

Solutions to Quiz-2 (Group-A)

IEC102

Q1) Consider the circuit shown in Fig. Q1. The switch has been in position-1 for a very long time and is moved to position-2 at time  $t=0$ . Find  $i(t)$  for  $t>0$ . (Assume that the circuit is in steady state at  $t=0^-$ )

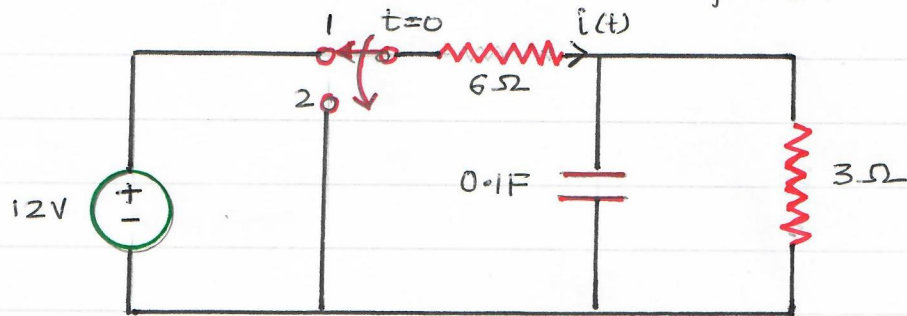
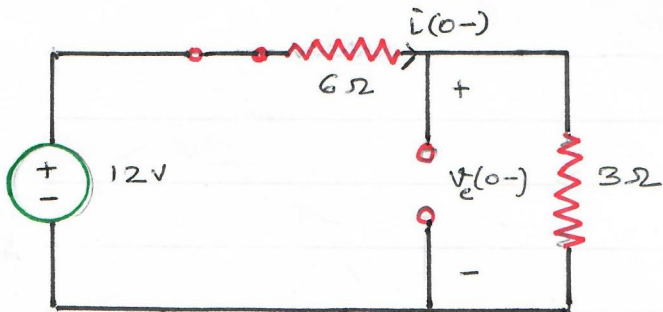


Fig. Q1

Sol.

Circuit at ~~start~~  $t=0^-$  (circuit is in steady state)

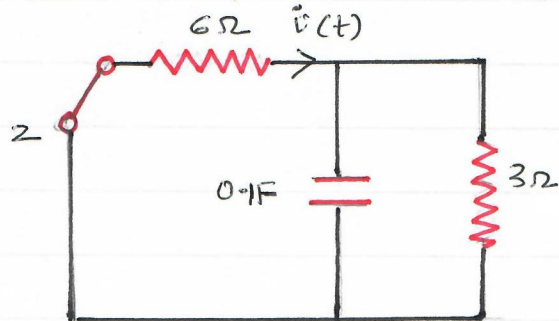


$$V_c(0^-) = 12 \times \frac{3}{6+3} = 12 \times \frac{3}{9} = 4V$$

$$= V_c(0) = V_c(0^+)$$

$$i(0^-) = \frac{12}{6+3} = \frac{4}{3} A$$

Circuit at  $t=0$



$$V_c(t) = A e^{-t/\tau}$$

$$\text{where } A = V_c(0) = 4V$$

$$\tau = R_{eq}C \quad R_{eq} = \frac{6 \times 3}{6+3} = 2\Omega$$

$$C = 0.1F$$

$$\tau = \frac{2 \times 0.1}{1} = 0.2s$$

$$\therefore V_c(t) = A e^{-t/\tau} = 4 e^{-5t}$$

$$-i(t) + C \frac{dV_c}{dt} + \frac{V_c}{3} = 0 \Rightarrow i(t) = C \frac{dV_c}{dt} + \frac{V_c}{3}$$

$$= 0.1 \times -20 e^{-5t} + \frac{4}{3} e^{-5t}$$

$$= -2 e^{-5t} + \frac{4}{3} e^{-5t} = -\frac{2}{3} e^{-5t}$$

Q2 If the switch in the network shown in Fig. Q2 closes at  $t=0$ , find  $V_o(t)$  for  $t>0$ . Assume that the circuit is in steady state at  $t=0^-$

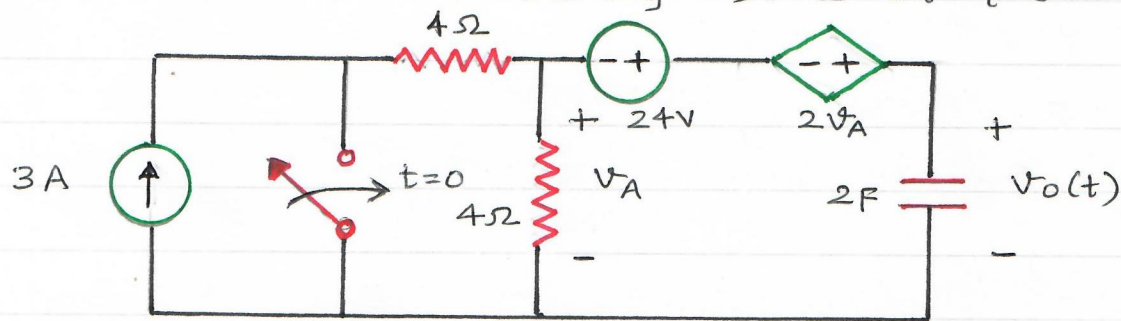
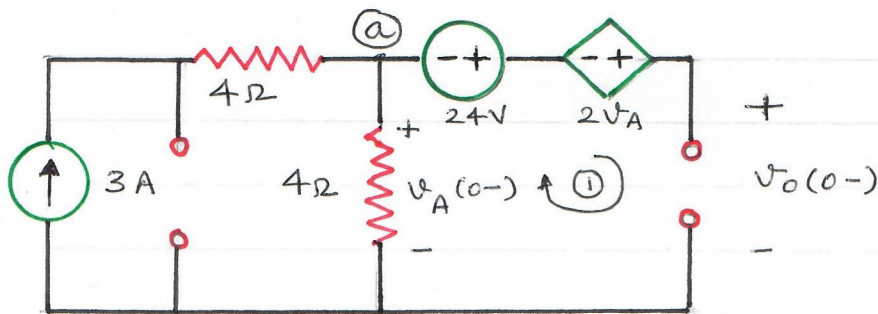


Fig. Q2

Sol.

The circuit at  $t=0^-$



Applying KCL at node (a)

$$-3 + \frac{V_A(0^-)}{4} = 0 \Rightarrow V_A(0^-) = 12V$$

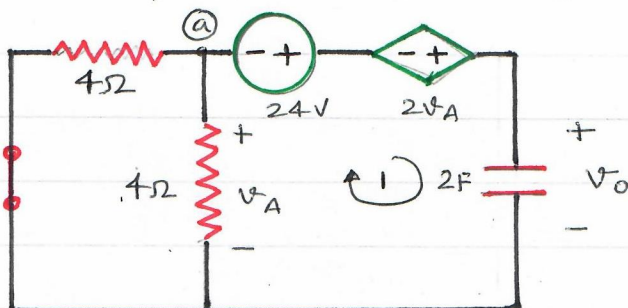
Applying KVL around loop (1)

$$-V_A(0^-) - 24 - 2V_A(0^-) + V_o(0^-) = 0$$

$$\Rightarrow V_o(0^-) = 3V_A(0^-) + 24 = 3 \times 12 + 24 = 36 + 24 = 60V$$

$V_o(0^-) = V_o(0) = V_o(0^+) = 60V$  ( $\because$  since it is the voltage across capacitor)

At  $t=0$ , the switch is closed and the circuit is



Applying KCL at node 'a'

$$\frac{V_A}{4} + \frac{V_A}{4} + 2 \frac{dV_o}{dt} = 0$$

$$\Rightarrow \frac{V_A}{2} = -2 \frac{dV_o}{dt}$$

$$V_A = -4 \frac{dV_o}{dt}$$

Applying KVL around loop ①

$$-V_A - 24 - 2V_A + V_0 = 0$$

$$\Rightarrow V_0 = 3V_A + 24$$

$$\Rightarrow V_0 = 3 \times -4 \frac{dV_0}{dt} + 24$$

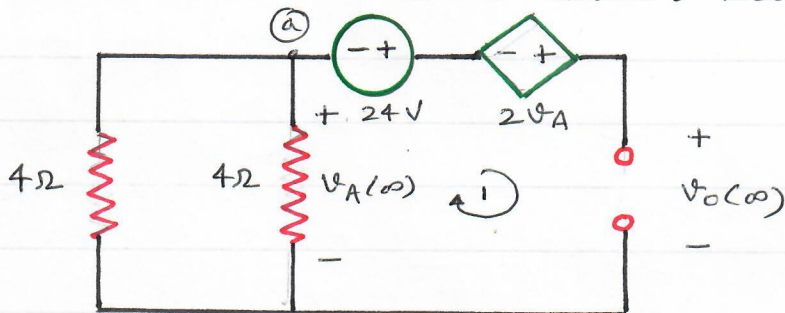
$$\Rightarrow 12 \frac{dV_0}{dt} + V_0 = 24$$

$$\Rightarrow \boxed{\frac{dV_0}{dt} + \frac{1}{12} V_0 = 2}$$

$$\therefore V_0(t) = K + A e^{-\frac{t}{12}}$$

$$V_0(\infty) = K$$

Circuit at  $t = \infty$  (the circuit will be in steady state)



Applying KCL at node (a)

$$\frac{V_A(\infty)}{4} + \frac{V_A(\infty)}{4} = 0$$

$$\Rightarrow V_A(\infty) = 0$$

Applying KVL around loop ①

$$-V_A(\infty) - 24 - 2V_A + V_0(\infty) = 0$$

$$\Rightarrow V_0(\infty) = 24 \text{ V}$$

$$V_0(\infty) = K = 24$$

$$\therefore V_0(t) = 24 + A e^{-\frac{t}{12}}$$

$$V_0(0) = 60 = 24 + A \Rightarrow A = 36$$

$$\therefore \boxed{V_0(t) = 24 + 36 e^{-\frac{t}{12}}}$$



Q3) Find  $V_C(t)$  for  $t > 0$  in the circuit shown in Fig. Q3  
 Given that the circuit is in steady state at  $t = 0^-$

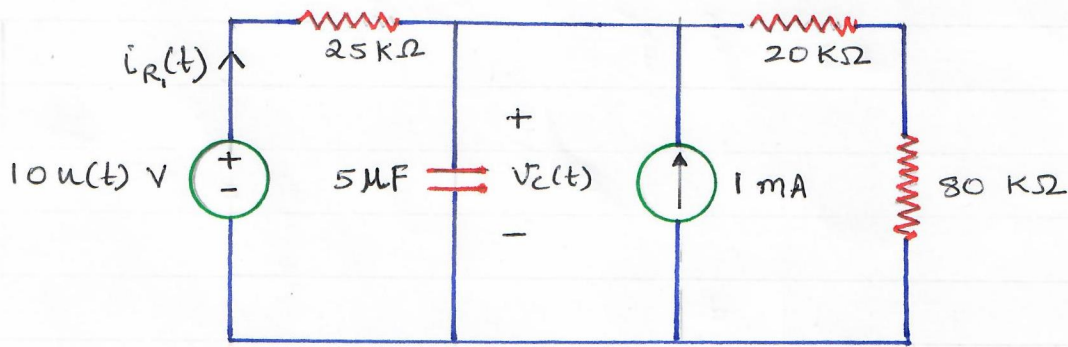
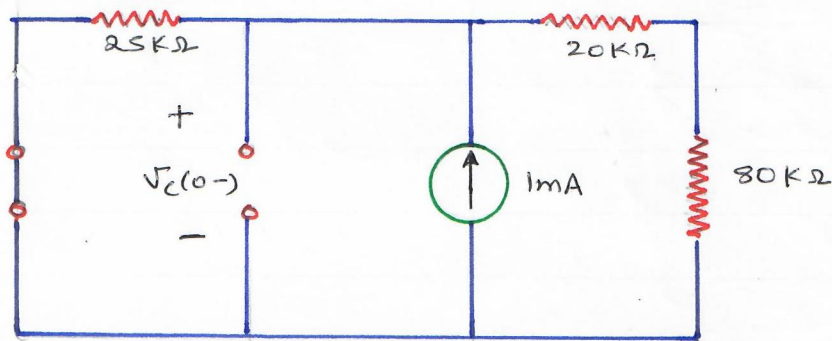


Fig. Q3

Sol. Lecture-07

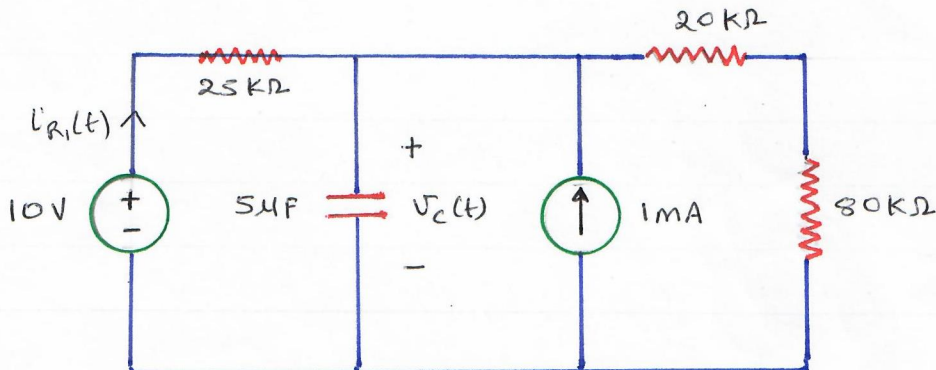
Circuit at  $t = 0^-$

Since the circuit is in steady state, capacitor acts as open circuit. and the voltage source  $10u(t) = 0$



$$V_C(0^-) = \frac{100K \times 10^{-3} \times 25K}{125K} = 20V \quad V_C(0^-) = V_C(0^+)$$

Circuit for  $t > 0$



$$V_C(t) = V_C(\infty) + (V_C(0) - V_C(\infty)) e^{-t/\tau}$$

where

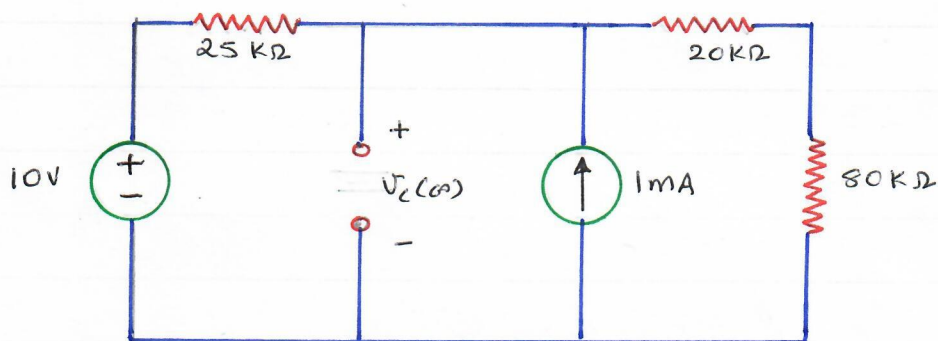
$$\tau = R_{eq} C$$

$$\begin{aligned}
 R_{eq} &= 25\text{K}\Omega \parallel (20+80)\text{K}\Omega \\
 &= 25\text{K}\Omega \parallel 100\text{K}\Omega \\
 &= 20\text{K}\Omega
 \end{aligned}$$

$$\begin{aligned}
 \therefore \tau &= R_{eq}C = 20 \times 10^3 \times 5 \times 10^{-6} \\
 &= 100 \times 10^{-3} = 0.1 \text{ sec}
 \end{aligned}$$

$$\begin{aligned}
 \therefore V_c(t) &= V_c(\infty) + (V_c(0) - V_c(\infty))e^{-t/0.1} \\
 &= V_c(\infty) + (V_c(0) - V_c(\infty))e^{-10t}
 \end{aligned}$$

circuit at  $t = \infty$  (the circuit will be in steady state)



$$\begin{aligned}
 V_c(\infty) &= \left(10 - \frac{10}{125\text{K}} \times 25\text{K}\right) + \frac{100\text{K} \times 10^{-3} \times 25\text{K}}{125\text{K}} \\
 &= 8 + 20 \\
 &= 28\text{V}
 \end{aligned}$$

$$\begin{aligned}
 \therefore V_c(t) &= 28 + (20 - 28)e^{-10t} \text{ V} \\
 &= 28 - 8e^{-10t} \text{ V} \quad \text{for } t \geq 0
 \end{aligned}$$

$$\begin{aligned}
 V_{R_1}(t) &= \frac{10 - V_c(t)}{25\text{K}} = \frac{10 - (28 - 8e^{-10t})}{25} \text{ mA} \\
 &= \frac{-18 + 8e^{-10t}}{25} = \frac{2}{25}(-9 + 4e^{-10t}) \text{ mA} \quad \text{for } t \geq 0
 \end{aligned}$$