

2106

(Syeeda shabnam miss)
(7A01/N)

Mark Distribution

Attendance → 10 %.

Mid Implementation → 20 %. (individual exam)

Final (written) → 20 %.

Viva + report → 10 %.

Viva + Experiments → 40 %.

* Lab Report → Every week

→ has viva

→ will be submitted at the end

* Attendance → V. V. Imp

* No partial marking

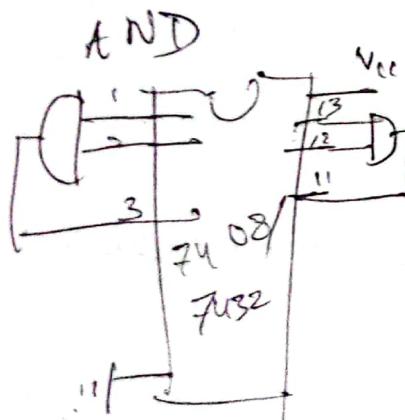
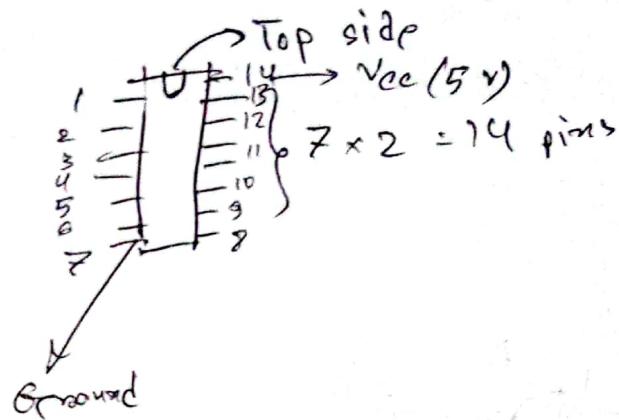
Microprocessors Data handbook

- BCB publication

7408 - AND

7432 - OR

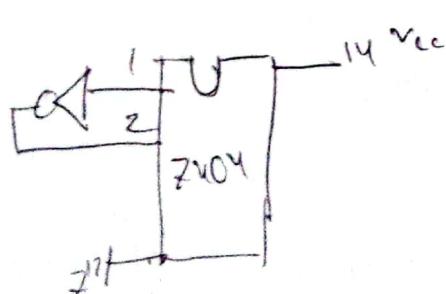
7404 - NOT



| | | AND | OR |
|---|---|---------|---------|
| A | B | 0 0 → 0 | 0 0 → 0 |
| 0 | 1 | 0 1 → 0 | 0 1 → 1 |
| 1 | 0 | 1 0 → 0 | 1 1 → 1 |
| 1 | 1 | 1 1 → 1 | 1 1 → 1 |

+ Four < 4, you need
1 IC

AND



| | | NOT |
|---|--|-------|
| A | | 0 → 1 |
| 1 | | 0 |

OR

NOT

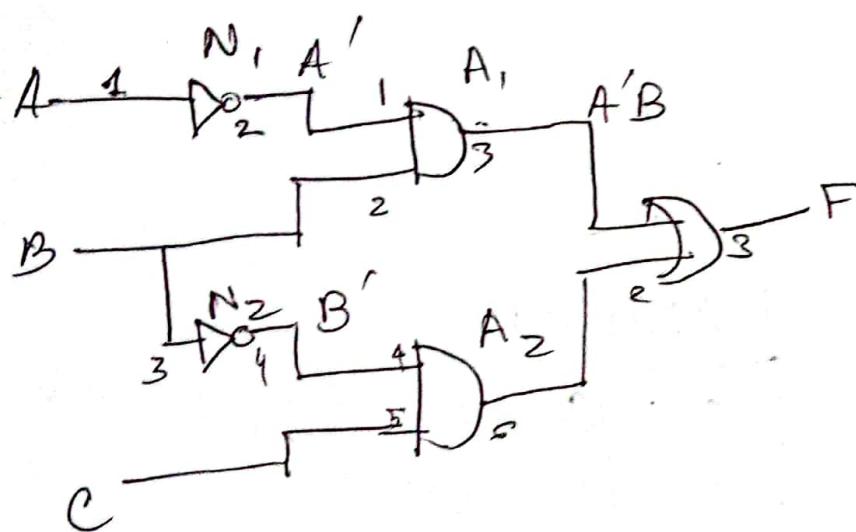
XOR

XNOR

Simplification

i) $F = A'B + B'C$

ii) $F = (A+B)'C + (B+C')' + AC'$



Report:

- Expt No.:
- Expt Name
- Objective:
- Truth table
- Circuit Diagram
- Required ICs
- IC pins connections in the diagram
- Discussion
(Obj & rev)

theory first → have later

Require: NOT gate (\neg icon)

i) N_1 → NOT gate (code)
Quantity - 1

ii) N_{Name} → Create name (code)

Quantity -

(Syeda Shabnam Miss)

Syeda

"Easiest one" ~ Miss

"Either a lot of marks or none"

"Conceptual course"

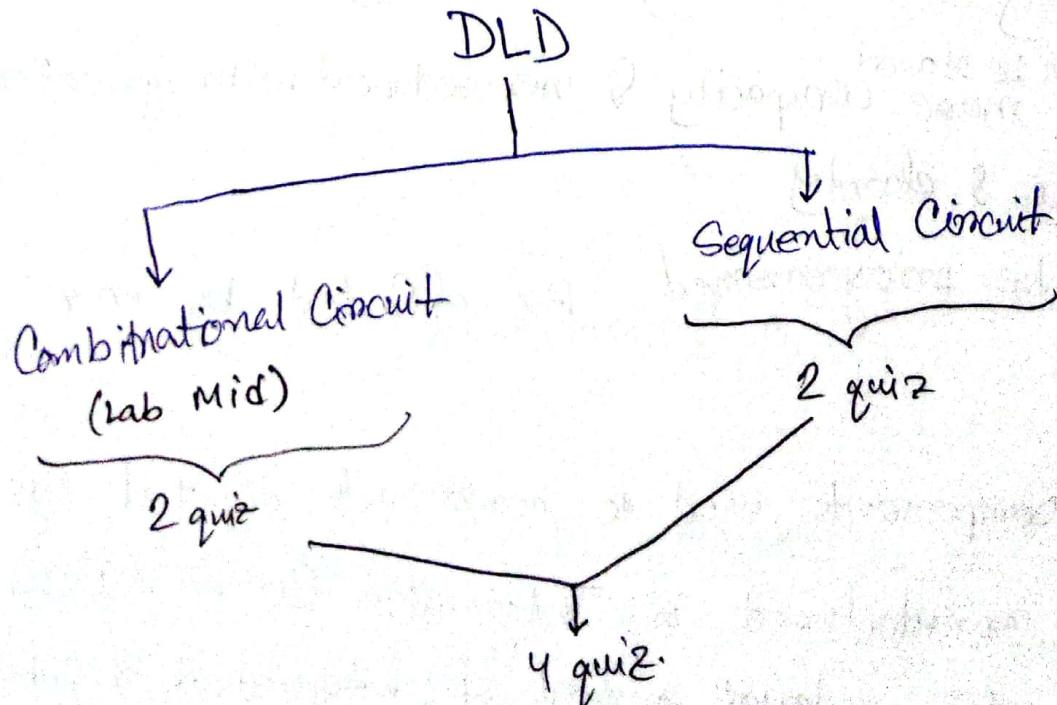
"Fully hardware related course"

"DLD is a core course"

"Book is essential. Slides are complementary"

Digital Design with an Introduction to the Verilog HDL
 (5th Edition). Mario, Michael D Ciletti
 ~ M. Monnais

*Strategy: Class → Book → Slide
 (Office) (Home) (Before next class)



* Lecture - 1:

□ Electronic Circuits can be divided into two categories:

i) Digital electronics :

- involves quantities with discrete values.

ii) Analog electronics :

- involves quantities with continuous values.

The term 'digital' is derived from the way computers performs operations; by computing digits.

□ Advantages:

- Digital data can be processed & transmitted more efficiently and reliably than Analog data.
- can be stored more compactly & reproduced with greater accuracy & clarity
- can be programmed, less effected by noise.

□ IC:

The components used to construct digital systems are manufactured in Integrated Circuit (IC) form.

- . IC contains a large amount of integrated digital circuit within a single package.

- **SST** (Small scale Integration)
- **MSI** (medium " ")
- **LST** (Large " ")
- **VLSI** (Very " " ")

Digital Logic is concerned with the interconnection among digital components and modules & is a term used to denote the design and analysis of digital systems.

* Digital electronics involves circuits & systems in which there are 2 possible states:

These states are presented by 2 different voltage levels;
A HIGH and Low LOW.

* In digital systems, combinations of 2 states called **codes**

* The 2 state number system is called **Binary** & its two digits are 0 and 1. A binary digit is called a bit

* HIGH = 1 = ON
LOW = 0 = OFF] v.v. Imp

The voltages used to represent a 1 & 0 are called logic gates.

* 0 - 15 in binary is **ESSENTIAL**

* Self Study (Important for i) Lateral II ii) Exams)

- Binary, Octal, Decimal, Hexadecimal number systems
- Conversion
- Binary addition, subtraction, multiplication
- 1's complement, 2's complement

Lect - 2 - 3: Introductionto Basic Logic operations& basic digital circuits* 1-2 mark question* Logic/Digital/switching circuit(ckt):

A ckt whose input & output ~~suz~~ signals are 2 state, either low or high voltages. The basic logic ckt's are

OR, AND and NOT gates.

* Basic because all gates can be derived logic ends up here when broken down

* Gate:

A logic ckt with one or more input signals but only one output signal (logic 1 or 0)

* Truth Table:

Shows all inputs & output possibilities for a logic ckt. The input words are listed in binary progression.

* Word:

A string of bits that represent a coded instruction or data

* For n inputs, there are 2^n input combinations. Truth table contains one row for each one input combination.

* The AND, OR, NOT, NAND, NOR, XOR, XNOR, operations: self study

* Universal Gates:

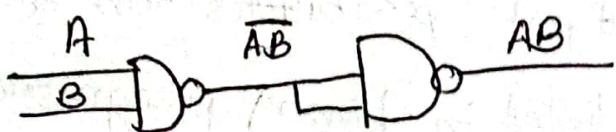
The only one type gates, either NAND or NOR are sufficient for the realization of any logical expression. For this, NAND and NOR gates are known as universal gates.

Realization of basic logic operations using NAND gate:

NOT operation:

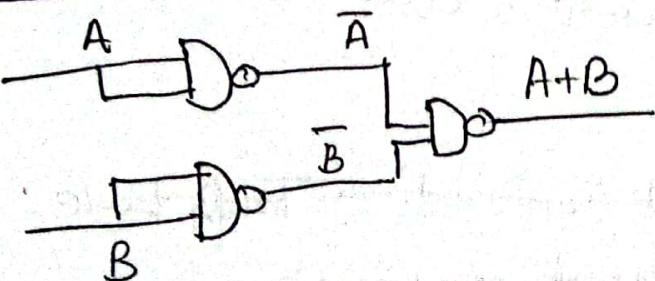


AND operation:



AND operation:

OR operation:



→ Realization of basic logic operations using NOR: self study

some formation

juts AND and OR are flipped

* NOR : Self Study

* Make operation \rightarrow using
NAND/NOR \rightarrow just draw
some figure

* XOR for more than two inputs:

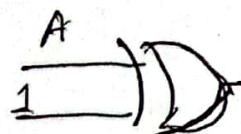
- If total no. of '1' is odd,
output will be 1

- If total no. of '1' is even,
output will be 0

opposite for X-NOR

* Realization of NOT operation:

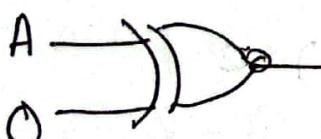
→ Using X-OR gate



$A \oplus B = \bar{A}B + A\bar{B} = \bar{A} \cdot 1 + 1 \cdot A = \bar{A}$; [the 1 makes it either odd or even]

$$A \oplus B = \bar{A}B + A\bar{B} = \bar{A} \cdot 1 + 1 \cdot A = \bar{A}$$

→ Using X-NOR gate:



$A \oplus B = A \odot B = A'B' + AB = \bar{A} \cdot 0 \cdot 1 + \bar{A} \cdot 0 = \bar{A}$; [the 0 makes it odd or even]

$$A \oplus B = A \odot B = A'B' + AB = \bar{A} \cdot 0 \cdot 1 + \bar{A} \cdot 0 = \bar{A}$$

Lecture 4 : Boolean Algebra and Logic

Simplification

* Introduced by George Boole in 1854

* Boolean : which is always either true or false.

Variable:

A symbol used to represent a logical quantity.

Any single variable can have a 1 or 0 value.

Complement:

The inverse of a variable which is indicated by a bar over the variable. If $A = 1$, then $\bar{A} = 0$

Literal:

A variable or its complement

* Laws of Boolean Algebra: + writing only once is enough

Commutative Law:

$$a) A + B = B + A$$

$$b) AB = BA$$

Associative Law:

$$a) A + (B + C) = (A + B) + C$$

$$b) A(BC) = (AB)C$$

* Distributive Law:

a) $A(B+C) = AB+AC$

b) $A+BC = (A+B)(A+C)$

* De-Morgan's Law/Theorem:

i) $\overline{AB} = \overline{A} + \overline{B}$ ii) $\overline{A+D} = \overline{A} \cdot \overline{B}$

* See the proofs from book

* Rules of Boolean Algebra:

1. $A+0 = A$

3. $A \cdot 0 = 0$

5. $A+A = A$

2. $A+1 = 1$

4. $A \cdot 1 = A$

6. $A+\overline{A} = 1$

7. $A \cdot A = A$

9. $\overline{\overline{A}} = A$

11. $A+\overline{A}B = A+B$

8. $A \cdot \overline{A} = 0$

10. $A+AB = A$

12. $(A+B)(A+C) = A+BC$

* See proofs

* Dual: Operators (AND/OR) & fixed values (1/0) will be changed but variables and NOT operator will ^{not} be changed AND and OR will change into each other. For example:

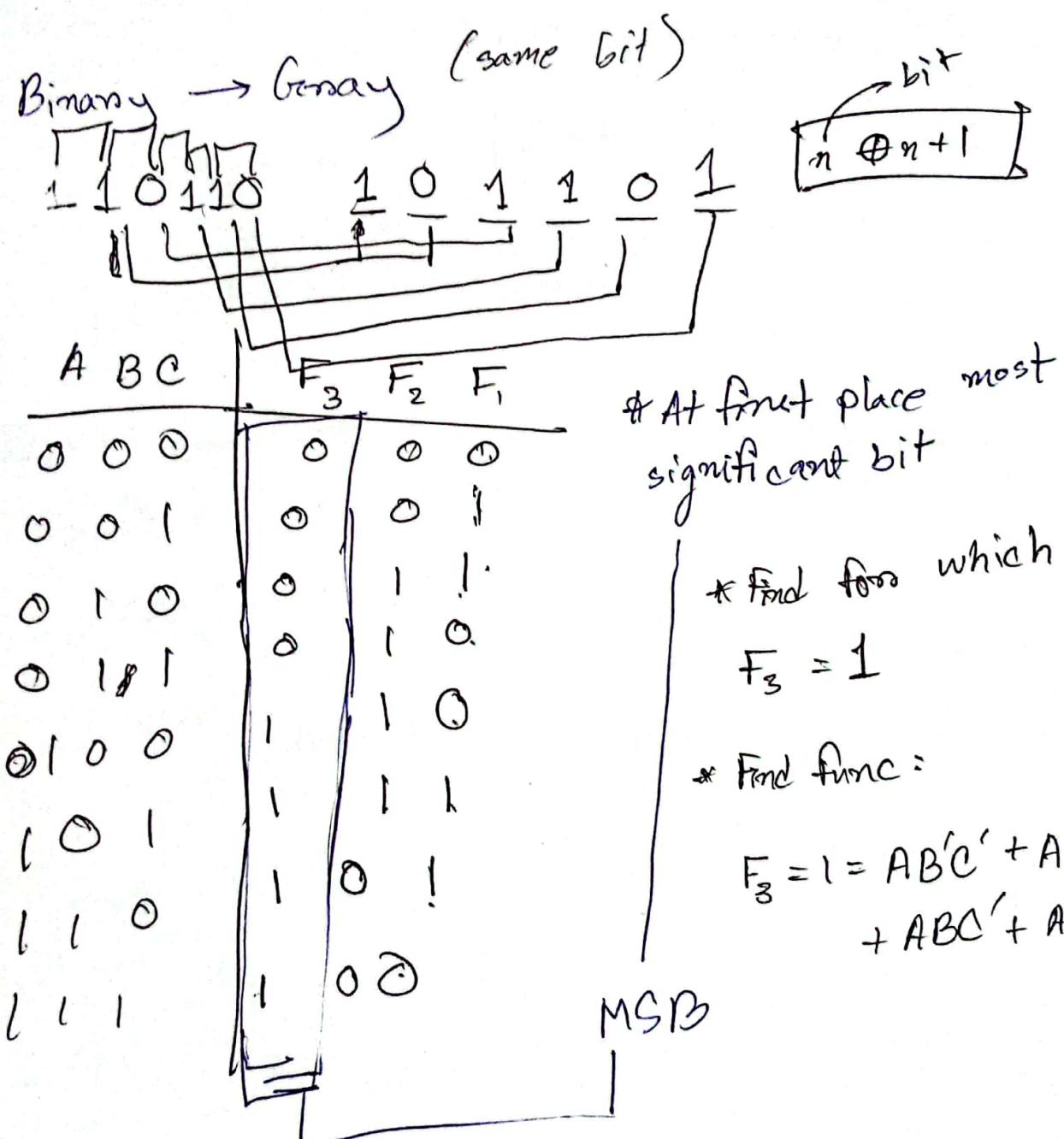
$$A+1 = 1$$

$$A \cdot 0 = 0$$

* Duality principle: + 2 mark SQ

- states that every algebraic expression deducible from the postulates of Boolean Algebra remains valid if the operations & identity elements are interchanged.

For example: if $A + \bar{A}$ is true, then $A \cdot \bar{A} = 0$ is also true

Gray Code:

Gray → Binary

$$\text{H}_2\text{O} \rightarrow \frac{1}{2} \text{H}_2 + \frac{1}{2} \text{O}_2$$

C-4 (w-3)

Example: Proof that $A + \bar{A}C = A + C$

$$\begin{aligned}
 \text{Sol'n: L.H.S} &= A + \bar{A}C \\
 &= (A + \bar{A})(A + C); \quad [\text{distributive law}] \\
 &= A + C; \quad [A + \bar{A} = 1] \\
 &= R.H.S \\
 \therefore \text{L.H.S} &= R.H.S. \quad (\text{proved})
 \end{aligned}$$

HW: Proof that:

$$\text{i}) x + xy = x(x + y)$$

$$\text{ii}) x(\bar{x} + y) = xy$$

Lecture 5: Simplification using Boolean Algebra

Example: Simplify $[AB(C + BD) + \bar{A}\bar{B}]C$

$$\begin{aligned}
 \text{Sol'n: } &[AB(C + BD) + \bar{A}\bar{B}]C \\
 &= (ABC + ABD + \bar{A}\bar{B})C; \quad [\text{distributive law}] \\
 &= (ABC + A \cdot 0 \cdot D + \bar{A}\bar{B})C; \quad [\because BB = 0] \\
 &= (ABC + 0 + \bar{A}\bar{B})C; \quad [\because x \cdot Q = 0] \\
 &= (ABC + \bar{A}\bar{B})C
 \end{aligned}$$

$$= A\bar{B}C \cdot C + \bar{A}\bar{B}C ; []$$

$$= A\bar{B}C + \bar{A}\bar{B}C$$

$$= \bar{B}C(A + \bar{A})$$

$$= \bar{B}C \cdot 1 ; [\because A + \bar{A} = 1]$$

$$= \bar{B}C$$

Q. H.W. Simplify:

$$AB + A(B+C) + B(B+C) \text{ using Boolean Algebra}$$

Boolean Functions:

An expression formed with binary variables,

AND, OR, NOT operations, parentheses and equal sign

Example: Implement $F_2 = x + y'z$ with basic gates.

↳ output will be either 1 or 0

* One of the ways to simplify is Boolean simplification

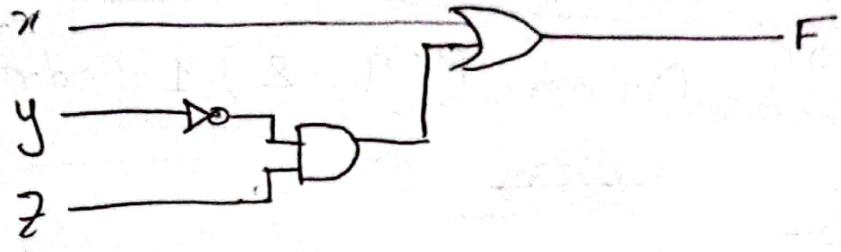
* Method of simplification will be mentioned in the exam

CSE 2105

Quiz #1 (Sec: A)

28/04/24 SUNDAY

on chap - 1 and 2



Input

| x | y | z | F |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

* short cut: Find the ones with 1

For $F = x + y^z$

1. if $x = 1$,
 $F = 1$

2. $y^z = 1$
 $\therefore y^z = 1 \& y = 0$
 $F = 1$

Lecture 6: Binary codes for the decimal digits

* Mainly used in encryption
■ BCD (Binary Coded Decimal) on 8, 4, 2, 1 code

| <u>Decimal</u> | <u>Binary</u> | <u>BCD</u> |
|--|---------------|--------------|
| 0 | 0000 | 0000 |
| 1 | 0001 | 0001 |
| : | : | : |
| 9 | 1001 | 1001 |
| : | : | : |
| 12 | 1100 | 00010010 |
| 15 | 1111 | 000110101011 |
| $(279)_{10} - \underline{\underline{00101011101001}} = \underline{\underline{111101010001}}$ | | |

Excess - 3:

| <u>Decimal</u> | <u>Binary</u> | <u>BCD</u> |
|----------------|---------------|------------|
| 0 | 0000 | 0011 |
| 1 | 0010 | 0101 |
| 2 | | 1100 |
| 3 | 1001 | |

q Add 3 to decimal

2, 4, 2, 1

Decimal

| | | | |
|-------|---|---|---|
| 2 | 4 | 2 | 1 |
| <hr/> | | | |
| 0 | 0 | 0 | 0 |

* [same as BCD
each bit has corresponding
weight]

0

1

5

5

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 |

* [maximum can be 9]

8, 4, -2, -1

Decimal

| | | | |
|-------|---|----|----|
| 8 | 4 | -2 | -1 |
| <hr/> | | | |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 |

* [yeah pretty much
same]

* [maximum can be 12]

② 2
6

Coding conversion (Example):

Decimal

| | | | |
|-------|---|---|---|
| 2 | 4 | 2 | 1 |
| <hr/> | | | |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 |

| | | | |
|-------|---|----|----|
| 8 | 4 | -2 | -1 |
| <hr/> | | | |
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 |

0

1

- * (i) Convert to Decimal
- * (ii) Convert to the other

Binary to Gray Conversion:

Binary: 10110

Gray: 11101

Either $1 \oplus 0 \oplus 1 \oplus 1 \oplus 0$

On $1+0+1+1+0$

Gray to Binary Conversion:

Gray: 11011

Binary: 10010

Bin: 1101101

Gray: 1010101

Binary: 1010110

Bin: 1100101

Error Detection:

* Parity Bit: + For Lab can be added anywhere
A parity bit is an extra bit included with a message to make the total number of 1's either odd (odd parity) or even (even parity)

Say message: 10110

* If we use odd parity, $P(\text{odd}) : 0$
" " " even " , $P(\text{even}) : 1$

* Parity checker

* Parity generator

> CHECKS parity
 $111 \rightarrow 1$, because it has odd number of 1's

MAKES parity

$111 \rightarrow 0$ is odd because it has odd number of 1's

Hamming Code: & V. V. Imp for EXAM

A hamming code can be used to detect & correct one bit change in an encoded code word.

* detects and the error

* finds the position of error

connections

Determine the single error, code for 101110110

Soln: The no. of data bits $n = 9$

So, the required no. of parity bits (p) can be obtained from the following formula $2^p \geq n + p + 1$

if $p = 1$, $2^1 \geq 9 + 1 + 1$ (false)

if $p = 2$, $2^2 \geq 9 + 2 + 1$ (false)

if $p = 3$, $2^3 \geq 9 + 3 + 1$ (false)

if $p = 4$, $2^4 \geq 9 + 4 + 1$ (true)

if $p = 4$, we have to send total $n + p = 9 + 4 = 13$

So, $p = 4$ and we have to send 13 bit message

Original message: 101110110

* Use p in 2^n bits

* Use bit positions that have 1

* As many bits as p_3

| Bit position | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|--------------|-------|-------|---|-------|---|---|---|-------|---|----|----|----|----|
| values | p_1 | p_2 | 1 | p_3 | 0 | 1 | 1 | p_4 | 1 | 0 | 1 | 1 | 0 |

To find the values of p_1 , p_2 , p_3 , p_4 , we have to

XOR the binary values of bit positions holding 1's

$$\begin{array}{r} 0011 \\ 0110 \\ 0111 \\ 1001 \\ 0100 \\ 0111 \\ \hline (0) 1001 \end{array} = 1100$$

$p_4\ p_3\ p_2\ p_1$

so, the sending message will be 001101110110
(cont.)

Cont.

Receiving End :

Case 1: If there is no error in transmitting, the receiver will get 001101110110.
to verify

~~→ p will be 1~~

XOR positions with 1

$$\begin{array}{r} 00\ 11\ (5) \\ 0\ 100\ (6) \\ 0\ 11\ 0\ (7) \\ \hline \cancel{1}\ 000\ \cancel{(8)}\ 1000\ (8) \\ 1\ 001\ \cancel{(9)} \\ 10\ 11\ \cancel{(10)} \\ \hline (\text{XOR})\ 1100\ (12) \\ \hline 0\ 000 \end{array}$$

So, there is no error in the received message

Case (2): Say, there is an error in the 6th position transmitting the receiver will get: 0011001110110.

To verify:

$$\begin{array}{r} 0011 \\ 0100 \\ 0111 \\ 1000 \\ 1001 \\ 10\cancel{0}\cancel{0} \\ \hline (\text{XOR}) & 1100 \\ \hline & 0110 \end{array}$$

So, there is an error at in the 6th position to get correct message, we have to flip the value of 6th position:

$$00110\underline{1}1110110$$

Chap - 2

Lecture 7: Standard forms of Boolean

Expressions

All Boolean expressions can be converted into either of 2 standard forms:

i) The Sum of Products (SOP): $\rightarrow \text{OR}$

Example: $ABC + CDE + B'CD'$

ii) The Product of Sums (POS): $\rightarrow \text{AND}$

Example: $(A' + B' + C')(A' + B')(A + B')$

* Standard Form: either SOP or POS

* Non Standard Form: $AB + C(D+E)$ etc

* $AB + CD + CE$

Canonical Form:

Boolean functions expressed as a SUM of minterms or product of maxterms.

- We can derive truth table from standard form

| Input | Output | Minterms | Max terms |
|-------------|--------|-----------------------------------|-------------------------------|
| $x \ y \ z$ | F | $x = 0 \ 1$ $y = 1$ $z = 1$ | $x = 0$ $y = 0$ $z = 2$ |
| 0 0 0 | 0 | $x'y'z' (m_0)$ | $(x+y+z)(M_0)$ |
| 0 0 1 | 1 | $x'y'z (m_1)$ | $(x+y+z') (M_1)$ |
| 0 1 0 | 0 | $x'yz' (m_2)$ | $(x+y'+z) (M_2)$ |
| 0 1 1 | 1 | $x'y.z (m_3)$ | $(x+y'+z') (M_3)$ |
| 1 0 0 | 1 | $xy'z' (m_4)$ | $(x'+y+z) (M_4)$ |
| 1 0 1 | 1 | $xy'z (m_5)$ | $(x'+y+z') (M_5)$ |
| 1 1 0 | 0 | $xyz' (m_6)$ | $(x'+y'+z) (M_6)$ |
| 1 1 1 | 0 | $-xyz$ | $(x'+y'+z') (M_7)$ |

By Minterms, $F = m_1 + m_3 + m_4 + m_5$
 $\Rightarrow \sum(1, 3, 4, 5) ; [function gives 1]$

$$\therefore F = x'y'z + x'y.z + xy'z + xyz'$$

By Maxterms, $F = M_0 \ M_2 \ M_6 \ M_7$
 $\Rightarrow \prod(0, 2, 6, 7) ; [function gives 0]$

$$\therefore F = (x+y+z) (x+y'+z) (x'+y+z') (x'+y'+z')$$

* Express the Boolean Function

$$F = A + B'C \text{ as sum of minterms}$$

Solⁿ: $F = A + B'C$

$$= A(B+B') + B'C(A+A'); [\because x+x' = 1]$$

$$= AB + AB' + AB'C + A'B'C$$

$$= AB(C+C') + AB'(C+C') + AB'C + A'B'C$$

$$= ABC + ABC' + AB'C + AB'C' + AB'C + A'B'C'$$

$$= ABC + ABC' + AB'C + AB'C' + A'B'C; [\because A+A=1]$$

④

$$= \sum(m_7 + m_6 + m_5 + m_4 + m_1)$$

$$= \Sigma(7, 6, 5, 4, 1)$$

* Express the Boolean function $F = xy + x'z$ as product of minterms

Solⁿ: $F = xy + x'z$

$$= (xy + x')(yz + z); [\because x+yz = (x+y)(x+z)]$$

$$= (x+x')(y+x')(x+z)(y+z)$$

$$= (x'+y)(x+z)(y+z)$$

$$= (x+y+z)(x+z+yy')(y+z+xx'); [A \cdot \bar{A} = 0]$$

$$= (x' + y + z)(x' + y + z')(x + y + z) + (x + y + z)$$

$$(x + y + z)(x + y + z')$$

$$= (x' + y + z)(x' + y + z')(x + y + z)(x + y' + z)$$

$$= M_0 M_1 M_2 M_3$$

$$= \overline{\Pi}(0, 2, 4, 5)$$

* H.W: Simplify:

i) $F(A, B, C) = \Pi(0, 2, 3)$

ii) $F(A, B, C) = \Sigma(1, 4, 5, 6, 7)$

* Converting SOP to POS } self study (Complement each others)
 * Converting POS to SOP }

$F(x, y, z) = 1$ means?

↳ always 1

↳ $F = \Sigma(0, 1, 2, 3, 4, 5, 6, 7)$

$F = \Pi(N; D)$

Lecture 8:

Karnaugh Map (K-MAP)

A table of n variables will have

Rule: (SOP) 2^n cells

1. Groups may not include any cell containing a zero.
2. Cells, ^{Groups} may be horizontal or vertical
3. Groups must contain 1, 2, 4, 8 or 2^n cells
4. Each group should be as large as possible
5. Each cell containing a ~~1~~ must be in at least one group.
6. Groups may overlap
7. Groups may wrap around the table
8. There should be as few groups as possible

2-variable K-Map

- self study.

Three variable K-map: * cell address matters

| A \ BC | 00 | 01 | 11 | 10 |
|--------|-------|-------|-------|-------|
| 0 | m_0 | m_1 | m_3 | m_2 |
| 1 | m_4 | m_5 | m_7 | m_6 |

Example: Simplify using K-map:

i) $F(A, B, C) = \Sigma(1, 2, 3, 5, 7)$ \rightarrow can be done in SOP
ii) $F(x, y, z) = \Sigma(3, 4, 6, 7)$ \rightarrow " "

i)

| A \ BC | B'C' | B'C | BC | BC' |
|--------|------|-----|----|-----|
| 0 | 00 | 01 | 11 | 10 |
| 1 | A' | 1 | 1 | 1 |

* Groups should be visible
→ use colors if necessary

$F = A'B + C$ * if group covers all rows or columns
it will not be counted

ii)

| x' \ yz' | y'z' | y'z | yz' |
|----------|------|-----|-----|
| 0 | 00 | 01 | 11 |
| 1 | 100 | 111 | 110 |

$$y'z + xz'$$

4-variable K-Map:

| AB \ CD | 00 | 01 | 11 | 10 |
|---------|----------|----------|----------|----------|
| 00 | m_0 | m_1 | m_3 | m_2 |
| 01 | m_4 | m_5 | m_2 | m_6 |
| 11 | m_{12} | m_{13} | m_{15} | m_{14} |
| 10 | m_8 | m_9 | m_{11} | m_{10} |

Example: Simplify using K-Map:

i) $F(x, y, z) = \sum(0, 1, 2, 4, 5, 6, 8, 9)$

ii) $F(A, B, C, D) = \sum(0, 2, 3, 5, 7, 8, 9, 10, 13, 15)$

$x + y$



For 2-variable

| A | B | $B=0$ | $B=1$ |
|-----------------------|---|-------|-------|
| $A = 0 \rightarrow 0$ | | m_0 | m_1 |
| $A = 1 \rightarrow 1$ | | m_2 | m_3 |

III

| | |
|----|----|
| 00 | 01 |
| 10 | 11 |

| A | B | F |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

| A | B | | |
|---|---|---|---|
| 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 |

For 3-variable:

→ we use gray code instead of binary order

| A | B | C | |
|---|---|---|--|
| 0 | 0 | 0 | |
| 0 | 0 | 1 | |

| A | B | C | F |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |

~~Q5~~ Q-7 (W-4)

K-Map Practices

02/03/24
(online)

| | | C | |
|----|----|-------|-------|
| AB | | 0 | 1 |
| A | 0 | m_0 | m_1 |
| | 1 | m_2 | m_3 |
| 1 | 00 | m_6 | m_7 |
| 1 | 10 | m_4 | m_5 |

| | | BC | 00 | 01 | 11 | 10 |
|---|---|-------|-------|-------|-------|-------|
| A | | 0 | m_1 | m_2 | m_4 | m_3 |
| Y | 0 | m_4 | m_5 | m_7 | m_6 | |
| | 1 | | | | | |

3 Variable Practice:

* Simplify $F = \sum m(1, 3, 4, 6)$

| | | BC | $B'C'$ | BC | BC' |
|----|---|----|--------|------|-------|
| A | | 00 | 01 | 11 | 10 |
| A' | 0 | | | | |
| | 1 | 1 | 1 | | 1 |

$$\therefore F = A' C + A C' \\ = A \oplus C$$

→ $A'0$ and C common

* ~~$\sum m$~~ $F = \sum m(0, 1, 2, 4, 6)$

| | | BC | $B'C'$ | BC | BC' |
|----|---|----|--------|------|-------|
| A | | 00 | 01 | 11 | 10 |
| A' | 0 | 1 | 1 | | 1 |
| | 1 | 1 | | | 1 |

$$\therefore F = C' + A'B'$$

* $F = \Sigma(1, 2, 3, 5, 7)$

| x' | y' | z' | z |
|------|------|------|-----|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |

$$F = z + x'y$$

$$\begin{array}{|c|c|} \hline 0 & 1 \\ \hline 0 & 5 \\ \hline 1 & 6 \\ \hline \end{array}$$

* $F(x, y, z) = \Sigma(3, 4, 6, 7)$

| x' | y' | z' |
|------|------|------|
| 0 | 0 | 0 |
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |
| 1 | 1 | 1 |

$$F = \bar{x}z + x\bar{z}$$

* Question may have Boolean expressions instead of F

- Place it directly OR
- Express in min terms

* $\sum P(w, x, y, z) = \Sigma(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$

| w' | x' | y' | z' |
|------|------|------|------|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 |

$$F = y' + w'z' + xz'$$

$$F(A, B, C, D) = \sum m(0, 1, 2, 5, 7, 8, 10, 13, 15)$$

~~WB~~ ~~CD~~

| | 00 | 01 | 11 | 10 |
|----|-----|-----|----|----|
| 00 | 1 | | | 1 |
| 01 | | 1 1 | | |
| 11 | | 1 1 | | |
| 10 | 1 1 | | | 1 |

$$F = \bar{B}D +$$

$$F(A, B, C, D) = \sum m(0, 1, 2, 5, 8, 9, 10)$$

~~WB~~ ~~CD~~

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| | | | |
| 0 | | | |
| 1 | 1 | 1 | 1 |

$$F = B'D' + B'C' + AC'D$$

$$F(A, B, C, D) = \bar{T}(3, 4, 6, 7, 11, 12, 13, 14, 15)$$

| | | CD | $C+D$ | $C'+D'$ | $C'+D$ | $C+D'$ |
|----|---------|----|-------|---------|--------|--------|
| | | 00 | 01 | 11 | 10 | 00 |
| AB | | 00 | | 0 | | |
| | $A+B$ | 00 | | | | |
| | $A+B'$ | 01 | 0 | 0 | 0 | |
| | $A'+B'$ | 11 | 0 | 0 | 0 | 0 |
| | $A'+B$ | 10 | | 0 | | |

$B'+D'$
 $F = (A'+B')(C'+D')(B'+D)$
 \downarrow
 $C'+D'$
 $\rightarrow A'+B'$

Dont Care:

In some cases, the output of function (1 or 0) is not specified for certain input combinations because the input combination never occurs or we don't care about the outputs of this particular combination. One specifies min terms for these functions are called don't care.

Notation: 'X' or 'd' \rightarrow Not necessary for grouping but can be used for convenience

Examples: Simplify using K-map:

$$i) F = \Sigma(1, 3, 7, 11, 15)$$

$$d = \Sigma(0, 2, 5) \rightarrow d \text{ on } X$$

$$ii) F = \Sigma(1, 3, 7)$$

$$d = \Sigma(0, 5)$$

$$\therefore F = CD + A'D$$

i)

| AB \ CD | 00 | 01 | 11 | 10 |
|---------|----|----|----|----|
| 00 | X | 1 | 1 | X |
| 01 | X | 1 | 1 | |
| 11 | | | 1 | 1 |
| 10 | | | | |

A'D

CD

+ Try to take
minimum
don't care

i)

| A | BC | 00 | 01 | 11 | 10 |
|---|----|----|----|----|----|
| 0 | X | 1 | 1 | | |
| 1 | | X | 1 | | |

$$\therefore F = C$$

Lecture 10:

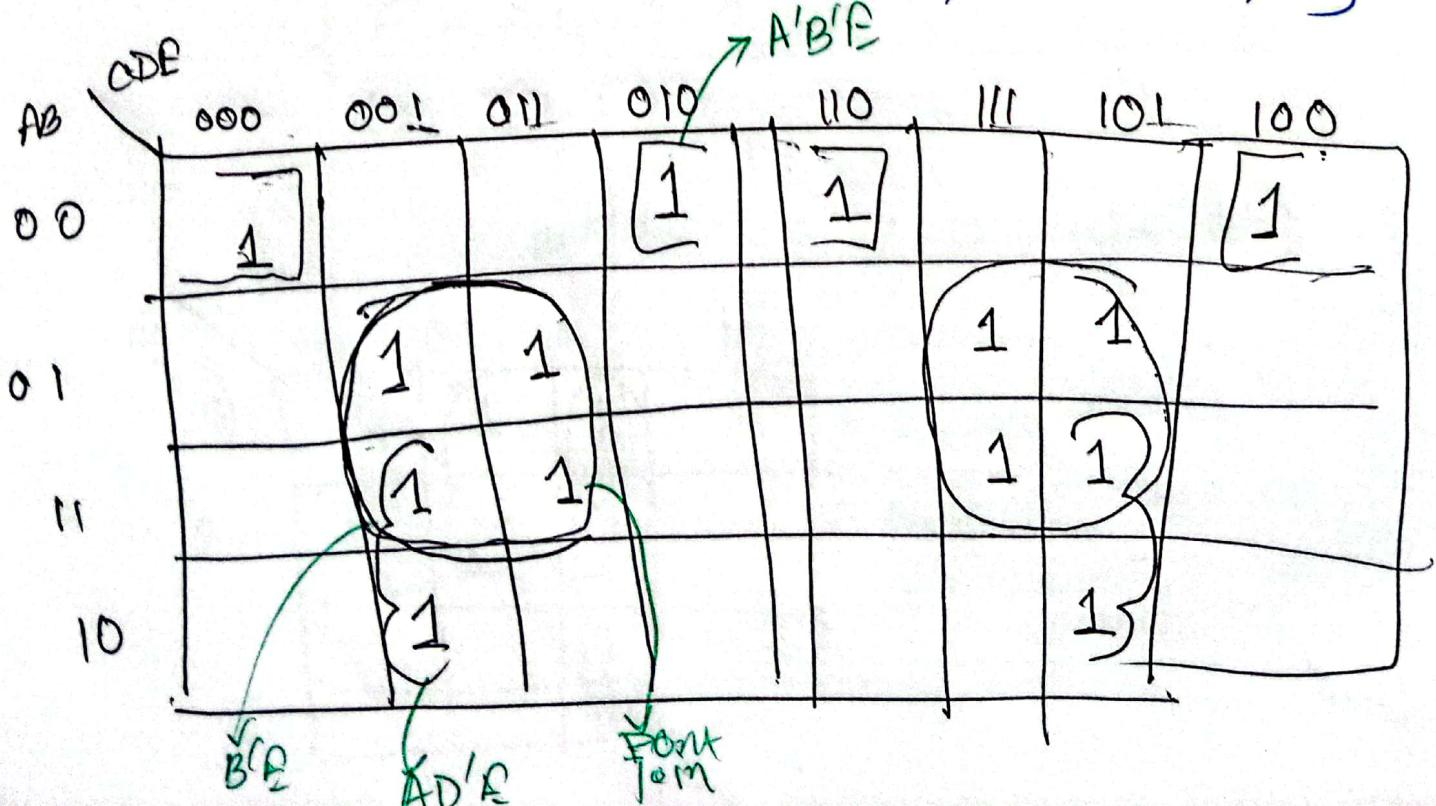
* Minmum Rule:

* 5 variable K-map

| | | i | | ii | | | | ii(iii) | | i(iii) | |
|----|----|-----|-----|-----|-----|-----|-----|---------|-----|--------|--|
| AB | | CDE | | | | | | | | | |
| | | 000 | 001 | 011 | 010 | 110 | 111 | 101 | 100 | | |
| 00 | 0 | 1 | 3 | 2 | | 6 | 7 | 5 | 4 | | |
| 01 | 8 | 9 | 11 | 10 | 14 | 15 | 13 | 12 | | | |
| 11 | 24 | 25 | 27 | 26 | | 30 | 31 | 29 | 28 | | |
| 10 | 16 | 17 | 19 | 18 | 22 | 23 | 21 | 20 | | | |

Example: Simplify using K-map

$$F = \Sigma(0, 2, 4, 6, 9, 11, 13, 15, 17, 21, 23, 27, 29, 31)$$



$$\therefore F = A'B'E + B'E + AD'E$$

7 variable K-map:

but not this

| ABC \ DEF | 000 | 001 | 011 | 010 | 110 | 111 | 101 | 100 |
|-----------|-----|-----|-----|-----|-----|-----|-----|-----|
| 000 | 0 | 1 | 3 | 2 | 6 | 7 | 5 | 4 |
| 001 | 8 | 9 | 11 | 10 | 14 | 15 | 13 | 12 |
| 011 | 24 | 25 | 27 | 28 | 30 | 31 | 29 | 28 |
| 010 | 16 | 17 | 19 | 18 | 22 | 23 | 21 | 20 |
| 110 | 48 | 49 | 51 | 50 | 54 | 55 | 53 | 52 |
| 111 | 56 | 57 | 59 | 58 | 62 | 63 | 61 | 60 |
| 101 | 40 | 41 | 43 | 42 | 46 | 47 | 45 | 44 |
| 100 | 32 | 33 | 35 | 34 | 38 | 39 | 37 | 36 |

Example: Simplify using K-Map:

$$F = \Sigma(6, 9, 13, 18, 19, 25, 27, 29, 41, 45, 57, 61)$$

| | 000 | 001 | 011 | 010 | 110 | 111 | 101 | 101 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 000 | | | | | (1) | | | |
| 001 | | (1) | | | | | (1) | |
| 011 | | (1) | | | | | | (1) |
| 010 | | | | (1) | (1) | | | |
| 110 | | | | | | | | |
| 111 | | (1) | | | | | (1) | |
| 101 | | | (1) | | | | | (1) |
| 100 | | | | | | | | |

$$F = CE'F + A'B'C'DEF + A'BD'E + A'BC'D'E$$

Q.W.

i) $F(A, B, C, D) = \Sigma(0, 1, 2, 8, 10, 11, 14, 15)$

ii) $F(A, B, C, D) = \bar{F}(5, 6, 7, 10, 11, 13, 14, 15)$

iii) $F(A, B, C, D) = A'B'D' + A'CD + A'BC$

and $d = A'BC'D + ACD + AB'D'$

| | CD 00 | CD' 01 | C'D 11 | CD 10 |
|---------|----------|-----------|-----------|----------|
| A'B' 00 | 1 | 1 | | 1 |
| A'B' 01 | | | | |
| AB 11 | | | 1 | 1 |
| AB' 10 | 1 | | 1 | 1 |

$$\therefore F = A'B'C'D' + B'D' + AC$$

$$i \rightarrow 4$$

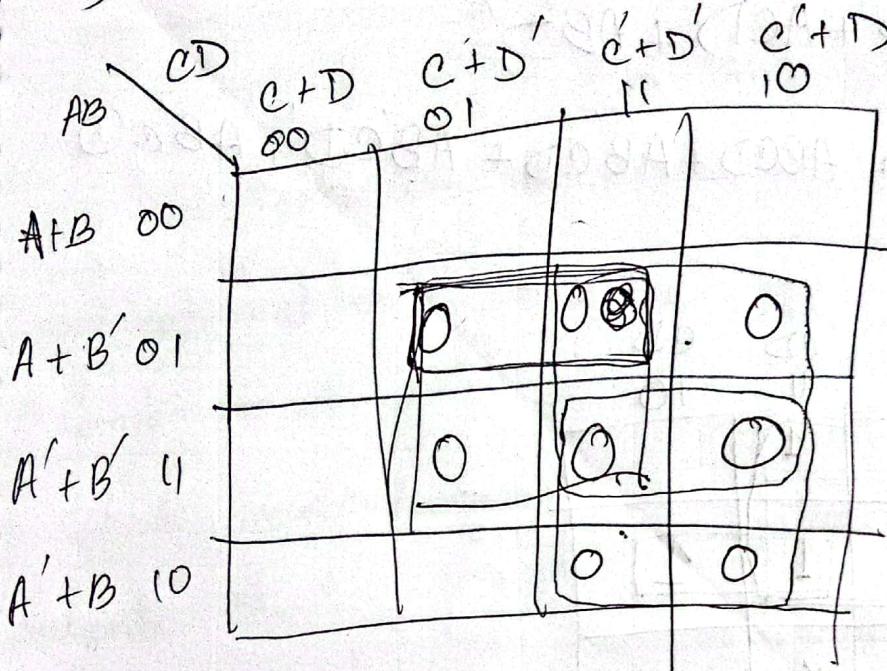
$$2 \rightarrow 3$$

$$4 \rightarrow 2$$

$$8 \rightarrow 1$$

$$\begin{aligned} Z^n & n - g + 1 \\ & 4 - 2 + 1 \end{aligned}$$

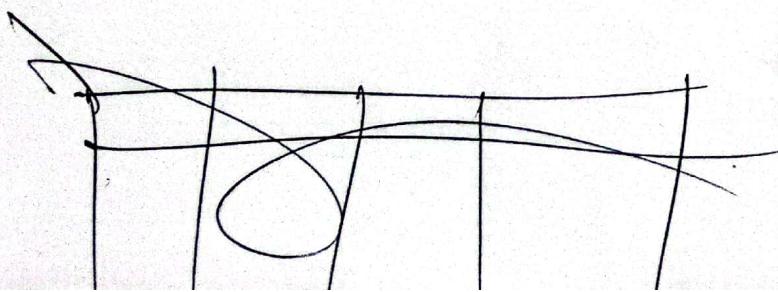
$$\text{ii) } F(A, B, C, D) = \prod (5, 6, 7, 10, 11, 13, 14, 15)$$



$$\therefore F = A'B (A+B'+D') (B'+C') (A'+C')$$

$$\begin{aligned} \text{iii) } F(A, B, C, D) &= A'B'D + A'CD + A'BC \\ &+ A'BC'D + ACD + A'B'D' \end{aligned}$$

$$\begin{aligned} F(A, B, C, D) &= A'B'D + A'CD + A'BC \\ &= A'B'D(C+C') + A'CD(B+B') + A'BC(D+D') \\ &= A'B'CD + A'B'C'D + A'BCD + A'B'CD \\ &+ A'BCD + A'BCD' \end{aligned}$$



$$= A'B'C'D + A'B'CD + A'BCD + A'B'CD + A'BCD + A'B'CD'$$

$$f(A, B, C, D) = A'B'C'D + ACD + AB'D'$$

$$= A'B'C'D + ABCD + AB'CD + AB'CD + AB'C'D$$

| $AB \backslash CD$ | $C'D'$ | $C'D$ | CD | CD' |
|--------------------|--------|-------|------|-------|
| $A'B'$ | 00 | 1 | 1 | |
| $A'B$ | 01 | X | 1 | 1 |
| AB | 11 | X | X | |
| AB' | 10 | | X | X |

$$F = A'D + A'BC$$

Lecture - 11

09/05/24

* Chapters - 4 : Combinational Logic

Logic circuits for digital systems may be
combinational or sequential
^{upto now} Circuits

* Combinational Circuits:

A combinational circuit consists of logic gates whose outputs at any time are ~~detected~~ determined directly from the present combination of inputs without regard to previous inputs. [Outputs depend entirely on inputs at that exact moment]

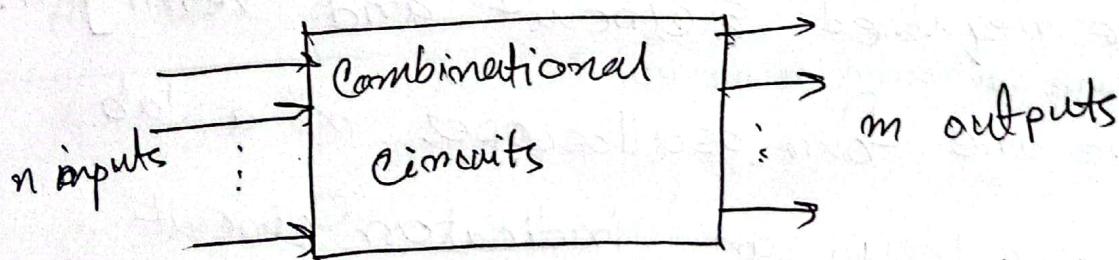


Fig.: Block diagram of combinational circuit

* Diff betⁿ seq comb and seq circs → will be explained in seq quiz

Design Procedure of Combinational Logic Circuits:

Given a problem statement :

- i) Determine the number of input variables and output variables. The input and output variables are assigned letter symbols.
- ii) Derive the truth table to define the required relationships between input and outputs.
- iii) Simplify the boolean expressions for each outputs. (K-Map)
- iv) Produce the required circuit and verify it.
(Pin numbers are not needed in theory)

Example : There are four oscilloscopes in a lab.

You have to design an indicator circuit to ~~determine~~ detect the condition when two or more of the oscilloscopes are malfunctioning.

Circuit Diagram

- Things we draw ~~is~~ with basic ~~names~~ mentions
- Detailed

Ex →

Let, working condition be 1

non-functioning " 0 " when 2_A bits will be 0

∴ The indicator will be 0

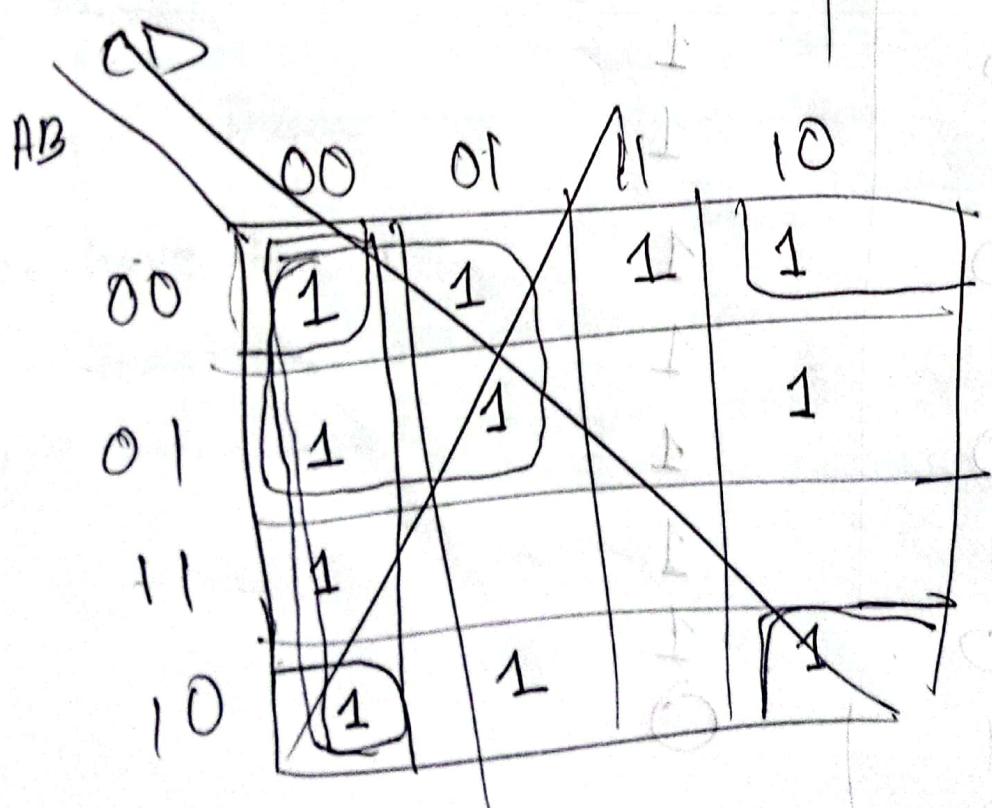
| A | B | C | D | F |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 |

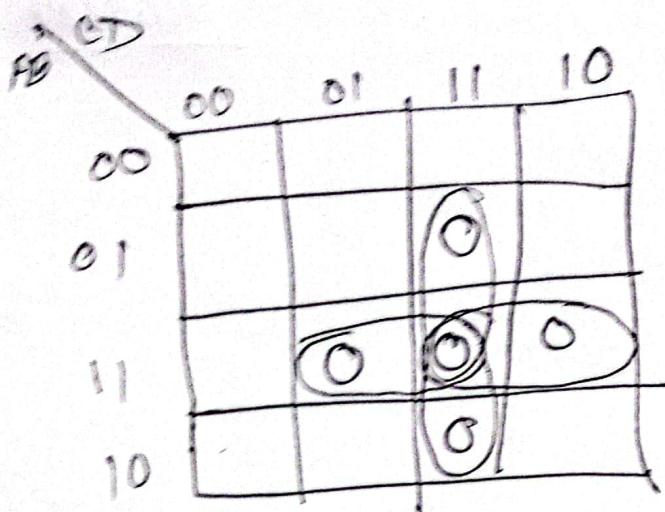
Block Diagram

- Not mentioned. Makes a block with name

- Not detailed

| A | B | C | D | F |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 |





AB

Binary Adder

* Half Adder :

- Adds 1 bit plus 1 bit
- Produces Sum & Carry

| <u>Input</u> | | | | |
|--------------|-----|----------|-----|-----|
| X | Y | \oplus | C | S |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 1 | 0 |

$$\therefore C = X \cdot Y$$

$$\begin{aligned} \therefore S &= X \cdot Y' + X' \cdot Y \\ &= X \oplus Y \end{aligned}$$

Cannot do : Carry becomes hard
 $A = \boxed{\begin{array}{|c|c|c|} \hline 1 & 1 & 0 \\ \hline \end{array}}$ 3 bit
 $B = \boxed{\begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline \end{array}}$

∴ Half Subtraction : $B - D$
 Borrow Diff

*Full Adder:

The full adder accepts 2 input bits & an input carry & generates a sum and an output carry

| X | Y | C _{in} (Carry of prev-bit) | C _{out} | S |
|---|---|---|------------------|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | - | 1 |
| 1 | 0 | 1 | - | 0 |
| 1 | 1 | 0 | - | 1 |
| 1 | 1 | 1 | - | 0 |

Carry = 1

A = 1101101 { 7 Addends

+ B = 1111101

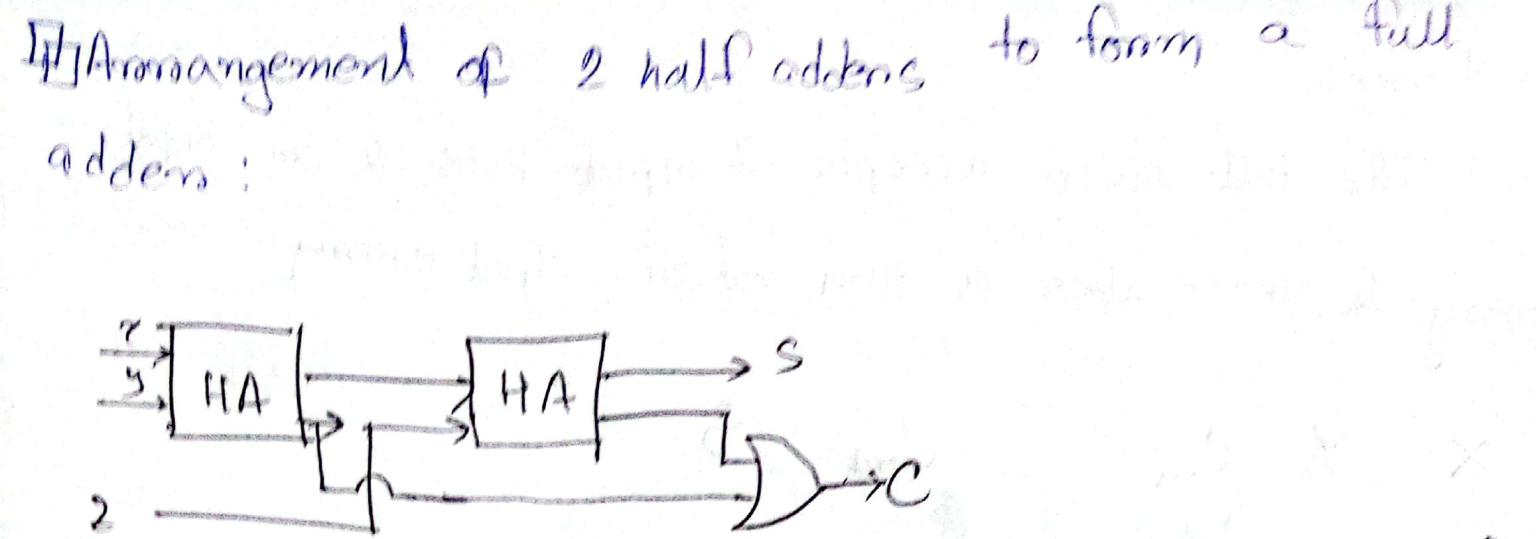


Fig: Diagram Block diagram of a full adder using 2 half adder.

Self Study:

- Half Subtraction [produces $x-y$]
- Full subtraction [produces $x-(y-z)$]
- Binary Subtractors

Magnitude Comparators

- A magnitude com A magnitude comparator is a combinational circuit that compares two numbers A and B and determines whether $A > B$, $A = B$ or $A < B$.

1 bit magnitude comparators:

| Input | | Outputs | | |
|-------|---|---------------|-------------------|-------------------------|
| X | Y | $F_1 (A > B)$ | $F_2 (A = B)$ | $F_3 (A < B)$ |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | $x'y' + \bar{x}y$ | $x'y$ $= x \oplus y$ |

2-bit, 3-bit, 4-bit Comparators:

2 bit

| X | Y | $F_1(A > B)$ | $F_2(A = B)$ | $F_3(A < B)$ |
|-------------------------|-------------------------|--------------|--------------|--------------|
| $\frac{x_2 x_1}{0 \ 0}$ | $\frac{y_2 y_1}{0 \ 0}$ | 0 | 1 | 0 |
| 0 0 | 0 1 | 0 | 0 | 1 |
| 0 0 | 1 0 | 0 | 0 | 1 |
| 0 0 | 1 1 | 1 | 0 | 0 |
| 0 1 | 0 0 | 0 | 1 | 0 |
| 0 0 1 0 | 0 0 | 0 | 1 | 0 |
| 0 1 | 0 0 | 1 | 0 | 0 |
| 1 1 | 0 1 | 1 | 0 | 0 |
| 1 1 | 1 0 | 1 | 0 | 0 |
| 1 1 | 1 1 | 0 | 1 | 0 |

[F_{1x} at home]

3-bit:

$$x = x_3 x_2 x_1$$

$$y = y_3 y_2 y_1$$

$$F_1(x > y) = \text{if } (x_3 > y_3) +$$

$$\text{if } (x_3 = y_3) \& \& (x_2 > y_2) +$$

$$\text{if } (x_3 = y_3) \& \& (x_2 = y_2) \& \& (x_1 > y_1)$$

$$= x_3 y_3' + (x_3 \odot y_3)(x_2 y_2') + (x_3 \odot y_3)(x_2 \odot y_2)$$

(from 0 1-bit)

$$F_2(x = y) = \text{if } (x_3 = y_3) \& \& (x_2 = y_2) \& \& (x_1 = y_1)$$

$$= (x_3 \odot y_3)(x_2 \odot y_2)(x_1 \odot y_1)$$

$$F_3(x < y) = \text{if } (x_3 < y_3) +$$

$$\text{if } (x_3 = y_3)(x_2 < y_2) +$$

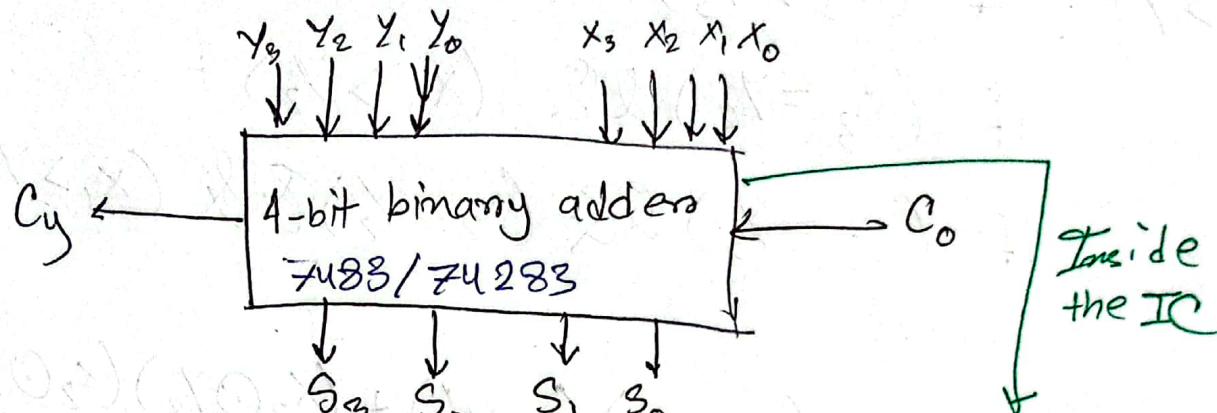
$$\text{if } (x_3 = y_3) > (x_2 = y_2)(x_1 < y_1)$$

$$= x_3' y_3 + (x_3 \odot y_3)(x_2' y_2) + (x_3 \odot y_3)(x_2 \odot y_2)(x_1' y_1)$$

* Draw Diagrams. ALWAYS

Binary Parallel Adder:

- produces the arithmetic sum of two binary numbers in parallel

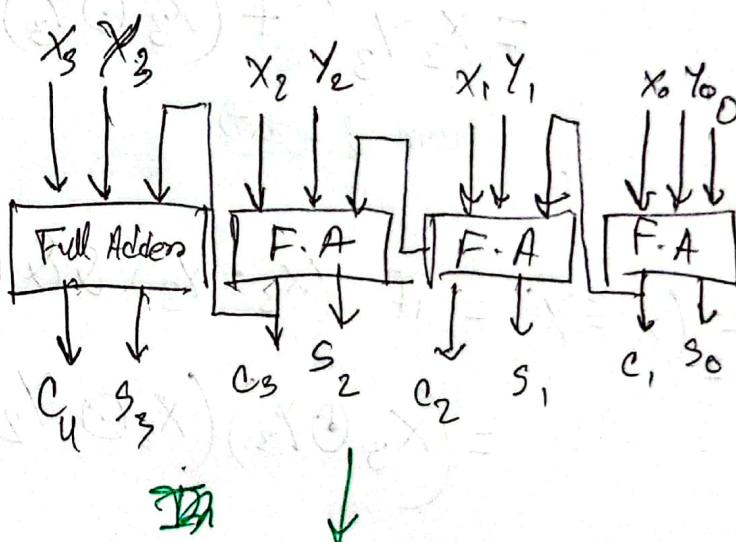


$$X = x_3 \ x_2 \ x_1 \ x_0$$

$$Y = y_3 \ y_2 \ y_1 \ y_0$$

$$x_3 \ x_2 \ x_1 \ x_0$$

$$+ y_3 \ y_2 \ y_1 \ y_0 \quad C_0 = 0$$



$$C_y \quad S_3 \ S_2 \ S_1 \ S_0$$

and and

$$C_2 \ C_1$$

(Carry is completed,
the next operation cannot
happen)

→ Ripple Effect

Carry Look Ahead Adder (CLA)

A method of speeding up the addition process by eliminating the ripple carry delay is called look ahead carry addition. In CLA, we define 2 new binary variables P_i and G_i , where,

$$P_i = A_i \oplus B_i, \quad G_i = A_i B_i$$

\rightarrow carry propagate \hookrightarrow carry generate
So, the output sum (S) and carry (C) of full adders can be expressed as,

$$S_i = P_i \oplus C_i, \quad C_{i+1} = G_i + C_i P_i$$

For 3-bit CLA adder:

$$S_1 = P_1 \oplus C_0 = \cancel{P_1 \oplus 0} = \cancel{P_1} \quad P_1 \quad A = A_1, A_2, A_3 \\ B = B_1, B_2, B_3$$

$$\therefore C_1 = P_1 = A \oplus B_1 \quad [\because P_1 = A_1 \oplus B_1]$$

$\therefore C_1$: initial carry = 0

$$\text{Similarly, } C_2 = G_1 + P_1 C_1 = A_1 B_1 + 0 = A_1 B_1$$

$$S_2 = P_2 \oplus C_2 = (A_2 \oplus B_2) \oplus C_2$$

$$C_3 = G_2 + P_2 = A_2 B_2 + (A_2 \oplus B_2) A_1 B_1$$

$$S_3 = P_3 \oplus C_3 = (A_3 \oplus B_3) \oplus [A_2 B_2 + (A_2 \oplus B_2) A_1 B_1]$$

$$C_4 = G_3 + P_3 C_3 = A_3 B_3 + (A_3 \oplus B_3) [A_2 B_2 + (A_2 \oplus B_2) A_1 B_1]$$

Q1) Design a BCD adder using Binary adder

input : two BCD digits A & B

output : A + B (in BCD)

A : 0 - 9

B : 0 - 9

A + B = BCD Sum

C-12(W-Z)

19/05/24

26/05/24

SUNDAY

Lecture
8-13

DLD Quiz - 02

#1. Design a BCD adder using binary adders & basic gates.

#2. Design a switch controlled 4-bit adder-subtractor circuit. When switch(s) = 0, Your circuit will perform $A+B$, when $s=1$, your circuit will perform $A-B$.

* Working Procedure is must
* Questions without truth table needs working procedure

BCD Adder: A

Adds two BCD digits in parallel and produces a sum also in BCD.

- The output sum cannot be greater than $(9+9+1) = 13$

| Sum (in decimal) | Sum (in binary) | Revised Sum ($C_{out} = 1$) (in BCD) |
|------------------|-----------------|---|
| 9 | 0 1 0 0 1 | 0 1 0 0 1 (9) |
| 10 | 0 1 0 1 0 | 1 0 0 0 0 (10) |
| 11 | 0 1 0 1 1 | 1 0 0 0 1 (11) |
| 12 | 0 1 1 0 0 | 1 0 0 1 0 (12) |
| 13 | 0 1 1 0 1 | 1 0 0 1 1 (13) |
| 14 | 0 1 1 1 0 | 1 0 1 0 0 (14) |
| 15 | 0 1 1 1 1 | 1 0 1 0 1 (15) |
| 16 | 1 0 0 0 0 | 1 0 1 0 1 (16) |
| 17 | 1 0 0 0 1 | 1 0 1 1 0 (17) |
| 18 | 1 0 0 1 0 | 1 0 1 1 1 (18) |
| 19 | 1 0 0 1 1 | 1 1 0 0 0 (19) |

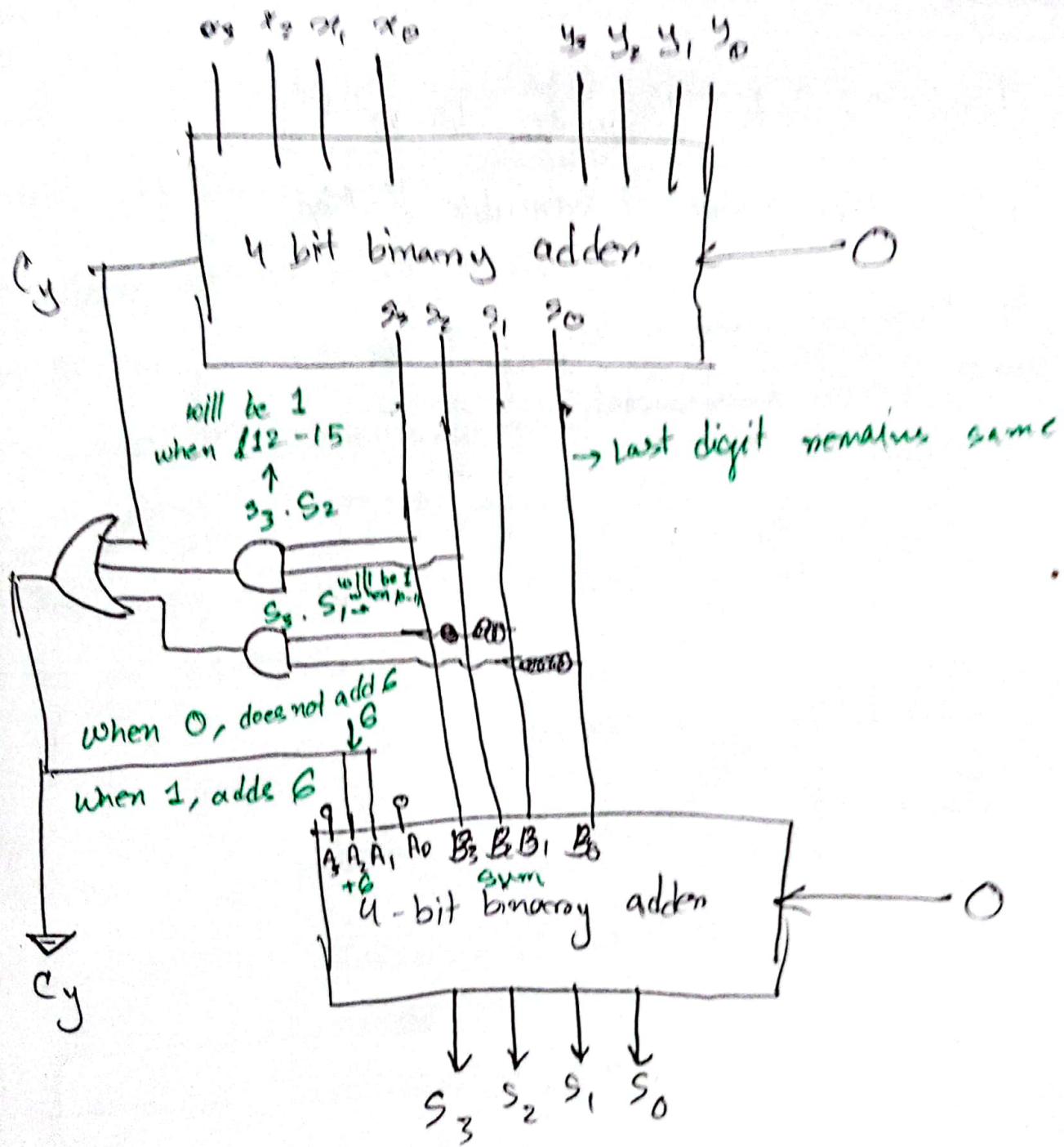


Fig: Block diagram of BCD adder using
binary adder.

For Book:

Emraan Shekhar : Gets 1 when 10^{-15}

then makes 4 variable K-Map

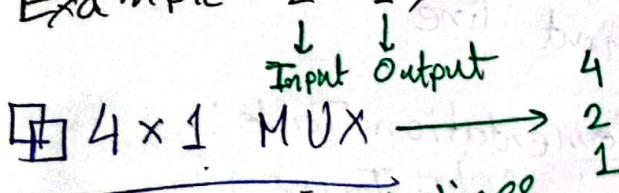
Emraan Shekhar : Gets 1 when 10^{-15}

Multiplexers / MUX

Data Selection:  same

- Selects one of many input lines (2^n) and directs it to its single output line. There are n selection lines whose bit combinations determine which input is selected.
- input (2^n), output (1), selector (n)
- The size of a MUX is specified by the number 2^n of its input lines the single output line.
- It also contains n selectors.

Example: 2×1 , 4×1 , 16×2 , 32×1 , MUX etc.

4×1 MUX  Function table:

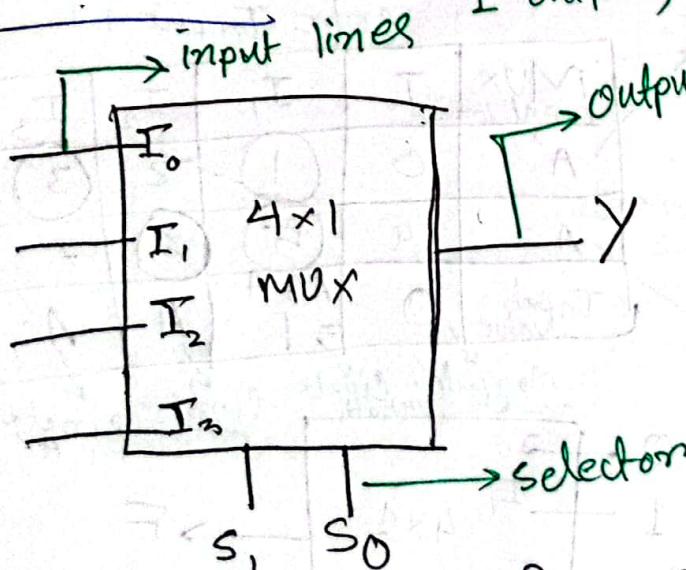


Fig : Block Diagram of a 4×1 MUX

| S_1 | S_0 | Y | Selectors determine what outcome is selected |
|-------|-------|-------|--|
| 0 | 0 | I_0 | |
| 0 | 1 | I_1 | |
| 1 | 0 | I_2 | |
| 1 | 1 | I_3 | |

Output equation:

$$Y = S_1 S_0' I_0 + S_1' S_0 I_1 \\ + S_1 S_0' I_2 + S_1' S_0 I_3$$

Logic/Circuit Diagram: See [YouTube](#)

Uses of MUX:

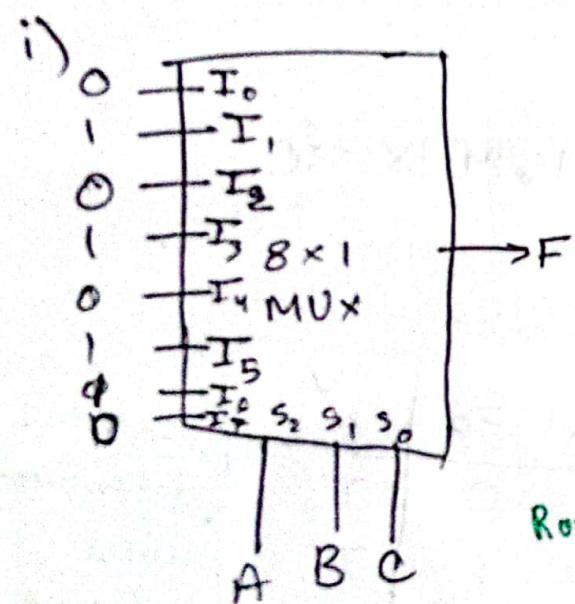
- Connecting two or more sources to a single destination
- It is useful for constructing a common bus system

Function Implementation Using MUX:

Example: Implement $F(A, B, C) = \sum(1, 3, 5, 6)$

using: i) 8×1 MUX, ii) 4×1 MUX

Soln:



$A = 0$
Row = No. of selectors as input + 2

$A = 1$

ii) Using 4×1 MUX
implementation table using
B, C as selections, A in
the input line

Implementation Table:

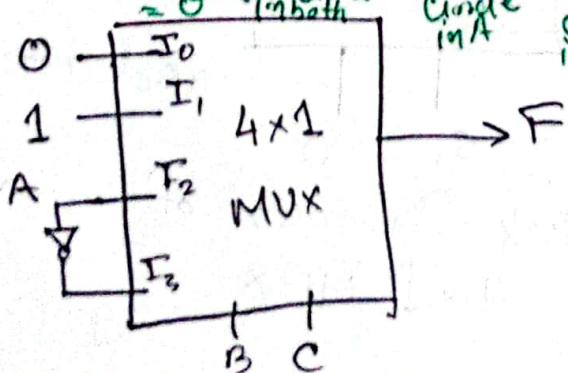
col = Input + 1

| MUX input line | I ₀ | I ₁ | I ₂ | I ₃ |
|----------------|----------------|----------------|----------------|----------------|
| A' | 0 | 1 | 2 | 3 |
| A | 4 | 5 | 6 | 7 |
| Input value | 0 | A + A | A | A' |

No circle in both

Circle in A

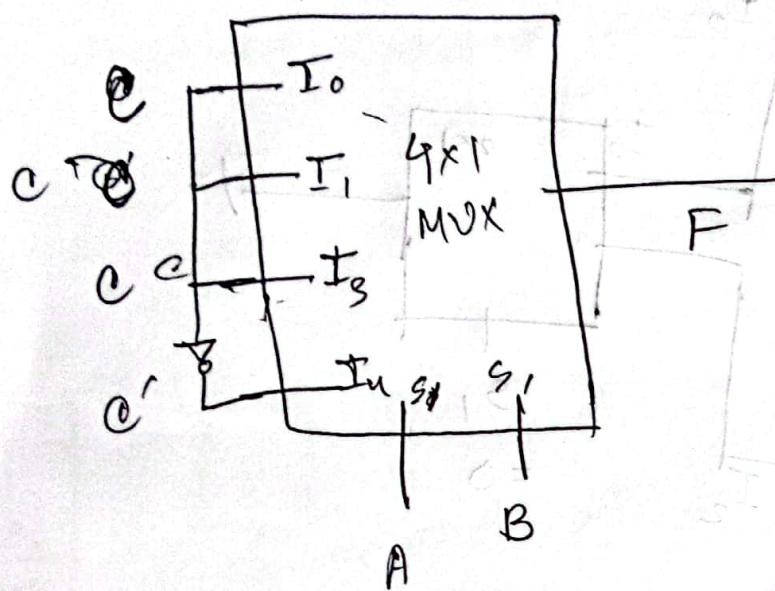
Circle in A'



C.W C.W:

Implement $F = \sum(1, 3, 5, 6)$ using 4×1 MUX
considering A, B as selectors & C as input line

* Quiz - 3 and 4 on Quiz W = 13, 14.

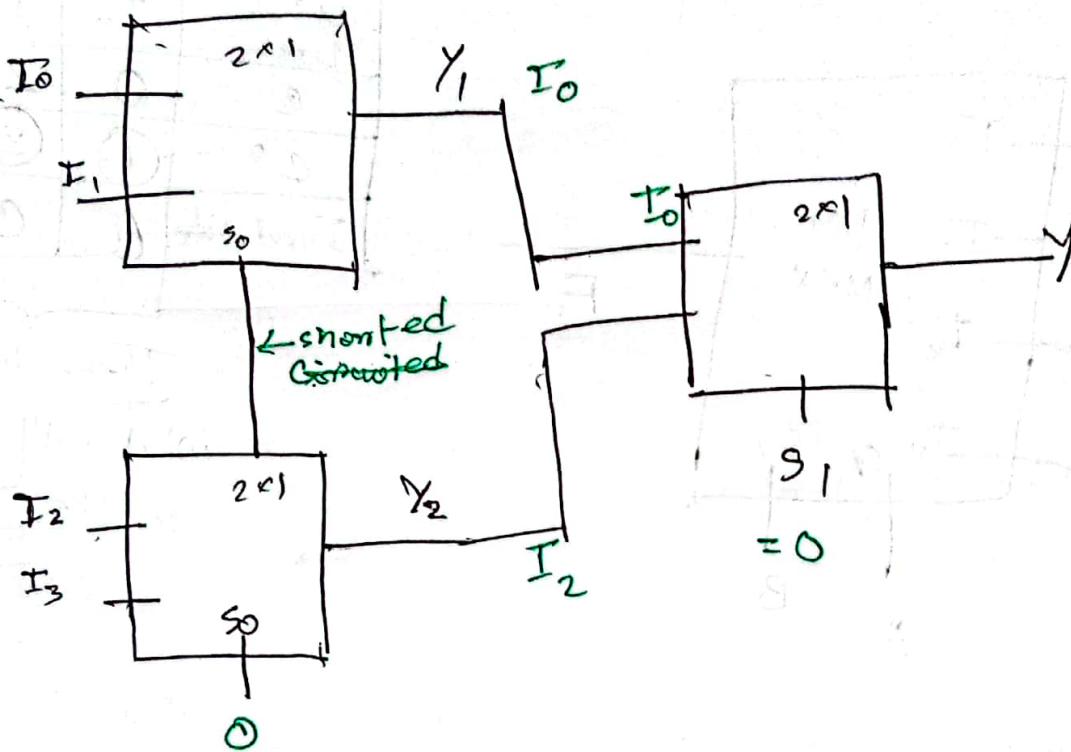


| MUX Line | I_0 | I_1 | I_2 | I_3 |
|-------------|-------|-------|-------|-------|
| c' | 0 | 2 | 4 | 6 |
| c | 1 | 3 | 5 | 7 |
| Input Value | A | C | C | c' |

These determine
what will be
in the Input

Multiplexer Expansion:

* 4x1 MUX using 2x1 MUX



* Description: Say different values yield different values

→ Not all values of have to be explained

_MUX

- Input line
- Output line
- Selectors
- Enable pin

⑩ Active High Enable:

The output is enabled when $E = 1$.

Notation: $E (5V)$

Active Low Enable:

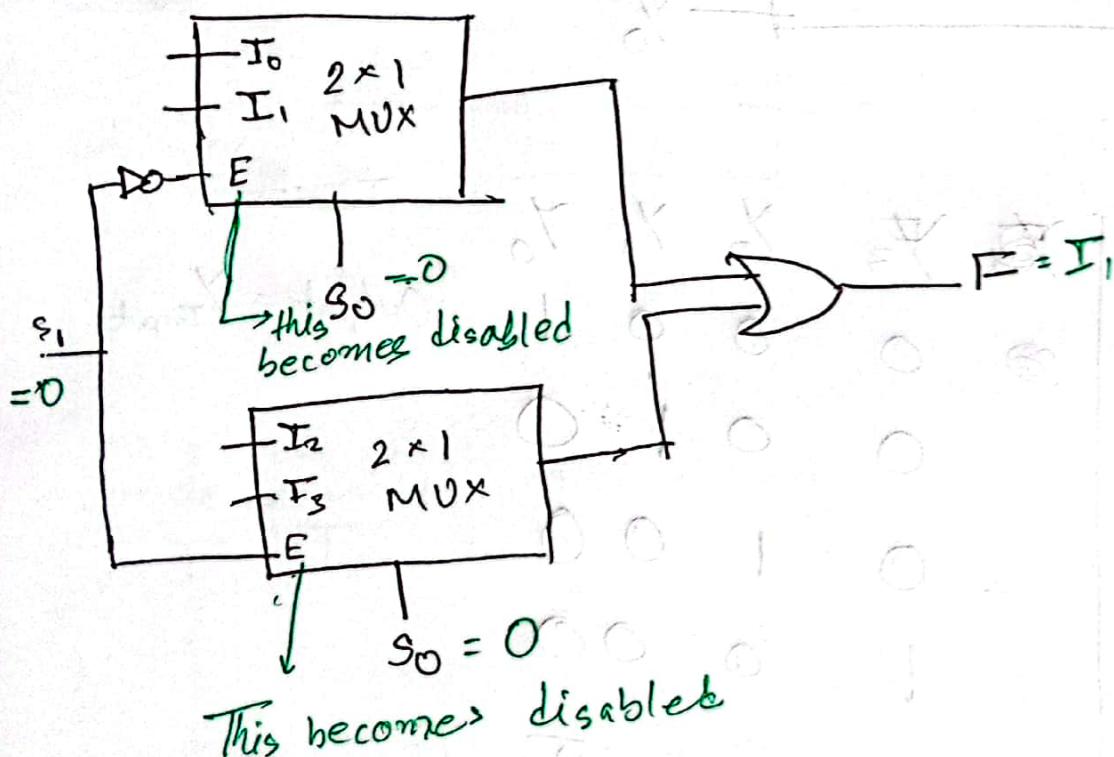
The output is enabled when $E = 0$.

Notation: $\bar{E} (\text{Gnd})$

$\hookrightarrow E \rightarrow$

Based on
type of MUX

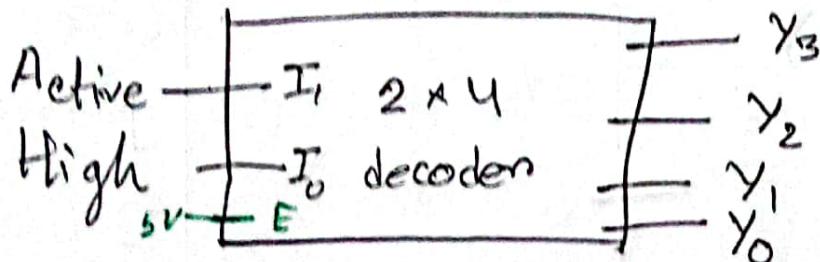
E_1, \bar{E}_2, E_3
 $\downarrow \quad \downarrow \quad \downarrow$
 $5V \quad 0V \quad 5V$



Decoder:

Converts binary information from n input lines to a maximum of 2^n unique output lines. [no selector]

* 2 to 4 lines Decoder:



Function Table:

| | | I_1 | I_0 | Y_3 | Y_2 | Y_1 | Y_0 | Input |
|---|---|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|--------------------|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | Y_{Input} |
| 0 | 1 | . | 0 | 0 | 1 | 0 | 0 | |
| 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | |

Output = Y_{Input}

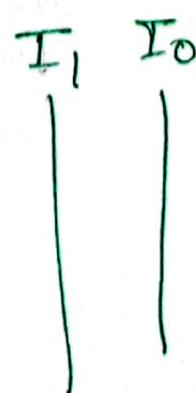
$$Y_3 = I_1 I_0$$

$$Y_2 = \overline{I}_1 I_0$$

$$Y_1 = I_1 \overline{I}_0$$

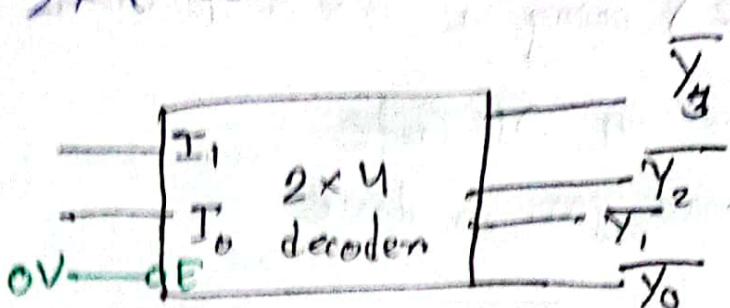
$$Y_0 = \overline{I}_1 \overline{I}_0$$

Circuit diagram
- draw yourself



Active low

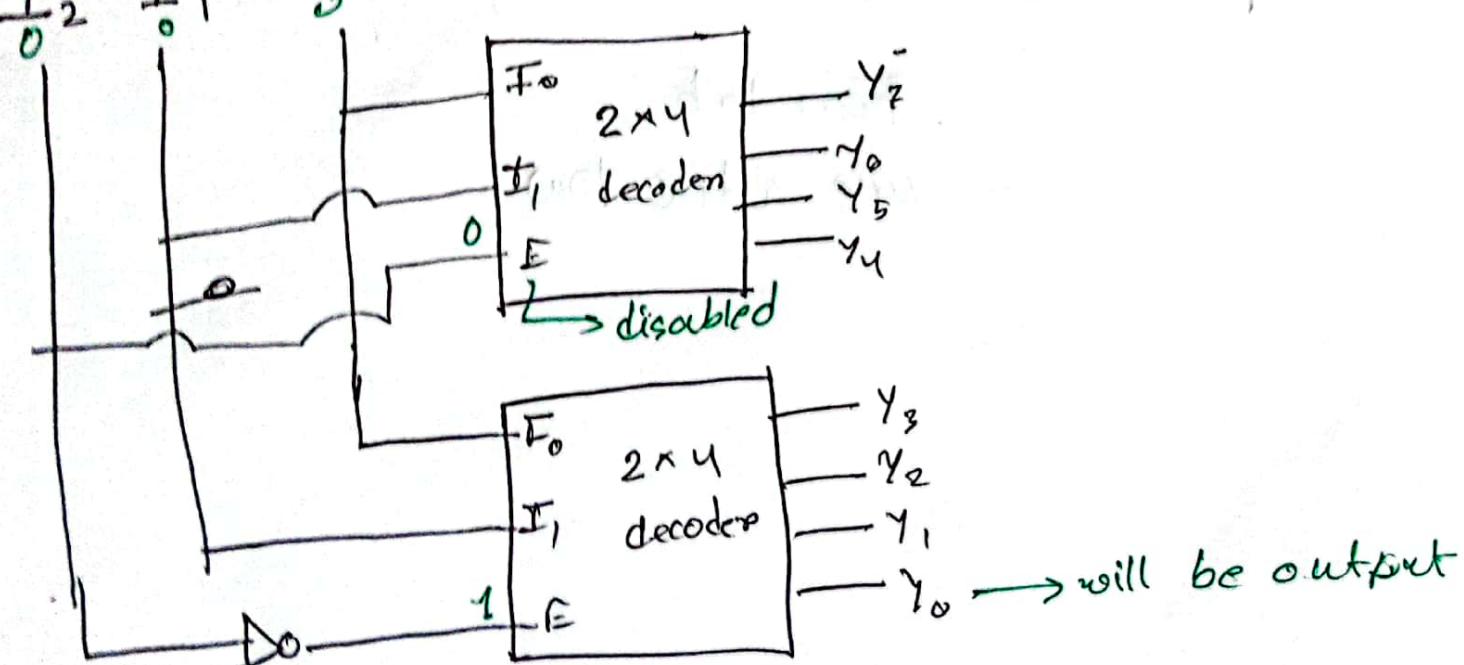
2x4 decoder



| I_1 | I_0 | \bar{Y}_3 | \bar{Y}_2 | \bar{Y}_1 | \bar{Y}_0 |
|-------|-------|-------------|-------------|-------------|-------------|
| 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 | 1 |

Design a 3×8 line decoder using two 2×4 decoders

→ use I_2 as enable pin

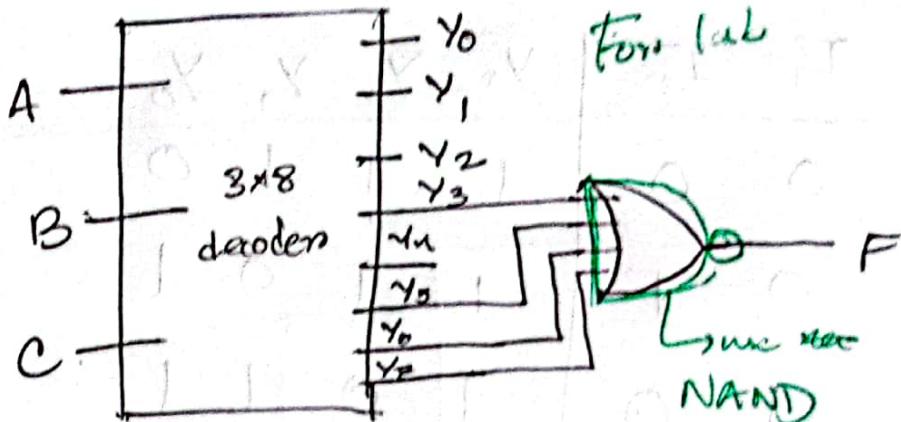


* Boolean Function implementation using Decoders

Implement $F = \sum(3, 5, 6, 7)$ using a 3×8 decoder
& basic gate

→ put in OR Gate
and job is done

| A | B | C | | F |
|---|---|---|--|---|
| 0 | 0 | 0 | | 0 |
| 0 | 0 | 1 | | 0 |
| 0 | 1 | 0 | | 0 |
| 0 | 1 | 1 | | 1 |
| 1 | 0 | 0 | | 0 |
| 1 | 0 | 1 | | 1 |
| 1 | 1 | 0 | | 1 |
| 1 | 1 | 1 | | 1 |



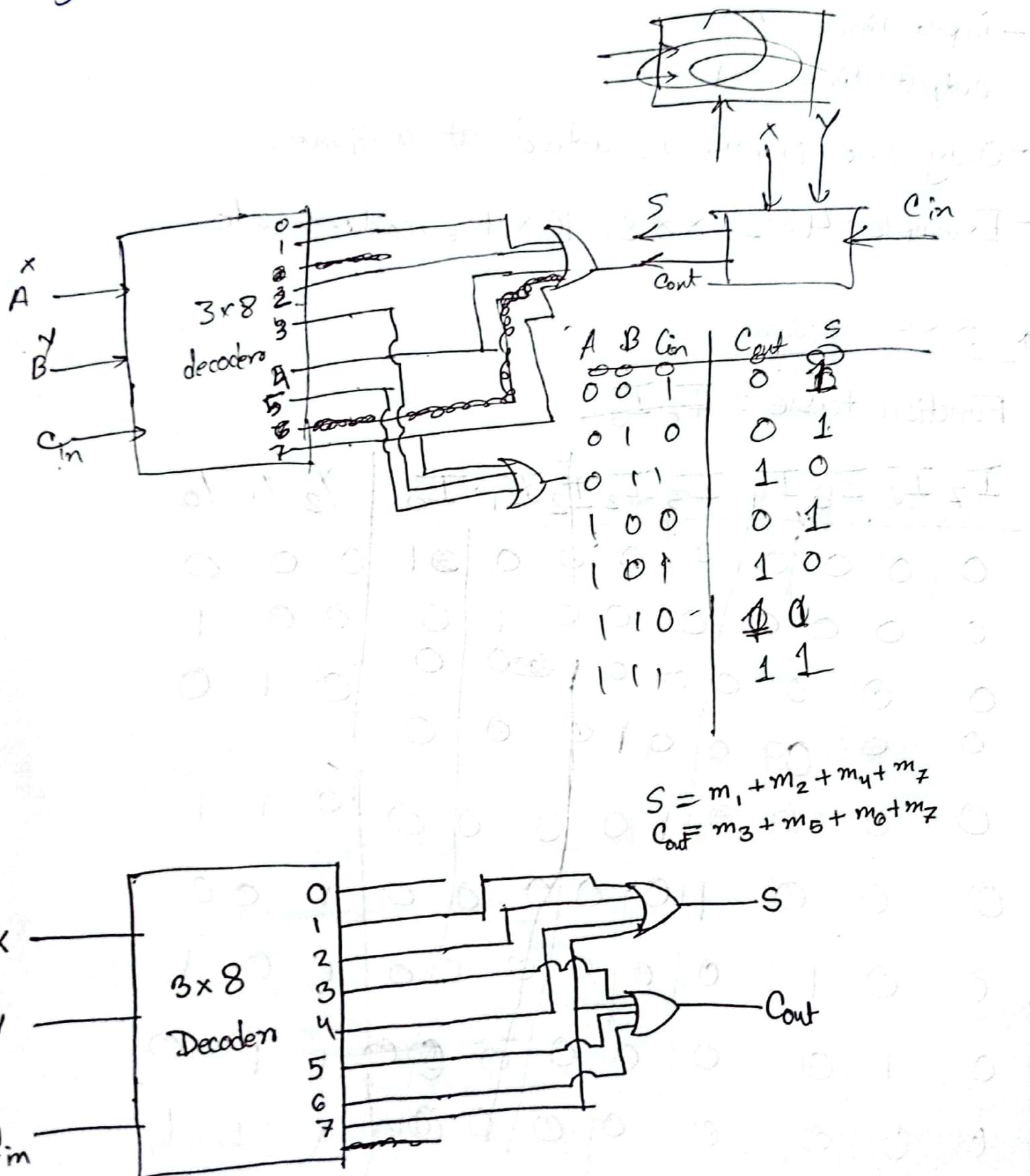
* For theory -

if nothing is presented,
use active high

For lab,
use active low

PW: Implement 1 bit full adder using 3×8 decoders

using 3×8 decoders & basic gates.



Encoder:

- Does reverse operation of decoders
- input lines: 2^n
- output lines: n
- Only one input is active at a time.
- Example: 4×2 , 8×3 , 16×4 encoders etc

* 8×3 encoder:Function table: ~~F₇ F₆ F₅~~

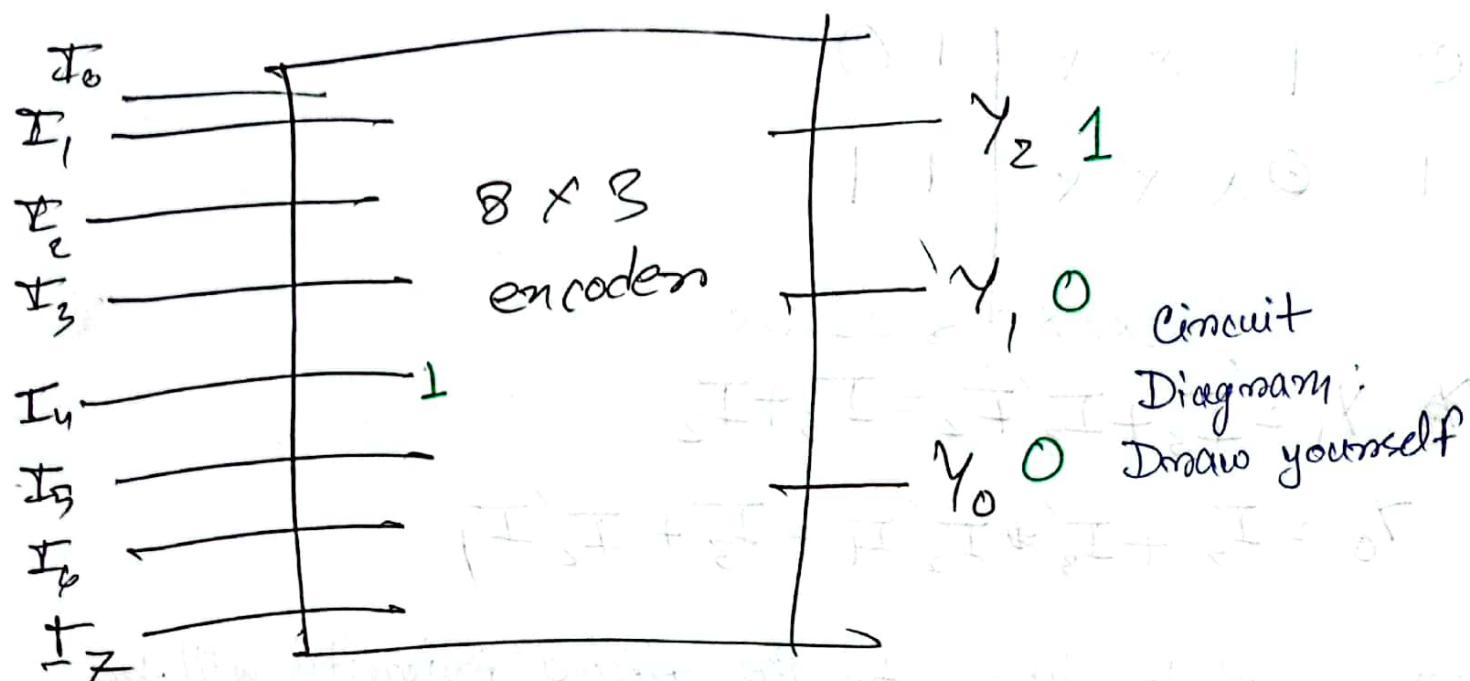
| I_7 | I_6 | I_5 | I_4 | I_3 | I_2 | I_1 | I_0 | y_2 | y_1 | y_0 |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |

Output Equations:

$$Y_2 = I_7 + I_6 + I_5 + I_4$$

$$Y_1 = I_7 + I_6 + I_3 + I_2$$

$$Y_0 = I_7 + I_5 + I_3 + I_1$$



Priority Encoders:

- Encoder with priority function. Multiple inputs may be true simultaneously, higher priority inputs gets the precedence.

* 4-input encoders:

Priority order: $I_3 > I_2 > I_1 > I_0$

Function table:

| I_3 | I_2 | I_1 | I_0 | y_1 | y_0 |
|-------|-------|-------|-------|-------|-------|
| 0 | 0 | 0 | 0 | x | x |
| 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | x | 0 | 1 |
| 0 | 1 | x | x | 1 | 0 |
| 1 | 0 | x | x | 1 | 1 |

$\rightarrow 1xxx \rightarrow 01xx$

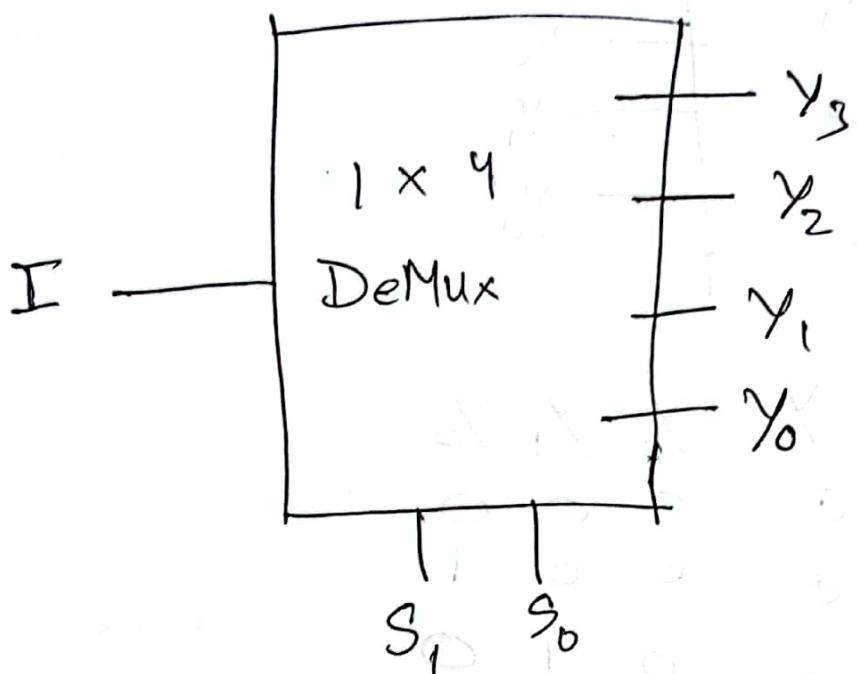
$$\therefore y_1 = I_3 + I_3' F_2 = I_3 + I_2$$

$$y_0 = I_3 + I_3' + I_2' F_1 = I_3 + I_2' I_1$$

* When multiple have 1, the selected priority will be selected.

* DeMultiplexen:

- A circuit receives information for from a single and directs it to one of 2^n possible output lines



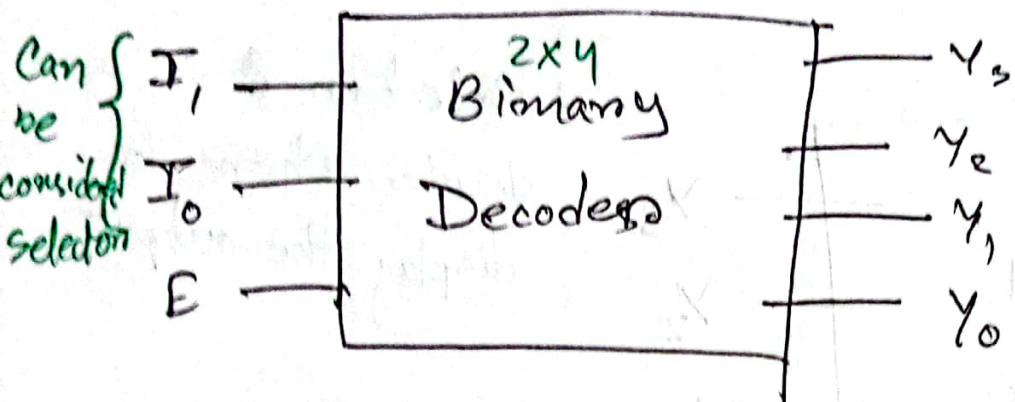
* Selection

decides where to display the output

Function Table:

| s_1 | s_0 | y_3 | y_2 | y_1 | y_0 | |
|-------|-------|-------|-------|-------|-------|--|
| 0 | 0 | 0 | 0 | 0 | I | |
| 0 | 1 | 0 | 0 | I | 0 | |
| 1 | 0 | 0 | I | 0 | 0 | |
| 1 | 1 | I | 0 | 0 | 0 | |

* A decoder with enable input can function as a demultiplexer



| E | I_1 | I_0 | y_3 | y_2 | y_1 | y_0 |
|---|-------|-------|-------|-------|-------|-------|
| 0 | x | x | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 0 | 0 |

* Decoders and DeMux have same IC

For fixed: Easy \rightarrow Use as
Hard \rightarrow Modify the Easy ones

Quine-McCluskey / Tabulation Method:

Simplify the following Boolean Function by using Tabulation method:

$$F(a, b, c, d) = \sum (0, 12, 3, 6, 2, 8, 9, 10, 14)$$

Soln: $F(a, b, c, d) = \sum (0, 1, 2, 5, 6, 7, 8, 9, 10, 14)$

$$= \sum(0000, 0001, 0010, 0101, 0110, 0111, \\ 1000, 1001, 1010, 1110)$$

Step 1: Determination of prime implicants (PIs):

| | | Column I | | | | Column - II with 3 bits same implicants | | | | Column - III 1 bit | | | |
|------------|---------|-----------|---|---|---|---|--------------|----------------|---|--------------------|---|---|---|
| | | a | b | c | d | a | b | c | d | a | b | c | d |
| Group 0: | a b c d | (0, 1) | 0 | 0 | - | ✓ | (0, 1, 8, 9) | - | 0 | 0 | * | | |
| Group 0: | 0 0 0 0 | (0, 2) | 0 | 0 | - | 0 | ✓ | (0, 2, 8, 10) | - | 0 | 0 | * | |
| Group 1: | 1 0 0 1 | (0, 8) | 0 | - | 0 | 0 | ✓ | (2, 6, 10, 14) | 0 | - | 1 | 0 | |
| 2 | 0 0 1 0 | (1, 5) | 0 | 0 | - | 0 | * | | | | | | |
| 8 | 1 0 0 0 | (2, 1, 9) | 0 | - | 0 | 1 | ✓ | | | | | | |
| Group 2: | 0 1 0 1 | (2, 6) | 0 | - | 0 | 1 0 | ✓ | | | | | | |
| 8 | 0 1 1 0 | (2, 10) | - | 0 | 0 | 1 0 | ✓ | | | | | | |
| 9 | 1 0 0 0 | (8, 9) | 1 | 0 | 0 | - | ✓ | | | | | | |
| 10 | 1 0 1 0 | (8, 10) | 1 | 0 | - | 0 | ✓ | | | | | | |
| Group 3: 7 | 0 1 1 1 | (8, 10) | 1 | 0 | - | 0 | ✓ | | | | | | |
| 14 | 1 1 1 0 | (5, 7) | 0 | 1 | - | 1 | * | | | | | | |
| | | (6, 7) | 0 | 1 | 1 | - | * | | | | | | |
| | | (6, 14) | - | 1 | 1 | 0 | * | | | | | | |
| | | (10, 14) | 1 | - | 1 | 0 | ✓ | | | | | | |

Step - 2: Selection of Prime Implicants:

Prime implicants:

$$a'c'd + a'b'd + a'bc + b'c' + b'd' + cd'$$

(1, 5)

(0, 1, 8, 9) (0, 2, 8, 10) (0, 2, 6, 10, 14)

Step - 2: Selection of Prime Implicants:

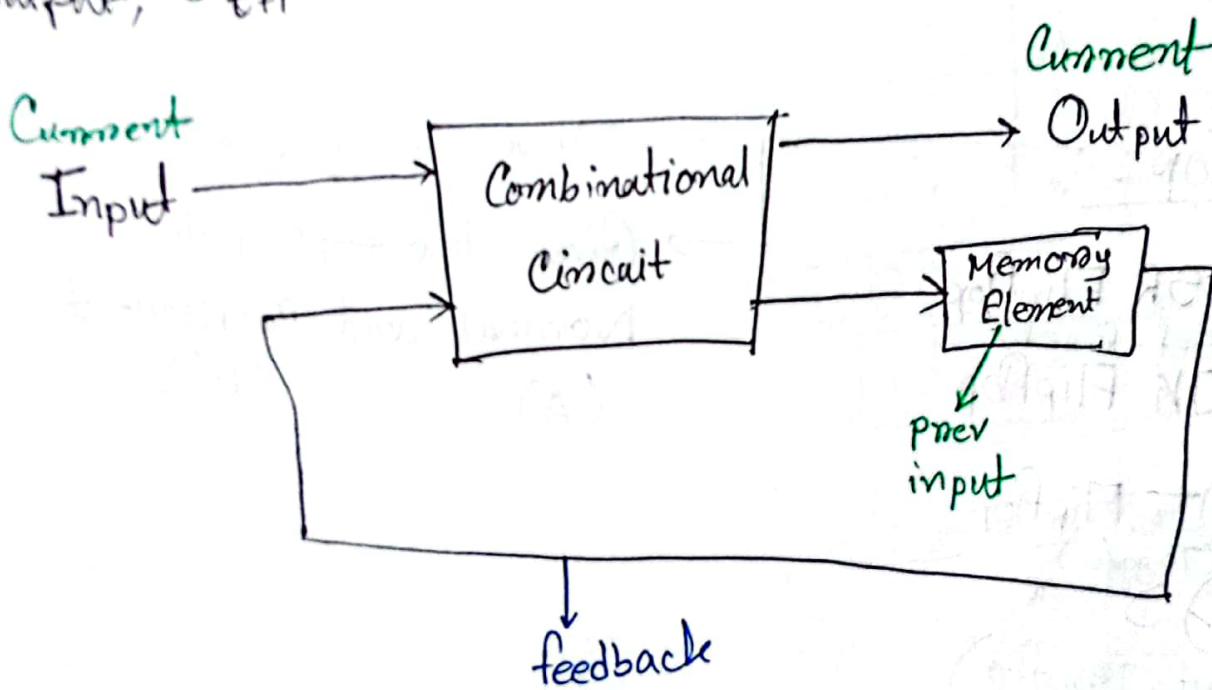
| PIs com | PIs | a | b | c | d | 0 | 1 | 2 | 5 | 6 | 7 | 8 | 9 | 10 |
|----------------|-----------|---|---|---|---|---|---|---|---|---|---|---|---|----|
| (0, 1, 8, 9) | 0 - 0 0 - | x | x | | | | | | | | | x | | |
| (0, 2, 8, 10) | - 0 - 0 - | x | | | | | x | | | | x | | x | |
| (2, 6, 10, 14) | - - 1 0 | | | | x | | | x | | | | x | | |
| (1, 5) | 0 - 0 1 | | x | | | | | | x | | | | | |
| (5, 7) | 0 1 - 1 | | | x | | | | x | | x | | | | |
| (10) | 0 1 1 - | | | | x | | | | x | x | | | | |

$$\text{Ans. } b'c' + cd' + a'b'd$$

Synchronous Sequential Logic

Current Output = $f(\text{previous output}, \text{current input})$

Next Output, $Q_{t+1} = f(\text{current output } Q_t, \text{input})$



* Flip Flop(FF).: Memory element.

Sequential Circuit

Asym
Asynchronous
changing sequence of
input changes order
of output

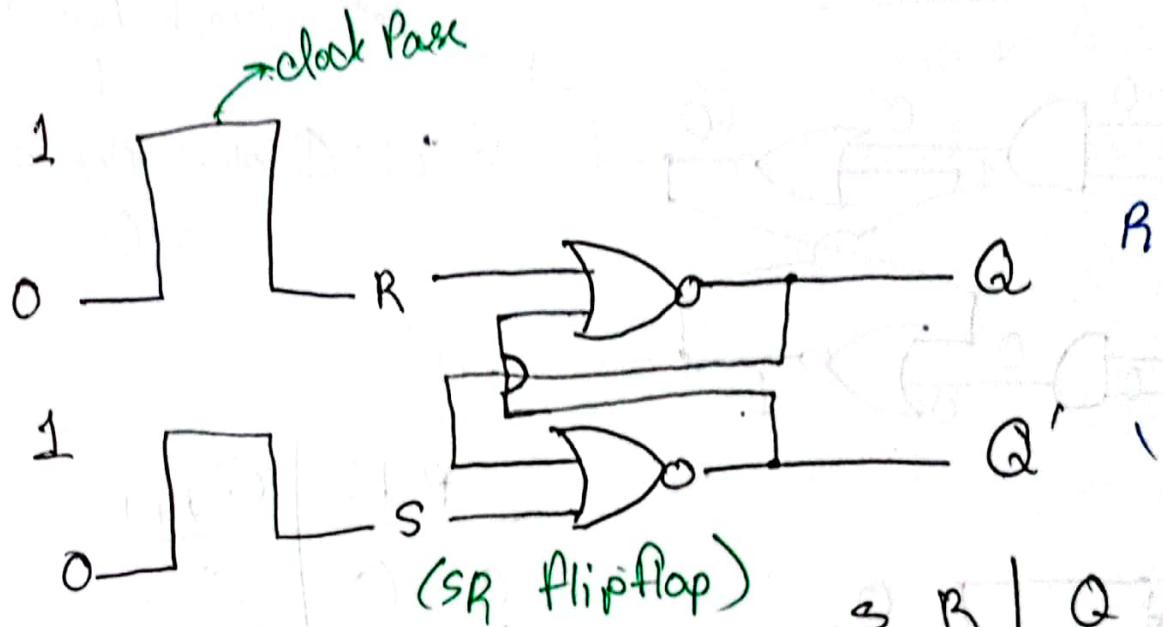
Synchronous
changing input time
changes output

Flip Flop

- i) SR Flipflop
(Set - Reset)
- ii) JK Flipflop
- iii) T Flipflop
(Toggle)
- iv) D
(Data Transfer)

→ Stores only 1 bit
→ Gives two outputs;
Normal and Complement
(A) (A')

* Basic Flip Flop: * Self Study: Construct using NAND



(SR flip flop)

base case
This is responsible
for the new
outputs

$$R : Q :: S : Q'$$

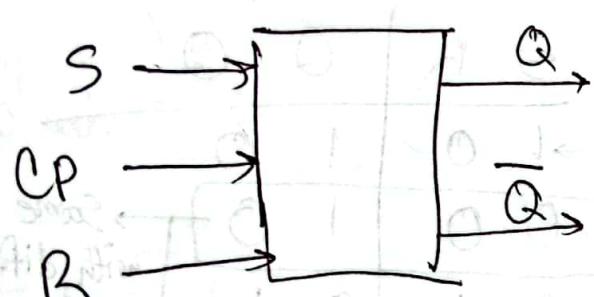
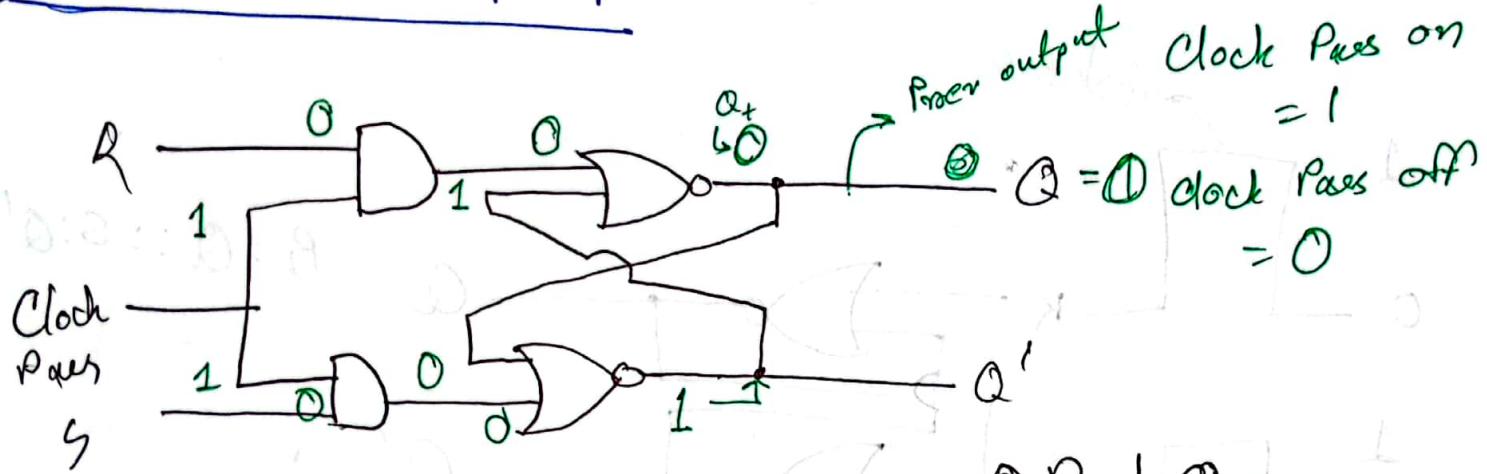
| S | R | Q | Q' |
|---|---|---|----|
| 1 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 1 | 1 | 0 | 0 |

Same in
with diff.
output
↓
Because of
previous
input

Invalid Condition

↳ For SR using
NAND, S and R
NOR, both cannot
be 1.

i. Clocked SR Flip Flop:



Characteristics Table

| Q_t | S | R | Q_{t+1} |
|-------|---|---|----------------|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | In determinate |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | In determinate |

| S | P | R | Q_{t+1} |
|---|---|---|-----------|
| 0 | 0 | 0 | Q |
| 0 | 1 | 0 | Q' |
| 1 | 0 | 0 | 1 |
| 1 | 1 | x | - |

→ Characteristics Table

$$Q_{t+1} = S + R'Q$$

Excitation Map Table: (SR)

* Using desired output
to find the required
input

| Q_+ | Q_{++1} | S | R |
|-------|-----------|---|---|
| 0 | 0 | 0 | X |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | X | 0 |

Using Characteristics Table

* Self Study: Memorize all flip flop Excitation Table
+ the rest of FFs

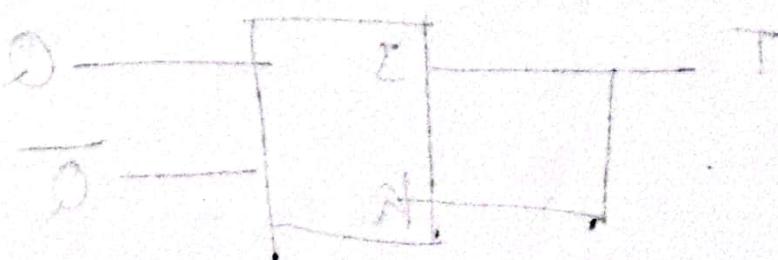
$$D^N + D^S = 1, D^N \neq 1$$

$$D \oplus T = D T + D' T = 1, D \neq T$$

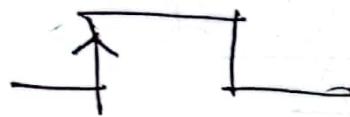
$$D^N + D^S < D$$

$$D T + D' T = 1$$

$$D \oplus T =$$



Triggering Flipflops:



* Triggering Point: When output changes.

\overrightarrow{CP} → positive edge

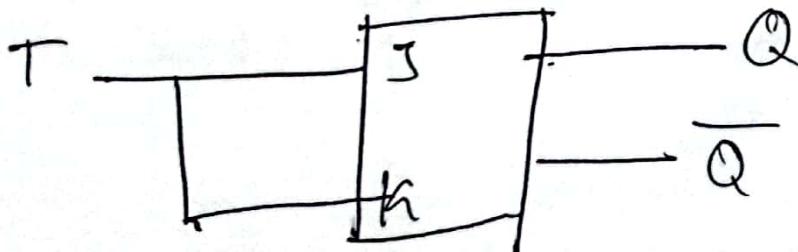
$\overrightarrow{\overline{CP}}$ → negative edge

Conversion:

* JK to T:

$$JK: Q_{++1} = JQ' + K'Q$$

$$T: Q_{++1} = TQ' + T'Q = T \oplus Q$$

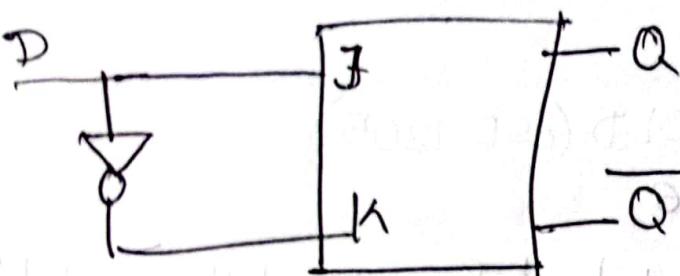


Hence,

$$\begin{aligned} Q_{++1} &= JQ' + K'Q \\ &= TQ' + T'Q \\ &= T \oplus Q \end{aligned}$$

* JK to D:

$$D : Q_{t+1} = D$$



$$Q_{t+1} = J\bar{Q}' + K'Q$$

$$= \cancel{J\bar{Q}'} +$$

$$= DQ' + (\bar{D})'Q$$

$$= DQ' + DQ$$

$$= D(Q' + Q)$$

$$= D\bar{Q}$$

Design Procedure of Sequential Ckt:

- i) Problem is stated \rightarrow text/state diagram/state table
 - ii) Obtain the state table
 - iii) State reduction \rightarrow Not required for DLD (CSE 1205)
 - iv) Assign binary values to each state.
 - v) Determine the ~~#~~ no. of FFs needed & assign letter symbol to each
 - vi) ~~Determine~~ ^{Choose} the type of FF to be used.
 - vii) Derive the Excitation map.
 - viii) Derive the ckt output function & FF input function.
 - ix) Draw the logic diagram.
- no. of bits = no. of FFs
 each IC has 2 bits
 7470 \rightarrow JK flipflop

Chap - 6

Registers & Counters

Counters: → keeps a count

A seqntl ckt that goes through a preordred sequence of states upon the application of input pulses is called a counter.

- Counting occurrences of an event
- Generates timing sequence
- Tracks elapsed time between events.

→ Binary Counter follows binary sequence

↳ n-bit binary counter needs n FFs and counts 0 to $2^n - 1$

↓

3 bit
binary
counter

3 FFs

$0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6 \rightarrow 7$

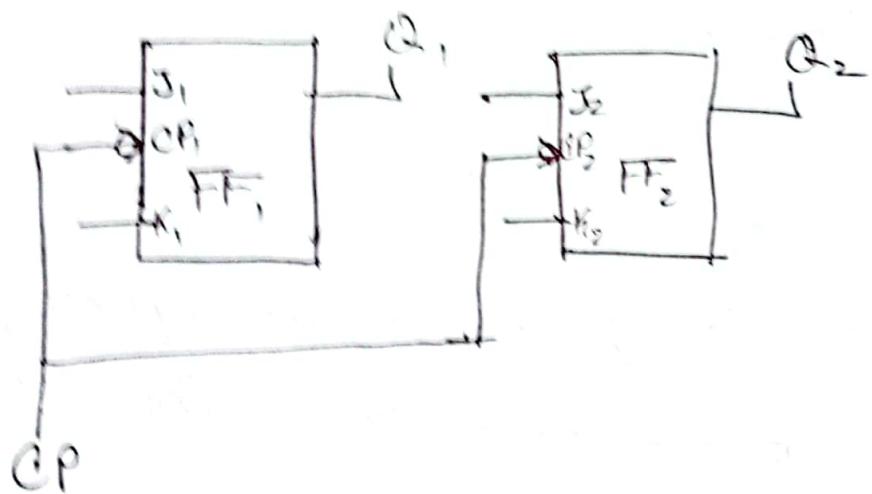
can start from anywhere
but must follow pattern.

Counter is of 2 types:

i) Synchronous → same clock pulse

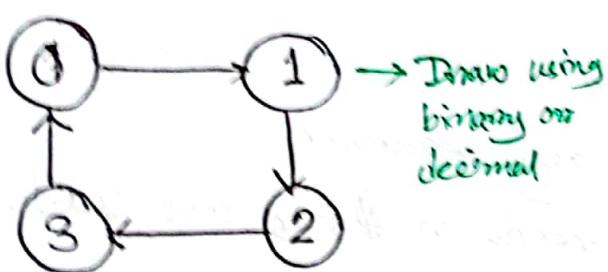
ii) Asynchronous → different clock pulse.

Q Design a 2-bit synchronous binary counter.



* $\overline{SD} \leftarrow 0$
Sets output to 1
 $\overline{RD} \leftarrow 0$
Sets output to 0

Keep them disabled by 5V to get different values



State Diagram

| Present State | | Next State | |
|---------------|-------|------------|-------|
| Q_2 | Q_1 | Q_2 | Q_1 |
| 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |

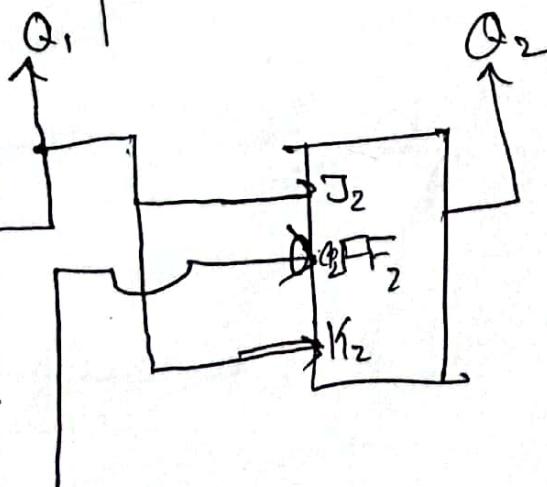
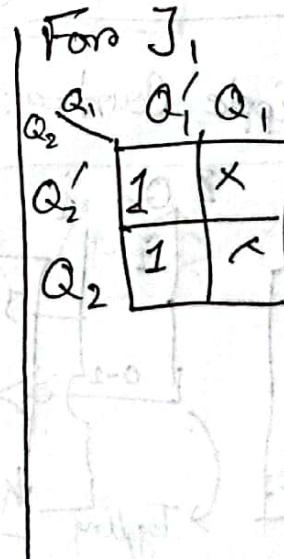
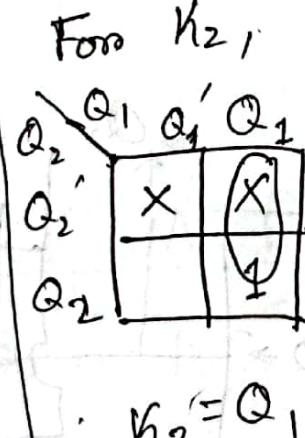
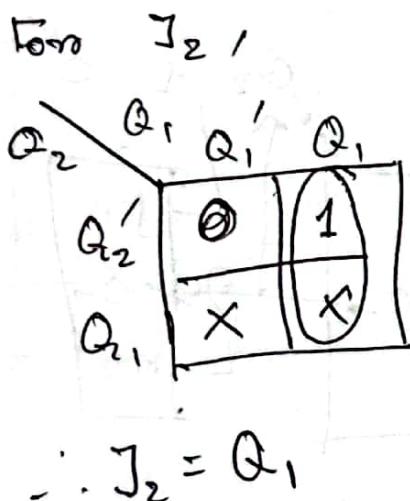
→ rule table for every UNIQUE combination/state

Excitation Table:

When making input eq's
ALWAYS consider present state

| Present State | | Next State | | FF | | | |
|---------------|-------|------------|--------|-------|-------|--------|-------|
| Q_2 | Q_1 | Q_2' | Q_1' | J_2 | K_2 | $-J_1$ | K_1 |
| 0 | 0 | 0 | 1 | 0 | x | 1 | x |
| 0 | 1 | 1 | 0 | 1 | x | x | 1 |
| 1 | 0 | 1 | 1 | x | 0 | 1 | x |
| 1 | 1 | 0 | 0 | x | 1 | x | 1 |

FF2



C-19 (n-11)

OVERVIEW

Asynchronous Counters

Different Clock Pcs

* Also known as Ripple Counter

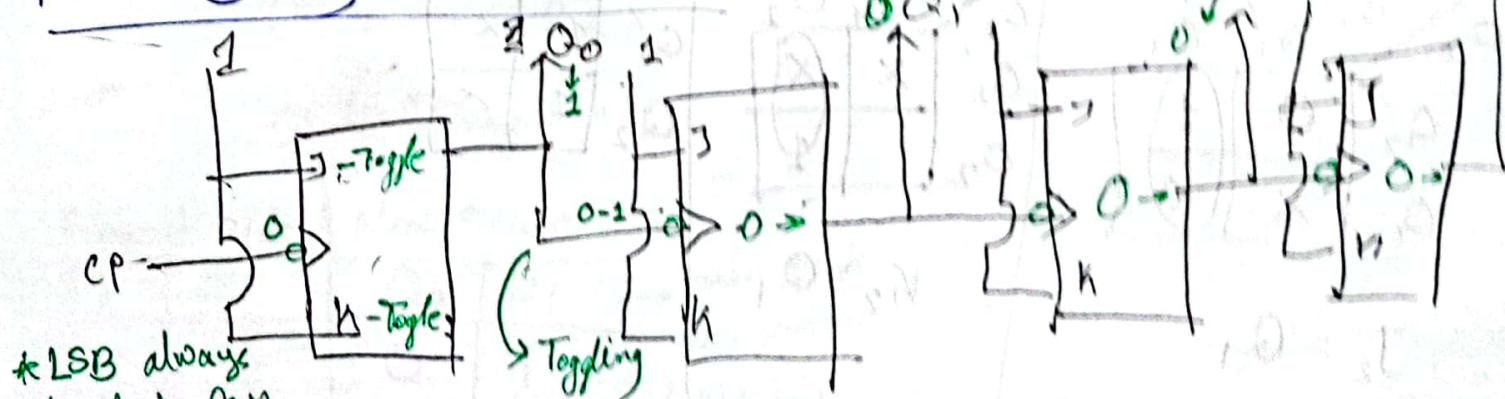
* Different Clock Pcs in FFs

→ Slower

- Not necessarily the same
- Use different method to solve
- Working Principle is necessary

Binary Ripple Counter : (slide is incomplete)

4 bit Binary Ripple Counter:



* LSB always gets clock-pcm

J.K Timing

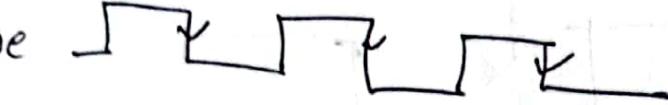
| J | K | Q _{t+1} |
|---|---|------------------|
| 0 | 0 | Q _{t+1} |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | Q _t |

* Timing Diagram: use scale

→ Draw CP

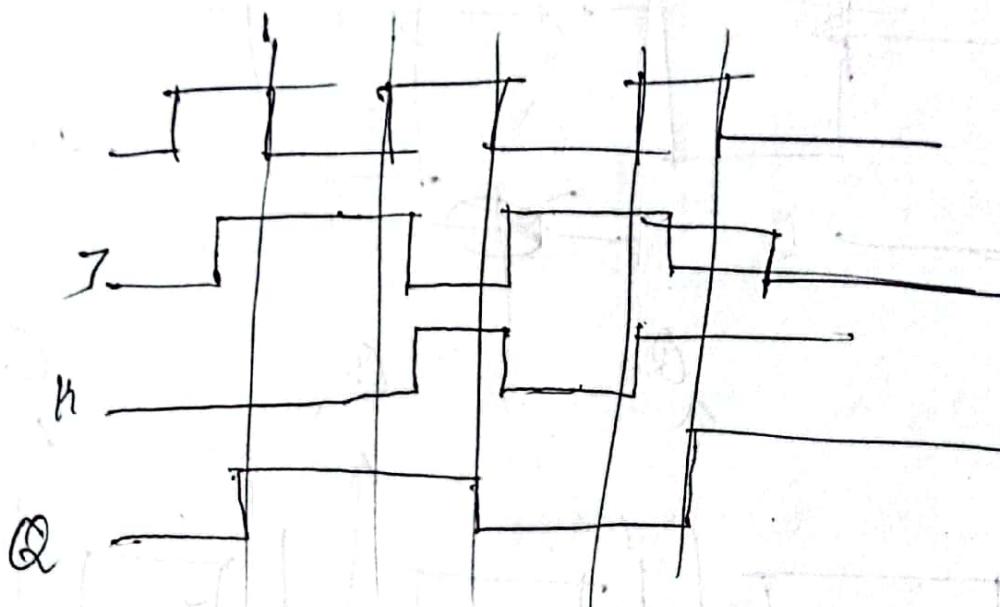


→ Identify where changes occur



→ change accordingly

→ check J and K

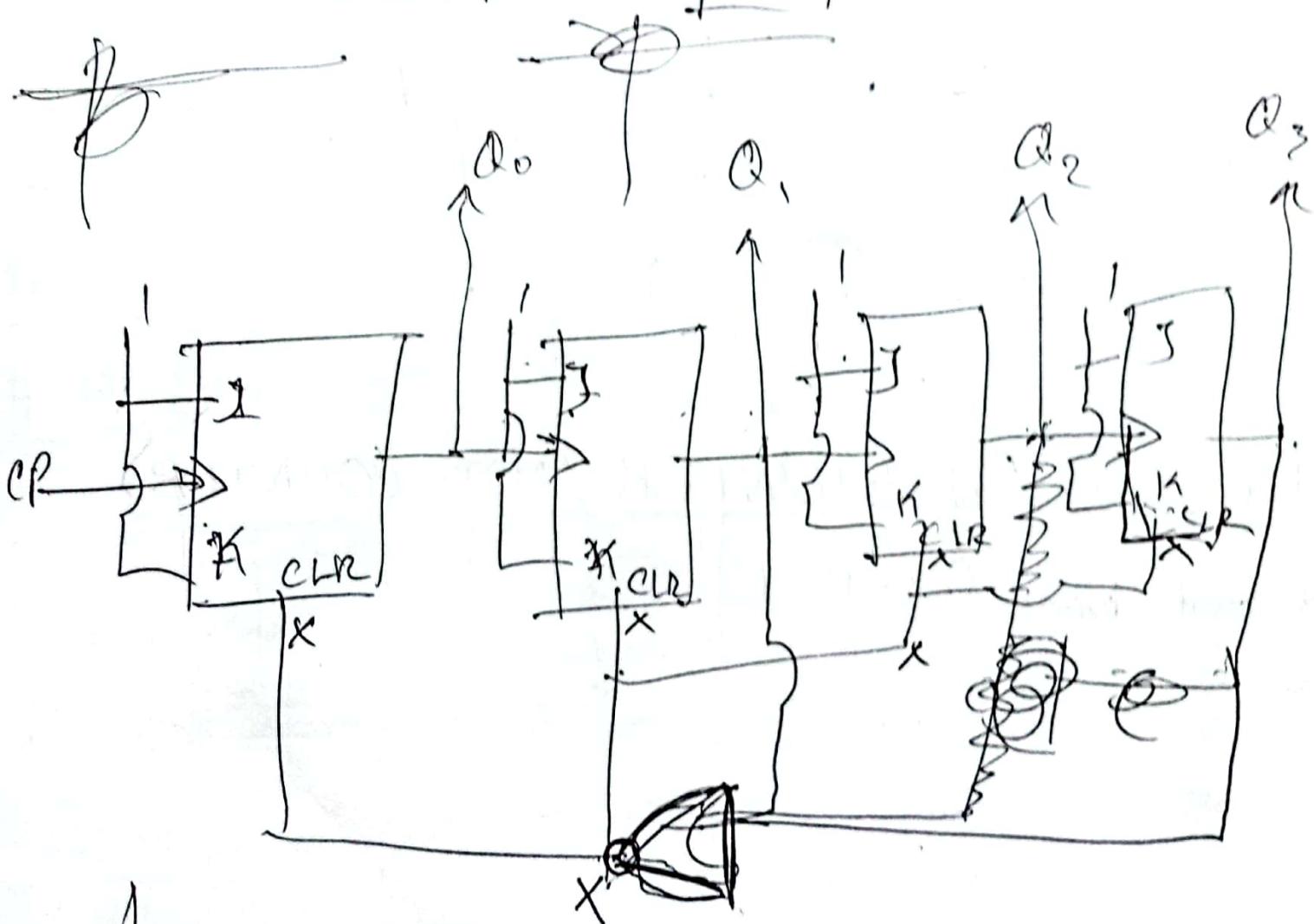
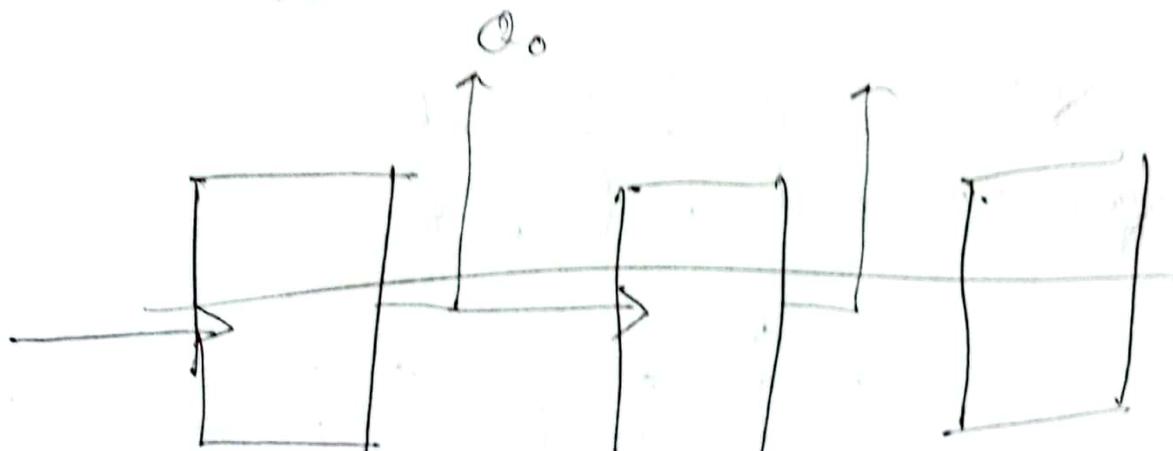


* MODULO N COUNTER (MOD COUNTER)

* Counts from 0 to $N - 1$

* MOD-10 Counter

$Q_3 Q_2 Q_1 Q_0$



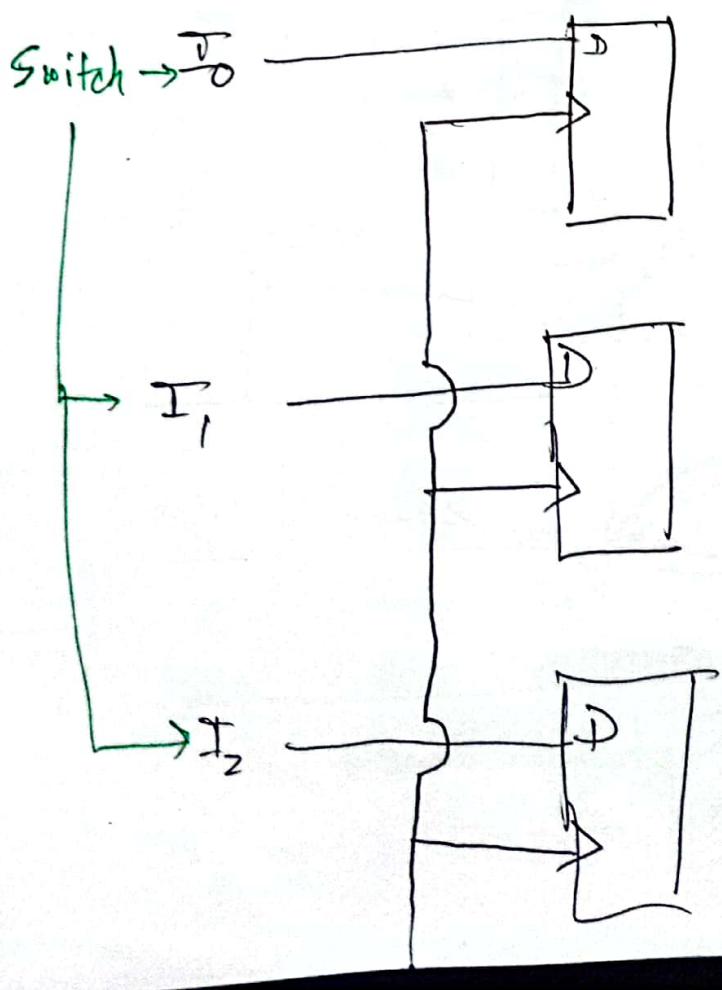
1000
1010
1011
1001

* Self Study: Do the decade converter maths

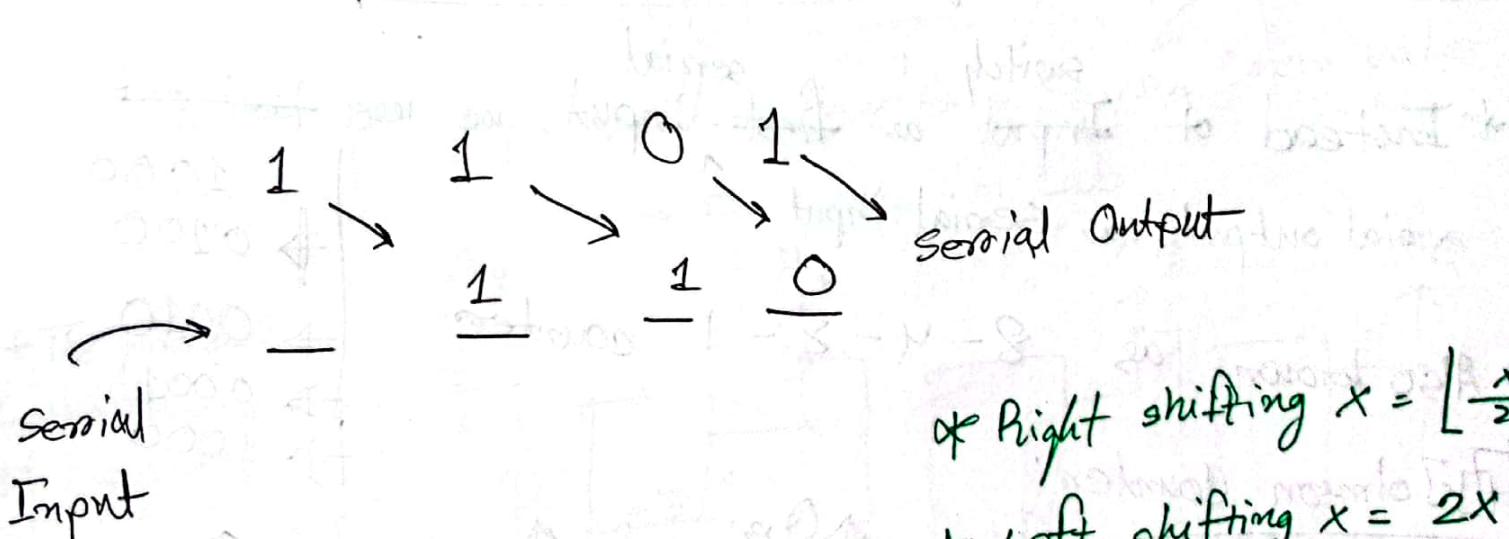
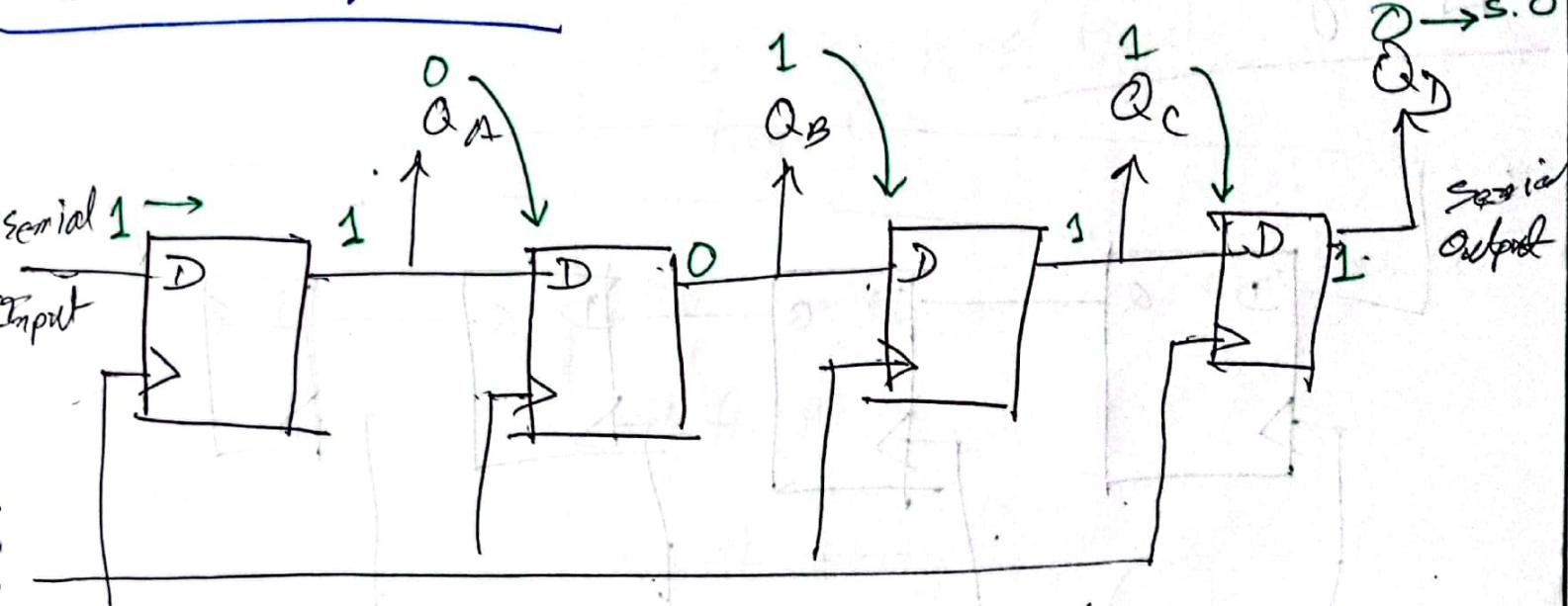
C-20(10-12)

07/07/24

Registers → will be in quiz-4



Shift Registers :

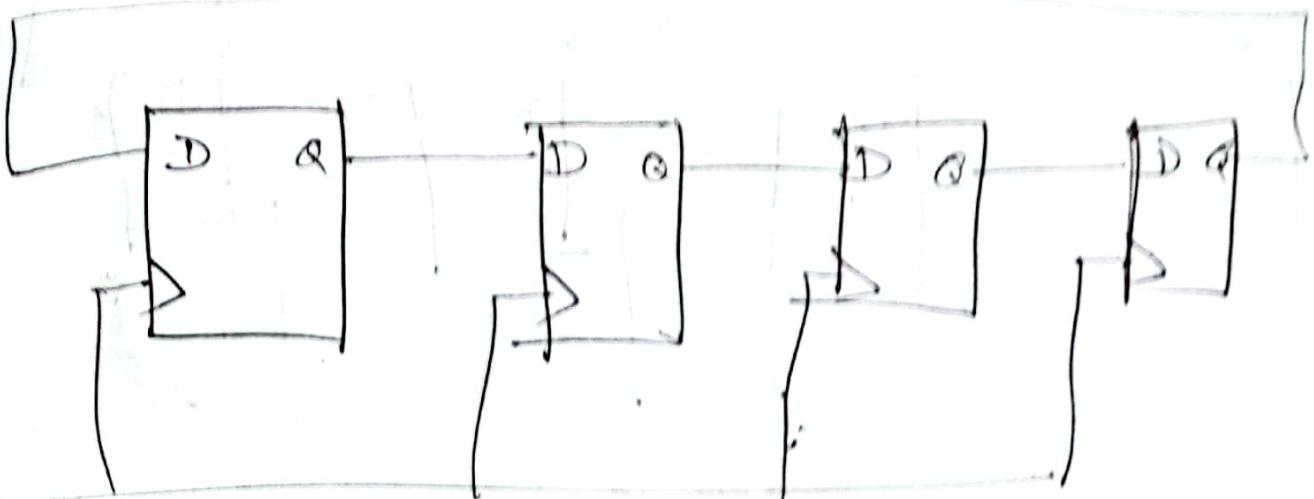


* Right shifting $x = \lfloor \frac{x}{2} \rfloor$

+ Left shifting $x = 2x$

* Serial Input can be whatever we want

Ring Counter:



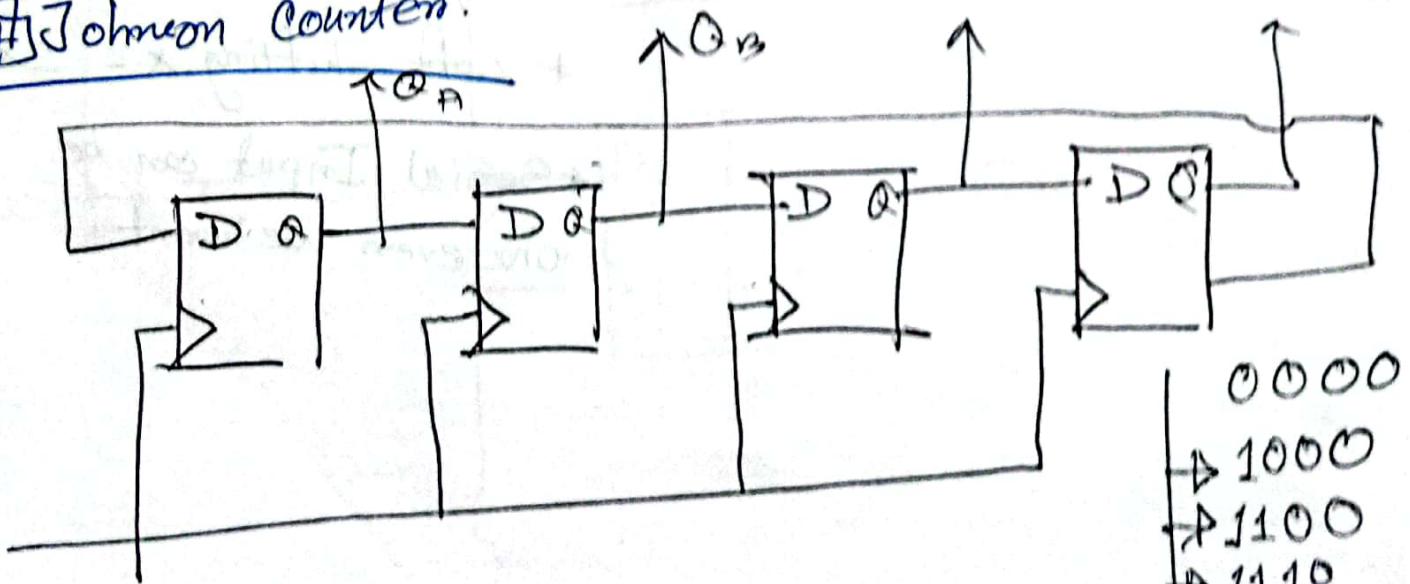
* Instead of switch input as first serial input, we use last out serial output as serial input

| |
|--------|
| 1000 |
| ↓ 0100 |
| ↓ 0010 |
| ↓ 0001 |
| ↓ 1000 |

8-4-2-1 counter

* Also known as

Johnson Counter:



| |
|--------|
| 0000 |
| ↓ 1000 |
| ↓ 1100 |
| ↓ 1110 |
| ↓ 1111 |
| ↓ 0111 |
| ↓ 0011 |
| ↓ 0001 |

Universal Shift Register

$S_1 \quad S_0$

Reg Op

Table not fixed

0 0

No change → Gives old result

0 1

Shift Right

1 0

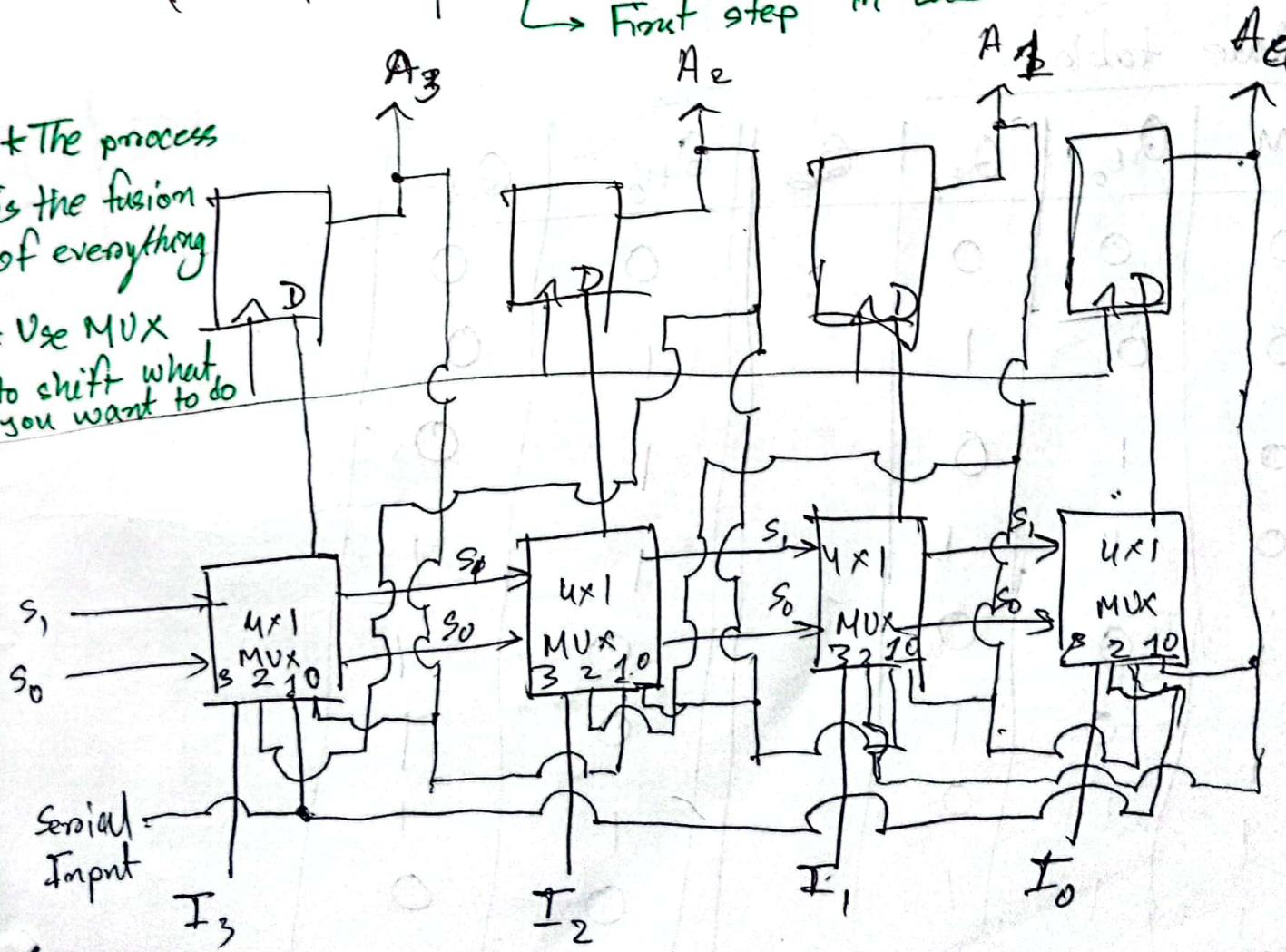
Shift Left

1 1

Parallel Load → Stores new value
↳ First step in Lab

* The process
is the fusion
of everything

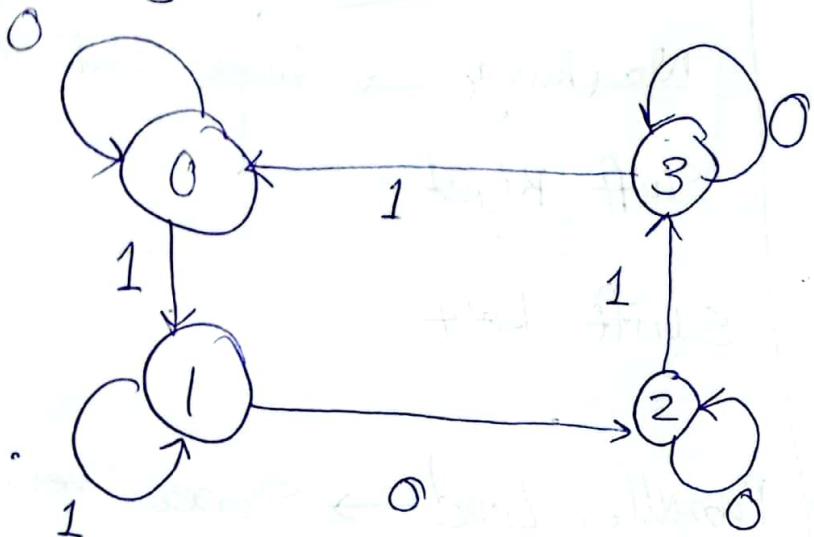
* Use MUX
to shift what
you want to do



10/07/24

C-21 (W-12)

Design a ^{synchronous} sequential circuit for the following problem (using JK flip-flop)



State table:

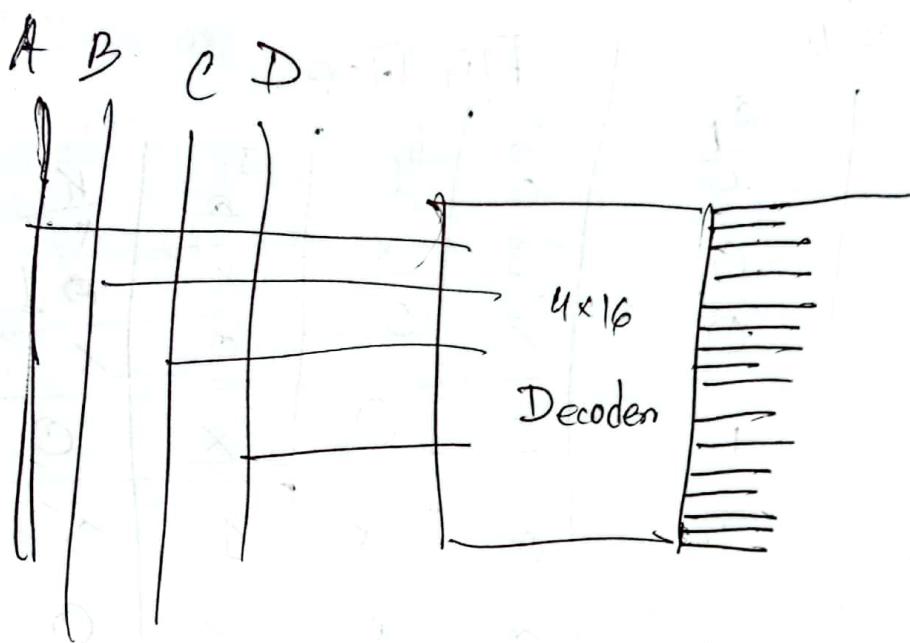
| M | Q_{t_0} | Q_{t_1} | Q_t | Q_{t+0} | Q_{t+1} |
|---|-----------|-----------|-----------------------------|-----------|-----------|
| 0 | 0 | 0 | | 0 | 0 |
| 0 | 0 | 1 | | 1 | 0 |
| 0 | 1 | 0 | | 1 | 1 |
| 0 | 1 | 1 | | 1 | 1 |
| i | 0 | 0 | | 0 | 1 |
| i | 0 | 1 | | 1 | 1 |
| i | 1 | 0 | | 1 | 0 |
| i | 1 | 1 | | 0 | 1 |

Excitation Table :

| Switch | Present State | | Next State | | Flip Flop | | | |
|--------|---------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| | M | Q ₀ | Q ₁ | Q ₀ | Q ₁ | J ₀ | K ₀ | J ₁ |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | x | 0 |
| 0 | 0 | 0 | 1 | 1 | 0 | 1 | x | 1 |
| 0 | 1 | 0 | 0 | 1 | 0 | x | 0 | x |
| 0 | 1 | 1 | 1 | 1 | 1 | x | 0 | x |
| 1 | 0 | 0 | 0 | 0 | 1 | 0 | x | 1 |
| 1 | 0 | 1 | 0 | 1 | 1 | 0 | x | 0 |
| 1 | 1 | 0 | 1 | 1 | 1 | x | 0 | 1 |
| 1 | 1 | 1 | 0 | 0 | 0 | 1 | x | 1 |

∴ J₀

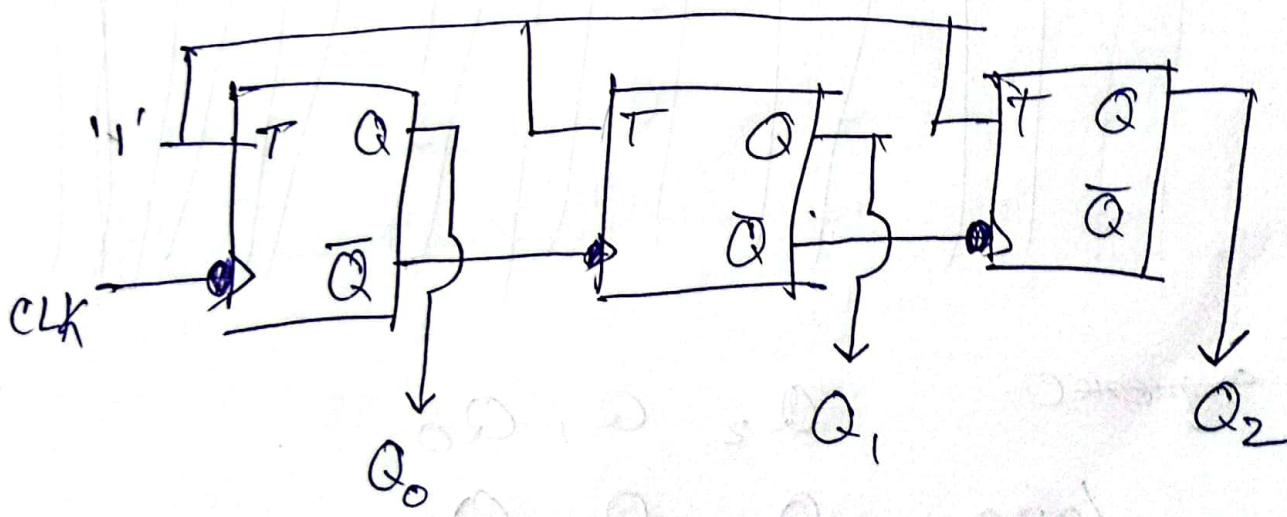
| M \ Q ₀ , Q ₁ | 0, 0 | 0, 1 | 1, 0 | 1, 1 |
|-------------------------------------|------|------|------|------|
| 0 | 1 | x | x | x |
| 1 | x | x | 1 | 1 |



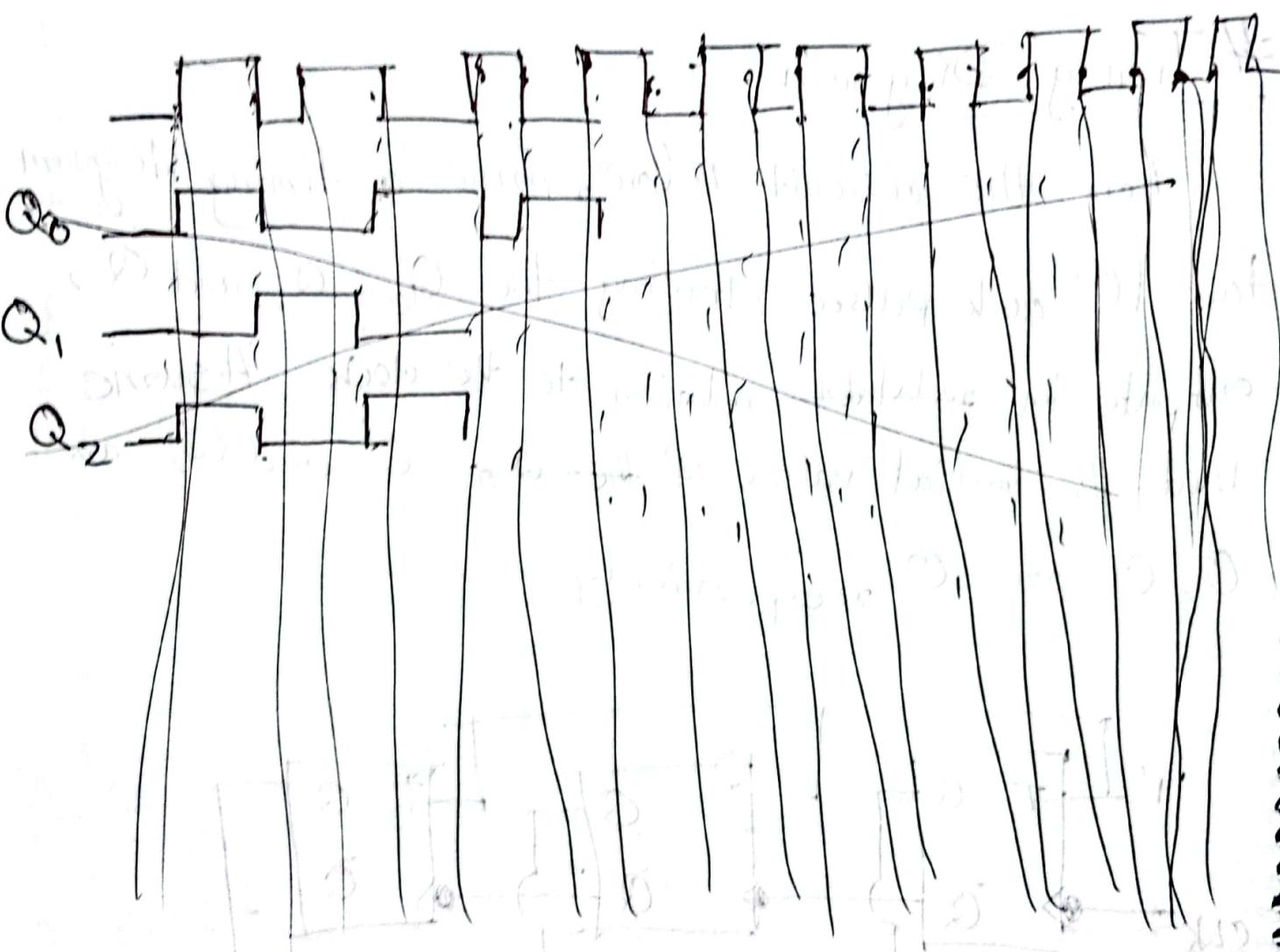
| Jn | A | B | C | D | P | Q | R | S | out |
|------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 0. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | - | - |
| 0 | 1 | 0 | 1 | 0 | 0 | 1 | - | - | - |
| 0 | 1 | 1 | 0 | 0 | 0 | 0 | - | - | - |
| 0 | 1 | 1 | 1 | 0 | 0 | - | - | - | - |
| 1 | 0 | 0 | 0 | 0 | 0 | - | - | - | - |

Timing Diagram

For the circuit below, draw a timing diagram for 10 clock pulses showing the Q_0 , Q_1 , and Q_2 outputs in ~~relative~~ relation to the clock. Assume that the initial values of the Q_0 , Q_1 , and Q_2 are 0, 0 and 0 respectively.



- (i) Notice the CLK (negative/positive)
- (ii) " " FF
- (iii) " " MSB and LSB
- (iv) Find the sequence first draw later.

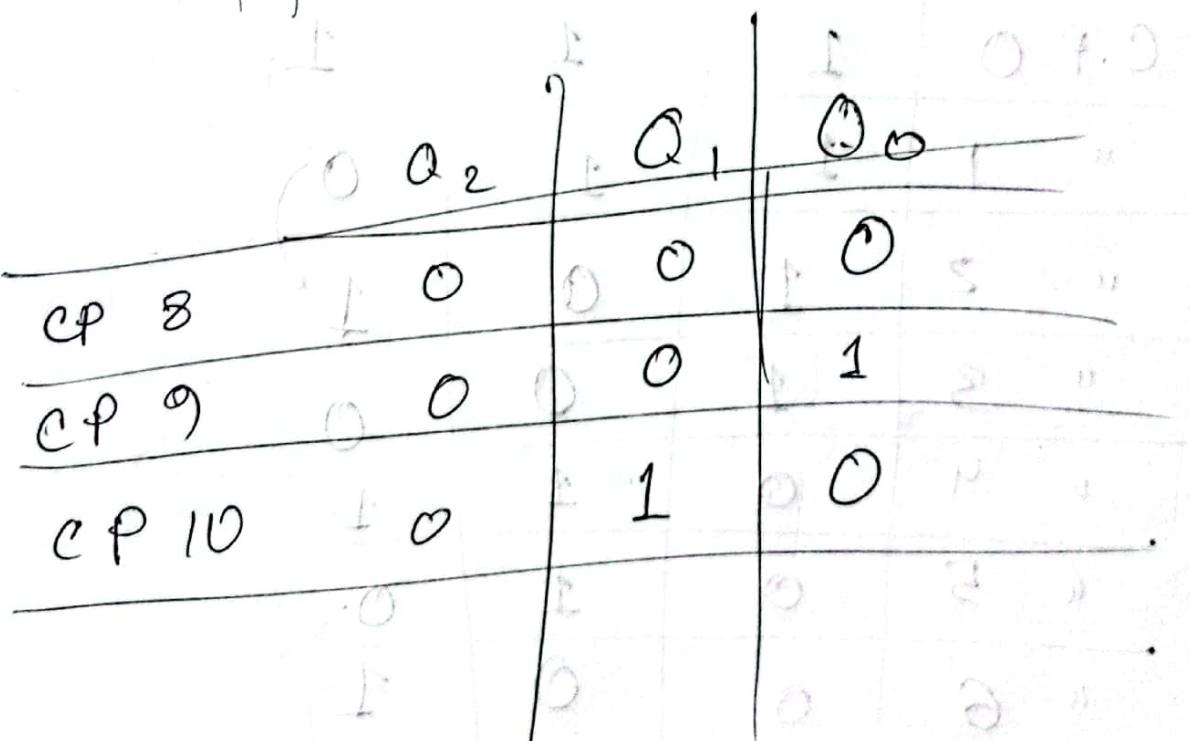
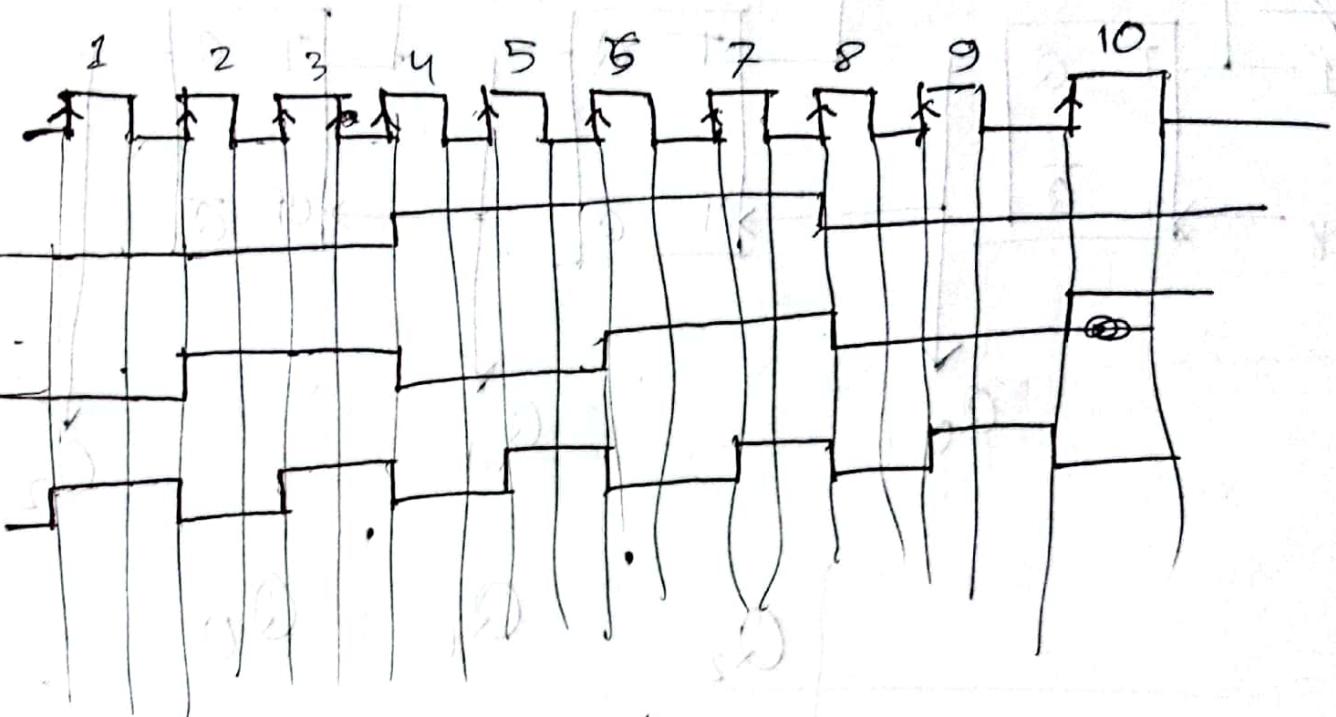


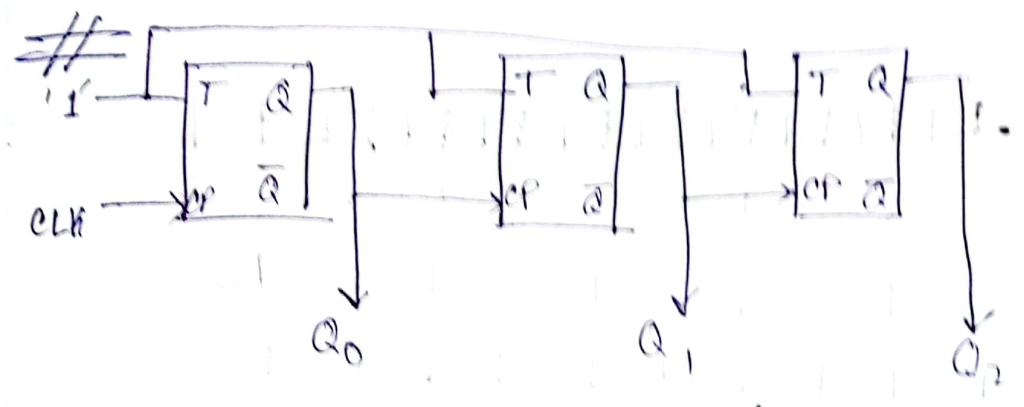
Output sequence

$Q_2 \quad Q_1 \quad Q_0$

CLK Initial /CP.O:

| CP 1 | 0 | 0 | 0 |
|------|---|---|---|
| CP 2 | 0 | 1 | 0 |
| CP 3 | 0 | 0 | 1 |
| CP 4 | 1 | 0 | 0 |
| CP 5 | 1 | 0 | 1 |
| CP 6 | 1 | 1 | 0 |
| CP 7 | 0 | 1 | 1 |





| C.P O | Q_2 | Q_1 | Q_0 |
|-------|-------|-------|-------|
| " 1 | 1 | 1 | 1 |
| " 2 | 1 | 0 | 1 |
| " 3 | 1 | 0 | 0 |
| " 4 | 0 | 1 | 1 |
| " 5 | 0 | 1 | 0 |
| " 6 | 0 | 0 | 1 |
| " 7 | 0 | 0 | 0 |
| " 8 | 1 | 1 | 1 |
| " 9 | 1 | 1 | 0 |
| " 10 | 1 | 0 | 1 |

Q_2
 Q_1
 Q_0

