Homework 2

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1 Part 1

1.1 Introduction

To compile code run ‘make -f Makefile‘. Refer to output.txt for the required results.

For part 1 of the homework we were tasked with writing multiple subroutines in LinAl.F90 we will use this file thought the rest of the semester. Thought the homework states to do the problems in different files it seemed easier to do them in one large drive file then to split them up.

1.2 Problem 2

In Problem we were tasked with writing 3 subroutines: Finding the trace of a matrix, finding the 2-norm of a vector, and writing a matrix in human readable format. For finding the trace of a matrix I used it on A.dat and it returned a value of 22 which is what I got when I manually calculated the trace by hand. Next to test my vector norm I used it on all of the columns of A.dat I received the below values which coincide with what I got using matlab. Finally in output.txt it can be verified that the matrix is in a human readable format.

Column #	2-Norm
1	10.95445115
2	10.39230485
3	13.11487705
4	9.48683298

1.3 Porblem 3

For problem 3 we programmed Gaussian Elimination with partial pivoting and Gaussian Elimination with partial pivoting backsubstitution. In output.txt I printed out A and B before and after Gaussian Elimination also prints the solution matrix X and calculates the norm of the error vector ($E = AX - B$) which was within the accuracy requested. Below is an approximation of the solution matrix which has been verified in matlab.

$$\begin{bmatrix} 1.33\times10^{-15} & 3.50 & .25 & -2.01 & 360.27 & 10.63 \\ 1.00 & -6.00 & -.50 & 33.40 & -1234.36 & -22.33 \\ 2.00 & -1.00 & 1.00 & -28.49 & 515.92 & 4.21 \\ -3.00 & 5.00 & -1.50 & 3.69 & 223.04 & 7.12 \end{bmatrix}.$$

The complete matrix with double precision is in output.txt. The below is an image of the norm of the columns of the error matrix these should be close to machine accuracy like requested in the homework.

```
The norm of column           1 is  9.06855828504677422E-015

The norm of column           2 is  8.18860042643344517E-015

The norm of column           3 is  3.14018491736755033E-016

The norm of column           4 is  8.67629882014001454E-014

The norm of column           5 is  5.57767926354882028E-013

The norm of column           6 is  1.24661781765118098E-014
```

1.4 Problem 4

In Problem 4 we created two subroutines the first was to decompose a square matrix into a LU form where the resulting matrix is both L and U as in the lecture notes Eq (2.55). The second subroutine is used to preform LU backsubstitution on matrix A and B. Once again using A.dat and B.dat to solve AX=B. We calculate the error matrix and find the norm or each of the columns and see that once again it is within machine accuracy. Below is the an approximate solution matrix the full solution matrix is once again in output.txt.

$$\begin{bmatrix} 6.66\times10^{-16} & 3.50 & .25 & -2.01 & 360.27 & 10.63 \\ 1.00 & -6.00 & -.50 & 33.40 & -1234.36 & -22.33 \\ 2.00 & -1.00 & 1.00 & -28.49 & 515.92 & 4.21 \\ -3.00 & 5.00 & -1.50 & 3.69 & 223.04 & 7.12 \end{bmatrix}.$$

Like problem 3 below is the norm of the columns of the error matrix (E=AX-B) we see that the norms are very close to machine accuracy.

```
The norm of column           1 is  1.83102671940889498E-015

The norm of column           2 is  9.15513359704447490E-016

The norm of column           3 is  9.15513359704447490E-016

The norm of column           4 is  2.51315357309141909E-014

The norm of column           5 is  6.99661678076977913E-013

The norm of column           6 is  5.40257841157140758E-015
```

1.5 Problem 5

The final problem of part one wanted us to use either the algorithm from problem 2 or problem 3 to solve for the equation of a plane. The standard form for the equation of a plane is $ax + by + cz = d$ where d can be any number other than 0 if the plane does not pass through the origin and 0 if it does. Since our points do not create a plane that passes through the origin we can let d be and real number. For my system of equations I used $d = -1$. The system I solved was

$$\begin{bmatrix} x_1 - x_2 & y_1 - y_2 & z_1 - z_2 \\ x_1 - x_3 & y_1 - y_3 & z_1 - z_3 \\ x_1 & y_1 & z_1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

We get this system by taking the three equations

$$ax_1 + by_1 + cz_1 = -d(1)$$

$$ax_2 + by_2 + cz_2 = -d(2)$$

$$ax_3 + by_3 + cz_3 = -d(3).$$

To get the first 2 equation in the matrix simply subtract (2) from (1) and (3) from (1) then the final equation is simply just given by (1). Here is the approximate solution vector when $d = -1$, $\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 3.90 \times 10^{-2} \\ .36 \\ 7.81 \times 10^{-2} \end{bmatrix}$. Below is the resulting plane plotted in matlab with the 3 points plotted along side the plane. Results were verified in matlab.