TOPIC NAME : ~~C-1~~ W - 1

DAY : C - 1

TIME :

DATE : 3 / 12 / 24

CSE 2201 : Numerical Methods

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* "Mostly Mathematical. But algo related"

* Books : * Balagurusamy – Numerical Methods

(Preferably Bought)

G. Shanker Rao – Numerical Analysis

* **Quizes:** ① from Moodle (1.2 PTSD yay)

GOOD LUCK

Bisection Method : (BG Ch-6)

$\rightarrow$ Bisection
$\rightarrow$ False Position
$\rightarrow$ Newton Raphson
$\rightarrow$ Secant
$\rightarrow$ Fixed point

* Used to find roots of Non-Linear Eq^n
  $\hookrightarrow$ exponent > 1

Find a root of $x^2 - 4x - 10 = 0$

$$\therefore x_{max} = \sqrt{\left(\frac{-4}{1}\right)^2 - 2\left(\frac{-10}{1}\right)}$$

$$= 6$$

$\therefore$ The root is between, -6 and 6

x	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
$f(x)$	50	35	22	11	2	-4	-10	-13	-14	-13	-10	-5	2

has one root $\longrightarrow$ intersected x-axis in between $\longleftarrow$ has another root

Let us take $x_1 = -2$ and $x_2 = -1$

$$\therefore x_0 = \frac{-2-1}{2} = -1.5$$

$$\therefore f(-1.5) = \cdots$$

* Take opposite sides in interval until they are roughly similar.

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Num Meth Lab

Marks

Attendance : 20%

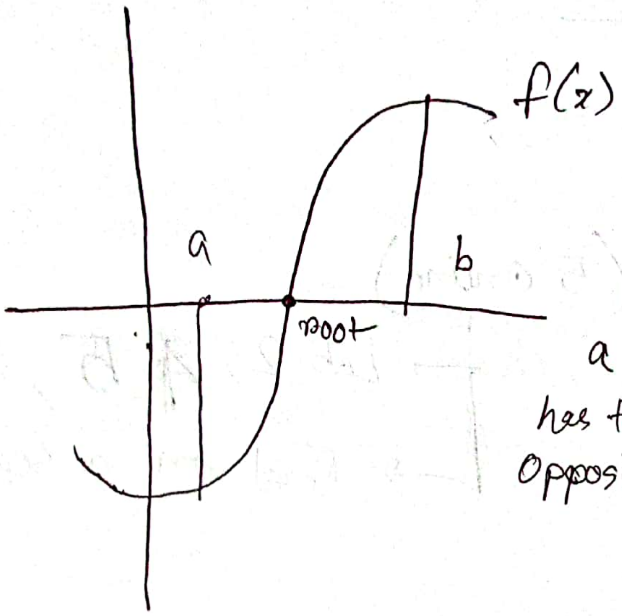
Online + Viva : 60% (5 Online)

Final : 20%

→ Lab 2, 4, 5, 7

→ Final on a separate date

Bisection Method



The root of $f(x)$ exists iff

$f(a) * f(b) \le 0$

- a and b has to be opposite signs
- if =, a or b will be root

Two types of methods:
i) Bracketing → fixed range
ii) Operand

* **Initial Guess:** Check if root exists ~~in~~ between a, b
* **Wrong Guess:** Find $\frac{a+b}{2}$, update guesses based on $\frac{a+b}{2}$. → for $c = +ve$, update b
$c = -ve$, " a

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Stop at,

$fabs(a-b) \longrightarrow 0.01$

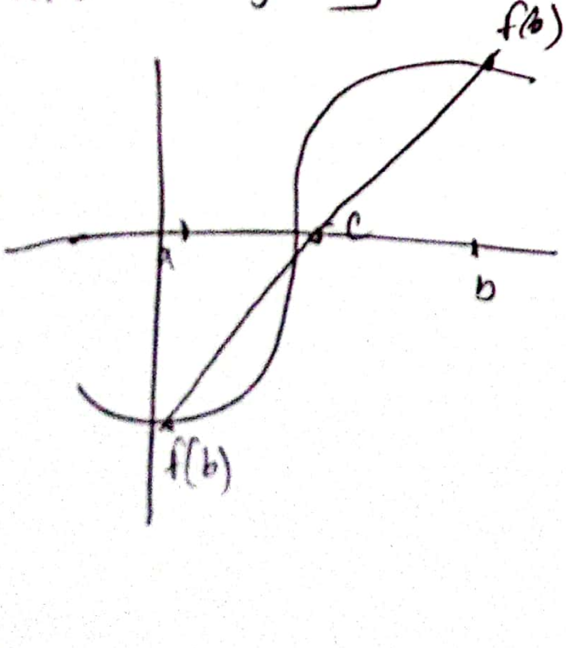
6.10 : Descartes Rule of Sign

Stopping/Loop continuation Criteria:

1. $f(root) >= 0.001$
2. ~~f(a~~ $fabs(a-b) >= 0.001$
3. $fabs((a-b)/a) >= 0.001$

Check Book (6.4)
4
5
6

[F] False position: Instead of middle point, find the point when joining $f(a)$ and $f(b)$ → more accurate and slower than bisection



$$root = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

GOOD LUCK

Roots of Non-Linear Eq^n

* Non Linear Polynomial

* Transcendental Equations: $(\text{constant})^x$

↳ $\frac{d}{dx}(a^x) = a^x \ln a$

⊞ Search Bracket :- Range for which all roots are available
- Largest possible roots.

For any polynomial:

$$a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_1 x + a_0 = 0,$$

Largest possible root $= -\frac{a_{n-1}}{a_n}$

Search Bracket $= \sqrt{\left(\frac{a_{n-1}}{a_n}\right)^2 - 2\left(\frac{a_{n-2}}{a_n}\right)}$

Stopping Criteria : [Check Book]

TOPIC NAME: DAY: TIME: DATE: / /

Bisection Method: // Algo from Book

// Convergance " "

:

$x_0 = (x_1 + x_2)/2;$

if ($f(x_0) == 0$)

```
{
    ~~print~~ cout << x0 << endl;
    break;
}
```

*If nothing is mentioned → do only 4 iterations

~~* For~~

* For False position → " " 3 "
* " Newton - Raphson → " " 6 "
* Secant → " " 4 "

False Position:

replace $x_0 = \frac{x_1 + x_2}{2}$

with $x_0 = x_1 - \frac{f(x_1)(x_2 - x_1)}{f(x_2) - f(x_1)}$

*degree of convergence is high (er than Bisection)

Derive the formula for Bisection/False position Method

TOPIC NAME: W-2 DAY: DC-3
TIME: DATE: 9/12/24

Newton Raphson - (6.8)

→ Check Ⓑ. $\boxed{\text{Example 6.6}}$

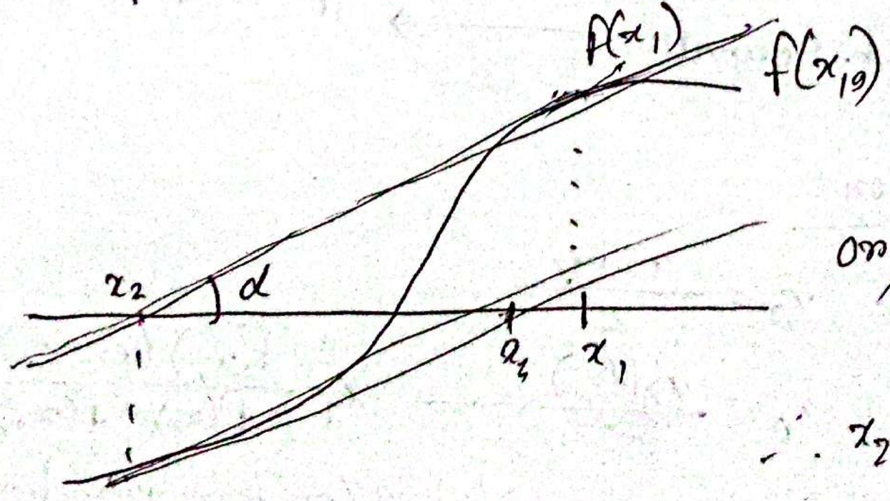
* Open end ~~has a~~ is uncontrollable
* Convergence NOT guaranteed

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$\vdots$

$$\boxed{x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}}$$

Formula Derivation



Here,

$$\tan\alpha = \frac{f(x)}{x_1 - x_2}$$

or, $$f'(x) = \frac{f(x)}{x_1 - x_2}$$

$$\therefore x_2 = x_1 - \frac{f(x)}{f'(x)}$$

CamScanner

TOPIC NAME ____ DAY: ____ TIME: DATE: / /

Q. Solve using Newton-Raphson with initial value of 0

visicinity

$x^2 - 3x + 2 = 0$

$\therefore f(x) = x^2 - 3x + 2$

$\therefore f'(x) = 2x - 3$

$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0 - \frac{f(0)}{f'(0)}$

$= 0.667$

$\therefore x_2 = 0.667 - \frac{f(0.667)}{f'(0.667)} = 0.9959$

$\vdots$

$\therefore x_6 = 0.999 - \frac{0.001}{-1.0002} = 1.$

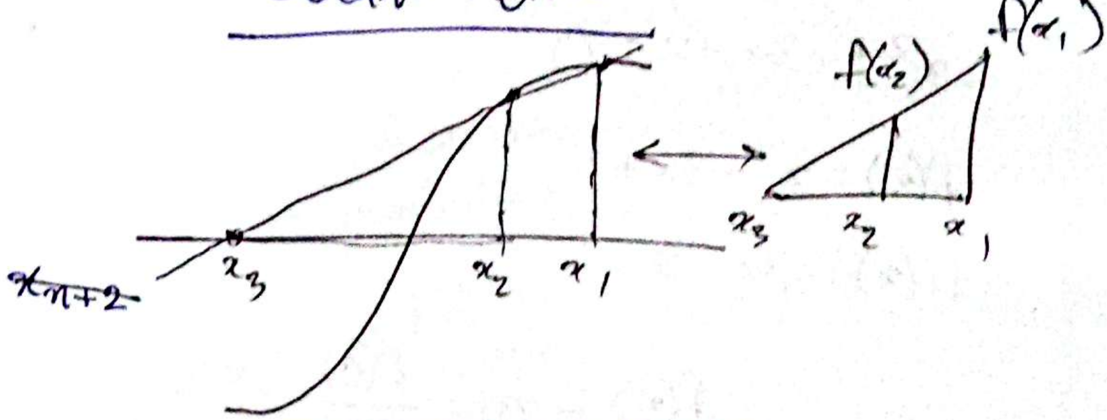
$\therefore f(x_6) = f(1) = 0,$

root = 1 (Ans)

Limitations/Weaknesses of Newton-Rhapson

* If it ~~does not con~~ diverges OR $f'(x) = 0$, the answer is that it will not have an answer in that initial value

Secant Method



$$\therefore \boxed{x_{n+2} = x_{n+1} - \frac{f(x_{n+1})(x_{n+1} - x_n)}{f(x_{n+1}) - f(x_n)}}$$

$$\updownarrow$$

$$\boxed{x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}}$$

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Comparison between Newton Rhapson and Secant

N-R	Sec
- Takes tangent to find x_2	- takes secant (using x_2 and x_1) to find x_3
- Probability of convergence is lower	- Probability of convergence is higher

All Possible Roots by Newton Method

Algo (in my own way): → divide the polynomial with $x - x_n$ and continue NR until all values are out

n = degree of eq^n

find root 1

while $n > 1$ → 2nd, Last root

find root (using Newton Rhapson method)

root x_n

Use synthetic division to ~~make~~ deflate eq^n

$n--$

$$Root = \frac{-a_0}{a_1}$$

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Synthetic Division: → Formula derivation

$$\frac{a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0}{x - x_n} = b_n x^n + b_{n-1} x^{n-1} + \dots + b_1 x + b_0 \, ; \; [b_n = 0]$$

$q(x)$

$\cancel{b_{i-1}}$

$$\boxed{\begin{aligned} b_n &= 0 \\ b_{i-1} &= a_i + x_n b_i \end{aligned}}$$

TOPIC NAME DAY TIME DATE

#6.15

Here,

$x_n = 3$

$a_3 = 1$, $a_2 = -7$, $a_1 = 15$, $a_0 = -9$

$\therefore b_3 = 0$

$\therefore b_2 = a_3 + b_3 x_n$

$= 1$

$\therefore b_1 = a_2 + b_2 x_n$

$= -7 + (1)(3)$

$= -4$

$\therefore b_0 = a_1 + b_1 x_n$

$= 3$

$\therefore$ New $eq^n \equiv x^2 - 4x + 3 = 0$

TOPIC NAME C - 4 DAY C-6
TIME DATE 22 / 12 / 24

* Descartes Rule of Sign

* Horner's Rule → 6.5 Evaluation of Polynomial

$2x^3 - 4x^2 + 3x + 6$ at $x = 2$

$\{(2x - 4)x + 3\}x + 6$

At $x = 2$,

$[\{2(2) - 4\}(2) + 3](2) + 6$

$= [(4 - 4)(2) + 3](2) + 6$

$= 3(2) + 6$

$= 12$

Use Pn method from book

TOPIC NAME : W-4 DAY : C-7
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Ch - 7

Direct Solution of Linear Equations

*** 7.2. Existence of Solution :**

- → Unique sol^n → intersecting ~~ten~~ lines
- → No " → parallel [diff const]
- → No unique / Infinite ~~slow~~ solutions → same eq^n
- * → ill conditioned (7.9) → almost same eq^n
  - → check / identify
  - → impact / effect **

*** Methodes**
- → Gauss Elimination
- → " with pivoting
- → " Jordan
- → Triangular Factorization / LU decomposition
- → Matrix inversion
- → Cramer's Rul[illegible]

TOPIC NAME : DAY : TIME : DATE : / /

#7.1. Solve using Gauss Elimination:

$$3x + 2y + z = 10$$
$$2x + 3y + 2z = 14$$
$$x + 2y + 3z = 14$$

→ Step 1: Make Upper triangle Matrix by Forward Elimination

→ Step 2: Back Substitution

#7.3. Solve using Gauss Elimination and Pivoting:

$$2x_1 + 2x_2 + x_3 = 6$$
$$4x_1 + 2x_2 + 3x_3 = 4$$
$$x_1 + x_2 + x_3 = 0$$

Swap highest magnitude at the top

GOOD LUCK

W-B4 C-8

24/12/24

* Gauss Jordan ⟶ already done ⟶ Example 7.4
[check book]

LU decomposition / Triangular Factorization:

$$\underset{\text{coefficient}}{A}\ \underset{\text{variable}}{X} = B \quad \text{matrix}$$

↓ divide into two matrices L and U

$LUX = B$ ⟶ Let,

$UX = Z$

$LZ = B$ → solve for Z

$UX = Z$ ⟶ solve for X

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \times \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

TOPIC NAME

$$L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{2}{3} & 1 & 0 \\ \frac{1}{3} & \frac{4}{5} & 1 \end{bmatrix} \qquad U = \begin{bmatrix} 3 & 2 & 1 \\ 0 & \frac{5}{3} & \frac{4}{3} \\ 0 & 0 & \frac{24}{15} \end{bmatrix}$$

$$\therefore LZ = \begin{bmatrix} 1 & 0 & 0 \\ \frac{2}{3} & 1 & 0 \\ \frac{1}{3} & \frac{4}{5} & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 14 \\ 14 \end{bmatrix}$$

$\therefore z_1 = 10$ (Ans)

$\therefore \frac{2}{3} z_1 + z_2 = 14$

$\therefore z_2 = 14 - \frac{20}{3} = \frac{22}{3}$ (Ans)

$\therefore \frac{1}{3} z_1 + \frac{4}{5} z_2 + z_3 = 14$

$\therefore z_3 = 14 - \frac{10}{3} - \frac{4 \times 22}{5 \times 3} = \frac{24}{5}$

Now,

$$Ux = z$$

or, $$\begin{bmatrix} 3 & 2 & 1 \\ 0 & \frac{5}{3} & \frac{4}{3} \\ 0 & 0 & \frac{24}{15} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ \frac{22}{3} \\ \frac{\cancel{2}72}{15} \end{bmatrix}$$

$$\therefore \frac{24}{15} z = \frac{72}{15}$$

$$\therefore z = 3 \text{ (Ans)}$$

$$\therefore \frac{5}{3} y + \frac{4}{3} z = \frac{22}{3}$$

$$\therefore \cancel{z =} \; y = \frac{3}{5} \times \left(\frac{22}{3} - \frac{4}{\cancel{3}} \times \cancel{3} \right)$$

$$\therefore y = 2 \text{ (Ans)}$$

$$\therefore 3x + 2y + z = 10$$

$$\therefore x = \frac{10 - 4 - 3}{3} = 1 \text{ (Ans)}$$

TOPIC NAME : W-5 DAY : C-9
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Cramer's Rule

$$X = \frac{D_x}{D} \longrightarrow D_x = \begin{bmatrix} const \\ const \\ const \end{bmatrix}$$

$$Y = \frac{D_y}{D}$$

$$Z = \frac{D_z}{D}$$

replace ith column with constant for D_i

Inversion Method

$Ax = B$

or, $A^{-1}Ax = A^{-1}B \longrightarrow$ left multiplication with A^{-1}

or, $\boxed{x = A^{-1}B}$

left multiplication $\neq$ right multiplication

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* Finding Inverse:

(i) For
$$\left[\begin{array}{ccc|ccc} a_{11} & a_{12} & a_{13} & 1 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 1 & 0 \\ a_{31} & a_{32} & a_{33} & 0 & 0 & 1 \end{array}\right],$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & a_{11}' & a_{12}' & a_{13}' \\ 0 & 1 & 0 & a_{21}' & a_{22}' & a_{23}' \\ 0 & 0 & 1 & a_{31}' & a_{32}' & a_{33}' \end{array}\right]$$

$\hookrightarrow A^{-1}$

$\therefore x_1 = a_{11}'b_1 + \cdots$

$x_2 = - - \cdot$

$x_3 = \cdot \cdot - \cdot$

(ii) $A^{-1} = \dfrac{\text{Adj}(A)}{\text{Det}(A)}$

GOOD LUCK

TOPIC NAME : DAY : TIME : DATE : / /

#

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 3 \\ -3 \\ 4 \end{bmatrix}$$

$$\therefore |A| = 3(-3+2) - 1(2+1) + 2(4+3)$$
$$= -3 - 3 + 14$$
$$= 8$$

$\because |A| \neq 0$, A^{-1} exists

For cofactor matrix, of A^T $\begin{bmatrix} 3 & 2 & 1 \\ 1 & -3 & 2 \\ 2 & -1 & 1 \end{bmatrix}$

$a_{11} = (-1)^{1+1} (-3+2)$
$= -1$

$a_{12} = (-1)^{1+2} (1-4)$
$= 3$

$a_{13} = (-1)^{1+3} (-1+6)$
$= 5$

$a_{21} = (-1)^{2+1} (2+1)$
$= -3$

$a_{22} = (-1)^{2+2} (3-2)$
$= 1$

$a_{23} = (-1)^{2+3} (-3-4)$
$= 7$

$a_{31} = (-1)^{3+1} (4+3)$
$= 7$

$a_{32} = (-1)^{3+2} (6-1)$
$= -5$

$a_{33} = (-1)^{3+3} (-9-2)$
$= -11$

GOOD LUCK

$$\therefore \text{Adj}(A) = \begin{bmatrix} -1 & 3 & \cancel{2}5 \\ 3 & 1 & 7 \\ 7 & -5 & -11 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{8}\begin{bmatrix} -1 & 3 & 5 \\ 3 & 1 & 7 \\ 7 & -5 & -11 \end{bmatrix}$$

TOPIC NAME :

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Ch-8 : Iterative Methods

- Jacobi Iteration Method
- Gauss Seidel Method
  - check book

Jacobi Iteration:

- Can diverge

$$5x + 2y + z = 12$$
$$x + 4y + 2z = 15$$
$$x + 2y + 5z = 20$$

$$\therefore x = \frac{12 - 2y - z}{5}$$

$$\therefore y = \frac{15 - x - 2z}{4}$$

$$\therefore z = \frac{20 - x - 2y}{5}$$

But x

$$x^{i+1} = \frac{\cdots y^{i} + z^{i}}{\cdots}$$

$$y^{i+1} = \frac{\cdots + x^{i} + z^{i}}{\cdots}$$

$$z^{i+1} = \frac{\cdots + y^{i} + x^{i}}{\cdots}$$

Ch-10 : Regression

~~Curve fit~~

- Curve Fitting: Regression
  - Not Exact fitting
- Curve Fitting: Interpolation
  - touches all point

* We minimize SSE

~~Linear~~ Reg

Curve Fitting Regression

- Straight line
- ~~PO~~ Power Function
- Polynomial
- Multiple Linear Regression

Derive the expression for a and b

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~~fon~~

for $y = a + bx$ and $y = ax^b \rightarrow$ use $\ln x$ and $\ln y$

$$b = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - (\sum x_i)^2}$$; n = no. of points/~~obs~~ observation

$\mapsto$ replace x_i with $\ln x_i$
y_i " $\ln y_i$

$$a = \frac{\sum y_i}{n} - b\frac{\sum x_i}{n} = \bar{y} - b\bar{x}$$

$$SSE = \sum(\epsilon)^2 = \sum(y_i - a - bx_i)^2$$

[Honestly, Just check book at this point]

TOPIC NAME: W-6 DAY: C-10
DATE: 6/1/25

Curve Fitting: **Interpolation**

$x_1 - x_0 = h$
$x_2 - x_1 = h$
$x_3 - x_2 = h$

Equidistant points →

* Finite difference operator : Δ, ∇, δ

forward " " : Δ

backward " " : ∇

central " " : δ

for,

$$h\begin{cases} x_0, & y_0 = f(x_0) \\ x_1; & y_1 = f(x_1) \end{cases} \quad \Delta y_0 = y_1 - y_0 \quad \Delta^2 y_0 = \Delta y_1 - \Delta y_0$$

$$h\begin{cases} x_1; & y_1 = f(x_1) \\ x_2, & y_2 = f(x_2) \end{cases} \quad \Delta y_1 = y_2 - y_1$$

$\therefore \Delta y_0 = \nabla y_1$

$\therefore \nabla y_0 = y_1 - y_0$

$\therefore \nabla y_3 = y_3 - y_2$

$\therefore \nabla^2 y_3 = \nabla y_3 - \nabla y_2$

$\nabla^3 y_3 = \nabla^2 y_3 - \nabla^2 y_2$

* $\Delta y_0 = y_1 - y_0$

$= \Delta f(x_0) = f(x_1) - f(x_0)$

$= f(x_0 + h) - f(x_0)$

$= \Delta f_0 = f_1 - f_0$

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Newton (& - Gregory's) Forward Interpolation Formula

" Backward " "

"

Central "

↓

~~Same~~ Equal Interval

Unequal Interval

↓

Newton Divided Difference Interpolation Formula

Lagrange Interpolation "

For inverse interpolation ⟶ Lagrange.

GOOD LUCK

* Check Interpolation from doc and then read from shankar Rao

* Occam's Razor Hypothesis

[5] Newton's Forward Interpolation:

* Multiple Eq^n can pass through same points

* Interpolation = Exact Fit
Errors = 0

$$y_x = \phi(x) = y_0 + \frac{u}{1!}\cdot\frac{\Delta y_0}{1} + \frac{u(u-1)}{2!}\cdot\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \dots +$$

$$u = \frac{x - x_0}{h}$$

value to find

x	$y =$	Δy	$\Delta^2 y$	$\Delta^3 y$
x_0	y_0	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$
x_1	y_1	Δy_1	$\Delta^2 y_1$	
x_2	y_2	Δy_2		
x_3	y_3			

Make sure h is constant

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$y_0 = 111.8034$

$\Delta y_0 = 0.0447$

$\Delta^2 y_0 = 0$

$u = 1.6$

$\therefore y_{(u)} = 111.8034 + (1.6)(0.0447)$

$= 111.87492$ (Ans)

~~food~~ 101 → Shankar

$\therefore u = \dfrac{0.05 - 0.00}{0.1} = 0.5$

$y_0 = 1$

~~$\Delta y_0 =$~~

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0	1				
		0.2214			
0.1	1.2214		0.049		
		0.2704		0.0109	
0.2	1.4918		0.0599		0.0083
		0.3303		0.0192	
0.3	1.8221		0.0731		
		0.4034			
0.4	2.225				

GOOD LUCK

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Now,

$$y_{0.5} = y_0 + \frac{u}{1!}\cdot\Delta y_0 + \frac{u(u-1)}{2!}\times\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0$$

$$+ \frac{u(u-1)(u-2)(u-3)}{4!}\times\Delta^4 y_0$$

$$= 1 + (0.5)(2.2214) + \frac{(0.5)(-0.5)}{2}(0.049)$$

$$+ \frac{(0.5)(-0.5)(-1.5)}{6}(0.0109) + \frac{(0.5)(-0.5)(-1.5)(-2.5)}{24}(0.0023)$$

$= 2.105$ (check later)

GOOD LUCK

TOPIC NAME: W-7 DAY: C-12
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* If $\phi(x) \to x$ lies near first values of the table, we use Newton Forward Value

* If it lies near last values, we use Newton Backward

* If it lies near center: Gauss Forward
" Backwards
Stirling
Bessel's Everett → X

Newton's Backward Interpolation:

$$\phi(x) = y_x = y_n + \frac{u \nabla y_n}{1!} + \frac{u(u+1)}{2!} \nabla^2 y_n + \frac{u(u+1)(u+2)}{3!} \nabla^3 y_n + \cdots$$

where,

$$u = \frac{x - x_n}{h}$$

TOPIC NAME: w-7 DAY: C12
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Gauss Forward Central Interpolation:

Use S

~~f(x~~

$$f(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_{-1} + \frac{(u+1)u(u-1)}{3!}\Delta^3 y_{-1}$$

$$+ \frac{(u+1)u(u-1)(u-2)}{4!}\Delta^4 y_{-2} + \frac{(u+2)(u+1)u(u-1)(u-2)}{5!}\Delta^5 y_{-2}$$

~~Gauss~~

Gauss Backward:

$$y = f(x) = y_0 + \frac{u}{1!}\Delta y_{-1} + \frac{u(u+1)}{2!}\Delta^2 y_{-1} + \frac{(u-1)u(u+1)}{3!}\Delta^3 y_{-2}$$

$$+ \frac{(u-1)u(u+1)(u+2)}{4!}\Delta^4 y_{-2} + \frac{(u-2)(u-1)u(u+1)(u+2)}{5!}\Delta^5 y_{-3}$$

Stirling: $\dfrac{\text{Gauss Forward} + \text{Gauss Backward}}{2} = f(x)$

TOPIC NAME: 10 - 8 DAY: 13 TIME: DATE: 26 / 1 / 25

Quiz - 3: Ch - 7, 8 → Check Matrix Inversion + Cramer's Rule } from sheet

Regression (Ch - 10)

→ Next Tuesday (2?/2/25)

Finites Differences: Interpolation with ~~dif~~ unequal Interval

→ Newton Divided

→ Lagrange's

Newton's Divided Differences:

$$\Delta = \frac{y_1 - y_0}{x_1 - x_0} \quad \Delta^2 = \frac{\Delta y_1 - \Delta y_0}{x_2 - x_0} \quad \Delta^3 = \frac{\Delta^2 y_1 - \Delta^2 y_0}{x_3 - x_0}$$

x	$f(x)$	$\Delta f(x)$ $f[x_0, x_1]$	$\Delta^2 f(x)$ $f[x_0, x_1, x_2]$	$\Delta^3 f(x)$ $f[x_0, x_1, x_2, x_3]$	$\Delta^4 f(x)$ $f[x_0, x_1, x_2, x_3, x_4]$
5	150	121	24	1	0
7	392	265	32		
11	1452	457	46		
13	2366	917			
21	9702				

TOPIC NAME: DAY: TIME: DATE: / /

$$y = f(x) = f[x_0] + (x - x_0) f[x_0, x_1] \rightarrow \Delta$$
$$+ (x - x_0)(x - x_1) f[x_0, x_1, x_2] \rightarrow \Delta^2$$
$$+ (x - x_0)(x - x_1)(x - x_2) f[x_0, x_1, x_2, x_3] \rightarrow \Delta^3$$

Lagrange's:

$$y = f(x) = \frac{(x - x_1)(x - x_2) \cdots (x - x_n)}{(x_0 - x_1)(x_0 - x_2) \cdots (x_0 - x_n)} y_0$$

$$+ \frac{(x - x_0)(x - x_2) \cdots (x - x_n)}{(x_1 - x_0)(x_1 - x_2) \cdots (x_1 - x_n)} y_1$$

$$+ \frac{(x - x_0)(x - x_1)(x - x_3) \cdots (x - x_n)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3) \cdots (x_2 - x_n)} y_2$$

Inverse Interpolation:

y is given — using y to find x

$x = ?$

Replace x with y in lagrange's formula:

$$x = \frac{(y-y_1)(y-y_2)(y-y_3)\cdots(y-y_n)}{(y_0-y_1)(y_0-y_2)(y_0-y_3)\cdots(y_0-y_n)}x_0$$

$$+ \frac{(y-y_0)(y-y_2)(y-y_3)\cdots(y-y_n)}{(y_1-y_0)(y_1-y_2)(y_1-y_3)\cdots(y-y_n)}x_1$$

$+ \cdots$

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E : Shift Operator | δ: Central Difference Operator

$E y_0 = y_1$ | μ : Averaging Operation

⋮

$E y_i = y_{i+1}$ | $E^n y_i = y_{i+n}$

$\Delta y_i = y_{i+1} - y_i$

$\nabla y_i = y_i - y_{i+1}$

$\delta y_i = y_{i+\frac{1}{2}} - y_{i-\frac{1}{2}}$; $i = \frac{p}{q}$

$\mu y_i = \frac{1}{2}\left[y_{i+\frac{1}{2}} + y_{i-\frac{1}{2}}\right]$; $i = \frac{p}{q}$

#SR → 4.1, 4.2, 4.3* ⟶ 97 pg

↳ polynomial passing ~~three~~ through n points has degree $(n-1)$

GOOD LUCK

TOPIC NAME: W-10 DAY: C-15
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* Find the missing value (4.1, 4.2, 4.3)

↳ the error in the table (3.24 → 83)

There will be one error, we use the next

#	x	0	1	2	3	4	5	6
	y				$y+\epsilon$			

0	1
1	2
2	33
3	$254+\epsilon$
4	
5	3126
6	7777

$+\epsilon$ . . . . . . $\Delta^5 y$

TOPIC NAME DAY TIME DATE / /

Quiz #4
- derivation
  - Newton Forward/Backward
  - Gauss Forward/Backward
  - Stirling
  - Newton Divided
  - Lagrange
  - Relation between operators
- math

Next Week Tuesday 18/02/25

TOPIC NAME : DAY : TIME : DATE : / /

Numerical Differentiation

Ch - 11

Forward Difference Quotient, $f'(x) = \dfrac{f(x+h) - f(x)}{h}$

Backward " " , $f'(x) = \dfrac{f(x) - f(x-h)}{h}$ better

Central " " , $f'(x) = \dfrac{f(x+h) - f(x-h)}{2h}$

→ use this if nothing is mentioned

only when function given

e.g. $f(x) = x^2$

Ex - 11.1, 11.2, 11.3 → Skip error analysis

11.4

11.5, 11.6 → tabular

TOPIC NAME : DAY
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* 2nd Order Derivative :

$$f''(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

* for a known function, we can also find the error → find using formula
find using hand/head

For tabulation ones check book for formula

TOPIC NAME w-10 C-16

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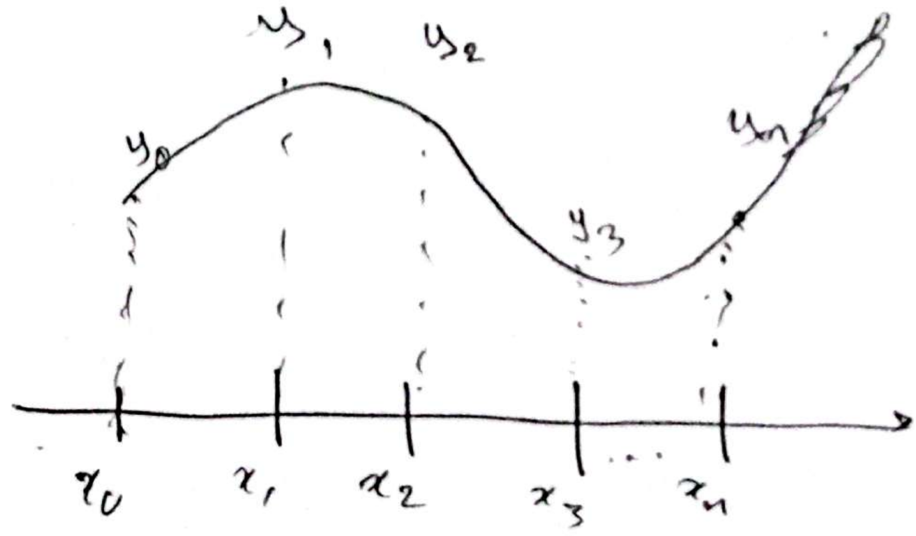
Numerical Integration - Ch-9

* Derive the General Quadrature Formula:
* * " " " Trapezoidal Rule from GQF ($n=1$)
  ↳ any num of interval (at least 6)
* * " " " Simpson's 1/3rd Rule " ($n=2$)
  ↳ even number of interval
* " " " 3/8 th rule ($n=3$)
  ↳ at least multiple of 3
* " " " weddle's rule ($n=6$)
  ↳ multiple of 6

decrease



TOPIC NAME

Trapezoidal $\Rightarrow I = \frac{h}{2}\left[(y_0 + y_n) + 2(y_1 + y_2 + y_3 + \cdots + y_{n-1})\right]$

for $n = 6$, $I = \frac{h}{2}\left[(y_0 + y_6) + 2(y_1 + y_2 + y_3 + y_4 + y_5)\right]$

Simpson's 1/3rd $\Rightarrow I = \frac{h}{3}\left[(y_0 + y_n) + 4(y_1 + y_3 + y_5 + \cdots) + 2(y_2 + y_4 + \cdots)\right]$

for $n = 6$, $I = \frac{h}{3}\left[(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)\right]$

Simpson's 3/8th $\Rightarrow I = \frac{3h}{8}\left[(y_0 + y_n) + 3(y_1 + y_2 + y_4 + y_5 + \cdots) + 2(y_3 + y_6 + y_9 \cdots)\right]$

for $n = 6$, $I = \frac{3h}{8}\left[(y_0 + y_6) + 3(y_1 + y_2 + y_4 + y_5) + 2y_3\right]$

Weddle's Rule $\Rightarrow I = \frac{3h}{10}\left[(y_0 + y_n) + (y_2 + y_4 + y_8 + y_{10} + y_{14} + \cdots) + 5(y_1 + y_5 + y_7 + y_{11} + y_{13} + \cdots) + 6(y_3 + y_9 + y_{15} + \cdots) + 2(y_6 + y_{12} + y_{18} + \cdots)\right]$

for $n = 6$, $I = \frac{3h}{10}\left[(y_0 + y_6) + 5(y_1 + y_5) + 6(y_3)\right]$

GOOD LUCK

Romberg Integration:

→ find ~~$\frac{y}{2}$~~ $h, \frac{h}{2}, \frac{h}{4}$

$I(h) \quad I(h/2) \quad I(h/4)$

merge

$I(h, \frac{h}{2}) \quad I(\frac{h}{2}, \frac{h}{4})$

$I(h, \frac{h}{2}, \frac{h}{4})$

~~$I(h, \frac{h}{2})$~~

$$I(x, y) = \frac{1}{3}\left[4I\left(\frac{h}{2}\right) - I(h)\right]$$

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Ch-4 → 4.7 * 4.9 * * *

+ Errors
↓
derive + proof

Approx and Errors

* Taxonomy of Errors

~~* Significant digits~~

* Significant Digits:

* All non-zeroes
* If 0 → (i) in between digits
  (ii) Trailing zeroes : We found using calculation
  3.50
  (iii) Leading zeroes ARE NOT counted

$$\underbrace{0}_{\text{Not significant}}.\underbrace{12}_{\text{significant}} \times 10^{-3} = 0.\underbrace{000}_{\text{Not significant}}12$$

(iv) Integers ending with 0 : $\underbrace{4500}_{\text{significant}}.0$

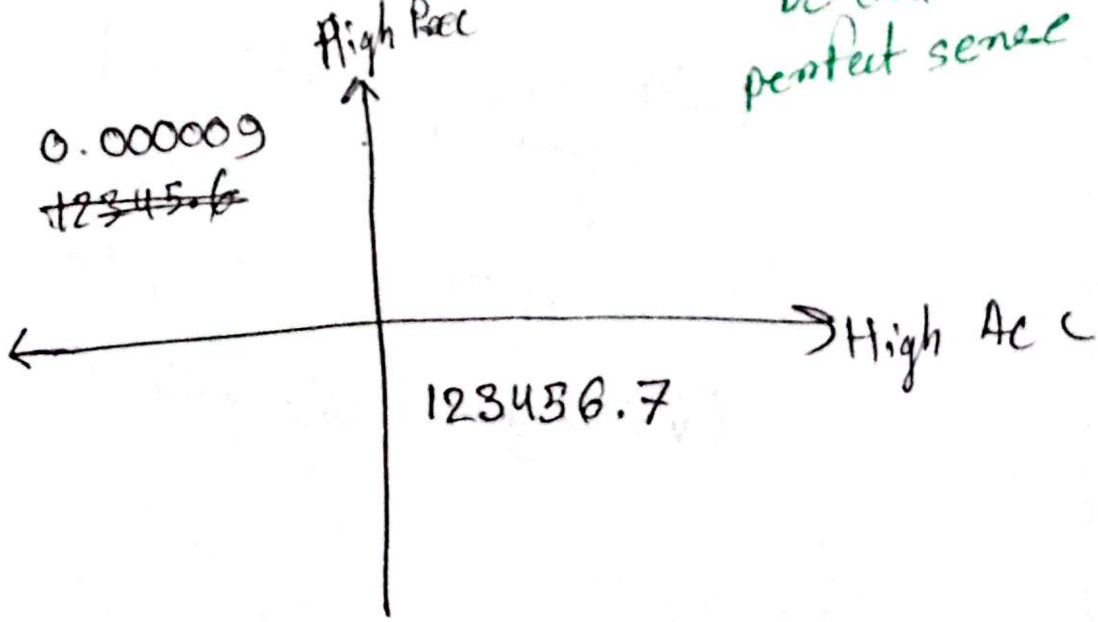
Ex 4.1, 4.2, 4.4

Accuracy: No. of significant digits

Precision: No. of decimal points

		Precision	Accuracy
4.3201	→	10^{-4} or 4	5 sd
4.32	→	10^{-2} or 2	3 sd
3600	→		Not allowed
3600.00	→		6 sd

bc that makes perfect sense



Error:

Inherent Errors:

→ Data/Emperical Error:

- Instrument error

→ Conversion/Representation Error

$0.1 \times 2 = 0.2 - 0$

$0.2 \times 2 = 0.4 - 0$

$0.4 \times 2 = 0.8 - 0$

$0.8 \times 2 = 1.6 - 1$

$0.6 \times 2 = 1.2 - 1$

$0.2 \times 2 = 0.4 - 0$

.

.

$\therefore 0.1 = 0.00011\ 0011\ 0011 \ldots$

Prove that Symmetric Roundoff errors $= \frac{1}{2}$ Chopping error

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4.4 (67)

752.6833

= 0 7526 835

Trunction Error:

~~Err~~ Error from taking a part from an infinite series

Modelling Error:

Error from ignoring things to make any models

Blunders:

Human error

Absolute Error:

$|x_t - x_a|$

Relative Error:

$\left|\frac{x_t - x_a}{x_a}\right|$ or $\left|\frac{x_t - x_a}{x_t}\right|$

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[7] Machine Epsilon :

Error from capacity of machine

* Prove that machine absolute - - -

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Error Propagation → most imp

$X_t \rightarrow$ true value of x

$x_a \rightarrow$ ~~at~~ approximate value of x

$\therefore x_t = x_a + e_x$

$y_t = y_a + e_y$

* Check proofs

error of $x_t + y_t \le |e_x| + |e_y|$

" $x_t - y_t \le |e_x| + |e_y|$

" $x_t * y_t \le |x_a y_a| \left(\left| \frac{e_x}{x_a} \right| + \frac{e_y}{y_a} \right)$

" $x_t / y_t \le \left| \frac{x_a}{y_a} \right| \left(\left| \frac{e_x}{x_a} \right| + \left| \frac{e_y}{y_a} \right| \right)$

$e_{r,xy} = |e_{r,x}| + |e_{r,y}|$

$e_{r\frac{x}{y}} = |e_{r,x}| + |e_{r,y}|$

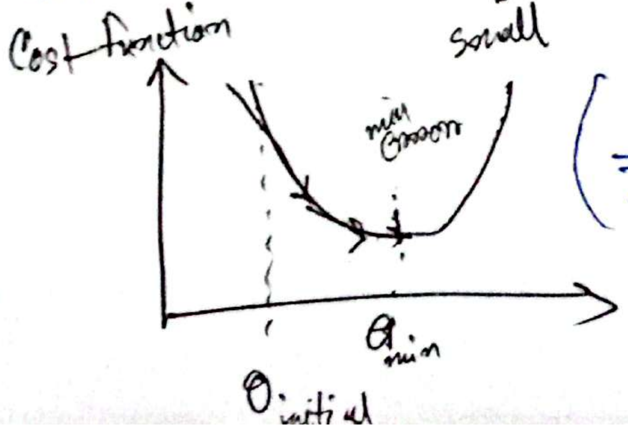
$e_{r,x \pm y} \le \left| \frac{x_a}{x_a \pm y_a} \right| |e_{r,x}| + \left| \frac{y_a}{x_a \pm y_a} \right| |e_{r,y}|$

Regression Analysis (using Gradient Descent)

- Derive the expression for:
  - linear regression
  - gradient descent algorithm
    - Multiple linear Regression
    - polynomial regression with degree 2, 3
      - ~~using~~ with variable conversion
      - without "   "
- Math → Update θ_0, θ_1 for ...

learning rate/step size

$$\text{new } \theta_i = \text{old } \theta_i - \underbrace{\alpha}_{\text{small}} \cdot \frac{\partial}{\partial \theta_i}(\text{cost function})$$



$\left(\frac{\partial}{\partial x}(\text{cost}), \frac{\partial}{\partial z}(\text{cost})\right)$ → Gradient

$$\frac{1}{2m}\sum_{i=1}^{m}(\theta_0 + \theta_1 x - y)^2$$

↳ cost function

TOPIC NAME

Final:

Part 1:

- def^n
- derivation**
- descriptive

Algo / short Q

Part 2:

- math / solve

Part 3:

- Regression Analysis

TOPIC NAME : W-12

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* θ Update :

Model : Linear $y = \theta_0 + \theta_1 x$.

Mult Linear → 2var $= \theta_0 + \theta_1 x_1 + \theta_2 x_2$

3var : $\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3$

⋮

Polynomial → 2 degree : $y = \theta_0 + \theta_1 x + \theta_2 x^2$

3 degree : $y = \theta + \theta_1 x + \theta_2 x^2 + \theta_3 x^3$

Rule :

$$\text{new } \theta_i = \text{old } \theta_i - \alpha \frac{\partial}{\partial \theta_i} (\text{cost function})$$

Costfunction : $\frac{1}{2m} \sum_{i=1}^{m} (\text{error})^2$

GOOD LUCK

CamScanner