Déduction naturelle

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Ce docuement contient des coquilles, des erreurs et des fautes d'orthographe. Merci de me les signaler par github ou par message discord.

Contents

1 Cours

1.1 Modus Ponens

$$\dfrac{\dfrac{}{(p \to q), p \vdash (p \to q)}\,\text{ax} \qquad \dfrac{}{(p \to q), p \vdash p}\,\text{ax}}{(p \to q), p \vdash q}\,{\to_e}\ p$$

1.2 Modus Tollens

$$\dfrac{\dfrac{\dfrac{\text{(Modus Ponens)}}{(p \to q), \neg q, p \vdash q} \qquad \dfrac{}{(p \to q), \neg q, p \vdash \neg q}\,\text{ax}}{(p \to q), \neg q, p \vdash \bot}\,{\neg_e}\ q}{(p \to q), \neg q \vdash \neg p}\,{\neg_i}$$

1.3 Syllogisme disjonctif

$$\dfrac{\dfrac{}{(p \vee q), \neg p \vdash (p \vee q)}\,\text{ax} \qquad \dfrac{\dfrac{\dfrac{}{(p \vee q), \neg p, p, \neg q \vdash \neg p}\,\text{ax} \quad \dfrac{}{(p \vee q), \neg p, p, \neg q \vdash p}\,\text{ax}}{(p \vee q), \neg p, p, \neg q \vdash \bot}\,\neg_e\ p}{(p \vee q), \neg p, p \vdash q}\,\bot \qquad \dfrac{}{(p \vee q), \neg p, q \vdash q}\,\text{ax}}{(p \vee q), \neg p \vdash q}\,\vee_e\ p, q$$

1.4 Syllogisme barbara

$$\dfrac{\dfrac{}{(p \to q), (q \to r), p \vdash (q \to r)}\,\text{ax} \qquad \dfrac{}{(p \to q), (q \to r), p \vdash q}\,\text{(Modus Ponens)}}{(p \to q), (q \to r), p \vdash r}\,\to_e\ q$$
$$\dfrac{}{(p \to q), (q \to r) \vdash (p \to r)}\,\to_i$$

1.5 Syllogisme Festino

$$\dfrac{\dfrac{}{(p \to \neg q), q, p \vdash \neg q}\,\text{(Modus Ponens)} \qquad \dfrac{}{(p \to \neg q), q, p \vdash q}\,\text{ax}}{(p \to \neg q), q, p \vdash \bot}\,\neg_e\ q$$
$$\dfrac{}{(p \to \neg q), q \vdash \neg p}\,\neg_i$$

1.6 Syllogisme Cesare

$$\dfrac{\dfrac{}{(p \to \neg q), (r \to q), r, p \vdash \neg q}\,\text{(Modus Ponens)} \qquad \dfrac{}{(p \to \neg q), (r \to q), r, p \vdash q}\,\text{(Modus Ponens)}}{(p \to \neg q), (r \to q), r, p \vdash \bot}\,\neg_e\ q$$
$$\dfrac{\dfrac{}{(p \to \neg q), (r \to q), r \vdash \neg p}\,\neg_i}{(p \to \neg q), (r \to q) \vdash (r \to \neg p)}\,\to_i$$

1.7 Tiers Exclu

$$\dfrac{\dfrac{\dfrac{}{\neg(\neg p \vee p), p \vdash \neg(\neg p \vee p)}\,\text{ax} \qquad \dfrac{\dfrac{}{\neg(\neg p \vee p), p \vdash p}\,\text{ax}}{\neg(\neg p \vee p), p \vdash (\neg p \vee p)}\,\vee_i^d}{\neg(\neg p \vee p), p \vdash \bot}\,\neg_e\ (\neg p \vee p)}{\neg(\neg p \vee p) \vdash \neg p}\,\neg_i \qquad \dfrac{}{\neg(\neg p \vee p) \vdash p}\,\text{(Idem)}$$
$$\dfrac{}{\vdash (\neg p \vee p)}\,\neg_i \qquad \dfrac{}{\neg(\neg p \vee p) \vdash \bot}\,\neg_e$$

2 Exercices

2.1 Exo 13.1

$$\dfrac{\dfrac{\dfrac{}{\forall x.\varphi \vdash \forall x.\varphi}\,\text{ax}}{\forall x.\varphi \vdash \varphi}\,\forall_e\ x}{\forall x.\varphi \vdash \exists x.\varphi}\,\exists_i$$

2.2 Exo 13.2

$$\dfrac{\dfrac{}{\exists z.\forall x.\varphi(z,x) \vdash \exists z.\forall x.\varphi(z,x)}\,\text{ax} \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{\exists z.\forall x.\varphi(z,x), \forall x.\varphi(z,x) \vdash \forall x.\varphi(z,x)}\,\text{ax}}{\exists z.\forall x.\varphi(z,x), \forall x.\varphi(z,x) \vdash \varphi(z,y)}\,\forall_e\ [y \to x]}{\exists z.\forall x.\varphi(z,x), \forall x.\varphi(z,x) \vdash \exists x.\varphi(x,y)}\,\exists_i\ [x \to y]}{\exists z.\forall x.\varphi(z,x), \forall x.\varphi(z,x) \vdash \forall y.\exists x.\varphi(x,y)}\,\forall_i}{\exists z.\forall x.\varphi(z,x) \vdash \forall y.\exists x.\varphi(x,y)}\,\exists_e\ x, \varphi(z,x)$$