Homework 2

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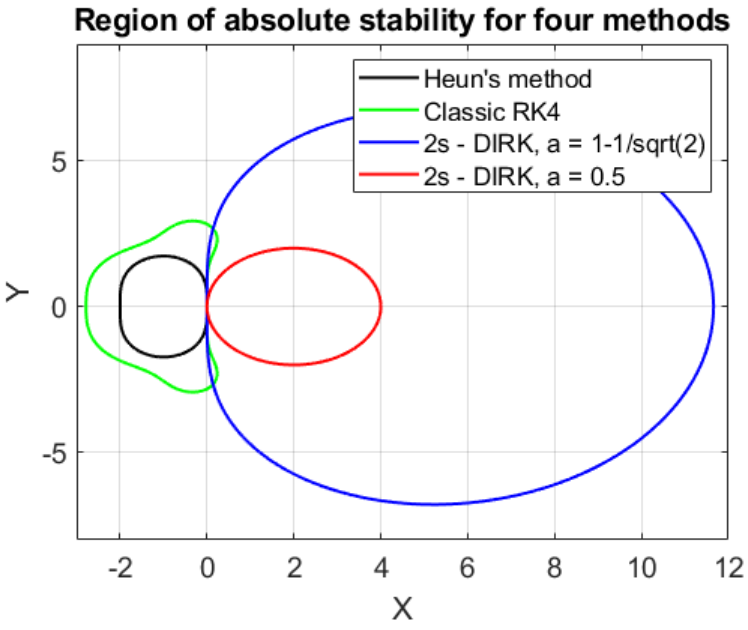
Problem 1-3

Submitted as a hand written pdf attached at the end of this report.

Problem 4

Problem 4 we were asked to plot the region of absolute stability for the following methods Predictor-corrector method (Heun's method), Classic 4th order Runge-Kutta method (RK4) and the 2s-DIRK method with $\alpha = 1 - 1/\sqrt{2}$ and $\alpha = -.5$. We derived the stability function for the first 2 methods in problem 1 and the stability function for 2s-DIRK method was derived in problem 2. Below is a figure of the 4 contours



Problem 5

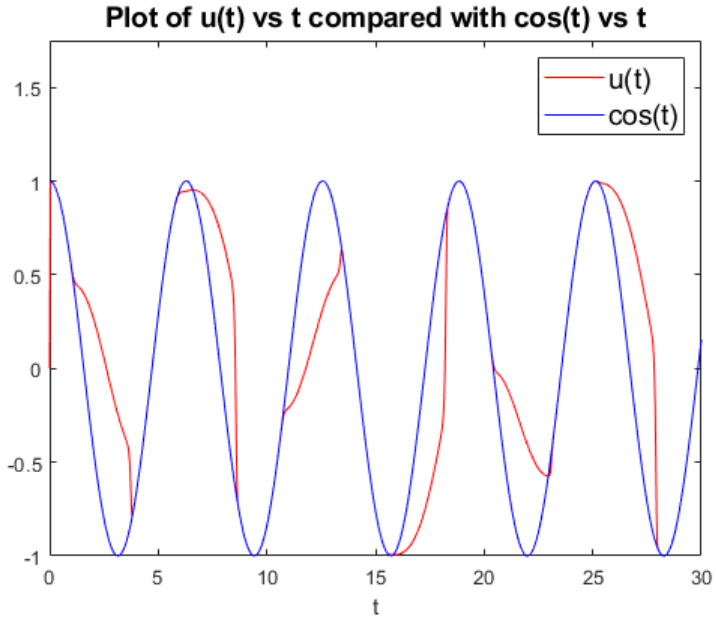
In problem 5 we were asked to solve an IVP to $T = 30$ using the 2s-DIRK method using $\alpha = 1 - 1/\sqrt{2}$.

$$u' = -(0.5 + \exp(20\cos(1.3t)))\sinh(u - \cos(t))$$

$$u(0) = 0$$

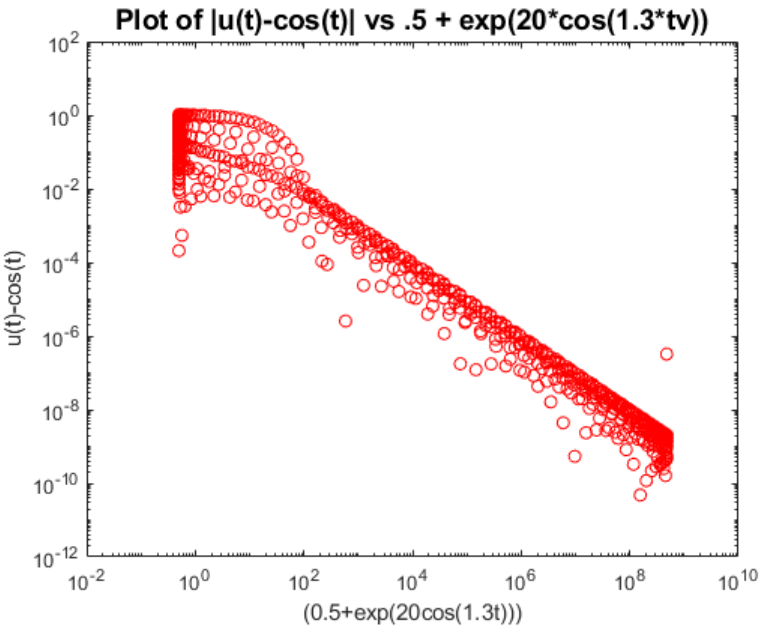
Part 1

In part 1 using a step size of $h = 2^{-5}$ we plot the numerical solution u(t) vs t and cos(t) vs t in the same figure inserted below. We can see that the numerical solution does not closely follow cos(t).



Part 2

For part 2 we were asked to plot $|u(t) - \cos(t)|$ vs $(0.5 + \exp(20\cos(1.3t)))$ for $t \in (0, 30]$. Below is a figure of the plot. From the plot we can see that for a horizontal value of 10^5 or greater $u(t)$ follows $\cos(t)$ under 10^{-6} accuracy.



Problem 6

In Problem 6 we once again solve the IVP from problem 5 this time using different step-sizes $h = \frac{1}{2^3}, \frac{1}{2^4}, ..., \frac{1}{2^8}$. We will calculate numerical error estimation for both 2s-DIRK method with $\alpha = 1 - 1/\sqrt{2}$ and backward euler method.

Part 1

Part 1 asks us to calculate and plot the numerical estimation for both methods when $h = \frac{1}{2^5}$. Below is a figure of the plot.

Part 2

In part 2 we followed part one except we plotted for $h = \frac{1}{2^7}$ insead of $h = \frac{1}{2^5}$.