CSE 411: Complexity and Advanced Algorithms
Monsoon 2018
IIIT Hyderabad

In Class Quiz 1, Due: August 27, 2018

Each question is for 5 points.

1. Define the following complexity classes and state an example of a problem in that class. (a) co-NLogSpace, and (b) NP.

2. Consider a non-deterministic TM M for a language L that has only two execution paths on some input w. One such path leads M to accept w and another leads M to reject. Answer the following questions.

   (a) What is the output of M on input w? Why?

   (b) Suppose now that Krishna claims that given M, a non-deterministic machine for $\overline{L}$ can be built as follows. The machine M' runs M on input w and accepts if M rejects w and rejects if M accepts. What is the flaw in this machine M'. Explain. (note: $\overline{L}$ is the complement of L and hence L is in co-NP.)

3. State Savitch's theorem. Which of the following can be deduced by using Savitch's theorem alone. Answer with a brief justification.

   (a) PSPACE = NPSPACE.

   (b) co-NLogSpace $\subsetneq P$

4. Consider the language of well-formed paranthesis. (Example: (), (()), ()(),). Show that one can verify if a string of paranthesis is well-formed in LogSpace.

Complexity and Advanced Algorithms
Quiz -2

October 8, 2018

Each question is for FIVE (5) points.

1. Explain what is accelerated cascading in brief.
2. Define the work complexity of a parallel algorithm. When is a parallel algorithm called optimal.
3. Briefly describe the parallel search algorithm and arrive at its time and work complexity when using p processors.
4. How quickly can you solve the problem of finding the Boolean-OR of n bits in the CRCW model and in the CREW model. Explain briefly.
5. Recall the standard binary tree traversal method for obtaining the prefix sum of n numbers. Is that algorithm a EREW algorithm? If not, can it be made to run as an EREW algorithm without losing on the time and work complexity in the asymptotic sense? Explain.

CSE 411: Complexity and Advanced Algorithms
Monsoon 2018
IIIT Hyderabad

Mid Exam 1

Instructions:

- The paper is for a duration of 90 minutes and 50 maximum points.
- The paper is spread over TWO pages and FIVE problems. Please verify and make sure that your question paper is printed properly.
- No clarification shall be provided during the exam. You are free to make suitable assumptions if you need to, but the points awarded will depend on the nature of the assumptions.
- Answer all questions and avoid being verbose.

Problem I. Basic Knowledge. Answer each question with a brief justification. Each question carries **TWO** points.

1. Define what is meant by a space-constructible function. Show that the function $f(n) = n \cdot \lfloor \log n \rfloor$ is space-constructible.
2. Define the class P^{poly}.
3. When do we say that a language L_1 Cook reduces to another language L_2. How is this reduction different from Turing reduction?
4. Write the following mathematical statements as quantified formulae.
   (a) For every positive integer $n \geq 2$, there exist four tuples a, b, c, and d such that $a^n + b^n = c^n + d^n$.
   (b) Every even integer can be expressed as the sum of two odd integers.
5. Which of the following statements are true. Justify briefly.
   a. NP = co-NP
   b. NSPACE = PSPACE
   c. NL = co-NL
   d. co-NL $\subseteq$ P

(2x5=10 Points)

Problem II. Define the class co-NP along with an example problem in that class. Consider the problem HAPPY-TUPLE = $\{(m, n) | m > n > 1$ are integers, there is a prime factor p of m between n and $m\}$. Is the problem HAPPY-TUPLE in NP $\cap$ co-NP. Justify your answer.

(3 + 7 =10 Points)

Problem III. Define terms LogSpace and NLogspace. Check if the following problems have algorithms that run in logarithmic space, deterministic or non-deterministic if required. If so, provide such an algorithm. If not, justify why.

1. Given a square matrix whose entries are from $\{0, 1\}$, find whether the matrix has a determinant of 0.
2. Given an array A of n integers, find the element of A that repeats the most times. In case of tie, you can report the element with the lowest index.

(2+4+4=10 Points)

Problem IV. Define what is meant by sparse langauges. Which of the languages below are sparse. Justify your answers.

1. The set of all simple, undirected graphs whose degree is a fixed constant.
2. $L = \{w | \ w$, interepreted as a natural number is a Mersenne prime $\}$. For instance, if $w = 0111$, then $w \in L$ since $7 = 2^3 - 1$ and 7 is prime. A number n is called a Mersenne prime if n is of the form $2^p - 1$ for a prime p.

(2+4+4=10 Points)

Problem V. The purpose of this problem is to eventually realize that the equivalence of NL and co-NL is not an accident, but a piece of the bigger puzzle. In fact, most space-based complexity classes are closed under complementation. For instance, we will prove now that NSPACE = co-NSPACE. Use the following hints to complete the proof.

a. State what is a configuration of a TM M with respect to an input w.

b. Understand how many nodes are there in the configuration graph $G_{M,w}$ for a TM M on input w.

c. Set up the proof of NSPACE = co-NSPACE by posing the proof as a graph non-reachability problem that can be checked in NSPACE. We are not completing other parts of the proof such as compteleness of the reachability and non-reachability languages wrt NSPACE and co-NSPACE respectively.

You are free to pick any other approach for the actual proof (part c.). You should answer parts (a) and (b) still.

(1+2+7=10 Points)