1. Consider a three-level single particle system with six microstates with energies 0, ε, ε, 2ε, 2ε, 2ε. What is the mean energy of the system if it is in equilibrium with a bath at temperature T? In the region where $\beta\varepsilon \to 0$, what will the graph of heat capacity of the system as a function of ε look like at a constant temperature?

Ans. $U = \frac{\sum_j E_j e^{-\beta E_j}}{\sum_j e^{-\beta E_j}} = \frac{\varepsilon.2e^{-\beta\varepsilon}+2\varepsilon.3e^{-2\beta\varepsilon}}{1+2e^{-\beta\varepsilon}+3e^{-2\beta\varepsilon}} = \varepsilon.\frac{2e^{-\beta\varepsilon}+6e^{-2\beta\varepsilon}}{1+2e^{-\beta\varepsilon}+3e^{-2\beta\varepsilon}}$

heat capacity, $C_V = \frac{\partial U}{\partial T} = -\frac{\beta}{T}\frac{\partial U}{\partial \beta} = -\frac{\beta\varepsilon}{T}.\left[-\varepsilon.\frac{2e^{-\beta\varepsilon}+12e^{-2\beta\varepsilon}}{1+2e^{-\beta\varepsilon}+3e^{-2\beta\varepsilon}} + \varepsilon.\frac{\left(2e^{-\beta\varepsilon}+6e^{-2\beta\varepsilon}\right)\left(2e^{-\beta\varepsilon}+6e^{-2\beta\varepsilon}\right)}{(1+2e^{-\beta\varepsilon}+3e^{-2\beta\varepsilon})^2}\right] = \text{const}.\varepsilon^2$

$\therefore$ the graph will look like a parabola.

2. State briefly the **difference** in **assumptions** made by Einstein and Debye in developing the theory for heat capacity of solid crystals.

Ans. Einstein : all atoms in different lattice points oscillate with the same frequency

Debye : There is a continuous distribution of the frequency of oscillations of the normal modes of the lattice vibrations starting from 0 to a cutoff frequency.

3. Explain qualitatively why the pressure of an ideal Fermi gas is different from that of the classical ideal gas. Mention also if it is lower or higher.

Ans. For the classical ideal gas, the molecules occupy continuous energy states and there is no restriction on how many molecules may be in a certain energy state. For the Fermi gas, there is a restriction that only one molecule may be in a certain energy state. This results in a 'quantum' repulsive interaction that increases the pressure of the gas.

4. How will the density of states of an ideal gas like system change if its volume is doubled?

Ans. density of states $\omega(\varepsilon) \propto V^N$

$\therefore \frac{\omega_{2V}(\varepsilon)}{\omega_V(\varepsilon)} = \frac{(2V)^N}{V^N} = 2^N$

5. Obtain the value for: $\frac{\Theta_{x,H_2}}{\Theta_{x,HD}}$, for x=r(rotational) at high temperatures, without using the Tables.

Ans. The equilibrium bond distance and the force constant is determined by electronic effects, so it will be the same for both H_2 and D_2. But the reduced masses will change. The symmetry number for both is 2.

$\frac{\Theta_{x,H_2}}{\Theta_{x,HD}} = \frac{I_{HD}}{I_{H_2}} = \frac{\mu_{HD}}{\mu_{H_2}} = \frac{\frac{m_H.m_D}{m_H+m_D}}{\frac{m_H}{2}} = \frac{\frac{2}{3}}{\frac{1}{2}} = \frac{4}{3}$