Instructor:Dr. Indranil Chakrabarty

Date: NOVEMBER 22, 2016

Time: 3 Hours (9:00 - 12:00)

End-Semester Examination

Total Marks: 100

Instructions:

- Instruction 1 : No Calculator or Log Table is allowed in the examination hall.
- Instruction 2 : This is not an open book exam.
- Instruction 3 : If any question is wrong, then full marks will be allotted.
- Instruction 4 : Notations will have their usual meanings unless otherwise specified.

Group A: Answer All Questions

Q 1. (Fill in the blank) Any two spanning trees for a graph have the .... of .... [2]

Q 2. (Fill in the blank) $n^3 + \ldots$ is divisible by 3 whenever n is......... [2]

Q 3. Is there a full binary tree that has 10 internal vertices and 13 terminal vertices. [2]

~~**Q 4.** The set $P(a,b,c)$ is partially ordered with respect to subset relation. Find a chain of length 3 in $P(a,b,c)$. [2]~~

Q 5. (Say True or False) For any two sets A and B, $A-(A\cap B)=A-B$. [2]

Q 6. (Say True or False) If $p \geq 1$, then $z^{p-1}G(z)$ is a generating function. [2]

Q 7. Let $P(x)$ denote the statement $x>3$. What are the truth values of $P(4)$ and $P(2)$. [2]

Q 8. What are the equivalence classes of 0 and 1 for congruence modulo 4. [2]

Q 9. (Say True or False) Every non-trivial tree T has atleast two vertices of degree 1. [2]

Q 10. (Say True or False) Any permutation can be expressed as a product of finite number of disjoint cycles. [2]

Group B: Answer All Questions

Q 1. Show that when a connected weighted graph is input to Kruskal's algorithm, then the output is a minimum spanning tree(b) Let G be a connected graph with n vertices out of which there are k loops and m parallel edges. Design an algorithm to find the shortest distance between two vertices. (c) Give an example of a relation that is reflexive, symmetric, anti symmetric and transitive.

[4+4+2=10]

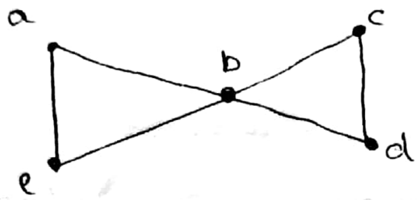
Q 2. (a) Obtain the partial fraction decompositions and identify the sequence having the expression $\frac{5+2z}{1-4z^2}$ as a generating function. (b) If $f: X \to Y$ be one-to-one then $f^{-1}Y \to X$ is one to one. (c) Show that the following statements constitute a valid argument: " If A works hard, then either B or C will enjoy. If B enjoys, then A will not work hard. If D enjoys, then C will not. Therefore if A works hard, D will not enjoy".

[4+2+4=10]

Q 3. (a) In a poset $(S, \preccurlyeq)$ if a subset $\{a,b\}$ of S has a least upper bound (l.u.b) and greatest lower bound (g.l.b), then show that they are unique. (b) Show that in a distributive lattice $(L, \preccurlyeq)$ with elements 1 and 0, evry element has at most one compliment. (c) The set $A=\{1,2,3,4,6,8,18,24,48\}$ ordered by divides relation. Draw the Hasse diagram of the corrosponding set and also construct topological sorting for this set.

[2+3+5=10]

Q 4. (a) Show that in a graph G, if vertices v and w are part of a circuit in G and one edge is removed from the circuit, then there still exists a trail from v to w in G. Let A be the adjacency matrix for K_3. Use mathematical induction to prove that for each positive integer n, all entries along the main diagonal of A^n are equal to each other. Find out whether the following graph has a hamiltonian circuit or not? [4+4+2=10]



Group C: Answer All Questions

Q 1. Prove that if G is a graph that has a vertex of degree k and H is isomorphic to G, then H has a vertex of degree k. Draw four non isomorphic graph with six vertices, two of degree 4 and four of degree 3. [2.5+2.5=5]

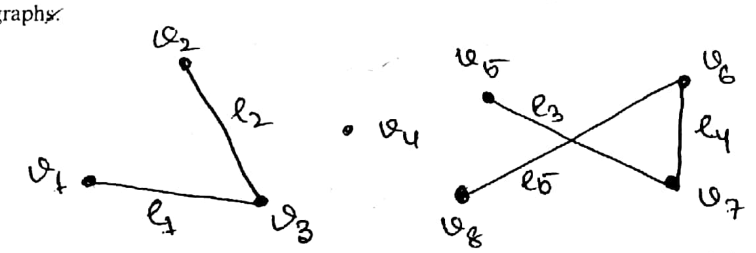
Q 2. Solve the recurrence relation: $a_n = 2a_{n-1} - 2a_{n-2}, a_0 = 1, a_1 = 2$.During a month with 30 days , a football team plays at least one game a day, but not more than 45 games. Show that there must be period of some number of consecutive days during which the team must play exactly 14 games [3+2=5]

Q 3. Find a formula for $\frac{1}{2} + \frac{1}{6} + \ldots + \frac{1}{n(n+1)}$ [5]

Q 4. If a graph has an Euler circuit, then evry vertex of the graph has positive even degree. Let R and S be two relations on X , then show that if R is reflexive then $R \cup S$ is also reflexive. [3+2=5]

Q 5. Show that evry planar graph G can be coloured with 5 colours. If $A_1, A_2, \ldots, A_m$ and P imply Q, then show that $A_1, A_2, \ldots, A_m$ imply $P \to Q$. [3+2=5]

Q 6. Show that in any graph there are even number of vertices of odd degree. Find all the connected components of the following graphs. [3+2=5]



Q 7. Consider the 'divides' relation defined on the set $A = \{1, 2, 2^2, 2^3, \ldots, 2^n\}$, where n is non negative integer. Prove that the relation is total order relation on A. Draw the Hasse diagram for this relation for $n = 4$. [3+2=5]

Q 8. Show that $f : R \to R$ defined by $f(x) = 2x - 3$ is a bijection and find its inverse. Compute $f^{-1} * f$ and $f * f^{-1}$. For a set having n elements, out of the total permutations, $\frac{n!}{2}$ are odd..... Justify this statement. [4+1=5]