Karlqvist Perpendicular and Longitudnal Head Equations

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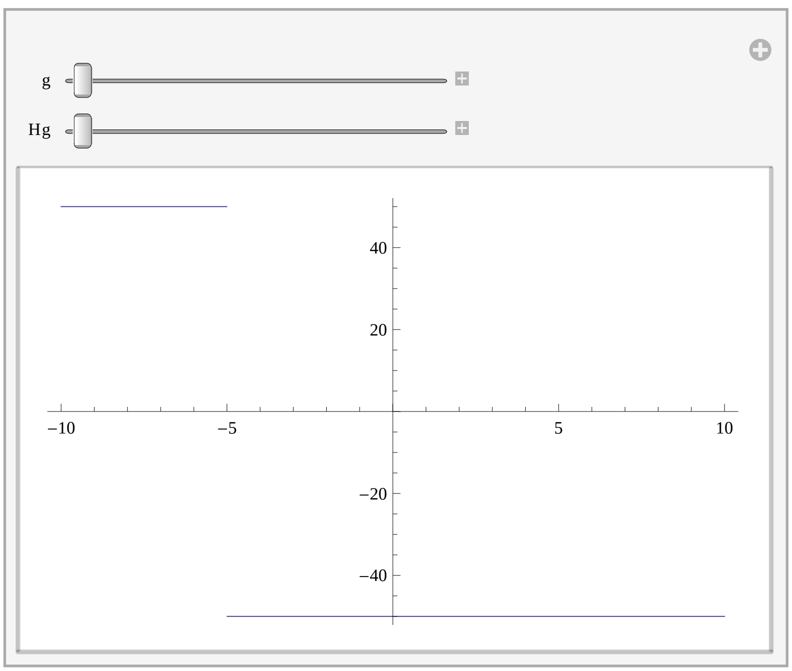
Magnetic Potential Deep accros the head gap:

```
Φ[x_] :=
 Piecewise[{{-Hg x,  -g / 2 <= x <= g / 2}, {-g Hg / 2, x >= g / 2}, {g Hg / 2,  x < -g / 2}}]
```

In[15]:=
```
Manipulate[Plot[
  Piecewise[{{-Hg x,  -g / 2 <= x <= g / 2}, {-g Hg / 2, x >= g / 2}, {g Hg / 2,  x < -g / 2}}],
  {x, -10, 10}], {g, -10, 10}, {Hg, -10, 10}]
```

Out[15]=



Attempts to integrate:

$$\text{Int1} = \text{Integrate}\Big[\frac{\text{y1}}{\pi}\ \frac{\Phi[\text{x}]}{(\text{x1}-\text{x})^2+\text{y1}^2},$$
$$\{\text{x}, -\text{Infinity}, \text{Infinity}\}, \text{Assumptions} \to \text{x1} > 0\ \&\&\ \text{y1} > 0\Big]$$

$$\begin{cases} -\frac{\text{g Hg}\left(\pi-2\,\text{ArcCot}\left[\frac{2\,\text{y1}}{\text{g}-2\,\text{x1}}\right]+2\,\text{ArcTan}\left[\frac{2\,\text{y1}}{\text{g}-2\,\text{x1}}\right]\right)}{4\,\pi} & \text{g} \le 0 \\ -\frac{1}{4\,\pi}\,\text{Hg}\left(\text{g}\,\pi-2\,\text{g}\,\text{ArcCot}\left[\frac{2\,\text{y1}}{\text{g}-2\,\text{x1}}\right]+4\,\text{x1}\,\text{ArcCot}\left[\frac{2\,\text{y1}}{\text{g}-2\,\text{x1}}\right]+\right. & \text{True} \\ \quad \left.4\,\text{x1}\,\text{ArcCot}\left[\frac{2\,\text{y1}}{\text{g}+2\,\text{x1}}\right]-2\,\text{g}\,\text{ArcTan}\left[\frac{2\,\text{y1}}{\text{g}+2\,\text{x1}}\right]-4\,\text{y1}\,\text{ArcTanh}\left[\frac{4\,\text{g}\,\text{x1}}{\text{g}^2+4\,(\text{x1}^2+\text{y1}^2)}\right]\right) & \end{cases}$$

This does not give the cleare solution mathematica doesn t know so one has to do following

$$\int_{-\infty}^{-g/2}\frac{\text{y1}}{\pi}\,\frac{\Phi[\text{x\_}]}{(\text{x1}-x)^2+\text{y1}^2}\,dx+\int_{-g/2}^{g/2}\frac{\text{y1}}{\pi}\,\frac{\Phi[\text{x\_}]}{(\text{x1}-x)^2+\text{y1}^2}\,dx+\int_{g/2}^{\infty}\frac{\text{y1}}{\pi}\,\frac{\Phi[\text{x\_}]}{(\text{x1}-x)^2+\text{y1}^2}\,dx$$

$$\text{a1} = \text{Integrate}\Big[\frac{\text{y1}}{\text{Pi}}\ \frac{\text{g Hg}\ /\ 2}{(-\text{x}+\text{x1})^2+\text{y1}^2},$$
$$\{\text{x}, -\text{Infinity}, -\text{g}/2\}, \text{Assumptions} \to \text{x1} > 0\ \&\&\ \text{y1} > 0\Big]\ //\ \text{Expand}$$

$$\frac{\text{g Hg}}{4}-\frac{\text{g Hg ArcCot}\left[\frac{2\,\text{y1}}{\text{g}+2\,\text{x1}}\right]}{2\,\pi}$$

$$\text{a2} = \text{Integrate}\Big[\frac{\text{y1}}{\text{Pi}}\ \frac{-\text{g Hg}\ /\ 2}{(-\text{x}+\text{x1})^2+\text{y1}^2},$$
$$\{\text{x}, \text{g}/2, \text{Infinity}\}, \text{Assumptions} \to \text{x1} > 0\ \&\&\ \text{y1} > 0\Big]\ //\ \text{Expand}$$

$$-\frac{\text{g Hg}}{4}+\frac{\text{g Hg ArcCot}\left[\frac{2\,\text{y1}}{\text{g}-2\,\text{x1}}\right]}{2\,\pi}$$

$$\mathbf{a3 = Integrate\left[\frac{y1}{Pi}\ \frac{-Hg\ x}{(-x + x1)^2 + y1^2},\ \{x,\ -g/2,\ g/2\},\ Assumptions \to x1 > 0\ \&\&\ y1 > 0\right]}$$

$$\text{ConditionalExpression}\Big[-\frac{1}{2\pi}\ \text{Hg}\ \Big(2\ \text{x1 ArcTan}\left[\frac{g - 2\ \text{x1}}{2\ \text{y1}}\right] + 2\ \text{x1 ArcTan}\left[\frac{g + 2\ \text{x1}}{2\ \text{y1}}\right] + \text{y1}\ \left(\text{Log}\left[g^2 - 4\ g\ \text{x1} + 4\ \left(\text{x1}^2 + \text{y1}^2\right)\right] - \text{Log}\left[g^2 + 4\ g\ \text{x1} + 4\ \left(\text{x1}^2 + \text{y1}^2\right)\right]\right)\Big),$$
$$\left(\text{x1 Im}\left[\frac{1}{g}\right] \neq \text{y1 Re}\left[\frac{1}{g}\right]\ ||\ 1 + 2\ \text{y1 Im}\left[\frac{1}{g}\right] + 2\ \text{x1 Re}\left[\frac{1}{g}\right] < 0\ ||\ 2\ \text{y1 Im}\left[\frac{1}{g}\right] + 2\ \text{x1 Re}\left[\frac{1}{g}\right] > 1\right)\ \&\&$$
$$\left(\text{x1 Im}\left[\frac{1}{g}\right] + \text{y1 Re}\left[\frac{1}{g}\right] \neq 0\ ||\ 1 + 2\ \text{y1 Im}\left[\frac{1}{g}\right] < 2\ \text{x1 Re}\left[\frac{1}{g}\right]\ ||\ 1 + 2\ \text{x1 Re}\left[\frac{1}{g}\right] < 2\ \text{y1 Im}\left[\frac{1}{g}\right]\right)\ \&\&$$
$$\left(\frac{\text{x1}^2\ \text{Im}[g]^2}{\text{y1}^2\ \text{Re}[g]^2} == 1\ ||\ \text{y1}^2 \geq \frac{\text{x1}^2\ \text{Im}[g]^2}{\text{Re}[g]^2}\ ||\ \frac{2\ \text{x1}}{\text{Re}[g]} \geq 1\ ||\ 1 + \frac{2\ \text{x1}}{\text{Re}[g]} \leq 0\right)\Big]$$

But this last expression doesn t seem to bee correct, I thing i need to use another method suggested by Hanz from wolfram.

$$\mathbf{j1 = \frac{1}{\pi} y1\ Integrate\left[\frac{g\ Hg\ /\ 2}{(-x + x1)^2 + y1^2},\ x,\ Assumptions \to x1 > 0\ \&\&\ y1 > 0\right]}$$

$$\frac{g\ \text{Hg ArcTan}\left[\frac{x - \text{x1}}{\text{y1}}\right]}{2\pi}$$

$$\mathbf{j2 = \frac{1}{\pi} y1\ Integrate\left[\frac{-\ Hg\ x}{(-x + x1)^2 + y1^2},\ x,\ Assumptions \to x1 > 0\ \&\&\ y1 > 0\right]}$$

$$-\frac{\text{Hg y1}\ \left(\frac{\text{x1 ArcTan}\left[\frac{x - \text{x1}}{\text{y1}}\right]}{\text{y1}} + \frac{1}{2}\ \text{Log}\left[(x - \text{x1})^2 + \text{y1}^2\right]\right)}{\pi}$$

$$\mathbf{j3 = \frac{1}{\pi} y1\ Integrate\left[\frac{-g\ Hg\ /\ 2}{(-x + x1)^2 + y1^2},\ x,\ Assumptions \to x1 > 0\ \&\&\ y1 > 0\right]}$$

$$-\frac{g\ \text{Hg ArcTan}\left[\frac{x - \text{x1}}{\text{y1}}\right]}{2\pi}$$

$$\mathbf{t1 = Simplify[(j1\ /.\ x \to -g/2) - (j1\ /.\ x \to -\infty),\ y1 > 0]\ //\ Expand}$$

$$\frac{g\ \text{Hg}}{4} - \frac{g\ \text{Hg ArcTan}\left[\frac{g + 2\ \text{x1}}{2\ \text{y1}}\right]}{2\pi}$$

```
t3 = Simplify[(j3 /. x → ∞) - (j3 /. x → g / 2), y1 > 0] // Expand
```

$$-\frac{g\,Hg}{4}+\frac{g\,Hg\,\text{ArcTan}\left[\frac{g-2\,x1}{2\,y1}\right]}{2\,\pi}$$

```
t2 = Collect[(j2 /. x → g / 2) - (j2 /. x → -g / 2) // Factor, y1] /.
  Log[u_] - Log[v_] → Log[u / v]
```

$$\frac{Hg\left(2\,x1\,\text{ArcTan}\left[\frac{-\frac{g}{2}-x1}{y1}\right]-2\,x1\,\text{ArcTan}\left[\frac{\frac{g}{2}-x1}{y1}\right]\right)}{2\,\pi}+\frac{Hg\,y1\,\text{Log}\left[\frac{\left(-\frac{g}{2}-x1\right)^2+y1^2}{\left(\frac{g}{2}-x1\right)^2+y1^2}\right]}{2\,\pi}$$

```
t1 + t2 + t3 // Simplify
```

$$\frac{1}{2\,\pi}Hg\left((g-2\,x1)\,\text{ArcTan}\left[\frac{g-2\,x1}{2\,y1}\right]-(g+2\,x1)\,\text{ArcTan}\left[\frac{g+2\,x1}{2\,y1}\right]+y1\,\text{Log}\left[\frac{\frac{1}{4}\,(g+2\,x1)^2+y1^2}{\frac{1}{4}\,(g-2\,x1)^2+y1^2}\right]\right)$$

$$-\partial_g\frac{1}{2\,\pi}$$
$$Hg\left((g-2\,x1)\,\text{ArcTan}\left[\frac{g-2\,x1}{2\,y1}\right]-(g+2\,x1)\,\text{ArcTan}\left[\frac{g+2\,x1}{2\,y1}\right]+y1\,\text{Log}\left[\frac{\frac{1}{4}\,(g+2\,x1)^2+y1^2}{\frac{1}{4}\,(g-2\,x1)^2+y1^2}\right]\right)$$

$$-\frac{1}{2\,\pi}Hg\left(\frac{g-2\,x1}{2\left(1+\frac{(g-2\,x1)^2}{4\,y1^2}\right)y1}-\frac{g+2\,x1}{2\left(1+\frac{(g+2\,x1)^2}{4\,y1^2}\right)y1}+\right.$$
$$\left(y1\left(\frac{1}{4}\,(g-2\,x1)^2+y1^2\right)\left(\frac{g+2\,x1}{2\left(\frac{1}{4}\,(g-2\,x1)^2+y1^2\right)}-\frac{(g-2\,x1)\left(\frac{1}{4}\,(g+2\,x1)^2+y1^2\right)}{2\left(\frac{1}{4}\,(g-2\,x1)^2+y1^2\right)^2}\right)\right)\Big/$$
$$\left.\left(\frac{1}{4}\,(g+2\,x1)^2+y1^2\right)+\text{ArcTan}\left[\frac{g-2\,x1}{2\,y1}\right]-\text{ArcTan}\left[\frac{g+2\,x1}{2\,y1}\right]\right)$$

Last expression is what looks like book s̈
The next expression is also correct,

```
t1 + t2 + t3 // FullSimplify
```

$$\frac{1}{2\,\pi}$$
$$Hg\left((g-2\,x1)\,\text{ArcCot}\left[\frac{2\,y1}{g-2\,x1}\right]-(g+2\,x1)\,\text{ArcCot}\left[\frac{2\,y1}{g+2\,x1}\right]+y1\,\text{Log}\left[1+\frac{8\,g\,x1}{(g-2\,x1)^2+4\,y1^2}\right]\right)$$

Derivatives:

$$\Phi[\text{x1\_}, \text{y1\_}] := \frac{1}{2\pi} \text{Hg}\left((g - 2\,x1)\,\text{ArcTan}\left[\frac{g - 2\,x1}{2\,y1}\right] - (g + 2\,x1)\,\text{ArcTan}\left[\frac{g + 2\,x1}{2\,y1}\right] + y1\,\text{Log}\left[\frac{\frac{1}{4}\,(g + 2\,x1)^2 + y1^2}{\frac{1}{4}\,(g - 2\,x1)^2 + y1^2}\right]\right);$$

```
H[x1_] := -D[Φ[x1, y1], x1];
H[x1]
```

$$-\frac{1}{2\pi}\text{Hg}\left(-\frac{g - 2\,x1}{\left(1 + \frac{(g-2\,x1)^2}{4\,y1^2}\right)y1} - \frac{g + 2\,x1}{\left(1 + \frac{(g+2\,x1)^2}{4\,y1^2}\right)y1} + \left(y1\left(\frac{1}{4}\,(g - 2\,x1)^2 + y1^2\right)\left(\frac{g + 2\,x1}{\frac{1}{4}\,(g - 2\,x1)^2 + y1^2} - \frac{(-g + 2\,x1)\left(\frac{1}{4}\,(g + 2\,x1)^2 + y1^2\right)}{\left(\frac{1}{4}\,(g - 2\,x1)^2 + y1^2\right)^2}\right)\right) \Big/ \left(\frac{1}{4}\,(g + 2\,x1)^2 + y1^2\right) - 2\,\text{ArcTan}\left[\frac{g - 2\,x1}{2\,y1}\right] - 2\,\text{ArcTan}\left[\frac{g + 2\,x1}{2\,y1}\right]\right)$$

```
Simplify[%31]
```

$$\frac{\text{Hg}\left(\text{ArcTan}\left[\frac{g-2\,x1}{2\,y1}\right] + \text{ArcTan}\left[\frac{g+2\,x1}{2\,y1}\right]\right)}{\pi}$$

$$\text{Hx}[\text{x1\_}, \text{y1\_}] = \frac{\text{Hg}\left(\text{ArcTan}\left[\frac{g-2\,x1}{2\,y1}\right] + \text{ArcTan}\left[\frac{g+2\,x1}{2\,y1}\right]\right)}{\pi};$$

```
Hy[x1_, y1_] = -D[Φ[x1, y1], y1];
```

```
Hy[x1, y1]
```

$$-\frac{1}{2\pi}\,\mathrm{Hg}\left(-\frac{(g-2\,x1)^2}{2\left(1+\frac{(g-2\,x1)^2}{4\,y1^2}\right)y1^2}+\frac{(g+2\,x1)^2}{2\left(1+\frac{(g+2\,x1)^2}{4\,y1^2}\right)y1^2}+\left(y1\left(\frac{1}{4}(g-2\,x1)^2+y1^2\right)\left(\frac{2\,y1}{\frac{1}{4}(g-2\,x1)^2+y1^2}-\frac{2\,y1\left(\frac{1}{4}(g+2\,x1)^2+y1^2\right)}{\left(\frac{1}{4}(g-2\,x1)^2+y1^2\right)^2}\right)\right)\Big/\left(\frac{1}{4}(g+2\,x1)^2+y1^2\right)+\mathrm{Log}\left[\frac{\frac{1}{4}(g+2\,x1)^2+y1^2}{\frac{1}{4}(g-2\,x1)^2+y1^2}\right]\right)$$

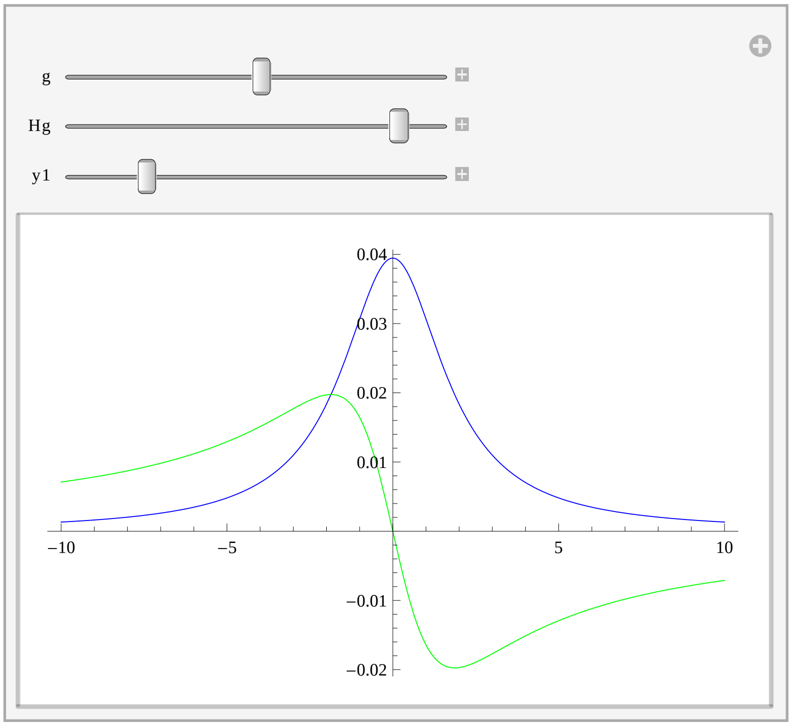
```
Simplify[%40]
```

$$-\frac{\mathrm{Hg}\,\mathrm{Log}\left[\frac{\frac{1}{4}(g+2\,x1)^2+y1^2}{\frac{1}{4}(g-2\,x1)^2+y1^2}\right]}{2\pi}$$

$$\mathbf{Hy[x1\_,\ y1\_]\ :=\ -\frac{Hg\ Log\left[\frac{\frac{1}{4}(g+2\,x1)^2+y1^2}{\frac{1}{4}(g-2\,x1)^2+y1^2}\right]}{2\pi};}$$

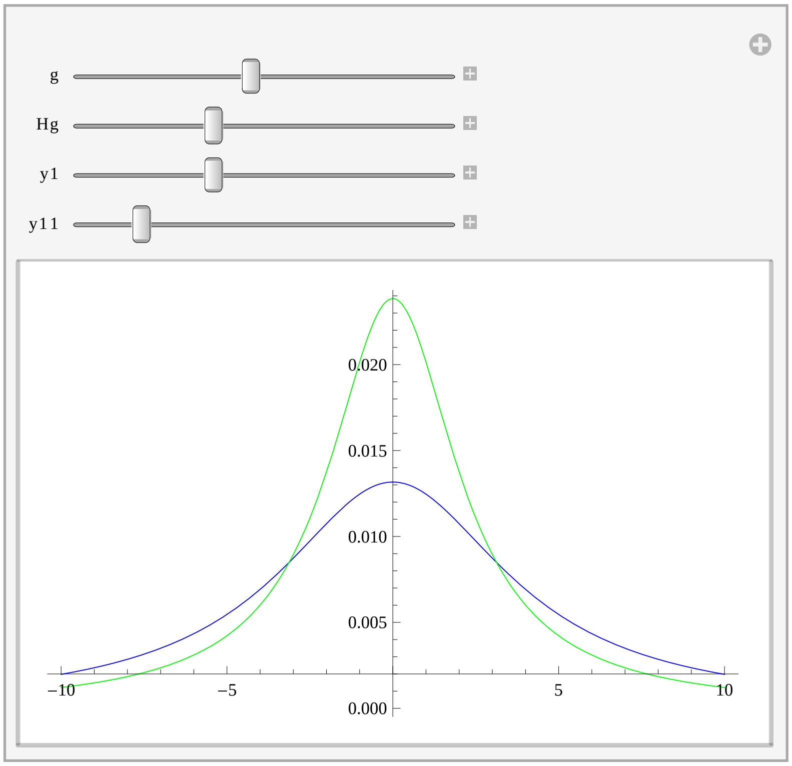
Plots:

```
Manipulate[
 Show[{Plot[Hg (ArcTan[(g-2 x1)/(2 y1)] + ArcTan[(g+2 x1)/(2 y1)])/π, {x1, -10, 10}, PlotStyle → Blue],
   Plot[-(Hg Log[(1/4 (g+2 x1)^2+y1^2)/(1/4 (g-2 x1)^2+y1^2)])/(2 π), {x1, -10, 10}, PlotStyle → Green]},
  PlotRange → Automatic], {g, -8, 8}, {Hg, -1, 1}, {y1, 0, 10}]
```

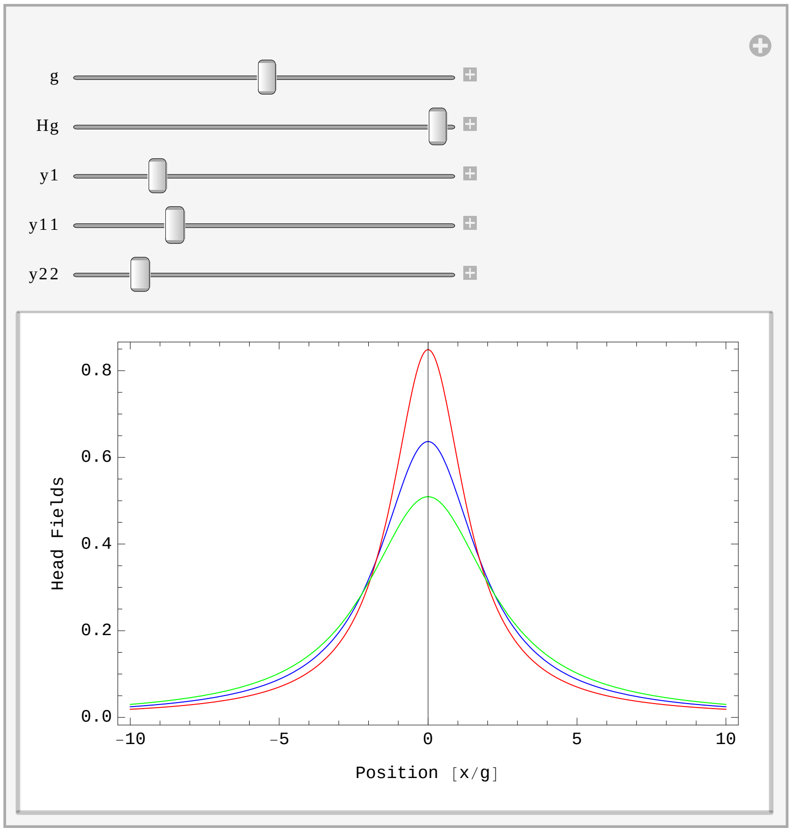


Book also compares two values for g for each type of recording so let’s try that in manipulate:

```
Manipulate[
 Show[{Plot[Hg (ArcTan[(g-2 x1)/(2 y1)] + ArcTan[(g+2 x1)/(2 y1)])/π, {x1, -10, 10}, PlotStyle → Blue],
   Plot[Hg (ArcTan[(g-2 x11)/(2 y11)] + ArcTan[(g+2 x11)/(2 y11)])/π, {x11, -10, 10}, PlotStyle → Green]},
  PlotRange → Automatic], {g, -8, 8}, {Hg, -1, 1}, {y1, 1, 10}, {y11, 1, 10}]
```

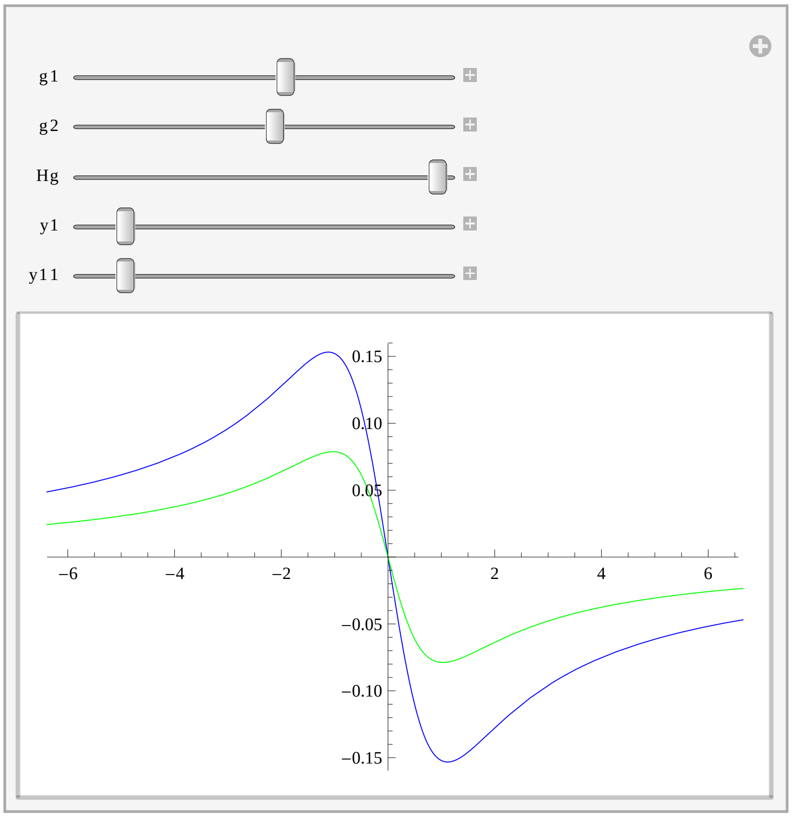



```
Manipulate[
 Show[{Plot[Hg (ArcTan[(g-2 x1)/(2 y1)] + ArcTan[(g+2 x1)/(2 y1)])/π, {x1, -10, 10}, PlotStyle → Blue],
   Plot[Hg (ArcTan[(g-2 x22)/(2 y22)] + ArcTan[(g+2 x22)/(2 y22)])/π, {x22, -10, 10}, PlotStyle → Red],
   Plot[Hg (ArcTan[(g-2 x11)/(2 y11)] + ArcTan[(g+2 x11)/(2 y11)])/π, {x11, -10, 10}, PlotStyle → Green]},
  PlotRange → Automatic, Frame → True,
  FrameLabel → {"Position [x/g]", "Head Field"}, FormatType → StandardForm],
 {g, -8, 8}, {Hg, -1, 1}, {y1, 0.1, 10}, {y11, 0.1, 10}, {y22, 0.1, 10}]
```



For Different g , Different y

```
Manipulate[Show[{Plot[-Hg Log[(1/4 (g1+2 x1)^2+y1^2)/(1/4 (g1-2 x1)^2+y1^2)]/(2 π), {x1, -10, 10}, PlotStyle → Blue],
   Plot[-Hg Log[(1/4 (g2+2 x11)^2+y11^2)/(1/4 (g2-2 x11)^2+y11^2)]/(2 π), {x11, -10, 10}, PlotStyle → Green]},
  PlotRange → Automatic], {g1, -8, 8}, {g2, -8, 8},
 {Hg, -1, 1}, {y1, 0, 1}, {y11, 0, 10}]
```



For Same g , Different y

```
Manipulate[Show[{Plot[- Hg Log[(1/4 (g+2 x1)^2+y1^2)/(1/4 (g-2 x1)^2+y1^2)] / (2 π), {x1, -10, 10}, PlotStyle → Blue],
  Plot[- Hg Log[(1/4 (g+2 x11)^2+y11^2)/(1/4 (g-2 x11)^2+y11^2)] / (2 π), {x11, -10, 10}, PlotStyle → Green]},
  PlotRange → Automatic], {g, -8, 8}, {Hg, -1, 1}, {y1, 0, 0.8}, {y11, 0, 0.8}]
```