Midsem: Probability and Statistics (50 Marks)

Instruction:

- Please state reasons wherever applicable.
- Use precise mathematical arguments, no speeches.

Each question: 6 marks

1. Consider a random variable X with the following pdf. Find mean and variance for X.
$$f_X(x) = \begin{cases} 0.5\lambda e^{-\lambda x}, & x \geq 0 \\ 0.5\lambda e^{\lambda x}, & x < 0 \end{cases}$$

2. Let X be a Uniform $U[0,1]$ random variable. Let $Y = e^{2X}$. FInd pdf and cdf of Y.

3. The joint probability mass function of the discrete random variables X and Y are given by $p_{X,Y}(x,y) = \frac{1}{2^{x+y}}$, $x = 1, 2., \ldots$ and $y = 1, 2, \ldots$.
   (a) Find the expression for the marginal pmf $p_X(x)$ and $p_Y(y)$.
   (b) Find $E[XY]$ and determine if X and Y are independent.

4. The joint pdf of random variables X and Y is given by $f_{X,Y}(x,y) = \lambda e^{-\lambda x - y}$, $x \geq 0, y \geq 0, \lambda > 0$.
   (a) Find the expressions for the marginal pdf's $f_X(x)$ and $f_Y(y)$
   (b) Find the joint cdf $F_{X,Y}(x,y)$. Are X and Y independent? Give reasons.

5. Let X, Y and Z be independent exponential random variables with parameters λ_1, λ_2 and λ_3. Let $W = min(X, Y, Z)$. Find the cdf and pdf of W.

Each question: 10 marks

1. Let $Y = aX^2 + b$ where X is a continuous random variable. Derive the expression for the CDF and pdf of Y in terms of the pdf of X.

2. Let X be a uniform random variable with support $[a, b]$, . Let Y be Poisson random variable with parameter λ. Derive the expression for the mean and variance of X and Y.