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MATH

* Matrix ⟶ Fourier ⟶ Vectors

*Matrix : . . . , Rank of matrix, Normal and Canonical form, sol^n of linear eq^ns, Matrix polynomial, Eigenvalue, Eigenvectors

*Vector : . . . triple product, linear dependence and independence of vectors, ~~Di~~ calculus, . . . , Green's, Stokes and Gauss theorem.

*Fourier Analysis: Fourier series,

* Book : H.K. Das ⟶ ~~Advang~~ Advanced Engineering Mathematics

Matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{n1} & a_{n2} & \cdots & a_{mn} \end{bmatrix}$$

#

Given that,

$$A_1 = \begin{bmatrix} 12 & 23 & 34 \\ 49 & 15 & 16 \\ 27 & 18 & 19 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 48 & 33 & 7 \\ 18 & 39 & 8 \\ 12 & 14 & 43 \end{bmatrix}$$

(i) $2A_1 + 3A_2 = 2\begin{bmatrix} 12 & 23 & 34 \\ 49 & 15 & 16 \\ 27 & 18 & 19 \end{bmatrix} + 3\begin{bmatrix} 48 & 33 & 7 \\ 18 & 39 & 8 \\ 12 & 14 & 43 \end{bmatrix}$

$$= \begin{bmatrix} 24 & 46 & 68 \\ 98 & 30 & 32 \\ 54 & 36 & 38 \end{bmatrix} + \begin{bmatrix} 144 & 99 & 21 \\ 54 & 117 & 24 \\ 36 & 42 & 129 \end{bmatrix}$$

0 #

$$= \begin{bmatrix} 168 & 145 & 89 \\ \cancel{142}\,152 & 147 & 56 \\ 90 & 78 & 167 \end{bmatrix}$$

(Ans)

$$\text{(ii) } 3A_1 - 4A_2 = 3\begin{bmatrix} 12 & 23 & 34 \\ 49 & 13 & 16 \\ 27 & 18 & 19 \end{bmatrix} - 4\begin{bmatrix} 48 & 33 & 7 \\ 18 & 39 & 8 \\ 12 & 14 & 43 \end{bmatrix}$$

$$= \begin{bmatrix} -156 & -63 & 74 \\ 75 & -111 & 16 \\ 33 & -2 & -115 \end{bmatrix}$$

$$\text{(iii) } A_1 \times A_2 = \begin{bmatrix} 1398 & 1769 & 1730 \\ 2814 & 2426 & 1151 \\ 1848 & 1859 & 1150 \end{bmatrix}$$

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$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$. Find $A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$

$\therefore |A| = \cancel{3(-3)(1)+(-3)(}\ 1$

$\cancel{= 3\{(-3)(1) - (4)(-1)\} -}$

$\therefore$ For co factors,

$A_{11} = (-1)^{1+1}\{(-3)(1) - (4)(-1)\}$

$= 1$

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H.W :

1. $\begin{bmatrix} 2 & 5 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$

2. $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 4 \\ 1 & 1 & 2 \end{bmatrix}$

3. $\begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$

Find A^{-1}

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Rank of a Matrix

+ The rank of a matrix is said to be the non-zero rows of a matrix

i) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ R - 3

#;) Find the rank of a matrix

ii) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 4 \\ 0 & 0 & 0 \end{bmatrix}$ R - 2

iii) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 4 \\ 0 & 0 & 0 \end{bmatrix}$ R - 2

Gauss - Elimination Method:

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Find the rank of the matrix:

$$\begin{bmatrix} 1 & 4 & 5 \\ 2 & 6 & 8 \\ 3 & 7 & 22 \end{bmatrix}$$

step 1 is to make it 1 0 0
step 2 " " " make it 0 1
step 3 is to

$$= \begin{bmatrix} 1 & 4 & 5 \\ 0 & -2 & -2 \\ 0 & -5 & 7 \end{bmatrix} \quad \begin{array}{l} R_2 - 2R_1 \rightarrow R_2 \\ R_3 - 3R_1 \rightarrow R_3 \end{array}$$

$$= \begin{bmatrix} 1 & 4 & 5 \\ 0 & 1 & 1 \\ 0 & -5 & 7 \end{bmatrix} \quad -\frac{1}{2} \times R_2 \rightarrow R_2$$

$$= \begin{bmatrix} 1 & 4 & 5 \\ 0 & 1 & 1 \\ 0 & 0 & 12 \end{bmatrix} \quad R_2 \times 5 + R_3 \rightarrow R_3$$

$$= \begin{bmatrix} 1 & 4 & 5 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad R_3 \times \frac{1}{12} \rightarrow R_3$$

R-3

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#2. $\begin{bmatrix} -1 & 2 & 3 & -2 \\ 2 & -5 & 1 & 2 \\ 3 & -8 & 5 & 2 \\ 5 & -12 & -1 & 6 \end{bmatrix}$

$= \begin{bmatrix} 1 & -2 & -3 & 2 \\ 2 & -5 & 1 & 2 \\ 3 & -8 & 5 & 2 \\ 5 & -12 & -1 & 6 \end{bmatrix} \quad R_1 \times -1 \longrightarrow R_1$

$= \begin{bmatrix} 1 & -2 & -3 & 2 \\ 0 & -1 & 7 & -2 \\ 0 & -2 & 14 & -4 \\ 0 & -2 & 14 & -4 \end{bmatrix} \quad \begin{matrix} R_2 - 2R_1 \longrightarrow R_2 \\ R_3 - 3R_1 \longrightarrow R_3 \\ R_4 - 5R_1 \longrightarrow R_4 \end{matrix}$

$= \begin{bmatrix} 1 & -2 & -3 & 2 \\ 0 & 1 & -7 & 2 \\ 0 & -2 & 14 & -4 \\ 0 & -2 & 14 & -4 \end{bmatrix} \quad R_2 \times -1 \longrightarrow R_2$

$= \begin{bmatrix} 1 & -2 & -3 & 2 \\ 0 & 1 & -7 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} 2R_2 + R_3 \longrightarrow R_3 \\ 2R_2 + R_4 \longrightarrow R_4 \end{matrix}$

$\therefore R-2$

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#3 $\begin{bmatrix} 3 & -4 & -1 & 2 \\ 1 & 7 & 3 & 1 \\ 5 & -2 & 5 & 4 \\ 9 & -3 & 7 & 7 \end{bmatrix}$

$= \begin{bmatrix} 1 & 7 & 3 & 1 \\ 3 & -4 & -1 & 2 \\ 5 & -2 & 5 & 4 \\ 9 & -3 & 7 & 7 \end{bmatrix} R_2 \Leftrightarrow R_1$

$= \begin{bmatrix} 1 & 7 & 3 & 1 \\ 0 & -25 & -10 & -1 \\ 0 & -37 & -10 & -1 \\ 0 & -66 & -20 & -2 \end{bmatrix}$ $R_2 - 3R_1 \longrightarrow R_2$
$R_3 - 5R_1 \longrightarrow R_3$
$R_4 - 9R_1 \longrightarrow R_4$

$= \begin{bmatrix} 1 & 7 & 3 & 1 \\ 0 & 1 & 0.4 & 0.04 \\ 0 & -37 & -10 & -1 \\ 0 & -66 & -20 & -2 \end{bmatrix}$ $R_2 \times -\frac{1}{25} \longrightarrow R_2$

$= \begin{bmatrix} 1 & 7 & 3 & 1 \\ 0 & 1 & 0.4 & 0.04 \\ 0 & 0 & 4.8 & 0.048 \\ 0 & 0 & 6.4 & 0.64 \end{bmatrix}$ $R_3 + 37R_2 \longrightarrow R_3$
$R_4 + 66R_2 \longrightarrow R_4$

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$$= \begin{bmatrix} 1 & 7 & 3 & 1 \\ 0 & 1 & 0.4 & 0.04 \\ 0 & 0 & 1 & 0.01 \\ 0 & 0 & 6.4 & 0.64 \end{bmatrix} \quad R_3 \times \frac{1}{4.8} \longrightarrow R_3$$

$$= \begin{bmatrix} 1 & 7 & 3 & 1 \\ 0 & 1 & 0.4 & 0.04 \\ 0 & 0 & 1 & 0.01 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_4 - 16 \times R_2 \longrightarrow R_4$$

$R_1 - 3$

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1. $$\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix} \quad R_1 \Longleftrightarrow R_2$$

$$= \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix} \quad \begin{matrix} R_2 - 2R_1 \longrightarrow R_2 \\ R_3 - 3R_1 \longrightarrow R_3 \\ R_4 - 6R_1 \longrightarrow R_4 \end{matrix}$$

$$= \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 1 & \frac{3}{5} & \frac{7}{5} \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix} \quad R_2 \times \frac{1}{5} \rightarrow R_3$$

$$= \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 1 & \frac{3}{5} & \frac{7}{5} \\ 0 & 0 & \frac{33}{5} & \frac{22}{5} \\ 0 & 0 & \frac{33}{5} & \frac{22}{5} \end{bmatrix} \quad \begin{matrix} R_3 - 4R_2 \longrightarrow R_3 \\ R_4 - 9R_2 \longrightarrow R_4 \end{matrix}$$

$$= \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 1 & \frac{3}{5} & \frac{7}{5} \\ 0 & 0 & 1 & \frac{2}{3} \\ 0 & 0 & \frac{33}{5} & \frac{22}{5} \end{bmatrix} \quad R_3 \times \frac{5}{33} \longrightarrow R_3$$

$$= \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 1 & \frac{3}{5} & \frac{7}{5} \\ 0 & 0 & 1 & \frac{2}{3} \\ 0 & 0 & 0 & 0 \end{bmatrix} - R_3 \times \frac{33}{5} + R_4 \longrightarrow R_4$$

$\therefore$ Rank $= 3$ (Ans)

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2. $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ $R_2 - 2R_1 \rightarrow R_2$
$R_3 - 3R_1 \rightarrow R_3$

$= \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ $R_3 - R_2 \rightarrow R_3$

$\therefore$ Rank = 2

3.

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3. $\begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{bmatrix}$

$= \begin{bmatrix} \cancel{3}1 & -\frac{1}{3} & \frac{2}{3} \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{bmatrix} \quad R_1 \times \frac{1}{3} \rightarrow R_1$

$= \begin{bmatrix} 1 & -\frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 8 \\ 0 & 0 & 4 \end{bmatrix} \quad \begin{matrix} R_2 + 6R_1 \rightarrow R_2 \\ R_3 + 3R_1 \rightarrow R_3 \end{matrix}$

$= \begin{bmatrix} 1 & -\frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 4 \\ 0 & 0 & 8 \end{bmatrix} \quad R_2 \Leftrightarrow R_3$

$= \begin{bmatrix} 1 & -\frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 1 \\ 0 & 0 & 8 \end{bmatrix} \quad R_2 \times \frac{1}{4} \rightarrow R_2$

$= \begin{bmatrix} 1 & -\frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad R_3 \times \cancel{\frac{1}{8}}\ \cancel{-}\ 8R_2 \rightarrow R_3$

$\therefore$ Rank $= 2$

4. $$\begin{bmatrix} 2 & 4 & 3 & -2 \\ -3 & -2 & -1 & 4 \\ 6 & -1 & 7 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & \frac{3}{2} & -1 \\ -3 & -2 & -1 & 4 \\ 6 & -1 & 7 & 2 \end{bmatrix} \quad R_1 \times \frac{1}{2} \rightarrow R_1$$

$$= \begin{bmatrix} 1 & 2 & \frac{3}{2} & -1 \\ 0 & 4 & \frac{7}{2} & 1 \\ 0 & -13 & -2 & 8 \end{bmatrix} \quad \begin{matrix} R_2 + 3R_1 \rightarrow R_2 \\ R_3 - 6R_1 \rightarrow R_3 \end{matrix}$$

$$= \begin{bmatrix} 1 & 2 & \frac{3}{2} & -1 \\ 0 & 1 & \frac{7}{2} & 4 \\ 0 & 8 & -2 & -13 \end{bmatrix} \quad C_2 \Longleftrightarrow C_4$$

$$= \begin{bmatrix} 1 & 2 & \frac{3}{2} & -1 \\ 0 & 1 & \frac{7}{2} & 4 \\ 0 & 0 & -30 & -45 \end{bmatrix} \quad R_3 - 8R_2 \rightarrow R_3$$

$$= \begin{bmatrix} 1 & 2 & \frac{3}{2} & -1 \\ 0 & 1 & \frac{7}{2} & 4 \\ 0 & 0 & 1 & \frac{3}{2} \end{bmatrix} \quad R_4 \times -\frac{1}{30} \rightarrow R_4$$

$\therefore$ Rank = 3 (Ans)

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⑨ Solution of ~~the~~ Simultaneous Equations

(Using Gauss Elimination)

Find the solution of (by using Gauss Elimination)

$$x - y + 2z = 3$$
$$x + 2y + 3z = 5$$
$$3x - 4y - 5z = -13$$

In matrix form,

$$\begin{bmatrix} 1 & -1 & 2 \\ 1 & 2 & 3 \\ 3 & -4 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ -13 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 1 \\ 0 & -1 & -11 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ -22 \end{bmatrix} \quad \begin{matrix} R_2 - R_1 \longrightarrow R_2 \\ R_3 - 3R_1 \rightarrow R_3 \end{matrix}$$

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$$= \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & -3 \\ 0 & -11 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ -22 \end{bmatrix} \quad C_2 \Leftrightarrow C_3$$

$$= \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & -3 \\ 0 & 0 & -34 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} \quad R_3 + 11R_2 \longrightarrow R_3$$

$$= \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} \quad R_3 \times -\frac{1}{34} \longrightarrow R_3$$

This is wrong

~~DOND~~

DONT do column shifting

$$= \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & \frac{1}{3} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ \frac{2}{3} \\ 2 \end{bmatrix}$$

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$\therefore x - y + 2z = 3 \quad \ldots \ldots (i)$

$y + \frac{z}{3} = \frac{2}{3}$

or, $3y + z = 2 \quad \ldots \ldots (ii)$

$\therefore z = 2 \quad \ldots \ldots (iii)$

From (ii),

$3y = 2 - 2 = 0$

$\therefore y = 0$ (Ans)

From (i),

$x - 0 + 4 = 3$

$\therefore x = -1$ (Ans)

$\therefore (x, y, z) = (-1, 0, 2)$ (Ans)

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(For Gauss - Jordan):

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} \quad \begin{matrix} R_2 \cancel{x} - R_3 \times \frac{1}{3} \rightarrow R_2 \\ R_1 - 2R_3 \longrightarrow R_1 \end{matrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} \quad R_2 + R_1 \longrightarrow R_1$$

$\therefore x = -1$

$y = 0$

$z = 2$

Find using Gauss Jordan,

$$2x_1 + x_2 + 2x_3 + x_4 = 6$$
$$6x_1 - 6x_2 + 6x_3 + 12x_4 = 36$$
$$4x_1 + 3x_2 + 3x_3 - 3x_4 = -1$$
$$2x_1 + 2x_2 - x_3 + x_4 = 10$$

$x_1 = 2$
$x_2 = 1$
$x_3 = -1$
$x_4 = 3$

In matrix form,

$$\begin{bmatrix} 2 & 1 & 2 & 1 \\ 6 & -6 & 6 & 12 \\ 4 & 3 & 3 & -3 \\ 2 & 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 6 \\ 36 \\ -1 \\ 10 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \frac{1}{2} & 1 & \frac{1}{2} \\ 6 & -6 & 6 & 12 \\ 4 & 3 & 3 & -3 \\ 2 & 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 3 \\ 36 \\ -1 \\ 10 \end{bmatrix} \quad R_1 \times \frac{1}{2} \rightarrow R_1$$

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$$= \begin{bmatrix} 1 & \frac{1}{2} & 1 & \frac{1}{2} \\ 0 & -9 & 0 & 9 \\ 0 & 1 & -1 & -5 \\ 0 & 1 & -3 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 3 \\ 18 \\ -13 \\ 4 \end{bmatrix} \quad \begin{array}{l} R_2 - 6R_1 \to R_2 \\ R_3 - 4R_1 \to R_3 \\ R_4 - 2R_1 \to R_4 \end{array}$$

$$= \begin{bmatrix} 1 & \frac{1}{2} & 1 & \frac{1}{2} \\ 0 & 1 & 0 & -1 \\ 0 & 1 & -1 & -5 \\ 0 & 1 & -3 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -13 \\ 4 \end{bmatrix} \quad R_2 \times -\frac{1}{9} \to R_2$$

$$= \begin{bmatrix} 1 & \frac{1}{2} & 1 & \frac{1}{2} \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -1 & -4 \\ 0 & 0 & -3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -11 \\ 6 \end{bmatrix} \quad \begin{array}{l} R_3 - R_2 \to R_3 \\ R_4 - R_2 \to R_4 \end{array}$$

$$= \begin{bmatrix} 1 & \frac{1}{2} & 1 & \frac{1}{2} \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & -3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 11 \\ 6 \end{bmatrix} \quad R_3 \times -1 \to R_3$$

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$$=\begin{bmatrix}1 & \frac{1}{2} & 1 & \frac{1}{2}\\ 0 & 1 & 0 & -1\\ 0 & 0 & 1 & 4\\ 0 & 0 & 0 & 13\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}=\begin{bmatrix}3\\-2\\11\\39\end{bmatrix} \quad R_4+3R_3\to R_4$$

$$=\begin{bmatrix}1 & \frac{1}{2} & 1 & \frac{1}{2}\\ 0 & 1 & 0 & -1\\ 0 & 0 & 1 & 4\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}=\begin{bmatrix}3\\-2\\11\\3\end{bmatrix} \quad R_4\times\frac{1}{13}\to R_4$$

$$=\begin{bmatrix}1 & \frac{1}{2} & 1 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}=\begin{bmatrix}\frac{3}{2}\\1\\-1\\3\end{bmatrix} \quad \begin{matrix}R_3-4R_1\to R_3\\ R_2+R_1\to R_2\\ R_1-\frac{1}{2}R_2\to R_1\end{matrix}$$

$$=\begin{bmatrix}1 & \frac{1}{2} & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}=\begin{bmatrix}\frac{5}{2}\\1\\-1\\3\end{bmatrix} \quad R_1-R_3\to R_1$$

$$=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}=\begin{bmatrix}2\\1\\-1\\3\end{bmatrix} \quad R_1-\frac{1}{2}R_2\to R_1$$

$\therefore x_1 = 2$

$x_2 = -1$

$x_3 = 1$

$x_4 = 3$

(Ans)

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#3. ~~Giv~~ Given equations,

$$3x_1 + 2x_3 + 2x_4 = 0$$

$$-x_1 + 7x_2 + 4x_3 + 9x_4 = 0$$

$$7x_1 - 7x_2 - 5x_4 = 0$$

In matrix form,

$$\begin{bmatrix} 3 & 0 & 2 & 2 \\ -1 & 7 & 4 & 9 \\ 7 & -7 & 0 & -5 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 & 2 & 2 \\ -1 & 7 & 4 & 9 \\ 7 & -7 & 0 & -5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & \frac{2}{3} & \frac{2}{3} \\ -1 & 7 & 4 & 9 \\ 7 & -7 & 0 & -5 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_1 \times \frac{1}{3} \rightarrow R_1$$

$$= \begin{bmatrix} 1 & 0 & \frac{2}{3} & \frac{2}{3} \\ 0 & 7 & \frac{14}{3} & \frac{29}{3} \\ 0 & -7 & -\frac{14}{3} & -\frac{29}{3} \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} R_2 + R_1 \rightarrow R_2 \\ R_3 - 7R_1 \rightarrow R_3 \end{matrix}$$

$$= \begin{bmatrix} 1 & 0 & \frac{2}{3} & \frac{2}{3} \\ 0 & 1 & \frac{2}{3} & \frac{29}{21} \\ 0 & -7 & -\frac{14}{3} & -\frac{29}{3} \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_2 \times \frac{1}{7} \rightarrow R_2$$

$$= \begin{bmatrix} 1 & 0 & \frac{2}{3} & \frac{2}{3} \\ 0 & 1 & \frac{2}{3} & \frac{29}{21} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_3 + 7R_2 \rightarrow R_3$$

$\therefore$ Let, $x_3 = b$ and $x_4 = a$.

$\therefore$

4. $x_1 + 2x_2 - x_3 = 1$

$3x_1 - 2x_2 + 2x_3 = 2$

$7x_1 - 2x_2 + 3x_3 = 5$

In matrix form,

$$\begin{bmatrix} 1 & 2 & -1 \\ 3 & -2 & 2 \\ 7 & -2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & -1 \\ 0 & -8 & 5 \\ 0 & -16 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix} \quad \begin{matrix} R_2 - 3R_1 \rightarrow R_2 \\ R_3 - 7R_1 \rightarrow R_3 \end{matrix}$$

$$= \begin{bmatrix} 1 & 2 & -1 \\ 0 & -8 & 5 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \quad R_3 - 2R_2 \rightarrow R_3$$

$$= \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & -\frac{5}{8} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{1}{8} \\ 0 \end{bmatrix} \quad R_2 \times -\frac{1}{8} \rightarrow R_2$$

Let, $x_3 = k$

$\therefore -8x_2 + 5x_3 = -1$

or, $-8x_2 + 5k = -1$

$\therefore x_2 = \frac{1+5k}{8}$

$\therefore x_1 + 2x_2 - x_3 = 1$

or, $x_1 = 1 - 2x_2 + x_3$

$= 1 - \frac{(5k+1)}{4} + k$

$= \frac{4 - 5k - 1 + 4k}{4}$

$= -\frac{k - 3}{4}$ (Ans)

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$$\# \begin{bmatrix} 2 & -6 & -2 & -3 \\ 5 & -13 & -4 & -7 \\ 1 & 4 & 1 & 2 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$$= \left[\begin{array}{cccc|cccc} 2 & -6 & -2 & -3 & 1 & 0 & 0 & 0 \\ 5 & -13 & -4 & -7 & 0 & 1 & 0 & 0 \\ 1 & 4 & 1 & 2 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{cccc|cccc} 1 & -4 & -1 & -2 & 0 & 0 & -1 & 0 \\ 2 & -6 & -2 & -3 & 1 & 0 & 0 & 0 \\ 5 & -13 & -4 & -7 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \quad \begin{array}{l} R_3 \times -1 \rightarrow R_3 \\ R_1 \Leftrightarrow R_3 \\ R_2 \Leftrightarrow R_3 \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & -4 & -1 & -2 & 0 & 0 & -1 & 0 \\ 0 & 2 & 0 & 1 & 1 & 0 & 2 & 0 \\ 0 & 7 & 1 & 3 & 0 & 1 & 5 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \quad \begin{array}{l} R_2 - 2R_1 \rightarrow R_2 \\ R_3 - 5R_1 \rightarrow R_3 \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & -4 & -1 & -2 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 1 & 0 \\ 0 & 7 & 1 & 3 & 0 & 1 & 5 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \quad R_2 \times \frac{1}{2} \rightarrow R_2$$

$$= \left[\begin{array}{cccc|cccc} 1 & -4 & -1 & -2 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 1 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & -\frac{7}{2} & 1 & -2 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & -\frac{1}{2} & 0 & -1 & 1 \end{array}\right] \quad \begin{array}{l} R_3 - 7R_2 \to R_3 \\ R_4 - R_2 \to R_4 \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & -4 & -1 & -2 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 1 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & -\frac{7}{2} & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & -2 & 2 \end{array}\right] \quad R_4 \times 2 \to R_4$$

$$= \left[\begin{array}{cccc|cccc} 1 & -4 & -1 & -2 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & -4 & 1 & -3 & 1 \\ 0 & 0 & 0 & 1 & -1 & 0 & -2 & 2 \end{array}\right] \quad R_3 + \frac{1}{2}R_4 \to R_3$$

$$= \left[\begin{array}{cccc|cccc} 1 & -4 & -1 & 0 & -2 & 0 & -5 & 4 \\ 0 & 1 & 0 & 0 & 1 & 0 & 2 & -1 \\ 0 & 0 & 1 & 0 & -4 & 1 & -3 & 1 \\ 0 & 0 & 0 & 1 & -1 & 0 & -2 & 2 \end{array}\right] \quad \begin{array}{l} R_2 - \frac{1}{2}R_4 \to R_2 \\ R_1 + 2R_4 \to R_4 \end{array}$$

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$$= \left[\begin{array}{cccc|cccc} 1 & -4 & 0 & 0 & -6 & 1 & -8 & 5 \\ 0 & 1 & 0 & 0 & 1 & 0 & 2 & -1 \\ 0 & 0 & 1 & 0 & -4 & 1 & -3 & 1 \\ 0 & 0 & 0 & 1 & -1 & 0 & -2 & 2 \end{array}\right] \qquad R_4 + R_3 \rightarrow R_1$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -2 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 2 & -1 \\ 0 & 0 & 1 & 0 & -4 & 1 & -3 & 1 \\ 0 & 0 & 0 & 1 & -1 & 0 & -2 & 2 \end{array}\right] \qquad R_1 + 4R_2 \rightarrow R_1$$

Types of Systems of Linear Equations

- Consistent
  - Unique
  - Infinitely many sol^n
    [has to follow ratio]

    $$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

    $$\begin{bmatrix} & \text{any} & \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \vdots \\ x \end{bmatrix} = \begin{bmatrix} \vdots \\ 0 \end{bmatrix}$$
- Inconsistent
  $0 = a$

  $$\begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \end{bmatrix} = \begin{bmatrix} a \end{bmatrix}$$

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\# 1. Find ~~fo~~ what value of k, the following ~~St~~ system has

i) No solution ii) Infinitely many solutions.

$$2x - 3y + 6z - 5t = 3$$
$$y - 4z + t = 1$$
$$4x - 5y + 8z + 9t = k$$

In Matrix form,

$$\left[\begin{array}{cccc|c} 2 & -3 & 6 & -5 & 3 \\ 0 & 1 & -4 & 1 & 1 \\ 4 & -5 & 8 & 9 & k \end{array}\right]$$

$$= \left[\begin{array}{cccc|c} 1 & -\frac{3}{2} & 3 & -\frac{5}{2} & \frac{3}{2} \\ 0 & 1 & -4 & 1 & 1 \\ 4 & -5 & 8 & 9 & k \end{array}\right] \quad R_1 \times \frac{1}{2} \rightarrow R_1$$

$$
= \left[\begin{array}{cccc|c}
1 & -\frac{3}{2} & 3 & -\frac{5}{2} & \frac{3}{2} \\
0 & 1 & -1 & 1 & 1 \\
0 & 1 & -4 & 1 & k-6
\end{array}\right] \quad R_3 - 4R_1 \to R_3
$$

$$
= \left[\begin{array}{cccc|c}
1 & -\frac{3}{2} & 3 & -\frac{5}{2} & \frac{3}{2} \\
0 & 1 & -4 & 1 & 1 \\
0 & 0 & 0 & 0 & k-7
\end{array}\right] \quad R_3 - R_2 \to R_3
$$

~~∴ It will have no soult solution if~~

~~$0 = k-7$~~

For (i),

$k-7 \neq 0$

$\therefore k \neq 7$

For (ii),

$k-7 = 0$

$\therefore k = 7$

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#2. Determine for what values of λ and μ of the following equations has,
i) No sol^n ii) Unique sol^n iii) Many sol^n

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu$$

In matrix form,

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda-1 & \mu-6 \end{array}\right] \quad \begin{array}{l} R_2 - R_1 \to R_2 \\ R_3 - R_1 \to R_3 \end{array}$$

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$$= \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda - 3 & \mu - 10 \end{array}\right] \quad R_3 - R_2 \rightarrow R_3$$

For (i),

$$\lambda - 3 = 0$$

$$\therefore \lambda = 3$$

and,

$$\mu - 10 \neq 0$$

$$\therefore \mu \neq 10$$

For (ii),

$$\lambda - 2 \neq 0$$

$$\therefore \lambda \neq 3$$

~~and,~~ and, $\mu - 10 = 0$. | $\mu \neq 10$ | $\mu \in \mathbb{R}$

$$\therefore \mu = 10$$

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For (iii),

$\lambda - 3 = 0$

$\therefore \lambda = 3$

and,

$\mu - 10 = 0$

$\therefore \mu = 10$

#3. Find the value of a and b here,
i) No sol^n ii) Infinite sol^n iii) Unique sol^n

$$3x - 2y + z = b$$
$$5x - 8y + 9z = 3$$
$$2x + y + a = -1$$

In Matrix form,

$$\left[\begin{array}{ccc|c} 3 & -2 & 1 & b \\ 5 & -8 & 9 & 3 \\ 2 & 1 & a & -1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & -\frac{2}{3} & \frac{1}{3} & \frac{b}{3} \\ 5 & -8 & 9 & 3 \\ 2 & 1 & a & -1 \end{array}\right] \quad R_1 \times \frac{1}{3} \rightarrow R_1$$

$$= \left[\begin{array}{ccc|c} 1 & -\frac{2}{3} & \frac{1}{3} & \frac{b}{3} \\ 0 & -\frac{14}{3} & \frac{22}{3} & 3 - \frac{5b}{3} \\ 0 & \frac{7}{3} & a - \frac{2}{3} & -1 - \frac{2b}{3} \end{array}\right] \quad \begin{array}{l} R_2 - 5R_1 \rightarrow R_2 \\ R_3 - 2R_1 \rightarrow R_3 \end{array}$$

$a - \frac{2}{3} + \frac{11}{3}$ $= a - \frac{13}{3}$ $-\frac{7}{3} \times -\frac{11}{7}$ $= \frac{11}{3}$ $-\frac{7}{3} \times \frac{-3}{14} = \frac{1}{2}$

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$$= \left[\begin{array}{ccc|c} 1 & -\frac{2}{3} & \frac{1}{3} & \frac{b}{3} \\ 0 & 1 & -\frac{11}{7} & -\frac{3}{14}\left(3 - \frac{5b}{3}\right) \\ 0 & \frac{7}{3} & a - \frac{2}{3} & -1 - \frac{2b}{-3} \end{array}\right] \quad R_2 \times -\frac{3}{14} \rightarrow R_2$$

$$= \left[\begin{array}{ccc|c} 1 & -\frac{2}{3} & \frac{1}{3} & \frac{b}{3} \\ 0 & 1 & -\frac{11}{7} & -\frac{3}{14}\left(3 - \frac{5b}{3}\right) \\ 0 & 0 & a - \frac{11}{3} & -1 - \frac{2b}{-3} + \frac{1}{2}\left(3 - \frac{5b}{3}\right) \end{array}\right] \quad R_3 - \frac{7}{3}R_2 \rightarrow R_3$$

For (i),

$a - \frac{11}{3} = 0$

$\therefore a = \frac{11}{3}$

and, $-1 + \frac{2b}{3} + \frac{3}{2} - \frac{5b}{6} \neq 0$

or, $\frac{1}{2} - \frac{b}{6} \neq 0$

or, $\frac{b}{6} \neq \frac{1}{2}$

$\therefore b \neq 3$ (Ans)

For (ii),

$a - \frac{11}{3} = 0$

$\therefore a = \frac{11}{3}$

and, $-1 + \frac{2b}{3} + \frac{3}{2} - \frac{5b}{6} = 0$

$\therefore b = 3$

For (iii),

$a - \frac{11}{3} \neq 0$

$\therefore a \neq \frac{11}{3}$

$b \in \mathbb{R}$

$a\bar{b} = c$

$a = b^{-1}c$

TOPIC NAME : W-4 DAY: C-7 TIME: DATE: 23/12/24

Cayley ~~Clay~~ - Hamilton Theorem

(a) Characteristics Matrix :

For a given square matrix A, $A - \lambda I$ matrix is called characteristica matrix, where λ is scalar and I is identity matrix

$$[A - \lambda I]$$

(b) Characteristics Polynomial :

The determinant $|A - \lambda I|$ will give a polynomial.

(c) Characteristics Equation :

$$|A - \lambda I| = 0$$

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Let,

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{bmatrix}$$

$$\text{C. Polynomial} = |A - \lambda I| = \begin{vmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{vmatrix}$$

$$= (2-\lambda)\{(3-\lambda)(2-\lambda) - (1)(2)\}$$
$$- 2\{(2-\lambda)(1) - (1)(1)\}$$
$$+ \{(1)(2) - (3-\lambda)(1)\}$$

$$= (2-\lambda)\{6 - 5\lambda + \lambda^2 - 2\} - 2(2-\lambda-1)$$
$$+ (2 - 3 + \lambda)$$
$$= \lambda^3 - 7\lambda^2 + 11\lambda + 5$$

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C. Roots : $\lambda = 1, 1, 5$

or, Eigen Values

Find the characteristics roots of

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

$$\therefore |A - \lambda I| = \begin{vmatrix} 6-\lambda & -2 & 2 \\ -2 & 3-\lambda & -1 \\ 2 & -1 & 3-\lambda \end{vmatrix}$$

$$= (6-\lambda)\{(3-\lambda)(3-\lambda) - (-1)(-1)\}$$
$$+ 2\{(-2)(3-\lambda) - (-1)(2)\}$$
$$+ 2\{(-2)(-1) - (3-\lambda)(2)\}$$

$$= (6-\lambda)\{9 - 6\lambda + \lambda^2 - 1\}$$
$$+ 2\{-6 + 2\lambda + 2\} + 2\{2 - 6 + 2\lambda\}$$

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$$= 48 - 36x + 6x^2 - 8x + 6x^2 - x^3$$

$$\cancel{-8} + 4x \cancel{+4} \cancel{+4} - 12 + 4x$$

$$= -x^3 + 12x^2 - 37x + 32 \rightarrow -36x$$

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C.H Theorem: 1º

Every square matrix satisfies its own characteristics equation.

If A is square matrix then,

$$\lambda^3 - 2\lambda^2 + 3\lambda - 4 = 0$$ be its equation then,

according to cayley Hamilton theorem,

$$A^3 - 2A^2 + 3A - 4I = 0$$

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// Find inverse of

$$A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$$

Sol^n: Characteristics equation of A is,

$$|A - \lambda I| = 0$$

or, $$\begin{vmatrix} 1-\lambda & 2 & -2 \\ 1 & 1-\lambda & 1 \\ 1 & 3 & -1-\lambda \end{vmatrix} = 0$$

or, $(1-\lambda)\{(1-\lambda)(-1-\lambda) - 3\}$

$-2\{(-1-\lambda) - 1\} - 2\{3 - 1 + \lambda\} = 0$

or, $(1-\lambda)\{\lambda^2 - 4\} - 2\{-2-\lambda\} - 2\{2+\lambda\} = 0$

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or, $\lambda^2 - 4 - \lambda^3 + 4\lambda + 4 + 2\lambda - 4 - 2\lambda = 0$

or, $-\lambda^3 + \lambda^2 + 4\lambda - 4 = 0$

or, $\lambda^3 - \lambda^2 - 4\lambda + 4 = 0$

Using Caylay - Hamilton theorem,

$$A^3 - A^2 - 4A + 4I = 0$$

or, $A^2 - A - 4I + 4A^{-1} = 0$

$\therefore A^{-1} = \frac{1}{4}\left[-A^2 + A + 4I\right]$

$$= \frac{1}{4}\left\{\begin{bmatrix} -1 & 2 & -2 \\ -3 & -6 & 2 \\ -3 & -2 & -2 \end{bmatrix} + \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}\right\}$$

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$$= \begin{bmatrix} 1 & 1 & -1 \\ -\frac{1}{2} & -\frac{1}{4} & \frac{3}{4} \\ -\frac{1}{2} & \frac{1}{4} & \frac{1}{4} \end{bmatrix} \text{(Ans)}$$

H.W. Find A^{-1}

1. $A = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$

2. $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$

3. $A = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$

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1. Given that,

$$A = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$$

∴ Characteristic equation of A is,

$$|A - \lambda I| = 0$$

or, $\begin{vmatrix} 4-\lambda & 3 & 1 \\ 2 & 1-\lambda & -2 \\ 1 & 2 & 1-\lambda \end{vmatrix} = 0$

or, $(4-\lambda)\{(1-\lambda)^2+4\} - 3\{\overset{4-2\lambda}{\cancel{2(1-\lambda)+2}}\} + (4-1+\lambda) = 0$

or, $(4-\lambda)(1-2\lambda+\lambda^2+4)\overset{-12+6\lambda}{\cancel{-6(1-\lambda)}}\cancel{-6}+3+\lambda = 0$

or, $\cancel{\overset{4+16}{20} - 8\lambda + 4\lambda^2 - \lambda + 2\lambda^2 - \lambda^3 - 6 + 6\lambda - 6\;3 + \lambda = 0}$

or, $\cancel{-\lambda^3 + 6\lambda^2 - 2\lambda + 11 = 0}$

or, $\cancel{\lambda^3 - 6\lambda^2 + 2\lambda - 11 = 0}$

$= 4 - 8\lambda + 4\lambda^2 + 16 - \lambda + 4\lambda^2 - \lambda^3 - 9 + 7\lambda = 0$

$= 11 - 2\lambda$

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Using Caylay - Hamilton Theorem, we get,

$$A^3 - 6A^2 + 2A - 11I = 0$$

or, $A(A^2 - 6A + 2I - 11A^{-1}) = 0$

or, $A^2 - 6A + 2I - 11A^{-1} = 0$

or, $A^{-1} = \dfrac{A^2 - 6A + 2I}{11}$ Supposed to be 6

$$= \frac{1}{11} \times \left\{ \begin{bmatrix} 23 & 17 & -1 \\ 8 & 3 & -2 \\ \boxed{9} & 7 & -2 \end{bmatrix} - \begin{bmatrix} 24 & 18 & 6 \\ 12 & 6 & -12 \\ 6 & 12 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \right\}$$

$$= \frac{1}{11} \begin{bmatrix} \square & -1 & -7 \\ -4 & \square & 10 \\ 3 & -5 & \square \end{bmatrix}$$

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1. Given that,

$$A = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$$

$\therefore$ The characteristic equation will be,

$$|A - \lambda I| = 0$$

or, $$\begin{vmatrix} 4-\lambda & 3 & 1 \\ 2 & 1-\lambda & -2 \\ 1 & 2 & 1-\lambda \end{vmatrix} = 0$$

or, $(4-\lambda)\{(1-\lambda)(1-\lambda) - (-2)(2)\}$
$- 3\{\cancel{(-2)(2)}\ 2(1-\lambda) - (-2)(1)\}$
$+ 1\{(2)(2) - (1-\lambda)(1)\} = 0$

or, $(4-\lambda)\{1-\lambda-\lambda+\lambda^2+4\} - 3\{2-2\lambda+2\} + (4\ \cancel{} - 1+\lambda) = 0$

or, $(4-\lambda)(5-2\lambda+\lambda^2) - 3(4-2\lambda) + (3+\lambda) = 0$

or, $20-8\lambda+4\lambda^2-5\lambda+2\lambda^2-\lambda^3-12+6\lambda+3+\lambda = 0$

or, $11 - 6\lambda + 6\lambda^2 - \lambda^3 = 0$

or, $\lambda^3 - 6\lambda^2 + 6\lambda - 11 = 0$

Using Caylay - Hamilton theorem, we get,

$$A^3 - 6A^2 + 6A - 11I = 0$$

or, $A(A^2 - 6A + 6I - 11A^{-1}) = 0$

or, $A^2 - 6A + 6I - 11A^{-1} = 0$

$$\therefore A^{-1} = \frac{1}{11}\{A^2 - 6A + 6I\}$$

$$= \frac{1}{11}\left(\begin{bmatrix} 23 & 17 & -1 \\ 8 & 3 & -2 \\ 9 & 7 & -2 \end{bmatrix} - \begin{bmatrix} 24 & 18 & 6 \\ 12 & 6 & -12 \\ 6 & 12 & 6 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} \right)$$

$$= \frac{1}{11} \times \begin{bmatrix} 5 & -1 & -7 \\ -4 & 3 & 10 \\ 3 & -5 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{5}{11} & -\frac{1}{11} & -\frac{7}{11} \\ -\frac{4}{11} & \frac{3}{11} & \frac{10}{11} \\ \frac{3}{11} & -\frac{5}{11} & -\frac{2}{11} \end{bmatrix}$$ (Ans)

2. Given that,

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

$\therefore$ The characteristic equation will be,

$$|A - \lambda I| = 0$$

or, $\begin{vmatrix} 2-\lambda & 1 & 1 \\ 0 & 1-\lambda & 0 \\ 1 & 1 & 2-\lambda \end{vmatrix} = 0$

or, $(2-\lambda)\{(1-\lambda)(2-\lambda)\} \overset{-0}{\cancel{-1\{1\}}} + 1\{-(1-\lambda)\} = 0$

or, $(2-\lambda)\{2-\lambda-2\lambda+\lambda^2\} \cancel{+1}0 + \lambda \cancel{-1} = 0$

or, $(2-\lambda)(2-3\lambda+\lambda^2) + \lambda \overset{-1}{} = 0$

or, $4 - 6\lambda + 2\lambda^2 - 2\lambda + 3\lambda^2 - \lambda^3 + \lambda \overset{-1}{} = 0$

or, $3 - 7\lambda + 5\lambda^2 - \lambda^3 = 0$

$\therefore \lambda^3 - 5\lambda^2 + 7\lambda - 3 = 0$

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Using Caylay-Hamilton theorem,

$$A^3 - 5A^2 + 7A - 3I = 0$$

or, $A(A^2 - 5A + 7I - 3A^{-1}) = 0$

or, $A^2 - 5A + 7I - 3A^{-1} = 0$

$\therefore A^{-1} = \frac{1}{3}(A^2 - 5A + 7I)$

$$= \frac{1}{3}\left(\begin{bmatrix} 5 & 4 & 4 \\ 0 & 1 & 0 \\ 4 & 4 & 5 \end{bmatrix} - \begin{bmatrix} 10 & 5 & 5 \\ 0 & 5 & 0 \\ 5 & 5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}\right)$$

$$= \frac{1}{3}\begin{bmatrix} 2 & -1 & -1 \\ 0 & 3 & 0 \\ -1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\ 0 & 1 & 0 \\ -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \end{bmatrix}$$ (Ans)

3. Given that,

$$A = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

The characteristic equation ~~f~~ of A will be,

$$|\cancel{\lambda} A - \lambda I| = 0$$

or, $\begin{vmatrix} 3-\lambda & 1 & 1 \\ -1 & 5-\lambda & -1 \\ 1 & -1 & 3-\lambda \end{vmatrix} = 0$

or, $(3-\lambda)\{(5-\lambda)(3-\lambda)-(-1)(-1)\} - \{(-1)(3-\lambda)-(-1)(1)\}$
$+ \{(-1)(-1)-(5-\lambda)\} = 0$

or, $(3-\lambda)\{15-5\lambda-3\lambda+\lambda^2-1\} - (\lambda-3+1) + (1-5+\lambda) = 0$

or, $(3-\lambda)(\lambda^2-8\lambda+14) - (\lambda-2) + (-4+\lambda) = 0$

or, $3\lambda^2 - 24\lambda + 42 - \lambda^3 + 8\lambda^2 - 14\lambda - \lambda + 2 + \lambda - 4 = 0$

or, $-\lambda^3 + 11\lambda^2 - 38\lambda + 40 = 0$

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$\therefore \lambda^3 - 11\lambda^2 + 38\lambda - 40 = 0$

~~Replaci~~

Using Cayley-Hamilton theorem, we get,

$$A^3 - 11A^2 + 38A - 40I = 0$$

or, $A(A^2 - 11A + 38I - 40A^{-1}) = 0$

or, $A^2 - 11A + 38I - 40A^{-1} = 0$

$$\therefore A^{-1} = \frac{1}{40}(A^2 - 11A + 38I)$$

$$= \frac{1}{40} \times \left(\begin{bmatrix} 9 & 7 & 5 \\ -9 & 25 & -9 \\ 7 & -7 & 11 \end{bmatrix} - \begin{bmatrix} 33 & 11 & 11 \\ -11 & 55 & -11 \\ 11 & -11 & 33 \end{bmatrix} + \begin{bmatrix} 38 & 0 & 0 \\ 0 & 38 & 0 \\ 0 & 0 & 38 \end{bmatrix} \right)$$

$$= \frac{1}{40} \times \begin{bmatrix} 14 & -4 & -6 \\ 2 & 8 & 2 \\ -4 & 4 & 16 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{7}{20} & -\frac{1}{10} & -\frac{3}{20} \\ \frac{1}{20} & \frac{1}{5} & \frac{1}{20} \\ -\frac{1}{10} & \frac{1}{10} & \frac{2}{5} \end{bmatrix} \text{(Ans)}$$

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TOPIC NAME: 10-5 C-8

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Verify C-H and find A^{-1}

$$A = \begin{bmatrix} -2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

The characteristics eq^n will be,

$$|A - \lambda I| = 0$$

or, $\begin{vmatrix} -2-\lambda & -1 & 2 \\ -1 & 2-\lambda & -1 \\ 1 & -1 & 2-\lambda \end{vmatrix} = 0$

or, $(-2-\lambda)(4-4\lambda+\lambda^2-1) + (\lambda-2+1) + 2(1-2+\lambda) = 0$

or, $(-2-\lambda)(3-4\lambda+\lambda^2) + \lambda - 1 - 2 + 2\lambda = 0$

or, $-6+8\lambda-2\lambda^2-3\lambda+4\lambda^2-\lambda^3-3+3\lambda = 0$

or, $-9+8\lambda+2\lambda^2-\lambda^3 = 0$

$\therefore \lambda^3 - 2\lambda^2 - 8\lambda + 9 = 0$

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Replacing x with A, we get,

$$A^3 - 2A^2 - 8A + 9I = O$$

L.H.S

$$= A^3 - 2A^2 - 8A + 9I$$

$$= \begin{bmatrix} -11 & -12 & 18 \\ -10 & 19 & -20 \\ 10 & -18 & 21 \end{bmatrix} - 2\begin{bmatrix} 7 & -2 & 1 \\ -1 & 6 & -6 \\ 1 & -5 & 7 \end{bmatrix} - 8\begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} -11-14+16+9 & -12+4+8 & 18-2-16 \\ -10+2+8 & 19-12-16+9 & -20+12+8 \\ 10-2-8 & -18+10+8 & 21-14-16+9 \end{bmatrix}$$

$$= O$$

$$= R.H.S$$

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$\therefore L.H.S = R.H.S$

~~$\therefore$ Caylay~~ (Proved)

Now,

$$A^3 - 2A^2 - 8A + 9I = 0$$

or, $A(A^2 - 2A - 8I + 9A^{-1}) = 0$

or, $A^2 - 2A - 8I + 9A^{-1} = 0$

$\therefore A^{-1} = -\frac{1}{9}(A^2 - 2A - 8I)$

$$= -\frac{1}{9}\left(\begin{bmatrix} 7 & -2 & 1 \\ -1 & 6 & -6 \\ 1 & -5 & 7 \end{bmatrix} - \begin{bmatrix} -4 & -2 & 4 \\ -2 & 4 & -2 \\ 2 & -2 & 4 \end{bmatrix} - \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}\right)$$

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$$= \frac{-1}{9}\begin{bmatrix} 3 & 0 & -3 \\ 1 & -6 & -4 \\ -1 & -3 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{3} & 0 & \frac{1}{3} \\ -\frac{1}{9} & \frac{2}{3} & \frac{4}{9} \\ \frac{1}{9} & \frac{1}{3} & \frac{5}{9} \end{bmatrix}$$

$$A^4 = 2\begin{bmatrix} -11 & -12 & 18 \\ -10 & 19 & -20 \\ 10 & -18 & 21 \end{bmatrix} + 8\begin{bmatrix} 7 & -2 & 1 \\ -1 & 6 & -6 \\ 1 & -5 & 7 \end{bmatrix}$$

$$-9\begin{bmatrix} -2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 52 & -31 & 26 \\ -19 & 68 & -79 \\ 19 & -67 & 80 \end{bmatrix}$$ (Ans.

#
$$A = \begin{bmatrix} 3 & 2 & 4 \\ 4 & 3 & 2 \\ 2 & 4 & 3 \end{bmatrix}$$

Char

$$|A - \lambda I| = 0$$

or, $$\begin{vmatrix} 3-\lambda & 2 & 4 \\ 4 & 3-\lambda & 2 \\ 2 & 4 & 3-\lambda \end{vmatrix} = 0$$

or, $(3-\lambda)(9-6\lambda+\lambda^2-8)$
$- 2(12-4\lambda-4)$
$+ 4(16-6+2\lambda) = 0$

or, $(3-\lambda)(1-6\lambda+\lambda^2) - 2(8-4\lambda) + 4(10+2\lambda) = 0$

or, $3 - 18\lambda + 3\lambda^2 - \lambda + 6\lambda^2 - \lambda^3 - 16 + 8\lambda + 40 + 8\lambda = 0$

or, $27 - 3\lambda + 9\lambda^2 - \lambda^3 = 0$

or,

Using CH Theorem,

$$27I - 3A + 9A^2 - A^3 = 0$$

or, $A(27A^{-1} - 3I + 9A - A^2) = 0$

or, $A^{-1} = \frac{1}{27}(3I - 9A + A^2)$

$$= \frac{1}{27}\left(\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 27 & 18 & 36 \\ 36 & 27 & 18 \\ 18 & 36 & 27 \end{bmatrix} + \begin{bmatrix} 25 & 28 & 28 \\ 28 & 25 & 28 \\ 28 & 28 & 25 \end{bmatrix}\right)$$

$$= \begin{bmatrix} \frac{1}{27} & \frac{10}{27} & -\frac{8}{27} \\ -\frac{8}{27} & \frac{1}{27} & \frac{10}{27} \\ \frac{10}{27} & -\frac{8}{27} & \frac{1}{27} \end{bmatrix}$$

TOPIC NAME : W – 5 DAY : C – 9
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Eigen Value

Characteristice Vectors or Eigen Vectors

Let, A be an $n\times n$ square matrix and X, Y are two non zero ~~comman~~ ~~of~~ column ~~vec~~ matrices

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix}$$

$$Y = AX$$

Using CHT, $Y = \lambda X$

Corresponding to each characteristic root λ, we have a corresponding non zero vector X which satisfies the equation $[A - \lambda I]X = 0$

The non zero vectors X is called characteristic vector or eigen vector

TOPIC NAME

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

$|A - \lambda I| = 0$

or, $\begin{vmatrix} 1-\lambda & 0 & -1 \\ 1 & 2-\lambda & 1 \\ 2 & 2 & 3-\lambda \end{vmatrix} = 0$

or, $(1-\lambda)(6 - 2\lambda - 3\lambda + \lambda^2 - 2)$
$- 1(2 - 4 + 2\lambda) = 0$

or, $(1-\lambda)(4 - 5\lambda + \lambda^2) + 2 + 4 - 2\lambda = 0$

or, $\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$

$\therefore \lambda = 1, 2, 3$

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$\therefore$ Eigen vector corresponding to eigen value $\lambda = 1$ is

$$[A - \lambda I]\, x = 0$$

or,
$$\begin{bmatrix} 1-1 & 0 & -1 \\ 1 & 2-1 & 1 \\ 2 & 2 & 3-1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 0 & 0 & -1 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore x_3 = 0$

$\therefore \cancel{x_2 = x_1}$ $x_1 + y + z = 0$

$x_1 + x_2 = 0$

$\therefore x_1 = -x_2$

Let,

$y = k$

$\therefore x_1 = -k$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -k \\ k \\ 0 \end{bmatrix}$$

$$\therefore X_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \text{ (Ans)}$$

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When $\lambda = 3$, the corresponding eigen vector is,

$$\begin{bmatrix} -2 & 0 & -1 \\ 1 & -1 & 1 \\ 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore -2x - z = 0$

$\therefore z = \frac{-x}{2} \quad \ldots (i)$

$x - y + z = 0 \quad \ldots (ii)$

$2x + 2y = 0 \quad (iii)$

$\therefore -x = y$

Now,

$x + x - \frac{x}{2} = 0$

or, $\frac{3}{2}x = 0$

$\therefore x = 0$

$\therefore y = 0$

$\therefore z = 0$

Let,

$x = k$

$\therefore z = -2k$

$\therefore y = -k$

$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} k \\ -k \\ -2k \end{bmatrix}$

$\therefore X_3 = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}$ (Ans)

$A = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$

$\therefore$

$$\begin{vmatrix} 1-\lambda & 1 & 2 \\ -1 & 2-\lambda & 1 \\ 0 & 1 & -1-\lambda \end{vmatrix} = 0$$

or, $(1-\lambda)(-2-2\lambda+\lambda+\lambda^2-1) - (1+\lambda) + 2(-1) = 0$

or, $(1-\lambda)(-3-3\lambda+\lambda^2) - 1 - \lambda - 2 = 0$

or, $-3 - 3\lambda + \lambda^2 + 3\lambda + 3\lambda^2 - \lambda^3 - 1 - \lambda - 2 = 0$

~~or, $6 - \lambda + 4\lambda^2 - \lambda^3 = 0$~~

$\therefore \lambda^3 - 2\lambda^2 - \lambda + 2 = 0$

$\therefore \lambda = -1, 1, 2$

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Putting $\lambda=-1$ in $[A-\lambda I]X=0$, we get,

$$\begin{bmatrix} 2 & 1 & -2 \\ -1 & 3 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore 2x+y-2z=0$

$-x+3y+z=0$

$y=0$

$\therefore 2x+y-2z=0$

or, $x=z$

Let, $x=k$

$z=k$

$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} k \\ 0 \\ k \end{bmatrix} = k\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$\therefore X_{-1} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

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Putting $\lambda = 1$ in $[A - 2I]x = 0$, we get,

$$\begin{bmatrix} 0 & 1 & -2 \\ -1 & 1 & 1 \\ 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore y - 2z = 0$

$\therefore -x + y + z = 0$

$\therefore y - 2z = 0$

Let, $y = k$

$\therefore z = \frac{k}{2}$

$\therefore x = k + \frac{k}{2} = \frac{3}{2}k$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{3}{2}k \\ k \\ \frac{k}{2} \end{bmatrix} = \frac{k}{2}\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$$\therefore X_1 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

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$$\vdots$$

$$-x + y - 2z = 0$$
$$-x + z = 0$$
$$y + z = 0$$

$$\therefore y = -z$$

$$\therefore -x - z - 2z = 0$$

$$\therefore x = -3z$$

Let,

$$z = k$$

$$\therefore x = -$$

$$X_2 = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$$

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H.W Eigen vectors of

Q. A = 1. $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

2. $\begin{bmatrix} -2 & 5 & 4 \\ 5 & 7 & 5 \\ 4 & 5 & -2 \end{bmatrix}$

3. $\begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 6 \\ 1 & 0 & 5 \end{bmatrix}$

TOPIC NAME: W-5

C-B10

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Find the eigenvalues and corresponding eigenvector of

$$A = \begin{bmatrix} -2 & 5 & 4 \\ 5 & 7 & 5 \\ 4 & 5 & -2 \end{bmatrix}$$

Sol:

$$\begin{vmatrix} -2-\lambda & 5 & 4 \\ 5 & 7-\lambda & 5 \\ 4 & 5 & -2-\lambda \end{vmatrix} = 0$$

or, $-(2+\lambda)(-14-7\lambda+2\lambda+\lambda^2-25)$
$-5(-10-5\lambda-20)+4(25-28+4\lambda)=0$

or, $-(2+\lambda)(-39-5\lambda+\lambda^2)+5(30+5\lambda)+4(-3+4\lambda)=0$

or, $-(-78-10\lambda+2\lambda^2-39\lambda-5\lambda^2+\lambda^3)+150+25\lambda-12+16\lambda=0$

or, $-\lambda^3+3\lambda^2+49\lambda+78+138+41\lambda=0$

$\lambda = -3, 6, 12$

or,

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The characteristic matrix corresponding to the eigen-values $\lambda = -3$ is

$$\begin{bmatrix} 1 & 5 & 4 \\ 5 & 10 & 5 \\ 4 & 5 & 1 \end{bmatrix} = 0$$

$\therefore x + 5y + 4z = 0$

$\therefore 5x + 10y + 5z = 0$

$\therefore 4x + 5y + z = 0$

Using cross multiplication we get,

$$\frac{x}{25 - 40} = \frac{y}{20 - 5} = \frac{z}{10 - 25} = k$$

or, $\dfrac{x}{-15} = \dfrac{y}{15} = \dfrac{z}{-15} = k$

$\therefore x = 1$

$\therefore y = -1$

$\therefore z = 1$

$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

GOOD LUCK

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$\lambda = -6$.

$$\begin{bmatrix} 4 & 5 & 4 \\ 5 & 13 & 5 \\ 4 & 5 & 4 \end{bmatrix} = 0$$

$4x + 5y + 4z = 0$

$5x + 13y + 5z = 0$

$$\frac{x}{25 - 52} = \frac{y}{20 - 20} = \frac{z}{52 -}$$

⋮

$$X_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

⋮

$$X = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

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//

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$\vdots$

$$\begin{vmatrix} 2-\lambda & -1 & 1 \\ -1 & 2-\lambda & -1 \\ 1 & -1 & 2-\lambda \end{vmatrix} = 0$$

or, $(2-\lambda)(4-4\lambda+\lambda^2-1) + (\lambda-2+1) + (1-2+\lambda) = 0$

or, $(2-\lambda)(3-4\lambda+\lambda^2) + \lambda - 1 + \lambda - 1 = 0$

or, $6 - 8\lambda + 2\lambda^2 - 3\lambda + 4\lambda^2 - \lambda^3 + 2\lambda - 2 = 0$

or, $-\lambda^3 + 6\lambda^2 - 9\lambda + 4 = 0$

or, $\lambda^3 - 6\lambda^2 + 9\lambda - 4 = 0$

$\therefore \lambda = 1, 1, 4$

~~or, $(\lambda-4)$~~

or, $(\lambda-4)$

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The - - - $\lambda = 4$,

$$\begin{bmatrix} -2 & -1 & 1 \\ -1 & -2 & -1 \\ 1 & -1 & -2 \end{bmatrix} = 0$$

$\therefore -2x - y + z = 0$

$\therefore -x - 2y - z = 0$

$\therefore x - y - 2z = 0$

$$\frac{x}{2+1} = \frac{y}{\cancel{2+1}\ -3}$$

$$\therefore X_1 = \begin{bmatrix} x_1 \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

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$\lambda = 1,$

$$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix} = 0$$

or, $$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

$\therefore x_1 - x_2 + x_3 = 0$

$\therefore x_1 = x_2 - x_3$

Let,

$x_2 = k_1$

$x_3 = k_2$

(a) Let, $k_1 = 1, k_2 = 1$

$\therefore x_1 = 0$

$$\therefore X_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

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(b) Let, $k_1 = 2$ and $k_2 = 1$

$\therefore \; x_1 = 1$

$\therefore \; X_2' = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

Let,

$X_3 = \begin{bmatrix} l \\ m \\ n \end{bmatrix}$

As, X_3 is orthogonal to X_1 since the given matrix is symmetric

$$\begin{bmatrix} 1, & -1, & 1 \end{bmatrix} \begin{bmatrix} l \\ m \\ n \end{bmatrix} = 0$$

$\therefore \; l - m + n = 0$

As X_3 is also orthogonal to X_2 since the given matrix is symmetric

$$\begin{bmatrix} 0, & 1, & 1 \end{bmatrix} \begin{bmatrix} l \\ m \\ n \end{bmatrix} = 0$$

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$\therefore m + n = 0$

Using cross multiplication,

$$\frac{l}{-1 \cdot -1} = \frac{m}{-1} = \frac{n}{1} = k$$

$\therefore l = 2\cancel{k}$

$\therefore m = 1$

$\therefore n = -1$

$$\therefore X_3 = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

$(3-\lambda)(2-\lambda)$

$= 6-3\lambda+\lambda^2$

TOPIC NAME

#2
$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

$(2-\lambda)(6-5\lambda+\lambda^2)+(-3)=0$

or, $12-10\lambda+2\lambda^2-6\lambda+5\lambda^2-\lambda^3-3=0$

$\therefore \lambda^3-7\lambda^2+16\lambda-9=0$

$\lambda = 1, 3, 3$

c. $\lambda = 1$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore x_1+x_3=0$ | $2x_2=0$ | $X_1=\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ Ans.

$\therefore x_1=-x_3$ | $\therefore x_2=0$

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$\lambda = 3$

$$X_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Let, $X_3 = \begin{bmatrix} l \\ m \\ n \end{bmatrix}$

X_3 is orthogonal ~~in~~ to X_1

$$\begin{bmatrix} -1, & 0, & 1 \end{bmatrix} \begin{bmatrix} l \\ m \\ n \end{bmatrix} = 0$$

$\therefore -l + \cancel{m}\ 0\ n = 0$

X_3 is orthogonal to X_2

$$\begin{bmatrix} 1, & 1, & 1 \end{bmatrix} \begin{bmatrix} l \\ m \\ n \end{bmatrix} = 0$$

$\therefore l + m + n = 0$

$$\frac{l}{-1} = \frac{m}{1+1} = \frac{n}{-1} = k$$

$\therefore l = 1, \quad m = -2, \quad n = 1$

TOPIC NAME: w-6 DAY: C-11
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\# Find the eigenvalu~~e~~ ~~vector~~ of H.W

$$A = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & -2 \\ 0 & 3 & -1 \\ 0 & 1 & -1 \end{bmatrix}$$

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Vector HK Das → 5

PTSD.png

* When finding out volume from area, we use scalar triple product:

$$\vec{a}.(\vec{b}\times\vec{c}) = \vec{b}.(\vec{c}\times\vec{a}) = \vec{c}.(\vec{a}\times\vec{b})$$

↳ if 0, they are coplanar

* Geometrical Interpretation

PTSD_admission.png

* ~~Admission Phase~~ Differentiation of Vector

* Directional vector → Page 384 math MUST ✓ ✓

** Ex 5.6 → 1, 2, 3, 5

* Gradient

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* Gradient : $\nabla \varphi = \hat{i}\dfrac{\partial \varphi}{\partial x} + \hat{j}\dfrac{\partial \varphi}{\partial y} + \hat{k}\dfrac{\partial \varphi}{\partial z}$

+ Divergence : $\nabla \cdot \varphi = \dfrac{\partial \varphi_x}{\partial x} + \dfrac{\partial \varphi_y}{\partial y} + \dfrac{\partial \varphi_z}{\partial z}$

+ Curly : $\nabla \times \varphi = \hat{i}\left(\dfrac{\partial \varphi_z}{\partial y} - \dfrac{\partial \varphi_y}{\partial z}\right)$

$$- \hat{j}\left(\dfrac{\partial \varphi_x}{\partial z} - \dfrac{\partial \varphi_z}{\partial x}\right)$$

$$+ \hat{z}\left(\dfrac{\partial \varphi_y}{\partial x} - \dfrac{\partial \varphi_x}{\partial z}\right)$$

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#16. $\nabla\phi = \left(\hat{i}\,.\frac{\partial}{\partial x} + \hat{j}\,\frac{\partial}{\partial y} + \hat{k}\,\frac{\partial}{\partial z}\right)(3x^2y - y^2z^2)$

H.W : 17, 18, 19 $\rightarrow$ 404

GOOD LUCK

C-12

Q-G 6.1.25

Directional Derivatives

D.D:

If $\phi(x, y, z) = c$ represents a family of surfaces for different values of the curve. On differentiating ϕ,

we get, $d\phi = 0$ → Gradient

$$d\phi = \boxed{\nabla\phi} \cdot dr = 0$$

$\nabla\phi$ and dr are perpendicular. dr is in the direction of tangent to the given surface.

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Defn : The component of $\nabla \varphi$ in the direction of a vector $\vec{d}$ is equal to zero, which is called the directional derivative of of φ in the direction of $\vec{d}$.

#1. The temperature at any point in space is given by $T = xy + yz + zx$. Determine the derivative of T in the direction of $3\hat{i} - 4\hat{k}$ at $(1, 1, 1)$

Soln : $\nabla T = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)(xy + yz + zx)$

$= (y+z)\hat{i} + (x+z)\hat{j} + (y+x)\hat{k}$

At $(1, 1, 1)$, $\nabla T = 2\hat{i} + 2\hat{j} + 2\hat{k}$

Directional derivative at $(1, 1, 1)$ in the direction of $(3\hat{i} - 4\hat{k})$

$= (2\hat{i} + 2\hat{j} + 2\hat{k}) \cdot \dfrac{3\hat{i} - 4\hat{k}}{\sqrt{3^2+4^2}} = \dfrac{(2)(3) + (2)(0) + (2)(-4)}{5}$

$= -\dfrac{2}{5}$ (Ans)

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#3. For $\phi = \frac{x}{x^2+y^2}$, find the magnitude of the directional derivative along a line making an angle 30° with positive x axis at $(0, 2)$

Solⁿ: $\nabla \phi = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y}\right)\frac{x}{x^2+y^2}$

$$= \hat{i}\cdot\frac{y^2-x^2}{(x^2+y^2)^2} - \hat{j}\frac{2xy}{(x^2+y^2)}$$

At $(0, 2)$, derivative $= \hat{i}\cdot\frac{4}{16} - \hat{j}\cdot\frac{2.4\times 0}{16^2}$

$$= \frac{1}{4}\hat{i}$$

Directional derivative in the direction

$$\vec{OA} = \vec{OB} + \vec{BA}$$

$$= \hat{i}\cos 30^\circ + \hat{j}\sin 30^\circ$$

$$= \frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$$

$$= \frac{\hat{i}}{4}\cdot\left(\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right) = \frac{\sqrt{3}}{8} \text{ (Ans)}$$

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~~#Q. Find the~~

H.w (pic in phone)

+ # Find the d.d of V^2 where $\vec{v} = xy^2\hat{i} + zy^2 + xz^2\hat{k}$

, at (2, 0, 2) in the direction of the outward normal to the sphere $x^2+y^2+z^2=14$

#4. Find the derivative of $f(x, y, z) = x^2y^2z^2$ at (1, 1, -1) in the tangent to the curve $x = e^t$, $y = 2\sin t + 1$, $z = t - \cos t$ at $t=0$

Soln: $f(x, y, z) = x^2y^2z^2$

$\nabla f = - - .$

$= 2xy^2z^2\hat{i} + 2yx^2z^2\hat{j} + 2zx^2y^2\hat{k}$

At (1, 1, -1), $\nabla f = 2\hat{i} + 2\hat{j} - 2\hat{k}$

At $t = 0$; $x = e^0 = 1$

$y = 2\sin 0^\circ + 1 = 1$

$z = 0 - \cos 0^\circ = -1$

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$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$= e^{t} + (2\sin t + 1)\hat{j} + (t - \cos t)\hat{k}$$

Tangent $\frac{d\vec{r}}{dt} = e^{t}\hat{i} + 2\cos t\hat{j} + (1+\sin t)\hat{k}$

At $t = 0$, $d\vec{r} = \hat{i} + 2\hat{j} + \hat{k}$

$\therefore$ unit vector $= \frac{\hat{i}+2\hat{j}+\hat{k}}{\sqrt{1^2+2^2+1}} = \frac{\hat{i}+2\hat{j}+\hat{k}}{\sqrt{6}}$

$\therefore$ The d.d is,

$$y = (2\hat{i} + 2\hat{j} - 2\hat{k}) \cdot \frac{\hat{i}+2\hat{j}+\hat{k}}{\sqrt{6}}$$

$$= \frac{4}{\sqrt{16}} \text{ (Ans)}$$

GOOD LUCK

TOPIC NAME : W-6 DAY : C-13
TIME : DATE : 9 / 1 /25

#1. Find the directional derivative of the divergenge of $\rightarrow$ 5.16 - to 5.19

$f(x,y,z) = xy\hat{i} + xy^2\hat{j} + z^2\hat{k}$ in $(2,1,2)$ in the direction of the outer normal sphere $x^2+y^2+z^2=9$

Sol^n: $f(x,y,z) = xy\hat{i} + xy^2\hat{j} + z^2\hat{k}$

Divergence $\nabla.f = \left(\frac{\partial}{\partial x}\hat{i} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)(xy\hat{i} + xy^2\hat{j} + z^2\hat{k})$

Not the same.

$\therefore \phi = y + 2xy + 2z$

Therefore $\nabla\phi = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)(y + 2xy + 2z)$

$= 2y\hat{i} + (1+2x)\hat{j} + 2\hat{k}$

At $(2,1,2)$, $\nabla\phi = 2\hat{i} + 5\hat{j} + 2\hat{k}$

Normal to sphere $= \nabla$ sphere $= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)(x^2+y^2+z^2-9)$

$= 2x\hat{i} + 2y\hat{j} + 2z\hat{k}$

At $(2,1,2) = 4\hat{i} + 2\hat{j} + 4\hat{k}$

D.D along normal at $(2,1,2) = (2\hat{i}+5\hat{j}+2\hat{k}) \cdot \frac{4\hat{i}+2\hat{j}+4\hat{k}}{\sqrt{4^2+2^2+4^2}}$

$= \frac{13}{3}$ (Ans.

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Divergence:

$$\text{div } F = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

if div F : 0 ⟶ Solenoidal function

$\nabla F = 0$ ⟶ continuity eqn or conservation of mess

Find the value of n for which $r^n \cdot \vec{r}$ is solenoidal.

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

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#(a) Show that $N = (x+3y)\hat{i} + (y-3z)\hat{j} + (z-2z)\hat{k}$

is solenoidal.

Sol^n :

$$\text{div} F = \frac{\partial (x+3y)}{\partial x} + \frac{\partial (y-3z)}{\partial y} + \frac{\partial}{\partial z}(x-2z)$$

$$= 1 + 1 - 2$$

$$= 0.$$

$\therefore F$ is solenoidal

(showed)

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Curl : $\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$

$$= \hat{i}\left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}\right)$$

$$- \hat{j}\left(\frac{\partial F_3}{\partial x} - \frac{\partial F_1}{\partial z}\right)$$

$$+ \hat{k}\left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)$$

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2. A fluid motion is given by -

$$V = (y\sin z - \sin x)\hat{i} + (x\sin z + 2yz)\hat{j} + (xy\cos z + y^2)\hat{k}$$

Curl and velocity potential $\rightarrow V = \nabla\phi$

$$\text{Curl } V = \vec{\nabla} \times \vec{V} = ~~\hat{i}\left(xy\cos z + y^2\right)\frac{\partial}{\partial z}~~$$

$$~~= \hat{i}\left\{(-xy\sin z) -\right.~~$$

$$= \hat{i}(x\cos z + 2y - x\cos z - 2y)$$

$$+ \hat{j}$$

$$\vdots$$

$$= 0$$

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$\therefore$ The motion is irrotational.

So, $V = \nabla \varphi$ where φ is called velocity potential

$$d\varphi = \frac{\partial \varphi}{\partial x}dx + \frac{\partial \varphi}{\partial y}dy + \frac{\partial \varphi}{\partial z}dz$$

$$= \left(\hat{i}\frac{\partial \varphi}{\partial x} + \hat{j}\frac{\partial \varphi}{\partial y} + \hat{k}\frac{\partial \varphi}{\partial z}\right) \cdot \left(\hat{i}\,dx + \hat{j}\,dy + \hat{k}\,dz\right)$$

$$= \nabla \varphi \,.\, d\vec{r}$$

$$= \left[(y\sin z - \sin x)\hat{i} + (x\sin z + 2yz)\hat{j} + (xy\cos z + y^2)\hat{k}\right] \cdot (\hat{i}dx + \hat{j}dy + \hat{k}dz)$$

$$= (y\sin z - \sin x)dx + (x\sin z + 2yz)dy + (xy\cos z + y^2)dz$$

$=$

$\Rightarrow$ Third line from first term's integration

$$= d(xy\sin z) + d(\cos x) + d(y^2 z)$$

$$\therefore \varphi = \int d(xy\sin z) + \int d(\cos x) + \int d(y^2 z)$$

$$= xy\sin z + \cos x + y^2 z + c$$

GOOD LUCK

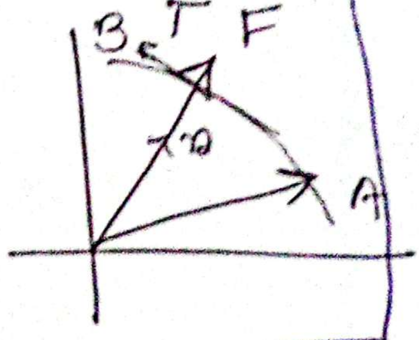
~~Int~~ Vector Integration

Integration of vectors or Line Integral:

Let $\vec{F}(x, y, z)$ be a vector function and a curve AB. Line integral of a vector function $\vec{F}$ along AB is defined as integral of the component of $\vec{F}$ along tangent to the curve

~~line I~~

$$\therefore \text{Line Integral} = \int_C \left(F \cdot \frac{dr}{ds}\right) ds = \int_C F \cdot d\vec{r}$$



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If a force $\vec{F} = 2x^2y\hat{i} + 3xy\hat{j}$ displaces a particle in xy plane from (0, 0) to (1.4) along curve $y = 4x^2$. Find work done.

Soln:

Work done $= \int_C F \cdot dr$

$= \int_C (2x^2y\hat{i} + 3xy\hat{j}) \cdot (dx\hat{i} + dy\hat{j})$

$= \int_C (2x^2y\,dx + 3xy\,dy)$

Now,

$y = 4x^2$

$\therefore dy = 8x\,dx$

$\therefore \int_C F\,dr = \int_0^1 (2x^2)(4x^2)dx + (3x)(4x^2)(8x\,dx)$

$= \int_0^1 (8x^4dx + 96x^4dx)$

$= 104\int_0^1 x^4dx$

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$$= 104 \times \left[\frac{x^5}{5}\right]_0^1$$

$$= 104 \times \frac{1}{5}$$

$$= 20.8 \text{ (Ans)}$$

#2. Compute $\int_C \vec{F}\, dr$ where $\vec{F} = \frac{\hat{i}\, y - \hat{j}\, x}{x^2+y^2}$

where C is the circle $x^2+y^2=1$

Sol^n:

Here,

replacing x with $\cos\theta$

and y " $\sin\theta$,

$\therefore dx = \cancel{d}\sin\theta\, d$

$\therefore dy = d\cos\theta\, d\theta$

we get,

$$\vec{F} = \frac{\hat{i}\sin\theta - \hat{j}\cos\theta}{1} = \hat{i}\sin\theta - \hat{j}\cos\theta$$

$$-d\sin\theta\,\hat{i} + d\cos\theta\,\hat{j}$$

$$\therefore \int_C \vec{F}\, dr = \int_0^{2\pi} \left(\hat{i}\,\overset{\cos\theta}{\cancel{\sin\theta}} - \hat{j}\,\overset{\sin\theta}{\cancel{\cos\theta}}\right) . \left(\cancel{d\theta\,\hat{i} + d\theta\,\hat{j}}\right)$$

$$= \int_0^{2\pi} \left(\overset{\cos\theta}{\cancel{\sin\theta}}\,\overset{(-d\sin\theta)}{\cancel{d\theta}} - \overset{\sin\theta}{\cancel{\cos\theta}}\,\overset{(d\cos\theta)}{\cancel{d\theta}}\right)$$

$$\cancel{= \left[-\cos\theta\right]_0^{2\pi} - \left[\sin\theta\right]_0^{2\pi}}$$

$$\cancel{= -1 + 1 - 0 + 0}$$

$$= \int_0^{2\pi} (\hat{i}\sin\theta - \hat{j}\cos\theta)(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j})\,d\theta$$

$$= \int_0^{2\pi} -(\sin^2\theta + \cos^2\theta)\,d\theta$$

$$= -1\int_0^{2\pi} d\theta$$

$$= -1(2\pi - 0)$$

$$= -2\pi \quad \text{(Ans)}$$

H.W

1 Show that $V = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$ is conservative. Find ϕ, $V = \text{grad}\,\phi$. Find work done by V in moving particle from $(1, -2, 1)$ to $(3, 1, 4)$.

Ans: $\phi = x^2y + xz^3$

$V = 202$

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Surface Integral :

Replace Line integral : curve → surface

tangent → normal

$AB \longrightarrow S$

$$\boxed{\hat{\eta} = \frac{\text{grad}\,\vec{F}}{|\text{grad}\,\vec{F}|} \qquad ds = \frac{dx\,dy}{(\hat{\eta}\cdot\hat{k})} = \iint_S (\vec{F}\cdot\hat{n})\,ds}$$

if 0, F is said to be a solenoidal vector

point function

Volume Integral:

Let $\vec{F}$ be a vector point function and volume V enclosed by closed surface.

$$\boxed{\therefore \text{Volume Integral} = \iiint_V F\,dv}$$

$\vec{F} = 2z\hat{i} - x\hat{j} + y\hat{k}$

$x=0,\ y=0,\ x=2,\ y=4,\ z=x^2,\ z=2$

Sol^n:

$$\iiint F\,dv = \int_0^2 dx \int_0^4 dy \int_{x^2}^{2} (2z\hat{i} - x\hat{j} + y\hat{k})\,dz$$

$$= \int_0^2 dx \int_0^4 dy \left\{ \left[z^2\right]_{x^2}^{2}\hat{i} - \left[xz\right]_{x^2}^{2}\hat{j} + \left[yz\right]_{x^2}^{2} \right\}$$

$$= \int_0^2 dx \int_0^4 dy \left\{ (\cancel{x0}\,4 - x^4)\hat{i} - (2x - x^3)\hat{j} + (2y - x^2 y)\hat{k} \right\}$$

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$$= \int_0^2 dx \int_0^4 (4 - x^4 - 2x + x^3 + 2y - x^2 y)\, dy$$

$$= -\int_0^2 dx \left[-4y\; x^4 y + 2xy + x^3 y + y^2 - \frac{x^2 y^2}{2} \right]_0^4$$

$$= \int_0^2 dx \left\{ (-16 + 0) + 8x + 4x^3 + 16 - 8x^2 \right\}$$

$$= -\int_0^2 (-\cancel{16} + 8x + 4x^3 + \cancel{16} - 8x^2)\, dx$$

$$= -4 \int_0^2 (x^3 - 2x^2 + x)$$

$\vdots$

Ans. $\frac{32}{15}(3\hat{i} + 5\hat{k})$

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Green's Theorem:

If $\phi(x, y)$, $\psi(x, y)$, $\frac{\partial \phi}{\partial y}$ and $\frac{\partial \psi}{\partial x}$ continuous over a region bounded by simple closed curve c in x-y plane, then

$$\oint_c (\phi dx + \psi dy) = \iint_R \left(\frac{d\psi}{dx} - \frac{\partial \phi}{\partial y}\right) dx dy$$

#1. Evaluate $\oint_c (x^2 + xy) dx + (x^2+y^2) dy$

Sol^n: $y = \pm 1$, $x = \pm 1$

Here,

$$\phi = x^2 + xy$$

$$\psi = x^2 + y^2$$

Now,

$$\frac{\partial \psi}{\partial x} = 2x$$

$$\frac{\partial \phi}{\partial y} = x$$

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$$= \int_{-1}^{1}\int_{-1}^{1} (2x - x)\,dx\,dy$$

$$= \int_{-1}^{1} \int_{-1}^{1} (x\,dx)\,dy$$

$$= \int_{-1}^{1} \left[\frac{x^2}{2}\right]_{-1}^{1} dy$$

$$= \int_{-1}^{1} 0 \cdot dy$$

$$= 0 \text{ (Ans)}$$

#2. Evaluate $\int_C (x^2y\,dx + x^2dy)$ where C is the boundary of a triangle with vertices $(0,0), (1,0), (1,1)$

Soln: Here,

$$\phi = x^2y$$

$$\psi = x^2$$

$$\therefore \frac{\partial \phi}{\partial y} = x^2$$

$$\therefore \frac{\partial \psi}{\partial x} = 2x$$

Here,

x limit $= 0 \rightarrow 1$

y " $= 0 \rightarrow x$

Now,

$$\iint_R \left(\frac{\partial \psi}{\partial x} - \frac{\partial \phi}{\partial y}\right) = \int_0^1 \left\{\int_0^x (2x - x^2)\,dy\right\} dx$$

$$= \int_0^1 (2x - x^2)\,dx \,.\, x$$

$$= \int_0^1 (2x^2 - x^3)\,dx$$

$$= 2\left[\frac{x^3}{3}\right]_0^1 - \left[\frac{x^4}{4}\right]_0^1$$

$$= \frac{2}{3} - \frac{1}{4} = \cdot - \cdot$$

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Evaluate $\oint_C (y-\sin x)dx + \cos x\, dy$ where C is the plane triangle by the lines ~~y=~~ $y=0$, $x=\frac{\pi}{2}$ and $y=\frac{2x}{\pi}$

Here,

$$\phi = y - \sin x$$

$$\psi = \cos x$$

$$\frac{\partial \phi}{\partial y} = \cancel{-s}\ 1$$

$$\frac{\partial \psi}{\partial x} = -\sin x$$

$$\int_0^{\frac{\pi}{2}} \int_0^{\frac{2}{\pi}x} \{(1+\sin x)\,dy\}\,dx = \int_0^{\frac{\pi}{2}} (1+\sin x) \}_0 \times \frac{2\pi}{x}$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{2\pi}{x} + \frac{2\pi}{x}\sin \right) =$$

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$$\frac{\cancel{\cos x}\sin x}{= \cancel{\cos x}}$$

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Using rectangle,

$$\frac{1}{2}\int_0^{\frac{\pi}{2}}\int_0^1 (1+\sin x)\,dy\,dx$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} (1+\sin x)(x)\,dx$$

$$= \frac{1}{2}\left\{\int_0^{\frac{\pi}{2}} (x)\,dx + \int_0^{\frac{\pi}{2}} (x\sin x)\,dx\right\}$$

$$= \frac{1}{2} \times \left[\frac{\pi}{2} + \left\{\left[x\sin x\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \cancel{\sin x}\ \cancel{x}\ \sin x\,dx\right\}\right]$$

$$= \frac{1}{2} \times \frac{\pi}{2} + \frac{\pi}{2} + 0$$

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#1. $F = \sin y\,\hat{i} + x(1+\cos y)\,\hat{j}$

Evaluate $\int_C F\,dr$ where c is the circular path given by $x^2+y^2=a$

Ans.

$$\int_C F\,dr = \int_C (\sin y\,\hat{i} + x(1+\cos y)\,\hat{j})(\hat{i}\,dx + \hat{j}\,dy)$$

$$= \int_C \sin y\,dx + x(1+\cos y)\,dy$$

Here,

$\phi = \sin y \quad \therefore \frac{\partial \phi}{\partial y} = \cos y$

$\psi = x(1+\cos y) \quad \therefore \frac{\partial \psi}{\partial x} = 1+\cos y$

$$\therefore \iint_S \left(\frac{\partial \psi}{\partial x} - \frac{\partial \phi}{\partial y}\right)dx\,dy = \iint_S (1+\cos y - \cos y)\,dx\,dy$$

$$= \iint_S dx\,dy$$

To convert dxdy to polar we use rdrdθ

$$= \int_0^{2\pi}\int_0^{a} r\,dr\,d\theta$$

$$= \int_0^{2\pi} a^2\,d\theta = \cancel{2}a^2 \cdot \frac{\pi^2}{\cancel{2}} = \pi a^2$$

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Stoke's Theorem :

Surface Integral of the component of curl $\vec{F}$ along the normal to the surface S, taken over S bounded by curve C is equal to the integral of point function $\vec{F}$ taken along the closed surface.

$$\boxed{\begin{aligned}\oint_C F \cdot dr &= \iint_S \text{curl}\, F \cdot \hat{\eta}\, ds \\ &= \iint_S (\vec{\nabla} \times \vec{F})\, \hat{\eta}\, ds\end{aligned}}$$

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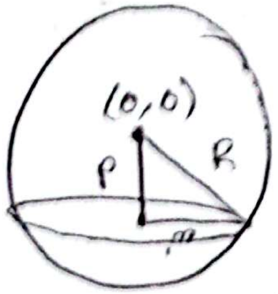
Use stokes theorem to evaluate   Do 10.1

$$\int_C (y\,dx + z\,dy + x\,dz)$$

where C is the curve of the intersection $x^2+y^2+z^2=a^2$ and $z+x=a$

Soln.



$$\therefore r^2 = R^2 - p^2 = a^2 - \frac{a^2}{2} = \frac{a^2}{2}$$

By applying stoke's theorem,

$$\int_C (y\,dx + z\,dy + x\,dz) = \iint_S (\vec{\nabla} \times \vec{F})\,\hat{n}\,ds$$

$$= \oint_C (y\hat{i} + z\hat{j} + x\hat{k}).(i\,dx + j\,dy + k\,dz)$$

$$= \int (y\hat{i} + z\hat{j} + x\hat{k})\,dr$$

$$= \iint_S \vec{\nabla} \times (y\hat{i} + z\hat{j} + x\hat{k})\,\hat{n}.\,ds$$

$$\therefore \vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & z & x \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial x}{\partial y} - \frac{\partial}{\partial z}\right) - \hat{j}\left(\frac{\partial x}{\partial x} - \frac{\partial y}{\partial y}\right) + \hat{k}\left(\frac{\partial z}{\partial x} - \frac{\partial y}{\partial y}\right)$$

$$= -(\hat{i}+\hat{j}+\hat{k})$$

$$\hat{n} = \frac{\nabla\phi}{|\nabla\phi|} = \frac{\left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)(x+z-a)}{|\nabla\phi|}$$

$$= \frac{\hat{i}+\hat{k}}{\sqrt{1+1}} = \frac{\hat{i}}{\sqrt{2}} + \frac{\hat{k}}{\sqrt{2}}$$

Now,

$$\iint_s (\vec{\nabla}\times\vec{F})\cdot\hat{n}\,ds = \iint_s -(\hat{j}+\hat{i}+\hat{k})\cdot\left(\frac{\hat{i}}{\sqrt{2}} + \frac{\hat{k}}{\sqrt{2}}\right)ds$$

$$= \iint_s -\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)ds = -\sqrt{2}\iint_s ds$$

$$= -\frac{\pi a^2}{\sqrt{2}}$$

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Steps:

1. Find F
2. Curl F
3. Find $\hat{n}$
4. $(\vec{\nabla} \times \vec{F}) \cdot \hat{n}$
5. Integration
6. Find Limit

Gauss Theorem (R/n between surface and Volume integral):

The surface integral of a normal component of a vector function of F taken around a closed surface S is equal to the integral of the divergence F taken over the volume V enclosed by the surface S.

Mathematically,

$$\iint_S \vec{F} \cdot \hat{n}\, ds = \iiint_V \operatorname{div} \vec{F}\, dv$$

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Example 105:

Solⁿ: $\iint_S A\, ds = \iiint \operatorname{div} A\, dv$

$$= \iiint (\nabla \cdot A)\, dv$$

$$= \iiint \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)(x^3\hat{i} + y^3\hat{j} + z^3\hat{k})\, dv$$

$$= \iiint (3x^2 + 3y^2 + 3z^2)\, dv$$

$$= 3\iiint (x^2 + y^2 + z^2)\, dv$$

Here,

$x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

$z = r\cos\theta$

$$= 3\iiint (r^2\sin^2\theta\cos^2\phi + r^2\sin^2\theta\sin^2\phi + r^2\cos^2\theta)\, dr\, d\theta\, d\phi\; r^2$$

$$= 3\iiint \{\sin^2\theta(\cos^2\phi + \sin^2\phi) + \cos^2\theta\}\, dr\, d\theta\, d\phi \cdot r^4$$

$$= 3\iiint (\sin^2\theta + \cos^2\theta)\, dr\, d\theta \cdot r^4\, d\phi$$

$$= 3\int_0^a \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} r^4\, d\theta\, d\phi\, dr$$

$$\int_{0}^{\frac{\pi}{2}} \sin\theta \, d\theta$$

$$-\cos\theta$$

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$$\cancel{= 12 \int_0^a \int_0^{\frac{\pi}{2}} \frac{\left(\frac{\pi}{2}\right)^5}{5} \, d\phi \, dr}$$

$$\cancel{= 12 \int_0^a \int_0^{\frac{\pi}{2}}}$$

$$= 24 \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \frac{a^5}{5} \, d\theta \, d\phi$$

$$= \frac{12 a^5}{5} \int_0^{\frac{\pi}{2}} \frac{\pi}{2} \, d\phi$$

$$= \frac{6\pi^2 a^5}{5}$$

GOOD LUCK

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Fourier Transform

Fourier Series $\xrightarrow{\text{Next}}$ Fourier Transform

Expressing a periodic function, ~~int~~ using sine and cosine within an interval

→ If $f(t)$ repeats: $f(t) = f(t+T) = f(t+2T) = \dots$

*Periodic Function:

A function is said to be periodic of period l if $f(t + \underbrace{l}_{T}) = f(t)$

*Fourier Series:

Consider $f(x)$ for the interval $(-l, l)$ is a periodic function of period $2l$. Then, the fourier series of $f(x)$ is,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$$

where,

$$a_0 = \frac{1}{l}\int_{-l}^{l} f(x)\,dx$$

l can be replaced with π where $\int_0^{2\pi}$ instead of $\int_{-l}^{l}$

$$a_n = \frac{1}{l}\int_{-l}^{l} f(x)\cos\frac{n\pi x}{l}\,dx$$

$$b_n = \frac{1}{l}\int_{-l}^{l} f(x)\sin\frac{n\pi x}{l}\,dx$$

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854 :

Given that,

$$f(x) = x + x^2 \; ; \; [-\pi, \pi]$$

Let,

$$x + x^2 = \frac{a_0}{2} + a_1\cos x + a_2\cos 2x + \dots + b_1\sin x + b_2\sin 2x + \dots$$

Now,

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx = \frac{1}{\pi}\int_{-\pi}^{\pi}(x + x^2)\,dx$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2} + \frac{x^3}{3}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left(\frac{\pi^2}{2} - \frac{\pi^2}{2} + \frac{\pi^3}{3} + \frac{\pi^3}{3}\right)$$

$$= \frac{\cancel{3\pi} + 2\pi^2}{\cancel{0}\,3}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos\frac{n\pi x}{\pi}\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi}(x + x^2)\cos nx\,dx$$

$$= \frac{1}{\pi}\Bigg[(x + x^2)\frac{\sin(nx)}{n} + (1+2x)\left(\frac{\cos nx}{n^2}\right) + (2)\left(-\frac{\sin nx}{n^2}\right)\Bigg]_{-\pi}^{\pi}$$

$x + x^2$	$\cos nx$
$1 + 2x$	$\frac{\sin nx}{n}$
2	$-\frac{\cos nx}{n^2}$
0	$-\frac{\sin nx}{n^3}$
	$\frac{\cos nx}{\dots}$

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$$= \frac{1}{\pi}\Big\{ (\pi+\pi^2)\frac{\sin\pi x}{\pi} - (1+2\pi)\frac{\cos\pi x}{\pi^2} + \frac{2\sin\pi x}{\pi^3}$$

$$- (-\pi+\pi^2)\frac{\sin(-\pi)x}{-\pi} - (1-2\pi)\frac{\cos(-\pi)x}{\pi^2} + 2\frac{\sin(-\pi)x}{-\pi^3}\Big\}$$

$$= \frac{1}{\pi}\left(-\frac{1+2\pi}{\pi^2} + \frac{1-2\pi}{\pi^2}\right) =$$

Ex -1, 2

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#871:

$\boxed{876 \to Ex - 1 - 5}$

$$\frac{a_0}{2} = \frac{1}{\pi}\left[\int_{-\pi}^{-\frac{\pi}{2}}(-1)dx + \int_{\frac{\pi}{2}}^{\pi}(1)dx\right]$$

$$= \frac{1}{\pi}\left\{-\cancel{\left[-\frac{\pi}{2}+\pi\right]} + \cancel{\left[\pi - \frac{\pi}{2}\right]}\right\}$$

$$\therefore a_0 = 0$$

$$a_n = \frac{1}{\pi}\left\{\left[\frac{-\sin nx}{n}\right]_{-\frac{\pi}{2}}^{-\pi} + \left[\frac{\sin nx}{n}\right]_{\frac{\pi}{2}}^{\pi}\right\}$$

$$= \frac{1}{\pi}(0) = 0$$

$$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{-\frac{\pi}{2}} \sin nx\,dx + \int_{\frac{\pi}{2}}^{\pi} \sin nx\,dx\right]$$

$$= \frac{1}{\pi}\left\{\left[-\cos x\right]_{-\pi}^{-\frac{\pi}{2}} + \left[-\cos x\right]_{\frac{\pi}{2}}^{\pi}\right\} = \frac{1}{\pi}\left(\cos\frac{\pi}{2} - \cancel{\cos\pi - \cos\pi+\dots}\right.$$

$$=$$

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Even funct : $f(-x) = f(x)$

Odd funct : $f(-x) = -f(x)$

$\cos(-x) = \cos x$

Parseval's Formula :

$$\int_{-c}^{c} [f(x)]^2 \, dx = c \left\{ \frac{1}{2} a_0^2 + \sum_{n=1}^{\infty} (a_n^2 + b_n^2) \right\}$$

* Even doesnt matter (as much)

Example is more imp

$$c_0 = \frac{a_0}{2} = \frac{1}{2} \frac{1}{l} \int_0^{2l} f(x) \, dx$$

$$c_n = \frac{1}{2l} \int_0^{2l} f(x) e^{-\frac{in\pi x}{l}} dx$$

$$c_{-n} = \frac{1}{2l} \int_0^{2l} f(x) e^{\frac{in\pi x}{l}} dx$$

$$f(x) = c_0 + \sum c_n e^{\frac{in\pi x}{l}} + \sum c_{-n} e^{\frac{-in\pi x}{l}}$$

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883 : 12:

Given that,

$$f(x) = |x| \; ; \; [-2 < x < 2]$$

$$\therefore f(x) = \begin{cases} -x & \cancel{0 < 2} \; -2 < x < 0 \\ x & 0 < x < 0 \end{cases}$$

Here, $f(-x) = |-x| = |x| = f(x) \longrightarrow \therefore f(x)$ is even $\therefore b_n = 0$

$$\therefore a_0 = \frac{1}{c}\int_{-c}^{c} f(x)\,dx = \frac{1}{2}\left[\int_{-2}^{0}(-x)\,dx + \int_{0}^{2} x\,dx\right]$$

$$\frac{\cancel{0}2 - (-2)}{2} = 2$$

$$= \frac{1}{2}\left[-\left(\cancel{\frac{4}{2}}0 - \frac{4}{2}\right) + \left(\frac{4}{2} - 0\right)\right]$$

$$= \frac{1}{2}(2+2) = 2$$

$$\therefore a_n = \frac{1}{\cancel{2}}\left[\int_{-2}^{0}\left(-x\cos\frac{n\pi x}{2}\right)dx + \int_{0}^{2} x\cos\frac{n\pi x}{2}\right] \qquad x \quad \cos\frac{n\pi x}{2}$$

$$1 \quad \frac{\sin\frac{n\pi x}{2}}{2n}$$

$$= \frac{1}{2}\Big[\qquad\qquad 0 \quad -\frac{\cos\frac{n\pi x}{\cancel{2}}}{n^2}$$

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889 : Ex-2 :

Given

$$f(x) = 2x - 1 \quad \text{when } 0 < x < 1$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} b_n \sin(n\pi x)$$

$$\therefore f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} b_n \sin(n\pi x)$$

Here,

$$a_0 = \frac{1}{1}\int_0^1 (2x-1)\,dx = \left[\frac{\cancel{2}x^2}{\cancel{2}} - x\right]_0^1$$

$$= (0 - 0) = 0$$

$$b_n = \frac{1}{1}\int_0^1 (2x-1)\sin\left(\frac{n\pi x}{1}\right)dx \; ;$$

$2x-1$	$\sin(n\pi x)$
2	$-\frac{\cos(n\pi x)}{n\pi}$
0	$-\frac{\sin(n\pi x)}{n^2\pi^2}$

$$= \left[(2x-1)\left(-\frac{\cos n\pi x}{n}\right) + (2)\left(-\frac{\sin(n\pi x)}{n}\right)\right]_0^1$$

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$$= \cancel{\left\{(2-1)(-1)^n + (2)(-0)\right\} (-1)}$$

$$= (2-1)\left(\frac{-\cos n\pi}{n\pi}\right) + \frac{2}{n^2\pi^2}\sin n\pi + \frac{1}{\pi n}\cos 0^\circ$$

$$- \frac{2}{n^2\pi^2}\sin 0^\circ$$

$$= \frac{-1}{n\pi}(-1)^n + \frac{1}{\pi n}$$

$$= \frac{2}{n\pi} \quad \text{when odd}$$

$$= 0 \quad \text{" even}$$

$$\therefore f(x) = \frac{2}{\pi}\sin \pi x + 0 + \frac{2}{3\pi}\sin 3\pi x + \text{—} (4x)$$

Integral Transform

$$F[f(x)] = F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx$$

↳ complex part of laplace

Fourier Integral $\neq$ Fourier Transform

$$f(x) = \int_0^{\infty} \int_{-\infty}^{\infty}$$

$$f(x) = \frac{2}{\pi} \int_0^{\infty} \int_0^{\infty} f(t) \sin ut \sin ux \, du \, dt \rightarrow \text{sine integral}$$

$$f(x) = \frac{2}{\pi} \int_0^{\infty} \int_0^{\infty} f(t) \cos ut \cos ux \, du \, dt \rightarrow \text{cosine integral}$$

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#954-2:

Sine int transformation of $f(x)$ is,

$$f(x) = \frac{2}{\pi}\int_0^\infty \sin(\lambda x)dx \int_0^\infty f(t)\sin(\lambda t)dt$$

$$= \frac{2}{\pi}\int_0^\infty \sin(\lambda x)dx \int_0^\infty e^{-\beta t}\sin(\lambda t)dt$$

G

$e^{-\beta t}$ $\sin\lambda t$

$\beta e^{\beta t}$ c

$$= \frac{2}{\pi}\int_0^\infty \sin(\lambda x)dx \left[\left[-e^{-\beta t}\frac{\cos\lambda t}{\lambda}\right]_0^\infty - \int_0^\infty (-\beta e^{-\beta t})\left(-\frac{\cos\lambda t}{\lambda}\right)dt\right]$$

$$= \frac{2}{\pi}\int_0^\infty \sin(\lambda x)dx \times \left[\left(0+\frac{1}{\lambda}\right) - \frac{\beta}{\lambda}\int e^{-\beta t}\cos\lambda t\, dt\right]$$

$$= \frac{2}{\pi}\int_0^\infty \sin(\lambda x)dx \left(\frac{1}{\lambda} - \frac{\beta}{\lambda}\int_0^\infty e^{-\beta t}\cos\lambda t\, dt\right)$$

$$\cancel{= \frac{2}{\pi}\int_0^\infty \frac{1}{\lambda}\sin\lambda x - \frac{\beta^2}{\lambda^2}\int_0^\infty e^{-\beta t}\sin\lambda t\, dt}$$

$$= \frac{2}{\pi}\int_0^\infty \sin($$

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Now,

$$\int_0^{\infty} \underset{u}{e^{-\beta t}} \, \underset{dv}{\cos(\lambda t)\,dt}$$

$$= \left(e^{-\beta t} \cdot \frac{\sin(\lambda t)}{\lambda}\right)_0^{\infty} - \int_0^{\infty} \frac{\sin(\lambda t)}{\lambda}\left(-\beta e^{-\beta t}\right) dt$$

$$= (0-0) + \frac{\beta}{\lambda}\int_0^{\infty} \sin \lambda t \, e^{-\beta t} dt$$

$$\cancel{= \frac{\beta}{\lambda} I}$$

Now,

$$I = \frac{2}{\pi}\int_0^{\infty} \cancel{2x} \sin(\lambda x)\,d\lambda \left(\frac{1}{\lambda} - \frac{\beta}{x} \cdot \frac{\beta}{x}\int_0^{\infty} \sin \lambda t \, e^{-\beta t} dt\right)$$

$$= \frac{2}{\pi}\int_0^{\infty} \sin(\lambda x)\,d\lambda \cdot \frac{1}{\lambda} - \frac{\beta^2}{x^2} I$$

or,

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#954-3:

Given that,

$$f(x) = e^{-kx}$$

$$\therefore f(t) = e^{-kt}$$

$$\therefore f(x) = \frac{2}{\pi}\int_0^\infty \cos(ux)\,du \int_0^\infty f(t)\cos(ut)\,dt$$

$u = \cos\omega t \quad dv = e^{-kt}dt$

$du = -\omega\sin\omega t \quad v = -\frac{e^{-kt}}{k}$

$$\therefore e^{-kx} = \frac{2}{\pi}\int_0^\infty \cos(ux)\,du \int_0^\infty e^{-kt}\cos\omega t\,dt$$

$$= \frac{2}{\pi}\int_0^\infty \cos(\omega x)\,du \times \left[\left(-\frac{e^{-kt}}{k}\cos\omega t\right)_0^\infty - \int_0^\infty \frac{e^{-kt}}{k} \cdot -\omega\sin\omega t\right]$$

$$= \frac{2}{\pi}\int_0^\infty \cos(\omega x)\,du \left\{\left(0 + 1\right) + \frac{\omega}{k}\int_0^\infty e^{-kt}\sin\omega t\right\}$$

$$= \frac{2}{\pi}\int_0^\infty \cos(\omega x)\,du \left[1 + \right.$$

$$= \frac{2}{\pi}\int_0^\infty \cos(\omega x)\,d\omega \left[\frac{e^{-kt}}{}\right.$$

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Fourier Transform

$\cancel{f(x)=}$

$$F(s) = \frac{1}{\sqrt{2\pi}} \int_{\infty}^{\infty} f(t)\, e^{ist}\, dt$$

$\cancel{F(s=}$

Fourier Sine and Cosine:

$$\cancel{f(x) = \frac{2}{\pi} \int_0^{\infty} \sin(sx)\, dx}$$

$$F(s) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(t) \sin(st)\, dt$$

$$F(s) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(t) \cos(st)\, dt$$

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#95 — 4

1) $f(x) = \begin{cases} 1 & |x| < a \\ 0 & |x| > a \end{cases}$

F.T of $f(x)$ is,

$$F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-a}^{a} 1 \cdot e^{isx} dx$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{e^{isx}}{is} \right]_{-a}^{a}$$

$$= \frac{1}{\sqrt{2\pi}} \cdot \frac{1}{is} \left(e^{isa} - e^{-isa} \right)$$

$$= \frac{1}{\sqrt{2\pi}} \cdot \frac{2}{s} \left(\frac{e^{isa} - e^{-isa}}{2i} \right)$$

$$= \sqrt{\frac{2}{\pi}} \cdot \frac{1}{s} \sin sa$$

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#5

Given,

$$f(x) = \begin{cases} 1-x^2 & \text{if } |x| \le 1 \\ 0 & \text{if } |x| > 0 \end{cases}$$

$$\therefore F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx$$

$$= \sqrt{\frac{1}{2\pi}} \int_{-1}^{1} (1-x^2) e^{isx} dx$$

$$= \sqrt{\frac{1}{2\pi}} \left(\int_{-1}^{1} e^{isx} dx - \int_{-1}^{1} x^2 e^{isx} dx \right);$$

$x^2 \quad e^{isx}$

$2x \quad \frac{e^{isx}}{is}$

$2 \quad \frac{e^{isx}}{i^2 s^2}$

$0 \quad \frac{e^{isx}}{i^3 s^3}$

$$= \sqrt{\frac{1}{2\pi}} \left[\left[\frac{e^{isx}}{is} \right]_{-1}^{1} - \left\{ \left[x^2 \frac{e^{isx}}{is} \right]_{-1}^{1} - \left[2x \frac{e^{isx}}{i^2 s^2} \right]_{-1}^{1} + \left[2 \cdot \frac{e^{isx}}{i^3 s^3} \right]_{-1}^{1} \right\} \right]$$

$$\cancel{= \frac{1}{\sqrt{2\pi}} \frac{1}{is} \cdot e^{is} - \frac{1}{is}}$$

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$$= \sqrt{\frac{1}{2\pi}} \left[\left(\frac{e^{is}}{is} - \frac{e^{-is}}{is}\right) - \left\{ \left(\frac{e^{is}}{is} - \frac{e^{is}}{is}\right) - 2\frac{e^{is}}{(is)^2} + 2\cdot\frac{e^{is}}{(is)^2} + 2\cdot\frac{e^{is}}{(is^3)} - 2\frac{e^{-is}}{(is)^3} \right\} \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left\{ \frac{2}{s^2}\left(e^{is} + e^{-is}\right) + \frac{2}{is^3}\left(e^{is} - e^{is}\right) \right\}$$

$$= \frac{1}{\sqrt{2\pi}} \left\{ \frac{2}{s^2} \times 2 \times \left(\frac{e^{is} + e^{-is}}{2}\right) + \frac{2}{is^3} \times 2i \left(\frac{e^{is} - e^{-is}}{2i}\right) \right\}$$

$$\cancel{= \frac{1}{\sqrt{2\pi}} \, 4 \cos}$$

$$= \frac{1}{\sqrt{2\pi}} \times \frac{4}{s^3} \left\{ \cancel{\cos} - s\cos s + \sin s \right\}$$

$$= \frac{2\sqrt{2}}{s^3\sqrt{\pi}} \left(-s\cos s + \sin s\right) \text{ (Ans)}$$

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#6 (9 56):

Given that,

$$f(x) = e^{-ax}$$

Sine transform of $f(x)$ is given by:

$$F(s) = \sqrt{\frac{2}{\pi}}$$

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7(957):

$$\cancel{F(s)}\ F_s\left(\frac{1}{x}\right) = \sqrt{\frac{2}{\pi}}\int_0^\infty \frac{\sin sx}{x}\,dx$$

$$= \sqrt{\frac{2}{\pi}}\int_0^\infty \frac{\sin\theta}{\frac{\theta}{s}}\,\frac{d\theta}{s}$$

why?

$$= \sqrt{\frac{2}{\pi}}\int_0^\infty \frac{\sin\theta}{\theta}\,d\theta = \sqrt{\frac{2}{\pi}}\left(\frac{\pi}{2}\right)$$

$$\boxed{\int_0^\infty \frac{\sin x}{x}\,dx = \frac{\pi}{2} \qquad \int_0^\infty \frac{\cos x}{x}\,dx = 0}$$

$$= \sqrt{\frac{\pi}{2}}$$

GOOD LUCK

9(~~9~~57):

$$f(x) = \frac{e^{-ax}}{x}$$

$$\therefore F(s) = \sqrt{\frac{2}{\pi}} \int_0^\infty \frac{e^{-ax}}{x} \cancel{dx} \sin sx \, dx$$

Differentiating w.r.t. s, we get,

$$\frac{d}{ds}(F(s)) = \sqrt{\frac{2}{\pi}} \int_0^\infty \frac{e^{-ax}}{x} (x \cos sx) \, dx$$

$$= \sqrt{\frac{2}{\pi}} \int_0^\infty e^{-ax} \cos sx$$

$$= \sqrt{\frac{2}{\pi}} \cdot \frac{a}{s^2 + a^2}$$

Taking integration with respect to s, we get,

$$F(s) = \sqrt{\frac{2}{\pi}} \int \frac{a}{s^2+a^2} = \sqrt{\frac{2}{a}} \tan^{-1}\frac{s}{a} + c$$

Putting $s = 0$, we get,

$0 = c$

$$\therefore F(s) = \sqrt{\frac{2}{\pi}} \tan^{-1}\frac{s}{a}$$

(Ans)

#11 (958)

$$f(x) = \begin{cases} x & \text{for } 0 < x < \frac{1}{2} \\ 1-x & \text{for } \frac{1}{2} < x < 1 \\ 0 & \text{"} \quad x > 1 \end{cases}$$

Fourier cosine transform :

$$F(s) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \cos(sx)\, dx$$

$$= \sqrt{\frac{2}{\pi}} \int_0^{\frac{1}{2}} x\cos(sx)\, dx + \sqrt{\frac{2}{\pi}} \int_{\frac{1}{2}}^{1} (1-x) \cos sx\, dx$$

$$\cancel{= \sqrt{\frac{2}{\pi}} \int_0^{\frac{1}{2}} \left(x \cdot \frac{s}{} \right.}$$

x	$1-x$	$\cos x$
1	-1	$\frac{\sin sx}{s}$
0	0	$-\frac{\cos sx}{s^2}$

$$= \sqrt{\frac{2}{\pi}} \left[x \cdot \frac{\sin sx}{s} + \frac{\cos sx}{s^2} \right]_0^{\frac{1}{2}} + \sqrt{\frac{2}{\pi}} \left[(1-x)\left(\frac{\sin sx}{s}\right) - \frac{\cos sx}{s^2} \right]_{\frac{1}{2}}^{1}$$

$$= \sqrt{\frac{2}{\pi}} \left(\cancel{\frac{1}{2} \times \frac{\sin \frac{1}{2}s}{s}} + \frac{\cos\frac{s}{2}}{s^2} - \frac{1}{s^2} - \frac{\cos(s)}{s^2} - \cancel{\frac{1}{2} \times \frac{\sin(\frac{s}{2})}{s}} + \frac{\cos\frac{s}{2}}{s^2} \right)$$

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$$= \sqrt{\frac{2}{\pi}}\left(\frac{2\cos\frac{s}{2}}{s^2} - \frac{\cos s}{s^2} - \frac{1}{s^2}\right)$$

14 – 15 NOT NECESSARY

H.W: 16, 10, 11, 12, 13

Ex 1 (962):

$$f(x) = \begin{cases} 1 & 0 < x < \pi \\ 0 & x > \pi \end{cases}$$

~~Cosine integral~~ $= \sqrt{\frac{2}{\pi}} \int_0^{\pi} \cos sx \, dx$

Sine integral, $f(x) = \frac{2}{\pi}\int_0^{\infty}\int_0^{\infty} \sin ut \sin ux \, d$

GOOD LUCK

TOPIC NAME : W-11 DAY C-23
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Properties of the Fourier Transform

1) Linear

2) Change of scale :

$$F(f(ax)) = \frac{1}{a} F\left(\frac{s}{a}\right)$$

3) Shifting Property :

$$F\{f(x-a)\} = e^{isa} F(s)$$

~~4) F{~~ 4) $F\left\{e^{iax} f(x)\right\} = F(s+a)$

5) Modulation Theorem:

$$F\{f(x) \cos ax\} = \frac{F(s+a) + F(s-a)}{2}$$

6) if $F\{f(x)\} = F(s)$; then

$$F\{x^n f(x)\} = (-i)^n \frac{d^n}{ds^n} F(s)$$

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7) $F\{f'(x)\} = -is\,F(s)$ if $f(x) \to 0$ as $x \to \pm\infty$

condition always follows

8) $F\left\{\int_a^x f(x)\,dx\right\} = \frac{1}{(-is)} F\{f_1'(x)\}$

Convolution :

~~f(x)(g~~

$$f(x)g(x) = \int_{-\infty}^{\infty} f(x)\, g(x-u)\, du$$

The fourier transform of the convolution of f(x) and g(x) is the product of their Fourier transforms. i.e.,

$$F[f(x) \cdot g(x)] = F[f(x)] \cdot F[g(x)]$$

GOOD LUCK

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Parseval's Identity for Fourier Transform :

i) $\int_{-\infty}^{\infty} F(s)\bar{G}(s)\,ds = \int_{-\infty}^{\infty} f(x)\bar{g}(x)\,dx$

$\bar{G}(s)$ and $\bar{g}(x)$ are conjugates

ii) $\int_{-\infty}^{\infty} [F(s)]^2\,ds = \int_{-\infty}^{\infty} |f(x)|^2\,dx$ $\longrightarrow$ $|a+ib| = \sqrt{a^2+b^2}$

For cosine : $\frac{2}{\pi}\int_0^{\infty} |F_c(s)|^2\,dx = \int_0^{\infty} |f(x)|^2\,dx$

$\frac{2}{\pi}\int_0^{\infty} F_c(s)\cdot G_c(s)\,ds = \int_0^{\infty} f(x)\cdot g(x)\,dx$

For sine : $\frac{2}{\pi}\int_0^{\infty} |F_s(s)|^2\,dx = \int_0^{\infty} |f(x)|^2\,dx$

$\frac{2}{\pi}\int_0^{\infty} F(s)\cdot G_s(s)\,ds = \int_0^{\infty} f(x)\cdot g(x)\,dx$

GOOD LUCK™

u dv

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(968) → 16 :

Fourier cosine transform of e^{-x} is,

$$F_c(s) = \sqrt{\frac{2}{\pi}} \int_0^\infty e^{-x} \cos(sx) dx \;; \quad u = \cos(sx) \quad dv = e^{-x} dx$$
$$\therefore du = -s\sin sx \quad v = -e^{-x}$$

$$= \sqrt{\frac{2}{\pi}} \left\{ \left[-e^{-x} \cos(sx) \right]_0^\infty - \int_0^\infty -e^{-x} (-s\sin sx) dx \right\}$$

$$= \sqrt{\frac{2}{\pi}} \left(0 + 1\right) -$$

$\vdots$

$$= \frac{1}{1+s^2} \cdot \sqrt{\frac{2}{\pi}}$$

By Parseval's identity, we get,

$$\frac{2}{\pi} \int_0^\infty [F_c(s)]^2 ds = \int_0^\infty |f(x)|^2 dx$$

or, $$\frac{2}{\pi} \int_0^\infty \left| \frac{1}{(s^2+1)} \right|^2 ds = \int_0^\infty |e^{-x}|^2 dx = \int_0^\infty e^{-2x} dx$$

$$= \left[\frac{e^{-2x}}{-2} \right]_0^\infty = \frac{1}{2} \times \frac{\pi}{2}$$

$$\therefore \int_0^\infty \frac{1}{(s^2+1)^2} ds = \frac{\pi}{4}$$

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DONT DO : Ex - 23, 22, 25

GOOD LUCK™

TOPIC NAME : W-12

DAY : C-24

TIME :

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Fourier Transform of Derivatives

$$F\left(\frac{\partial^2 u}{\partial x^2}\right) = (-is)^2 F\{u(x)\} = -s^2 \bar{u}$$

$$F_c\{f'(x)\} = \sqrt{\frac{2}{\pi}}\, f(0) + sF_s(s)$$

$$F_s\{f'(x)\} = -sF_c(s)$$

#Imp : ~~975~~ → 978 ~~30 — 31~~ 31 – 32
~~(26 — 28)~~

GOOD LUCK

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26 (975): Let, $u(x, t)$ be the temp - - .

We know, the heat equation is,

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2} \longrightarrow \text{Heat Eq}^n$$

The initial and boundary conditions are:

- $u = 0$ when $t = 0$ for all x $(x \geq 0)$
- $u = u_0$ when $x = 0$ for all t
- u is finite for all x and for all t

???

Multiplying

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31 : (980) :

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

We apply Fourier Sine Transform,

$$\int_0^\infty \frac{\partial u}{\partial t} \sin sx \, dx = \int_0^\infty \frac{\partial^2 u}{\partial x^2} \sin sx \, dx$$

removes $\sqrt{\frac{2}{\pi}}$

or, $$\frac{\partial}{\partial t} \int_0^\infty u \sin sx \, dx = -s^2 \bar{u}(s) + s u(0)$$

or, $$\frac{\partial \bar{u}}{\partial t} + s^2 \bar{u} = 0$$

$$\bar{u} = A e^{-s^2 t}$$

$$\bar{u} = \bar{u}(s,t) = \int_0^\infty u(x,t) \sin sx \, dx$$

$$\bar{u}(s, 0) = \int_0^\infty$$

GOOD LUCK

TOPIC NAME : C-13 DAY : C-25

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33(983):

Given that,

$u(x,0) = 0$ for $x \geq 0$

$\frac{\partial u}{\partial x}(0,t) = -a$

$u(x,t)$ is bounded

~~Given that~~,

Here, $\frac{\partial u}{\partial x}$ at $x=0$ is given

$\therefore$ We take cosine transform.

$$F_c\left(\frac{\partial u}{\partial t}\right) = F_c\left(k\frac{\partial^2 u}{\partial x^2}\right)$$

or, $\frac{d\bar{u}}{dt} = k\left(-s^2\bar{u} - \sqrt{\frac{2}{\pi}} \cdot \frac{\partial u}{\partial x}(0,t)\right)$

$$= -ks^2\bar{u} + \cancel{\sqrt{\frac{2}{\pi}}ka}\ \sqrt{\frac{2}{\pi}}ka$$

$$\therefore \frac{d\bar{u}}{dt} + ks^2\bar{u} - \sqrt{\frac{2}{\pi}}ka = 0$$

TOPIC NAME

$\because$ This is linear in $\bar{u}$. #Ex-14.3

$$\therefore \bar{u}e^{ks^2t} = \int \sqrt{\frac{2}{\pi}} ka\, e^{ks^2t} dt$$

$$= \sqrt{\frac{2}{\pi}} ka \int e^{ks^2t} dt$$

$$= \sqrt{\frac{2}{\pi}} ka \cdot \left(\frac{e^{ks^2t}}{ks^2} + c\right)$$

$$\therefore \bar{u}(s,t) = \sqrt{\frac{2}{\pi}} \cdot \frac{a}{s^2} + c \cdot e^{-ks^2t}$$

Since $u(x,0) = 0$,

$$\therefore \bar{u}(s,0) = 0$$

$$\therefore \bar{u}(s,0) = \sqrt{\frac{2}{\pi}} \frac{a}{s^2} + c \cdot 1$$

$$\text{or, } c = -\sqrt{\frac{2}{\pi}} \cdot \frac{a}{s^2}$$

$$\therefore \bar{u}(s,t) = \sqrt{\frac{2}{\pi}} \cdot \frac{a}{s^2}\left(1 - e^{-ks^2t}\right)$$

By inversion theorem,

$$u(x,t) = \frac{2}{\pi} \cdot a \int_0^\infty \frac{1 - e^{-ks^2t}}{s^2} \cos sx\, dx \quad (\text{Ans})$$

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~~Ex-1~~

Finite Fourier

H.W → 34, 35, 40, 41

Sine:

$$F_s(p) = \bar{f}_s(p) = \int_0^l f(x) \sin \frac{p\pi x}{l} dx$$

Inverse:

$$f(x) = \frac{2}{l} \sum_{p=l}^{\infty} \bar{f}_s(p) \sin \frac{p\pi x}{l}$$

Cosine:

$$F_c(p) = \bar{f}_c(p) = \int_0^l f(x) \cos \frac{p\pi x}{l} dx$$

Inverse:

$$f(x) = \frac{1}{l} \bar{f}_c(0) + \frac{2}{l} \sum_{p=l}^{\infty} \bar{f}_c(p) + \cos \frac{p\pi x}{l}$$

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Ex - 39 (993):

Given that

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}; \quad \begin{matrix} u(0,t)=0 \\ u(4,t)=0 \\ u(x,0)=2x \end{matrix} ; \left[\begin{matrix} 0 < x < 4, \\ t > 0 \end{matrix} \right]$$

Taking sine transform, we get,

$$\int_0^4 \frac{\partial u}{\partial t} \sin\frac{p\pi x}{4}\, dx = \int_0^4 \frac{\partial^2 u}{\partial x^2} \sin\frac{p\pi x}{4}\, dx$$

$$= F_s\left(\frac{\partial^2 u}{\partial x^2}\right)$$

$$= -\frac{p^2\pi^2}{4^2}\bar{u}_s(p) + \frac{p\pi}{4}\left[u(0,t) - (-1)^p u(4,t)\right]$$

$$=$$

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Vectors (+ Matrix)

Linear dependence and Independence

scalar multiplication

#1 Determine whether the vectors $v_1(1,-2,3)$, $v_2=(5,6,-1)$ and $v(3,2,1)$ are linearly de independent or dependent in $\mathbb{R}^3$?

Sol^n: The linear independence or dependence of these vectors is determined by whether the vector equation

$$k_1 v_1 + k_2 v_2 + k_3 v_3 = 0$$ can be satisfied

that all are not zero.

$$\text{or, } k_1(1,-2,3) + k_2(5,6,-1) + k_3(3,2,1) = (0,0,0)$$

Therefore,

$$k_1 + 5k_2 + 3k_3 = 0$$

$$-2k_1 + 6k_2 + 2k_3 = 0$$

$$3k_1 - k_2 + k_3 = 0$$

Thus, we need to determine whether the system has non trivial solution or not.

The augmented matrix is,

$$\begin{bmatrix} 1 & 5 & 3 \\ -2 & 6 & 2 \\ 3 & -1 & 1 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

or, $\begin{bmatrix} 1 & 5 & 3 \\ 0 & 16 & 8 \\ 0 & -16 & -8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$; $\cancel{2\times R}$ $R_2 = R_2 + 2R_1$, $R_3 = R_3 - 3R_1$

or, $\begin{bmatrix} 1 & 5 & 3 \\ 0 & 16 & 8 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$; $R_3 = R_3 + R_2$

$\therefore$ Let,

$$k_3 = t$$

Hence,

$$16k_2 + 8k_3 = 0$$

or, $k_2 = -\frac{8}{16}k_3 = -\frac{1}{2}t$

and, $k_1 + 5k_2 + 3k_3 = 0$

or, $k_1 - \frac{5}{2}t + 3t = 0$

$\therefore k_1 = -\frac{1}{2}t$

Since, the system has non trivial solution, therefore, the vectors are linearly dependent

i.e. $v_3 = \frac{1}{2}v_1 + \frac{1}{2}v_2$

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#3. Determine whether the vectors $V_1=(1,2,2,-1)$, $V_2=(4,9,9,-4)$, $V_3=(5,8,9,-5)$ are linearly dependent or independent?

Sol^n: -

$$k_1V_1 + k_2V_2 + k_3V_3 = 0$$

$$\text{or, } k_2(4,9,9,-4) + k_1(1,2,2,-1) + k_3(5,8,9,-5) = 0$$

Therefore,

$$k_1 + 4k_2 + 5k_3 = 0$$
$$2k_1 + 9k_2 + 8k_3 = 0$$
$$2k_1 + 9k_2 + 9k_3 = 0$$
$$-k_1 - 4k_2 - 5k_3 = 0$$

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The augmented matrix is,

$$\begin{bmatrix} 1 & 4 & 5 \\ 2 & 9 & 8 \\ 2 & 9 & 9 \\ -1 & -4 & -5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

or,
$$\begin{bmatrix} 1 & 4 & 5 \\ 0 & 1 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}; \quad \begin{aligned} R_2 &= R_2 - 2R_1 \\ R_3 &= R_3 - 2R_1 \\ R_4 &= R_4 + R_1 \end{aligned}$$

or,
$$\begin{bmatrix} 1 & 4 & 5 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}; \quad R_3 = R_3 - R_2$$

$\therefore k_3 = 0$

$\therefore k_2 - 2k_3 = 0$

$\therefore k_2 = 0$

$k_1 + 4k_2 + 5k_3 = 0$

$\therefore k_1 = 0$