Differential Equations quiz 1

Version 1

1.

A thermometer is removed from a room where the temperature is 70° F and is taken outside, where the air temperature is 10° F. After one-half minute the thermometer reads 50° F. What is the reading of the thermometer at $t = 1$ min? How long will it take for the thermometer to reach 15° F?

2.

Two large containers A and B of the same size are filled with different fluids. The fluids in containers A and B are maintained at 0° C and 100° C, respectively. A small metal bar, whose initial temperature is 100° C, is lowered into container A. After 1 minute the temperature of the bar is 90° C. After 2 minutes the bar is removed and instantly transferred to the other container. After 1 minute in container B the temperature of the bar rises 10°. How long, measured from the start of the entire process, will it take the bar to reach 99.9° C?

3.

Constant-Harvest Model A model that describes the population of a fishery in which harvesting takes place at a constant rate is given by

$$\frac{dP}{dt} = kP - h,$$

where k and h are positive constants.

(a) Solve the DE subject to $P(0) = P_0$.

(b) Describe the behavior of the population $P(t)$ for increasing time in the three cases $P_0 > h/k$, $P_0 = h/k$, and $0 < P_0 < h/k$.

(c) Use the results from part (b) to determine whether the fish population will ever go extinct in finite time, that is, whether there exists a time $T > 0$ such that $P(T) = 0$. If the population goes extinct, then find T.

4. Consider the initial value problem y'=ty(4−y)/3,y(0)=y0.(a) Determine how the behavior of the solution as t increases depends on the initial value y0.(b) Suppose that y0=0.5. Find the time T at which the solution first reaches the value 3.98

5. The value of y0 for which the solution of the equation y’ – y = 1 + 3sin(t), y(0) = y0 remains finite as t -> inf is?

6. A population of species satisfies y’ = (0.5 + sin(t))*(y/5). If y(0) = 1. The time T at which the population has doubled is?

Version 2

1. Consider the set of solutions u1 and u2 which are a fundamental set of solutions for the differential equation u'' + q(t) u'+ p(t) u = 0 . Given that

$$u3 = p1u1 + p2u2$$
$$u4 = s1\,u1 + s2u2,$$

where s1,s2, p1, and p2 are any constants. Show that W [u3, u4] = (p1s2 - p2s1) W [u1, u2]. Do u3 and u4 also form a fundamental set of solutions? Why or why not?

2. Let u1 and u2 be two solutions of a differential equation y'' + p(t) y' + q(t) y = 0. If both these solutions have a point of inflection at the same point, say at t0 in an interval J , then show that they cannot be a fundamental set of solutions on J unless both p(t) and q(t) are zero at t0.

3. A model for the population P(t) in a suburb of a large city is given by the initial-value problem

$$dP/dt = P(0.1-(10^{-}7).P),\ P(0)=5000$$

where t is measured in months. What is the limiting value of the population? At what time will the population be equal to one-half of this limiting value?