Performance modeling for Computer systems
(Endsem)

Instructions:

- There are 2 parts to the exam, Part A and Part B.
- Part B is to be answered by only those students who have missed quiz/midsem.
- Part A is for 90 mins, no extra time.
- Additional 30 mins for students answering part B.

1 Part A: Each question is for 6 marks

1. Consider an amusement park where customers arrive according to a Poisson process with rate 5. Suppose the entry ticket costs Rs 2500. Let $R(t)$ denote the amount earned by the ticket seller by time t. Then find the revenue rate defined by $\lim_{t\to\infty} \frac{E[R(t)]}{t}$.

2. Consider an $M/M/2/2$ system where jobs arrive according to a Poisson process with rate λ. The two servers have different speeds μ_1 and μ_2 and service time at server i is exponential with rate $\mu_i, i = 1, 2$. When both servers are free, an arriving job chooses one of the servers uniformly at random. Model this as a Markov chain, obtain its stationary distribution. Also draw the transition diagram.

3. Consider X_n to be i.i.d random variables such that $P(X_n = 1) = 0.5$ and $P(X_n = 2) = 0.5$. Let $N = min\{n : X_1 + X_2 + \ldots + X_n = 10\}$. Show that N is a stopping time. Use Wald's lemma to obtain $E[N]$.

4. For an $M/M/1$ queue, let random variable S denote the steady-state (at stationarity) time spent in the system by an arriving customer. Derive the distribution of S.

5. State the criteria for classifying state i of a DTMC as recurrent (null and positive) and transient based on $F_{ii}, \mu_{ii}, f_{ii}^n$ and $\sum_n p_{ii}^n$.

2 Part B: Each question is for 5 marks

1. Consider a Poisson process with rate λ. Derive an expression for the renewal function $m(t)$ and its Laplace transform $\bar{m}(s)$.

2. For a CTMC, prove the Chapman Kolmogorov equation that $P(t+l) = P(t)P(l)$. Further show that $P(l) = P(t)P(l-t)$ for $0 < t < l$.