

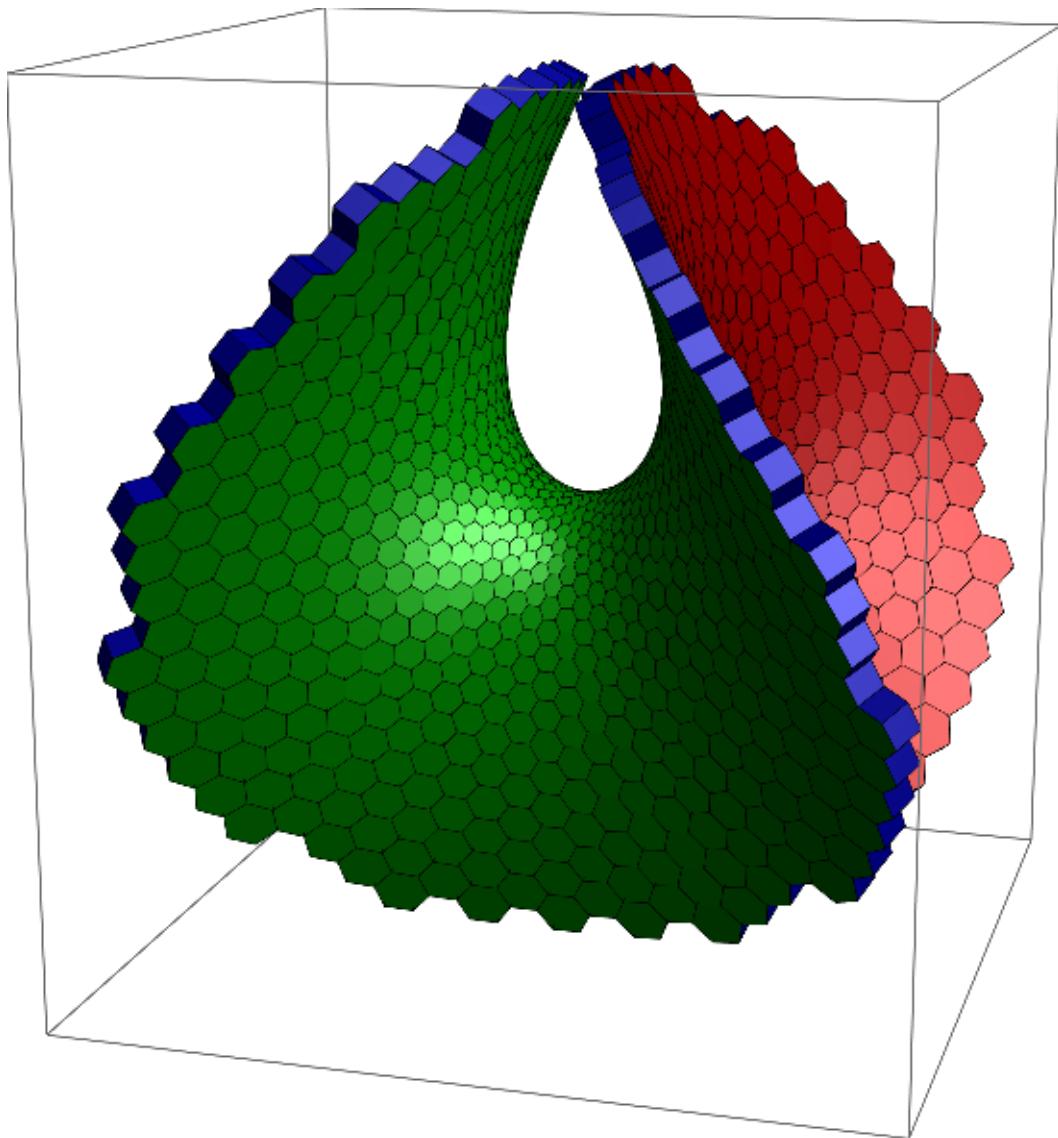
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In[5]:= ClearAll[hexhex2]
hexhex2[surface_, thickness_, planehex_] :=
Module[{surfacenormal, normals, midlayer, toplayer,
bottomlayer, topidx, bottomidx, mantles, B, p0, regmemb},
Quiet[Block[{X}, surfacenormal = X \!> Evaluate[Norm[Cross @@
Transpose[D[surface[{X[[1]], X[[2]]}], {{X[[1]], X[[2]]}, 1}]]]]];
midlayer = Map[surface, planehex, {2}];
normals = Map[surfacenormal, planehex, {2}];
toplayer = midlayer + 0.5 thickness normals;
bottomlayer = midlayer - 0.5 thickness normals;
topidx = Partition[Append[Range[6], 1], 2, 1];
bottomidx = Reverse /@ topidx;
mantles = Join[ArrayReshape[
toplayer[[All, Flatten[topidx]]], {Length[toplayer], 6, 2, 3}], ArrayReshape[
bottomlayer[[All, Flatten[bottomidx]]], {Length[bottomlayer], 6, 2, 3}], 3];
{toplayer, Flatten[mantles, 1], bottomlayer}];

In[7]:= ClearAll[hexhex]
hexhex[surface_, thickness_, meshsize_, region_] :=
Module[{hex0, centers0, centers, m, n, shifts, planehex, surfacenormal, normals,
midlayer, toplayer, bottomlayer, topidx, bottomidx, mantles, B, p0, regmemb},
B = BoundingRegion[DiscretizeRegion[region]];
p0 = RegionCentroid[B];
regmemb = RegionMember[region];
hex0 = Table[meshsize {Cos[Pi k / 3.], Sin[Pi k / 3.]}, {k, 0, 5}];
shifts =
MovingAverage[(2. meshsize) Table[{Cos[Pi k / 3.], Sin[Pi k / 3.]}, {k, -1, 1}], 2];
{m, n} = Max /@ Transpose[{Ceiling[Abs[LinearSolve[shifts, B[[2]] - p0]]],
Ceiling[Abs[LinearSolve[shifts, {B[[1, 2]], B[[2, 1]]} - p0]]]}];
centers0 = Plus[Flatten[Outer[List, Range[-m, m], Range[-n, n]], 1].shifts,
ConstantArray[p0, (2 m + 1) (2 n + 1)]];
centers = Pick[centers0, regmemb /@ centers0];
planehex = Outer[Plus, centers, hex0, 1];
Quiet[Block[{X}, surfacenormal = X \!> Evaluate[Norm[Cross @@
Transpose[D[surface[{X[[1]], X[[2]]}], {{X[[1]], X[[2]]}, 1}]]]]];
midlayer = Map[surface, planehex, {2}];
normals = Map[surfacenormal, planehex, {2}];
toplayer = midlayer + 0.5 thickness normals;
bottomlayer = midlayer - 0.5 thickness normals;
topidx = Partition[Append[Range[6], 1], 2, 1];
bottomidx = Reverse /@ topidx;
mantles =
Join[ArrayReshape[toplayer[[All, Flatten[topidx]]], {Length[toplayer], 6, 2, 3}],
ArrayReshape[bottomlayer[[All, Flatten[bottomidx]]],
{Length[bottomlayer], 6, 2, 3}], 3];
{toplayer, Flatten[mantles, 1], bottomlayer}]

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In[9]:= surface = Quiet[Block[{X, z, f, g}, f = z I - 1;
g = z I - z;
x I Evaluate[N@ComplexExpand[
{Re[Integrate[f[z] (1 - g[z]^2) / 2, z]], Re[Integrate[I f[z] (1 + g[z]^2) / 2,
z]], Re[Integrate[f[z] g[z], z]]} /. {z → (X[[1]] + I X[[2]])}]]]];
meshsize = 0.025;
thickness = 0.1;
region = Disk[{0., 0.}, 1.6];
data = hexhex[surface, thickness, meshsize, region];
Graphics3D[{Specularity[White, 30], EdgeForm[{Thin, Black}], Darker@Darker@Red,
Polygon[data[[1]]], Darker@Darker@Blue, Polygon[data[[2]]],
Darker@Darker@Green, Polygon[data[[3]]]}, Lighting → "Neutral"]
```



```
c = t \[Pi] (2 + Cos[5 t]) / 3 {Cos[t], Sin[t]};
region = Module[{pts, edges, B}, pts = Most@Table[c[t], {t, 0., 2. Pi, 2. Pi / 2000}];
edges = Append[Transpose[
{Range[1, Length[pts] - 1], Range[2, Length[pts]]}], {Length[pts], 1}];
BoundaryMeshRegion[pts, Line[edges]]];
data = hexhex[surface, thickness, 0.5 meshsize, region];
Graphics3D[{Specularity[White, 30], EdgeForm[{Thin, Black}], Darker@Darker@Red,
Polygon[data[[1]]], Darker@Darker@Blue, Polygon[data[[2]]],
Darker@Darker@Green, Polygon[data[[3]]]}, Lighting \[Rule] "Neutral"]
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