

POPL Monsoon 2018 Final Exam

POPL Instructors

2018-11-19 Mon

1 IF+/ language (60 points)

Refer to the IF+/ language specified at the end of this question paper.

1.1 Type safety (15 points)

State the type safety theorems (preservation and progress) for the the IF+/ language.

1.2 Language extension (45 points)

Extend the IF+/ language with a $<$ (denoting the less than operator on numbers) construct. Show how the following get extended: semantic domains, syntax, rewrite rules, reduction rules, argument type mismatch judgements and the type system.

2 Determinacy (60 points)

State and prove that the reduction rules of IF+/ form a deterministic transition system: i.e., for any expression e , e reduces to atmost one expression. [Hint: show $e \rightarrow e'$ and $e \rightarrow e''$ implies $e' = e''$.] State clearly the induction scheme employed. Also, clearly state the reason for each step in the proof. To save time, you may restrict your proof to rules that deal only with the numbers, addition, and the if construct in the syntax.



3 CPS Transformations (30 points)

Transform the following direct style programs into continuation passing style.

3.1 Hemachandra-Fibonacci function

```
(define (hem n)
  (cond
    [(= n 0) 0]
    [(= n 1) 1]
    [else (+ (hem (- n 1)) (hem (- n 2)))])])
```



3.2 Reduce

```
(define (reduce x0 f us)
  (if (null? us) x0
      (f (reduce x0 f (rest us))
          (first us))))
```

4 AST evaluation (30 points)

Assume the top-level continuation is the identity function. Assume, further, that +, sub1 and * are part of the language's syntax, i.e., keywords. Assume that the top-level environment contains closures corresponding to the following two functions:

```
(define (f n)
  (+ 4 (g (sub1 n))))
```

```
(define (g m)
  (* m 5))
```

Draw an AST annotation diagram illustrating the evaluation of the expression below, assuming the evaluation is done in continuation passing style:

```
(+ 3 (f 2))
```

```
(+ 3 (f 2))
```

Recap of the IF+/ language

Semantic domains and Syntax

$v ::=$

n

b

$n ::=$

$b ::=$

true

false

$+: [NUM, NUM] \rightarrow NUM$

$/: [NUM, NUM \setminus \{0\}] \rightarrow NUM$

Addition function

Division function

VAL Value

NUM_VAL Number Value

BOOL_VAL Boolean Value

NUM. Number

BOOL. Boolean

TRUE_VAL True value

FALSE_VAL False value

$e ::=$

\bar{n}

\bar{b}

$e \boxed{+} e$

$e \boxed{/} e$

if e e

$e \in EXP$

EXP

NUM_LIT

BOOL_LIT

PLUSEXP

DIVEXP

IFEXP

IF+/ Expression

Number Literal

Boolean literal

Plus Expression

Div Expression

If Expression

Values and Rewrite rules

$e \in VAL$

$\bar{n} \in VAL$ NUM

$\bar{b} \in VAL$ BOOL

$\hookrightarrow \subseteq EXP \times EXP$

$\bar{n}_1 \boxed{+} \bar{n}_2 \hookrightarrow \overline{n_1 + n_2}$ PLUS

$\bar{n}_1 \boxed{/} \bar{n}_2 \hookrightarrow \overline{n_1 / n_2}$ $n_2 \neq 0$ DIV

if $\overline{\text{true}}$ e $e' \hookrightarrow e$ IF-TRUE

if $\overline{\text{false}}$ e $e' \hookrightarrow e'$ IF-FALSE

Semantics: Reduction and Simplification

$$\rightarrow \subseteq EXP \times EXP$$

$$\frac{e \mapsto e'}{e \rightarrow e'} \text{ REWRITE}$$

$$\frac{e_1 \rightarrow e'_1}{\text{if } e_1 \ e_2 \ e_3 \rightarrow \text{if } e'_1 \ e_2 \ e_3} \text{ IF_TEST}$$

$$\rightarrow^* \subseteq EXP \times EXP$$

$$\frac{e_1 \rightarrow e'_1}{e_1 \boxed{+} e_2 \rightarrow e'_1 \boxed{+} e_2} \text{ P_LEFT}$$

$$\frac{e_1 \ \overline{VAL} \quad e_2 \rightarrow e'_2}{e_1 \boxed{+} e_2 \rightarrow e_1 \boxed{+} e'_2} \text{ P_RIGHT}$$

$$\overline{e \rightarrow^* e} \text{ REF}$$

$$\frac{e_1 \rightarrow e_2}{e_1 \rightarrow^* e_2} \text{ RED}$$

$$\frac{e_1 \rightarrow e_2 \quad e_2 \rightarrow^* e_3}{e_1 \rightarrow^* e_3} \text{ TRANS}$$

$$\frac{e_1 \rightarrow e'_1}{e_1 \boxed{/} e_2 \rightarrow e'_1 \boxed{/} e_2} \text{ D_LEFT}$$

$$\frac{e_1 \ \overline{VAL} \quad e_2 \rightarrow e'_2}{e_1 \boxed{/} e_2 \rightarrow e_1 \boxed{/} e'_2} \text{ D_RIGHT}$$

expressions stuck due to DIV/0

$$e \text{ DIV/0}$$

$$\overline{n_1 \boxed{/} 0} \text{ DIV/0} \text{ ZERO}$$

$$\frac{e_1 \text{ DIV/0}}{e_1 \boxed{+} e_2 \text{ DIV/0}} \text{ PLUS-LEFT}$$

$$\frac{e_1 \ \overline{VAL} \quad e_2 \text{ DIV/0}}{e_1 \boxed{+} e_2 \text{ DIV/0}} \text{ PLUS-RIGHT}$$

$$\frac{e_1 \text{ DIV/0}}{e_1 \boxed{/} e_2 \text{ DIV/0}} \text{ DIV-LEFT}$$

$$\frac{e_1 \ \overline{VAL} \quad e_2 \text{ DIV/0}}{e_1 \boxed{/} e_2 \text{ DIV/0}} \text{ DIV-RIGHT}$$

$$\frac{e_1 \text{ DIV/0}}{\text{if } e_1 \ e_2 \ e_3 \text{ DIV/0}} \text{ IF}$$

$e_1 \ll e_2$

2 ()
3 2 4

Expressions stuck due to Argument Type mismatch

$\frac{}{e \text{ ATM}}$	
$\frac{\overline{b_1} \boxed{+} \overline{n_2} \text{ ATM}}{\text{BOOL+NUM}}$	
$\frac{\overline{b_1} \boxed{+} \overline{b_2} \text{ ATM}}{\text{BOOL+BOOL}}$	
$\frac{\overline{n_1} \boxed{+} \overline{b_2} \text{ ATM}}{\text{NUM+BOOL}}$	
$\frac{\overline{b_1} \boxed{/} \overline{n_2} \text{ ATM}}{\text{BOOL/NUM}}$	
$\frac{\overline{b_1} \boxed{/} \overline{b_2} \text{ ATM}}{\text{BOOL/BOOL}}$	
$\frac{\overline{n_1} \boxed{/} \overline{b_2} \text{ ATM}}{\text{NUM/BOOL}}$	
$\frac{\frac{e_1 \text{ ATM}}{e_1 \boxed{+} e_2 \text{ ATM}}}{e_1 \text{ VAL } e_2 \text{ ATM}} \text{ PLUS-LEFT}$	
$\frac{\frac{e_1 \text{ VAL } e_2 \text{ ATM}}{e_1 \boxed{+} e_2 \text{ ATM}}}{e_1 \text{ VAL } e_2 \text{ ATM}} \text{ PLUS-RIGHT}$	
$\frac{\frac{e_1 \text{ ATM}}{e_1 \boxed{/} e_2 \text{ ATM}}}{e_1 \text{ VAL } e_2 \text{ ATM}} \text{ DIV-LEFT}$	
$\frac{\frac{e_1 \text{ VAL } e_2 \text{ ATM}}{e_1 \boxed{/} e_2 \text{ ATM}}}{e_1 \text{ VAL } e_2 \text{ ATM}} \text{ DIV-RIGHT}$	
$\frac{}{\text{if } \overline{n_1} \ e_2 \ e_3 \text{ ATM}} \text{ IF-NUM}$	
$\frac{\frac{e_1 \text{ ATM}}{\text{if } e_1 \ e_2 \ e_3 \text{ ATM}}}{\text{if } e_1 \ e_2 \ e_3 \text{ ATM}} \text{ IF-ATM}$	

Types: syntax and static semantics

$\tau \in \text{TYPE}$	$e : \tau \subseteq \text{EXP} \times \text{TYPE}$	$\frac{e_1 : \text{num } e_2 : \text{num}}{e_1 \boxed{-} e_2 \text{ num}} \text{ PLUS}$
$\tau ::=$		$\frac{e_1 : \text{num } e_2 : \text{num}}{e_1 \boxed{/} e_2 \text{ num}} \text{ DIV}$
num	NUMTYPE	Number Type
bool	BOOLTYPE	Boolean Type
		$\frac{e_1 : \text{bool } e_2 : \tau \ e_3 : \tau}{\text{if } e_1 \ e_2 \ e_3 : \tau} \text{ IF}$