Min-Max Optimization in GANs and Convergent Optimization Algorithms

Research Proposal for Numerical Algorithm

Spring 2025

Abstract

Here we look at numerical algorithm called gradient ascent and gradient descent as in generative models like GAN. GANs have become one of the most popular generative models for image generation.

1 Timelines

1. Make sure that you selected this project and mentioned your name in sheet. Link: Google Sheet Numerical Algorithms
2. ChatGPT or any other coding assistant is allowed as much as you want!
3. Read the material of the project selected, and run the basic starter code, if it exists.
   - did you understand the code? if not, then read it again!
4. Phase-1 evaluation on 25/26 March. Did you mention this in sheet? If not, then do it!
   - Done with evaluation? Was something suggested during evaluation? Did you clearly figured out? If not, then contact instructor for clarification!
5. Project objectives are well defined! Continue working.
6. Phase-2 evaluation 10 April. Are you doing OK?
   - You are at a stage where you are midway, setup baselines, and entered research phase.
7. Final evaluation 7th or 8th May
   - Introduce your work, objectives, and your outcomes. Time 15 minutes. Keep the slides clear, one line per item. Use overleaf and beamer! Select presentation format: Overleaf presentation template. Then click on open as template. This creats your own copy.
8. Most projects are doable on google colab. Link: Google Colab. If your code requires longer runs, then do a checkpoint to save the intermediate data, then run it again.
9. Submission: Report 5 pages, codes.
10. Do you need more GPU resources? Let us know ASAP. Dom kae sure that you have a base code running on colab for small dataset.

2 Introduction

Generative Adversarial Networks (GANs) Wiki: GAN have gained immense popularity in deep learning due to their ability to generate realistic data. The training process of GANs is inherently formulated as a min-max optimization problem, where a generator and a discriminator compete against each other. However, solving min-max problems (Min-Max-1, Min-Max-2) efficiently is challenging, especially in high-dimensional spaces. Another set of challenges are when we work on manifolds, see this advanced work here (my work) Min-Max on manifold.

However, for this project, we look at simpler work. The recent work by Keswani et al. [1] presents a dimension-independent and convergent optimization method for min-max problems, which can significantly impact GAN training and other adversarial learning settings. Your goal is to have algorithmic understanding, they gave algorithm in paper.

Min-Max shows up in GANs, you work will be only to replace the min-max solver, no need to understand the GANs. For that, you can have a lok at codes here.

3 Starter Codes

The background readings are be done from the following

- Coursera GAN course, Coursera GAN Course

4 Min-Max Optimization in GANs

GANs are defined by a two-player game

$$\min_G \max_D \quad \mathbb{E}_{x\sim p_{\text{data}}}[\log D(x)] + \mathbb{E}_{z\sim p_z}[\log(1 - D(G(z)))] \tag{1}$$

where

- G is the generator trying to synthesize data indistinguishable from real data.
- D is the discriminator trying to distinguish between real and fake data.
- The loss function represents a min-max game between G and D.

The optimization problem is challenging because standard gradient-based methods often fail due to oscillations and non-convergence.

5 Gradient Ascent-Descent Method

A common approach to solving min-max problems is gradient ascent-descent (GDA)

$$\theta_{t+1} = \theta_t - \eta \nabla_\theta f(\theta_t, \phi_t), \quad \phi_{t+1} = \phi_t + \eta \nabla_\phi f(\theta_t, \phi_t) \tag{2}$$

where:

- θ represents parameters of the generator.
- ϕ represents parameters of the discriminator.
- η is the learning rate.

This method suffers from slow convergence, instability, and dimensionality dependence.

6 Convergent and Dimension-Independent Min-Max Optimization

The paper by Keswani et al. [1] introduces a min-max optimization algorithm that improves convergence in high-dimensional settings by:

- Introducing a new momentum-based update rule to stabilize GDA.
- Ensuring dimension-independence, making it robust for deep learning tasks.
- Proving theoretical convergence guarantees even in adversarial settings.

This approach can be particularly useful for stabilizing GAN training, avoiding mode collapse, and ensuring consistent learning dynamics.

7 Research Directions

Based on the insights from the paper, we propose several research directions:

7.1 Applying Dimension-Independent Min-Max Optimization to GANs

- Implement the proposed algorithm of [1] in standard GAN architectures (e.g., DCGAN, StyleGAN) using Courera GAN codes.
- Compare convergence speed and stability against standard optimizers (e.g., Adam, RMSProp).
- Evaluate improvements in generated sample quality using FID and Inception Score.

7.2 Extending to Other Adversarial Learning Problems

- Apply the optimization method [1] to adversarial training in robust machine learning.
- Test the algorithm on reinforcement learning problems involving adversarial agents.
- Study its impact on meta-learning and generative modeling beyond GANs.

8 Conclusion

Min-max optimization is fundamental to training GANs, yet traditional methods face convergence issues. The recent work on dimension-independent optimization presents promising theoretical and practical improvements. Future research can focus on applying these techniques to adversarial deep learning, improving GAN training, and extending optimization strategies to broader machine learning applications.

References

[1] V. Keswani, O. Mangoubi, S. Sachdeva, N. K. Vishnoi, "A Convergent and Dimension-Independent Min-Max Optimization Algorithm," 2023.