

PMR Forces in 2D (x , z)

Basic Formulas and Definitions:

$$\vec{F}_{\text{mag}} = \mu_0 (\vec{m}_{\text{eff}} \cdot \vec{\nabla}) \vec{H}$$

$$\vec{H} = H_x(x, z) \hat{x} + H_z(x, z) \hat{z}$$

Fields calculated from Lytvinov and Kryder

$$H_x = \frac{-8M_r}{\pi^2} \sum_n \left[\frac{1}{n} \cos\left(\frac{n\pi x}{a}\right) \left(1 - e^{-\frac{n\pi\delta}{a}}\right) e^{-\frac{n\pi z}{a}} \right]$$

$$H_z = \frac{-8M_r}{\pi^2} \sum_n \left[\frac{1}{n} \sin\left(\frac{n\pi x}{a}\right) \left(1 - e^{-\frac{n\pi\delta}{a}}\right) e^{-\frac{n\pi z}{a}} \right]$$

$$\vec{F}_{\text{mag}} = F_{\text{mag},x}(x, z) \hat{x} + F_{\text{mag},z}(x, z) \hat{z}$$

general formulation of effective dipole moment method (Furlani)

$$F_{\text{mag},x}(x, z) = \mu_0 V_p f(H) \left[H_x(x, z) \frac{\partial H_x(x, z)}{\partial x} + H_z(x, z) \frac{\partial H_x(x, z)}{\partial z} \right]$$

$$F_{\text{mag},z}(x, z) = \mu_0 V_p f(H) \left[H_x(x, z) \frac{\partial H_z(x, z)}{\partial x} + H_z(x, z) \frac{\partial H_z(x, z)}{\partial z} \right]$$

$$\vec{F}_{\text{mag}} = \mu_0 V_p (\vec{M}_{\text{eff}} \cdot \vec{\nabla}) \vec{H}$$

In general

$$f(H) = \begin{cases} \frac{3(\chi_p - \chi_f)}{(\chi_p - \chi_f) + 3} & H < \left(\frac{(\chi_p - \chi_f) + 3}{3\chi_p}\right) M_p \\ \frac{M_p}{H} & H \geq \left(\frac{(\chi_p - \chi_f) + 3}{3\chi_p}\right) M_p \end{cases} \quad (|\chi_f| \ll 1), \quad H = |\vec{H}|$$

$$\chi_f = \frac{\mu_p}{\mu_0 - 1} \quad \text{susceptibility of the fluid}$$

The intrinsic magnetic susceptibility of the particle, i.e. $M_p = \chi_p H_{\text{in}}$ where H_{in} field inside the particle different from H by demagnetizing field $H_{\text{in}} = H - N_d M_p$ and for spherical particle $N_d = 1/3$. The value of χ_p can be obtained from measured M v H curve (hysteresis) but after M is plotted as a function of H in which case $M_p = \chi_a H$ where χ_a is apparent susceptibility. The two values of susceptibility are related as

$$\chi_p = \frac{\chi_a}{(1 - N_d \chi_a)} \text{ reduces to } \chi_p = \frac{3\chi_a}{(3 - \chi_a)}$$

$$\text{let for a spherical particle} \quad H_{\text{Demag}} = \frac{M_p}{3}$$

$$f(H) = \begin{cases} 3 & H < \frac{M_p}{3} \\ \frac{M_p}{H} & \text{otherwise} \end{cases}$$

a = period spacing in nanometers [nm]

M_r = remanent magnetization in amperes/meter [A]/[m]

δ = height above the media in nanometers [nm]

μ_p = permeability for a particle

μ_0 = permiability of free space

$$V_p = \text{volume of a particle in namometers cubed } [\text{nm}]^3, \quad V_p = \frac{4}{3} \pi r^3$$

r = particle radius [nm]

$$\rho = \text{particle density } [g]/[\text{cm}]^3 \text{ or } [g]/[10^7 \text{ nm}]^3$$

H_z : Field in z direction

H_x : Field in x direction

H_{ZZ} : z component of Hz Field Gradient

H_{ZX} : x component of Hz Field Gradient

H_{XZ} : z component of Hx Field Gradient

H_{XX} : x component of Hx Field Gradient

(*r=200nm* pattern case)

$$Mr = 6.5 \times 10^5;$$

$$\mu_0 = 4 \pi 10^{-7};$$

$$\rho = 5.175;$$

$$V_p[r_] = \frac{4}{3} \pi r^3;$$

$$fH[H_, Mp_] := If\left[H < \frac{Mp}{3}, 3, \frac{Mp}{H}\right];$$

$$Hx[x_, z_] := \left(-\frac{8 Mr}{\pi^2}\right) \sum \left[\frac{1}{n} \cos\left(\frac{n \pi x}{a}\right) \left(1 - \text{Exp}\left[-\frac{n \pi \delta}{a}\right]\right) \text{Exp}\left[-\frac{n \pi z}{a}\right], \{n, 1, 50, 2\}\right]$$

$$Hz[x_, z_] := \left(\frac{8 Mr}{\pi^2}\right) \sum \left[\frac{1}{n} \sin\left(\frac{n \pi x}{a}\right) \left(1 - \text{Exp}\left[-\frac{n \pi \delta}{a}\right]\right) \text{Exp}\left[-\frac{n \pi z}{a}\right], \{n, 1, 50, 2\}\right]$$

$$HgXX[x_, z_] := \text{Evaluate}[D[Hx[x, z], x]];$$

$$HgXZ[x_, z_] := \text{Evaluate}[D[Hx[x, z], z]];$$

$$HgZX[x_, z_] := \text{Evaluate}[D[Hz[x, z], x]];$$

$$HgZZ[x_, z_] := \text{Evaluate}[D[Hz[x, z], z]];$$

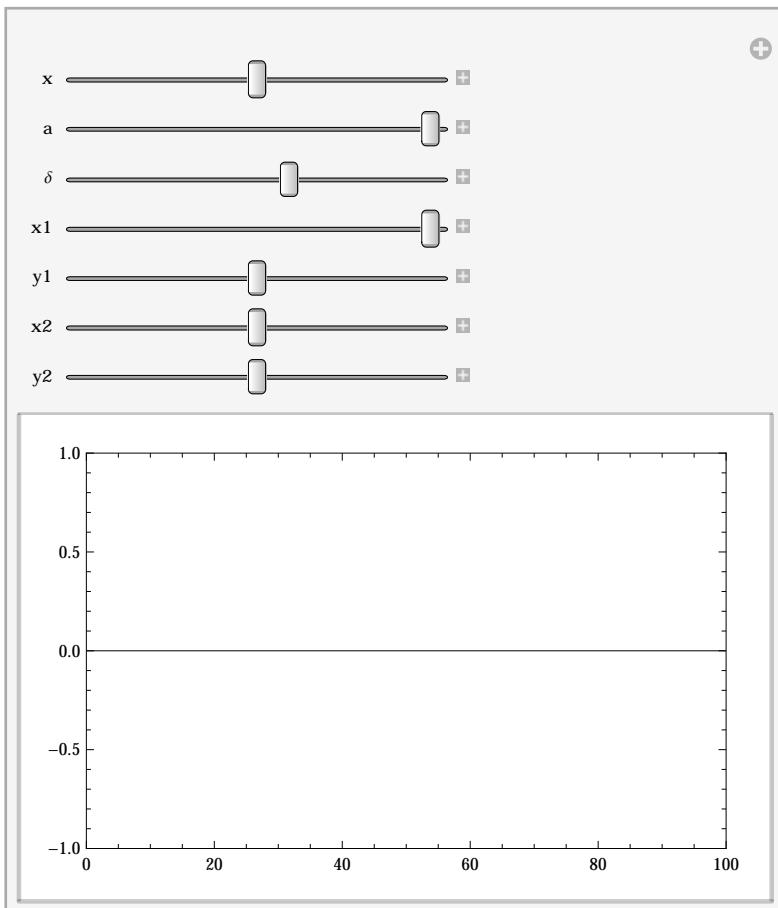
$$FmagX[x_, z_, r_, Mp_] :=$$

$$\mu_0 Vp[r] fH[\text{Sqrt}[Hx[x, z]^2 + Hz[x, z]^2], Mp] (Hx[x, z] HgXX[x, z] + Hz[x, z] HgXZ)$$

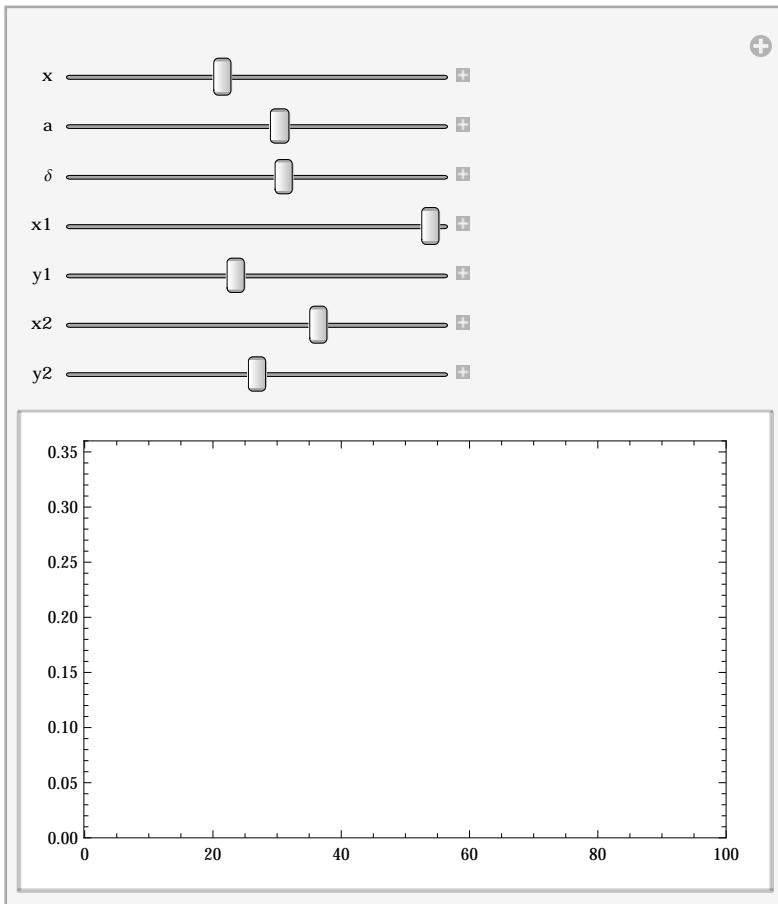
$$FmagZ[x_, z_, r_, Mp_] := \mu_0 Vp[r] fH[\text{Sqrt}[Hx[x, z]^2 + Hz[x, z]^2], Mp]$$

$$(Hx[x, z] HgZX[x, z] + Hz[x, z] HgZZ)$$

```
Manipulate[Plot[\mu0 Hz[x, z], {z, 1, 100},  
 PlotRange -> {{x1, y1}, {x2, y2}}, Frame -> True], {x, 0, 20}, {a, 1, 1500},  
 {\delta, 1, 50}, {x1, -100, 100}, {y1, -1, 1}, {x2, -100, 100}, {y2, -100, 100}]
```



```
Manipulate[Plot[\mu0 Hz[x, z], {z, 1, 100},  
 PlotRange -> {{x1, y1}, {x2, y2}}, Frame -> True], {x, 0, 20}, {a, 1, 1500},  
 {\delta, 1, 50}, {x1, -100, 100}, {y1, -1, 1}, {x2, -1, 1}, {y2, -100, 100}]
```



a = 1500

1500

δ = 50

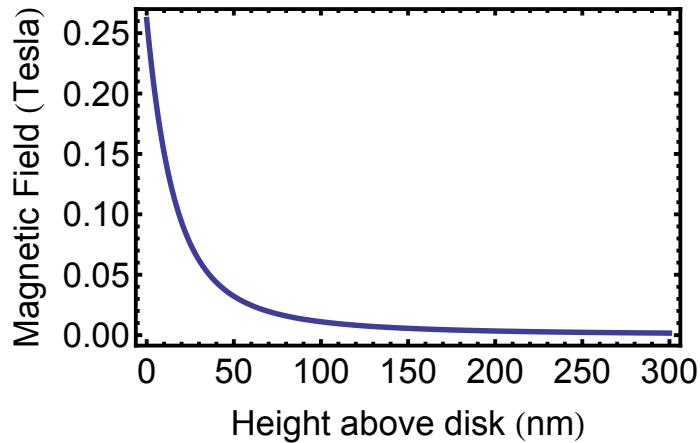
50

x = 10

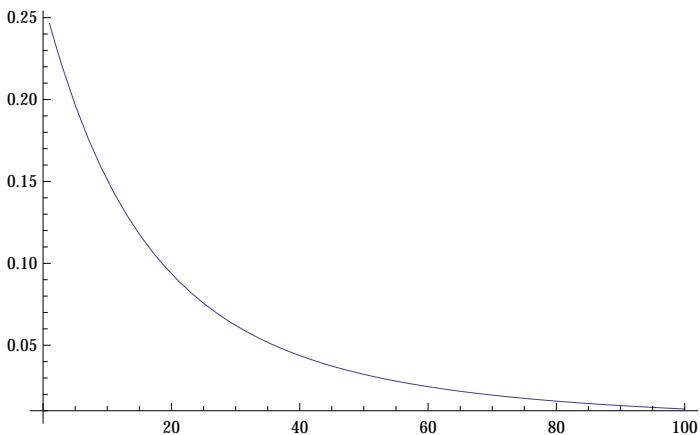
10

Clear[a, x, δ]

```
Plot[ $\mu_0$  Hz[10, z], {z, 0, 300}, PlotRange -> All, Frame -> True,
FrameLabel -> {"Height above disk (nm)", "Magnetic Field (Tesla)" },
PlotStyle -> {{Thickness -> 0.01}}, GridLines -> None,
FrameStyle -> {{Directive[Thick, FontFamily -> "Arial", Black, 18],
Directive[Thick, FontFamily -> "Arial", Black, 18],
Directive[Thick, FontFamily -> "Arial", Black, 18],
Directive[Thick, FontFamily -> "Arial", Black, 18]}]}
```

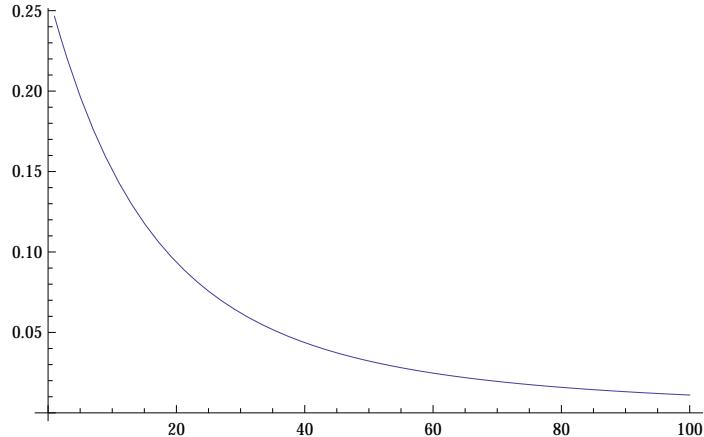


```
Plot[ $\mu_0 \left( \frac{8 Mr}{\pi^2} \right) \sum \left[ \frac{1}{n} \sin \left[ \frac{n \pi x}{a} \right] \left( 1 - \text{Exp} \left[ -\frac{n \pi \delta}{a} \right] \right) \text{Exp} \left[ -\frac{n \pi z}{a} \right]$ , {n, 1, 50, 2}], {z, 1, 100}, PlotRange -> All]
```

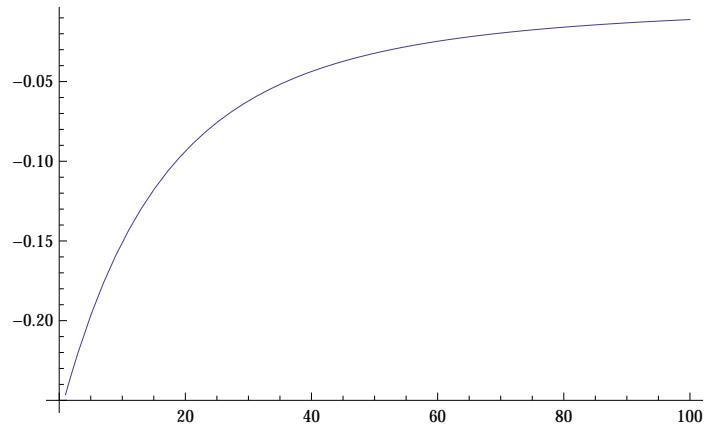


```
Plot[ $\left( \frac{8 Mr}{\pi^2} \right) \sum \left[ \frac{1}{n} \sin \left[ \frac{n \pi z}{a} \right] \left( 1 - \text{Exp} \left[ -\frac{n \pi \delta}{a} \right] \right) \text{Exp} \left[ -\frac{n \pi z}{a} \right]$ , {n, 1, 50, 2}]
```

```
Plot[μ0 (8 Mr / π2) Sum[1/n Sin[n π 10 / 1500] (1 - Exp[-n π 50 / 1500]) Exp[-n π z / 1500], {n, 1, 50, 2}], {z, 1, 100}, PlotRange → Automatic]
```



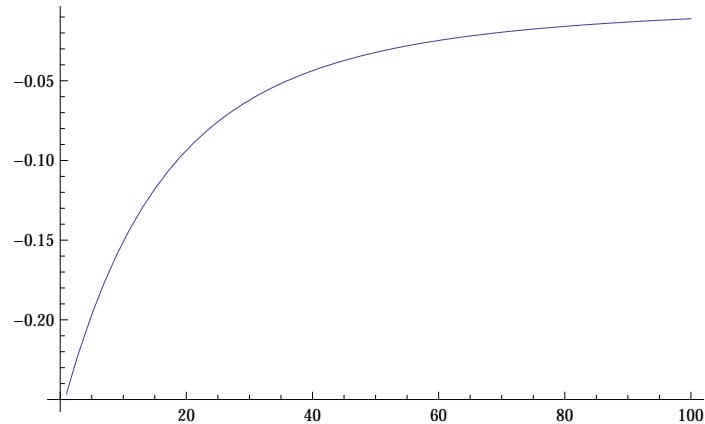
```
Plot[-μ0 (8 Mr / π2) Sum[1/n Sin[n π 10 / 1500] (1 - Exp[-n π 50 / 1500]) Exp[-n π z / 1500], {n, 1, 50, 2}], {z, 1, 100}, PlotRange → All]
```



```

Plot[μ0 (-(8*6.5*10^5)/π^2) *
  Sum[((1/n)*Sin[(n*Pi*10)/1500]*(1-Exp[-Pi*(n/1500)*50])*
    Exp[-Pi*(n/1500)*z]), {n, 1, 50, 2}], {z, 1, 100}, PlotRange -> All]

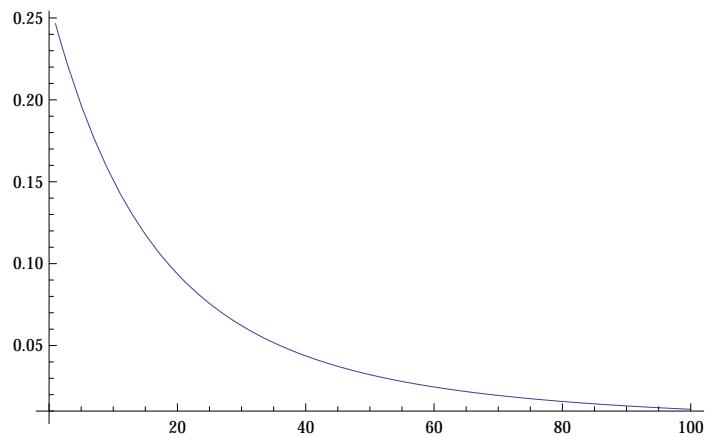
```



```

Plot[μ0 (8*6.5*10^5/π^2) *
  Sum[((1/n)*Sin[(n*Pi*10)/1500]*(1-Exp[-Pi*(n/1500)*50])*
    Exp[-Pi*(n/1500)*z]), {n, 1, 50, 2}], {z, 1, 100}, PlotRange -> All]

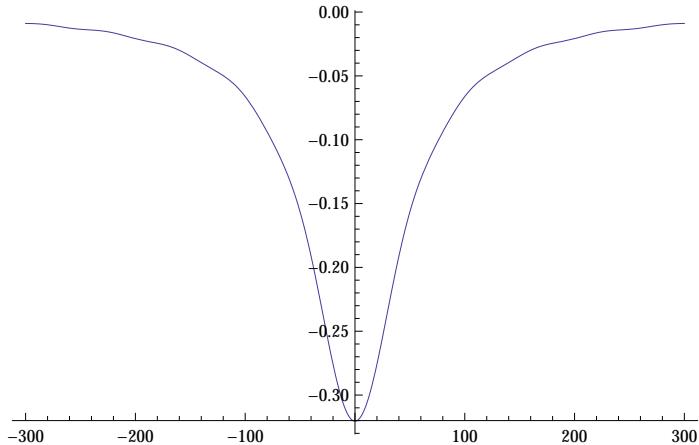
```



```

Plot[μ0 (-(8 * 6.5 * 105) / π2) *
  Sum[((1 / n) * Cos[(n * Pi * x) / 1500] * (1 - Exp[-Pi * (n / 1500) * 50]) *
    Exp[-Pi * (n / 1500) * 30]), {n, 1, 50, 2}], {x, -300, 300}, PlotRange -> All]

```



```

Plot[μ0 (8 mr / π2) *
  Sum[((1 / n) * Sin[(n * Pi * 10) / a] * (1 - Exp[-Pi * (n / a) * δ]) * Exp[-Pi * (n / a) * z]), {n, 1, 50, 2}], {z, 1, 100}]

```

