Introduction to Algorithm Engineering
Spring 2024
End Semester Exam
IIIT Hyderabad

30-APRIL-2024, 3:00 PM to 5:00 PM

Instructions:

- The exam is for a duration of 120 minutes and 60 points.
- Answer legibly. Answer all parts of a question contiguously.
- No clarifications shall be given in the exam. If you have to make any assumptions, state those assumptions clearly. Points awarded will depend on the nature of the assumptions.
- The question paper has 9 (NINE) questions spread across 3 (THREE) pages. Make sure you have all the pages printed properly, or ask for a replacement copy.
- All questions test all course outcomes equally.

1 Basic Knowledge

Answer each question briefly. Each question is for four points. The section is for 20 points.

Problem 1. Define the notion of a cover set of edges for triangles. Find a cover set of edges for triangles in the graph shown in Figure 1. **(2+2=4 Points)**
(4 Points)

Problem 2. Define what is meant by a t-spanner of a graph. How many edges are added in Phase 2 of the Baswana and Sen algorithm. Explain your answer briefly. **(2+2=4 Points)**

Problem 3. Describe some of the important characterstics of a cache oblivious algorithm. Recall the simple recursive matrix multiplication algorithm. Write a simple pseudocode for this algorithm and derive the asymptotic number of cache misses of this algorithm. **(1+1+2=4 Points)**

Problem 4. Define betweenness-centrality. Find the betweenness-centrality of vertex c in the graph in Figure 2.

Problem 5. Write a short paragraph on your course project. Include details such as problem statement, current progress, and plans. **(4 Points)**

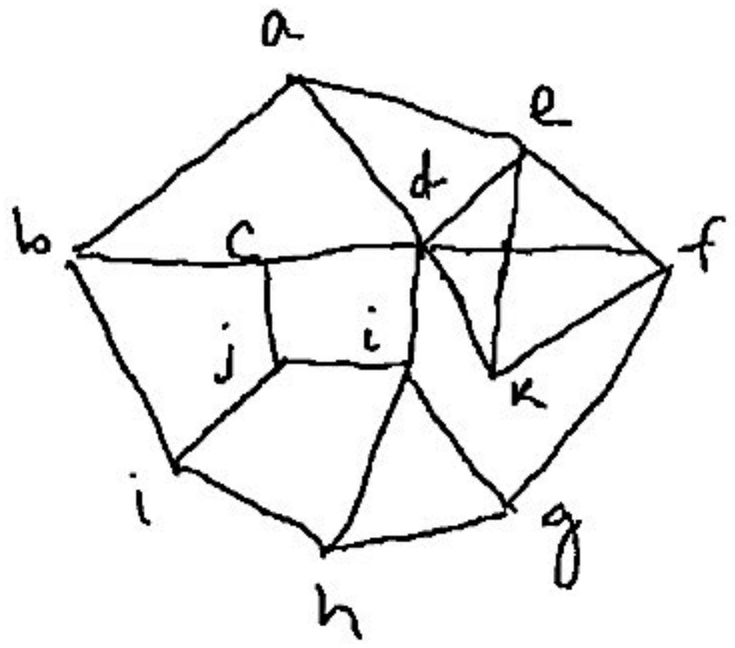

Figure 1: Figure for Question 1.

2 Long Answer Questions

Answer the following questions providing suitable justifications whereever needed. The section is for 40 points.

Problem 6.

Define what is meant by the biconnected components of a graph. Recall that in class we discussed two algorithms for idnetifying the biconnected components of a graph, the Jen-Schmidt algorithm, and the Tarjan-Vishkin algorithm. Answer the following questions.

1. How do these algorithms identify, or can be used to identify, bridges in a graph? Illustrate the mechanims with pseudocode and example.

2. What data structures would you use to run the two algorithms and what is the run time and space needed by the two algorithms with the choice of your data structures.

(2+4+4=10 Points)

Problem 7.

Define what is meant by the strongly connected components of a graph along with an example. Describe in brief the recursive algorithm to obtain the strongly connected components of a graph by including the main claims and their proofs. **(2+2+6=10 Points)**

(4 Points)

Problem 8. Describe in brief what is meant by cache-aware algorithms. Consider multiplying two square matrices A and B of dimensions $n \times n$ using the following method. The matrices are stored in row-major order. To obtain $A \times B$, we first transpose B into B^T. Then, we read rows of A and rows of B^T and perform the required multiplications. Answer the following questions.

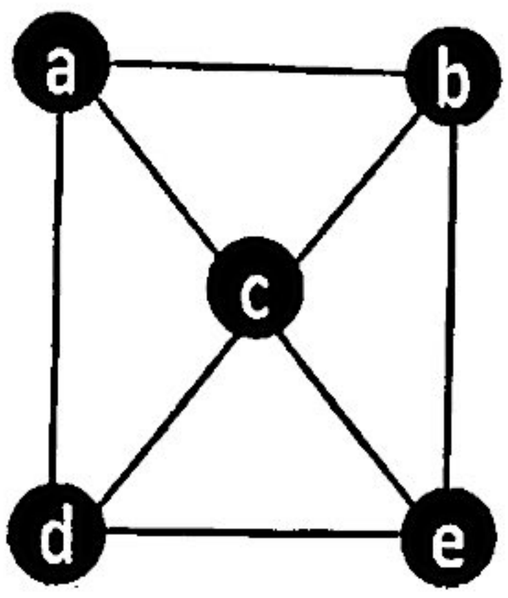

Figure 2: Figure for Question 4.

1. Write a small pseudocode for the above method.

2. Using relevant parameters, analyze the number of cache misses incurred by the algorithm.

3. Compare your answer with the number of cache misses incurred by the (a) naive matrix multiplication algorithm that uses three nested loops, and (b) the optimized matrix multiplication that uses multiple nested loops as discussed in class.

(2+4+2+2=10 Points)

Problem 9. Consider the setting where a graph on n vertices is represented using an $n \times n$ adjacency matrix. Answer the following questions where the answer should rely on using matrix operations.

1. How do we count the number of paths from a vertex i to vertex j in the graph.

2. How do we count the number of triangles in the graph.

3. Extend your answer from the second part to counting the number of cycles of length 4.

(3+4+3=10 Points)