CSE 411: Complexity and Advanced Algorithms
Monsoon 2019
IIIT Hyderabad

Mid Exam 1

Instructions:

- The paper is for a duration of 90 minutes and 50 maximum points.
- The paper is spread over TWO pages and FIVE problems. Please verify and make sure that your question paper is printed properly.
- For questions that ask for an algorithmic solution, points awarded will depend on the time complexity of the algorithm.
- No clarification shall be provided during the exam. You are free to make suitable assumptions if you need to, but the points awarded will depend on the nature of the assumptions.
- Answer all questions and avoid being verbose.

Problem I. Basic Knowledge. Answer each question with a brief justification. Each question carries **TWO** points.

1. Define what is meant by a time-constructible function. Show that the function $f(n) = n^2$ is time-constructible.
2. Define the class IP of languages.
3. When do we say that a language L_1 Cook reduces to another language L_2. How is this reduction different from a Turing reduction?
4. State Savitch's theorem. Will the theorem work for space functions that are $o(\log n)$. Justify your answer.
5. Which of the following statements are true. Justify briefly.
   a. NP = P^{poly}.
   b. NSPACE = PSPACE
   c. NL = co-NL
   d. co-NL $\subseteq$ P

(2x5=10 Points)

Problem II. Define the class co-NP along with an example problem in that class. Consider the problem HAPPY-TUPLE = $\{(m,n) | m > n > 1$ are integers, there is a prime factor

p of m between n and m}. Is the problem HAPPY-TUPLE in NP ∩ co-NP. Justify your answer.

(3 + 7 =10 Points)

Problem III. Define terms LogSpace and NLogspace. Check if the following problems have algorithms that run in logarithmic space, deterministic or non-deterministic if required. If so, provide such an algorithm. If not, justify why.

1. Given a graph G show that it is not bipartite.
2. Given an array A of n integers, find the element of A that repeats the most times. In case of tie, you can report the element with the lowest index.

(2+4+4=10 Points)

Problem IV. Define what is meant by sparse languages. Which of the languages below are sparse. Justify your answers.

1. The set of all positive integer tuples of the form a, b, c such that $a^2 + b^2 = c^2$.
2. $L = \{w|$ w, interpreted as a natural number is a Mersenne prime $\}$. A number n is called a Mersenne prime if n is of the form $2^p - 1$ for a prime p For instance, if $w = 0111$, then $w \in L$ since $7 = 2^3 - 1$ and 3, 7 are prime.

(2+4+4=10 Points)

Problem V.

Define the term PH (Polynomial Hierarchy). Show the following are true:

- $P^{NP} \subseteq NP^{NP} \cap co - NP^{NP}$
- $NP \cup co - NP \subseteq P^{NP}$

(3+4+3=10 Points)