Homework 2

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Problem 1-3

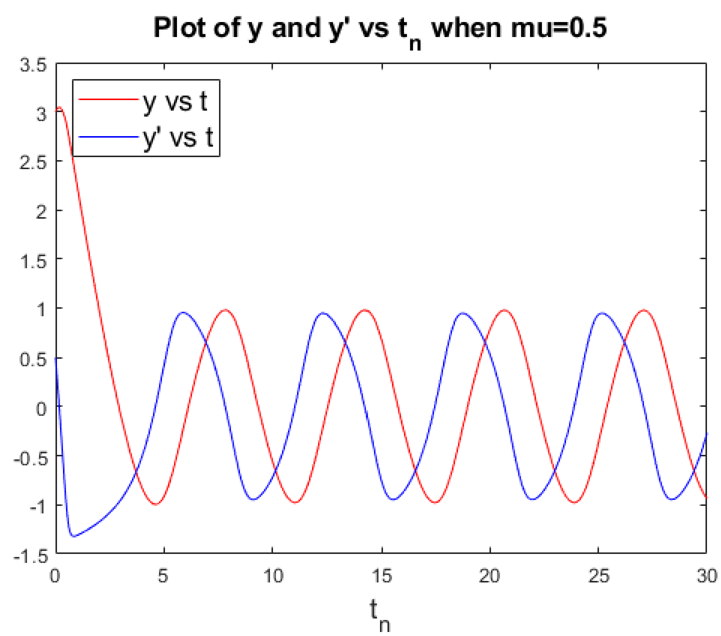
Submitted as a hand written pdf attached at the end of this report.

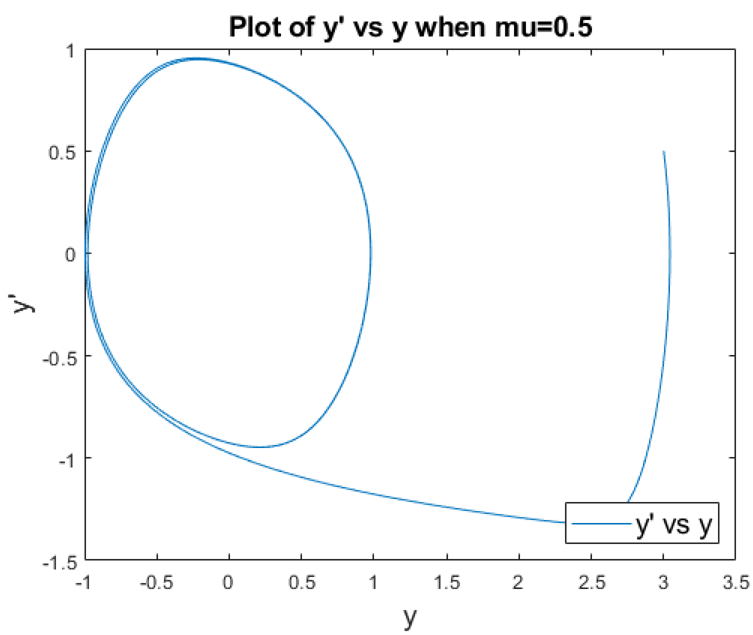
Problem 4

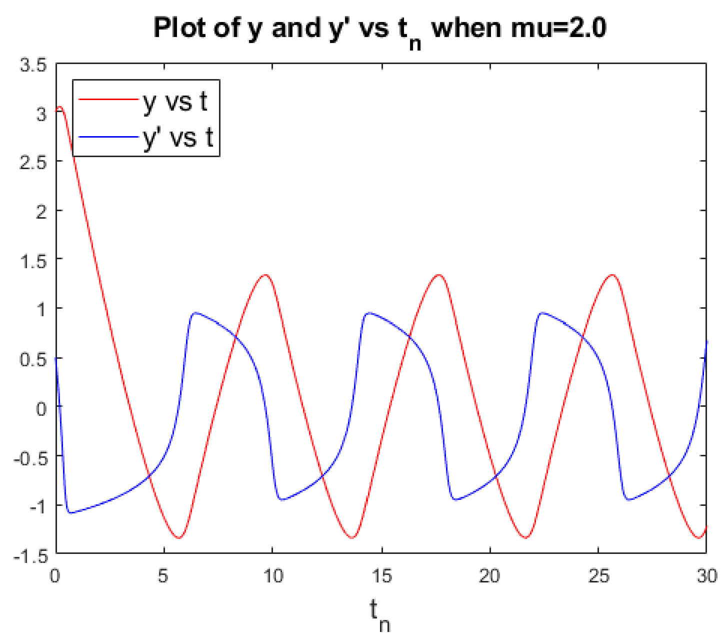
In problem 4 we implemented classic 4th order Runge-Kutta method to solve the following IVP

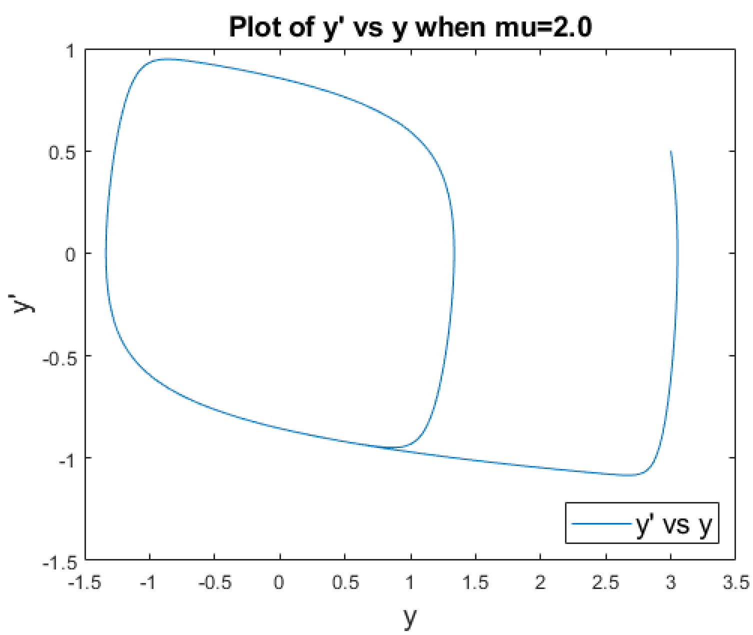
$$y'' - \mu(2 - \exp(y'^2))y' + y = 0$$
$$y(0) = y_0, \quad y'(0) = v_0$$

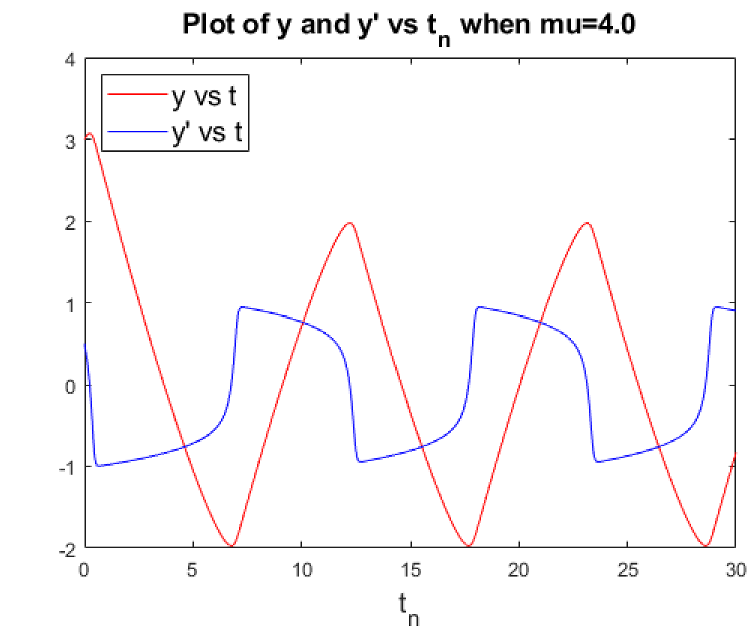
using $y_0 = 3, v_0 = 0.5$ and $h = 0.025$. Before we can use RK4 we convert the IVP to a first order system. We are asked to solve the IVP to T = 30 for 3 different μ values being $\mu = 0.5, 2, 4$. Below are 6 figures grouped by μ one is y(t) vs t and y'(t) vs t and the other is y'(t) vs y(t)

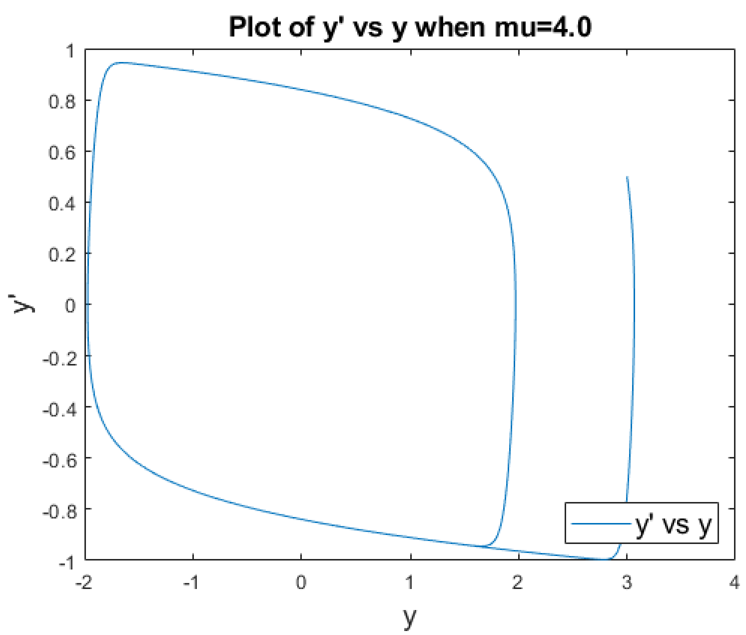

Plot of y and y' vs t_n when mu=0.5
y vs t
y' vs t
t_n


Plot of y' vs y when mu=0.5
y' vs y
y'
y


Plot of y and y' vs t_n when mu=2.0
y vs t
y' vs t
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
0
5
10
15
20
25
30
t_n


Plot of y' vs y when mu=2.0
1
0.5
0
-0.5
-1
-1.5
y'
y' vs y
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
3
3.5
y



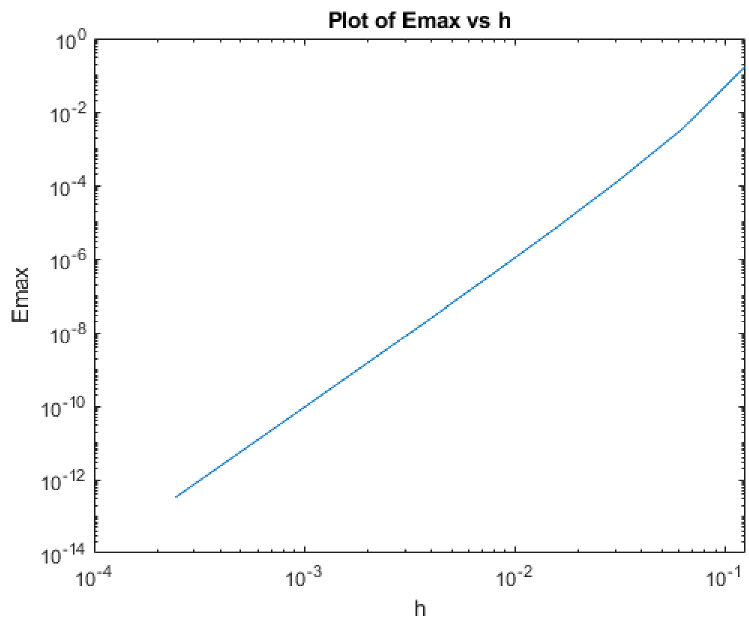


Problem 5

In problem 5 we were asked to solve the IVP in problem 4 using $y_0 = 3$ and $v_0 = 0.5$ but this time $\mu = 4$ and we vary the time stepsize $h = \frac{1}{2^3}, \frac{1}{2^4}, ..., \frac{1}{2^{13}}$ We used the numerical solutions to estimate the error in the numerical solutions.

Part 1

In part 1 we look at the max error for each time step size. Below is the plot of E_{max} vs h

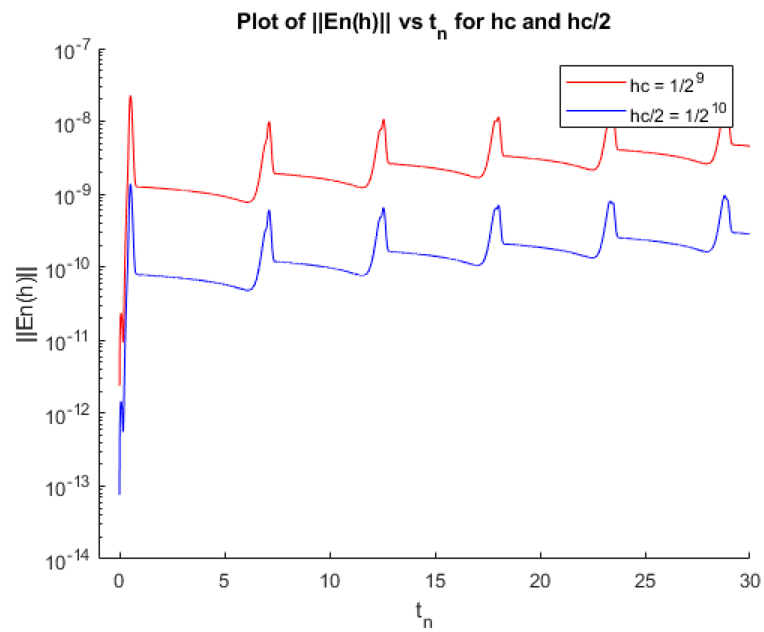


Part 2

In part 2 we needed to find a step-size such that $E_{max}(h) < 5 \times 10^{-8}$. that value was

$$h_c = \frac{1}{2^9}$$

Below is the plot of $\|E_n(h)\|$ vs t_n with time step size h_c and $\frac{h_c}{2}$

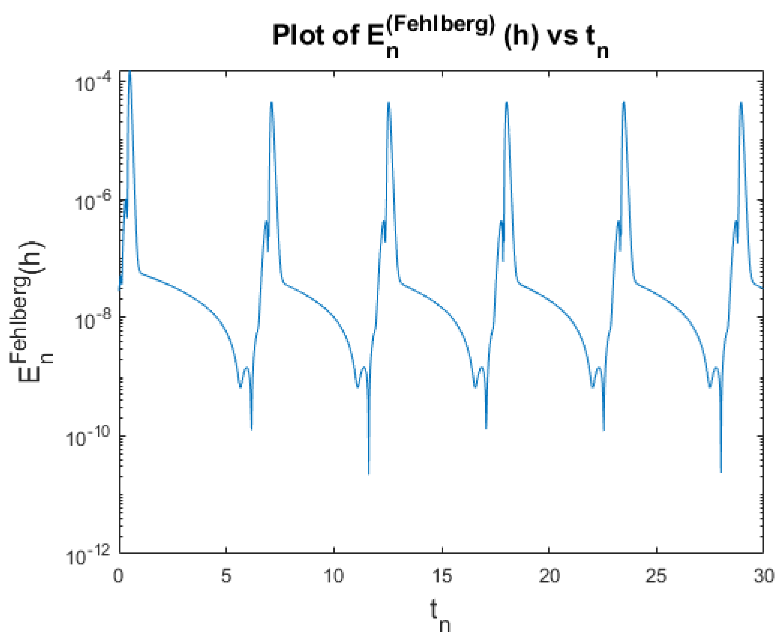


Problem 6

In Problem 6 we program the fehlberg 45 method. Using this method we once again solve the IVP to $T = 30$ from problem 4. We use $y_0 = 3, v_0 = 0.5\mu = 4$ and our step size $h = 0.025$.

Part 1

Part 1 asks us to calculate $E_n^{Fehlberg}(h)$ and make a plot of $E_n^{Fehlberg}(h)$ vs t_n. The plot of that is below.

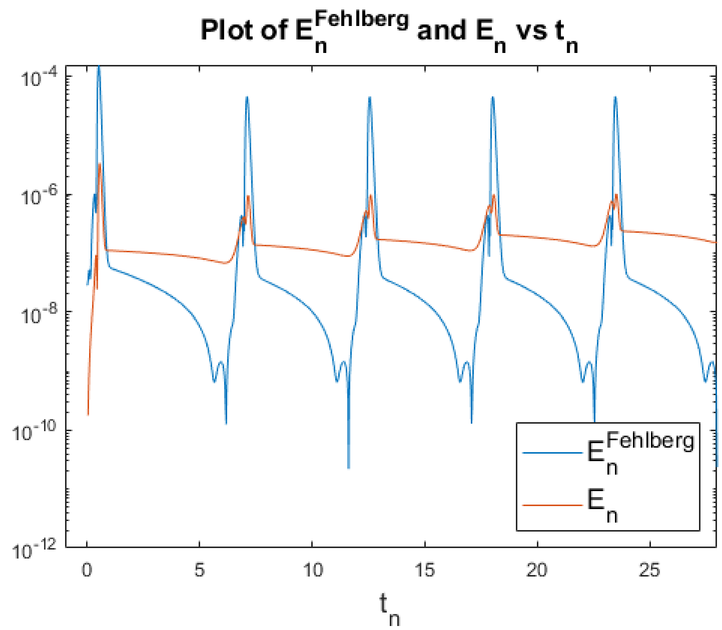


Part 2

Finally in part 2 we used

$$E_n(h) = \frac{1}{1-(0.5)^5} \| w_n(h) -_{2n} (\frac{h}{2}) \|$$

to calculate the error then we plot $E_n^{Fehlberg}(h)$ vs t_n and $E_n(h)$ vs t_n in one figure to compare them. from the figure below we can see that E_n is more consistent with error than $E_n^{Fehlberg}(h)$ is.



Plot of $E_n^{Fehlberg}$ and E_n vs t_n
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
0
5
10
15
20
25
t_n
$E_n^{Fehlberg}$
E_n