Homework 8

Anthony Falcon

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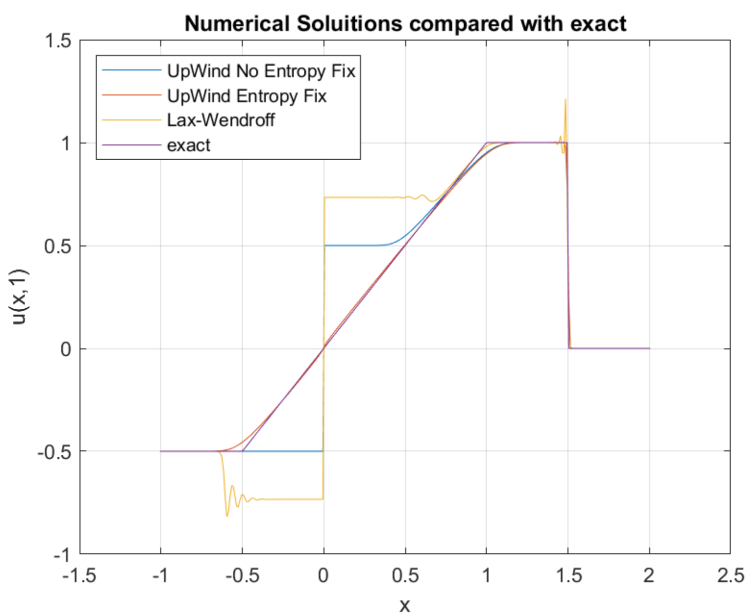
Problem 1

We consider the IVP of Burgers' equation. Using the finite volume discretization and the artificial boundary conditions $u_0^n = u_1^n$ and $u_{N+1}^n = u_N^n$. We implement the following 3 methods: Upwind method 1, Upwind method 2, Lax-Wendroff. Using N=300 and $r = \Delta t/\Delta x = 0.5$.

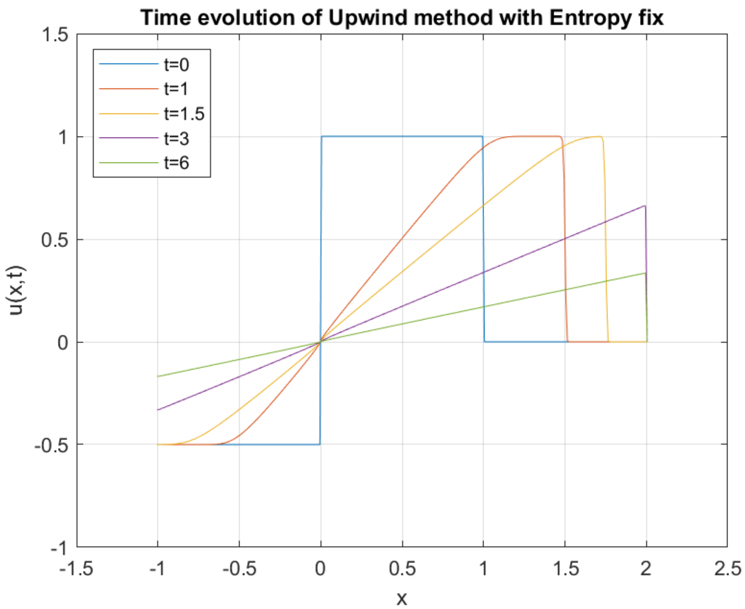
Part 1

Part 1 simply asks us to plot the exact solution along with the three numerical solutions at $t = 1$. Shown below. We can see that Lax-Wendroff deviates the most from the exact solution.



Part 2

Part 2 asks us to only look at Upwind method 2 and plot the numerical solution at $t = 0, t = 1, t = 1.5, t = 3, t = 6$. We are also asked if the characteristic boundaries ever go into the computational domain? Since there is a positive slope at the left boundary the left characteristic boundary goes into the computational domain.

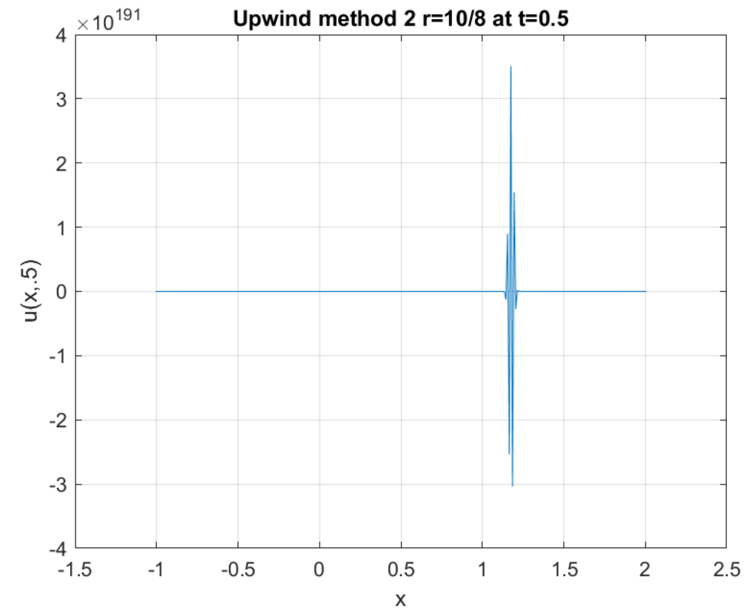


Problem 2

Problem 2 we continue with the IVP from problem 1 and only using upwind method 2.

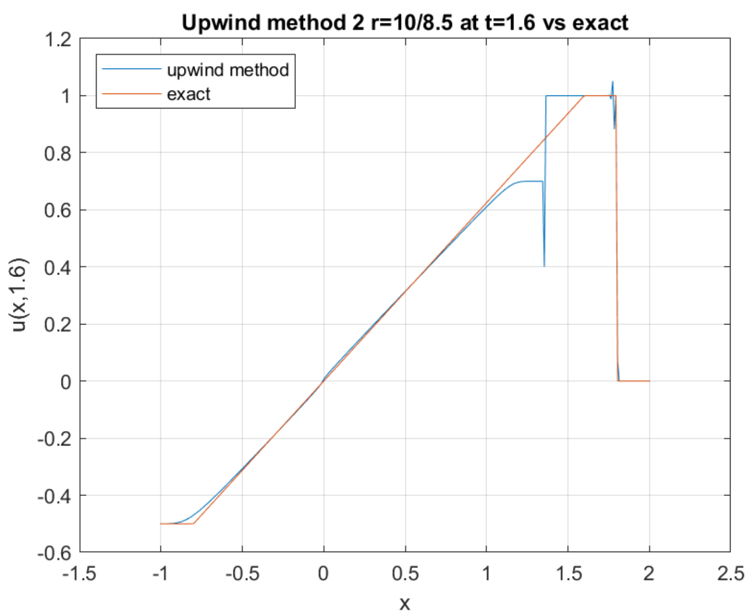
Part 1

Part 1 asks us to use $r = 10/8$ and plot the numerical solution at $t = 0.5$.



Part 2

Part 2 asks us to use $r = 10/8.5$ and plot the exact solution vs the numerical solution at $t = 1.6$.

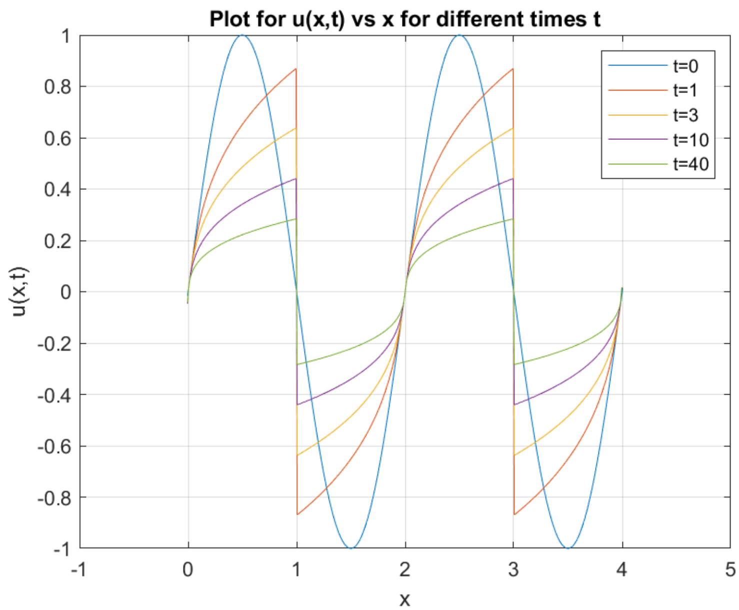


Problem 3

Problem 3 we consider the IVP of conservation law and we use periodic boundary conditions $u_0^n = u_N^n$ and $u_{N+1}^n = u_1^n$. We use N=400 and r=0.5.

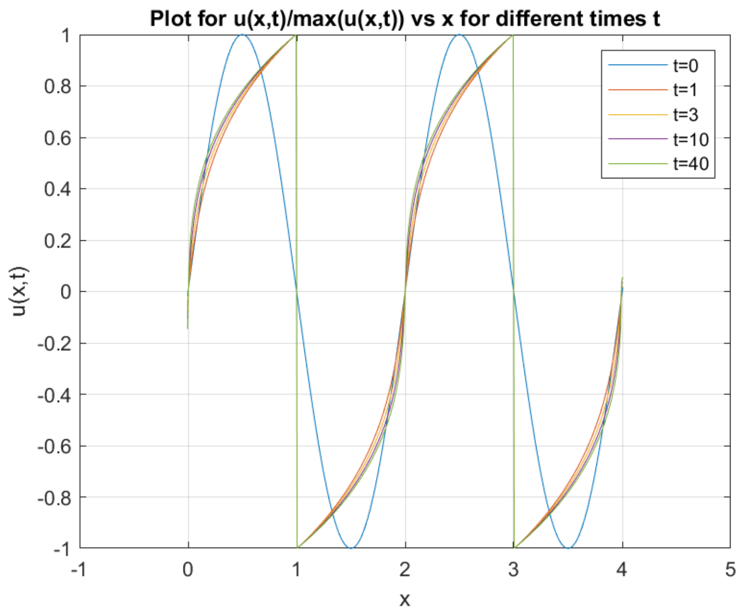
Part 1

Part 1 we plot u(x,t) vs x at t =0,1,3,10,40.



Part 2

Part 2 we need to plot u(x,t)/max(u(x,t)) vs x at t=0,1,3,10,40. From the graph below we can see that u(x,t) vs x has a similar shape for large t.

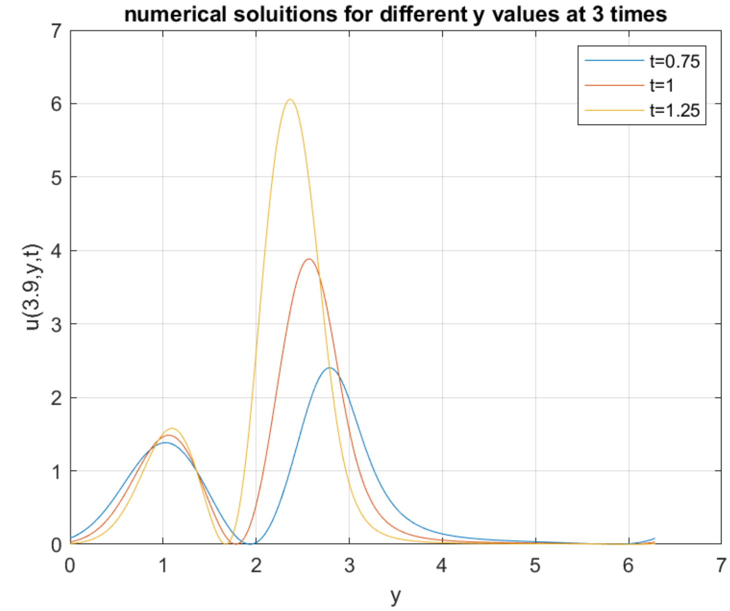


Problem 4

Problem 4 asks us to solve a 2D IVP using method of charachteristics.

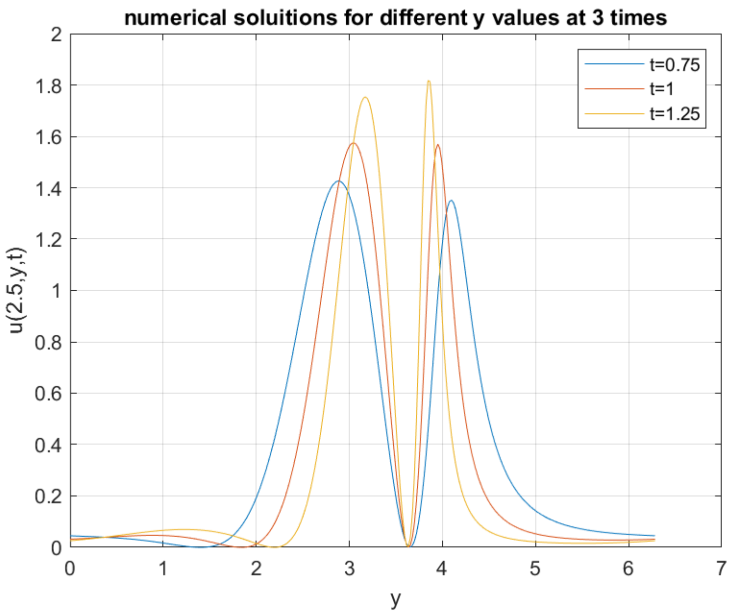
Part 1

Part 1 we set $x_1 = 3.9$ using that we calculate and plot $u(x_1, y, T)$ vs y for $T = .75, 1, 1.25$



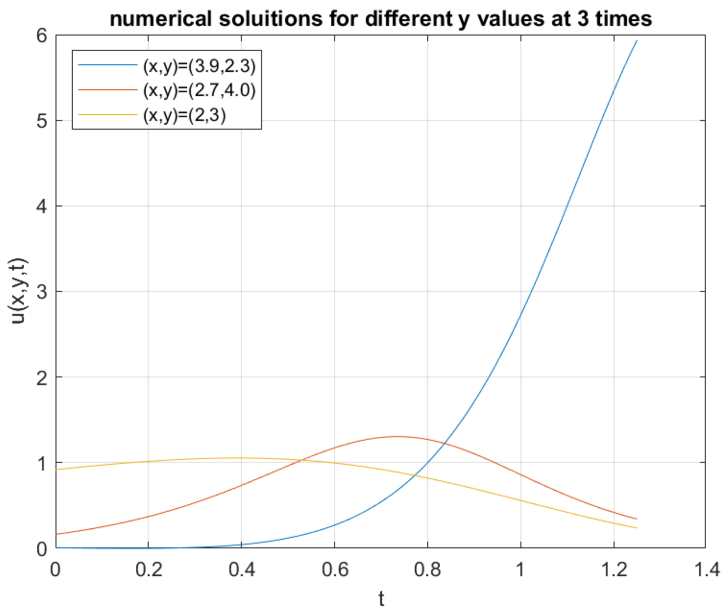
Part 2

Next in part 2 we set $x_1 = 2.5$ and repeat the process as in part 1.



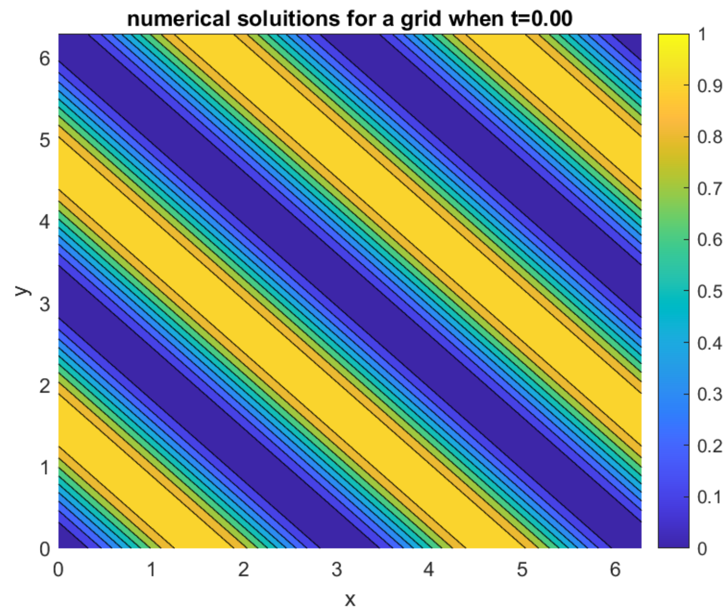
Problem 5

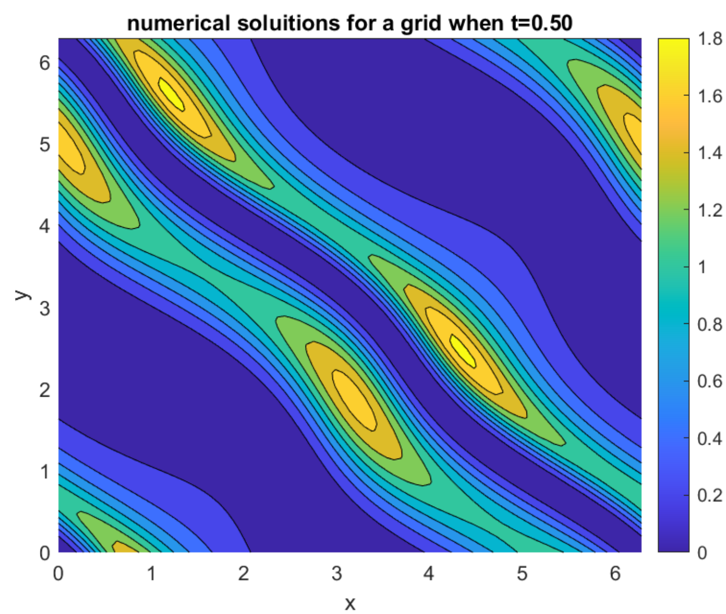
We continue with the 2D IVP and we set $(x_1, y_1) = (3.9, 2.3), (2.7, 4), (2, 3)$ for each of the points we calculate $u(x_1, y_1, t)$ as a function of t for $t \in [0, 1, 25]$. We plot $u(x_1, y_1, t)$ vs t for each of the 3 sets of points.

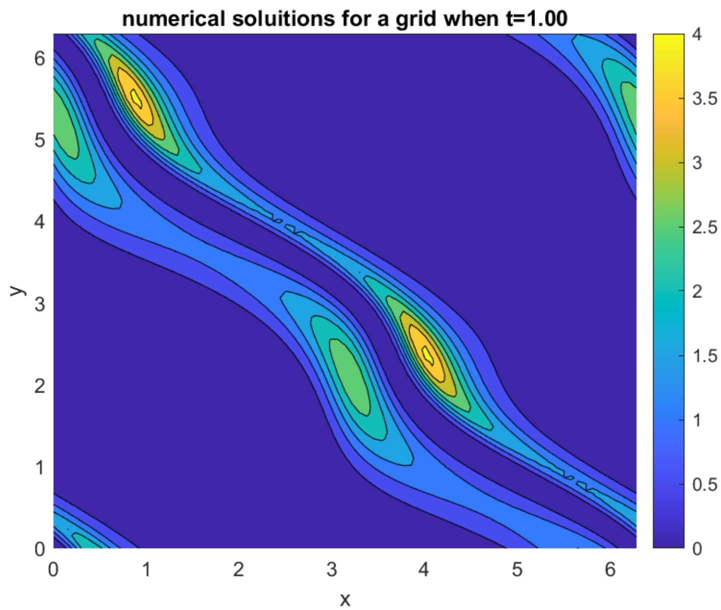


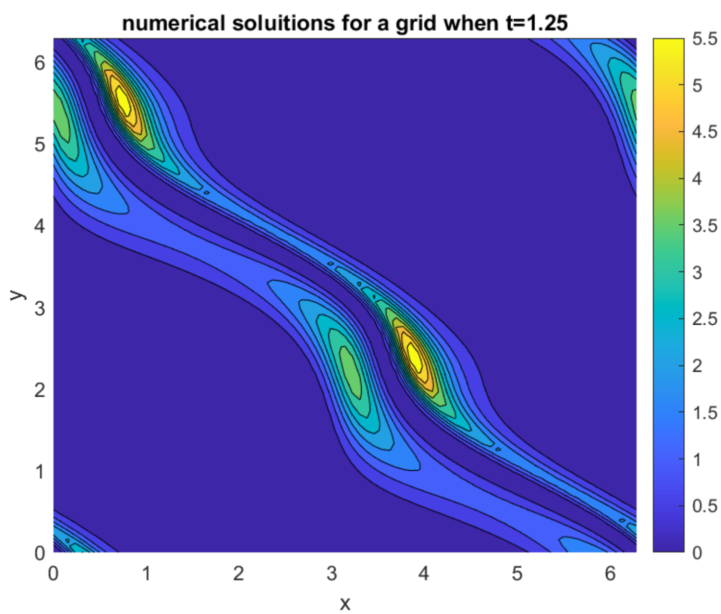
Problem 6

Again we continue with problem 4 we plot a grid of points $(x, y) \in [0, 2\pi]$ for 4 different t values $t = 0, .5, 1, 1.25$ using contourf with a color bar the 4 plots are below.


numerical soluitions for a grid when t=0.00
y
x
0
1
2
3
4
5
6
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9


numerical soluitions for a grid when t=0.50
y
x
0
1
2
3
4
5
6
0.2
0.4
0.6
0.8
1.2
1.4
1.6
1.8


numerical soluitions for a grid when t=1.00
y
x
0
1
2
3
4
5
6
0
0.5
1
1.5
2
2.5
3
3.5
4


numerical soluitions for a grid when t=1.25
y
x
0
1
2
3
4
5
6
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5