In the Name of God

Foundations of Neuroscience

Homework_1 Report

Statistical Hypothesis testing

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1- Statistical Hypothesis testing:

1.1-

Null hypothesis: Somatosensory neurons activity aren`t correlated with skin temperature.

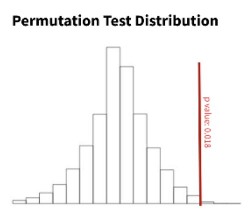
Alternate hypothesis: Somatosensory neurons activity are correlated with skin temperature.

1.2-

If the distribution of the null hypothesis is not normal, we can`t do the t-test and as we don`t have the null hypothesis distribution, we can`t do the t-test.

1.3-

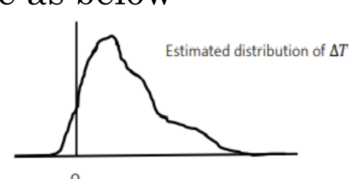
We divide our data`s from this experiment in two parts and calculate the mean of each and the difference is called ΔY^*. Then again, we do permutation on the hole sample and do the separation in two groups again. We perform this permutation for a large time and we now we have a distribution of this differences. We check the value ΔY^* in the resulted distribution and calculate the p-value and check our hypothesis.



1.4-

This test is like permutation test but we do all that process just for n randomly sampled numbers of the grouped data. The result is again a distribution which we can calculate the **one-tailed** p-value as below:

$$\text{P-value}=\frac{\#\Delta T<0}{\#\Delta T}$$



1.5- Our tests should be two-tailed because we don`t no if the temperature increase or decrease the neuron activity. So bootstrap which is a resampling test is not good for this hypothesis checking.

2- T-test:

2.1-

<u>Alternate hypothesis:</u> Vinca manor will decrease the height of sunflowers comparing to the standard mean which is 15.7 cm.

<u>Null hypothesis:</u> Vinca manor won`t decrease the height of the sunflowers and our sample mean will be equal or more than standard mean which is 15.7 cm.

2.2-

Using a One-sample T-test, we will check our null hypothesis. One-sample t-test will be done using "ttest" function in Maltab:

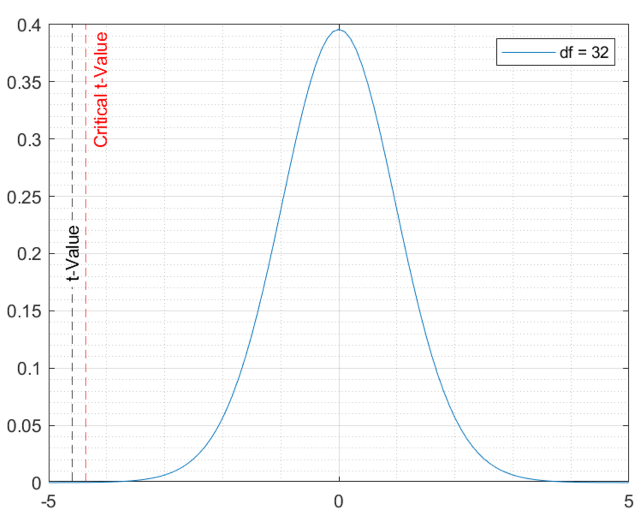
```
[h,p,ci,stats] = ttest(measuredHeights,meanHeight);

tValue = stats.tstat;
```

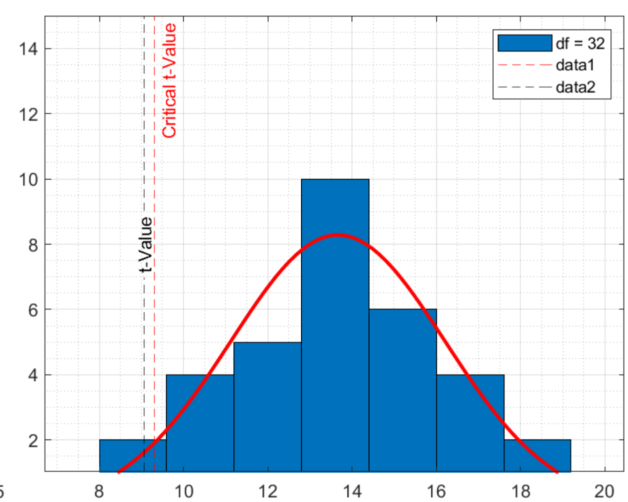
results:

t-value = -4.5990, p-value = 6.3485e^-05

This p-value will gives us the critical t-value: -4.3577 (using "tinv" function)

Now we plot histogram of our data and the t-distribution of the given degree of freedom:

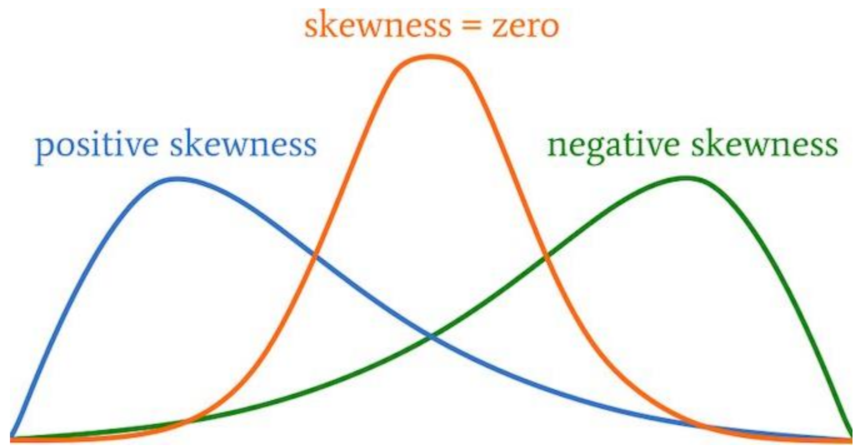




3- Excess kurtosis:

3.1-

Skewness: In statistics, skewness is a measure of asymmetry of the distribution of a group of real values. Skewness can be negative, zero, positive and undefined. Negative skewness means that the tail is on the left of the distribution, zero skewness means that both tails on both sides are balanced and the distribution is symmetric, positive skewness means that the tail is on the right of the distribution.

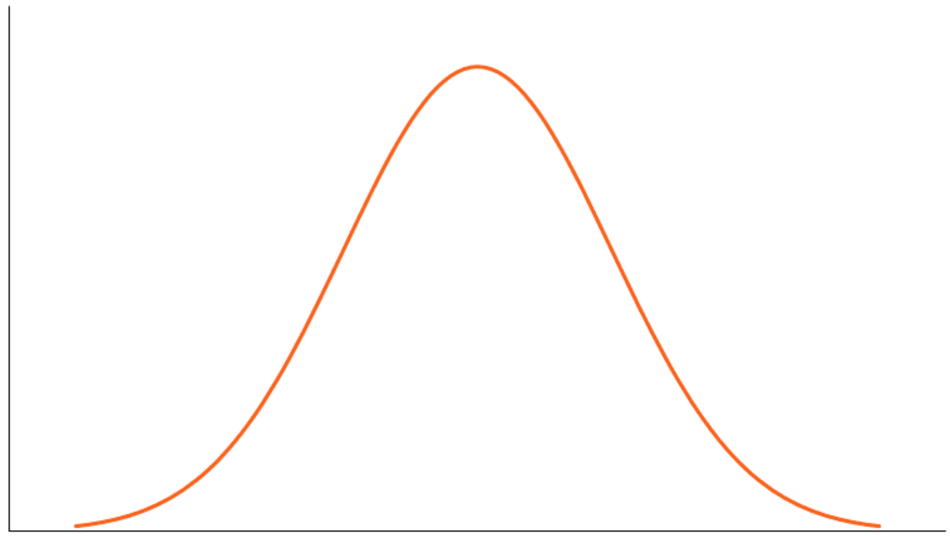


Kurtosis: Like skewness, kurtosis tells us about the shape of the distribution. Kurtosis measures how heavily the tails of a distribution is compare to a normal distribution. Kurtosis value can be positive, negative and zero. Kurtosis can be confused with skewness, kurtosis measures the heaviness of the both tails but skewness tells us about the symmetry of the given distribution. Excess kurtosis is used to compare the kurtosis of a distribution to the kurtosis of a normal distribution. Excess kurtosis can be calculated as follows:

Excess kurtosis = kurtosis – 3;

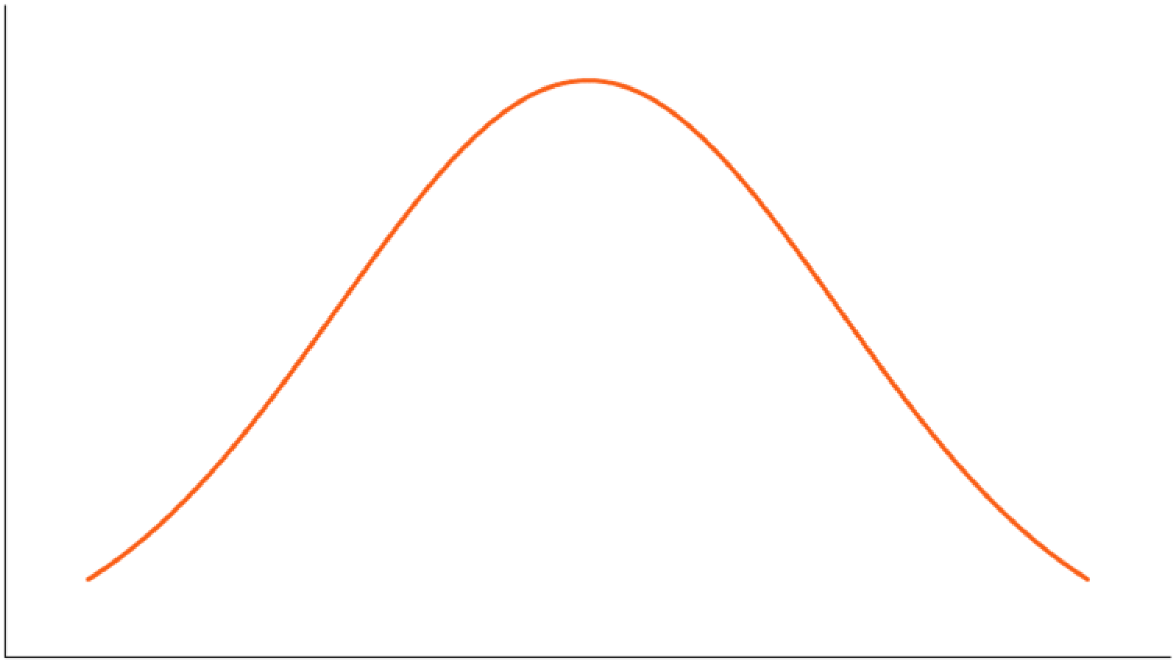
3 is the kurtosis of a normal distribution.

A zero excess kurtosis means that our distribution follows a normal distribution:

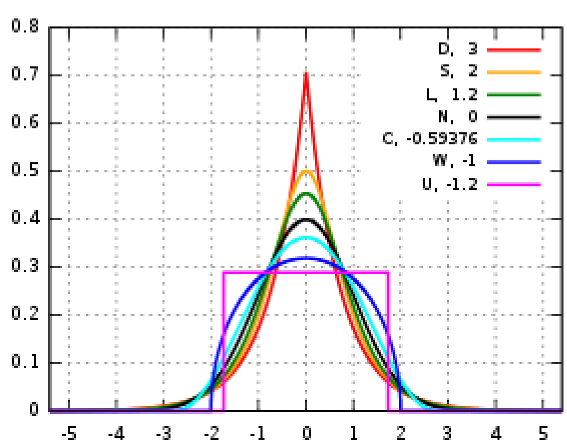


A positive excess kurtosis, shows that our distribution has heavily tails and a sharp peak:

A negative excess kurtosis tells us that our distribution is flatter and the tails are less heavy:



In the plot bellow, you can see distributions with high positive to low negative excess kurtosis and as you see, zero excess kurtosis indicates a normal distribution and distributions with more than zero excess kurtosis are sharp and distributions with negative excess kurtosis are flatter.



3.2-

Because if a distribution is symmetric, skewness tells us there is no difference with a normal distribution but it could be a sharper or flatter distribution than the normal distribution and skewness can`t measure that. So, kurtosis is needed, too!

3.3-

Given the distribution X as bellow:

$$X \sim \mathcal{N}(\mu, \sigma^2)$$

$E\,[e^{tX}] = \int_{-\infty}^{\infty} e^{tx} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2} \mathrm{dx} = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^{2+tx}} dx =$

$\frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(\frac{x-(\mu-\sigma^2 t)}{\sigma})^2 + \frac{-\mu^2}{2\sigma^2} + \frac{\sigma^2}{2}(\frac{\mu}{\sigma^2}+t)^2} dx = \int_{-\infty}^{\infty} \mathcal{N}(\mu - \sigma^2 t, \sigma^2) e^{\frac{-\mu^2}{2\sigma^2} + \frac{\sigma^2}{2}(\frac{\mu}{\sigma^2}+t)^2} dx =$

$e^{\frac{-\mu^2}{2\sigma^2} + \frac{\sigma^2}{2}(\frac{\mu}{\sigma^2}+t)^2} = e^{\frac{-\mu^2}{2\sigma^2} + \frac{\sigma^2}{2}(\frac{\mu}{\sigma^2}+t)^2} = e^{\mu t + \frac{\sigma^2 t^2}{2}}$

3.4-

$E\,[X^n] = \frac{d^n E\,[e^{tX}]}{dt^n}|_{t=0}$ and $E\,[e^{tX}] = e^{\mu t + \frac{\sigma^2 t^2}{2}}$

G (Gaussian distribution) = Kurt [X] – 3 $= E\,[(\frac{X-\mu}{\sigma})^4] - 3 = \frac{d^4 E\,[e^{t(\frac{X-\mu}{\sigma})}]}{dt^4}|_{t=0}$- 3 =

$\frac{d^4 e^{\mu t + \frac{\sigma^2 t^2}{2}}}{dt^4}|_{t=0}$- 3 = $\mu^4 + 6\mu^2\sigma^2 + 3\sigma^4 - 3 \xRightarrow{\mu=0,\ \ \sigma=1}$ 3-3 = 0

3.5-

$E\,[(\frac{X-\mu}{\sigma})^4] = \frac{d^4 E\,[e^{tX}]}{dt^4}|_{t=0} = \frac{d^4 e^{\mu t + \frac{\sigma^2 t^2}{2}}}{dt^4}|_{t=0} = \mu^4 + 6\mu^2\sigma^2 + 3\sigma^4$

Cause as we see, a distribution with mean and sigma closer to zero, will have an excess kurtosis closer to zero, too.

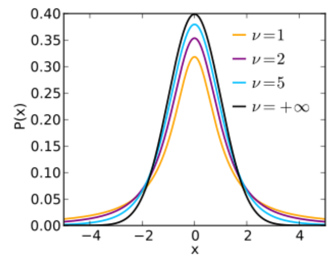
Using Jarque-Bera test, we use both values of skewness and excess kurtosis and test if our distribution is normal or not:

$$JB = \frac{n}{6}\left(S^2 + \frac{1}{4}(K-3)^2\right)$$

s is the skewness, k-3 is the excess kurtosis, n is if JB is zero our distribution is gaussian.

3.6-

t-distributions with degree of freedom more than 4 are sharper in peak and have heavier tails, and their excess kurtosis is calculated throw this equation:



$$\frac{6}{df-4}$$

As we see, a t-distribution with df of 5 has a excess kurtosis equals to 6 and this means it is not a gaussian at all. A t-distribution with df of 6 has a excess kurtosis of 3 which it indicates that it is more gaussian comparing to a t-distribution with df of 5. As the df increases, the excess kurtosis of the t-distribution gets closer to 0 which it means they get closer to a gaussian distribution. For example, a t-distribution with df of infinity (excess kurtosis = 0) is a gaussian.