesmerPrecision.h` will not be necessary. Tests that we have run on a variety of 32 and 64 bit LINUX architectures have shown that the MESMER executable may be unreliable if compiled with g++ compilers earlier than version 4.1.0, and with optimization flags greater than `-O2`. Thus, we recommend that users test their compiled code against the test suite detailed below.

If the user installs both TinyXML and QD themselves with no complicated changes to the standard install, then the MESMER `Makefile` in `/src` shouldn't need any alteration, as it is presently set up with the options that most users will require. To install the main executable, all that should be required is to go to the `/src` folder and do

```
make
```

or

```
make install
```

The latter command will copy the executable to the `/bin` folder after a successful compilation. If you need to recompile MESMER, you can also use the command

```
make remake
```

This will remove the previous built object files in `/src` folder and do a clean recompile/installation. This is useful when there is a clock skew between the local computer and remote cluster; hence all files on the remote cluster will be recompiled regardless of the time attributes on files. Note that if you don't do a `make install`, then the executable will reside in `/src` and won't be copied to `/bin`.

Should problems occur in compiling MESMER, or if you used more complicated ways of installing TinyXML and QD, we include a brief discussion of what must be in place to compile the Main MESMER executable. Additional guidance may be found within the comments in the `Makefile` itself. Compilation of the main MESMER executable requires linking with TinyXML and QD. To be sure this is done correctly, verify that the MESMER `Makefile` refers to the correct location of `libqd.a` and `tinyxml.a` within the LIBS field. If you successfully compiled the QD and Tinyxml packages, then paste something like this into its appropriate location with the MESMER `Makefile`:

```
LIBS:= ../tinyxml/tinyxml.a /usr/local/newqd/lib/libqd.a
```

where the paths of `libqd.a` and `tinyxml.a` may be relative or absolute.

Similarly, in the INCS section of the `Makefile`, specify the absolute or relative location of both the `tinyxml/` and the `/include` folder.

```
INCS := -I../tinyxml/ -I/usr/local/qd/include
```

Where the `/include` folder specifies where `qd_real.h` and `dd_real.h` live. So long as the LIBS and INC sections of the `Makefile` are correct, then you should be able to carry on with compilation of the main MESMER executable using `make` or `make install` as discussed above.

#### 2.3.2.4. Running on Linux/UNIX/Mac

Calling MESMER in UNIX/Linux/Mac systems is similar to Windows systems. The only difference is the name of the executable (`mesmer.exe` in Windows and `mesmer` in Linux). Assuming that you have done a `make install` so that the executable resides in `/bin`, then from the directory where `filename.xml` is located, one can call MESMER in UNIX/Linux systems by typing:

```
./~path/bin/mesmer filename.xml
```

where `~path` specifies the location of the `bin/` folder on your machine. If you would rather call MESMER by simply typing

```
mesmer filename.xml
```

without having to specify the executable path every time, then you have to export the directory in which the `mesmer` executable resides to the appropriate environment variables. One can see a list of the input options that may be specified with the MESMER executable by typing

```
-?
```

after the MESMER executable.

#### 2.3.3. Testing MESMER on Windows and Linux/UNIX/Mac

Following MESMER compilation and/or installation, it is a good idea to perform some tests to ensure that your executable gives similar answers to the test jobs that we used during development. Tiny differences from those in the test jobs are probably OK, and likely arise from numerical issues due to OS/architecture combinations; however, larger errors are a cause

for concern and you should check your compilation sequence. Under Windows, enter DOS command line mode, go to the `MesmerQA/` directory, and type the following command:

`QA`

This command executes a script called `QA.bat` which runs MESMER for each test system included within each file in the `MesmerQA` directory. Each system included in the test suite has a folder in which its input file is located. The input files for each test system are specified within the section of this manual detailing the test suite. Within the folder for each test system is another folder called `baselines/`, into which the output from the `QA.bat` script is copied as `test.test`. The `baselines/` directory also includes a file called `mesmer.test`, which contains the output obtained by the developers for the corresponding test input file. The user needs to verify that the results generated by their executable in `test.test` are nearly identical to those in `mesmer.test` for each system. If the user carries out any MESMER development, then all changes to the code should be checked against the `mesmer.test` baselines. Additional instructions for operating the `QA` command in developer's mode are included within comments that are written in the `QA.bat` script.

For Linux/UNIX/Mac, we have not yet written a script that does the QA tests automatically. At present, the user will have to test their executable against the baselines manually – i.e., enter the folder for each system, execute MESMER with the appropriate QA input file, and check the output against the results in `mesmer.test`.

#### 2.3.4. MESMER command line

All of MESMER's chemistry input and much of the program control is in the XML-formatted datafile described in the next section. The command line interface offers some options which mostly concern the location of files and only a few display/control tasks. The interface is the same in Windows and Linux. Typing

```
mesmer -?
```

will display the complete set of options for the command line. In normal execution of MESMER you would use:

```
mesmer infile.xml -o outfile.xml
```

If `outfile.xml` already existed, it would be renamed `outfile_prev.xml`, and any existing file of this name will be deleted. If you simply type

```
mesmer infile.xml
```

then output is written to `mesmer_out.xml` and there is the same single layer of buffering (i.e., renaming and deleting protocol) as with an explicitly named output file.

### 2.3.5. MESMER environment variables

The environment variable `MESMER_AUTHOR` can be set so that the user's name appears in the metadata section of the output XML file.

`MESMER_DIR` specifies the directory containing the executable, and is also where MESMER looks first for the files `defaults.xml` and `librarymols.xml`. If `MESMER_DIR` is not set, MESMER looks for these files in the directory two levels up from the current directory. This is appropriate when the current directory is one of the test system directories provided with the MESMER distribution.

On UNIX, Linux, and Mac platforms, `MESMER_DIR` can be used to simplify running MESMER from any directory, without having to specify the location of the MESMER executable every time:

```
mesmer filename.xml
```

To do this, set `MESMER_DIR` to the location of the MESMER executable by

```
MESMER_DIR="\user\username\Mesmer"
```

```
export MESMER_DIR
```

If you want the `MESMER_DIR` variable set every time you login to your machine, then add the above two lines to the relevant login scripts. The files `defaults.xml` and `librarymols.xml` also need to be in the same directory. If the user is running using the PBS command `qsub`, then the above two lines are usually placed in the beginning of the `qsub` script file, which indicates that the commands are executed by the login shell every time the shell is initiated.

Under Windows, an entry in the `PATH` variable provides the location of `mesmer.exe`. If the Windows installer has been used, the `MESMER_AUTHOR`, `MESMER_DIR`, and `PATH` environment variables will have been set appropriately. `MESMER_AUTHOR` can be set temporarily, for the duration of a command window, by typing

```
set MESMER_AUTHOR name
```

### 3. MESMER data files

MESMER data files are in XML format and are intended to be more than a temporary means of transferring data to the program. They are more generally intended to be a representation of the chemical system – i.e., a set of reactions – which may (eventually) be used by other applications. Running MESMER produces an output file which is an augmented input file – it has all the original information together with additional data calculated by MESMER and the default values of parameters that were not explicitly specified. Consequently, any datafile can be used as input file.

The files can contain data in excess of that required by MESMER. For instance, they may contain chemical structure information, which is not used by MESMER, but helps to define the system unambiguously and can be used in the presentation of results. One of the reasons XML format was chosen because of the availability of tools and technologies for reformatting the data for presentation or reuse by other programs.

#### 3.1. Editing and Viewing Data Files

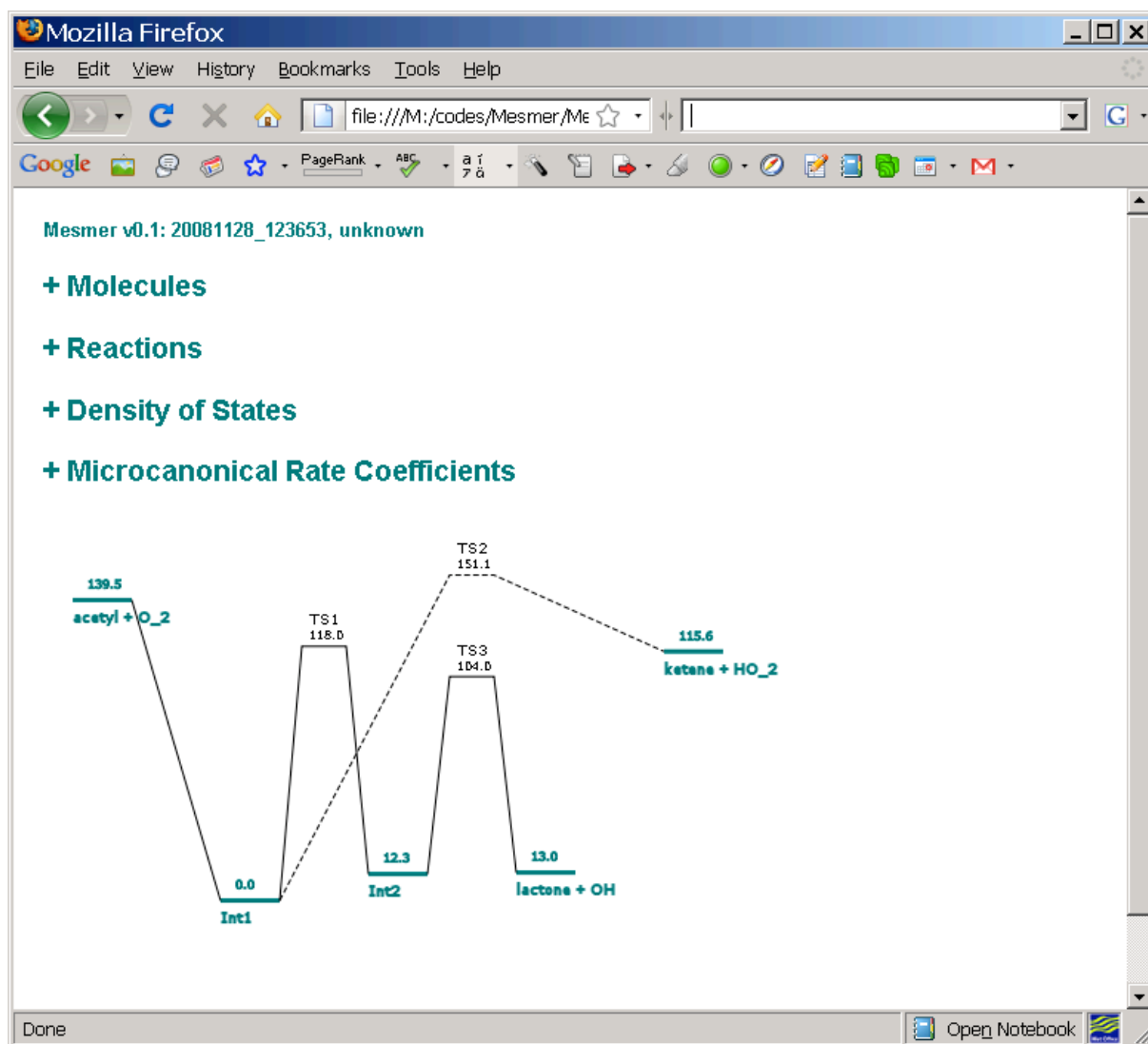
With a very basic knowledge of XML syntax, any ordinary text editor may be used to read and edit a MESMER XML data file, but their construction and viewing is facilitated by the use of a specialised XML editor. Many commercial editors are available. Free ones for Windows include:

- Microsoft XML Notepad, which hides the syntax but emphasises the tree structure
- The editor in VS2008, which is good for syntax checking, but is part of a large development system – probably an excessive download unless you intend to do development.
- Notepad++, which has the basic capability of expanding and collapsing the XML tree structure.

MESMER data files can be viewed in a more user friendly way with Firefox 3, as shown in Figure 3.1. For Firefox to translate the XML data file, it requires with some xsl formatting files – mesmerDiag.xsl and mesmer1.xsl – which need to be in a directory two levels above



the xml data file and are usually in the MESMER root directory. In order to use your version of Firefox 3 to view the MESMER input files, you need to alter one of the Firefox defaults as follows: (1) type `about:config` in the address bar. You will get a warning; receive it and carry on; (2) scroll down to `security.fileuri.strict_origin_policy`, right click on it, and change it to false; (3) restart Firefox. Now you can use it to view MESMER \*.xml input files.



**Figure 3.1** Viewing the potential energy surface of Acetyl+O<sub>2</sub> reaction by opening the XML input file in Firefox 3.

MESMER itself provides some assistance in constructing data files. If certain required items are not present, MESMER can insert them and prompt the user to check whether the inserted values are appropriate. See the tutorial “Constructing a Datafile from Gaussian output”.

## 3.2. The basics of the \*.xml input file

In the material that follows, we discuss the structure of MESMER input files. This discussion will be of significantly more benefit to the user if they use an XML editor to examine some of the sample input files included within the `MesmerQA` directory. Mesmer's input structure follows naturally from the XML data structure.

The MESMER data file has a top level element `<mesmer>`, and below it has the following sections, all of which have straightforward titles:

- (1) `moleculeList` specifies the molecules relevant to the ME system, as well as associated properties of the molecules
- (2) `reactionList` specifies the reactions relevant to the ME system to be modelled, and associated properties of the reactions
- (3) `me:conditions` specifies the conditions (e.g., temperature, pressure, bath gas) under which a particular ME model is to be run
- (4) `me:modelparameters` specifies the parameters relevant to the model (e.g., grain size and the maximum grain energy)
- (5) `me:control` specifies program options concerning the content of the output file;

Both `moleculeList` and `reactionList`, which define the chemistry of the system, are based on CML (Chemical Markup Language) and this is the default namespace. Several programs work with CML; the tutorial mentioned in the previous section uses [OpenBabel](#), to convert from other formats to CML. For more information about the CML schema can be found [here](#)

### 3.2.1. moleculeList

`moleculeList` is composed of each `molecule` involved in the ME system, and represents the distinct molecules or molecular configurations involved in the ME system. A `molecule` may represent one of the following: (1) an individual reactant in an association process, (2) an association product (i.e., an adduct), (3) an isomer, (4) a transition state, (5) an individual product from a dissociation reaction, or (6) a bath gas.

Each `molecule` includes the following:

- (1) An `id` attribute, which is used to identify the molecule in other portions of the input file (e.g., the `reactionList`). An id of a molecule can be arbitrary as long as it is composed of ASCII characters, but it must be distinct from that of other molecules;
- (2) A `description` attribute, which is available for the user to add their own comments regarding a particular `molecule` should they so choose;
- (3) An `atomarray`, which is not necessary for the calculation to proceed, but which is part of the CML. The elements are each individual `atom` within the `molecule`. In the case that a `molecule` is no more than a single atom, as is often true for a reactant in an association reaction, a product in a dissociation reaction, or a bath gas, then `atomarray` has only a single constituent `atom`. Similar to molecule, each `atom` has the following attributes: an `id`, which is used to identify the `atom` in other portions of the input (e.g., the `bondarray`), and an `elementType`;
- (4) A `bondarray`, whose elements are each `bond` between the constituent `atoms` in a `molecule`. Each `bond` has the following attributes: `atomRefs2`, which specifies atoms that are bonded (using the `id` attribute of `atom`) and `order`, which specifies the bond order. In the case that a `molecule` is no more than a single atom, it does not have a `bondarray`. It is important to note that a MESMER calculation requires neither an `atomarray` nor a `bondarray` for a successful calculation. However, these are features of the CML on which MESMER input syntax is based, and further applications may be available for interpreting these data structures.
- (5) `spinMultiplicity` is an attribute on `molecule`;
- (6) A `propertyList` array, which includes the following:
  - a. `me:DOSMethod`, which specifies the method for MESMER to use in calculating rotational density of states. Presently, classical and quantum mechanical approximations of the external rotational density of states are available, and they are specified with the text `ClassicalRotors` and `QMRotors`. Table 3.1 gives the classical and QM methods that are presently included in MESMER, and the manner in which the program recognizes the type of calculation to perform. Neither bath gas molecules nor products of sink reactions require calculation of the rotational state

density and thus do not require a specification of `me:DOSMethod`; however, specifying such a method will not affect program execution.

- b. The `me:reservoirSize` flag is only relevant if the reservoir state approximation is being used (see below) and applies only to isomer wells, and specifies the reservoir size for the reservoir state approximation. The lower bound of the reservoir is always the well bottom, and the upper bound of the reservoir is specified by the user in energy units. If a negative number is supplied, then the reservoir upper bound is located that far (in energy) below the lowest available reaction threshold for a particular well – typically a few  $kT$ . If a positive number is supplied, MESMER will determine the upper bound that far (in energy) above the well bottom. MESMER will apply a correction if the user input specifies a reservoir upper bound that is higher than the lowest barrier. The energy may be input in any of several units, which must be specified in `me:units`. The user must determine whether the results of reservoir grain approximation are in agreement with the full master equation and whether it is an appropriate approximation. The syntax below specifies a reservoir spanning the well bottom to 2 kJ mol<sup>-1</sup> beneath the lowest threshold.

```
<me:reservoirSize units="kJ/mol">-2.0</me:reservoirSize>
```

- c. If the molecule being modelled is a minimum energy crossing point (MECP) – i.e., a transition state for spin forbidden crossing, then data is required. Both WKB and LZ corrections require specification of the following:
- A root mean squared spin orbit coupling element, specified following `me:RMS_SOC_element` with `units="cm-1"`
  - The norm of the vector representing the gradient difference at the MECP, specified following `me:GradientDifferenceMagnitude` with `units="a.u./Bohr"`
  - The reduced mass for movement along the direction orthogonal to the crossing seam, specified following `me:GradientReducedMass` with `units="a.m.u."`
  - WKB transmission probabilities require one additional input: the geometric mean of the norms of the gradients on the two surfaces at

the MECP, specified using `me:AverageSlope` with `units="a.u./Bohr"`

Type of rotor	MESMER recognition criteria	Rotor type	DOS expression
not a rotor	no nonzero rotational constants available in the input	Classical	$\rho(E) = 0$
2d linear classical rotor	only one or two nonzero rotational constants provided in the input	Classical	$\rho(E) = \frac{1}{\sigma B}$
3d classical rotor	three rotational constants provided in the input	Classical	$\rho(E) = \frac{1}{\sigma} \sqrt{\frac{4E}{B_x B_y B_z}}$
Spherical top	One non zero rotational constant provided in the input	QM	For $A = B = C$ $E_r(J,K) = BJ(J+1)$ $g_{JK} = (2J+1)^2$
Oblate or near oblate symmetric top	Three rotational constants provided in the input where two of them are equal and greater than the third	QM	For $A = B > C$ $E_r(J,K) = BJ(J+1) + (C-B)K^2$ $J=0,1,2; K=0,\pm 1,\pm 2,\dots, \pm J$ $g_{JK} = 2J+1$
Prolate or near prolate symmetric top	Three rotational constants provided in the input where two of them are equal and smaller than the third	QM	For $A > B = C$ $E_r(J,K) = BJ(J+1) + (A-B)K^2$ $J=0,1,2; K=0,\pm 1,\pm 2,\dots, \pm J$ $g_{JK} = 2J+1$

Table 3.1: methods for calculating DOS that are presently available in MESMER. Note that the rotational constants have the unit of  $\text{cm}^{-1}$ .  $g_{JK}$  are the degeneracies of  $(J,K)$  energy levels.

- d. `property` elements of a `molecule`. Each `property` element has a `dictRef` attribute which specifies what type of `property` is subsequently defined. A `property` type which requires a list, such as vibrational frequencies and rotational constants, is specified in an `array`, whereas one which requires only a single number, such as zero point energy location or a symmetry number, is specified to be `scalar`. For `property` types which have associated units, (e.g., vibrational frequencies, rotational constants, and zero point energy) the units are specified as an attribute of `array` or `scalar`. Unitless quantities such as spin multiplicity, symmetry number, or frequency scale factor, are specified to be `scalar`. Table 3.2 gives the values of `dictRef` (i.e., property attributes) currently recognized by MESMER, specifies whether they are `scalar` or `array`, and specifies

the `units` presently available in MESMER. There are two important points to remember for specifying zero point energy:

- i. While units are available for input of a molecule's ZPE, the selected units must be consistent for every molecule in the input file with respect to some arbitrary reference energy.
- ii. For association and dissociation reactions, the respective sum of the reactant and product ZPEs for each molecule must equal the total energy of the products or reactants. That is, if the ZPE of A+B is equal to  $x$  kJ/mol for a particular system, there are infinite number of ways in which this may be specified so long as the ZPE of A added to the ZPE of B is equal to  $x$  kJ/mol.

<code>dictRef</code> value	scalar or array	Available input units
<code>me:ZPE</code>	Scalar	kJ/mol or kJ per mol cm-1 or wavenumber kcal/mol or kcal per mol Hartree or au
<code>me:rotConsts</code>	array	cm-1
<code>me:symmetryNumber</code>	scalar	unitless, no units specification
<code>me:frequenciesScaleFactor</code>	scalar	unitless, no units specification
<code>me:vibFreqs</code>	array	cm-1
<code>me:MW</code>	scalar	amu
<code>me:spinMultiplicity</code>	scalar	unitless, no units specification
<code>me:epsilon</code>	scalar	K, no units specification
<code>me:sigma</code>	scalar	Angstroms, no units specification
<code>me:deltaEDown</code>	scalar	cm-1

Table 3.2: values of `dictRef` recognized by MESMER, whether the associated input is scalar or an array, and the available units for the input values.

### 3.2.2. reactionList

`reactionList` is composed of each individual `reaction` that may occur in a ME system. Effectively, the `reactionList` specifies which `molecules` are connected via which transition state. A `reaction` may be one of the following: (1) an isomerization, (2) an association, or (3) a dissociation.

Each `reaction` may include the following:

- (1) An `id` attribute, which is used to identify the reaction;

- (2) An `active` attribute. If this is false the reaction is ignored by MESMER. This feature makes it possible to do calculations on only part of a complex reaction system. A `molecule` can also have this attribute, but this does not affect the calculation. Inactive reactions and molecules are shown either greyed or omitted (at the users choice) when the XML file is viewed in Firefox.
- (3) One or more of a `reactant` and one or more of a `product`. An isomerization reaction ( $A \rightarrow B$ ) has one `reactant` and one `product`, an association reaction ( $A+B \rightarrow C$ ) has two `reactants` and one `product`, a dissociation reaction ( $A \rightarrow B+C$ ) has one `reactant` and two `products`, irreversible unimolecular reactions ( $A \rightarrow C$ ) have one `reactant` and one or two `products`, and irreversible exchange reactions ( $A+B \rightarrow C+D$ ) have two `reactants` and two `products`. `reactions` may or may not have a `me:transitionState`, which is discussed further below. All `reactants`, `products`, and `me:transitionStates` are molecules, and have:
- A `ref`, which should be identical to the molecule `id` specified in `moleculeList`, and
  - A `me:type`, which specifies the `molecule`'s role in the ME model of the system. The possible values of `me:type` are given in Table 3.3.
- (4) If `me:type` is `excessReactant`, then `reaction` requires a value of the excess reactant number density associated with `me:excessReactantConc` (in units of molecules  $\text{cm}^{-3}$ );
- (5) A `me:tunnelling` specification, which indicates whether and what sort of tunnelling corrections are to be implemented for a particular reaction. Presently, the only tunnelling correction that has been implemented is one dimensional tunnelling through an asymmetric Eckart barrier, using the method described by Miller,<sup>1</sup> and specified in MESMER with `Eckart`;
- (6) A `me:crossing` specification for invoking spin forbidden corrections to RRKM theory. Presently, there are two methods available for calculating spin forbidden transmission coefficients: (1) a Landau Zener method specified with `LZ`, and (2) a WKB method specified with `WKB`. Unlike the former method, the latter includes

tunnelling corrections below threshold. Both methods are described by Harvey and Aschi, *Faraday Discuss.*, 2003(124), 129-143;

- (7) A `me:MCRMethod` specification, which indicates how microcanonical flux through the transition state is to be treated for a particular reaction. Presently, there are two methods for treating the transition state: (1) If `me:transitionState` is specified, then the `SimpleRRKM` method, which uses the standard, well known RRKM expression to calculate  $k(E)$ s, may be used. (2) If no transition state is specified, then `MesmerILT` may be used. This specifies that  $k(E)$ s are calculated using an inverse Laplace transform on  $k^\infty(T)$ s, which may be experimentally measured or theoretically calculated. This method is often convenient for reactions that do not have a well defined energy barrier;.

If `MesmerILT` is specified, then `reaction` requires three parameters from an Arrhenius fit to the  $k^\infty(T)$ s. The expression used by MESMER is:

$$k^\infty(T) = A^\infty \left( \frac{T}{T^\infty} \right)^{n^\infty} \exp(-E^\infty / RT)$$

where  $A^\infty$  is the pre-exponential factor (`me:preExponential`),  $E^\infty$  is the activation energy (`me:activationEnergy`) in units identical to those of the corresponding stationary point energies, and  $n^\infty$  accounts for curvature in the observed  $k^\infty(T)$ s (`me:nInfinity`).

<code>me:type</code> value	MESMER definition
<code>deficientReactant</code>	Deficient reactant in an association or exchange reaction; modelled using a pseudo first order bimolecular source term and assumed to be thermalized in the ME model. Presently, reversible association reactions and irreversible exchange reactions are available
<code>excessReactant</code>	Excess reactant in an association reaction; its large concentration relative to the deficient reactant allows a pseudo first order treatment of the association step
<code>modelled</code>	Molecule that may undergo relaxation via collisions with the bath gas
<code>transitionState</code>	Molecular configuration that represents the phase space flux bottleneck for a particular reaction
<code>sink</code>	product molecule that acts as an infinite sink (i.e., an irreversible loss) from a particular well modelled in the ME

Table 3.3: different values of `me:type` and their corresponding definitions in MESMER



### 3.2.3. me:conditions

The tree structure for `me:conditions` includes the following elements:

- (1) A specification of the `me:bathGas`, which is identical to the `id` attribute associated with the bath gas `molecule` in the `moleculeList`. The syntax should look something like:

```
<me:bathGas>He</me:bathGas>
```

- (2) `me:PTs`, which is used to specify the pressure (`me:P`) and temperature (`me:T`) at which the ME model is to be run. `me:PTs` includes the specific pressure and temperature pairs, `me:PTpair`, at which a particular master equation model is to be run. The number of `me:PTpairs` tells MESMER how many ME simulations to run. For example, three `me:PTpairs` would result in ME runs at three specified temperature and pressure pairs. Each `me:PTpair` requires temperature and pressure specification. Temperature is input in Kelvin, but the pressure may be input in any of several units, which must be specified in `me:units`. The units recognized by MESMER for pressure input include: (1) number density, which may be specified as `particles per cubic centimeter`, `number density`, or `PPCC`; (2) torr, specified as `Torr` or `mmHg`; (3) millibar, specified as `mbar`, (4) atmospheres, specified as `atm`, and (5) pounds per square inch, specified as `psi`.
- (3) `me:InitialPopulation`, which specifies the initial population for each species in order to get the time evolution of the system. If there is a source term (i.e., an association reaction), MESMER will automatically set its initial population to 1.0. If there is more than one source term, MESMER will set the population of the first source term it encountered to 1.0, leaving the populations of the other source terms nil. If there is no source term in the system, MESMER will set the population of the first isomer it encountered to 1.0. Otherwise, the user can specify the initial population of the species by using this element, and the syntax should look like:

```
<me:InitialPopulation>  
  <molecule ref="cyclopropene" me:population="1.0" />  
</me:InitialPopulation>
```

### 3.2.4. me:modelParameters

The tree structure for `me:modelParameters` includes the following elements:

- (1) `me:grainSize` includes a specification of the grain size to be used in partitioning the system phase space. It has an associated value and associated units. The units available are identical to those given in Table 2.2 for specification of ZPE. For a convergent solution of the ME utilizing the exponential down model, the grain size should be smaller than the  $\langle \Delta E_d \rangle$  values of any of the `modelled` molecules.
- (2) `me:energyAboveTheTopHill` specifies the energy range to be spanned by the grains, where the units are  $kT$ . The value of this parameter sets up a model wherein the energy grains span a region possessing  $x$   $kT$  of energy in excess of the highest energy stationary point. In general, the maximum grain energy in the ME should be at least 20  $kT$  above the highest energy molecular configuration, including reactants. If the maximum grain energy is too large, then numerical errors may result given that the probability for activating collisions is so small as to exceed machine precision.
- (3) `me:maxTemperature` applies if more than one `me:PTpair` has been specified in `me:conditions`. If `me:maxTemperature` is not specified, then each ME run at a particular `me:PTpair` will use the `me:energyAboveTheTopHill` value to determine the maximum grain energy – i.e., the maximum grain energy will be different at different temperatures. Using `me:maxTemperature` makes the maximum grain energy at each `me:PTpair` identical, choosing the maximum grain energy to be  $x$   $kT$  above the highest energy stationary point, where  $T$  is equal to the value set in `me:maxTemperature` regardless of the temperature specified in the `me:PTpair`.

### 3.2.5. `me:control`

The tree structure for `me:control` includes several elements, all of which determine the content of the output file and may be turned off and on by commenting or un-commenting any of the following items:

- `me:calculateRateCoefficientsOnly`, which makes MESMER only calculate TST rate coefficients without doing the diagonalization required by a full ME treatment.
- `me:eigenvalues`, which prints the eigenvalues obtained from diagonalization of the full system collision matrix. The integer included within this element

specifies how many eigenvalues to truncate in the printing. If *ngrn* is the number of grains in the collision operator, and the value in `me:eigenvalues` is *x*, then *ngrn* – *x* eigenvalues will be printed. *x* = 0 indicates that all of the eigenvalues should be printed.

- `me:gridSearch`, which does a grid search for parameters specified by the user.  $\chi^2$  for every point of condition may then be calculated against the experimental data provided by the user. Further instructions for executing a grid search are included below.
- `me:gridSearchWithPunch`, is the same as `me: gridSearch`, but additionally produces a \*.punch file, where each line prints a phenomenological rate for each grid search point. The rates are separated by a tab character, so that user can simply paste the whole file into a spreadsheet to examine it.
- `me:printCellDOS`, which prints out the cell DOS for the wells in the ME system
- `me:printCellTransitionStateFlux`, which prints out cell transition state fluxes.
- `me:printReactionOperatorColumnSums`, which prints column sums of the full, normalized collision operator.
- `me:printGrainBoltzmann`, which prints out the normalized equilibrium grain population of all wells in the ME system.
- `me:printGrainDOS`, which prints out the grain DOS for the wells in the ME system
- `me:printGrainkbE`, which calculates  $k(E)$ s for backward reactions specified in the `reactionList`.
- `me:printGrainkfE`, which calculates  $k(E)$ s for forward reactions specified in the `reactionList`.
- `me:printGrainedSpeciesProfile`, which prints out time evolution of every grain in the logarithmic time scale.
- `me:printGrainTransitionStateFlux`, which prints out grained transition state fluxes.

- `me:printReactionOperatorSize`, which prints out the reaction operator. The text within this element specifies how many lower-right rows/columns to generate for printing. For example, if one specifies 20, then the lower-right square matrix block containing 20 rows/columns will be printed. In addition, one can specify negative numbers: -1 for printing whole matrix, -2 for printing lower-right square matrix with  $\frac{1}{2}$  the number rows/columns compared to the whole reaction matrix, -3 for printing lower-right square matrix with  $\frac{1}{3}$  the number rows/columns compared to the whole reaction matrix.
- `me:printSpeciesProfile` which prints out time evolution of every species in the logarithmic time scale.
- `me:printTunnelingCoefficients`, which prints tunnelling coefficients for those reactions where tunnelling corrections are specified.
- `me:printTunnellingCoefficients`, same as the previous one.
- `me:printCrossingCoefficients`, which prints transmission coefficients for those reactions where spin forbidden RRKM theory is requested
- `me:testDOS`, which calculates and prints partition functions for the wells in the ME system using the grain DOS, cell DOS, and analytical forms.
- `me:testMicroRates`, which computes canonical rate coefficients at a range of temperatures using the grain DOS and the grain  $k(E)$ s.
- `me:testRateConstants`, which computes TST  $k(T)$ s for each reaction in the system.
- `me:useTheSameCellNumberForAllConditions`, which forces MESMER to use the same cell number (the highest cell number amongst all simulations) for all calculations. This option only applies for input files specifying multiple ME calculations.

The following elements control the display of the energy level diagram generated from the XML data.

- `me:hideInactive`, which removes molecules or reactions with the attribute `active="false"` from the diagram. There is a control on the diagram which allows this to be toggled.

- `me:diagramEnergyOffset` which adjusts the displayed energy values of species. For example,

`<me:diagramEnergyOffset>0</me:diagramEnergyOffset>`

makes the lowest energy species have a displayed energy of 0. This is almost essential when the energies are used directly from a computational chemistry program, because of the large offset that they have.

### 3.2.6. Summary Table: Molecular input variables in MESMER

The following table lists essential variables for molecules in the MESMER calculation. The different types of molecules in the table are bath gas (Bath), transition states (TS), Excess Reactants (ExcS), deficient reactants (DefS), sink molecules (Sink), and modelled molecules (MM). The variables in grey boxes are not input by user, but calculated by MESMER according to the input parameters.

Parameter	Bath	TS	ExcS	DefS	Sink	MM
atomic/molecular mass	•	•	•	•	•	•
$\sigma$ (sigma)	•					•
$\varepsilon$ (epsilon)	•					•
$B_{xyz}$ (rotational constants)		•	•	•	•	•
$\sigma_{\text{sym}}$ (rotational symmetry)		•	•	•	•	•
ZPE (zero point energy)		•	•	•	•	•
$\nu_f$ (vibrational frequencies)		•	•	•	•	•
Spin multiplicity		•	•	•	•	•
Electronic Excitation		•	•	•	•	•
Number of cells		• only for RRKM	•	•	•	•
Number of grains			•	•	•	•
Energy of grains			•	•	•	•
Initial Population				•	•	•
Equilibrium fraction				•	•	•
$\langle \Delta E \rangle_d$ (delta E down)						•
Collision frequency						•
Collision operator size						•
Initial grain populations						•

### 3.3. Additional facilities and examples

Below, we introduce the structure of some basic XML, and provide some examples for utilizing MESMER functionality that was not discussed above. The best place to look for examples is in MesmerQA folder, the components of which are discussed in detail below. The examples inside the MesmerQA folder cover *most* of the functionality available in MESMER, but not *all* of it. As MESMER grows, so will the material that follows.

#### 3.3.1. Basic XML Structure

A basic one atomic molecule should look like this in MESMER:

```

<molecule id="He">
  <propertyList>
    <property dictRef="me:epsilon">
      <scalar>10.2</scalar>
    </property>
    <property dictRef="me:sigma">
      <scalar>2.55</scalar>
    </property>
    <property dictRef="me:MW">
      <scalar units="amu">4.0</scalar>
    </property>
  </propertyList>
</molecule>

```

From this view one can see that all property elements are enclosed inside `propertyList` element, where the `<atomarray></atomarray>` element and `<bondarray></bondarray>` elements are not shown in this example. It is not necessary to include the `<atomarray></atomarray>` and `<bondarray></bondarray>` elements in a molecule, but in the future, MESMER may use the elemental data to check the credibility of the vibrational/rotational data provided by user, or even draw pictures of the molecular geometries on the PES.

Any complete XML element should be one of the following

```
<elementName parameterName="id"></elementName>
```

or

```
<elementName parameterName="id" />
```

where in the first expression, child elements can be inserted between two element bodies as

```

<elementName parameterName="id">
  <childElementName parameterName="id"></childElementName>
  <childElementName parameterName="id"></childElementName>
</elementName>

```

Be careful when converting the first expression to the second expression if you are editing on a text editor, especially on the location of the forward slash in the expression.

### 3.3.2. Comparing MESMER rate coefficients to experimental values

Users often want to compare some value of a calculated phenomenological rate coefficient to an experimental value. This is a straightforward task in MESMER. The example below is derived from the Acetyl + O<sub>2</sub> system, which is included in the test suite.

Every rate in MESMER is defined by two different references, ref1 and ref2, where ref1 is the reactant, and ref2 is the product. It is easy to identify ref1 and ref2 for any reaction in MESMER by running a preliminary calculation, and inspecting the output in the Bartis Widom phenomenological rate coefficient analysis in mesmer.test. All the rate coefficients are output in the format ref1 -> ref2, unless the rate coefficient is that for phenomenological loss, in which case it is specified as a loss reaction. For a loss reaction, ref1 and ref2 are identical, as in the example shown below, where we compared the experimentally measured rate coefficient for acetyl loss to the phenomenological rate coefficient calculated by MESMER. To get MESMER to output the square of the difference between a calculated and an experimental rate coefficient divided by the square of the error (i.e  $\chi^2$ ), enter the following:

```
<me:PTpair me:units="Torr" me:P="200.15" me:T="500" me:precision="double">
    <me:experimentalRate ref1="acetyl" ref2="acetyl" error="6.75E+03"></me:experimentalRate>
</me:PTpair>
```

where the experimental rate goes in between the `<>`, as discussed above. You can input an arbitrary number of experimental rates. If there are no experimental rates to compare with, simply remove the middle line:

```
<me:PTpair me:units="Torr" me:P="200.15" me:T="500" me:precision="double"></me:PTpair>
```

or even shorter

```
<me:PTpair me:units="Torr" me:P="200.15" me:T="500" me:precision="double" />
```

Since double is the default precision in MESMER, one can write

```
<me:PTpair me:units="Torr" me:P="200.15" me:T="500" />
```

instead.

### 3.3.3. Specifying Numerical Precision

As discussed above, double is the default MESMER precision; however, quad-double and double-double precisions can also be specified inside `<me:PTpair>` element, the syntax is

```
me:precision="quad-double"
me:precision="double-double"
```

OR

```
me:precision="qd"
me:precision="dd"
```



### 3.3.4. ME calculations over a Large Parameter Space

Often, a user is interested in running ME calculations over a large parameter space – for example, to explore the sensitivity of the computed results to a barrier height or an energy transfer parameter like  $\langle \Delta E \rangle_d$ . Performing lots of calculations over a parameter space is called a ‘grid search’ in MESMER, and is enabled by inserting an element

```
<me:gridSearch />
```

into the `<me:control></me:control>` element. In addition, the user must specify the particular parameters over which to search, the search range, and the step size. Presently, the parameters available for performing a grid search are zero-point energies,  $A^\infty$ ,  $n^\infty$ , and  $E^\infty$ , and  $\langle \Delta E \rangle_d$ . The parameter search range is specified as in the following example for  $A^\infty$ :

```
<me:preExponential lower="4.6e-12" upper="6.201e-12" stepsize="2.0e-13">6.00e-12</me:preExponential>
```

In this expression, there are three additional parameters for the `me:preExponential` element: the lower bound, upper bound, and step size over which to vary the parameter. The order of these three elements is not important. According to XML formatting protocol, the number `6.00e-12` within the element pair of `me:preExponential` still must be supplied (and for ZPE parameters, it is actually used to display the PES using Firefox), but MESMER will ignore it if the other three elements are correctly specified.

To estimate the time of running a grid search, one should see how long it takes to run one point. If there are 11 points in  $A^\infty$ , 13 points in  $n^\infty$ , and 9 points in zero-point energy to be evaluated, the task will have 1287 total points to calculate in parameter space. Say one point in precision double-double takes one minute to finish, then this grid search task will take approximately 21 hours and 30 minutes. Usually, in order to compare with the experimental results, a grid search task is done on several temperatures and pressures; therefore, it is ideal to divide a task into multiple tasks.

For grid searches, a useful command line option is the `-N` option, which when typed as

```
mesmer.out -N acetyl_O2_0003.xml
```

will prefix name all of the MESMER output files, \*.test file, \*.log file, and \*.xml using the `acetyl_O2_0003` prefix.

### 3.3.5. Temperature dependent $\langle \Delta E \rangle_d$

Often, one finds that  $\langle \Delta E \rangle_d$  depends on temperature – or one at least wants to investigate the very likely possibility that it might. In MESMER it is easy to set  $\langle \Delta E \rangle_d$  so that it depends on temperature. The functional form utilized in MESMER is

$$\langle \Delta E_d \rangle = K \left( \frac{T}{T_{\text{ref}}} \right)^n.$$

where  $T$  is temperature,  $T_{\text{ref}}$  is the reference temperature, and  $n$  is an exponent for introducing some nonlinearity into the expression. By default,  $n = 0$ , which means there is no temperature dependency for  $\langle \Delta E \rangle_d$ , so that  $\langle \Delta E \rangle_d = K$ . The user can change these values by the following syntax

```
<property dictRef="me:deltaEDown">  
  <scalar units="cm-1" referenceTemperature="298" exponent="1.0">150.0</scalar>  
</property>
```

This gives linear dependency of  $\langle \Delta E \rangle_d$  in temperature with  $T_{\text{ref}} = 298$ ,  $n = 1$ , and  $K = 150 \text{ cm}^{-1}$ .<sup>1</sup> A temperature dependent form of  $\langle \Delta E \rangle_d$  does not prevent a grid search on  $\langle \Delta E \rangle_d$ . For example, the following syntax will do a grid search on the value of  $K$ :

```
<property dictRef="me:deltaEDown">  
  <scalar units="cm-1" lower="140.0" upper="200.1" stepsize="10.0" referenceTemperature="298"  
    exponent="1.0">150.0</scalar>  
</property>
```

### 3.3.6. Unimolecular and Reverse ILTs

As indicated above, MESMER includes two different ways for calculating the microcanonical rate coefficients – through the use of the RRKM expression, or the use of the ILT technique. The ILT technique is particularly useful when there is no easily identifiable energetic barrier, such as often occurs in the case of radical-radical reactions or non-adiabatic reactions. ILT offers a mathematical formalism for deriving  $k(E)$ s from an Arrhenius fit to a set of  $k(T)$ s, which may be obtained from experiment or theory. There are three possible uses of the ILT in MESMER: (1) for association reactions, where the Arrhenius expression is for the association reaction of one deficient reactant combining with one excess reactant to make

a modelled molecule; (2) for isomerization reactions of one modelled molecule to another, where the Arrhenius expression is for the forward reaction; (3) for an irreversible reaction of one modelled molecule to one or two sink molecules, where the Arrhenius expression is for the forward reaction, and (4) for dissociation reactions of one modelled molecule to two sink molecules, where the Arrhenius expression is for the association of the two sink molecules. Implementation (1), which is the most common, we call the standard ILT. Implementations (2) and (3) are the unimolecular ILT, and (3) is the reverse ILT. The test file for the acetyl + O<sub>2</sub> reaction has examples of the standard ILT, so we do not discuss this here. The unimolecular ILT is identical to the standard ILT, except that the reaction replaces the two reactants with a single modelled molecule. Below, we provide syntax for the reverse ILT, which in this case corresponds to the dissociation of I<sub>2</sub>O<sub>2</sub> to OIO + I:

```
<reaction id="R2">
  <reactant>
    <molecule ref="IO_IO" me:type="modelled" />
  </reactant>
  <product>
    <molecule ref="OIO" me:type="sink" />
  </product>
  <product>
    <molecule ref="I" me:type="sink" />
  </product>
  <me:MCRMethod>MesmerILT</me:MCRMethod>
  <me:preExponential>1.4E-10</me:preExponential>
  <me:activationEnergy units="kJ/mol" reverse="true">0.0</me:activationEnergy>
  <me:TInfinity>298.0</me:TInfinity>
  <me:nInfinity>0.01</me:nInfinity>
</reaction>
```

The reverse ILT is requested with reverse="true" on the me:activationEnergy element, where the activation energy is referenced to the ZPE of OIO + I. The input Arrhenius data is for the OIO + I association reaction, but reverse="true" tells MESMER to use that data for calculating  $k(E)$ s of the forward dissociation reaction – i.e., I<sub>2</sub>O<sub>2</sub> → OIO + I. MESMER treats a typical dissociation reaction as irreversible – i.e., data for sink molecules is not required by MESMER (although names and ZPEs *are* required if you want a nice diagram in Firefox, and it's good for future bookkeeping of reaction data). However, for a reverse ILT, product data of the sink molecules is essential, and the MESMER will terminate if it is not provided. Finally, due to the mathematics of the ILT, it is necessary to point out that there are some constraints on: for the standard ILT,  $n^\infty$  must be greater than 1.5, while for the unimolecular and reverse ILT,  $n^\infty$  must be greater than zero. If a zero  $n^\infty$  is desired, setting it to a very small number will introduce minimal error.

### 3.3.7. Secondary input files

Another facility for making use of a library of molecules is to use *secondary input files*. These are XML files specified on the command line after the main XML file. For instance, in

```
mesmer maininput.xml sec1.xml sec2.xml -o outfile.xml
```

`sec1.xml` and `sec2.xml` are secondary input files. MESMER inserts their contents into the main XML. So if the secondary file contained:

```
<moleculeList>
  <molecule id="mol1"> ... </molecule>
  <molecule id="mol2"> ... </molecule>
</moleculeList>
```

the molecules would be inserted into the main `<moleculeList>`. Or a secondary file like:

```
<me:control>
  <me:printSpeciesProfile />
</me:control>
```

might be used to obtain a different output, without editing the main data file.

## 4. MESMER files explained

This section provides an explanation of important features of the more significant files in the source folder and produced during a calculation.

### 4.1. MESMER output files

Each MESMER calculation creates three output files. They are \*.test, \*.log and \*.xml;

#### 4.1.1. mesmer.test

Much of the mesmer.test file contains information that user chooses to print out in <me:control>. Here we review some of the most important items in this file and provide brief explanations.

##### 4.1.1.1. Partition Functions and State Densities

Test rovibronic density of states for: <molecule name>

This line is followed by columns like the following

T	qtot	sumc	sumg
200	4.38418e+006	4.4283e+006	4.40474e+006
300	5.15155e+007	5.21956e+007	5.2071e+007
400	5.69583e+008	5.78432e+008	5.77654e+008
500	6.05474e+009	6.15825e+009	6.15289e+009
600	6.12019e+010	6.17397e+010	6.16902e+010

which gives the canonical partition function of the named molecule. Column **qtot** is obtained through analytical approximation of the partition function while columns labelled **sumc** and **sumg** calculate the partition functions from summing the state averaged cell and grain densities, respectively.

Grain rovibronic density of states of <molecule name>

This section prints rovibronic state densities for the molecule in grains, where the first column is the mean grain energy in  $\text{cm}^{-1}$ .

#### 4.1.1.2. $k(E)$ s & Tunnelling Corrections

```
k_f(e) grains for <reaction name>
```

This section gives the forward microcanonical rates calculated from the grains.

```
k_b(e) grains for <reaction name>
```

This section gives the reverse microcanonical rates calculated from the grains.

```
Tunnelling coefficients for: <reaction name>
```

```
V0 = 11201.4, V1 = 10047.3, barrier0 = 9864, barrier1 = 8834, imFreq = 5.0605e+13
```

This section gives tunnelling coefficients of the reaction. V0 and V1 are forward and reverse classical barrier height of the reaction. Barrier0 and barrier1 are zero-point energy differences between the transition state and the respective reactant and product, with units in  $\text{cm}^{-1}$ . The data used to calculate transmission coefficients for spin forbidden RRKM theory is printed out in the \*.log file.

#### 4.1.1.3. Equilibrium Fractions

```
Eq fraction matrix:
```

```
{  
  -4.9732e+013      1      0  
      0  -0.0036887      1  
      1      1      1  
}
```

```
inverse of Eq fraction matrix:
```

```
{  
  -2.0108e-014 -2.0034e-014  2.0034e-014  
  2.0034e-014  -0.99632      0.99632  
  7.3899e-017  0.99632      0.0036751  
}
```

```
Equilibrium Fraction for Int1 = 0.996325
```

```
Equilibrium Fraction for Int2 = 0.00367513
```

```
Equilibrium Fraction for acetyl = 2.00339e-014
```

This section uses rovibronic partition functions of isomers and pseudo-isomers to calculate equilibrium constants. Consider a three well system: e.g.,  $A \rightleftharpoons B \rightleftharpoons C$  where the equilibrium constant for  $A \rightleftharpoons B$  is given by,  $k_{\text{eq}} = k_1 = \frac{Q_B^{\text{rve}}}{Q_A^{\text{rve}}}$  and that for  $B \rightleftharpoons C$  is given by

$k_{\text{eq}} = k_2 = \frac{Q_C^{\text{rve}}}{Q_B^{\text{rve}}}$ . Therefore, the relation is defined by the following three linear equations:

$$\begin{aligned}
-k_1 A + B &= 0 \\
-k_2 B + C &= 0 \\
A + B + C &= 1
\end{aligned}$$

**Equation 4.1**

which is a 3 by 3 matrix and a vector which satisfy the following:

$$\begin{vmatrix} -k_1 & 1 & 0 \\ 0 & -k_2 & 1 \\ 1 & 1 & 1 \end{vmatrix} \begin{vmatrix} A \\ B \\ C \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix}$$

**Equation 4.2**

The equilibrium fraction of each isomer (or pseudo isomer, in the case of a source term) may be obtained by inverting the matrix shown above, and taking the elements in the final column of the inverse. Any system, with an arbitrary number of wells and connections, may be described by such a Matrix. Irreversible channels are not included within the calculation of the equilibrium fractions.

#### 4.1.1.4. Eigenvalues

Total number of eigenvalues = 347

Eigenvalues

This section gives the eigenvalues of the reaction (system collision) operator, all of which should be less than or equal to zero. Within the energy grained Master Equation, collisional energy transfer in the grained phase space and inter-conversion between species is described using a set of coupled differential equations:

$$\frac{d}{dt} \mathbf{p} = \mathbf{M} \mathbf{p}$$

**Equation 4.3**

where  $\mathbf{p}$  is the population vector containing the populations of the energy grains for the  $i$ th isomer,  $n_i(E)$ , and  $\mathbf{M}$  is the matrix that determines population evolution due to collisional energy transfer and reaction. The discretized matrix  $\mathbf{M}$  is diagonalized, and the eigenpairs give a solution of the form:

$$\mathbf{p} = \mathbf{U} e^{\lambda t} \mathbf{U}^{-1} \mathbf{p}(0)$$

#### Equation 4.4

where  $\mathbf{p}(0)$  contains the initial conditions (i.e.,  $t = 0$ ) for each grain (i.e.,  $n_{iE}(0)$ ),  $\mathbf{U}$  is matrix of eigenvectors obtained from diagonalization of  $\mathbf{M}$ , and  $\lambda$  are the corresponding eigenvalues. For many chemical systems, one observes a separation in eigenvalues, with the bulk corresponding to very fast decay (i.e., a large absolute value), and a select few corresponding to slower decay (i.e., a small absolute value). The eigenvalues for fast decay correspond to the time scales for relaxation of activated intermediates and are often referred to as internal energy relaxation eigenvalues (IEREs). Those for slow decay correspond to the time scales for chemical reaction and are often referred to as chemically significant eigenvalues (CSEs). In general, for a system with  $n_1$  wells and  $n_2$  sources, the number of CSEs is equal to  $n_1 + n_2$ . If the system is conservative (i.e., it has no sinks/irreversible loss channels), then one of the CSEs will be zero within numerical precision.

#### 4.1.1.5. Species Profiles

Print time dependent species and product profiles

{

Timestep/s	acetyl	Int1	Int2	lactone
1e-011	0.999999	1.3743e-006	1.59562e-010	2.89049e-011
1.25893e-011	0.999998	1.72938e-006	2.51184e-010	3.21681e-011
1.58489e-011	0.999998	2.17595e-006	3.95081e-010	3.7701e-011
1.99526e-011	0.999997	2.73745e-006	6.20583e-010	4.90837e-011
2.51189e-011	0.999997	3.44324e-006	9.73003e-010	7.0918e-011
3.16228e-011	0.999996	4.33003e-006	1.52194e-009	1.13004e-010
3.98107e-011	0.999995	5.44372e-006	2.37345e-009	1.94525e-010
5.01187e-011	0.999993	6.84152e-006	3.68781e-009	3.51826e-010
6.30957e-011	0.999991	8.59463e-006	5.70456e-009	6.51742e-010

This section gives time dependent species and product profiles of the system. The first column is the time step in seconds, and all the following columns are the populations at the specific time step of that species. For intermediates, these are calculated by simply summing the time dependent solutions for each energy grain which correspond to a particular isomer. The normalized time dependent product profiles are obtained in a related, but slightly more indirect manner. The normalized total product yield,  $P(t)$ , is as follows:



$$P(t) = 1 - \sum_i \sum_E n_{iE}(t) dt$$

**Equation 4.5**

If only one product channel is available, then (4.5) provides information regarding the time dependent product yield; however, when there are  $p$  products, then the normalized yield of a specific product at time  $t$ ,  $P_{pi}(t)$ , may be written as:

$$P_{pi}(t) = \left[ \frac{\sum_t \left[ \sum_E k_{pi}(E) n_{iE}(t) dE \right] dt}{\sum_p \left[ \sum_t \left[ \sum_E k_{pi}(E) n_{iE}(t) dE \right] dt \right]} \right] \times P(t)$$

**Equation 4.6**

where  $n_{iE}(t)$  is the time dependent population of the energy grains spanning the entire state space of the  $i$ th well obtained from (4.5), and  $k_{pi}(E)$  are the microcanonical rate constants for formation of product  $p$  from isomer  $i$ .

#### 4.1.1.6. Phenomenological rate coefficients

##### Bartis Widom eigenvalue/eigenvector analysis

This section gives phenomenological rate coefficients (i.e.,  $k(T,P)$ ) derived from an eigenvalue-eigenvector analysis of the solution to (E1). The mathematical development of the Bartis -Widom technique implemented in MESMER is described by Robertson *et al.*,<sup>2</sup> and so will not be detailed here. Briefly, the basic idea is as follows: the phenomenological rates for an arbitrary interconnected kinetic system may be described identically to (E2), with the primary difference being the absence of a description for collisional relaxation kinetics – i.e., all the rate coefficients correspond to interconversion between species. This is the sort of approach generally used to interpret kinetics experiments: for a system of  $n$  species, the kinetics of the system may be described using an  $n \times n$  rate coefficient matrix  $\mathbf{K}$  representing  $n$  coupled first order differential equations, where the matrix element  $K_{ab}$  is the rate coefficient  $k_{b \rightarrow a}(T,P)$ . Diagonalization of this rate matrix yields a solution to the coupled differential equations in terms of  $n$  eigenvalues and  $n$  eigenvectors. The Bartis-Widom method exploits the separation between the IEREs and CSEs: assuming that the CSEs obtained from the diagonalization of  $\mathbf{M}$  (i.e., the full energy grained master equation, which includes a model for collisional relaxation) are identical those which would be obtained from

diagonalization of  $\mathbf{K}$ , then the phenomenological rate matrix  $\mathbf{K}$  may be obtained using simple matrix algebra. The Bartis-Widom analysis is a very powerful technique because it provides a global description of the time dependent kinetics in terms of  $n \times n$  rate coefficients, and in many cases, the phenomenological rate coefficient is the quantity of interest to be obtained from a ME calculation. However, the Bartis-Widom analysis relies on the separation between CSEs and IEREs. If these are not well separated by more than an order of magnitude, then MESMER will print a warning, and the user should proceed with caution because the Bartis Widom rate coefficients may not be reliable. In such cases, and as long as numerical precision is not an issue, the user may rely on the species profiles, since these do not require separation between CSEs and IEREs. When there is good separation (i.e., at least an order of magnitude) between the CSEs and the IEREs, then the species profiles printouts are identical to the species profiles that would be obtained from the phenomenological rate coefficients. MESMER prints out the following sections for any system, all of which are defined in Robertson *et al.*:

```
Z_matrix * Z_matrix^(-1):
```

The  $\mathbf{Z}$  matrix is identical to that described in eq (19) of Robertson *et al.* The matrix printed below this header is essentially a test of numerical accuracy, and should give the identity matrix within numerical precision.

```
Kr matrix:
```

The  $\mathbf{K}_r$  matrix is identical to that in Eq. (20) of Robertson *et al.*

```
First order & pseudo first order rate coefficients for isomerization rxns:
```

The pseudo and pseudo first order rate coefficients printed in this section are contained in the  $\mathbf{K}_r$  matrix with labels describing the interconversion to which they correspond.

```
Kp matrix:
```

The  $\mathbf{K}_p$  matrix is identical to that in eq (35) of Robertson *et al.*, and is only printed out for systems which have sinks / irreversible loss channels.

```
First order & pseudo first order rate coefficients for loss rxns:
```

The first order and pseudo first order rate coefficients printed in this section are contained in the  $\mathbf{K}_p$  matrix with labels describing the interconversion to which they correspond.

### 4.1.2. mesmer.log

This file contains information, warning and error messages generated during the calculation. Also, it records when default values of input parameters have been used and provides additional usage log of some variables when MESMER finishes calculation. It will be useful for anyone who is writing an input file from the beginning, to check this file for any missed arguments.

### 4.1.3. XML output

The XML output contains a copy of the input XML file together with:

- explicit entries for parameters where a default value was used;
- molecules, etc. from librarymols.xml and secondary input files that were used;
- calculated partition functions
- calculated microcanonical rate coefficients
- calculated Bartis-Widom rate coefficients
- calculated species/ time profiles
- metadata, including name of user etc.

The calculated values are a subset of those in mesmer.test. The last two groups of data are in an additional element `<analysis>` under `<mesmer>`.

XML data can be re-formatted for various purposes, and a more human-friendly text presentation and an energy level diagram has been provided for the MESMER XML files using XSLT. Currently, it is necessary to use Firefox 3. See section 3.1.

## 4.2. Other files

### 4.2.1. defaults.xml and librarymols.xml

Many of the parameters specified in the input file come with default values. These may be properties of molecules or reactions, methods and modelling parameters. If MESMER requires a value and it is not present in the input file, it reads a value from the file `defaults.xml`, inserts the entry into the output XML file and records these actions in `mesmer.log`. If the default value could be contentious, it flags the need for user checking by

making the entries in mesmer.log and the output XML file in uppercase letters. For instance, spinMultiplicity has a default value of 1, which is not flagged in uppercase, while the default value of  $\langle \Delta E \rangle_d$  of 130 cm<sup>-1</sup> is. The `defaults.xml` file can be edited if necessary.

It is good practice to specify all the reactants and products of a reaction in a `<molecule>` element with a `ref` attribute. If there is no molecule under `<moleculeList>` with an `id` matching the `ref`, or if the matching molecule has no content, or if it fails to initialize, MESMER searches in the file `librarymols.xml`. If successful, it copies the molecule into the output XML file. Molecules with an attribute `active=false` in this file are ignored, giving the opportunity to easily select alternatives.

Alternative names for molecules can be specified in `librarymols.xml`. for example:

```
<molecule id="oh" ref="OH">
```

Then, if a datafile referenced a molecule "oh", but had no complete specification of it, MESMER would insert the data from the molecule OH in `librarymols.xml`, but with an `id="oh"`, so that this name can continue to be used in the datafile. It would have an additional attribute `libId` to show its origin.

These procedures mean that the output XML file is explicit: if used as input for further runs, no default values will be used. The safest way of sharing data files between users, who might have adjusted the defaults, is to use an output XML file.

#### 4.2.2. source.dot and source.ps

These two files show the file dependency tree structure of the MESMER source code which is a guide for programmers to avoid repeated inclusion. File `source.dot` is produced by a PERL script `cinclude2dot.pl` (<http://flourish.org/cininclude2dot/>).

#### 4.2.3. mesmer1.xsl, mesmerDiag.xsl and switchcontent.xsl

These three files, located in the root folder, provide a browser interface to XML files, where xsl stands for extensible stylesheet language. The MESMER \*.xml input file refers to the \*.xsl stylesheets using a relative pathname. In order to view the \*.xml file in Firefox, one must verify that the XML input files refer to the correct location of the \*.xsl files. The default for the input files included in the QA directory is two levels down in the root folder of MESMER; however, if a \*.xml input file is in a different location, one can change the href path in second line of the XML file

```
<?xml-stylesheet type='text/xsl' href='../..//mesmer1.xsl'?>
```

so that it points to the correct location of the \*.xsl files.

## 5. Test Suite

As discussed previously, we have a range of test jobs included within MesmerQA, and we recommend that users to utilize them as templates for constructing their jobs. There is no one job that includes all the MESMER functionality discussed thus far, but taken together, these jobs do utilize most of MESMER's functionality. So users should be able to patch together their own MESMER input files using fragments of these input files. Below, we briefly describe each of the test jobs, and include a screen shot of the corresponding PES, visualized using Firefox to read the \*.xml input file. Note that some of the QA directories include additional input files beyond those discussed in this part of the manual. These files are in various stages of development by the MESMER authors. While they are not discussed explicitly below, there's no harm in experimenting with them!

## 5.1. N-Pentyl Isomerization

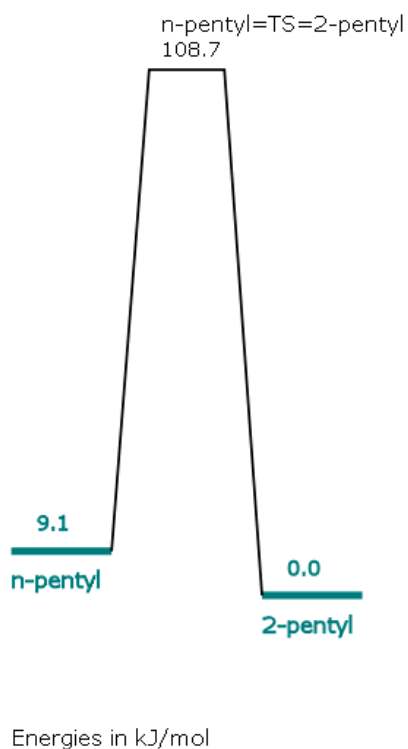


Figure 5.1: PES schematic for N-Pentyl Isomerization

The input file for this job is `pentyl_isomerization_test.xml`. This is a two well system for the isomerization of 2-pentyl over n-pentyl in an Argon bath gas using a standard RRKM treatment at the transition state.

## 5.2. Cyclopropene Isomerization + Reservoir State

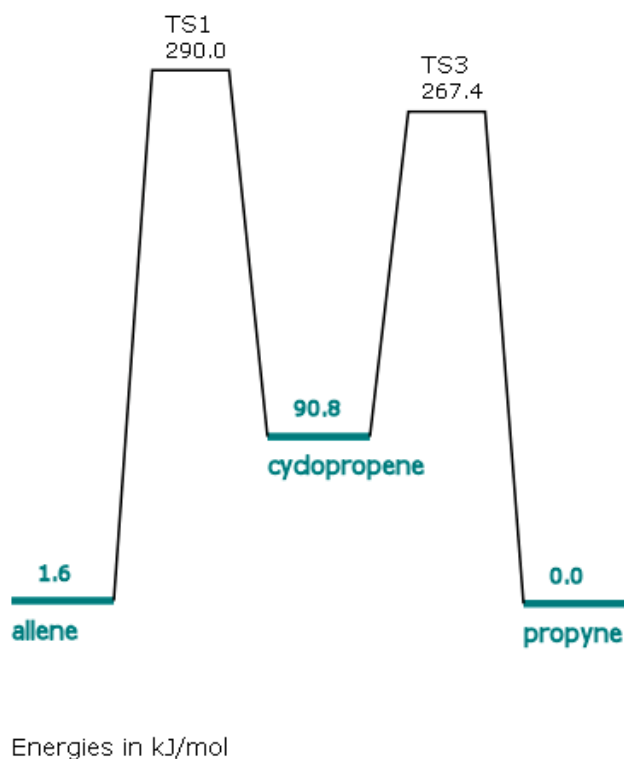


Figure 5.2: schematic of the PES used the cyclopropene isomerization test

The `Cyclopropene_isomerization_test.xml` input file is located within the folder ‘cyclopropene isomerization/’. This is a three well system for the isomerization of allene, cyclopropene, and propyne in a He bath gas using a standard RRKM treatment at both transition states.

The `Cyclopropene_isomerization_reservoir_state_test.xml` input file is located within the folder ‘cyclopropene isomerization reservoir state/’. This is a three well system for the isomerization of allene, cyclopropene, and propyne in a He bath gas using a standard RRKM treatment at both transition states; however, it includes reservoir states for each isomer.



### 5.3. H + SO<sub>2</sub>



Figure 5.3: PES schematic for H + SO<sub>2</sub> test system

The HSO<sub>2</sub>\_test.xml input file is located within the folder ‘HSO<sub>2</sub>/’. This system includes one well and a bimolecular source term, which is composed of one deficient and one excess reactant in an Ar bath gas using a standard RRKM treatment at TS1.

## 5.4. OH + C<sub>2</sub>H<sub>2</sub>

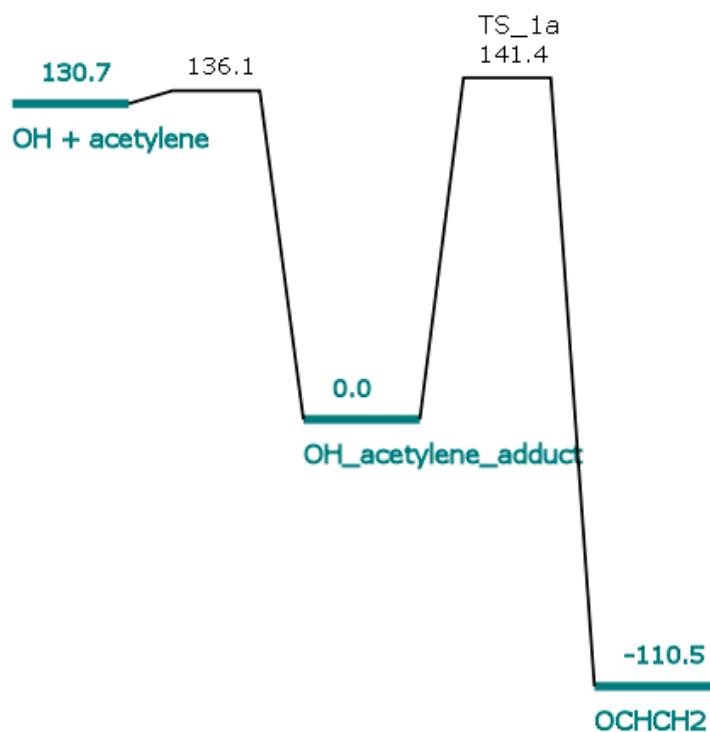


Figure 5.4: PES for OH + C<sub>2</sub>H<sub>2</sub> test job

The OH\_acetylene\_association\_test.xml input file is located within the folder ‘OH acetylene association/’. This system includes one well, a bimolecular source term, and one irreversible unimolecular channel via TS\_1a with an OCHCH<sub>2</sub> sink in an N<sub>2</sub> bath gas. TS\_1a is treated using standard RRKM theory, and the association TS is treated using a standard ILT.

## 5.5. $\text{O}_2 + \text{CH}_3\text{CO}$

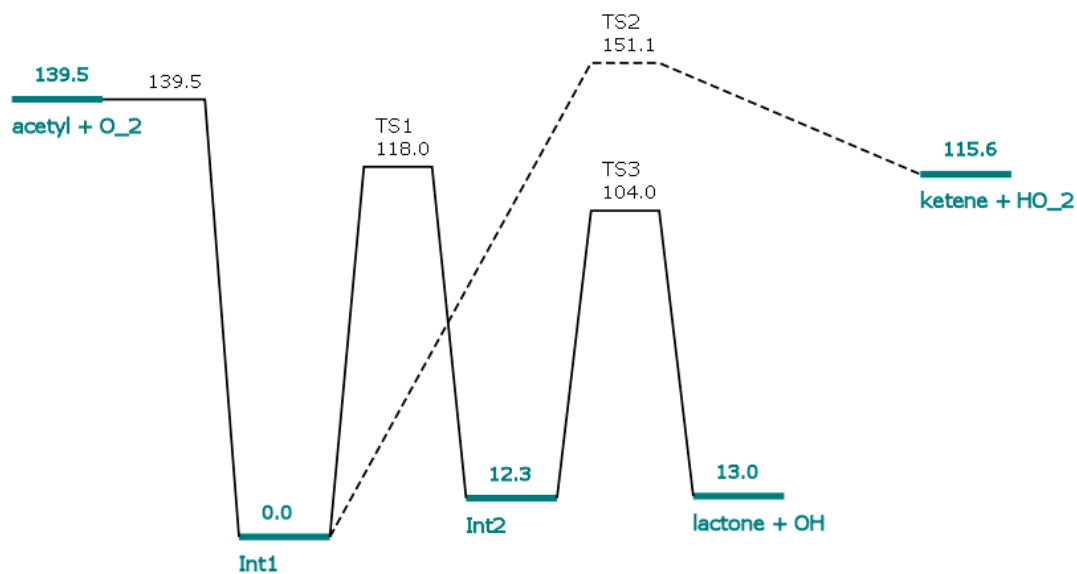


Figure 5.5: PES for  $\text{O}_2 + \text{CH}_3\text{CO}$

The `Acetyl_O2_association.xml` input file is located within the folder ‘Acetyl O2 association/’. This system includes two wells, a bimolecular source term, and two irreversible unimolecular dissociation channels via both TS2 and TS3, and He bath gas. TS1 is treated using standard RRKM theory with an Eckart tunnelling correction, the association TS is treated using a standard ILT, and all other TSs with standard RRKM theory.

## 5.6. i-propyl

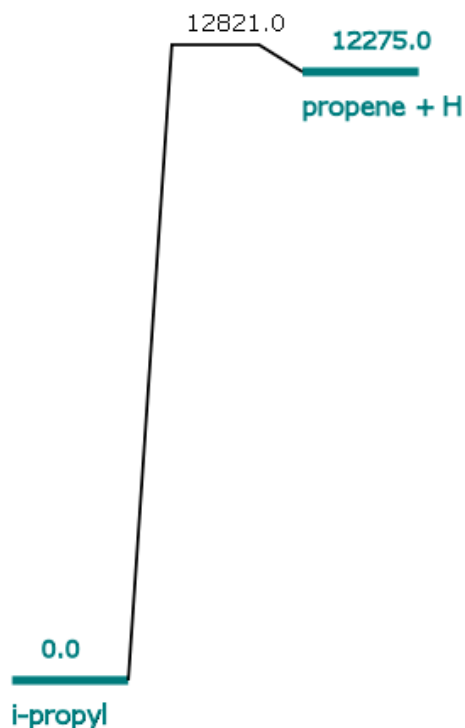


Figure 5.6: PES for i-propyl dissociation

The `ipropyl_test.xml` input file is located within the folder ‘i-propyl/’. This system includes one well, and an irreversible unimolecular dissociation channel in a He bath gas. The  $k(E)$ s for the irreversible dissociation channel are calculated using a reverse ILT of the propene + H association rate coefficients.

## 5.7. Benzene-OH Oxidation

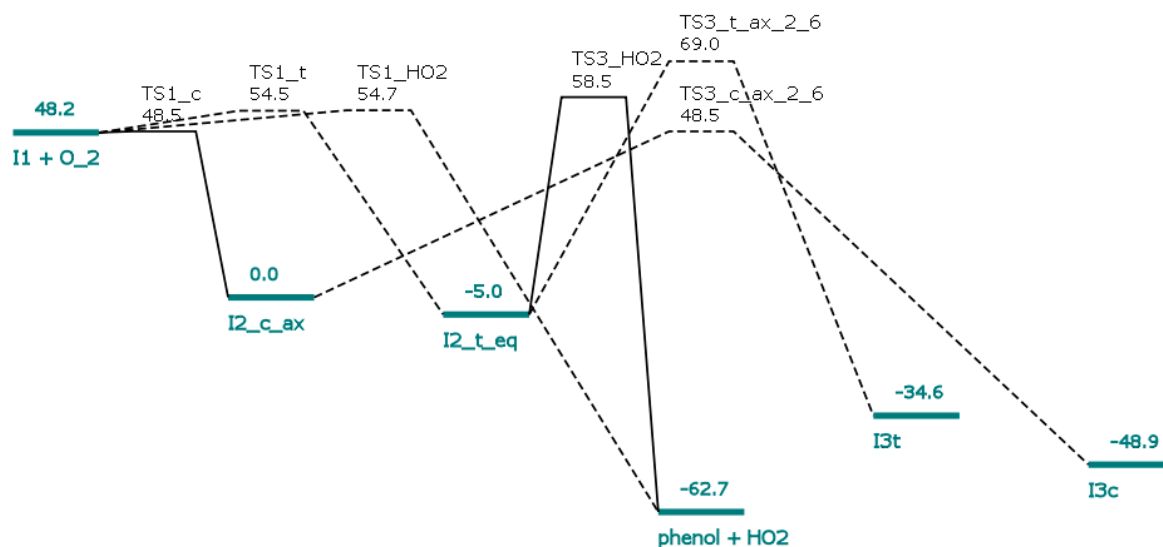


Figure 5.7: PES for benzene-OH + O<sub>2</sub>

The `benzene_oxidation_test.xml` input file is located within the folder 'benzene\_oxidation/'. This system includes a bimolecular source (I1 + O<sub>2</sub>), two wells (I2\_c\_ax and I2\_t\_eq), and four irreversible channels in N<sub>2</sub> bath gas. Two of the irreversible channels are unimolecular isomerizations (to I3t and I3c), one is a unimolecular dissociation (I2\_t\_eq → phenol + HO<sub>2</sub>), and one is an irreversible exchange reaction (benzene-OH + O<sub>2</sub> → phenol + HO<sub>2</sub>). The  $k(E)$ s for each channel are calculated using RRKM theory.

## 5.8. Spin Forbidden Test Systems

Input files that show how to invoke spin forbidden RRKM theory are include in the folder 'spin\_forbidden\_kinetics/'. This folder includes two different input files, one of which invokes Landau Zener corrections (LZ\_test.xml), and the other of which invokes WKB corrections (WKB\_test.xml), as described in Harvey and Aschi, *Faraday Discuss.*, 2003(124), 129-143. The modelled systems are simple isomerizations of singlet cyclopentyne to the lowest lying triplet via the DFT calculated minimum energy crossing point (MECP).

## 6. Adding Functionality to MESMER

We have written MESMER in a manner than anticipates future developments, so that as the functionality of MESMER increases, we expect that this portion of the manual to grow. For example, we can envision the implementation of more sophisticated models for the calculation of tunnelling corrections, spin forbidden transmission coefficients, collisional energy transfer, density of states calculations, non-equilibrium initial distributions, and microcanonical rate coefficients.

In order to anticipate such developments, MESMER has been written with the use of ‘plug-in’ classes – a useful polymorphic feature of object-oriented languages such as C++. Plug-in classes are concrete classes derived from abstract base classes, and may be added without changing any of the existing code. They have a single global instance, the constructor of which registers the class with the base class. A pointer to a derived plug-in class may be obtained by supplying the id (i.e., a string) of the derived class to the Find function on the base class.

It will likely be the case that new plug-in classes will require the user to supply new data in the input file, and the development philosophy that we have agreed to implement is that plug-in classes should read their own data. This will allow additions to the MESMER code in a modular and relatively painless fashion. Additionally, it will minimize the amount of code that needs to be changed and retain backward compatibility with existing data structures and input files.

### 6.1. Plug-in Classes

#### 6.1.1. Microcanonical Rates

The abstract base class for calculating density of states is `MicroRateCalculator`, which lives in `Microrate.h`. The derived concrete classes (i.e., the plug-in classes) offer different methods for calculating microcanonical rate coefficients as follows:

- `MesmerILT`, which includes standard ILT, unimolecular ILT, or reverse ILT
- `SimpleRRKM`, which uses the standard RRKM equation

- SimpleILT, which is not presently used for anything, and was initially used for testing purposes

### 6.1.2. Density of States

`DensityOfStatesCalculator` is the abstract base class for performing density of states calculations, and lives in `DensityOfStates.h`. The derived concrete classes (i.e., the plug-in classes) offer different methods for calculating densities of states as follows:

- `ClassicalRotor`, which includes classical approximations for external rotations
- `QMRotor`, which includes quantum approximations for calculate external rotations
- `HinderedRotorA`, which doesn't presently include any functionality, but will be used for a hindered rotor correction to the calculated densities of states. Because many such corrections exist, we have labelled this one 'A'; subsequent corrections may be labelled sequentially – i.e., B, C, etc...

### 6.1.3. Tunneling Corrections

The abstract base class for tunnelling corrections is `TunnelingCalculator`, which lives in `Tunelling.h`. The derived concrete classes (i.e., the plug-in classes) offer different methods for calculating tunnelling corrections as follows:

- `EckartCoefficients`, which approximate tunnelling using a one dimensional asymmetric Eckart potential

### 6.1.4. Spin Forbidden RRKM theory

The abstract base class for tunnelling corrections is `CrossingCalculator`, which lives in `Crossing.h`. The derived concrete classes (i.e., the plug-in classes) offer different methods for calculating tunnelling corrections as follows:

- `LandauZenerCrossingCoeff`, which calculates transmission coefficients using Landau Zener theory
- `WKBCrossingCoeff`, which calculates transmission coefficients that include the effects of tunnelling below threshold



### **6.1.5. Distribution Calculator**

The abstract base class for calculating the initial distributions within the grains is `DistributionCalculator`, which lives in `Distribution.h`. The derived concrete classes (i.e., the plug-in classes) offer different methods for the initial grain distribution as follows:

- `BoltzmannDistribution`, which uses a Boltzmann population for the initial grain distribution within a particular well

In the future, we hope to add a plug-in class that will calculate initial non-equilibrium grain distributions, based on energy partitioning models similar to the prior distribution.

### **6.1.6. Collisional Energy Transfer Models**

When MESMER was initially developed, the exponential down collisional energy transfer model was hard-wired into the code; however, we are presently changing the exponential down model so that it will be a plug-in class of a more general abstract base class.

## 7. MESMER FAQs

### Q. What is qtot? What is sumc? What is sumg?

This is discussed in section 4.1.1.1

### Q. What is conc?

It is the number density in units of particles cm<sup>-3</sup>

### Q. For an irreversible Reaction –either an isomerization or a dissociation reaction – do I need to input the data for the product?

Irreversible reactions should not require any data apart from the name of the sink molecule, and the sink molecule requires no more than a molecule id within the moleculeList portion of the input. So if HO<sub>2</sub> is a sink molecule, and you can't supply its molecular data, then all you need to supply for HO<sub>2</sub> in the moleculeList is the following:

```
<molecule id="HO_2"/>
```

### Q. What are the different types of reactions in MESMER and how are they defined?

MESMER has four types of reactions:

- AssociationReaction, which has two reactants (one excess and one deficient), and one product, which is a modelled molecule
- IrreversibleReaction, which has one modelled molecule reactant, and either one or two sink molecules for products
- IsomerizationReaction, which has one reactant and one product, each of which is a modelled molecule
- IrreversibleExchangeReaction, which has two reactants (one excess and one deficient), and two products, both of which are sink molecules

### Q. Do all the energies in a MESMER input file need to be in the same units?

For ZPEs, yes, they do. The ZPE units for each stationary point must be consistent, and if you need energies for any other part of the calculation (e.g., an ILT), then the units for those must be consistent with the molecular ZPEs.

**Q. What is the file naming convention of MESMER?**

This is described in section 2.0.

**Q. How can I tell if my calculation is suffering from numerical precision problems?**

MESMER uses numerical algorithms to solve the Master Equation, and thus is not immune from the sorts of numerical issues that arise in any numerical approach. In general, for any system, there will be a temperature and pressure regime where the results output by MESMER are not reliable; however, the onset of this unreliability varies from system to system. Certain portions of the MESMER output are more resistant to numerical problems than others: for example, the product yield profiles vs. time tend to be the least reliable, since their calculation involves a great deal of arithmetic. The rate coefficients output in the Bartis-Widom analysis tend to be reliable over a relatively larger range of temperatures and pressures. The only way to systematically determine whether MESMER results are reliable is to run ME calculations over a range of temperatures and pressures, plot the results of interest, and examine where the MESMER output looks nonsensical. For example, in many applications, rate coefficients are the desired output, so the user should plot fall off curves at several temperatures, and they will see where the results numerically blow up. MESMER includes some facilities for circumventing numerical issues, and they are detailed in this manual.

**Q. Do I have to specify an atom and a bond array?**

No, MESMER requires neither an atom nor a bond array to carry out a ME calculation; these data structures are elements of the chemical markup language upon which MESMER input syntax is based, and may offer useful functionality in the future (e.g., visualizing molecular structures in Firefox).

## 8. Theoretical Background

This section is not meant to be a thorough mathematical description of the Master Equation, but rather to provide broad overview of MESMER, and provide insight into some of the less straightforward details in MESMER.

For thorough reviews of the ME mathematical development in MESMER, we refer the readers to work by Pilling and Robertson,<sup>2-4</sup> and Miller and Klippenstein.<sup>5,6</sup> Useful discussions of numerical precision issues may be found in Gannon et al.<sup>7</sup> and references therein, and some discussion of the Standard ILT is found in Davies *et al.*<sup>8</sup>

### 8.1. Matrix Formulation of the EGME

The form of the EGME in MESMER is the one dimensional ME, wherein the total rovibrational energy of the system,  $E$ , is the independent variable. Indeed, other forms of the EGME consider the time dependent evolution of the system with respect to the total  $E$  as well as angular momentum,  $J$ . However, such 2 dimensional ME treatments are restricted in their application, given the difficulty of describing the transition probabilities wherein both  $E$  and  $J$  are coupled.<sup>6</sup> They may only be used to solve the ME in the collisionless limit, or for a system that has a single isomer.<sup>9,10</sup> Thus, the bulk of ME modelling for systems under conditions of relevance to atmospheric and combustion chemistry is restricted to a one dimensional ME.<sup>6</sup>

In the mathematical formulation of the one dimensional EGME, the population of rovibrational energy levels in different isomers (denoted by subscript  $i$ ) on the potential energy surface are lumped into energy grains, characterized by an average energy,  $E_i$ , and the population in each grain,  $n_i(E_i)$ , is described by a set of coupled differential equations that account for collisional energy transfer within each isomer as well as isomerization and dissociation:

$$\begin{aligned}
\frac{dn_i(E_i)}{dt} = & \omega \int_{E_{0_i}}^{\infty} P(E_i \leftarrow E_i') n_i(E_i') dE_i' - \omega n_i(E_i) \\
& - \sum_{j \neq i}^M k_{ji}(E_i) n_i(E_i) + \sum_{j \neq i}^M k_{ij}(E_j) n_j(E_j) \\
& - k_{Pi}(E_i) n_i(E_i) + K_{Ri}^{eq} k_{Ri}(E_i) \frac{\rho_i(E_i) e^{-\beta E}}{Q_i(T)} n_R n_m \\
& - k_{Ri}(E_i) n_i(E_i)
\end{aligned}$$

**Equation 8.1**

The first term in Eq 8.1 represents the probability that  $n_i(E_i)$  is populated by collisional energy transfer via activating/deactivating bath gas collisions.  $\omega$  is the Leonard-Jones collision frequency,<sup>11</sup> and  $P(E_i \leftarrow E_i')$  is the probability that collision with bath gas will result in a transition from a grain with energy  $E_i'$  to a grain with energy  $E_i$ . The second term represents the loss from grain  $E_i$  via collisional energy transfer. The third term represents the loss from grain  $E_i$  via reaction to give other isomers, denoted by subscript  $j$ .  $k_{ji}(E_i)$  is the microcanonical rate constant for loss from isomer  $i$  to isomer  $j$ . The fourth term represents the population of grain  $E_i$  by reactions from isomer  $j$  that give isomer  $i$ , the grains  $E_i$  and  $E_j$  spanning the same range energy range. The fifth term represents the rate of loss from grain  $E_i$  via dissociation to products, with  $k_{Pi}(E_i)$ , representing the corresponding rate of loss. Because re-association of the products of unimolecular dissociation is generally negligible on an experimental time scale, dissociation to products is often treated via an infinite sink approximation – i.e., re-association is not considered. The final two terms are associated with the so-called bimolecular source term. They only apply to those wells that are populated via bimolecular association. Assuming that the reactants are thermalized via bath gas collisions in a Boltzmann distribution, and that a pseudo-first order approximation is appropriate, then the sixth term and seventh term represent the rate at which two reactants associate to populate grain  $E_i$ , and the rate of loss from that grain via re-dissociation to reactants, respectively.  $k_{Ri}(E_i)$  represents the rate constant at which  $E_i$  re-dissociates to give reactants,  $R$ , and  $K_{Ri}^{eq}$  is the equilibrium constant between isomer  $i$  and the reactants.  $Q_i(T) = \sum \rho_i(E_i) e^{-\beta E}$ , which is the rovibrational partition function for the molecular species corresponding to isomer  $i$ .

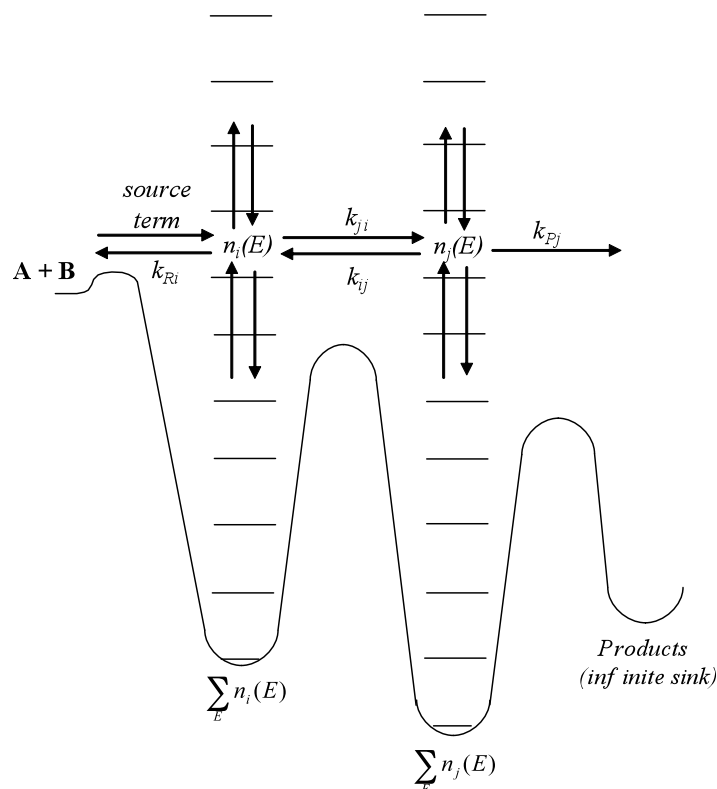


Figure 8.1: pictorial representation of Eq 1.9 for a two well system, composed of isomers  $i$  and  $j$ . The reactants,  $A + B$ , are connected to isomer  $i$ , and the product channel is connected to isomer  $j$ . This figure shows the terms of Eq 1.9 as they apply to one grain within each isomer,  $n_i(E_i)$  and  $n_j(E_j)$ .

Microcanonical rate coefficients for the unimolecular reactions that occur in each energy grain,  $k(E)$ , are calculated from the PES data pertaining to the reagents and transition states via the microcanonical transition state/RRKM theory expression:

$$k(E) = \frac{W(E)}{h\rho(E)}$$

**Equation 8.2**

where  $W(E)$  is the sum of rovibrational states at the optimized transition state geometry and  $\rho(E)$  is the density of rovibrational states of the isomer. As discussed by Baer and Hase,<sup>12</sup> RRKM theory depends on the assumption that the total phase space of a molecule is populated statistically. That is, the population density of molecules in phase space may be assumed uniform as the molecule moves from the reactant phase space, through the transition state dividing surface, and into the product phase space. This is equivalent to assuming that all regions of phase space are available to the molecule – i.e., that the time scale for energy randomization within a molecule is very fast with respect to the rate of reaction, maintaining a microcanonical ensemble. This common simplification, which is fundamental to ME analysis, is called the ergodicity assumption.

Eq. 8.2 is applicable for transition state dividing surfaces located at constrained geometry with a well defined energetic barrier. When the reaction in question is barrierless, a first principles determination of  $k(E)$  requires a variational approach – i.e., the  $k(E)$ s are calculated by minimizing  $W(E)$  on the PES in question. An alternative approach, available in Mesmer, is to calculate the  $k_{Ri}(E)$ s for barrierless association reactions using an inverse Laplace transform (ILT).<sup>8,13</sup> With this technique, the microcanonical dissociation  $k_{Ri}(E)$ s are determined from experimental measurements of the temperature dependent high pressure limiting rate coefficient for association. The microcanonical rate coefficients for dissociation are then determined via detailed balance, as indicated in Eq 8.1.

In general, the transition probabilities for collisional energy transfer in the EGME utilize the exponential down model:

$$P(E \leftarrow E') = C(E') \exp\left(-\frac{E'-E}{\langle \Delta E_d \rangle}\right)$$

**Equation 8.3**

where  $E' > E$ ,  $C(E')$  is a normalization constant, and  $\langle \Delta E \rangle_d$  is the average energy transferred per collision in a downward direction. The transition probabilities for energy transfer in the upward direction may be obtained from those calculated with Eq. 8.3 via detailed balance. The exponential down model is the most commonly used for describing collisional energy transfer probabilities. It derives from scattering theory, and it reflects the common sense notion that collisions which transfer lots of energy are less probable than those that transfer small amounts of energy.<sup>3</sup> Other models with different transition probability distributions have been proposed,<sup>3</sup> such as Gaussian models<sup>11</sup> and double exponential models.<sup>6</sup> In a recent review of ME techniques by Miller and Klippenstein, they noted that classical trajectory simulations as well as experimental data suggest that the exponential down model is not the most accurate for describing collisional transition probabilities.<sup>6</sup> Models that produce longer tails are more accurate.

However, nearly all those ME models published in the literature utilize the single exponential down model. Other functions, such as a double exponential model, feature more parameters and systematic techniques for assigning parameter values have not been established. Additionally, given the extensive use of the single exponential down model in the literature, a set of typical  $\langle \Delta E \rangle_d$  values has emerged. For example, at room temperature He bath gas tends to have  $\langle \Delta E \rangle_d$  values from  $\sim 120$ - $175 \text{ cm}^{-1}$ , while  $\text{O}_2$  and  $\text{N}_2$  bath gases tend

to have  $\langle \Delta E \rangle_d$  values of  $\sim 175\text{-}275\text{ cm}^{-1}$ . In general,  $\langle \Delta E \rangle_d$  is left as a variable parameter determined by fitting to experimental data, within the limits given above, and it usually shows a slight positive temperature dependence in one dimensional ME analyses. The origin of the temperature dependence is not entirely clear, although Miller suggested that this temperature dependence may correspond to rotational excitation.<sup>10</sup> Classical Trajectory calculations have identified the dependence of  $\langle \Delta E \rangle_d$  on the angular momentum of the target molecule. Experimentally, higher temperatures correspond to higher angular momentum states, and in the 1d ME, this is manifest as an effective increase in  $\langle \Delta E \rangle_d$ .<sup>10</sup>

The collisional energy transfer models discussed above assume that the transition probability depends only on the energy of the molecule, and not on its collisional history or its configuration. The same is true of the RRKM  $k(E)$ s. Such processes, where the probability that a system occupies a particular state, depends only on the immediately previous state and nothing else, are common to many fields, and are referred to as Markov processes. Indeed, the ME in Eq 8.1 is a stochastic differential equation of the Markov type, and its relationship with the more general field of stochastic probability theory has been discussed in detail by Pilling and Robertson.<sup>3</sup> So long as the time that a pair of molecules spends in collision is much smaller than the time between collisions, the molecular configuration in a collision event should not depend on the molecular configuration in the previous collision event. The energy transferred in a collision may be treated as depending only on the internal energy of the molecule, and not its configurational history – i.e., collisional energy transfer may be treated as a ‘random walk’, and the ME is applicable.<sup>3,6</sup>

The coupled stochastic differential equations represented by Eq 8.1 may be reformulated as:

$$\frac{d\mathbf{n}}{dt} = \mathbf{M}\mathbf{n}$$

**Equation 8.4**

where  $\mathbf{n}$  is a vector containing the populations of the grains for each isomer,  $n_i(E_i)$ , and  $\mathbf{M}$  is the matrix that describes collisional energy transfer, as well as reactive loss and gain for each grain. In the case where Eq 8.1 includes a pseudo-first order bimolecular source term to describe the fractional rates of population of the entrance well by the reactants, then the final element of  $\mathbf{n}$  corresponds to the time dependent population of the reactant that is not in



excess. Eq. 8.4 shows that the solution of the ME has been reduced to a standard eigenvalue problem, and diagonalization of  $\mathbf{M}$  yields the corresponding eigenpair solutions.

Approximating the time dependent grain populations as sums of exponential functions, and combining the eigenpair solutions with the appropriate initial conditions vector for describing the system at time zero, the time dependent grain evolution for a particular isomer,  $n_{iE}(t)$ , may be obtained. The solution to the ME may be written as:

$$\mathbf{n}(t) = \mathbf{U}e^{\mathbf{\Lambda}t}\mathbf{U}^{-1}\mathbf{n}(0)$$

**Equation 8.5**

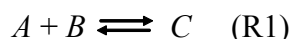
where  $\mathbf{n}(0)$  contains the initial conditions (i.e., at  $t = 0$ ) for each grain (i.e.,  $n_{iE}(0)$ ),  $\mathbf{U}$  is matrix of eigenvectors obtained from diagonalization of  $\mathbf{M}$ , and  $\mathbf{\Lambda}$  is a diagonal matrix of the corresponding eigenvalues. For a conserved system (i.e., one for which the previously discussed ‘infinite sink’ approximation has not been introduced) with  $S$  different chemical configurations (or wells), there will be  $S$  eigenpairs that are substantially smaller in absolute magnitude (i.e., they are less negative) than the other eigenvalues. The first eigenvalue, often referred to as  $\lambda_0$ , will be equal to zero, and the corresponding eigenvector gives the equilibrium Boltzmann distributions of the different isomers on the PES.<sup>3,5</sup> For systems that utilize the infinite sink approximation, diagonalization of  $\mathbf{M}$  does not yield an eigenvalue equal to zero. In both cases the  $S$  eigenvalues are often referred to as the ‘chemically significant’ eigenvalues. Along with their corresponding eigenvectors, they describe the time evolution of the system as it approaches equilibrium. The ‘chemically significant’ eigenvalues are those that determine the experimentally observed phenomenological rates measured in kinetics experiments, since they describe reaction and interconversion between the different molecular configurations on the PES.<sup>5,14</sup> The remaining eigenvalues – those that are much more negative than the chemically significant eigenvalues – correspond to collisional relaxation on very short time scales, and have been referred to as the internal energy relaxation eigenvalues (IERE).<sup>2,6</sup>

The formulation of the EGME in terms of grains essentially corresponds to expanding the solutions of Eq. 8.4 in a basis of delta functions whose maxima lie at the centre of the grain. In general, chemical problems are not concerned with the time evolution of every single grain for a particular isomer. However, in order to obtain a convergent solution to Eq. 8.4, the grain size must be smaller than  $\langle \Delta E \rangle_d$ , which is generally less than  $\sim 150 \text{ cm}^{-1}$ .

In general objective of EGME calculations is to determine quantities that can be compared to experimental observables: product yields, branching ratios, and rate coefficients, all of which may be a function of temperature and / or pressure. Extracting information about product yields and branching ratios is straightforward. The solutions to the EGME yield multi-exponential functions that describe the time dependent population in each grain, and the grain populations are normalized to unity,<sup>5</sup> and time dependent species concentrations are calculated by summing over the appropriate grains. Extraction of phenomenological rate coefficients from the EGME solution for comparison with experimentally determined rate coefficients is less straightforward. In general, two methods have been proposed: one by Klippenstein and Miller,<sup>9</sup> and one by Bartis and Widom.<sup>2,17</sup> In the Klippenstein and Miller approach, the rate coefficient for a particular transformation on the PES is obtained by taking the time derivative of the exponential expression that describes the  $n_{iE}(t)$  terms for a particular isomer. The Bartis and Widom approach defines a set of rate coefficients based on the eigenpairs of the chemically significant eigenvalues.

### 8.1.1. The Bimolecular Source Term

Provision is made within MESMER for bimolecular reaction to act as a source term, the main restriction being that one of the reactants must be in excess, so that the bimolecular association occurs under pseudo-first order conditions. A generic association reaction



has a forward association rate constant,  $k_a$ , and a backward dissociation rate constant,  $k_d$ . If the conditions are such that  $[A] \gg [B]$ . Then, R1 can be represented as,



where the forward rate coefficient is  $k_a' = k_a A$ . At equilibrium, the forward and reverse rates are equal:

$$k_a' x_B = k_d x_C$$

**Equation 8.6**

where  $x_B$  and  $x_C$  are the equilibrium fractions of  $B$  and  $C$ . Both  $k_a'$  and  $k_d$  are canonical rates of reaction and so depend on temperature, but each is related to their corresponding microcanonical rate constants  $k(E)$ . The canonical dissociation rate,  $k_d$ , is related to the microcanonical dissociation rates,  $k_d(E)$ s, through the following relation:

$$k_d = \sum_E k_d(E) \frac{f(E)}{Q_C}$$

**Equation 8.7**

where  $f(E) = \rho(E) \exp(-\beta E)$  and  $Q_C = \sum_E f(E)$ . Substituting Eq. 8.7 into 8.6 gives,

$$k_a' x_B = \sum_E k_d(E) \frac{f(E)}{Q_C} x_C$$

**Equation 8.8**

where the term  $f(E)x_C / Q_C$  represents the equilibrium fraction in grain  $E$  of adduct  $C$ , and the  $k_d(E)$ s in (3) describe transition from grains in  $C$  to the bimolecular source species,  $B$ .

In implementing the bimolecular source term, the symmetrization master equation matrix that follows detailed balance is exploited. The asymmetric ME transition matrix,  $\mathbf{M}$ , would require two sets of microcanonical rate coefficients: one that describes the transition from the bimolecular source term  $B$  to grains in  $C$ , and one that describes the transition from grains in  $C$  to the bimolecular source term  $B$ . However, for the symmetric matrix  $\mathbf{S}$ , we only need to calculate the  $k_d(E)$ s because the rows and columns that correspond to transition between grains in  $C$  and the source term  $B$  are identical by detailed balance. In general, matrix elements of the symmetric matrix  $\mathbf{S}$  are related to the matrix elements of the asymmetric matrix  $\mathbf{M}$  as follows:

$$S_{ij} = M_{ij} \left( \frac{f_j}{f_i} \right)^{1/2} = M_{i \leftarrow j} \left( \frac{f_j}{f_i} \right)^{1/2}$$

**Equation 8.9**

where  $f_i$  and  $f_j$  are the equilibrium fractions in grains  $i$  and  $j$ , respectively, and  $M_{ij}$  has been rewritten as  $M_{i \leftarrow j}$  to emphasize that matrix element  $M_{ij}$  corresponds to the transition  $i \leftarrow j$  in the physical system. If we say that a well has  $N$  grains, then the row and column that correspond to transitions involving the bimolecular source term have index  $N+1$ . The  $\mathbf{S}$  matrix elements for transition between grains in  $C$  and the bimolecular source term,  $B$ , are calculated using Eq. 8.9 and recognizing that the  $M_{i \leftarrow j}$  matrix elements are equivalent to  $k_d(E)$ . It follows that  $f_j$  is equivalent to the expression in (4), and  $f_i$  is equivalent to  $x_B$ . Substituting these into Eq. 8.9, we obtain the matrix elements in  $\mathbf{S}$ , which run from  $E = 1$  to  $E = N$ :

$$S_{N+1,E} = S_{E,N+1} = k_d(E) \left( \frac{f(E)}{Q_C} x_C \cdot \frac{1}{x_B} \right)^{1/2}$$

#### Equation 8.10

The final matrix element,  $S_{N+1,N+1}$ , is simply  $k_a'$ , the pseudo first order canonical loss rate constant of the bimolecular source term to all the grains in  $C$ . If the (R2) equilibrium constant,  $K$ , is as follows:

$$K = \frac{k_a'}{k_d} = \frac{x_C}{x_B}$$

#### Equation 8.11

then  $k_a'$  may be obtained by rearranging Eq. 8.11:

$$k_a' = K \cdot k_d = S_{N+1,N+1}$$

#### Equation 8.12

## 8.2. Other Methods for solving the master equation

### 8.2.1. The Reservoir State Approximation

This method assumes that significant portions of low energy molecular phase space are in a Boltzmann distribution throughout the course of the reaction. It is usually appropriate for grains which are more than a few  $kT$  below the lowest reaction threshold, and when the rate of collisional deactivation is faster than the rate of reaction (which is usually the case at moderate pressures). We have done extensive testing of the reservoir state approximation, and shown that it gives results nearly identical to the full ME over a range of conditions. The reservoir state approximation does not eliminate numerical problems *per se*, but it significantly truncates the size of the matrix that must be diagonalized. Since the bulk of a MESMER calculation is tied up in matrix diagonalization, the reservoir state approximation results in a far more efficient calculations (up to a factor of 30 faster!), especially in conjunction with increased precision arithmetic using the QD libraries installed with MESMER.

The reservoir state can be formulated by analogy with the treatment of the bimolecular source term. In effect, the bimolecular source term represents a collection of grains that are represented with one grain because we assume that these grains are always thermalized throughout the reaction. This saves computational effort, reducing the size of the matrix that we need to diagonalize, and it has been shown to be a good assumption so long as the reactants are thermalized throughout the course of the reaction. As long as the frequency of

non reactive collisions is substantially higher than the frequency of reactive collisions, then assuming the reactants to be thermalized is a good approximation.

By analogy then, a reservoir state approximation will be good when the transition probabilities between the reservoir and the high energy grains are very small. In this case, the probability of deactivating collisions will be much greater than activating collisions. Because we are often interested in using the ME to model pressure dependence in the region of a barrier, a reservoir state that lies significantly below the barrier should not significantly affect the stochastic behaviour that happens in the region of the barrier, and which is generally the behaviour that we want to capture with a ME treatment.

The implementation of the reservoir state is very similar to that of the bimolecular source term. There are only a few differences, and the problem is slightly simplified because the process is unimolecular, so concerns about pseudo first order conditions do not arise. Using the same scheme used to describe the bimolecular source term, we imagine the following reaction:



where the forward rate constant,  $k_a$ , now represents the rate constant for activation from the reservoir state,  $B$ , into the active state,  $C$ . The backward rate constant,  $k_d$ , represents the rate constant for deactivation from the active state,  $C$ , into the reservoir state,  $B$ . At equilibrium, the forward and reverse rates are equal:

$$k_a x_B = k_d x_C$$

**Equation 8.13**

where  $x_B$  and  $x_C$  are the equilibrium fractions of  $B$  and  $C$ . Both  $k_a$  and  $k_d$  are canonical rates of reaction and so depend on temperature, but each is related to their corresponding microcanonical rate constants  $k(E)$ . The canonical deactivation rate,  $k_d$ , is related to the microcanonical dissociation rates,  $k_d(E)$ , through the following relation:

$$k_d = \sum_E k_d(E) \frac{f(E)}{Q_C}$$

**Equation 8.14**

where  $f(E) = \rho(E) \exp(-\beta E)$  and  $Q_C = \sum_E f(E)$ . Now, the most significant difference between treating the bimolecular source term and the reservoir state is how we treat the  $k_d(E)$ s. In our treatment of the bimolecular source term, the  $k_d(E)$ s are obtained from either

ILT or RRKM methods. But we cannot use these techniques for treating  $k_d(E)$ s into the reservoir state. The  $k_d(E)$ s in this case correspond to the energy dependent rates at which species in grains within  $C$  are deactivated into the state  $B$ , and the most straightforward way to do this is to use the downward collision transition probabilities. To get the  $k_d(E)$ s for deactivation from a grain in  $C$  into the reservoir  $B$ , we must sum the normalized downward transition probabilities,  $P(i \leftarrow E)$ , for deactivation of a particular grain in  $C$  into *every possible grain in  $B$* . If  $B$  spans the energy range from  $E_0$  to  $E_t$  and  $C$  spans the energy range from  $E_{t+1}$  to  $E_\infty$ , then the  $k_d(E)$ s for deactivation of a grain  $E$  in  $C$  to the reservoir state  $B$  may be calculated as follows:

$$k_d(E) = \sum_{i=E_0}^{i=E_t} P(i \leftarrow E) \omega$$

**Equation 8.15**

where  $\omega$  is the collision frequency. Substituting Eq. 8.14 into 8.13, we obtain:

$$k_a x_B = \sum_E k_d(E) \frac{f(E)}{Q_C} x_C$$

**Equation 8.16**

where the term  $f(E)x_C / Q_C$  represents the equilibrium fraction in grain  $E$  of the active state  $C$ , and the  $k_d(E)$ s are calculated according to Eq. 8.15. Similar to the case of the bimolecular source term, the asymmetric ME transition matrix,  $\mathbf{M}$ , requires microcanonical rate coefficients that describe the transition from the reservoir  $B$  to the grains in  $C$ . In practice though, we only need to calculate the  $k_d(E)$ s because the symmetrized ME matrix,  $\mathbf{S}$ , needs to have reservoir row and column vectors that are identical by detailed balance (see Eq. 8.9). Let's say that an active state,  $C$ , has  $N$  grains, so that the row and column that correspond to transitions with the reservoir state have index  $N+1$ . Recognizing that the  $M_{i \leftarrow j}$  matrix elements are equivalent to  $k_d(E)$ , then  $f_j$  is equivalent to  $f(E)x_C / Q_C$  and  $f_i$  is equivalent to  $x_B$ . Substituting these into Eq. 8.9, we obtain the matrix elements in  $\mathbf{S}$  for the reservoir state, which run from  $E = 1$  to  $E = N$ :

$$S_{N+1,E} = S_{E,N+1} = k_d(E) \left( \frac{f(E)}{Q_C} x_C \cdot \frac{1}{x_B} \right)^{1/2}$$

**Equation 8.17**

The final matrix element,  $S_{N+1,N+1}$ , is simply  $k_a$ , the canonical loss rate constant of the reservoir state to all the activated grains in  $C$ . If the (R3) equilibrium constant,  $K$ , is as follows:

$$K = \frac{k_a}{k_d} = \frac{x_C}{x_B}$$

**Equation 8.18**

then  $k_a$  may be obtained by rearranging Eq. 8.18:

$$k_a = K \cdot k_d = S_{N+1,N+1}$$

**Equation 8.19**

### 8.2.2. The Contracted Basis Set Approach

The aim of this approach is to reduce the size of the final matrix to be diagonalized by using a basis set that is constructed from the diagonalization of the individual isomer collision energy transfer operators. It is similar in spirit to the basis set methods reported by Venkatas *et al*<sup>15,16</sup>. Consider a two well isomerization system, after symmetrization the overall master equation matrix can be written as:

$$\mathbf{M} = \begin{pmatrix} \omega(\mathbf{S}_A - \mathbf{1}) & \mathbf{0} \\ \mathbf{0} & \omega(\mathbf{S}_B - \mathbf{1}) \end{pmatrix} + \begin{pmatrix} -\mathbf{K}_A & \mathbf{K}' \\ \mathbf{K}' & -\mathbf{K}_B \end{pmatrix}$$

**Equation 8.20**

where the first term on the right hand side represents the collisional activation/deactivation process and the second represents reactive exchange. The first term on the right hand side is clearly block diagonal and each of these blocks can be diagonalized independently, generating two sets of eigenvectors which are orthogonal within each set and between each set. If these sets are denoted  $\mathbf{U}_A$  and  $\mathbf{U}_B$ , then the combined eigenvector matrix for the first term on the right hand side can be written as:

$$\mathbf{U} = \begin{pmatrix} \mathbf{U}_A & \mathbf{0} \\ \mathbf{0} & \mathbf{U}_B \end{pmatrix}$$

**Equation 8.21**

This matrix can then be applied as a similarity transform to give:

$$\mathbf{M}' = \mathbf{U}\mathbf{M}\mathbf{U}^T = \begin{pmatrix} \mathbf{\Lambda}_A & \mathbf{0} \\ \mathbf{0} & \mathbf{\Lambda}_B \end{pmatrix} + \begin{pmatrix} \mathbf{U}_A & \mathbf{0} \\ \mathbf{0} & \mathbf{U}_B \end{pmatrix} \begin{pmatrix} -\mathbf{K}_A & \mathbf{K}' \\ \mathbf{K}' & -\mathbf{K}_B \end{pmatrix} \begin{pmatrix} \mathbf{U}_A^T & \mathbf{0} \\ \mathbf{0} & \mathbf{U}_B^T \end{pmatrix}$$

Equation 8.22

While all the eigenvectors have been used in the above, this is not always necessary to converge the smallest eigenvalues of the overall matrix. If a smaller set is sufficient then there is the potential for increased computational efficiency.

### 8.3. Inverse Laplace Transform

As discussed above the inverse Laplace transform (ILT) can be used to obtain microcanonical rate coefficients from an existing Arrhenius form. In the following an outline is given of how the methods are applied for both the case where the Arrhenius form is for a unimolecular dissociation and for a bimolecular association.

#### 8.3.1. Unimolecular ILT

The canonical high pressure rate coefficient may be expressed as:

$$k(\beta) = \frac{1}{Q(\beta)} \int_0^{\infty} k(E) \rho(E) \exp(-\beta E) dE$$

Equation 8.23

Where  $\rho(E)$  is the rovibrational density of states and  $Q(\beta)$  is the corresponding canonical partition function. Substituting Eq. 8.2 for  $k(E)$  gives,

$$k(\beta) = \frac{1}{hQ(\beta)} \int_0^{\infty} W(E) \exp(-\beta E) dE$$

Equation 8.24

and rearranging gives:

$$hQ(\beta)k(\beta) = \int_0^{\infty} W(E) \exp(-\beta E) dE = L[W(E)]$$

Equation 8.25



$W(E)$  can be recovered by inverting the Laplace transform,

$$hL^{-1}[Q(\beta)k(\beta)] = W(E)$$

**Equation 8.26**

If  $k(\beta)$  can be represented by the modified Arrhenius expression:

$$k(\beta) = A_0 \left( \frac{\beta_0}{\beta} \right)^n \exp(-\beta E_a)$$

**Equation 8.27**

it follows that:

$$\frac{W(E)}{h} = k(E)\rho(E) = A_0 \beta_0^n L^{-1} \left[ Q(\beta) \frac{1}{\beta^n} \exp(-\beta E_a) \right]$$

**Equation 8.28**

Further progress can be made by applying the convolution theorem:

$$L^{-1}[Q(\beta)G(\beta)] = q \otimes g$$

**Equation 8.29**

where  $Q$  and  $q$ , and  $G$  and  $g$  are transform pairs, and  $\otimes$  denotes convolution. Solution of 8.25 is possible by recognizing that

$$L^{-1}[Q(\beta)] = \rho(E)$$

**Equation 8.30**

$$L^{-1}[G(\beta)] = L^{-1} \left[ \frac{1}{\beta^n} \exp(-\beta E_a) \right] = \frac{(E - E_a)^{n-1} u(E - E_a)}{\Gamma(n)}$$

**Equation 8.31**

where  $u(E - E_a) = 0$  if  $E < E_a$  and is unity otherwise, is the Heavyside step function.

Convolving the ILTs in Eq. 8.30 and 8.31 gives:

$$k(E)\rho(E) = \frac{A_0 \beta_0^n}{\Gamma(n)} \int_0^E d\tau \rho(E - \tau) (\tau - E_a)^{n-1} u(\tau - E_a)$$

### Equation 8.32

The units should be input follows:

$\rho(E)$  in states per  $\text{cm}^{-1}$

$A_0$  in molecules  $\text{cm}^{-3} \text{s}^{-1}$

$\beta_0 = \frac{1}{k_B T_0}$  where  $k_B$  is in units of  $\text{cm}^{-1} \text{K}^{-1}$ , and  $T_0$  in units of K.

### 8.3.2. The association ILT

A similar expression can be obtained for the case where the Arrhenius expression is for the high pressure association rate coefficient. Under high pressure conditions the forward and the reverse rate coefficients are related by the equilibrium constant as indicated by eq. 8.11:

$$k_d(\beta) = K_e(\beta)k_a(\beta)$$

### Equation 8.33

If  $k_a(\beta)$  has a modified Arrhenius form then the equivalent expression to Eq. 8.28 is,

$$\frac{W(E)}{h} = k(E)\rho(E) = A_0\beta_0^n L^{-1} \left[ Q(\beta)K_e(\beta) \frac{1}{\beta^n} \exp(-\beta E_a) \right]$$

### Equation 8.34

Solution of Eq. 8.34 is complicated by the appearance of translational degrees of freedom in the equilibrium constant, but otherwise proceeds as previously by exploiting the convolution theorem. The final result is (see ref. 8),

$$k(E)\rho(E) = \frac{A_0\beta_0^n}{\Gamma(n+1.5)} \left( \frac{2\pi\mu}{h^2} \right)^{3/2} \left( \frac{g_A g_B}{g_C} \right) \int_0^E d\tau \rho_R(E-\tau) (\tau - E_a - \Delta H_0^0)^{n+0.5} u(\tau - E_a - \Delta H_0^0)$$

### Equation 8.35

where  $\rho_R(E)$  is the convolved density of states for the associating pair,  $\Delta H_0^0$  is the enthalpy of reaction,  $\mu$  is the reduced mass of the system and  $g_X$  is the spin degeneracy of species X.

### 8.3.3. The C' constant in MESMER ILT

The constant C' that occurs in the MESMER implementation of ILT follows from the translational partition function,

$$Q_t = \left( \frac{2\pi mkT}{h^2} \right)^{3/2} V$$

**Equation 8.36**

where all quantities are in standard SI units. For ease of computation, it is useful to re-write  $Q_t$  in terms of the molar mass M (g/mol) and the reciprocal temperature  $\beta'$  expressed in wave numbers:

$$m = M / 10^3 L$$

**Equation 8.37**

$$\beta' = hc/kT$$

**Equation 8.38**

where L is Avogadro's number and c is the speed of light expressed in cm/s, otherwise all quantities are in SI units. Inserting Eqs. 8.37 and 8.38 into 8.36 gives:

$$Q_t = \left( \frac{2\pi Mc}{10^3 L \beta' h} \right)^{3/2} V = C \left( \frac{M}{\beta'} \right)^{3/2} V$$

**Equation 8.39**

where C is given by

$$C = \left( \frac{2\pi c}{10^3 L h} \right)^{3/2}$$

**Equation 8.40**

Volume is more conveniently expressed in cm<sup>3</sup> and to account for this Eq. 8.40 can be written as

$$Q_t = C' \left( \frac{M}{\beta'} \right)^{3/2} V'$$

**Equation 8.41**

where  $V'$  is the volume now expressed in cm<sup>3</sup> and the constant  $C'$  is given by,

$$C' = \left( \frac{2\pi c}{10^3 L h} \right)^{3/2} 10^{-6} = \left( \frac{2\pi c}{10^7 L h} \right)^{3/2}$$

**Equation 8.42**

Substituting in  $L = 6.02205 \times 10^{23}$ ,  $h = 6.62618 \times 10^{-34}$  Js and  $c = 2.997925 \times 10^{10}$  cm/s gives  $C' = 3.2433 \times 10^{20} \text{ (mol/g/cm)}^{3/2}$ . This is the value that is defined in the `constant.h` file of the MESMER source code.

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