

INTERNATIONAL INSTITUTE OF INFORMATION TECHNOLOGY-HYDERABAD

MID I, Monsoon 2018

TIME: 1.5 hrs

Maths-III

Marks: 50

Instructions

- Answer all Questions
- No Formula sheet is allowed

1. Estimate the bias in a pH meter. Data are collected on the meter by measuring the pH of a neutral substance (pH = 7). A sample of size 10 is taken with results given by 7.07, 7.00, 7.10, 6.97, 7.00, 7.03, 7.01, 7.01, 6.98, 7.08. [2]

2. If the probabilities that an automobile mechanic will service 3, 4, 5, 6, 7, or 8 or more cars on any given workday are, respectively, 0.012, 0.19, 0.28, 0.24, 0.10, and 0.07, what is the probability that he will service at least 5 cars on his next day at work? [2]

3. A coin is biased so that a head is twice as likely to occur as a tail. If the coin is tossed 3 times, what is the probability of getting 2 tails and 1 head? [3]

4. In a certain assembly plant, three machines, B1, B2 and B3, make 30%, 45%, and 25%, respectively, of the products. It is known from the past experience that 2%, 3% and 2% of the products made by each machine, respectively, are defective. Now suppose that a finished product is randomly selected. What is the probability that it is defective? [3]

5. The probability density function is given as follows:

$$f(x) = ax^2, 0 \leq x \leq 5, f(x) = 0, \text{ otherwise}$$

(a) What is the probability that a value selected at random from this distribution will be less than 2? [2]

(b) What is the probability if the random variable is in between 1 and 3? [1]

(c) What is the probability that the random variable will be larger than or equal to 4? [1]

(d) What is the probability that the random variable exceeds 6? [1]

6. Given the joint density function

$$f(x, y) = \begin{cases} \frac{x(1+3y^2)}{4} & 0 < x < 2, 0 < y < 1 \\ 0 & \text{Otherwise} \end{cases}$$

(a) Find marginal density function, g(x) [2]

(b) Find marginal density function, h(y) [2]

$$\int_0^1 \frac{x}{4} (1+3y^2) dy = \int_0^1 \left(\frac{x}{4} + \frac{3xy^2}{4} \right) dy = \frac{x}{4} + \frac{3x}{4} \cdot \frac{y^3}{3} \Big|_0^1 = \frac{x}{4} + \frac{x}{4} = \frac{2x}{4} = \frac{x}{2}$$

$$\int_0^2 \frac{x}{4} (1+3y^2) dx = \int_0^2 \frac{x}{4} dx + \int_0^2 \frac{3xy^2}{4} dx = \frac{1}{4} \cdot \frac{x^2}{2} \Big|_0^2 + \frac{3y^2}{4} \int_0^2 x dx = \frac{1}{4} (4) + \frac{3y^2}{4} (4) = \frac{12y^2+4}{4} = \frac{12}{4} \left(\frac{1}{2} \right) + \frac{4}{4} \left(\frac{1}{2} \right)$$

- (e) Find the conditional probability density function, $f(x/y)$ [2]
 (d) Find $P(\frac{1}{4} < X < \frac{1}{2} | Y = \frac{1}{3})$ [2]
 (e) Test the statistical dependence of random variables X and Y. [2]
 (f) Find the expected value of X, $E(X)$ [2]
 (g) Find the expected value of Y, $E(Y)$ [2]
 (h) Find the expected value of X and Y as $E(XY)$ [2]
 7. Suppose that the shelf life, in years, of a certain perishable food product package in cardboard containers is a random variable whose probability density function is given by:

$$f(x, y) = \begin{cases} e^{-x} > 0 \\ 0 & \text{Otherwise} \end{cases}$$

Let X_1 , X_2 , and X_3 represent the shelf lives for three of these containers selected independently, find $P(X_1 < 2, 1 < X_2 < 3, X_3 > 2)$. [4]

8. A fair coin is tossed 6 times; consider heads as success. Find

- (a) The probability that exactly two heads occur [3]
 (b) The probability of getting at least four heads [3]
 (c) The probability of getting no heads [2]

9. A random variable X has mean 40 and standard deviation as 5. Find the value of b for which $P(40 - b \leq X \leq 40 + b) \geq 0.95$ [3]

10. A group of 10 individuals is used for a biological case study. The group contains 3 people with blood type O, 4 with blood type A, and 3 with blood type B. What is the probability that a random sample of 5 will contain 1 person with blood type O, 2 people with blood type A, and 2 people with blood type B. [4]

x_1, x_2, \dots, x_n

$$E(X) = \int_0^{\infty} x \cdot f(x) dx \geq \int_a^{\infty} x \cdot f(x) dx \geq \int_a^{\infty} a \cdot f(x) dx$$

$$E(X) \geq a \cdot \int_a^{\infty} f(x) dx \quad x = x_i \quad g(x_j) = \int f(x, y) dy$$

$$\begin{aligned} \sigma^2 &= E(X^2) - [E(X)]^2 \\ &= E(X^2) - a^2 \int_a^{\infty} f(x) dx \\ &= E(X^2) - \mu^2 \\ &= E(X^2 + \mu^2 - 2\mu X) \\ &= E(X^2) + E(\mu^2) - 2\mu E(X) \\ &= E(X^2) + \mu^2 - 2\mu^2 \\ &= E(X^2) - \mu^2 \end{aligned}$$

INTERNATIONAL INSTITUTE OF INFORMATION TECHNOLOGY-HYDERABAD
MID II, Monsoon 2018
TIME: 1.5 hrs
Maths-III
Marks: 30

Instructions

- Answer all Questions
- No Formula sheet is allowed

Part - I (Complex Numbers)

1. Find x, y if $\frac{x}{1+i} + \frac{y}{2-i} = 2+4i$ [2.5]
2. Find the roots of $z^2 - (1-i)z + 7i - 4 = 0$ in the form $a+ib$ [2.5]
3. Show that $\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2) \pm 2\pi$ [2.5]
4. Find the modulus and argument of $z = (2-i)(1+3i)$ [2.5]

Part - II (Probability and Statistics)**II. Choose the Correct Option**

1. In a symmetric random walk of flipping of an unbiased coin, determine the probability of the particle taking the value of 1 in $n = 3$ trials. [2.5]

(a) $\frac{15}{8}$ (b) $\frac{8}{15}$ (c) $\frac{1}{3}$ (d) 15



2. Two random process $X(t)$ and $Y(t)$ are given as follows:

$$X(t) = A \sin(\omega t + \phi_1) \quad Y(t) = B \cos(\omega t + \phi_2)$$

The normalized cross-covariance is

(a) 1 (b) 0 (c) C_{XY} (d) ρ_{AB} [2.5]

3. If X and Y are two random variables with PDF as $f(X, Y) = \frac{1}{14} \left(5 - \frac{y}{2} - x\right)$ with $0 \leq x \leq 2$ and $0 \leq y \leq 2$.

- (i) The PDF $g(u, v)$, where $u = x + y$ and $v = \frac{y}{2}$ is [2.5]

(a) $g(u, v) = \frac{1}{7} \left(5 - \frac{y}{2} - x\right)$

(b) $g(u, v) = \frac{1}{7} (5 - v - u)$

$$\frac{1}{5} + \frac{1}{3}$$

$$2^2 - 2 - 4$$



$$36 \cdot \left(\frac{1}{2}\right)^3$$

$$\left(\frac{3}{8}\right)$$

$$\sin a \sin b = \frac{1}{2} [\cos(a+b) + \cos(a-b)]$$

$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{8}$
-3	-2	-1	1	2
$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$
6	1	2	3	



HTT
THT

(c) $g(u, v) = \frac{1}{7}(5 + v - u)$

(d) $g(u, v) = \frac{1}{14}(5 - v - u)$

(ii) The limits of u and v are given as

[1.0]

(a) $v \leq u \leq 2 + v$ and $0 \leq v \leq 2$

(b) $v \leq u \leq 2 + 2v$ and $0 \leq v \leq 2u$

(c) $2v \leq u \leq 2v + 2$ and $2 \leq v \leq u$

(d) $2v \leq u \leq 2v + 2$ and $0 \leq v \leq 1$

4. The probability of getting no mistakes in a page of a book is e^{-4} . The probability that a page of a book contains more than 2 mistakes is

[2.5]

(a) $1 - e^{-4}$ (b) $1 - 2e^{-4}$ (c) $1 - 13e^{-4}$ (d) $13e^{-4}$

5. Fill in the blanks, each carry 1.5 mark

[1.5X6=9]

i) Condition for the first order stationarity.....

ii) If X and Y are two random variables with PDF, $f(x, y)$, the transformation to $g(u, v)$, where u and v are functions of x and y is given by

iii) If X follows binomial distribution, with large number of trials as probability is close to zero, the variable can follow distribution with mean as

iv) If \bar{x} is the mean of a random sample of size n , from a population with mean μ and variance as σ^2 . As $n \rightarrow \infty$ the standard normal random variable will follow a form of.....

v) If X is a random variable following exponential distribution, the mean and variance are.....

vi) If x follows normal distribution with mean μ and standard deviation as σ , then 99.7% of the probability will be in the interval of $[\mu - 3\sigma, \mu + 3\sigma]$

Swarg

(18)

INTERNATIONAL INSTITUTE OF INFORMATION TECHNOLOGY-HYDERABAD

End Sem, Monsoon 2018

TIME: 3 hrs

Maths-III

Marks: 100

SH

SH

Probability and Statistics: Shaik Rehana

Maximum Marks: 60

Note:

1. All questions are compulsory
2. Notations have usual significance
3. Calculators are not allowed

Section A:

Each question carries 5 marks

[5X6=30]

1. Briefly explain Bayes theorem
2. Chebshev inequality
3. Central limit theorem
4. Random walk
5. Statistical dependence
6. Hypergeometric distribution

Section B:

1. There are two streams flowing, let A be the event that stream a is polluted, and B is the event that stream b is polluted. In a given day $P(A) = 2/5$ and $P(B) = 3/4$. The probability that at least one stream will be polluted in any given day is $4/5$. Determine

(a) Probability that stream a is also polluted given that stream b is polluted [2.5]

(b) Probability that stream b is also polluted given that stream a is polluted [2.5]

2. A box contain 25 strain gages, and 4 of them are known to be defective gages. If 6 gages were used in an experiment, what is the probability that there was one defective gages in the experiment? [5]

3. The height of earth dam must allow sufficient freeboard above the maximum reservoir level to prevent waves from washing over the top. The wind tide, in feet, above still-water level is

$$Z = \frac{F}{1400 d} V^2$$

Handwritten notes:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$
$$P(A|B) = \frac{P(A \cap B)}{P(A)}$$

Where V = wind speed in miles per hour; F = length of water surface over which the wind blows, in feet; d = average depth of lake along the length, in feet. If wind speed follows exponential distribution with mean speed v_0 , then determine the distribution of the tide Z . [10]

4. A fair coin is tossed and if heads come up, a sine wave $x_1(t) = \sin(5\pi t)$ is sent. If tails come up, then $x_2(t) = t$. The resulting random process $x(t)$ is an ensemble of $x_1(t)$. Find the mean and variance of the random process, for $t = 0, 0.5, 0.7$. [5]

5. A population consists of set, $S = \{4, 7, 10\}$ in equiprobable space. Random samples of size 2 are drawn with replacement.

(a) Compute the population mean and standard deviation. [2.5]

(b) Find the sampling distribution for the sample mean. [2.5]

$$n^k - \frac{n!}{(n-r)!r!}$$

Complex Analysis

Indranil Chakrabarty

November 15, 2018

Maximum marks: 40

Note:

1. All questions are compulsory.
2. Notations have usual significance

SECTION A:

Each question carries 5 marks.

1. Solve $\cos(z) = 1/2$ for z
2. For complex numbers z_i, w_i ($i = 1..n$) show that $|z_1 w_1 + \dots + z_n w_n| \leq \sqrt{|z_1|^2 + \dots + |z_n|^2} \sqrt{|w_1|^2 + \dots + |w_n|^2}$
3. $\int_C \frac{\sin(\pi z^2) + \cos(\pi z^2)}{(z-1)(z-2)} dz$ where C is the circle: $|z-i| = 3$
4. Find the harmonic conjugates of the following harmonic functions on C (Complex numbers): a) $u(x, y) = x^2 - y^2$ b) $u(x, y) = \sin x \cosh y$
5. Suppose f is an analytic function on a region (an open connected set) A and that $|f(z)|$ is constant on A . Show that f is constant on A .
6. Find the expansion of e^z around the point $i\pi$.

SECTION B:

Each question carries 5 marks.

1. Let $f : A \rightarrow C$ and $g : B \rightarrow C$ be analytic (A, B are open sets) and let $f(A) \subset B$. Then show that $g \circ f : A \rightarrow C$ defined by $(g \circ f)(z) = g(f(z))$ is analytic and $\frac{d}{dz}(g \circ f)(z) = g'(f(z)) \cdot f'(z)$. (where C is the set of all Complex numbers)
2. Consider the integration of the function e^{-z^2} around the rectangular contour Γ with vertices $\pm a, \pm a + ib$ and oriented positively. By letting $a \rightarrow \infty$ while keeping b fixed, show that $\int_{-\infty}^{\infty} e^{-x^2} e^{\pm 2i b x} dx = \int_{-\infty}^{\infty} e^{-x^2} \cos(2bx) dx = e^{-b^2} \sqrt{\pi}$