Homework 5

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Problem 1-2

Submitted as a hand written pdf attached at the end of this report.

Problem 3

In problem 3 we are asked to solve the folllowing IBVP of the heat equation

$$u_t = u_{xx},\ x \in (0, 2),\ t > 0$$

$$u(x, 0) = f(x),\ x \in (0, 2)$$

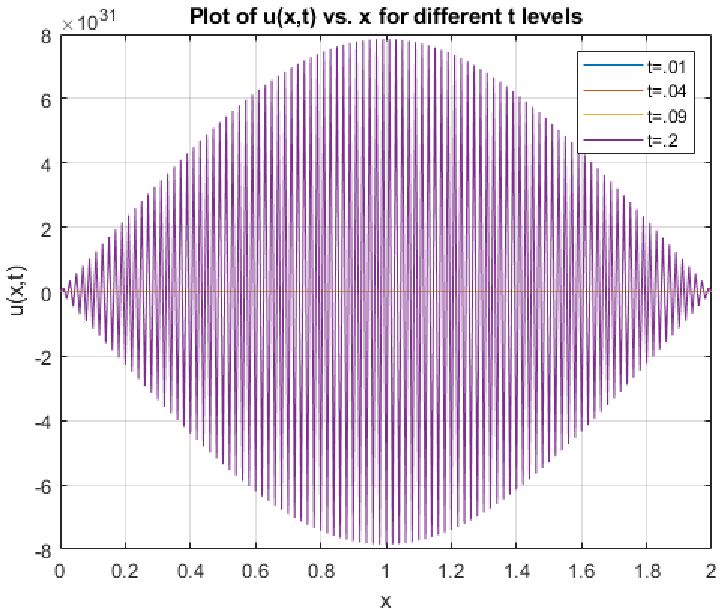
$$u(0, t) = g_L(t),\ u(2, t) = g_R(t)$$

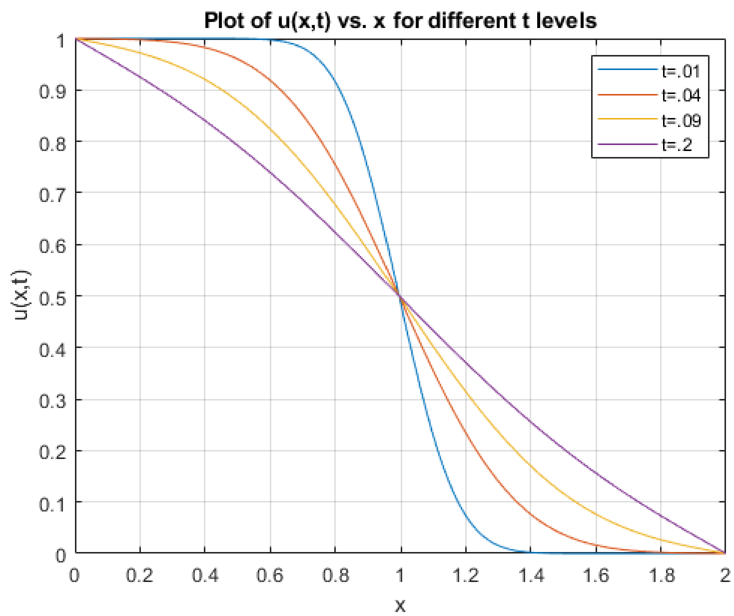
Where $g_L(t) = 1,\ g_R(t) = 0,\ f(x) = \{1,\ 0 < x < 1, \text{ and } 0,\ x \geq 1\ \}$. We are to implement FTCS to solve the IBVP to $T = 0.2$ using $\Delta x = 0.01$ and for 2 different values of Δt.

$$\Delta t = \frac{(\Delta x)^2}{2(.99)}$$

$$\Delta t = \frac{(\Delta x)^2}{2(1.01)}.$$

We are then asked to plot $u(x, t)$ vs x for $t = 0.01, 0.04, 0.09, 0.2$ for both Δt. Those figure are shown below.

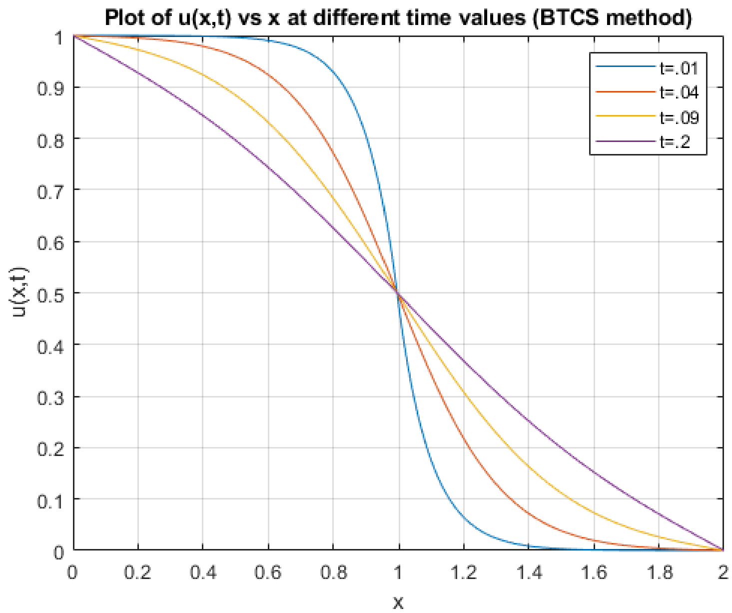


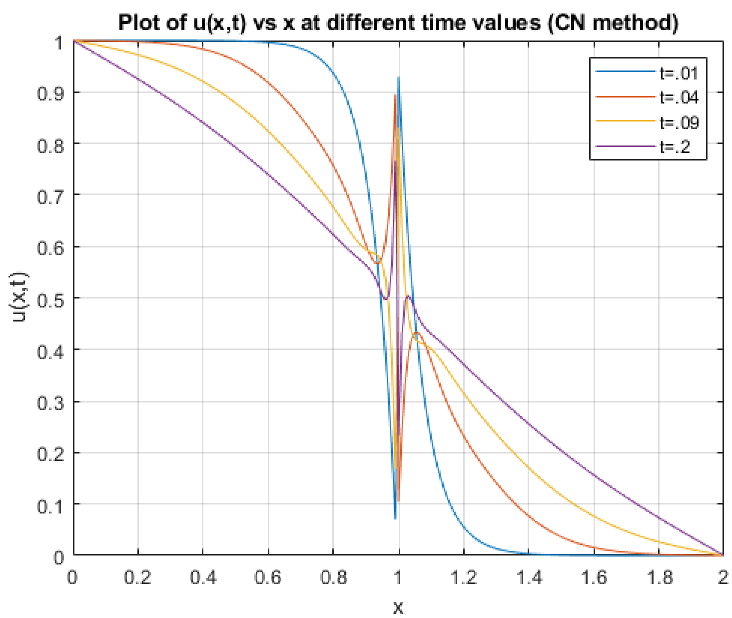


Problem 4

In problem 4 we implemented the BTCS method and C-N method to solve the same IBVP as in problem 3. We again plot $u(x,t)$ vs x at $t = 0.01, 0.04, 0.09, 0.2$

for both methods. From the figures below we can see that the C-N method becomes unstable around x=1.



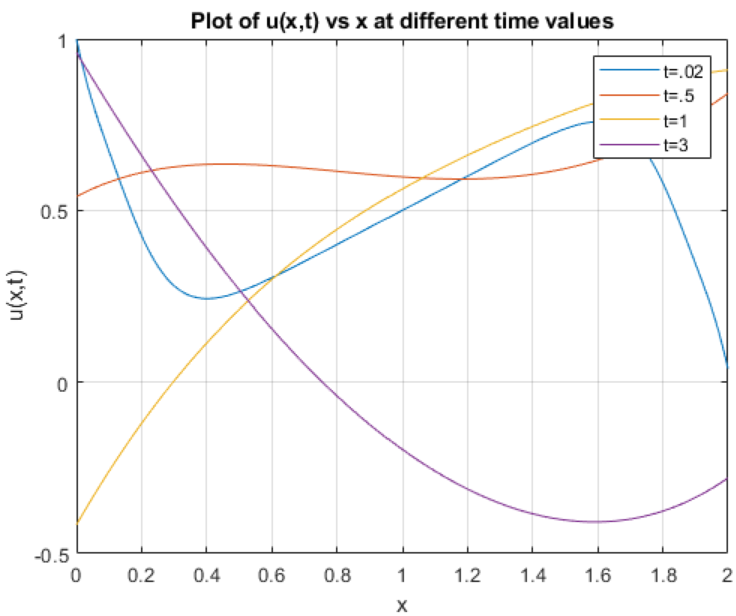


Problem 5

In Problem 5 we use the same IBVP as problem but this time we change the initial and boundary conditions to $f(x) = 0.5x$, $g_L(t) = cos(2t)$, $g_R(t) = sin(2t)$. We solve this IBVP using 2s-DIRK with $\alpha = 1-1/\sqrt{2}$. Where $T = 3$, $\Delta x = 0.01$ and $\Delta t = 0.01$

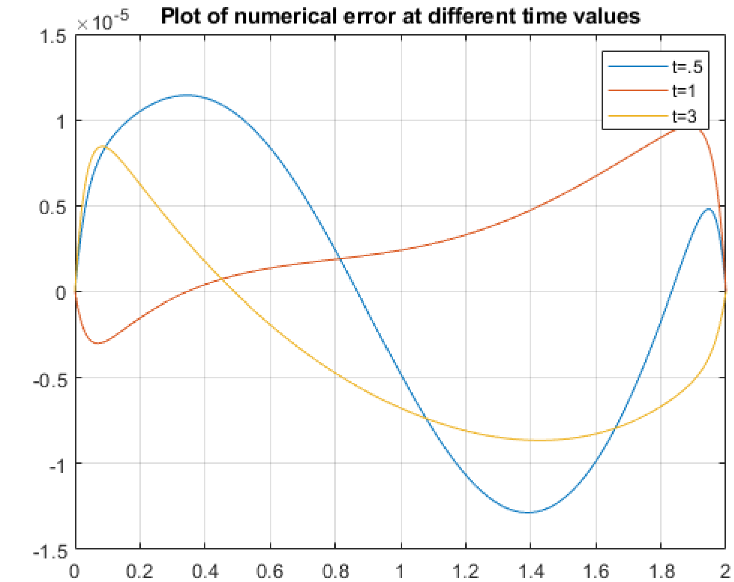
Part 1

For part one we were simply asked to plot the numerical solution $u(x,t)$ vs x at $t = 0.02, 0.5, 1, 3$ shown below.

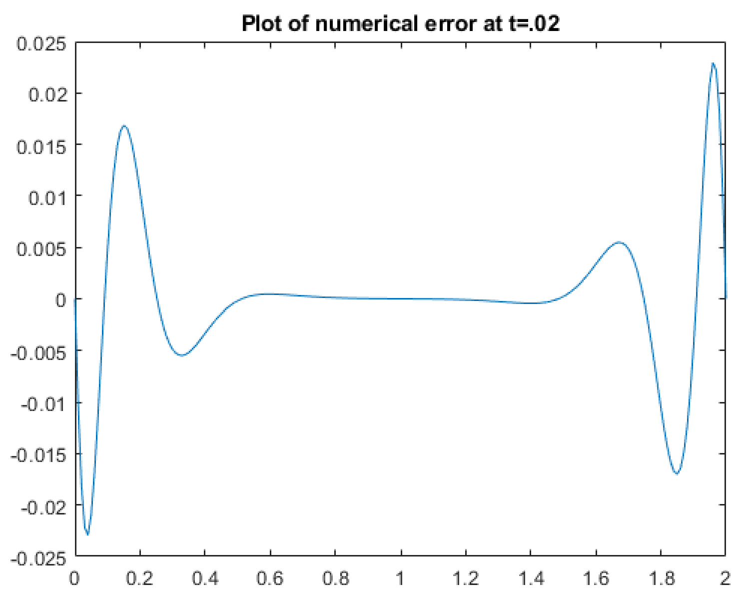


Part 2

In part 2 we estimated the error of the numerical solution from part 1 and plot it when $t = 0.5, 1, 3$ and then $t = 0.02$ in a separate figure.



Plot of numerical error at different time values
×10^-5
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
t=.5
t=1
t=3



Plot of numerical error at t=.02
0.025
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
-0.02
-0.025
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

Problem 6

Problem 6 we has to solve the following IBVP to $T = 3$ using the FTCS where $\Delta t = 4 \times 10^{-5}$ and $\Delta x = \frac{L}{N-0.5}$ $N = 200$. We then plot $u(x,t)$ vs x at $t = 0.02, 0.5, 1, 3$.

$$\begin{cases} u_t = u_{xx}, & x \in (0,\ L),\quad t > 0 \\ u(x,0) = p(x), & x \in (0,\ L) \\ u_x(0,t) - \alpha u(0,t) = 0, & u(L,t) = q(t) \end{cases}$$

where $L = 2,\ \alpha = 0.4,\ p(x) = (1-0.5x)^2,\ q(t) = 2\sin^2(t)$.