

26/2/20

Kripa Anne
20171159

IIT Hyderabad

DIFFERENTIAL EQUATIONS

Spring 2020 – Mathematics Elective IMA303

Instructor: B. S. Lakshmi Date: FEBRUARY 26, 2020

Time: 1H 30 Min (4 00 PM- 5 30 PM)

Mid-Semester Examination

Total Marks: 50

Instructions:

- Answer all questions. (✓)
- Doubts and Questions will not be entertained during the Exam (×)
- Class notes or books are not permitted. (×)
- Calculators are allowed. (✓)

Q 1. (a) Consider the initial value problem

$$y' = \frac{ty(4-y)}{3} \quad y(0) = y_0$$

- Determine how the behavior of the solution as t increases depends on the initial value y_0 .
 - Suppose that $y_0 = 0.5$. Find the time T at which the solution first reaches the value 3.98.
- (b) In the following differential equation verify whether the equation is exact. If not find an integrating factor (if possible) and solve it.

$$y + (2xy - e^{-2y})y' = 0$$

[6+7=13]

Q 2. (a) Suppose that the population $P(t)$ of a country satisfies the differential equation $\frac{dP}{dt} = kP(200 - P)$ with k a constant. Its population in 2000 was 100 million and was then growing at the rate of one million per year. Predict this country's population for the year 2060.

- (b) Two large containers A and B of the same size are filled with different fluids. The fluids in containers A and B are maintained at 0°C and 100°C , respectively. A small metal bar, whose initial temperature is 100°C , is lowered into container A. After 1 minute the temperature of the bar is 90°C . After 2 minutes the bar is removed and instantly transferred to the other container. After 1 minute in container B the temperature of the bar rises 10°C . How long, measured from the start of the entire process, will it take the bar to reach 99.9°C ?

[6+7=13]

Q 3. (a) Assume that y_1 and y_2 are a fundamental set of solutions of $y'' + p(t)y' + q(t)y = 0$ and let $y_3 = a_1y_1 + a_2y_2$ and $y_4 = b_1y_1 + b_2y_2$, where a_1, a_2, b_1 , and b_2 are any constants. Show that the Wronskian of (y_3, y_4)

$$W[y_3, y_4] = (a_1b_2 - a_2b_1)W[y_1, y_2].$$

Are y_3 and y_4 also a fundamental set of solutions? Why or why not?

- (b) Suppose that r_1 and r_2 are roots of $ar^2 + br + c = 0$ and that $r_1 \neq r_2$; then write down the solutions the differential equation $ay'' + by' + cy = 0$. Show that

$$\phi(t; r_1, r_2) = \frac{e^{r_2t} - e^{r_1t}}{r_2 - r_1}$$

is also a solution of the equation for $r_1 \neq r_2$. Then think of r_1 as fixed, obtain the second solution in the case of equal roots. (using limits)

[6+6=12]

Q 4. Consider the differential equation

$$ay'' + by' + cy = g(t),$$

where a, b , and c are positive

If $g(t) = d$, a constant, to what value does every solution of the equation approach as $t \rightarrow \infty$? What happens if $c = 0$? What if $b = 0$ also?

[12]