Homework 1

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Problem 1

Submitted as a hand written pdf attahed at the end of this report.

Problem 2

In problem 2 of the homework we are asked to numerically approximate the following integral using the composite trapezoidal method and composite Simpson's method.

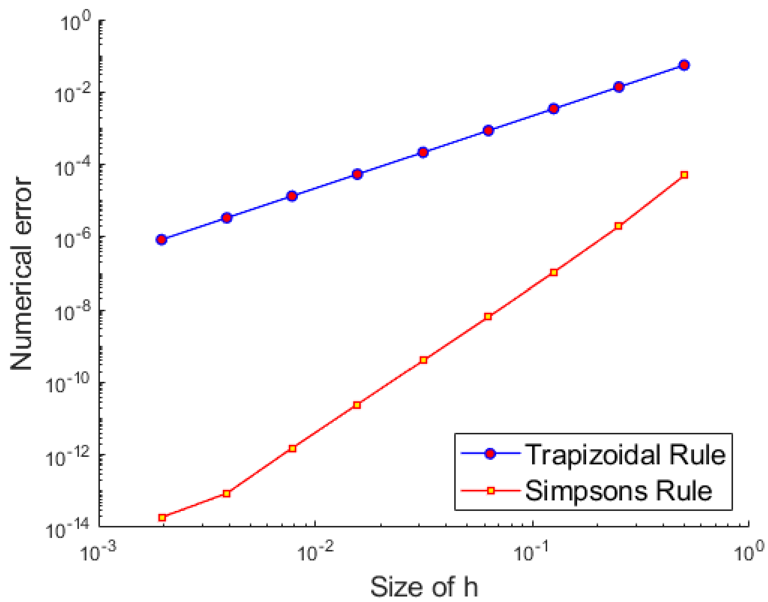
$$I \equiv \int_1^3 \sqrt{2 + \cos^3(x)} \exp(\sin(x)) dx$$

We had 9 different numerical resolutions at $N = 2^2, 2^3, 2^4, ...2^{10}$. Using matlab the numerical solution for $N = 2^{10}$ for composite trapezoidal rule and for composite Simpson's rule is the following:

$$A_{trap} = 5.948926538426832 \quad A_{simp} = 5.948926749149474$$

The difference between the two numbers is $2.107226e - 07$

Below is a plot comparing the error between trapezoidal rule and Simpson's rule



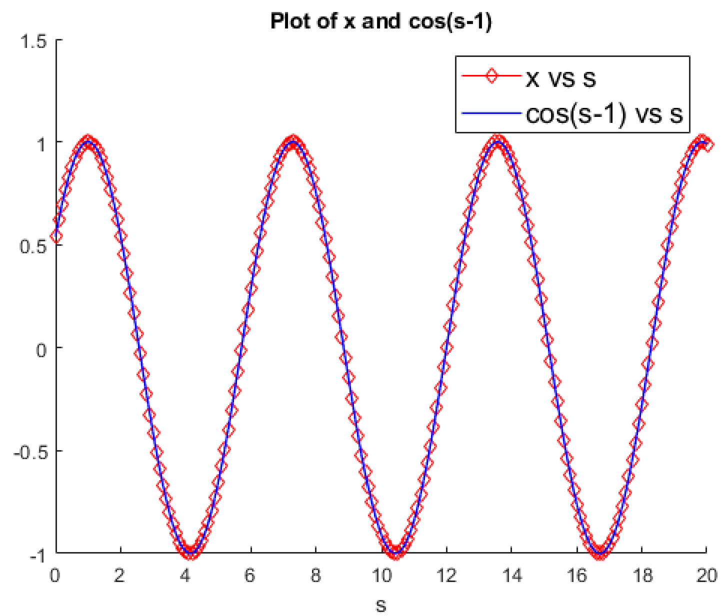
Problem 3

In problem 3 we use Newton's method to solve the following non-linear equation of x:

$$x - \alpha + \beta \sinh(x - \cos(s - 1)) = 0$$

We are asked to solve the equation for 201 values of s defined as $s = [0 : 0.1 : 20]$ and we let $\alpha = 0.9$ and $\beta = 50000$

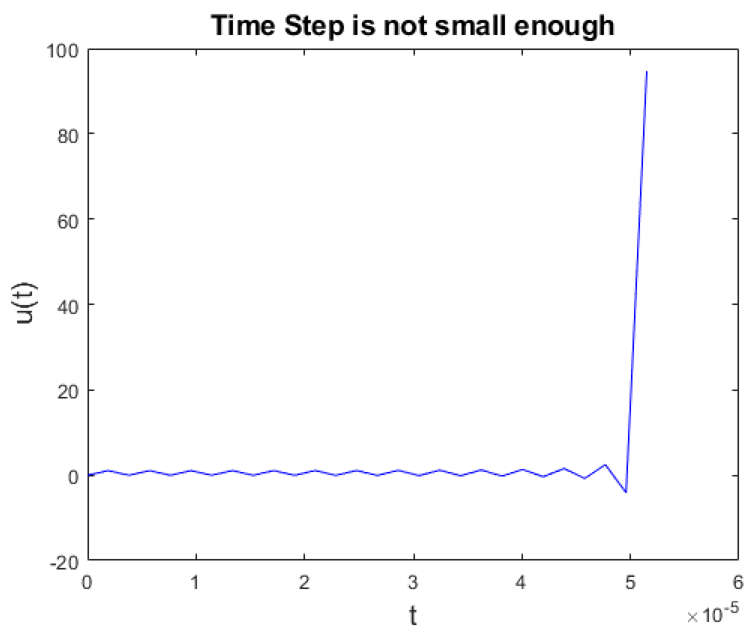
Below is a plot for x vs s compared with $\cos(s - 1)$ vs s

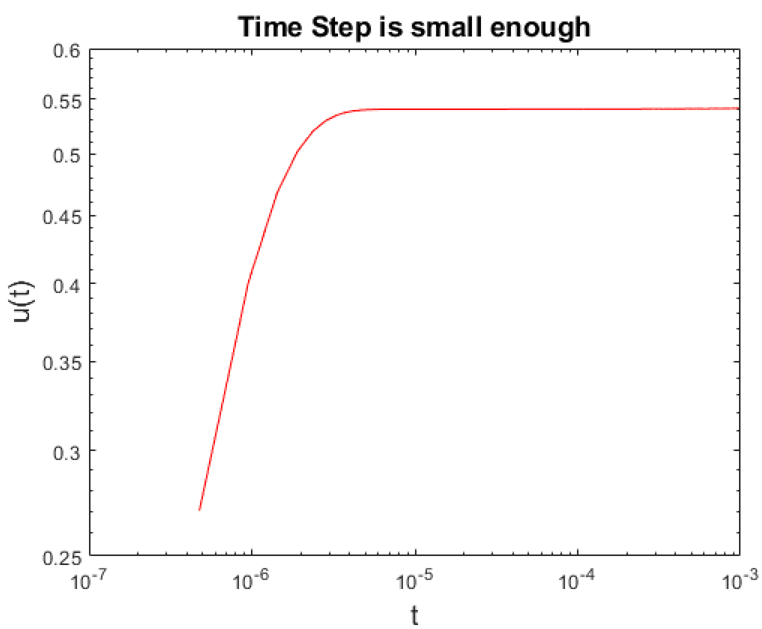


Problem 4

Part 1

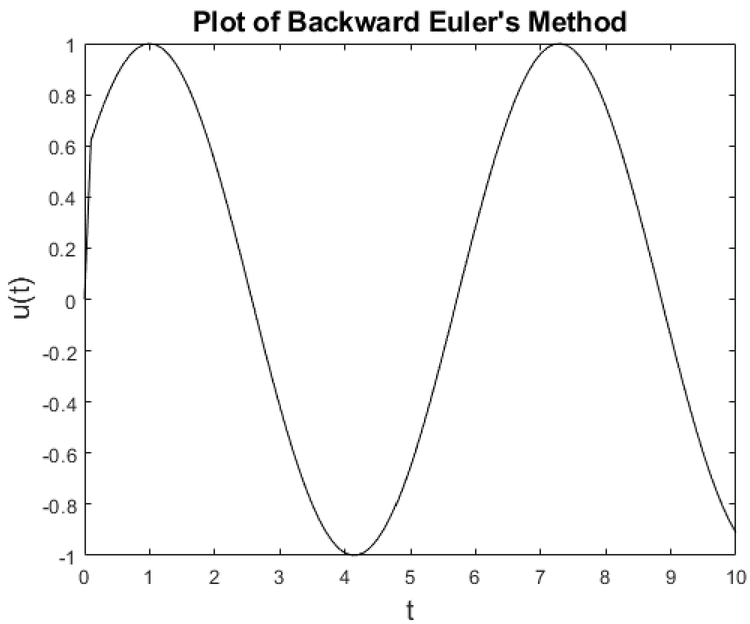
We were first asked to program Euler method to solve an IVP for a few step sized $h = 2^{-18}, 2^{-19}, 2^{-20}, 2^{-21}, 2^{-22}$. Testing out these different step sizes $h = 2^{-20}$ was the smallest step size such that the solution would remain bounded. Below is a graph when the solution is bounded and unbounded.





Part 2

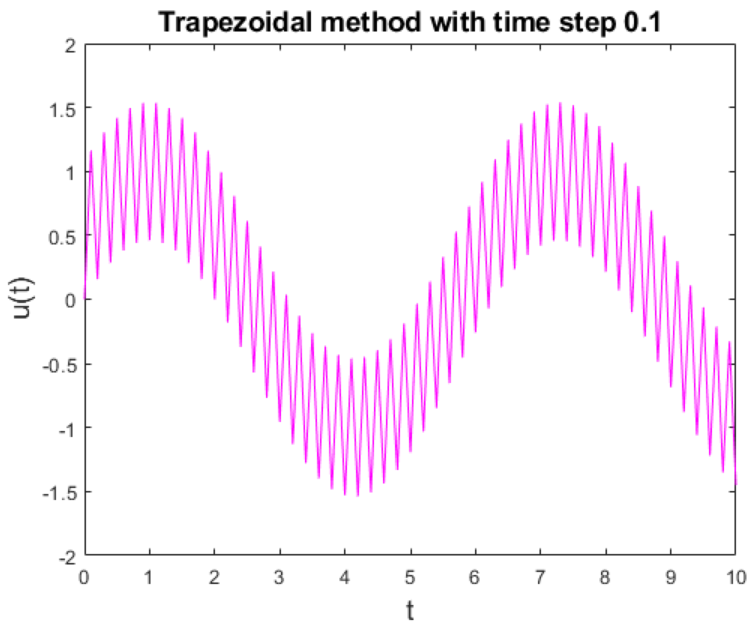
We were then asked to program backward Euler method to solve IVP and also to use Newton's method to help us solve the non-linear equation in each time step. Below is a plot of the solution for backward Euler method.



Problem 5

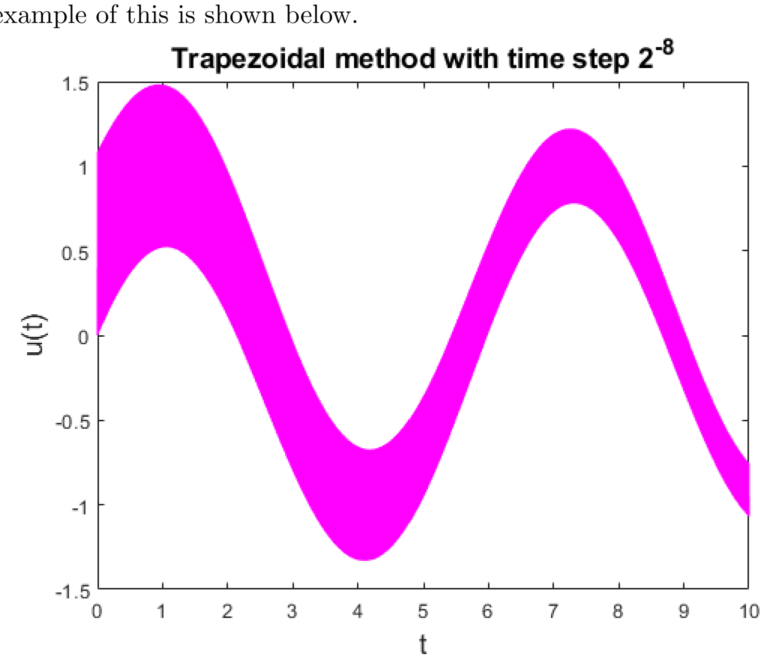
Part 1

In problem 5 we were asked to solve the same function in problem 4 but this time using the trapezoidal method and once again we use Newton's method to solve the non-linear method in each time step. Below is the plot of the solution vs. t. From the graph we see that the solution is bounded and there are also oscillations within the solution.



Part 2

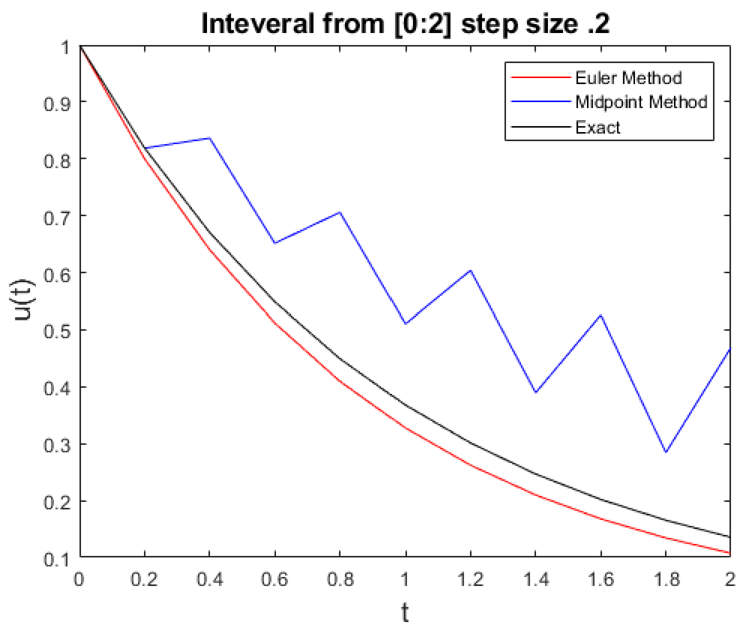
We reduced the step size and investigate what happens to the oscillations. As we decrease the step size we see the oscillations decrease quicker and quicker. An example of this is shown below.



Problem 6

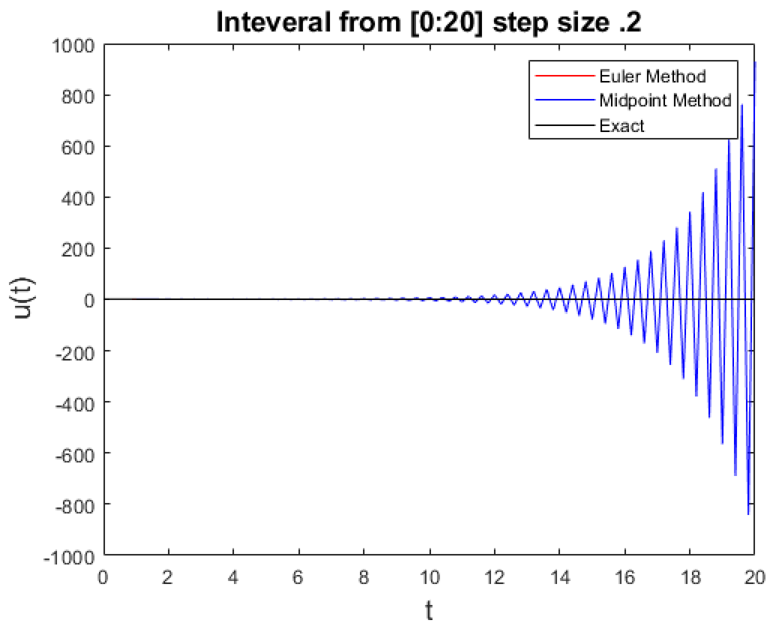
Part 1

In the first part we needed to program and compare Euler method and 2-step midpoint method over a small interval ($T = 2$) with the exact solution. Over a small interval the midpoint method is somewhat accurate but much less accurate than Euler over this time period. Below is the figure comparing the two functions with the exact solution.



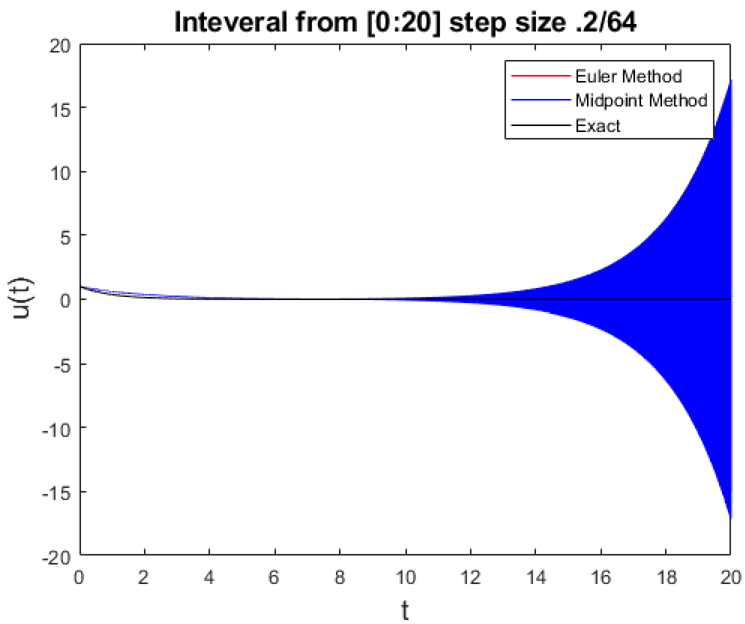
Part 2

Next we increase our interval to $T = 20$ we quickly see that the midpoint method starts to become not well behaved. In the following figure both methods and the exact solution is shown however, since midpoint method is not well behave Euler's method and the exact solution are covered. Below is the figure.



Part 3

Finally we decreased the time step size which significantly reduces the growth error of the midpoint method. Below is a figure when $h = .2/64$. We can see that when $h = .2$ $u(t)$ is getting close to 1000 in the final step and for $h = .2/64$ $u(t)$ is only approaching 17.


Inteveral from [0:20] step size .2/64
Euler Method
Midpoint Method
Exact
u(t)
t
20
15
10
5
0
-5
-10
-15
-20
0
2
4
6
8
10
12
14
16
18
20

Homework 1

Wednesday, March 31, 2021 4:22 PM

Problem 1 (Theoretical)

Suppose E_n satisfies the recursive inequality

$$E_{n+1} \le (1+Ch)E_n + h^2 \quad \text{for } n \ge 0$$
$$E_0 = 0$$

where $C > 0$ is a constant independent of h and n.

Derive that $\quad E_N \le \dfrac{e^{CT}-1}{C}h \quad$ for $Nh \le T$

1) $$(1+Ch)^{-(n+1)} E_{n+1} \le \left((1+Ch)E_n + h^2\right)(1+Ch)^{-(n+1)}$$

$$(1+Ch)^{-(n+1)} E_{n+1} \le (1+Ch)^{-n} E_n + h^2 (1+Ch)^{-(n+1)}$$

$$(1+Ch)^{-(n+1)} E_{n+1} - (1+Ch)^{-n} E_n \le h^2 (1+Ch)^{-(n+1)}$$

Next sum from $n=0$ to $n=N-1$ where $E_0 = 0$

we get

$$(1+Ch)^{-N} E_N \le h^2 \sum_{n=0}^{N-1} (1+Ch)^{-(n+1)}$$

From HW#0 $\displaystyle\sum_{n=0}^{N-1} r^{n+1} = r\frac{1-r^N}{1-r}$

$$\leq h^2 \cdot (1+ch)^{-1} \left(\frac{1-(1+ch)^{-N}}{1-(1+ch)^{-1}} \right)$$

$$\leq \frac{h}{c}\left(1-(1+ch)^{-N}\right)$$

Multiply by $(1+ch)^N$ and use $1+ch \leq e^{ch}$

$$E_N \leq \frac{h}{c}\left((1+ch)^N - 1\right) \leq \frac{h}{c}\left(e^{chN}-1\right) \leq \frac{e^{cT}-1}{c} h$$

where $T \geq Nh$

Hence we have

$$E_N \leq \frac{e^{cT}-1}{c} h \quad ∎$$