In the Name of God

Communication Systems

Computer Homework_1 Report

Multi-Path Channels &

Signal Reconstruction

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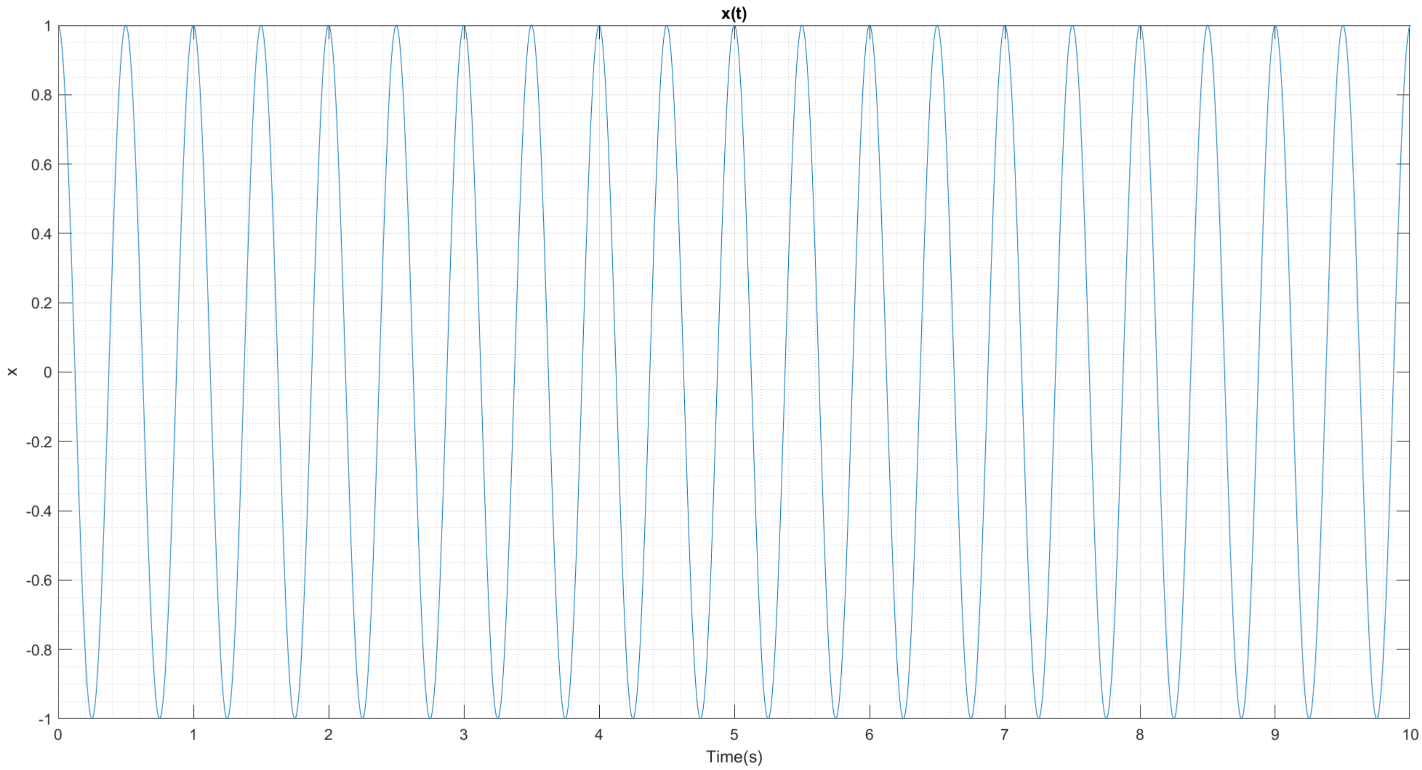
1- Multi-Path Channels:

1.1 -



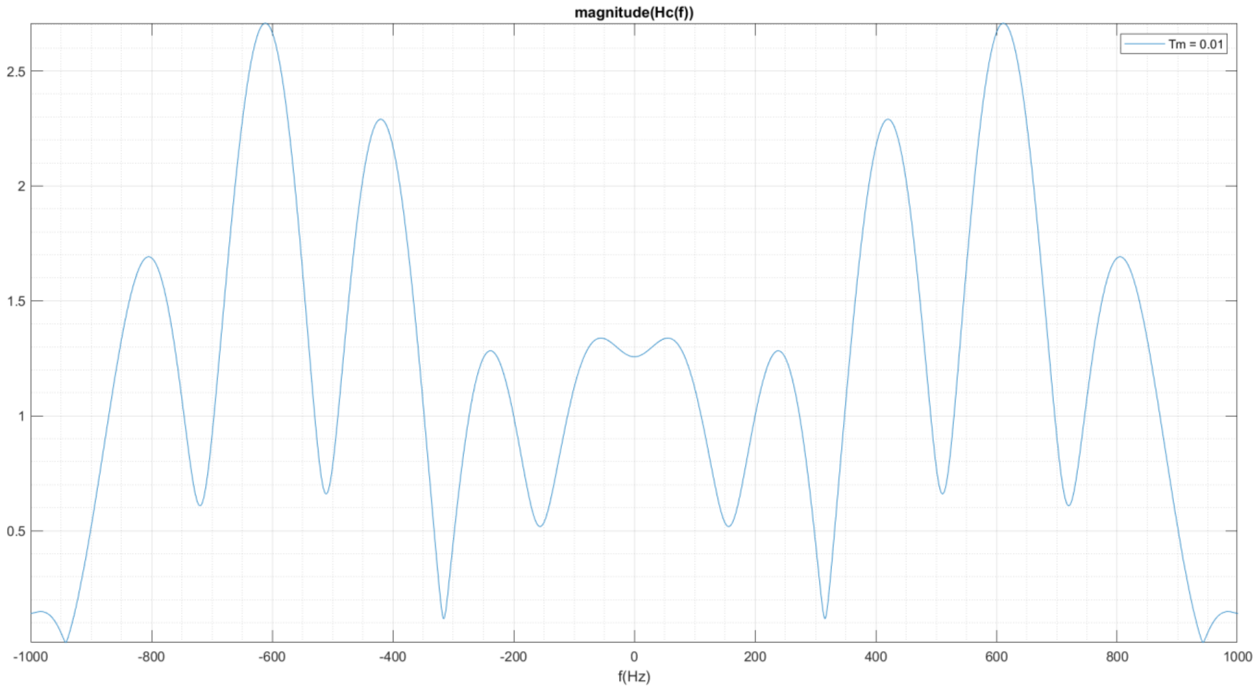
1.2 –

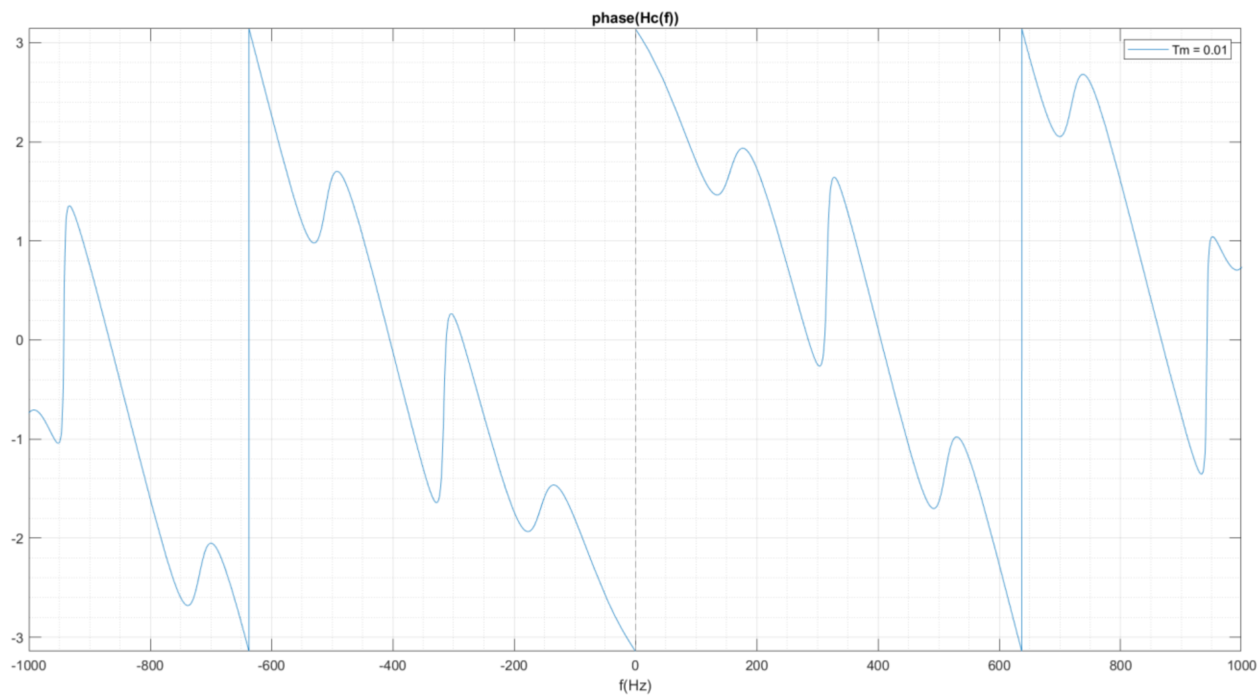
paramters:

```
mu = 0;
sigma = 0.5;
Tm = 0.01;
n = [1 10]; % length of a
a = normrnd(mu,sigma,n); % coeffiecients of impulses
tau = unifrnd(0,Tm,n); % shiftings of impulses
```

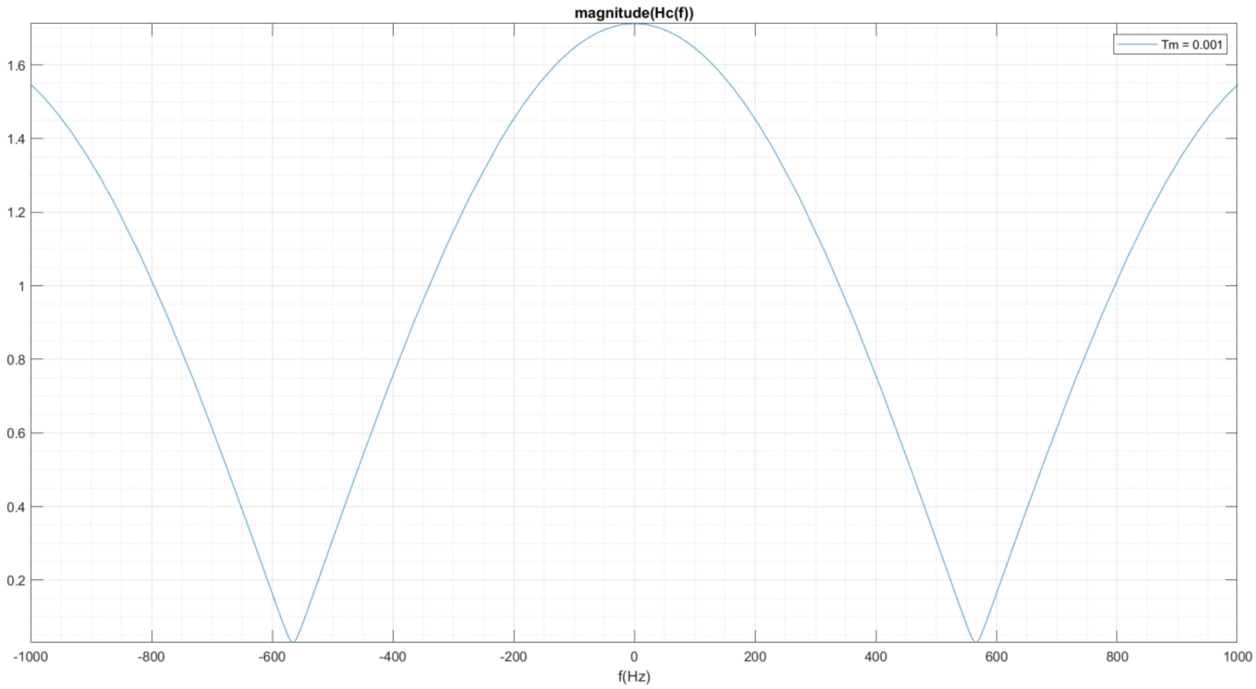
Hc is defined using a syms function and then using abs and angle, the magnitude and phase of the channel is drawn:

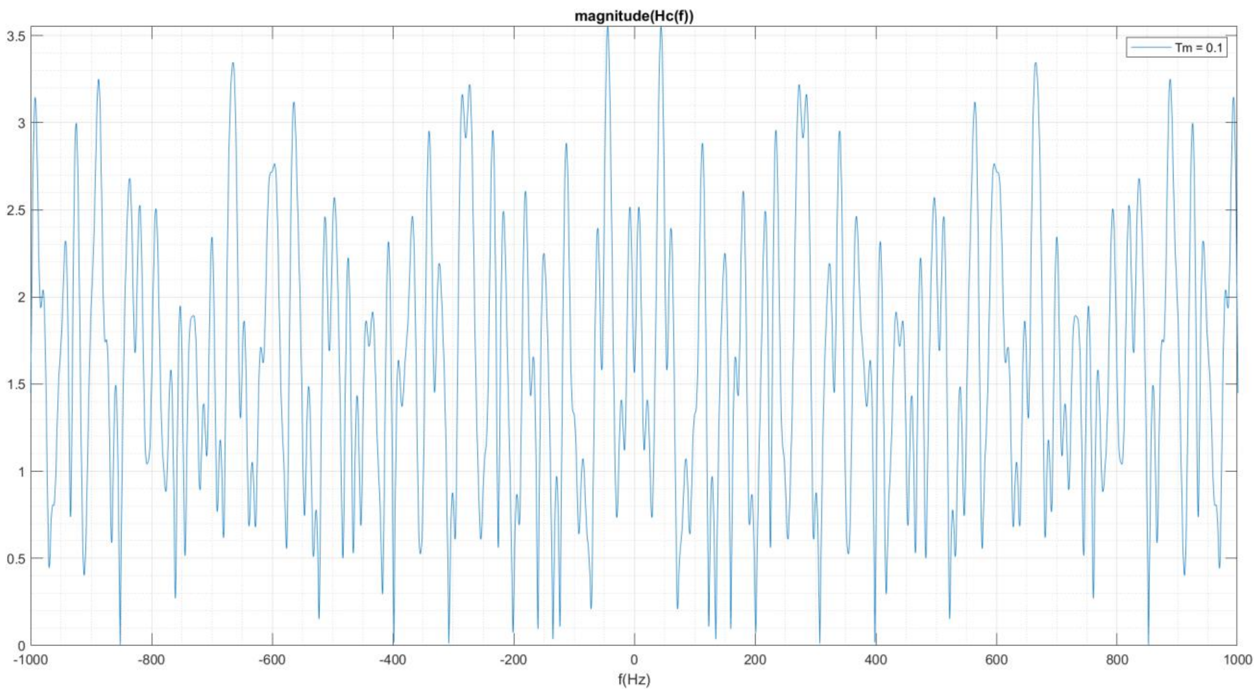
```
syms f;
syms HcF(f);
HcF(f) = sum(a.*exp(-1i*2*pi*f.*tau));
```


magnitude(Hc(f))
Tm = 0.01
f(Hz)


phase(Hc(f))
Tm = 0.01
f(Hz)

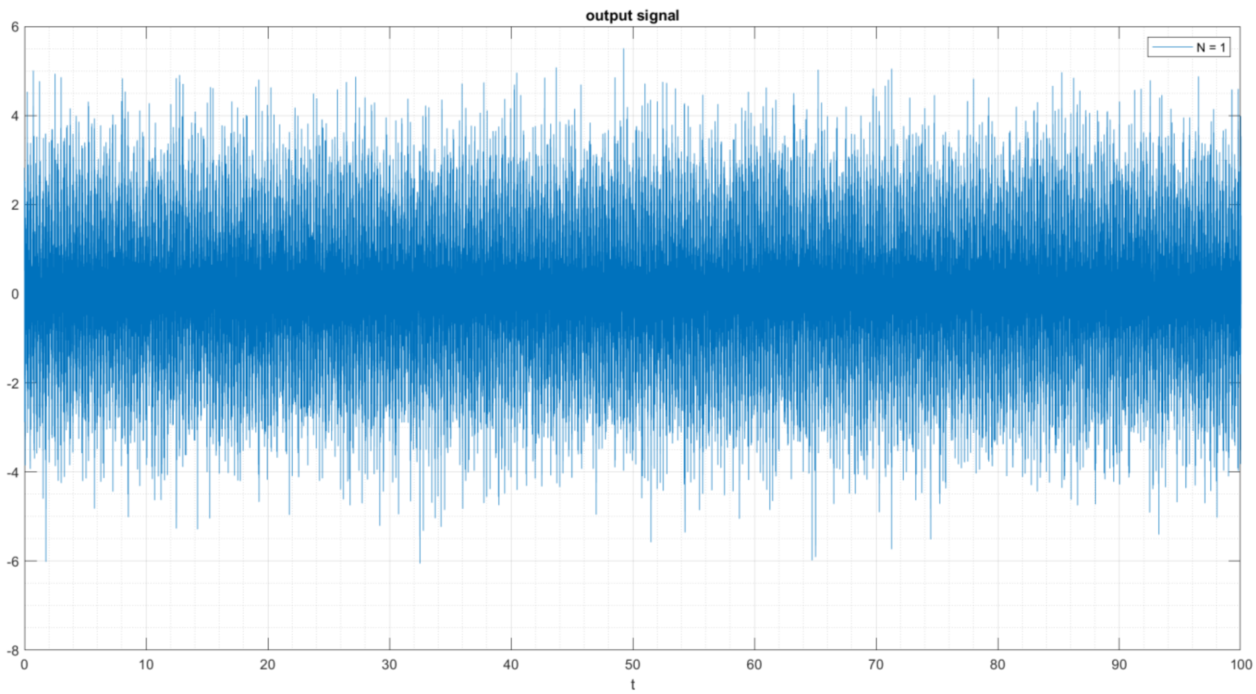
1.3 – By testing different Tm values, we find out that the distance between each two local minimums has a inverse relationship with Tm.

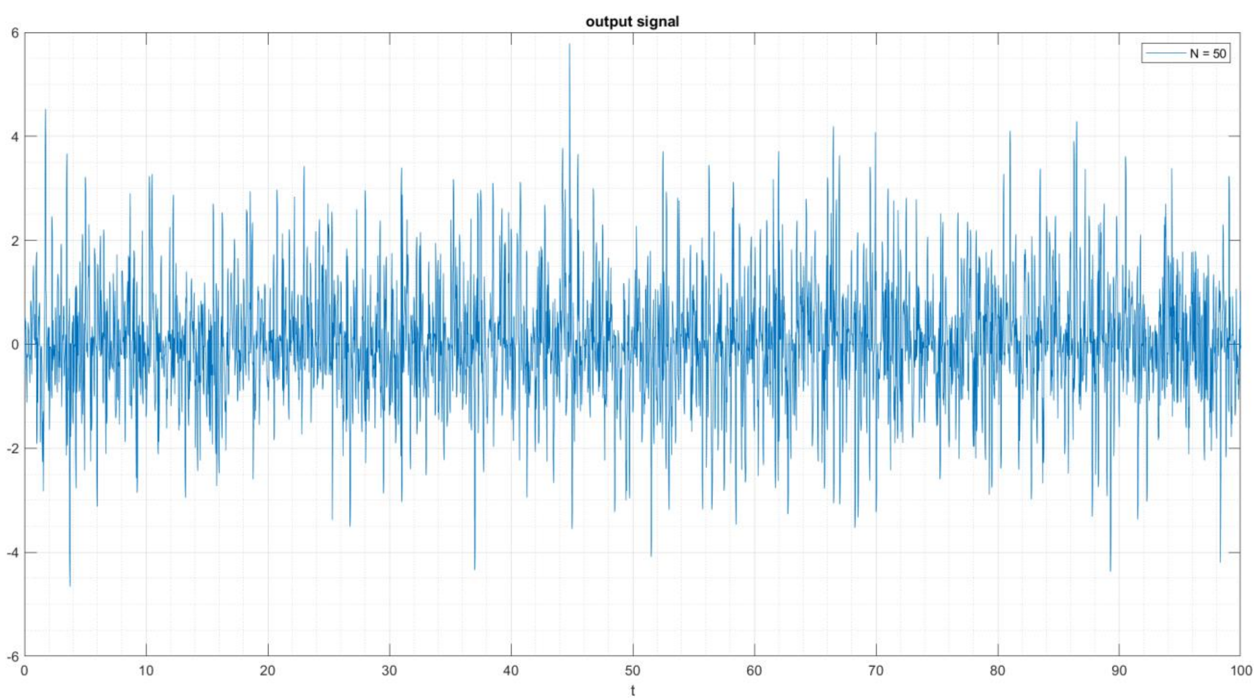


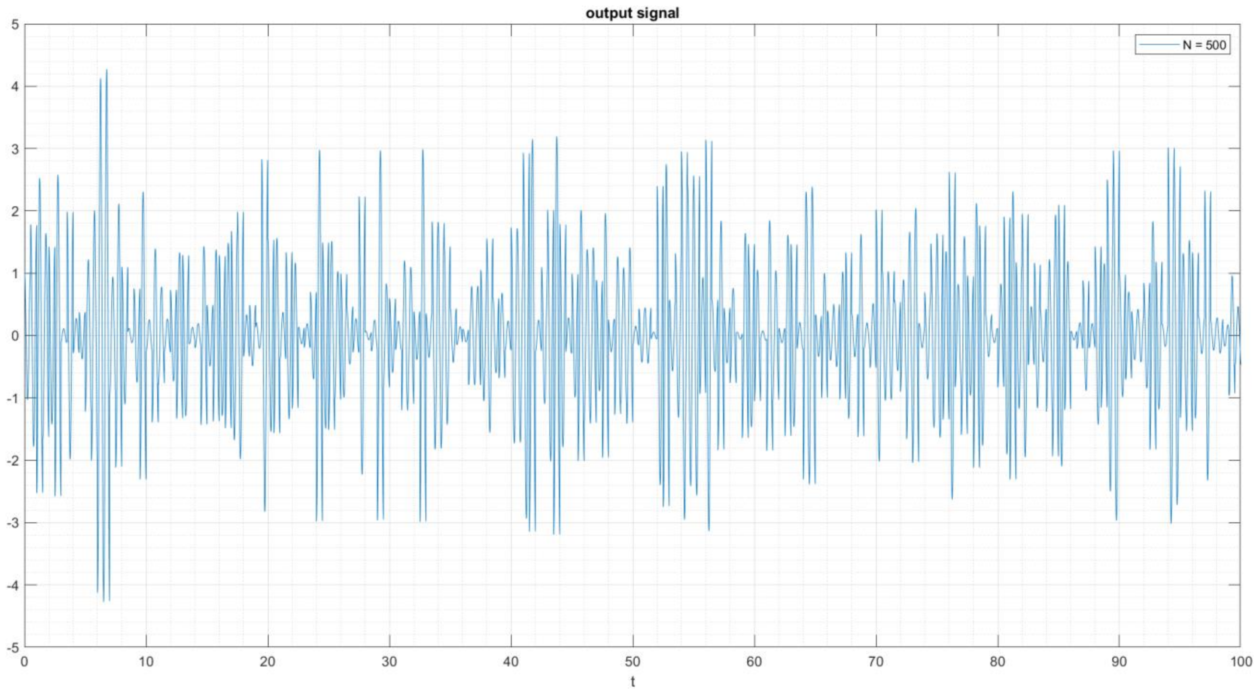


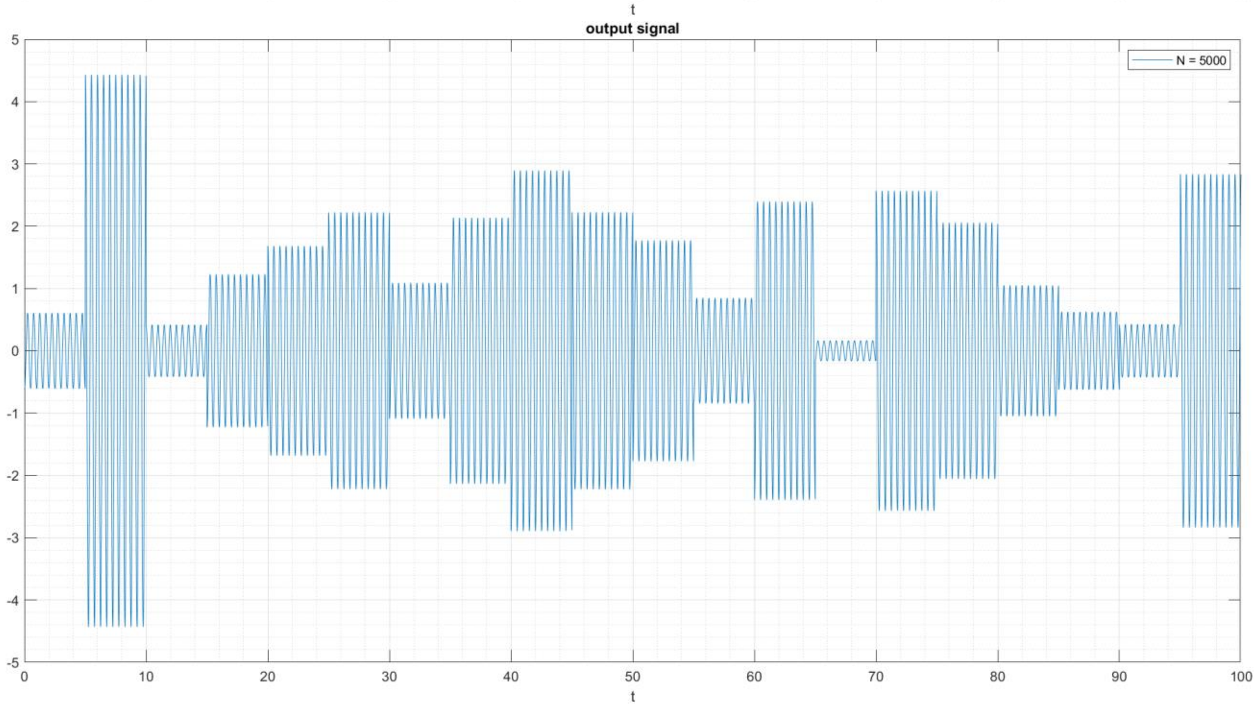
1.4 –

```
tau = (1/fs).*randi([0 floor(Tm*fs)],n); % samples
```






output signal
N = 500
t


output signal
N = 5000
t

2- Signal Reconstruction in multi path channels:

Using M-Tapped-Delay Line Equalizer, the output signal of the multi-path channel will be reconstructed:

2.1 –

$$H_{Eq}(f)H_C(f) = ke^{-j2\pi f t_0}$$

$$\rightsquigarrow H_{Eq}(f) = \frac{ke^{-j2\pi f t_0}}{\sum_{i=1}^{n} a_i e^{-j2\pi f \tau_i}} = \frac{1}{1+\sum_{i=2}^{N} k_i e^{-j2\pi f t_i}}$$

$$\rightsquigarrow \boxed{k = a_1 \quad , t_0 = \tau_1 \quad , \underset{i\geq 2}{k_i = \frac{a_i}{k}} \quad , \underset{i \geq 2}{t_i = \tau_i - t_0}}$$

according to Part 4: $(h(t) = \delta(t-5) + 0.4\,\delta(t-5.01))$

$$\boxed{k = 1 , \ t_0 = 5 , \ k_2 = \frac{0.4}{1} = 0.4 , \ t_2 = 5.01 - 5 = 0.01}$$

2.2 –

$$H_{Eq}(f) = \frac{1}{1+\sum_{i=2}^{n} k_i e^{-j2\pi f t_i}}$$

$\rightsquigarrow$ taylor series of $\frac{1}{1+x}$ when $|x| < 1$:

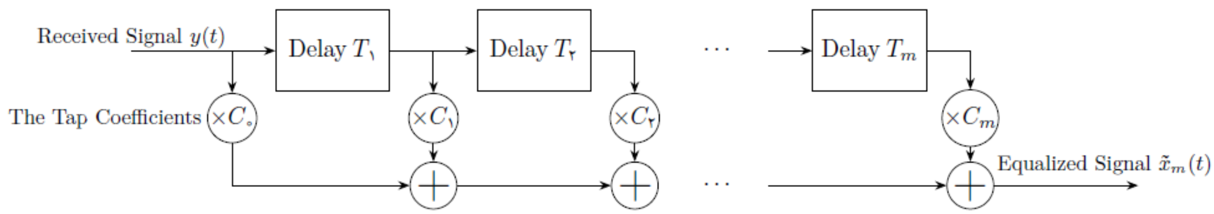
$$= 1 - x + x^2 - x^3 + x^4 - x^5 + \dots$$

$$= 1 - \sum_{i=2}^{n} k_i e^{-j2\pi f t_i} + \left(\sum_{i=2}^{n} k_i e^{-j2\pi f t_i}\right)^2 - \left(\sum_{i=2}^{n} k_i e^{-j2\pi f t_i}\right)^3 + \dots$$

$$\simeq C_0 + C_1 e^{-j2\pi f T_1} + C_2 e^{-j2\pi f T_2} + C_3 e^{-j2\pi f T_2} + \dots$$

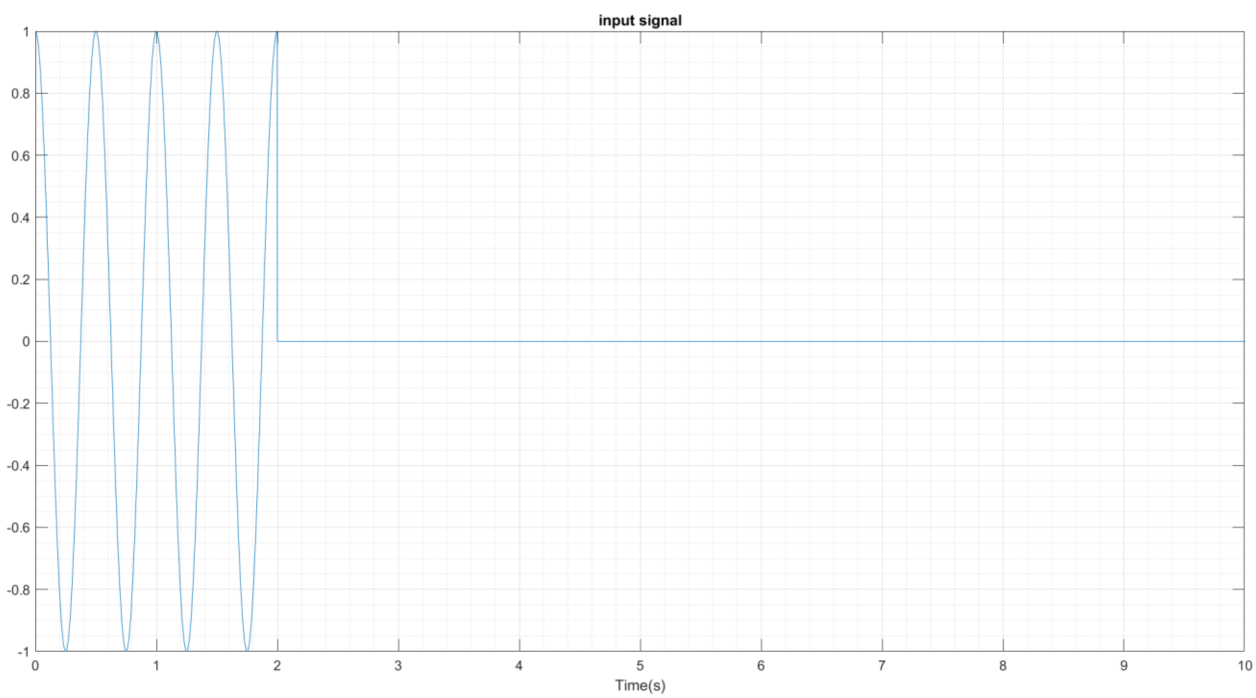
$$\xrightarrow{\mathcal{F}^{-1}} C_0 + C_1\,\delta(t-T_1) + C_2\,\delta(t-T_2) + C_3\,\delta(t-T_3) + \cdots$$

$\boxed{\checkmark}$ this formula approve the given structure

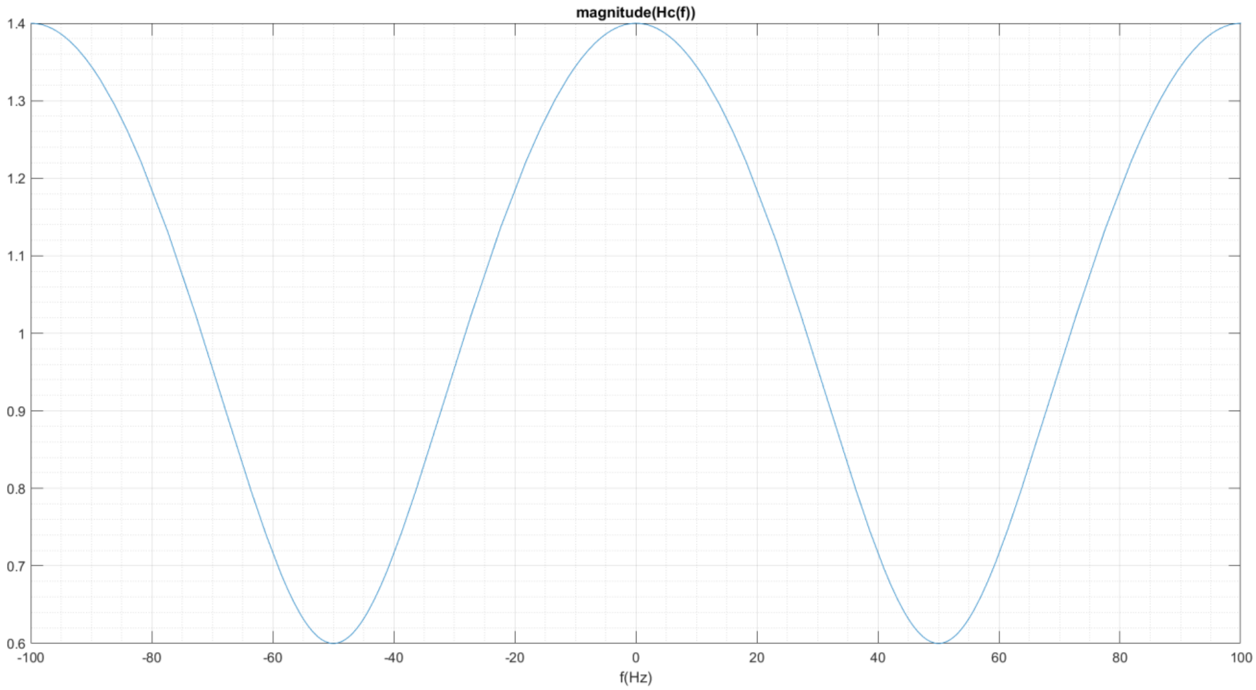


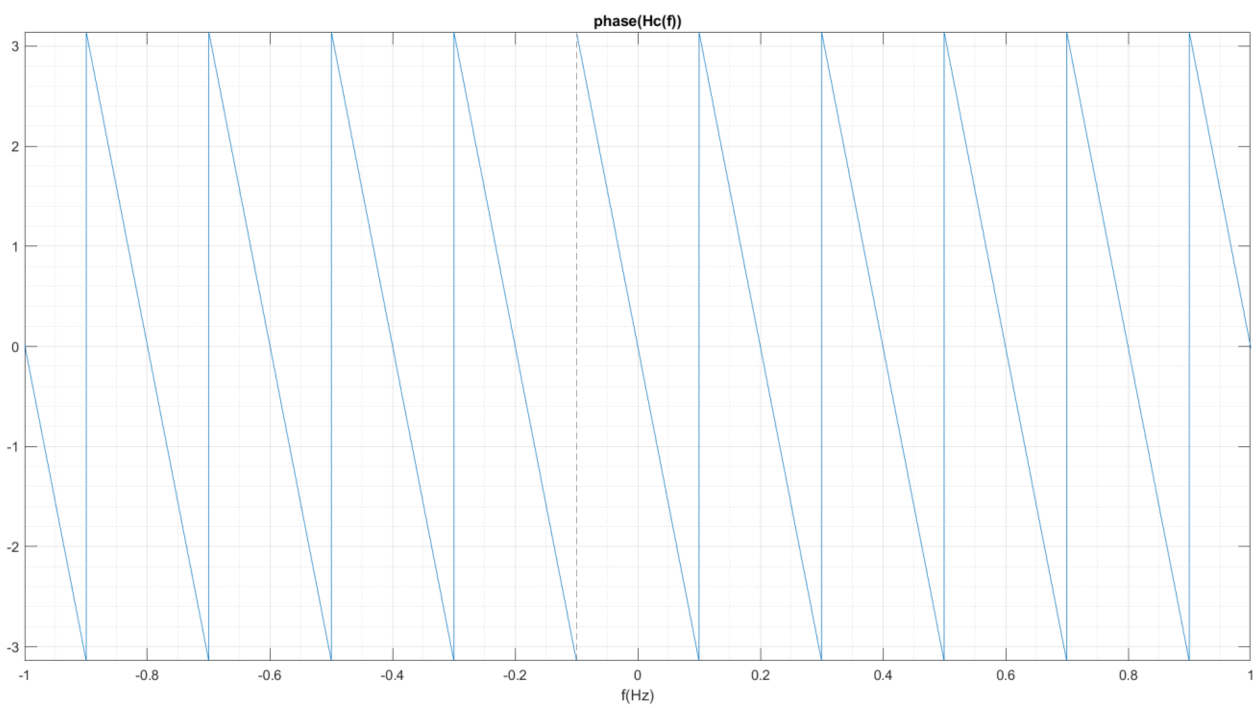
m–Tapped-Delay Line Equalizer :شکل ۱

2.3 –

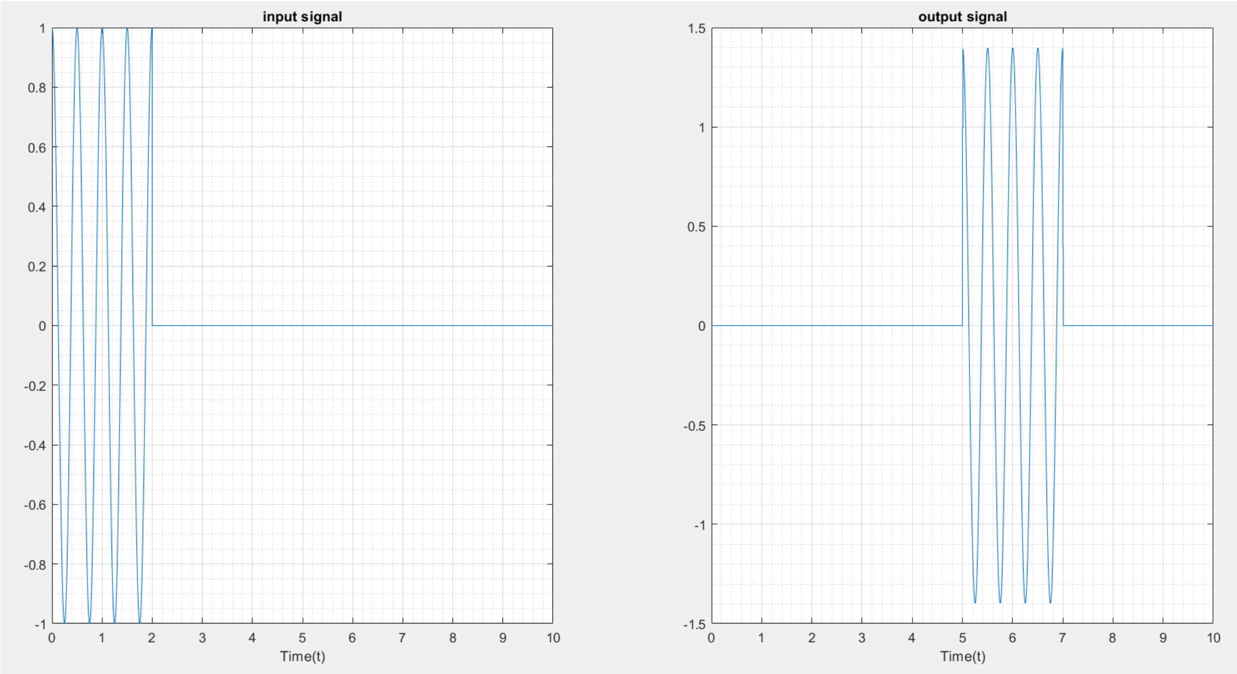


2.4 –


magnitude(Hc(f))
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
-100
-80
-60
-40
-20
0
20
40
60
80
100
f(Hz)


phase(Hc(f))
3
2
1
0
-1
-2
-3
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
f(Hz)

2.5 –



2.6 –

According to part 2.1, Ti and Ci are:

```
C(i+1) = ((-1)^i)*((k2)^i);
T(i) = t2*i;
```

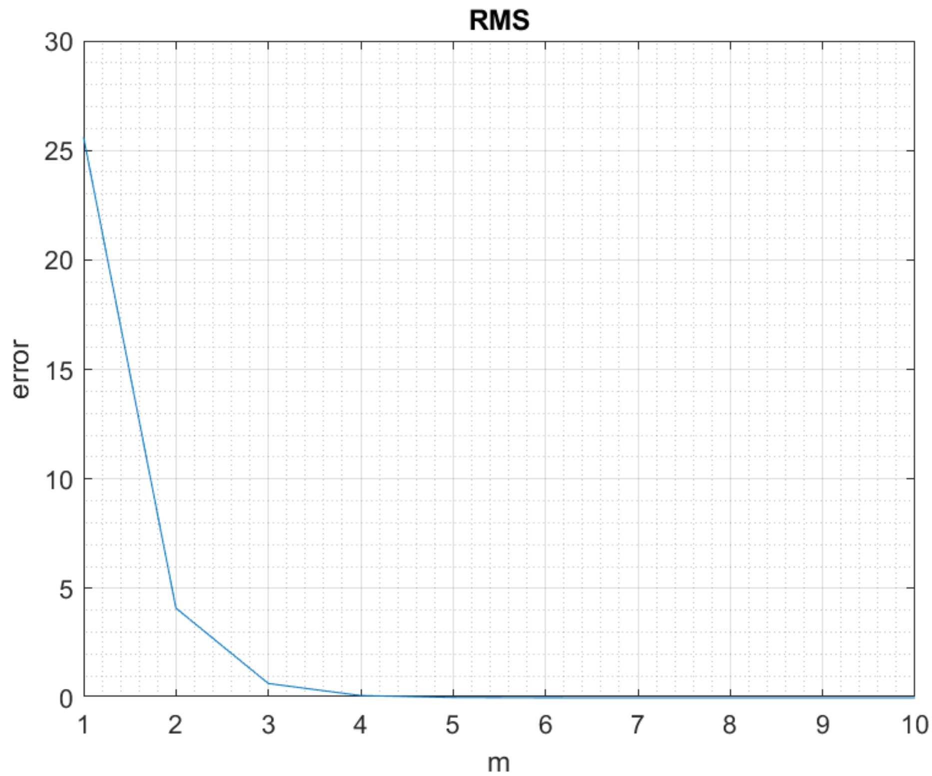
Ci:

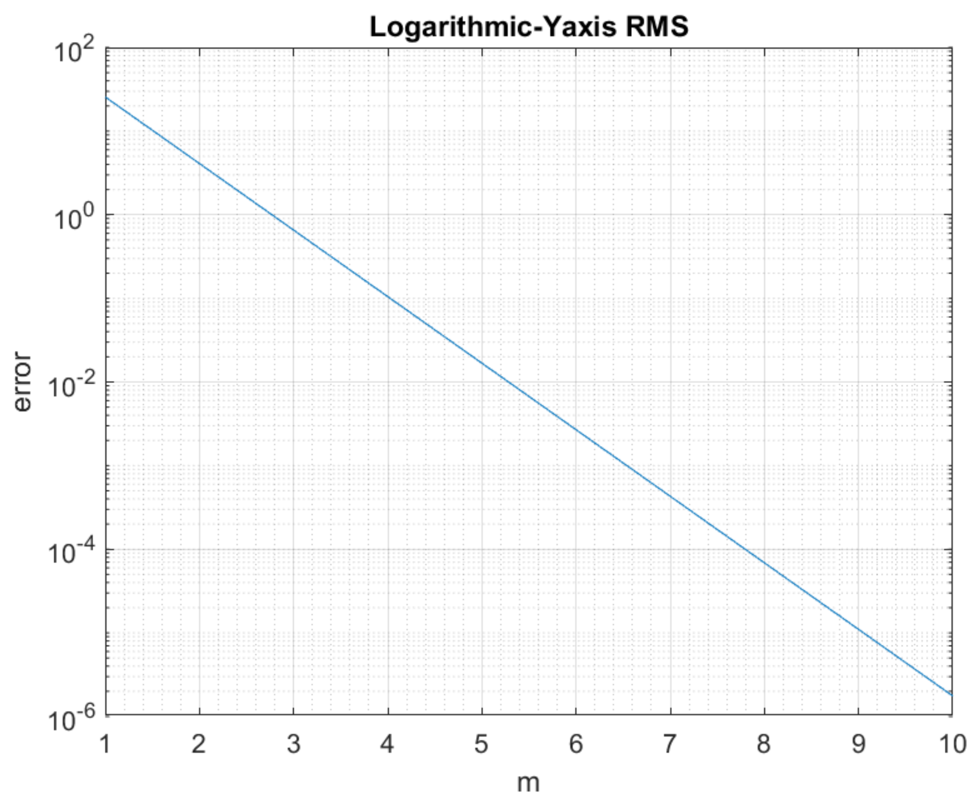
1	-0.4000	0.1600	-0.0640	0.0256	-0.0102	0.0041	-0.0016	6.5536e-04	-2.6214e-04	1.0486e-04

Ti:

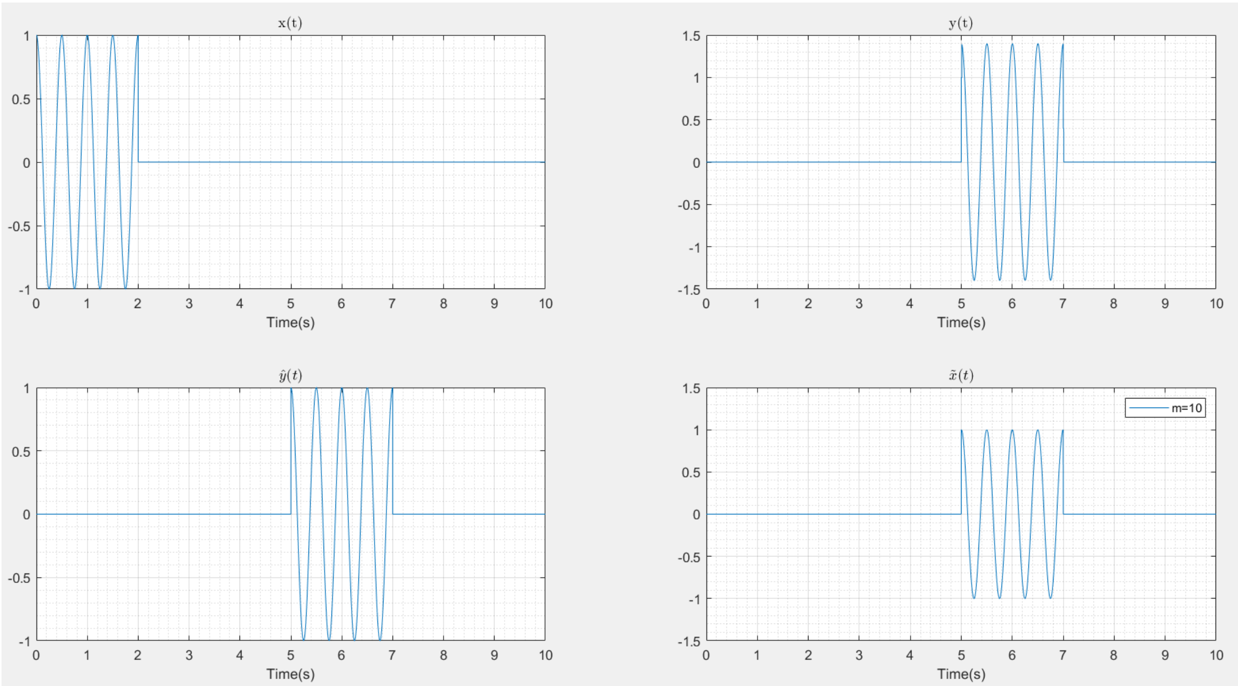
0.0100	0.0200	0.0300	0.0400	0.0500	0.0600	0.0700	0.0800	0.0900	0.1000

As you can see in RMS plots, by increasing m, a better signal would be reconstructed:

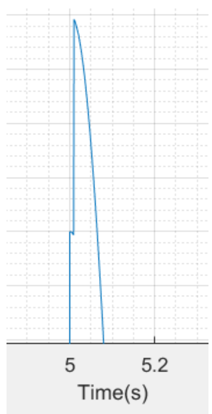

RMS
30
25
20
15
10
5
0
error
1
2
3
4
5
6
7
8
9
10
m


Logarithmic-Yaxis RMS
10^{2}
10^{0}
10^{-2}
10^{-4}
10^{-6}
error
1
2
3
4
5
6
7
8
9
10
m

2.7 –



As we see, y(t) which is the output of the multipath channel is destroyed because the domain is changed and if we zoom, there are some impulses on the signal:



but $\hat{y}$ and $\tilde{x}$ which are in order, the output of the ideal channel and reconstructed signal are the same and has a low RMS as we saw.