PMR Forces in 2D (x_,z_)

Basic Formulas and Definitions:

$$\vec{F}_{\text{mag}} = \mu_0 \left(\vec{m}_{\text{eff}} \cdot \vec{\nabla} \right) \vec{H}$$
$$\vec{H} = H_x(x,\, z)\, \hat{x} + H_z(x,\, z)\, \hat{z}$$

Fields calculated from Lytvinov and Kryder

$$H_x = \frac{-8\, M_r}{\pi^2} \sum_n \left[\frac{1}{n} \cos\left(\frac{\text{n}\pi\text{x}}{a}\right) \left(1 - e^{-\frac{\text{n}\pi\delta}{a}}\right) e^{-\frac{\text{n}\pi\text{z}}{a}} \right]$$
$$H_z = \frac{8\, M_r}{\pi^2} \sum_n \left[\frac{1}{n} \sin\left(\frac{\text{n}\pi\text{x}}{a}\right) \left(1 - e^{-\frac{\text{n}\pi\delta}{a}}\right) e^{-\frac{\text{n}\pi\text{z}}{a}} \right]$$
$$\vec{F}_{\text{mag}} = F_{\text{mag},x}(x,\, z)\, \hat{x} + F_{\text{mag},z}(x,\, z)\, \hat{z}$$

genaral formulation of effective dipole moment method (Furlani)

$$F_{\text{mag},x}(x,\, z) = \mu_0\, V_p\, f(H) \left[H_x(x,\, z)\, \frac{\partial H_x(x,z)}{\partial x} + H_z(x,\, z)\, \frac{\partial H_x(x,z)}{\partial z} \right]$$
$$F_{\text{mag},z}(x,\, z) = \mu_0\, V_p\, f(H) \left[H_x(x,\, z)\, \frac{\partial H_z(x,z)}{\partial x} + H_z(x,\, z)\, \frac{\partial H_z(x,z)}{\partial z} \right]$$

$$\vec{F}_{\text{mag}} = \mu_0\, V_P \left(\vec{M}_{\text{eff}} \cdot \vec{\nabla} \right) \vec{H}$$

In general

$$f(H) = \begin{cases} \frac{3\,(\chi_p - \chi_f)}{(\chi_p - \chi_f) + 3} & H < \left(\frac{(\chi_p - \chi_f) + 3}{3\, \chi_p}\right) M_p \\ \frac{M_p}{H} & H \geq \left(\frac{(\chi_p - \chi_f) + 3}{3\, \chi_p}\right) M_p \end{cases} \quad (|\chi_f| \ll 1) \quad , \quad H = |\vec{H}|$$

$\chi_{f = \frac{\mu_p}{\mu_0 - 1}}$ susceptibility of the fluild

The intrinsic magnetic susceptibility of the particle, i.e. $M_p = \chi_p\, H_{\text{in}}$ where H_{in} field inside the particle different from H by demagnetizing field $H_{\text{in}} = H - N_d\, M_p$ and for spherical particle $N_d = 1/3$. The value of χ_p can be obtaind from measured M v H cureve (hysteresis) but ofter M is plotted as a function of H in which case $M_p = \chi_a\, H$ where $\chi_a$is apparent susceptibility. The two values of susceptibility are related as $\chi_p = \frac{\chi_a}{(1 - N_d\, \chi_a)}$ reduces to $\chi_p = \frac{3\, \chi_a}{(3 - \chi_a)}$

let for a spherical particle $H_{\text{Demag}} = \frac{M_p}{3}$

$$f(H) = \begin{cases} 3 & H < \frac{M_p}{3} \\ \frac{M_p}{H} & \text{otherwise} \end{cases}$$

a = period spacing in nanometers [nm]
M_r = remanent magnetization in amperes/meter $[A]/[m]$
δ = height above the media in nanometers [nm]
μ_p= permeability for a particle
μ_0= permiability of free space
V_p= volume of a particle in namometers cubed $[\text{nm}]^3$, $\quad V_p = \frac{4}{3}\pi r^3$
r = particle radius [nm]
ρ = particle density $[g]/[\text{cm}]^3$or $[g]\Big/\left[10^7\ \text{nm}\right]^3$
H_z: Field in z direction
H_x: Field in x direction
HgZZ: z component of Hz Field Gradient
HgZX: x component of Hz Field Gradient
HgXZ: z component of Hx Field Gradient
HgXX: x component of Hz Field Gradient

```
In[22]:= Mr = 6.5 × 10^5;
μ0 = 4 Pi 10^-7;
ρ = 5.175;
(*r=200nm* pattent case*)
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In[25]:= Vp[r_] = 4/3 π r^3;
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In[26]:= fH[H_, Mp_,] := If[H < Mp/3, 3, Mp/H];
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In[27]:= Hx[x_, z_] := (-(8 Mr)/π^2) Sum[1/n Cos[(n π x)/a] (1 - Exp[-(n π δ)/a]) Exp[-(n π z)/a], {n, 1, 50, 2}]
Hz[x_, z_] := ((8 Mr)/π^2) Sum[1/n Sin[(n π x)/a] (1 - Exp[-(n π δ)/a]) Exp[-(n π z)/a], {n, 1, 50, 2}]
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```
In[29]:= HgXX[x_, z_] := Evaluate[D[Hx[x, z], x]];
HgXZ[x_, z_] := Evaluate[D[Hx[x, z], z]];
HgZX[x_, z_] := Evaluate[D[Hz[x, z], x]];
HgZZ[x_, z_] := Evaluate[D[Hz[x, z], z]];
```

```
In[33]:= FmagX[x_, z_, r_, Mp_] :=
  μ0 Vp[r] fH[Sqrt[Hx[x, z]^2 + Hz[x, z]^2], Mp] ( Hx[x, z] HgXX[x, z] + Hz[x, z] HgXZ)
FmagZ[x_, z_, r_, Mp_] := μ0 Vp[r] fH[Sqrt[Hx[x, z]^2 + Hz[x, z]^2], Mp]
   ( Hx[x, z] HgZX[x, z] + Hz[x, z] HgZZ)
```

Magnetic Field Plot of Hz in x and z direction:

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a = 1500;
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δ = 50;
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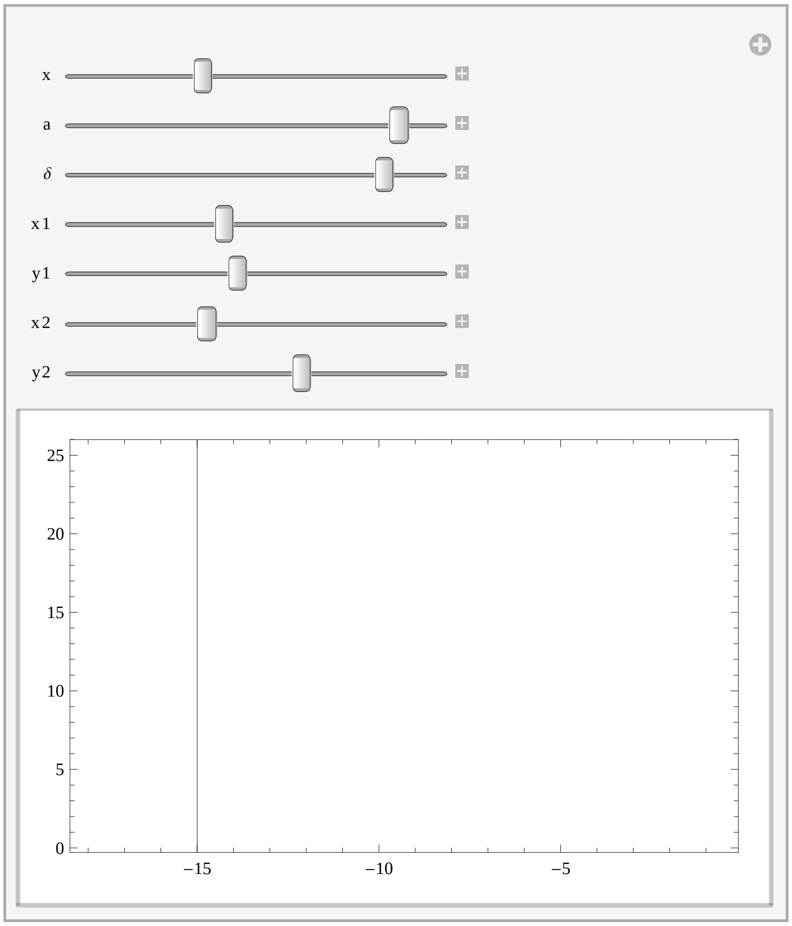
In[35]:=
```
Manipulate[Plot[μ0 Hz[x, z], {z, 1, 100},
  PlotRange → {{x1, y1}, {x2, y2}}, Frame → True], {x, 0, 20}, {a, 1, 1500},
 {δ, 1, 50}, {x1, -100, 100}, {y1, -1, 1}, {x2, -1, 1}, {y2, -100, 100}]
```

Out[35]=



In[36]:=
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Clear [a, x, δ]
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for fixed x and delta but changing a

In[37]:=
```
Manipulate[
 Plot[μ0 (8 Mr/π^2) Sum[1/n Sin[n π 10/a] (1 - Exp[-n π 50/a]) Exp[-n π z/a], {n, 1, 50, 2}],
  {z, 1, 100}, PlotRange → {{x1, y1}, {x2, y2}}, Frame → True],
 {a, 1, 1420}, {x1, -100, 100}, {y1, -1, 1}, {x2, -1, 1}, {y2, -100, 100}]
```

Out[37]=