Quiz-4
Data Structures

Set A

18th April — Time Allowed: **60** minutes

INSTRUCTIONS

1. This paper contains **Multiple choice questions** .
2. Marking Scheme

   Marking scheme format : (correct, unattempted, incorrect)

   MCQs (+4, 0, -1)

   True/False(+4, 0, -2)

   Fill-ups(+4, 0, 0)
3. Answers have to be written in the space provided besides the question .

Roll No : ______________________________

Seat No: ______________________________

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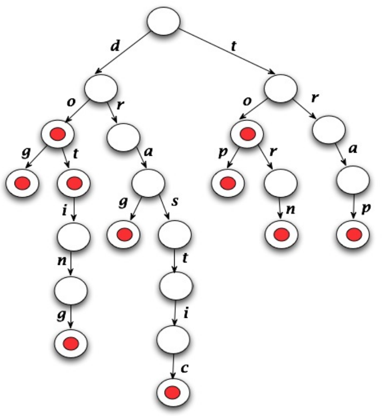
1. If all possible strings of length at most k (k > 0) having only lowercase English alphabet are inserted in a Trie, what is the total number of nodes in the Trie ?

   Ans : _____

2. Consider the following Trie :
   Lets denote the ending node of strings "drag" and "drastic" by A and B respectively. Which of the following strings when inserted in this Trie end on the node represented by LCA(A, B) ?

   (a) "dra"
   (b) "dr"
   (c) "d"
   (d) "drag"



3. Number of distinct strings in the Trie is always equal to the number of leaf nodes. State True / False
   Ans : ______

4. Inserting a string in a Trie is O(height of the Trie), where height of the Trie is maximum distance from Root node to any of the leaf nodes before insertion. State True / False.
   Ans : ______

5. Number of nodes at depth d (depth of root node == 0) is equal to the number of distinct characters at d-th position in all the strings (having length $\geq d$) that have been inserted in the Trie. State True / False.
   Ans : ______

6. Given a full binary tree of height H, and a node A at depth d (depth of root node == 0) , find out how many unordered pairs of nodes u, v (such that

u != v) exist such that LCA(u, v) == A ?
Ans : _____

7. Consider a tree, and its two nodes A and B , at depths d1 and d2 respectively (depth of root node == 0). Let LCA(A, B) = L and depth(L) = d3. Let S = { LCA(u, B) | u is an ancestor of A }. Assume a node is also an ancestor of itself. What is the cardinality of S ?

   (a) d3 + 1
   (b) d1 + d2 - d3 + 1
   (c) d1 + 1
   (d) d2 + 1

8. Consider the following pseudo-code of a function F that takes two arguments : nodes u, v that belong to a tree.

```
F(u, v):
       Initialize : l = 0, r = depth(v)
       while l <= r :
           d =  ( l + r ) / 2
           A = d-th ancestor of v
           if A is an ancestor of u :
            return A
           else :
            l = mid + 1
       return u
```

   Mark all the correct options:

   (a) F returns LCA(u, v) in some cases.
   (b) F returns LCA(u, v) in all cases.
   (c) F returns some ancestor of LCA(u, v) in all cases.
   (d) F returns ancestor of u in all cases.

9. If the first k nodes on the path from Root to node u are same as the first k nodes on the path from Root to node v, in a given Tree, the k-th node in any of these paths (from root to u or v) is always the LCA of u and v. State True / False
   Ans : _____

10. Given a tree, and two nodes u and v belonging to it. Any node A present in the path from u to v which satisfies the property, that both u and v are present in the subtree of A is LCA of u and v. State True / False.
    Ans : _____

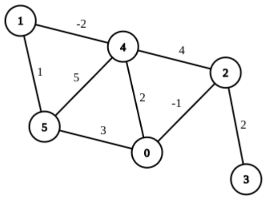
11. Consider a tree having N nodes and N - 1 edges ($N \geq 2$). Let us define a function F(u, v, R) which returns the LCA(u, v) when the tree is rooted at R. Given two nodes A and B (A != B), now consider the set S = { F(A, B, R) | R is any node in the tree }. What is the cardinality of S ?

    (a) N
    (b) 1
    (c) N/2
    (d) distance(A, B) + 1

12. Given a rooted tree, consider nodes A, B, C, D. Let E = LCA(A, B) and F = LCA(C, D) and G = LCA(E, F). Assume nodes A, B, C, D, E, F, G are all distinct.
    Mark all correct options :

    (a) LCA(A, C) == G
    (b) LCA(B, D) == G
    (c) LCA(A, C) == E
    (d) LCA(A, G) == G

13. Consider the following graph :



Mark all the correct statements for the given graph :

(a) Dijkstra's Algorithm can be used to find the single source shortest path from source vertex 0.

(b) Bellman Ford Algorithm can be used to find the single source shortest path from source vertex 0.

(c) Floyd Warshall Algorithm can be used to find all pair shortest path.

(d) None of these.

14. For a directed graph having V vertices and having no negative cycles, if instead of running the outer loop of Bellman Ford Algorithm for V - 1 iterations, it is run for V iterations, the distances computed by the algorithm may be wrong. State True / False.
Ans : _____

15.

```
DAG-BELLMAN-FORD(V, E, s):
   1. TOPOLOGICALLYSORT(V, E)
   2. INITIALIZE(V, E)
   3. for i in 1..|V|-1:
   4.    for each vertex u in V taken in topological order :
   5.      for each edge originating at u : (u, v) in E :
   6.         RELAX(u, v)
   7. if no edge was relaxed :
   8.    break
```

For an input graph which is a DAG, to the above function, how many times does the loop in line number 3 run ?
Mark all the correct options :

(a) 1

(b) 2
(c) $|V| - 1$
(d) Can't say, depends on the input graph.

16. Consider a connected undirected graph G(V, E) having all edge weights greater than 0. Dijkstra's algorithm is applied from source vertex s. Let d[i] store the shortest distance from s to vertex i. Let E' = { (u, v) | d[u] == d[v] + w(u, v) or d[v] == d[u] + w(u, v) }, such that (u, v) denotes the edge joining vertices u and v, and w(u, v) denote the edge weight of edge (u, v). Graph represented by G'(V, E') will always form a tree. State True / False.
Ans : _____

17. What is the worst case space complexity of Bellman Ford algorithm on an input Graph G(V, E) ?

    (a) $\theta(|V|)$
    (b) $\theta(|V| + |E|)$
    (c) $O(|V|)$
    (d) $O(|V| + |E|)$

    Note : We consider only the auxiliary space used for calculating the space complexity of an algorithm.

18. On applying Floyd Warshall Algorithm on a directed Graph G(V, E), it is found that $distance(u, u) < 0$ for some vertex u ∈ V. Mark all the correct options :

    (a) G has at least one edge with negative edge weight.
    (b) G has a negative cycle.
    (c) All edges going out of u have negative edge weights.
    (d) There exists an edge going out of u having negative edge weight.

19. Consider a connected undirected weighted graph G(V, E) with distinct edge weights, such that $|V|, |E| > 3$. Let S1, S2 be two non-empty mutually exclusive subsets of V, such that $S1 \cup S2 == V$.
Let E' = { (u, v) | (u, v) ∈ E and ((u ∈ S1 and v in ∈ S2) or (u ∈ S2 and v ∈ S1)) }
The claim that edge e = (u, v) such that $e \in E'$ and weight(e) is minimum, will belong to the Minimum Spanning Tree of G. State True / False.
Ans : _____

20. Given an undirected graph G(V, E), if you start the DFS from a node n $\in$ V, what is the worst complexity of the algorithm ?
Mark all the correct options :
    (a) $O(|V|)$
    (b) $O(|V| + |E|)$
    (c) $O(|V| * |E|)$
    (d) $O(|E|)$