Tutorial

Course: Dynamics & Control of Aerial Robots

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Lecture Timings

Monday & Wednesday: 2.00 pm -3.00pm

Tutorial Timings

Friday: 2.00 pm -3.00 pm

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Summary Equations of Motion

$$\dot{p}_n = (\cos\theta\cos\psi)u + (\sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi)v + (\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi)w$$

$$\dot{p}_e = (\cos\theta\sin\psi)u + (\sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi)v + (\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi)w$$

$$\dot{h} = u\sin\theta - v\sin\phi\cos\theta - w\cos\phi\cos\theta$$

$$\dot{u} = rv - qw - g\sin\theta + \frac{\rho V_a^2 S}{2\mathrm{m}}\left[C_X(\alpha) + C_{X_q}(\alpha)\frac{cq}{2V_a} + C_{X_{\delta_e}}(\alpha)\delta_e\right] + \frac{\rho S_{\text{prop}} C_{\text{prop}}}{2\mathrm{m}}\left[(k_{\text{motor}}\delta_t)^2 - V_a^2\right]$$

$$\dot{v} = pw - ru + g\cos\theta\sin\phi + \frac{\rho V_a^2 S}{2\mathrm{m}}\left[C_{Y_0} + C_{Y_\beta}\beta + C_{Y_p}\frac{bp}{2V_a} + C_{Y_r}\frac{br}{2V_a} + C_{Y_{\delta_a}}\delta_a + C_{Y_{\delta_r}}\delta_r\right]$$

$$\dot{w} = qu - pv + g\cos\theta\cos\phi + \frac{\rho V_a^2 S}{2\mathrm{m}}\left[C_Z(\alpha) + C_{Z_q}(\alpha)\frac{cq}{2V_a} + C_{Z_{\delta_e}}(\alpha)\delta_e\right]$$

$$\dot{\phi} = p + q\sin\phi\tan\theta + r\cos\phi\tan\theta$$

$$\dot{\theta} = q\cos\phi - r\sin\phi$$

$$\dot{\psi} = q\sin\phi\sec\theta + r\cos\phi\sec\theta$$

$$\dot{p} = \Gamma_1 pq - \Gamma_2 qr + \frac{1}{2}\rho V_a^2 Sb\left[C_{p_0} + C_{p_\beta}\beta + C_{p_p}\frac{bp}{2V_a} + C_{p_r}\frac{br}{2V_a} + C_{p_{\delta_a}}\delta_a + C_{p_{\delta_r}}\delta_r\right]$$

$$\dot{q} = \Gamma_5 pr - \Gamma_6(p^2 - r^2) + \frac{\rho V_a^2 Sc}{2J_y}\left[C_{m_0} + C_{m_\alpha}\alpha + C_{m_q}\frac{cq}{2V_a} + C_{m_{\delta_e}}\delta_e\right]$$

$$\dot{r} = \Gamma_7 pq - \Gamma_1 qr + \frac{1}{2}\rho V_a^2 Sb\left[C_{r_0} + C_{r_\beta}\beta + C_{r_p}\frac{bp}{2V_a} + C_{r_r}\frac{br}{2V_a} + C_{r_{\delta_a}}\delta_a + C_{r_{\delta_r}}\delta_r\right]$$

$$C_{p_0} = \Gamma_3 C_{l_0} + \Gamma_4 C_{n_0}$$
$$C_{p_\beta} = \Gamma_3 C_{l_\beta} + \Gamma_4 C_{n_\beta}$$
$$C_{p_p} = \Gamma_3 C_{l_p} + \Gamma_4 C_{n_p}$$
$$C_{p_r} = \Gamma_3 C_{l_r} + \Gamma_4 C_{n_r}$$
$$C_{p_{\delta_a}} = \Gamma_3 C_{l_{\delta_a}} + \Gamma_4 C_{n_{\delta_a}}$$
$$C_{p_{\delta_r}} = \Gamma_3 C_{l_{\delta_r}} + \Gamma_4 C_{n_{\delta_r}}$$

$$C_{r_0} = \Gamma_4 C_{l_0} + \Gamma_8 C_{n_0}$$
$$C_{r_\beta} = \Gamma_4 C_{l_\beta} + \Gamma_8 C_{n_\beta}$$
$$C_{r_p} = \Gamma_4 C_{l_p} + \Gamma_8 C_{n_p}$$
$$C_{r_r} = \Gamma_4 C_{l_r} + \Gamma_8 C_{n_r}$$
$$C_{r_{\delta_a}} = \Gamma_4 C_{l_{\delta_a}} + \Gamma_8 C_{n_{\delta_a}}$$
$$C_{r_{\delta_r}} = \Gamma_4 C_{l_{\delta_r}} + \Gamma_8 C_{n_{\delta_r}}.$$

Objective is to compute trim states and inputs when aircraft simultaneously satisfies three conditions:

- Traveling at constant speed V_a^*,
- Climbing at constant flight path angle γ^*,
- In constant orbit of radius R^*.

V_a^*, γ^*, and R^*, are inputs to the trim calculations.

$$
\begin{aligned}
\text{States:} \quad & x \triangleq (p_n, p_e, p_d, u, v, w, \phi, \theta, \psi, p, q, r)^\top \\
\text{Inputs:} \quad & u \triangleq (\delta_e, \delta_t, \delta_a, \delta_r)^\top
\end{aligned}
$$

For constant-climb orbit:

$\rightarrow$ speed of aircraft not changing $\rightarrow \dot{u}^* = \dot{v}^* = \dot{w}^* = 0$

$\rightarrow$ roll and pitch angles constant $\rightarrow \dot{\phi}^* = \dot{\theta}^* = \dot{p}^* = \dot{q}^* = 0$

Turn rate constant and given by

$$\dot{\psi}^* = \frac{V_a^*}{R^*}\cos\gamma^* \quad \rightarrow \quad \dot{r}^* = 0$$

Climb rate constant, and given by

$$\dot{h}^* = V_a^* \sin\gamma^*$$

Given parameters V_a^*, γ^*, and R^*, can specify $\dot{x}^*$ as

$$\dot{x}^* = (\dot{p}_n^* \ \dot{p}_e^* \ \dot{h}^* \ \dot{u}^* \ \dot{v}^* \ \dot{w}^* \ \dot{\phi}^* \ \dot{\theta}^* \ \dot{\psi}^* \ \dot{p}^* \ \dot{q}^* \ \dot{r}^* \ )^\top$$

Given parameters V_a^*, γ^*, and R^*, can specify $\dot{x}^*$ as

$$\dot{x}^* = \begin{pmatrix} \dot{p}_n^* \\ \dot{p}_e^* \\ \dot{h}^* \\ \dot{u}^* \\ \dot{v}^* \\ \dot{w}^* \\ \dot{\phi}^* \\ \dot{\theta}^* \\ \dot{\psi}^* \\ \dot{p}^* \\ \dot{q}^* \\ \dot{r}^* \end{pmatrix} = \begin{pmatrix} \text{[don't care]} \\ \text{[don't care]} \\ V_a^* \sin\gamma^* \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{V_a^*}{R^*}\cos\gamma^* \\ 0 \\ 0 \\ 0 \end{pmatrix} = f(x^*, u^*)$$

Problem of finding x^* and u^* such that $\dot{x}^* = f(x^*, u^*)$, reduces to solving nonlinear algebraic systems of equations.

Linearized Lateral Model

$$\begin{pmatrix} \dot{\bar{v}} \\ \dot{\bar{p}} \\ \dot{\bar{r}} \\ \dot{\bar{\phi}} \\ \dot{\bar{\psi}} \end{pmatrix} = \begin{pmatrix} Y_v & Y_p & Y_r & g\cos\theta^*\cos\phi^* & 0 \\ L_v & L_p & L_r & 0 & 0 \\ N_v & N_p & N_r & 0 & 0 \\ 0 & 1 & \cos\phi^*\tan\theta^* & q^*\cos\phi^*\tan\theta^* - r^*\sin\phi^*\tan\theta^* & 0 \\ 0 & 0 & \cos\phi^*\sec\theta^* & p^*\cos\phi^*\sec\theta^* - r^*\sin\phi^*\sec\theta^* & 0 \end{pmatrix} \begin{pmatrix} \bar{v} \\ \bar{p} \\ \bar{r} \\ \bar{\phi} \\ \bar{\psi} \end{pmatrix} + \begin{pmatrix} Y_{\delta_a} & Y_{\delta_r} \\ L_{\delta_a} & L_{\delta_r} \\ N_{\delta_a} & N_{\delta_r} \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \bar{\delta}_a \\ \bar{\delta}_r \end{pmatrix}$$

Lateral	Formula
Y_v	$\frac{\rho S b v^*}{4mV_a^*}\left[C_{Y_p}p^* + C_{Y_r}r^*\right] + \frac{\rho S v^*}{m}\left[C_{Y_0} + C_{Y_\beta}\beta^* + C_{Y_{\delta_a}}\delta_a^* + C_{Y_{\delta_r}}\delta_r^*\right] + \frac{\rho S C_{Y_\beta}}{2m}\sqrt{u^{*2}+w^{*2}}$
Y_p	$w^* + \frac{\rho V_a^* S b}{4m}C_{Y_p}$
Y_r	$-u^* + \frac{\rho V_a^* S b}{4m}C_{Y_r}$
Y_{δ_a}	$\frac{\rho V_a^{*2} S}{2m}C_{Y_{\delta_a}}$
Y_{δ_r}	$\frac{\rho V_a^{*2} S}{2m}C_{Y_{\delta_r}}$
L_v	$\frac{\rho S b^2 v^*}{4V_a^*}\left[C_{p_p}p^* + C_{p_r}r^*\right] + \rho S b v^*\left[C_{p_0} + C_{p_\beta}\beta^* + C_{p_{\delta_a}}\delta_a^* + C_{p_{\delta_r}}\delta_r^*\right] + \frac{\rho S b C_{p_\beta}}{2}\sqrt{u^{*2}+w^{*2}}$
L_p	$\Gamma_1 q^* + \frac{\rho V_a^* S b^2}{4}C_{p_p}$
L_r	$-\Gamma_2 q^* + \frac{\rho V_a^* S b^2}{4}C_{p_r}$
L_{δ_a}	$\frac{\rho V_a^{*2} S b}{2}C_{p_{\delta_a}}$
L_{δ_r}	$\frac{\rho V_a^{*2} S b}{2}C_{p_{\delta_r}}$
N_v	$\frac{\rho S b^2 v^*}{4V_a^*}\left[C_{r_p}p^* + C_{r_r}r^*\right] + \rho S b v^*\left[C_{r_0} + C_{r_\beta}\beta^* + C_{r_{\delta_a}}\delta_a^* + C_{r_{\delta_r}}\delta_r^*\right] + \frac{\rho S b C_{r_\beta}}{2}\sqrt{u^{*2}+w^{*2}}$
N_p	$\Gamma_7 q^* + \frac{\rho V_a^* S b^2}{4}C_{r_p}$
N_r	$-\Gamma_1 q^* + \frac{\rho V_a^* S b^2}{4}C_{r_r}$
N_{δ_a}	$\frac{\rho V_a^{*2} S b}{2}C_{r_{\delta_a}}$
N_{δ_r}	$\frac{\rho V_a^{*2} S b}{2}C_{r_{\delta_r}}$