Homework 4

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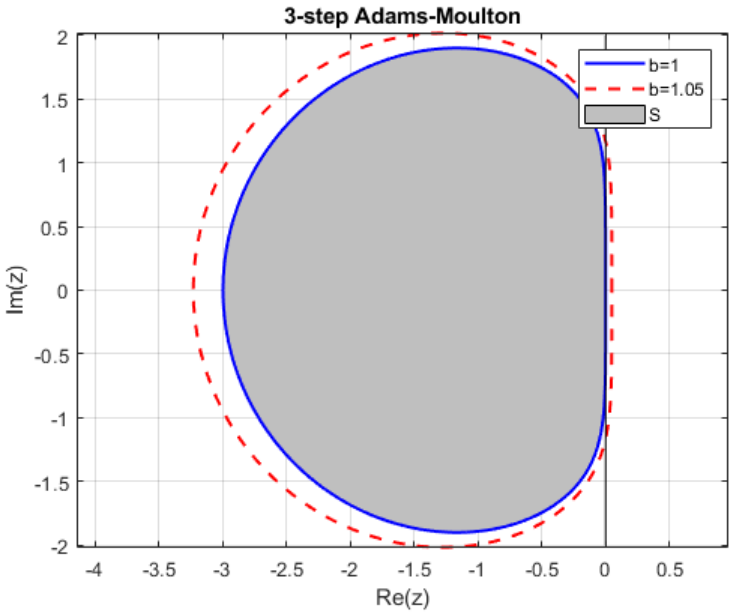
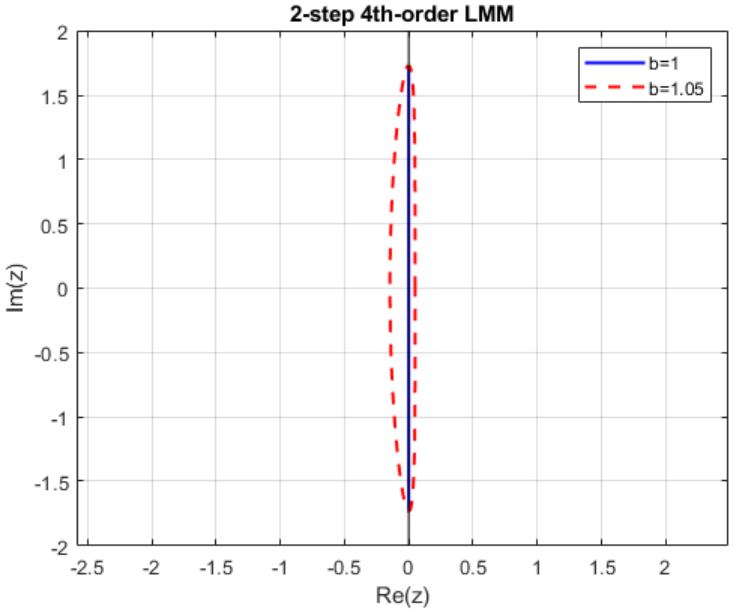
Problem 1

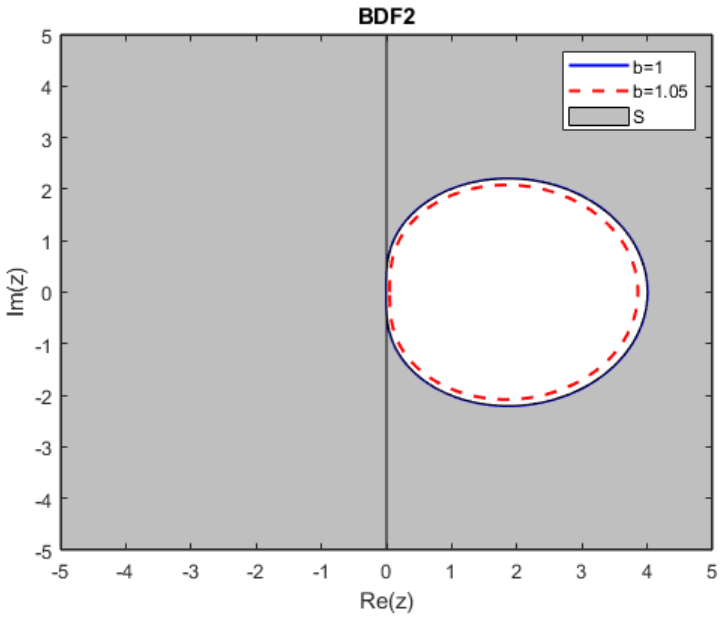
Submitted as a hand written pdf attached at the end of this report.

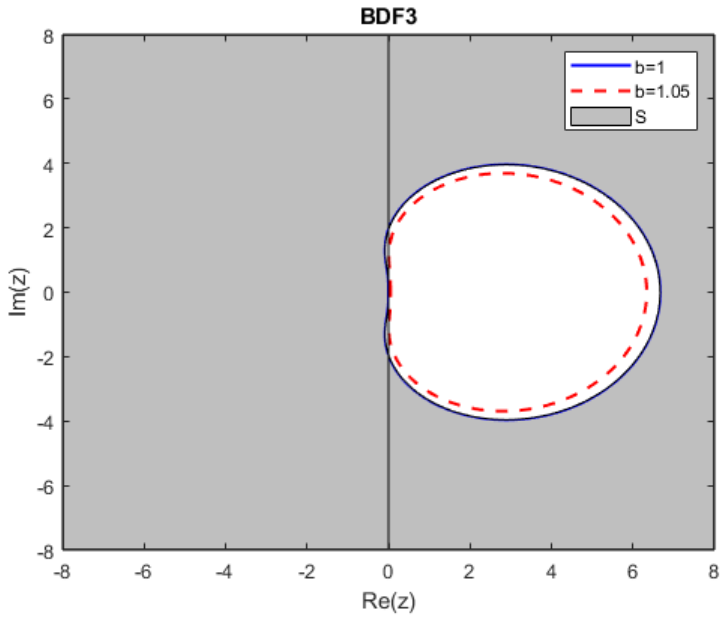
Problem 2

In problem 2 we are asked to plot the region of absolute stability for the following four methods 3-step Adams-Moulton,2-step 4th-order LMM, BDF2, and BDF3. Here are the results




2-step 4th-order LMM
b=1
b=1.05
Im(z)
Re(z)
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2


BDF2
b=1
b=1.05
S
Im(z)
Re(z)
5
4
3
2
1
0
-1
-2
-3
-4
-5
-5
-4
-3
-2
-1
0
1
2
3
4
5



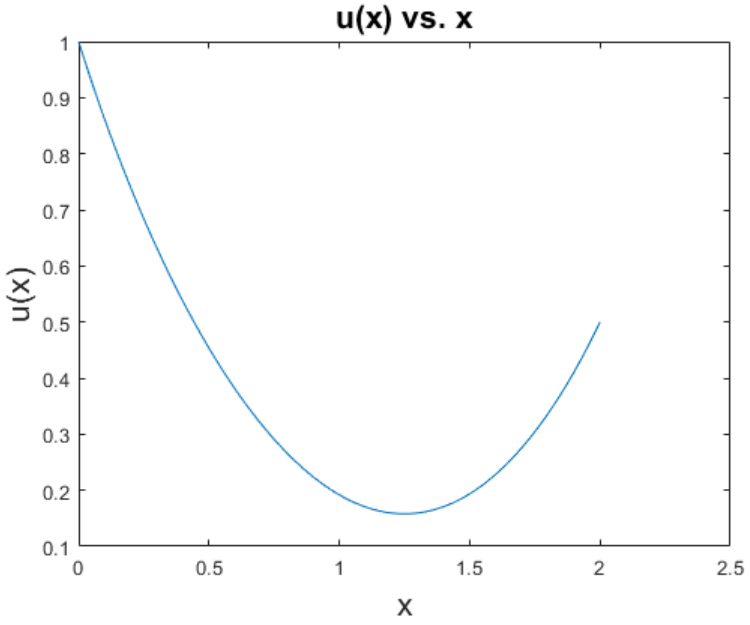
Problem 3

In problem 3 we implemented the shooting method to solve the following BVP

$$u'' - (1 + 0.5u'^2)u = \sin x$$

$$u(0) = 1,\ u(2) = 0.5$$

We started $v_0 = -1$ and the value of v found was $v = -1.4851$. Below is a plot of $u(t)$ vs. x.



Problem 4

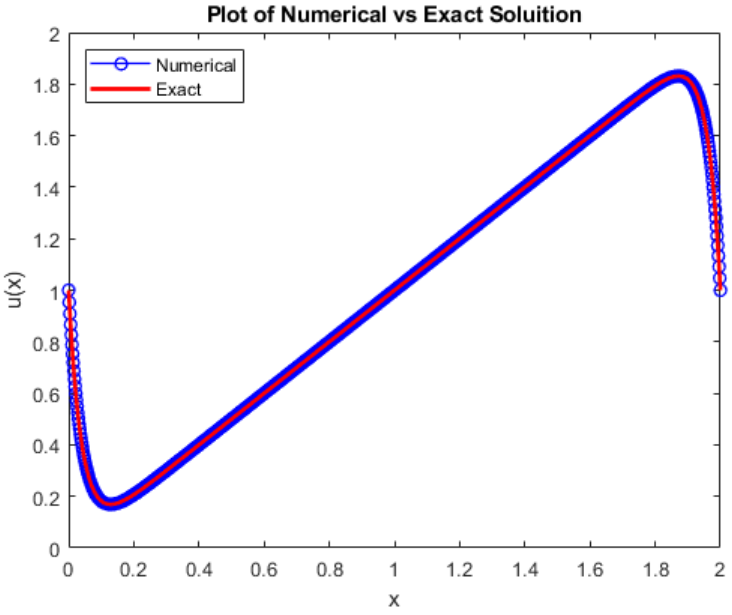
In Problem 4 we used the FDM to solve the following BVP

$$u'' - 625u = -625x$$

$$u(0) = 1,\ u(2) = 1$$

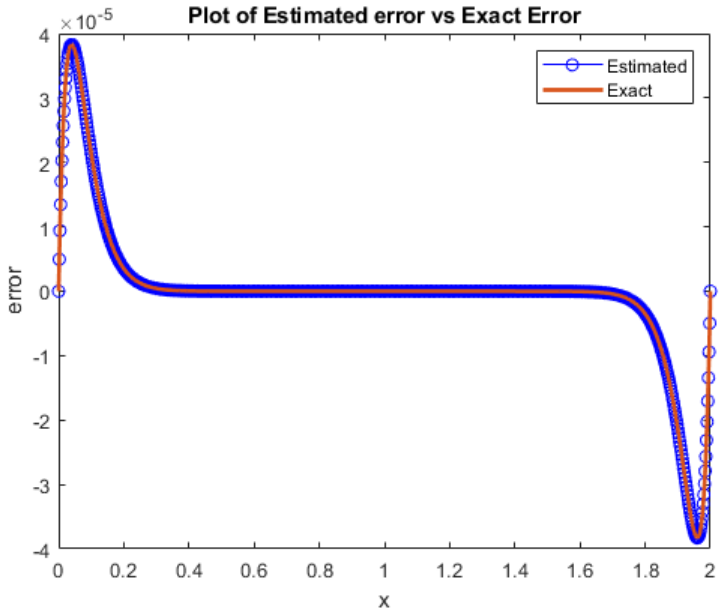
Part 1

For part one we were simply asked to plot the numerical solution $u(x)$ vs x and the exact solution $u(x)$ vs x on one figure shown below.



Part 2

In part 2 we estimated the error of the numerical solution from part 1 and plot it on the same figure as the exact error. Figure shown below.



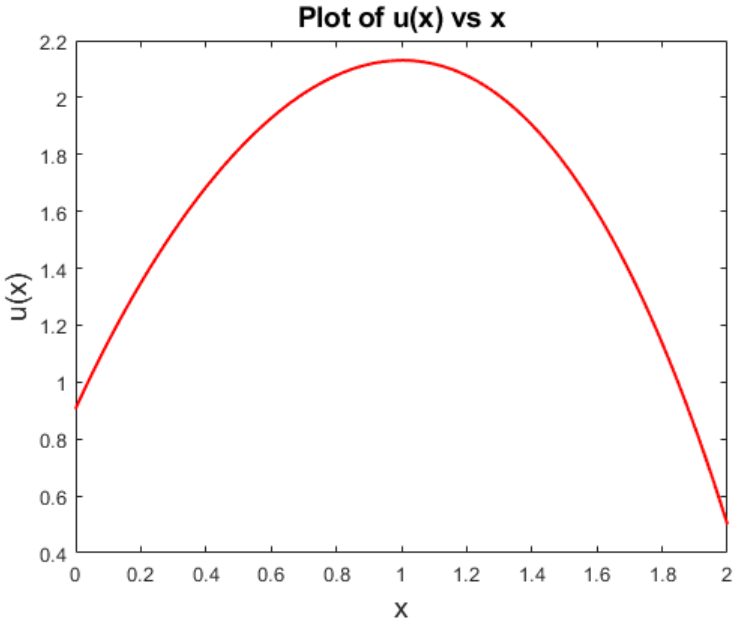
Problem 5

Problem 5 had us edit FDM to solve the following BVP

$$u'' - (1 + \exp(-\sin x))u = -5 - (\sin x)^2$$

$$u'(0) = 2.5,\ u(2) = 0.5$$

. The change was given to us in the homework problem so we only had to implement it in matlab. Below is a plot of numerical $u(x)$ vs x.



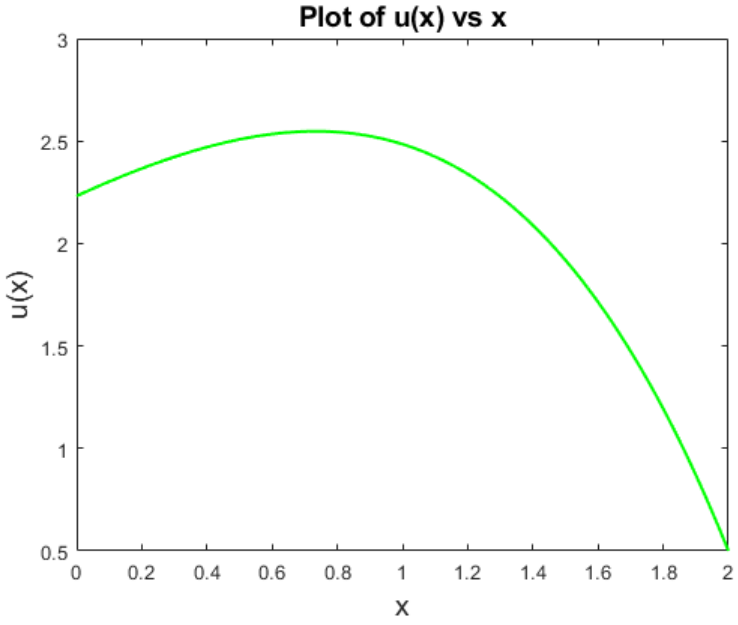
Problem 6

Problem 6 had us edit FDM similar to problem 5 to solve the following BVP

$$u'' - (1 + \exp(-\sin x))u = -5 - (\sin x)^2$$

$$u(0) - u'(0) = 1.5,\ u(2) = 0.5$$

. The deivation of the discretization for FDM is attached at the end of this report. Below is a plot of numerical $u(x)$ vs x.


Plot of u(x) vs x
u(x)
x
3
2.5
2
1.5
1
0.5
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2