

Science-2 (Mid Sem Exam)
(2024)

- ✓ Q1a. Consider the Compartmental models (Susceptible-Recovery-Infection) of epidemiology. If at any time t , the density of the susceptible, infected, and recovered population is captured by s, i , and r respectively, construct the associated differential equation of each compartment. Explain each term in one or two sentences. Here the rate of infection is β , rate of recovery is γ , and take $s(t) + i(t) + r(t) = 1$. 3
- ✓ Q1b. At initial phase of the disease spread, what should be the relation between β and γ such that the disease starts to propagate (exponentially)? 3
- ✓ Q2. For a certain class of distributions, it is possible to create pseudo-random numbers from uniformly distributed random numbers by finding a mathematical transformation (inverse transform method). If z (drawn from uniform distribution) is the random element chosen from 0 to 1, and the target distribution is $f = k e^{-\gamma k}$, then find the relation between γ and z . k is constant. 3
- ✓ Q3a. In 19th century, Robert Brown, a Scottish botanist, observed that pollen grains suspended in water, instead of remaining stationary or falling downwards, would trace out a random zig-zagging pattern. Using Langevin's approach, write down (with explanation) the equation of motion of one single pollen grain. 4
- Q3b. Specify which principles/axioms (of statistical thermodynamics) are required for solving the equation of motion of a large number of pollen grains.
Using these axioms, calculate the average square displacement ($\langle x^2 \rangle$) of a large number pollen grains.
2+3=5
- ✓ Q4a. The Predator-prey system consists of two kinds of animals. One of which preys on the other. If X symbolize prey, Y the predator, and A the food of the prey, we can write (this is also called as Lotka-Volterra model) following rate equations:
- $$\begin{aligned} X + A &\rightarrow 2X \quad \dots\dots(1) \\ X + Y &\rightarrow 2Y \quad \dots\dots(2) \\ Y &\rightarrow B \quad \dots\dots(3). \end{aligned}$$
- Construct the associated master equation for the model. k_1, k_2, k_3 are the rate constants of equations 1,2, and 3 respectively. 3
- ✓ Q4b. Write down the related ODE models for the abovementioned rate reactions (assuming A is constant). Explain each term in one to two sentences. 3

✓ Q5: If $f(x, y) = x^3 + x^2y^3 - 2y^2$, find $f_x(1, 2)$ and $f_y(2, 1)$. Show that $u(x, y) = e^x \sin y$ is a solution of Laplace Equation $\frac{\partial^2 u(x, y)}{\partial x^2} + \frac{\partial^2 u(x, y)}{\partial y^2} = 0$. 1+1+2=4

▶ Q6: If $f''(x_i) = \frac{f_{i+1} + f_{i-1} - 2f_i}{\Delta x^2}$, and if the heat conduction equation is $\alpha \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}$, then show that $T_i^{j+1} = T_i^j + (T_{i+1}^j + T_{i-1}^j - 2T_i^j) \alpha \frac{\Delta t}{\Delta x^2}$. 3