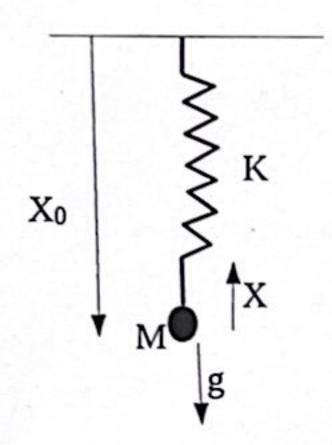
## Quiz 2

## (Science 1 : Classical Mechanics, each question carries 15 marks : $5 \times 3$ ) Time: 1 Hour

Q1.

The following mass M is attached to a spring of spring constant K and conducting a simple harmonic oscillation in the vertical direction in presence of gravity. The position at rest is given by  $X_0$  and the X is simple harmonic oscillation around  $X_0$ .

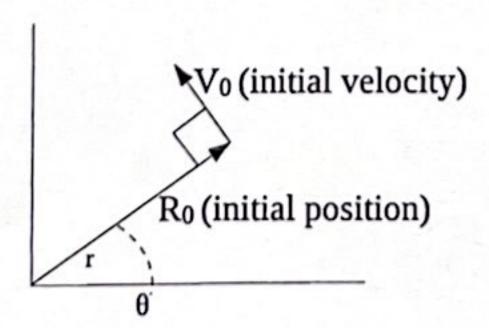


- I. Write down the expression for the kinetic energy and potential energy.
- II. Evaluate the Lagrangian L and Hamiltonian H of the system
- III. Write down the Lagrangian equation of motion for X.
- IV. Calculate the value of X<sub>0</sub> (position at rest) as function of M,g and K.
- V. If total energy is E, evaluate the frequency and amplitude of the oscillation.

Q2.

A particle of mass M is moving under a central potential given by V(r)=-k/r on a plane as shown below with center of force at the origin. An initial velocity of  $V_0$  is given to the system perpendicular to the initial position vector  $R_0$  as shown in the figure below.

The values are  $M=10^{24} kg$ ;  $k=10^{43} m^3 kg s^{-1}$ ;  $R_0=100\times 10^6 km$ ;  $V_0=10 km s^{-1}$ 



- I. Write down the Lagrangian  $L(\dot{r},\dot{\theta},r,\theta)$  of the system and the generalized momenta  $P_r$ ,  $P_{\theta}$  corresponding to r and  $\theta$ .
- II. Write down the Hamiltonian  $H(P_r, P_\theta, r, \theta)$
- III. Calculate the initial angular momentum A around the center and initial total energy E of the system
- IV. Show if the orbit would be bound or not bound and what is the maximum initial velocity  $V_0$  to get bound orbit
- V. Calculate the velocity  $V_0$  for a circular orbit of radius  $R_0$ .