

## **TASK:**

Consider a state lottery and the probability of winning the grand prize. The game is played by selecting a group of 6 numbers from  $\{1, 2, 3, \dots, 51\}$ , and the state selects a group of 6 numbers from the same set. You win the grand prize if all 6 of your numbers match the state's. The probability of winning is so low that some people are trying different ways of increasing their odds.

### **Tasks:**

1. Identify whether the probability of winning is discrete or continuous. Write a Python function that calculates the probability of winning the grand prize for the new game, given that a group of 6 numbers are selected from  $\{1, 2, 3, \dots, 51\}$ .
2. Alexandra buys 1 ticket every 24 hours of every 365 days per year for 80 years of her life. Each ticket costs \$5 and she selects her numbers independently from one ticket to the next. Write a Python program that calculates the probability of Alexandra winning the lottery at least one time and the amount of money she spends trying.
3. Amir decides to increase his chance of winning by organizing  $N$  of his friends to buy one ticket each for this week's drawing. Each friend buys a ticket independently, and the minimum value of  $N$  ( $N_{\min}(\epsilon)$ ) to ensure that the probability that there is at least one winner among the group is greater than or equal to  $\epsilon$  needs to be calculated for various values of  $\epsilon$ . Write a Python function that calculates the value of  $N_{\min}(\epsilon)$  for  $\epsilon = 10^{-5}$ ,  $10^{-3}$ ,  $0.1$ , and  $0.5$ .
4. If Amir has gathered enough friends to win with probability  $0.5$  using the above strategy, write a Python program that determines a way that they can increase their win probability above  $0.5$  without adding more people to the group.
5. The state lottery commissioner introduces a consolation prize: If exactly 3 of your 6 numbers match any 3 of the state's, you win a smaller prize. Write a Python program that calculates the probability of winning the consolation prize and the expected value of the consolation prize. Is this consolation prize likely to increase sales?