

# Déduction naturelle

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Ce document contient des coquilles, des erreurs et des fautes d'orthographe. Merci de me les signaler par github ou par message discord.

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## 1 Cours

### 1.1 Modus Ponens

$$\frac{\frac{}{(p \rightarrow q), p \vdash (p \rightarrow q)} \text{ ax} \quad \frac{}{(p \rightarrow q), p \vdash p} \text{ ax}}{(p \rightarrow q), p \vdash q} \rightarrow_e p$$

### 1.2 Modus Tollens

$$\frac{\frac{}{(p \rightarrow q), \neg q, p \vdash q} \text{ (Modus Ponens)} \quad \frac{}{(p \rightarrow q), \neg q, p \vdash \neg q} \text{ ax}}{\frac{(p \rightarrow q), \neg q, p \vdash \perp}{(p \rightarrow q), \neg q \vdash \neg p} \neg_i} \neg_e q$$

### 1.3 Syllogisme disjonctif

$$\begin{array}{c}
 \frac{}{(p \vee q), \neg p, p, \neg q \vdash \neg p} \text{ax} \quad \frac{}{(p \vee q), \neg p, p, \neg q \vdash p} \text{ax} \\
 \hline
 \frac{}{(p \vee q), \neg p, p, \neg q \vdash \perp} \perp \\
 \hline
 \frac{}{(p \vee q), \neg p \vdash (p \vee q)} \text{ax} \quad \frac{}{(p \vee q), \neg p, p \vdash q} \perp \quad \frac{}{(p \vee q), \neg p, q \vdash q} \text{ax} \\
 \hline
 (p \vee q), \neg p \vdash q \quad \vee_e p, q
 \end{array}$$

### 1.4 Syllogisme barbara

$$\begin{array}{c}
 \frac{}{(p \rightarrow q), (q \rightarrow r), p \vdash (q \rightarrow r)} \text{ax} \quad \frac{}{(p \rightarrow q), (q \rightarrow r), p \vdash q} \text{(Modus Ponens)} \\
 \hline
 \frac{}{(p \rightarrow q), (q \rightarrow r), p \vdash r} \rightarrow_e q \\
 \hline
 \frac{}{(p \rightarrow q), (q \rightarrow r) \vdash (p \rightarrow r)} \rightarrow_i
 \end{array}$$

### 1.5 Syllogisme Festino

$$\begin{array}{c}
 \frac{}{(p \rightarrow \neg q), q, p \vdash \neg q} \text{(Modus Ponens)} \quad \frac{}{(p \rightarrow \neg q), q, p \vdash q} \text{ax} \\
 \hline
 \frac{}{(p \rightarrow \neg q), q, p \vdash \perp} \neg_e q \\
 \hline
 \frac{}{(p \rightarrow \neg q), q \vdash \neg p} \neg_i
 \end{array}$$

### 1.6 Syllogisme Cesare

$$\begin{array}{c}
 \frac{}{(p \rightarrow \neg q), (r \rightarrow q), r, p \vdash \neg q} \text{(Modus Ponens)} \quad \frac{}{(p \rightarrow \neg q), (r \rightarrow q), r, p \vdash q} \text{(Modus Ponens)} \\
 \hline
 \frac{}{(p \rightarrow \neg q), (r \rightarrow q), r, p \vdash \perp} \neg_e q \\
 \hline
 \frac{}{(p \rightarrow \neg q), (r \rightarrow q), r \vdash \neg p} \neg_i \\
 \hline
 \frac{}{(p \rightarrow \neg q), (r \rightarrow q) \vdash (r \rightarrow \neg p)} \rightarrow_i
 \end{array}$$

### 1.7 Tiers Exclu

$$\begin{array}{c}
 \frac{}{\neg(\neg p \vee p), p \vdash \neg(\neg p \vee p)} \text{ax} \quad \frac{}{\neg(\neg p \vee p), p \vdash p} \text{ax} \\
 \hline
 \frac{}{\neg(\neg p \vee p), p \vdash \neg(\neg p \vee p)} \neg_e (\neg p \vee p) \\
 \hline
 \frac{}{\neg(\neg p \vee p), p \vdash \perp} \neg_i \quad \frac{}{\neg(\neg p \vee p) \vdash p} \text{(Idem)} \\
 \hline
 \frac{}{\neg(\neg p \vee p) \vdash \neg p} \neg_e \quad \frac{}{\neg(\neg p \vee p) \vdash \perp} \neg_i \\
 \hline
 \frac{}{\vdash (\neg p \vee p)} \neg_i
 \end{array}$$

## 2 Exercices

### 2.1 Exo 13.1

$$\frac{\frac{\overline{\forall x.\varphi \vdash \forall x.\varphi} \text{ ax}}{\forall x.\varphi \vdash \varphi} \forall_e x}{\forall x.\varphi \vdash \exists x.\varphi} \exists_i$$

### 2.2 Exo 13.2

$$\frac{\overline{\exists z.\forall x.\varphi(z, x) \vdash \exists z.\forall x.\varphi(z, x)} \text{ ax}}{\exists z.\forall x.\varphi(z, x) \vdash \forall y.\exists x.\varphi(x, y)} \text{ ax}$$

$$\frac{\frac{\overline{\exists z.\forall x.\varphi(z, x), \forall x.\varphi(z, x) \vdash \forall x.\varphi(z, x)} \text{ ax}}{\exists z.\forall x.\varphi(z, x), \forall x.\varphi(z, x) \vdash \varphi(z, y)} \forall_e [y \rightarrow x]}{\frac{\overline{\exists z.\forall x.\varphi(z, x), \forall x.\varphi(z, x) \vdash \exists x.\varphi(x, y)} \exists_i [x \rightarrow y]}{\exists z.\forall x.\varphi(z, x), \forall x.\varphi(z, x) \vdash \forall y.\exists x.\varphi(x, y)} \forall_i} \exists_e x, \varphi(z, x)$$