In the Name of God

Neuroscience of Learning, Memory, Cognition

Homework_1 Report

Hudgkin-Huxley Model

Leaky Integrate and Fire Neuron Model

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1- The Hudgkin-Huxley Model:

Hudgkin-Huxley model is a mathematical model for describing how action potentials happen in a neuron. Actually, it`s a set of differential equations as we can see below:

$$\frac{dv}{dt} = \left(g_{Na}(E_L - v) - g_{Na}m^{٣}h(v - E_{Na}) - g_k n^{٤}(v - E_K)\right) + I_{ext})/C_M)$$

$$\tau_n \frac{dn}{dt} = -n + n_\infty$$

$$\tau_m \frac{dm}{dt} = -m + m_\infty$$

$$\tau_h \frac{dh}{dt} = -h + h_\infty$$

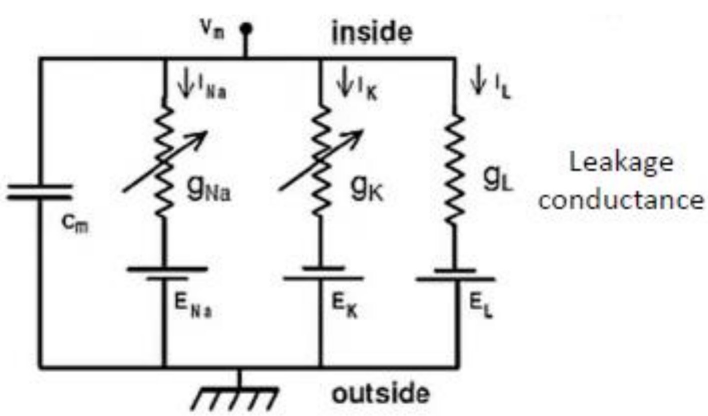
$$\tau_i = \frac{١}{\alpha_i + \beta_i} \quad . \ i_\infty = \frac{\alpha_i}{\alpha_i + \beta_i} ; i \in \{n.m.h\}$$

Which n, m, h is the probability of each sub-unit of Potassium being open and Activation and Inactivation of Sodium Channels.

α is the rate of transitions, from closed to open and

β is the rate of transitions, from open to closed.

Here`s the complete model of the Hudgkin-Huxley and we can see what are other elements in the equations in the model below:



For implementing this Model, at first we have to declare our variables like conductivities, Nernst potentials, etc:

```
Cm = 1; % Membrane capacitance in micro Farads
gNa = 120; % in Siemens, maximum conductivity of Na+ Channel
gK = 36; % in Siemens, maximum conductivity of K+ Channel
gl = 0.3; % in Siemens, conductivity of leak Channel
ENa = 55; % in mv, Na+ nernst potential
EK = -72; % in mv, K+ nernst potential
El = -49.4; % in mv, nernst potential for leak channel
vr = -60; % in mv, resting potential
```

Then we need 2 vectors for representing our voltage and current:

```
v = vr * ones(1,T); % Vector of output voltage
I = zeros(1,T); % in uA, external stimulus (external current)
```

Then we declare α and β:

```
u = vr - v; % u is just a variable which makes alpha and beta dependent on v
alpha_n = (.1 * u + 1)./(exp(1 + .1 * u) - 1) / 10;
beta_n = .125 * exp(u/80);
alpha_m = (u+25) ./ (exp(2.5+.1*u)-1)/10;
beta_m = 4*exp(u/18);
alpha_h = .07 * exp(u/20);
beta_h = 1 ./ (1+exp(3 + .1*u));
```

At the end, we need probabilities(n, m, h) initial values and a vector for each of them to put the values at each time in them:

```
% we have the intial conditions from the plots in the Homework paper in v =
vr which is -60 mv

n_initialCond = 0.32;
m_initialCond = 0.05;
h_initialCond = 0.59;
n = n_initialCond * ones(1,T);
m = m_initialCond * ones(1,T);
h = h_initialCond * ones(1,T);
```

Now how we can solve these equations? We`re going to use **Euler Method** for solving the equations.

- What is **Euler Method?**

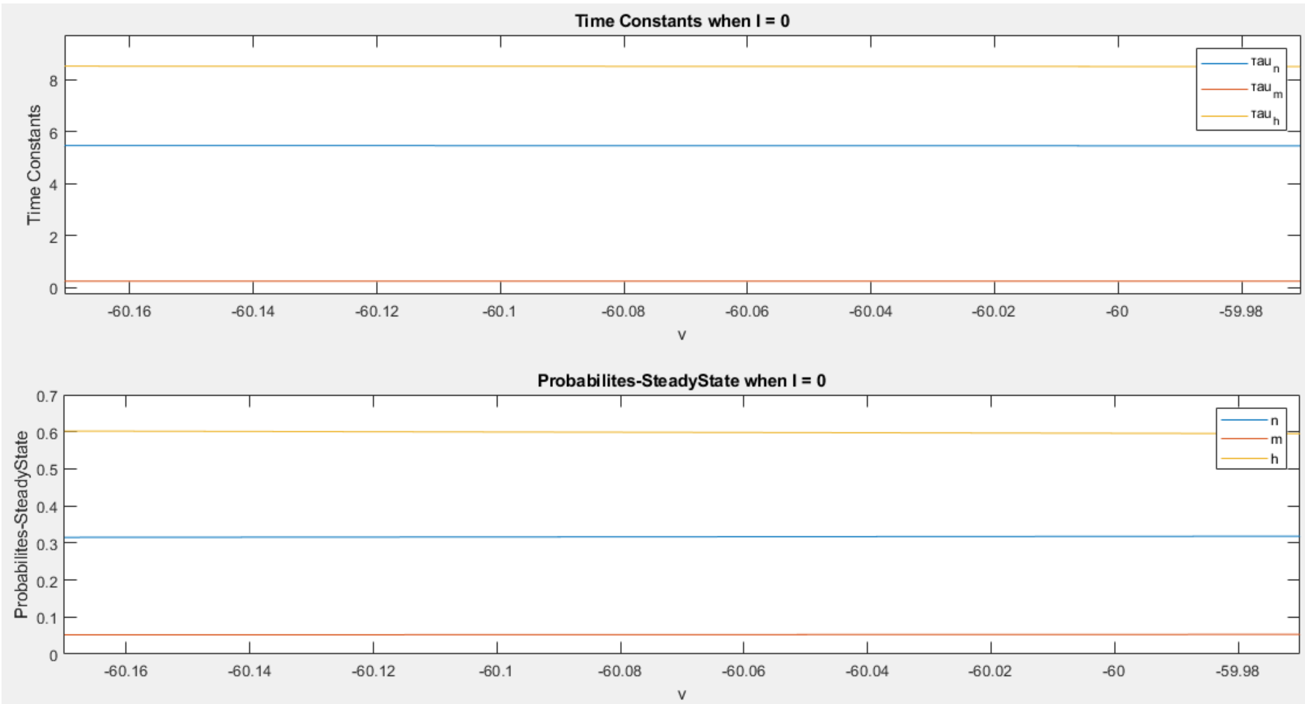
  Actually we determine variables, dv, dn, dm, dh and from the initial conditions, we can calculate the value of these derivatives at time zero (dv(1), dn(1), dm(1), dh(1)). Then we add these the next values of these vectors and we have v(2), n(2), m(2), h(2). So with the values of v/n/m/h(i) and dv/n/m/h(i), we can have the value of dv/n/m/h(i+1). Therefor all we need is a **for loop** to goes over the time and calculate our variables step by step and put their values, into their vector that we declared.

We calculate dv, dn, dm, dt by equation which came in the first page.

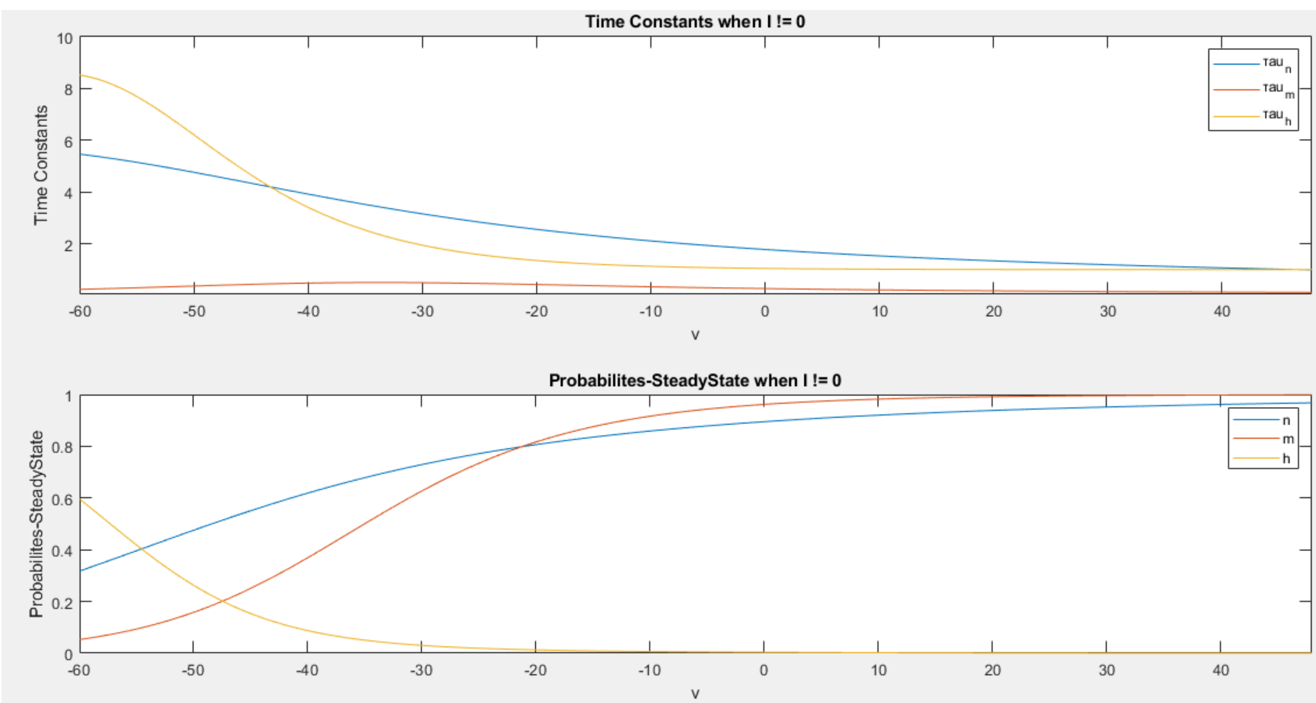
- Click here to access my code in github.

1 - Time Constants and Steady state values of Probabilities:

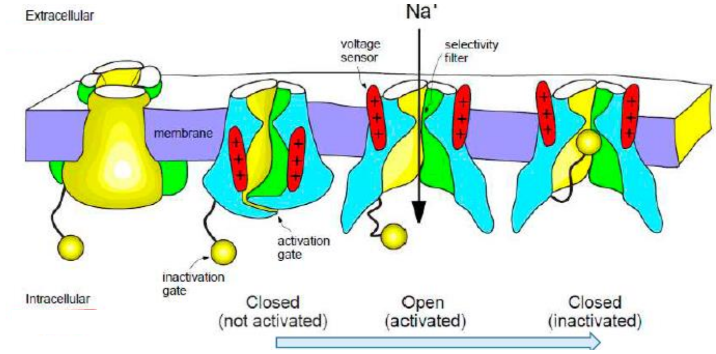
If I is equal to zero, which mean we have no input in our neuron, the values must be constant and have no change.

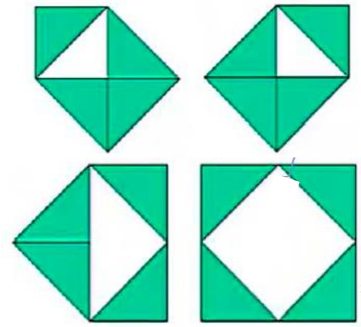


As we expected the values are constant. What if we have a non-zero input current?

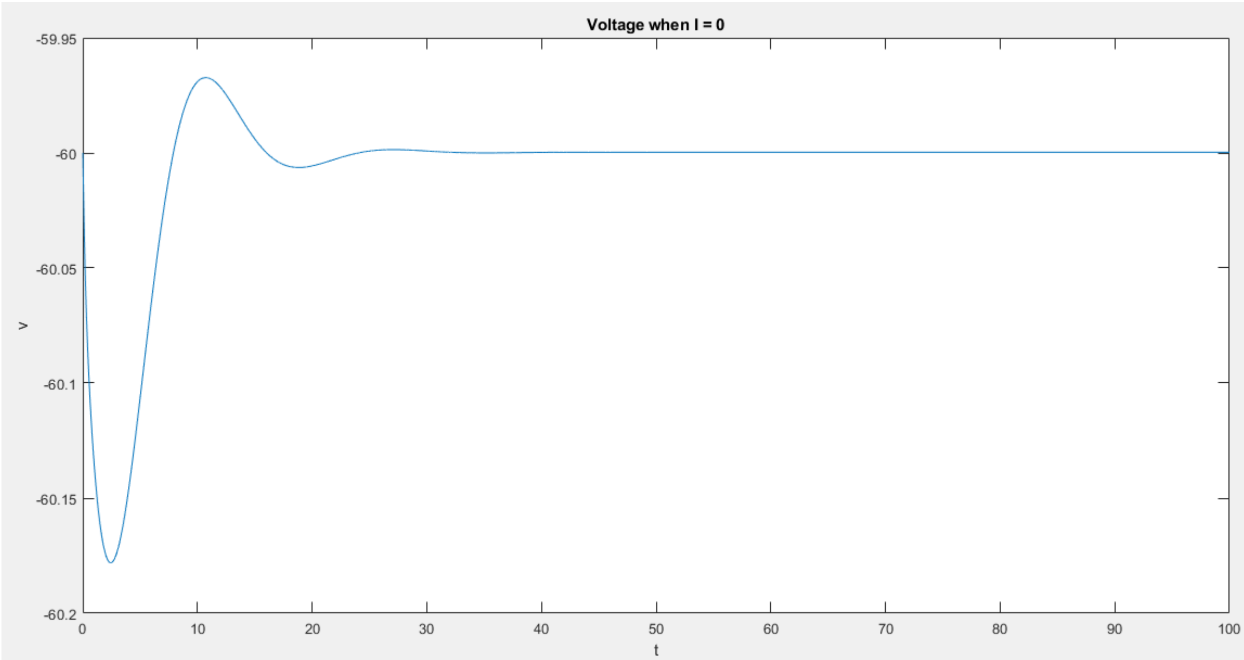


When I is zero and there is no input current, our neuron actually does nothing because there`s nothing to stimulate it and therefor, the probabilities and time constants are constant. But when we have input current, neuron is getting stimulated in its receptive field and then as we know at first, **Depolarization** happens. We see “m” that is the probability of Activation gates, getting open is highly increasing. Activation gates will be open extremely fast as we see the Time Constants of “m” is very low. In same time, “h” that is the probability of Inactivation gates not doing their job will decreases. That means, the Inactivation gates will do their job and inactive the Sodium channels. As we see the time constant of “m” is lower than the time constant of “h”, which tells us Activation gates do their job so fast and at the end, Inactivation gates will do theirs. About “n”, as we see it has a higher time constant than “m” that again means at first Sodium channels will be opened and after that we have Potassium channels. After the neuron spikes, the voltage of the neuron has increased and for going back to rest state, the voltage needs to get low and that`s what Potassium channels are for. During the spiking and voltage increasing, the probability of Potassium channels getting open will increase and they will make the neuron goes back to the resting potential. We call this **Repolarization**.

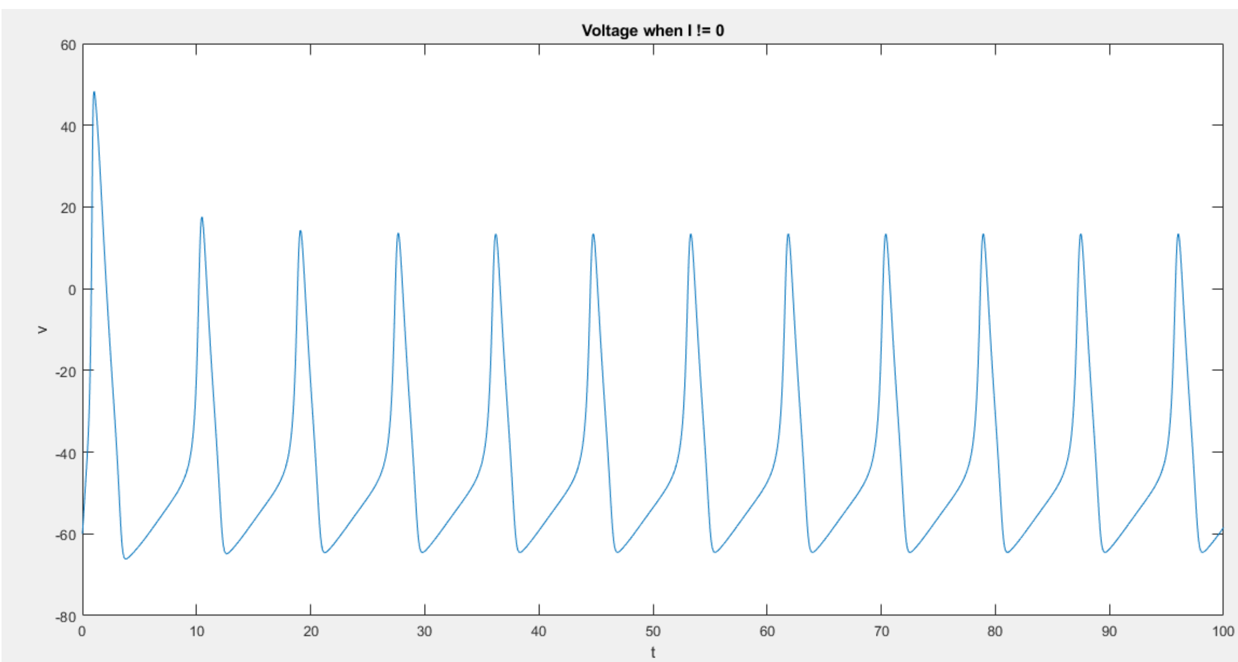




2 - Voltage:



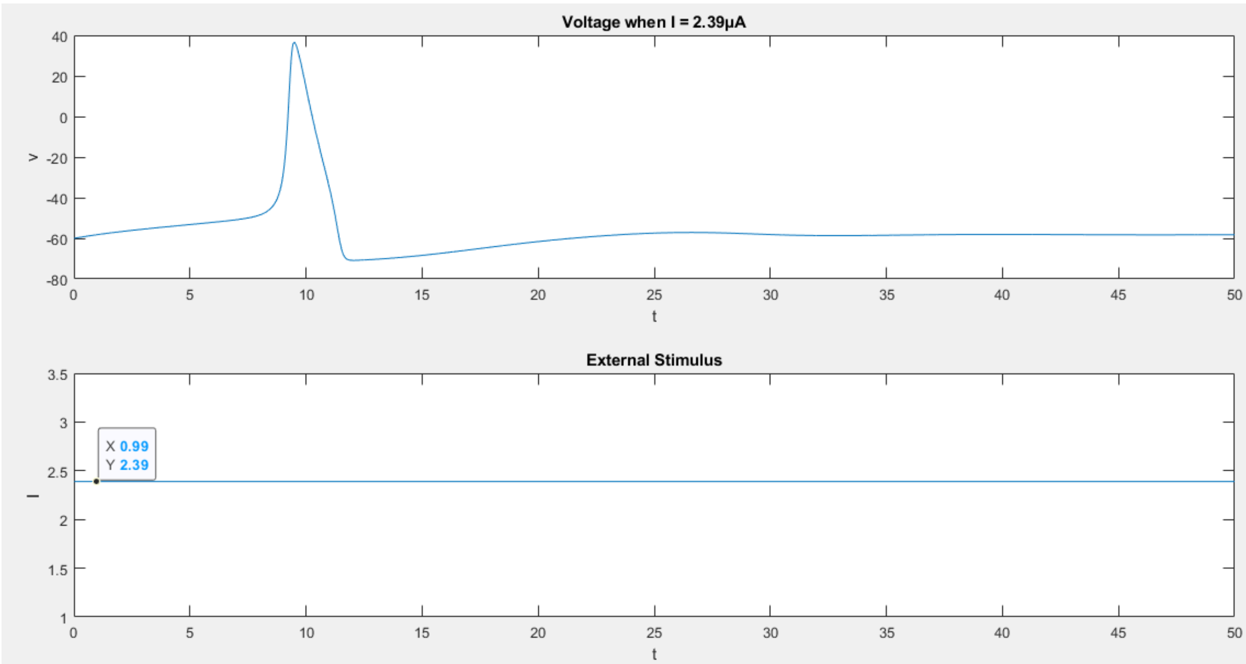
As we can see, when we have no input current, the neuron wont spike but if we have a current input:



As we see the neuron will spike.

3 - Minimum of input current needed for spiking:

The minimum current with 2 decimal places is **2.39** $\boldsymbol{\mu A}$ that neuron will spikes one with this current: (trial & error method)

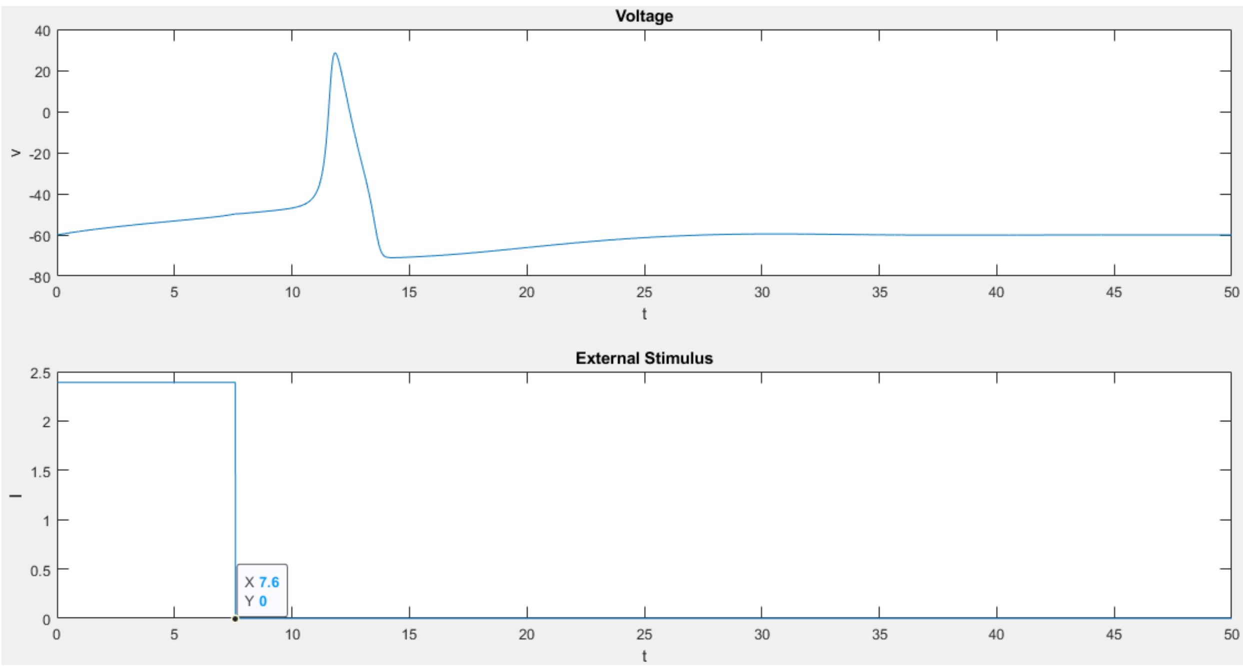


٤ – Time needed for at least one spike:

If we declare the current,

```
I =  [2.39 * ones(1,759), zeros(1,T-759)];
%in uA,external stimulus(external current)
```

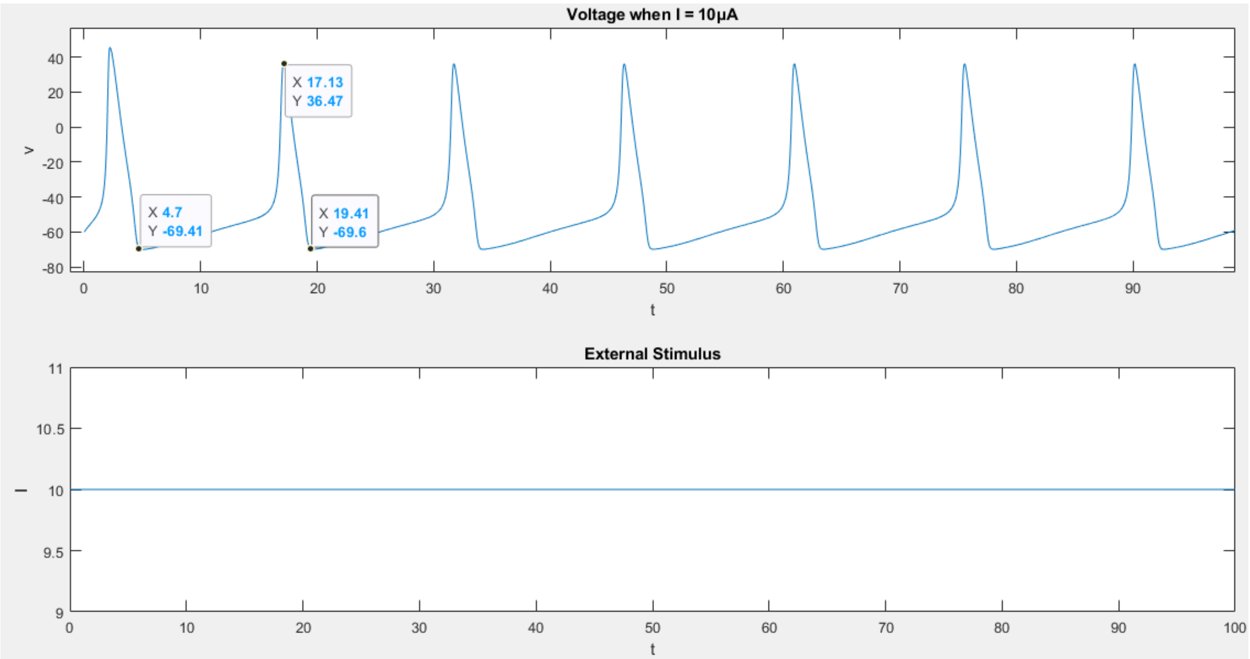
neuron will spike once. So the time of stimulation is about 759 time steps and each step is 0.01s. So the minimum time needed for stimulation is about **7.6ms** as we can see below:



5 – Action potential Domain and frequency for different Domains of External Stimulus:

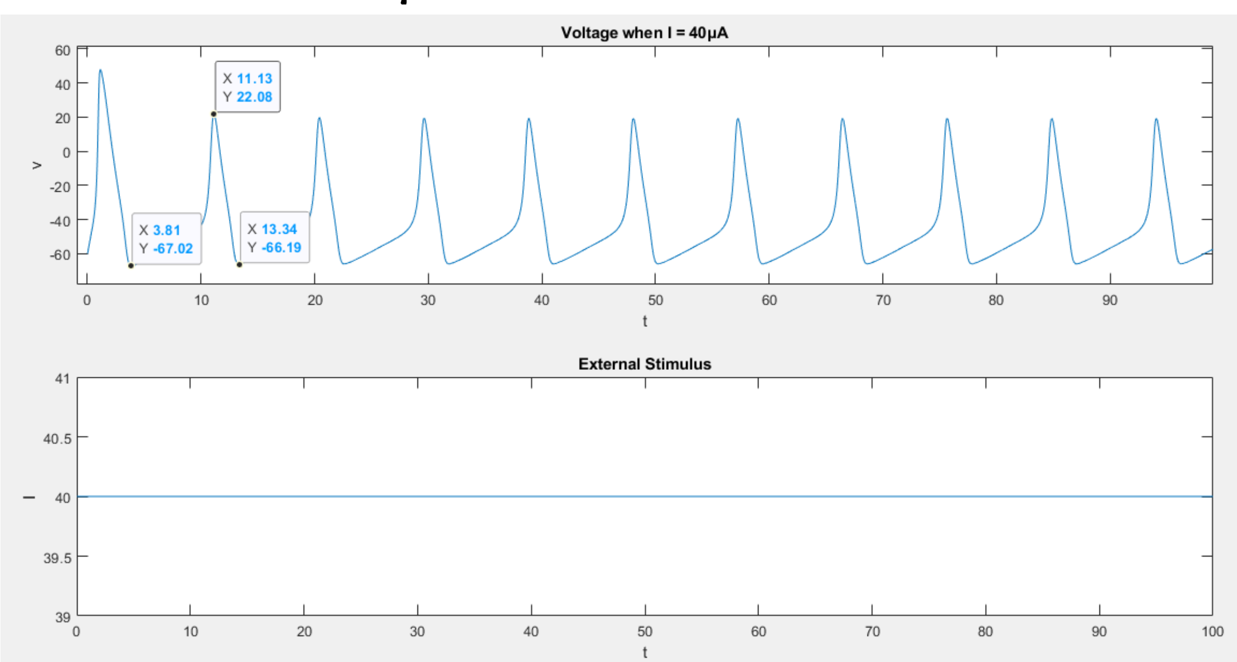
We expect low frequency spikes for low or very high external currents and high frequency spikes for something between these and when currents increases, domain decreases.

We start with $10\ \mu A$ current:



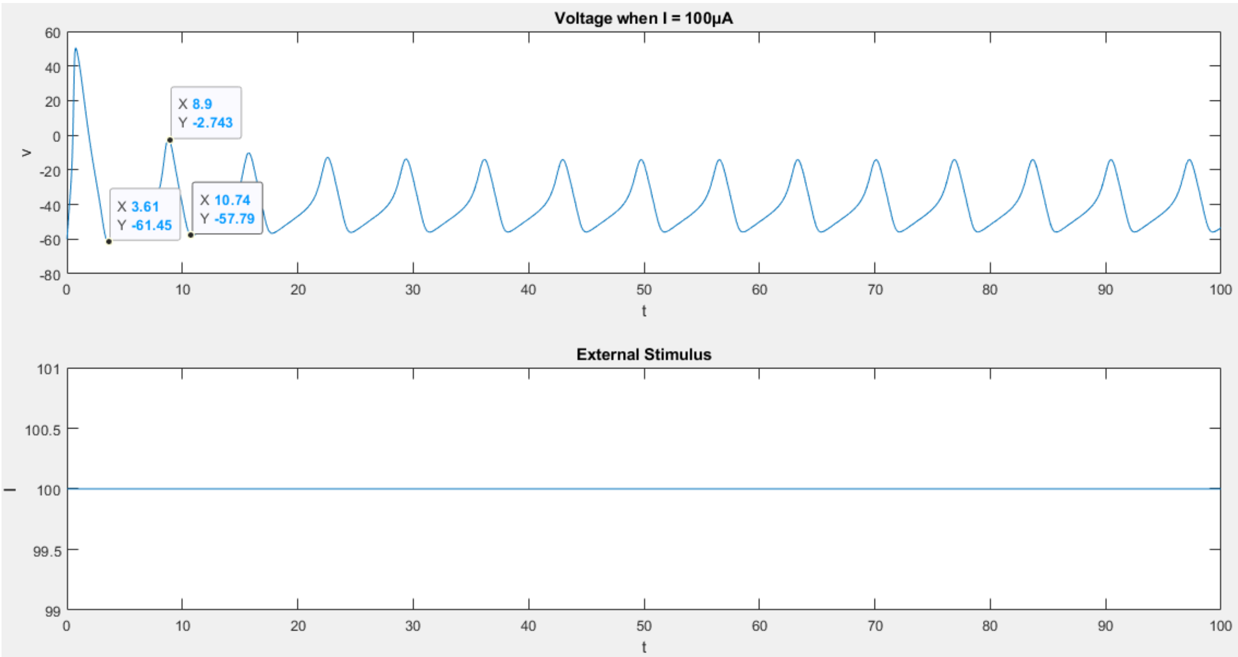
Here we can see spikes with the domain about **105** $\boldsymbol{mV}$ and the period about 15ms. So the frequency is about **67 Hz** .

When current is $40\ \mu A$:



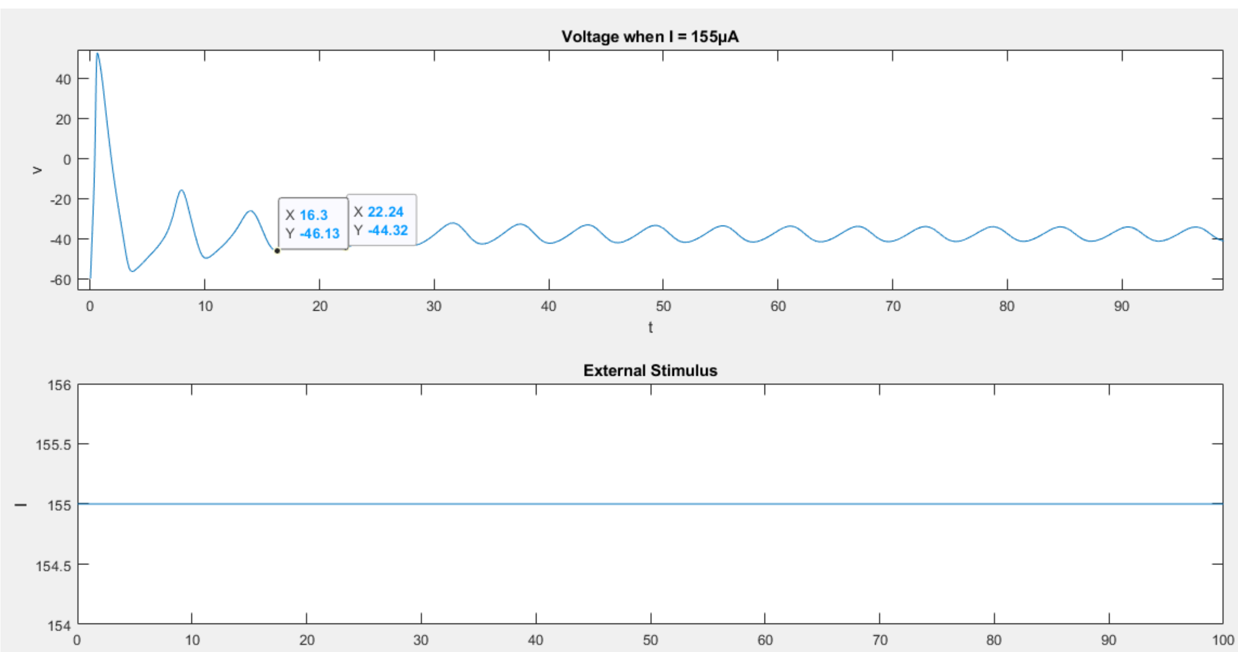
We can see the domain has decreases a little bit and it`s about **100 m$\boldsymbol{V}$** and the frequency has increases to **100 Hz**.

When current is 100 μA:



Domain ≅ **60 mV**, Frequency ≅ **140 Hz**

When current is **155 μA**:



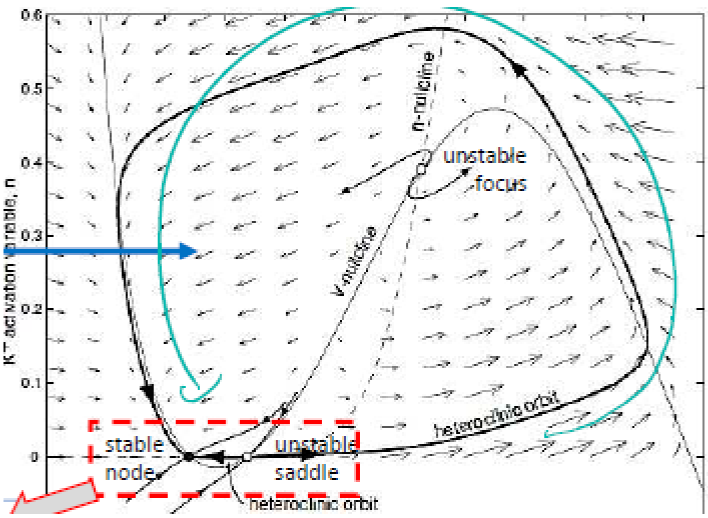
Frequency ≅ **170Hz** and the domain is so low.

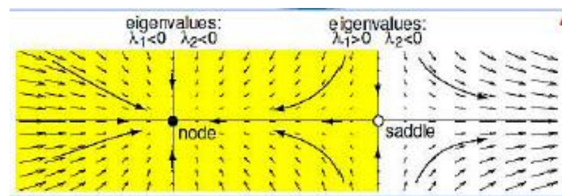
If we increase the current, neuron won`t spike periodic anymore.

So, **maximum frequency** is when I is in the maximum and it`s about **170Hz** and **maximum domain** is when I is near the minimum and it`s about **105 mV**.

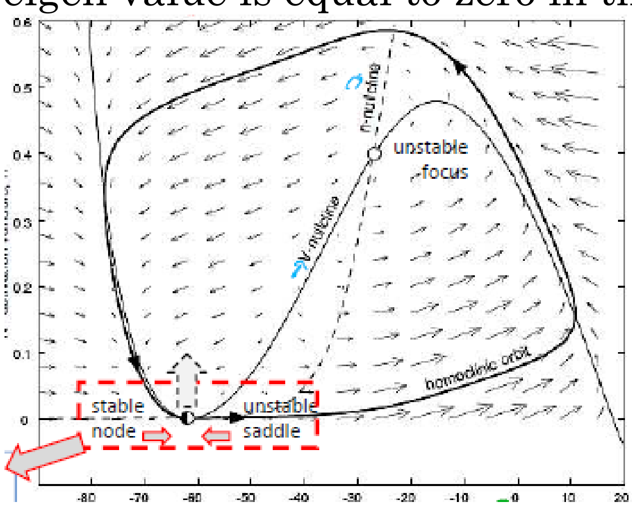
6 – External stimulus:

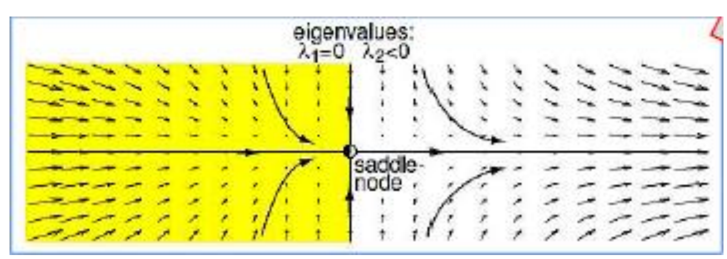
At low currents, we have a node and a saddle which with a little stimulation, our neuron move from node to saddle and it will spike once. (Like when the current was about 5 μA)



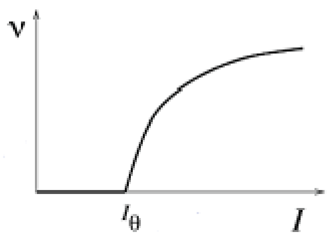


When we increase the current, V nullcline will go up and the 2 nullclines just touch each other in one point which node and saddle get mixed and **saddle-node bifurcation** happens. (one eigen value is equal to zero in this case)



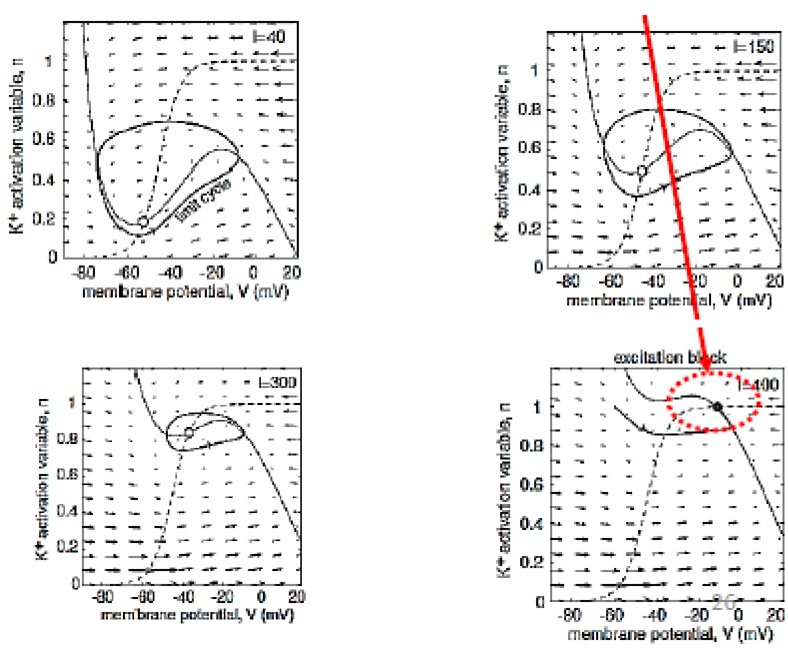


If we increase the current more, 2 nullclines won`t touch each other in any points but in an unstable focus and we won`t have node and saddle. In this case, a limit cycle attractor will be made and neuron will spikes periodic with a high rate. As we increase the current, the frequency will increase up to a limit:



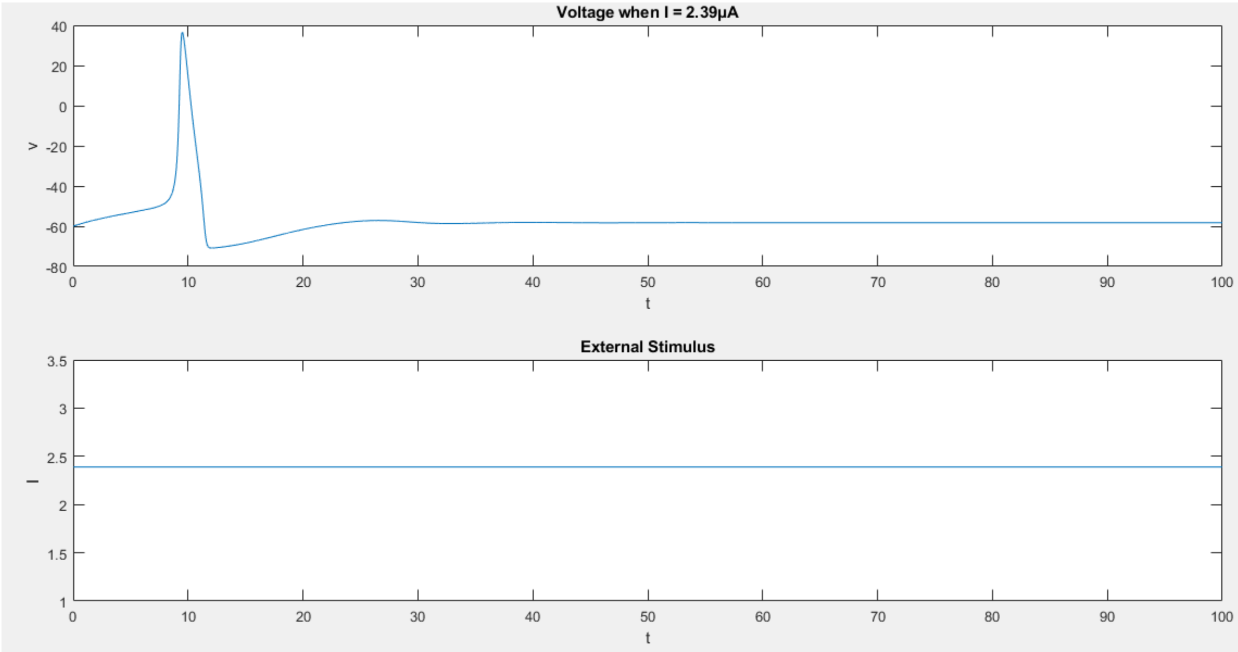
- V is the firing frequency

Again by increasing the current, V nullcline will go upper and upper until something called, **Excitation Block**, happens.

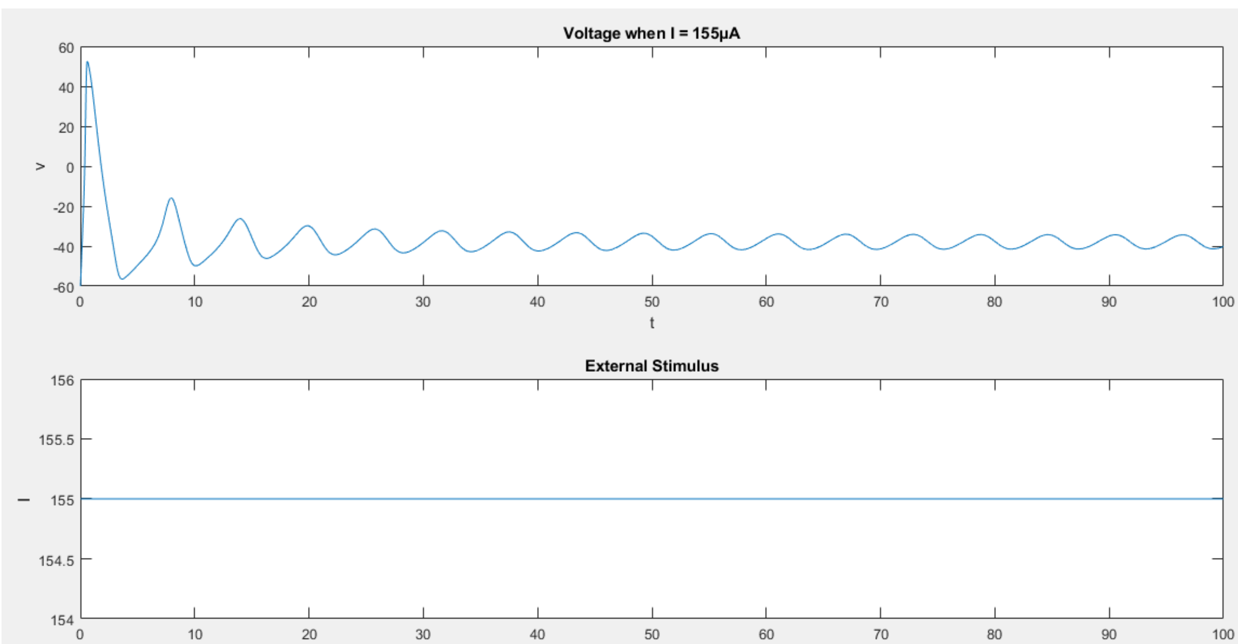


This is when the unstable fixed point will becomes stable and the limit cycle shrinks and spiking activity disapears.

- Minimum external stimulus:

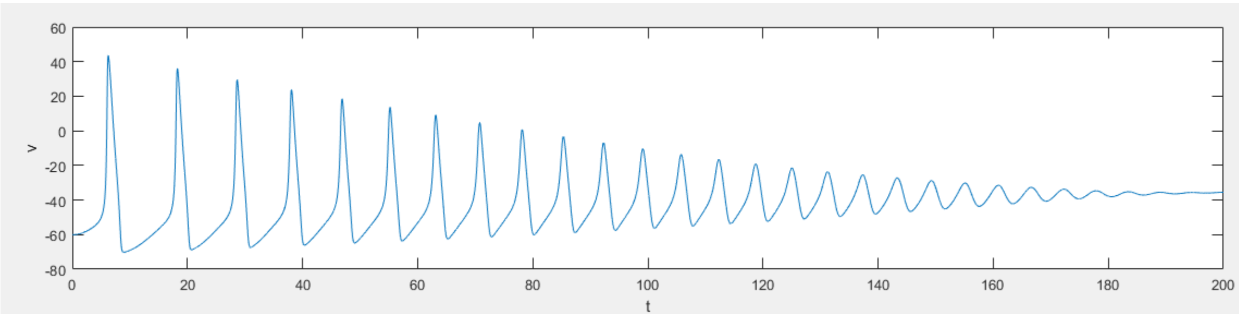


- Maximum external stimulus:



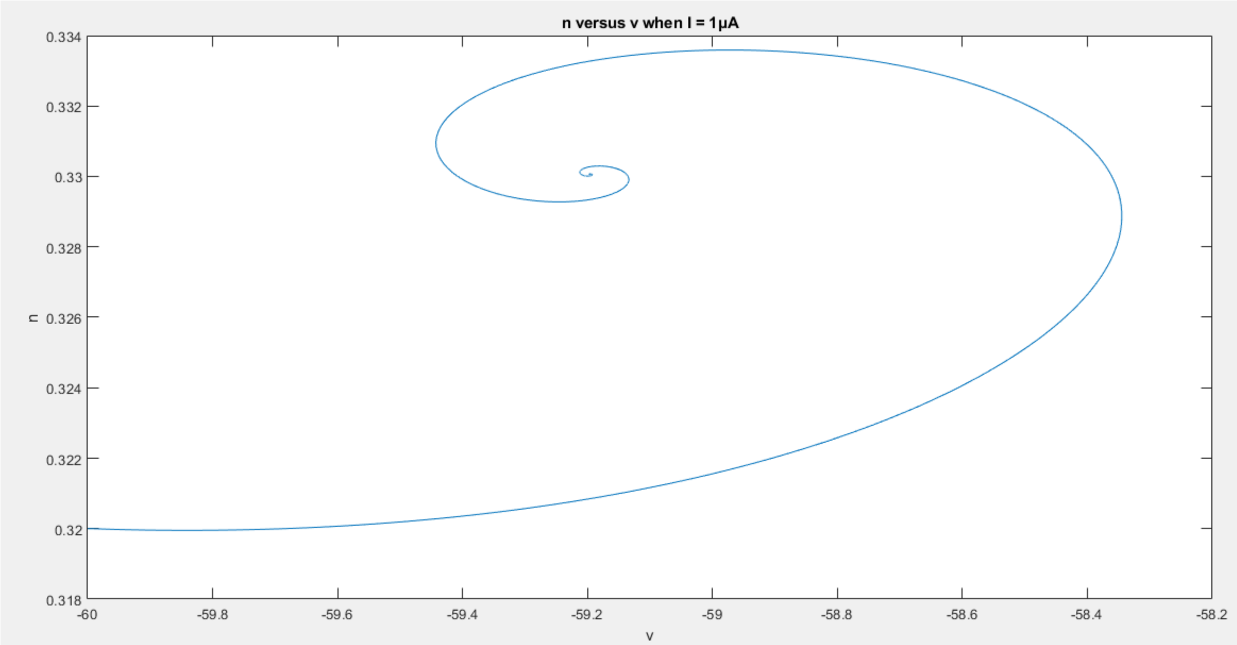
7 – A current input between min and max current that neuron spikes:

As we see, we have a current which starts from 0 μA and goes to 200 μA. At first when we`re at the minimum value, we have the maximum domain and a low frequency but as we go forward to the maximum value of current, the domain gets lower and lower and the frequency increases.



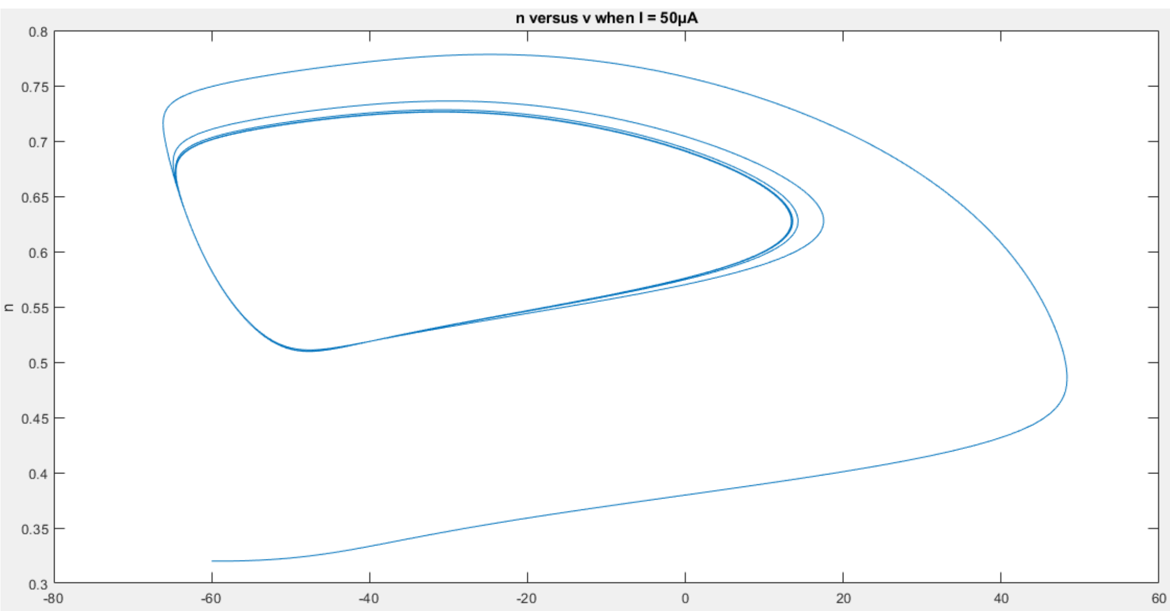
8 – n to v plot:

- Low current that neuron won`t spike:



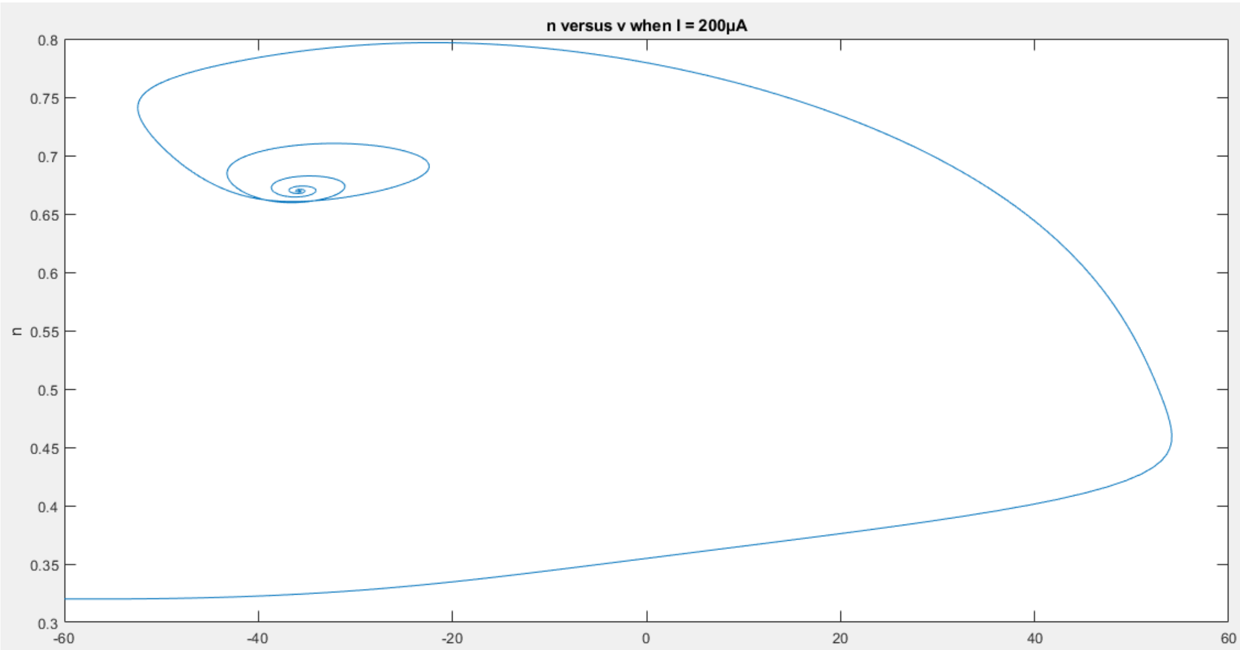
As we see value of n and v is almost constant and don`t change.

- Current that neuron will spike:



As we can see when voltage goes to its maximum, n increases and Potassium channels will open and voltage decreases and we can see this cycle that shows spiking in the plot.

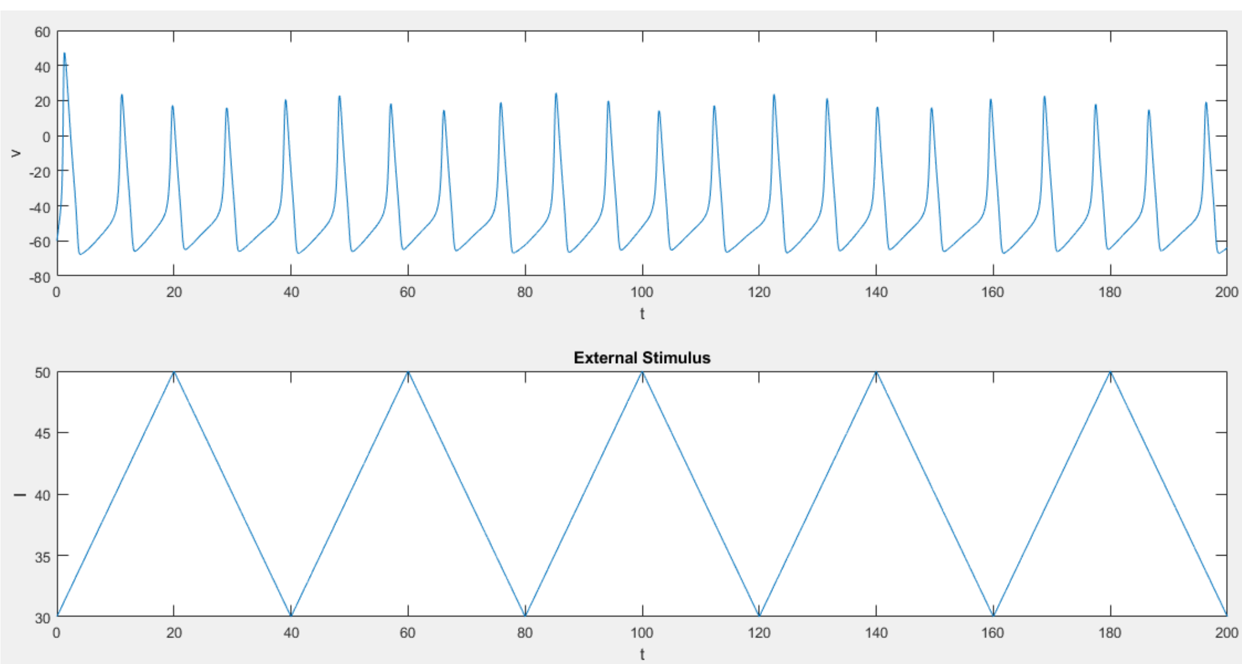
- High current that neuron won`t spike:



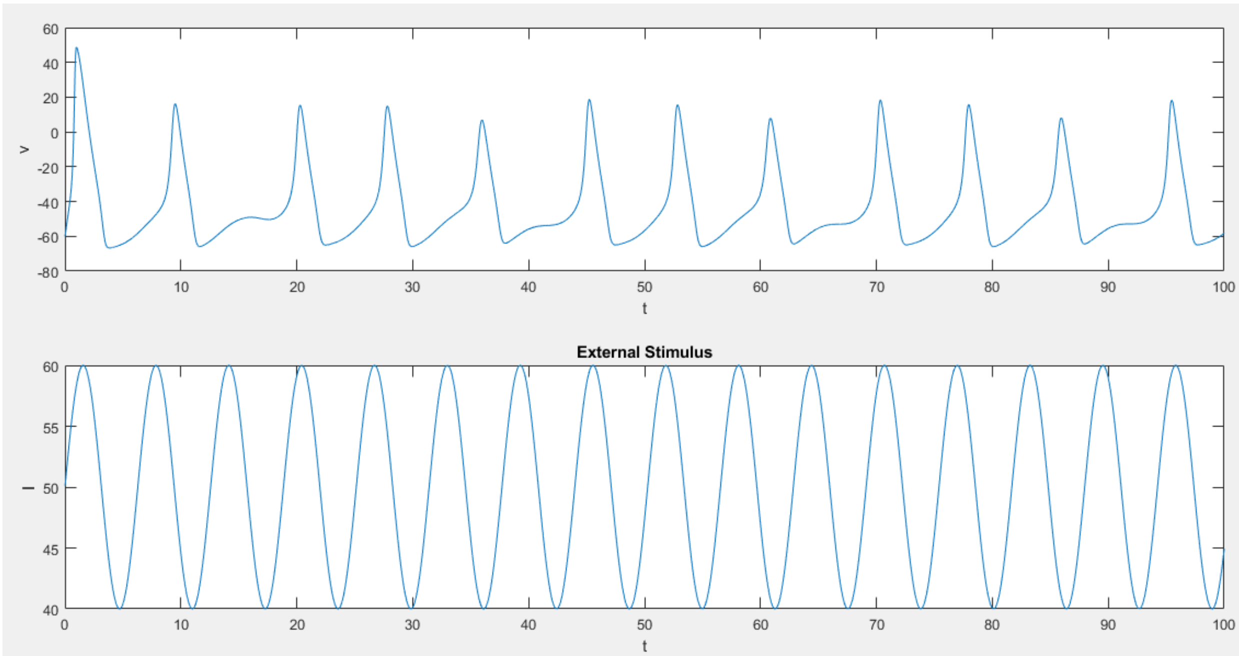
Neuron will just spike then quickly it goes to the stable fixed point which n and v will be constant like the plot (**Excitation Block**)

9 - Different types of external stimulus:

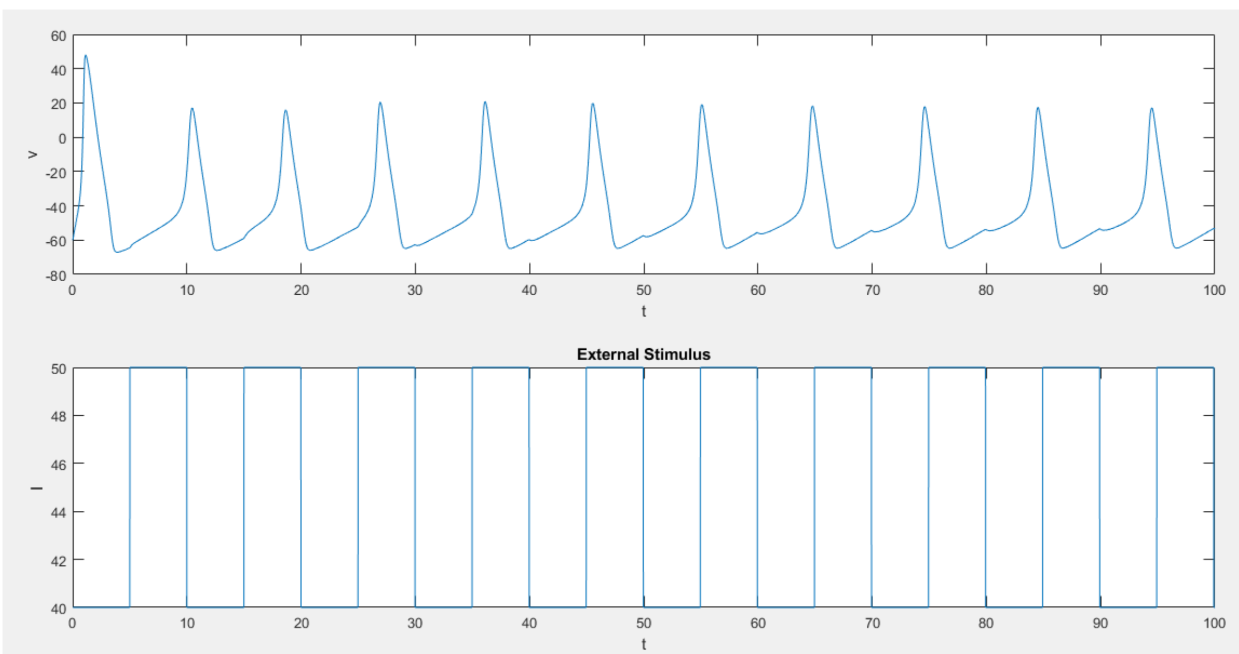
1 – Triangular:



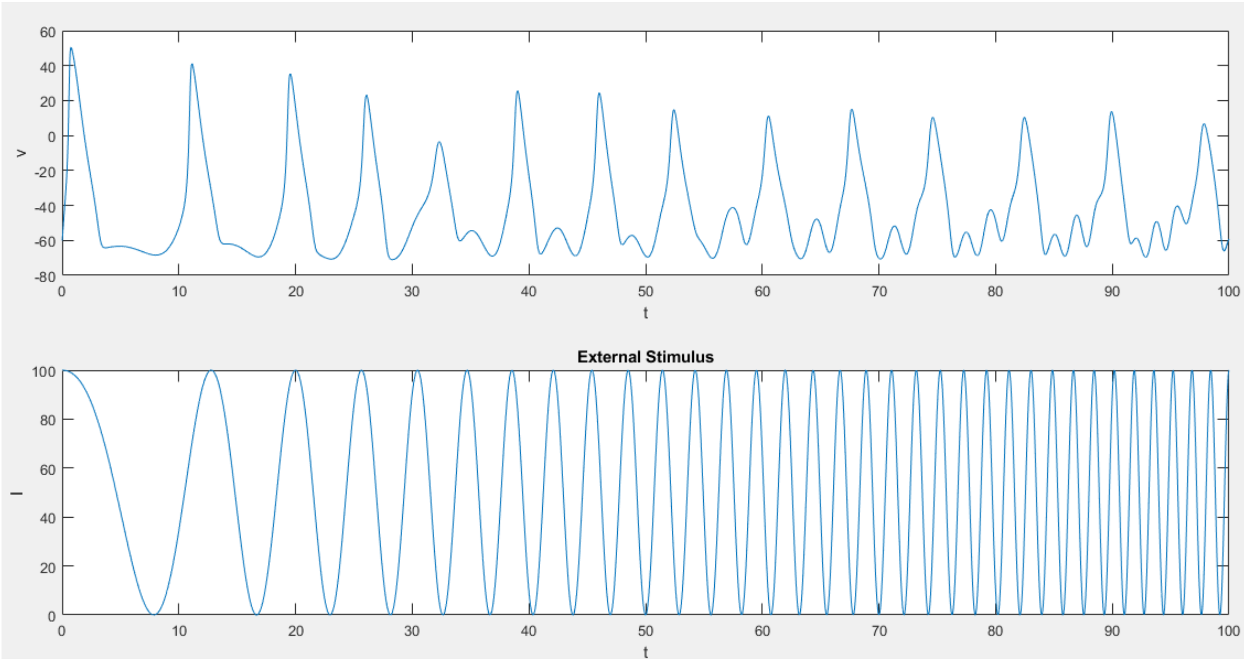
2 – Sin:



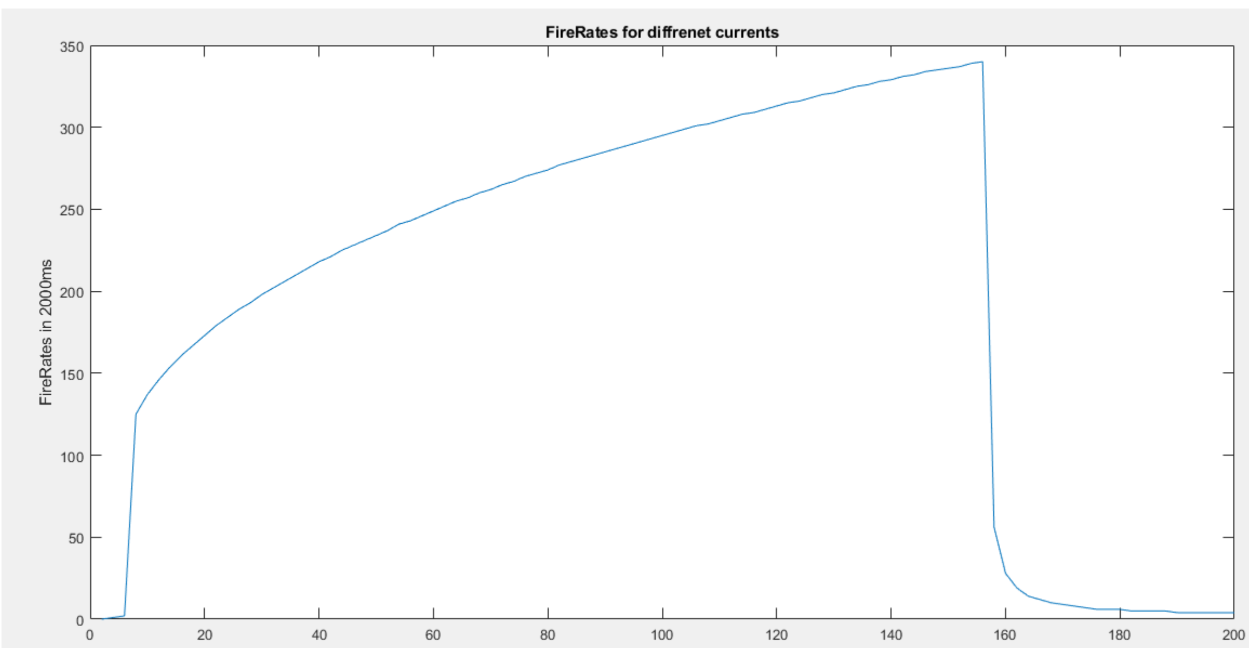
3 – Square pulse:



٤ – Chirp:



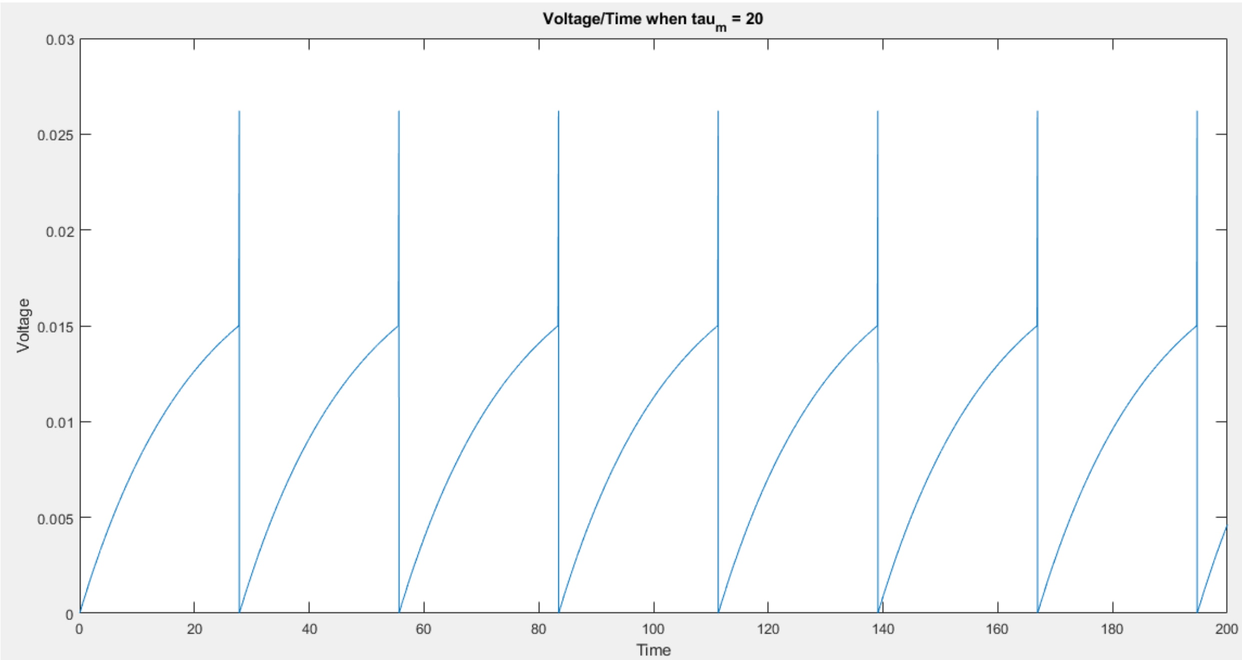
10 - Different external stimulus:

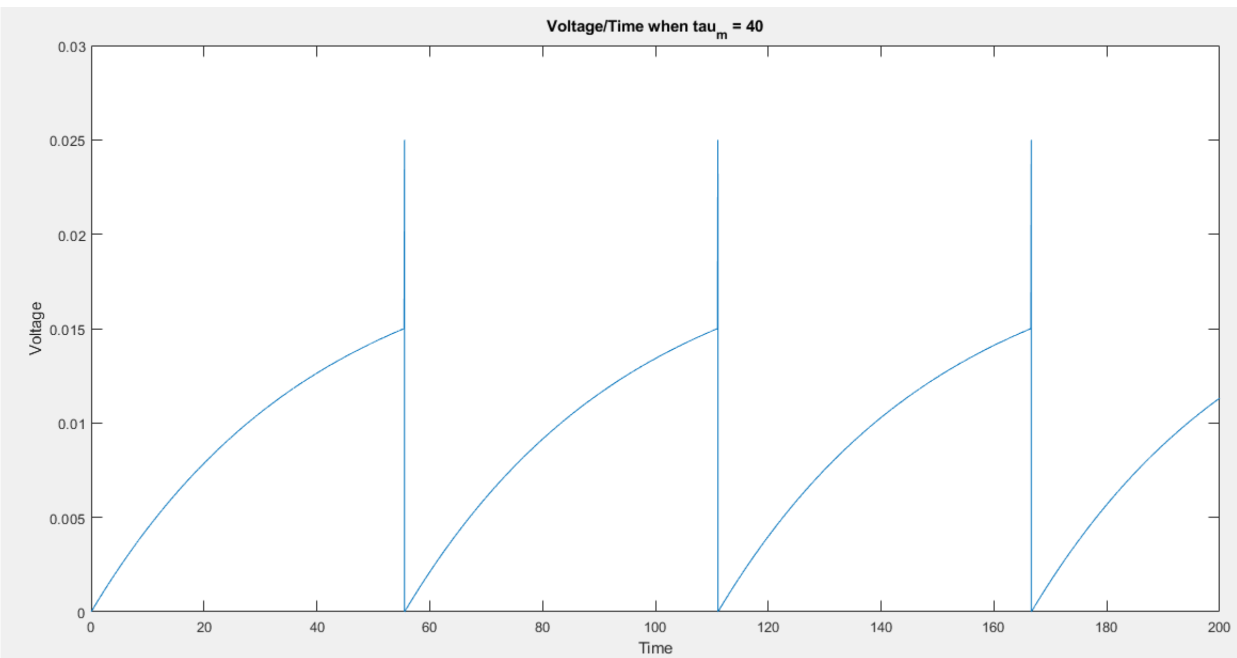


When I increases, fire rate increases too, But after we reach the maximum current value which we found in previous parts, fire rate decreases. I have explained the neuron behavior very detailed in part 6 which fully covers this part too.

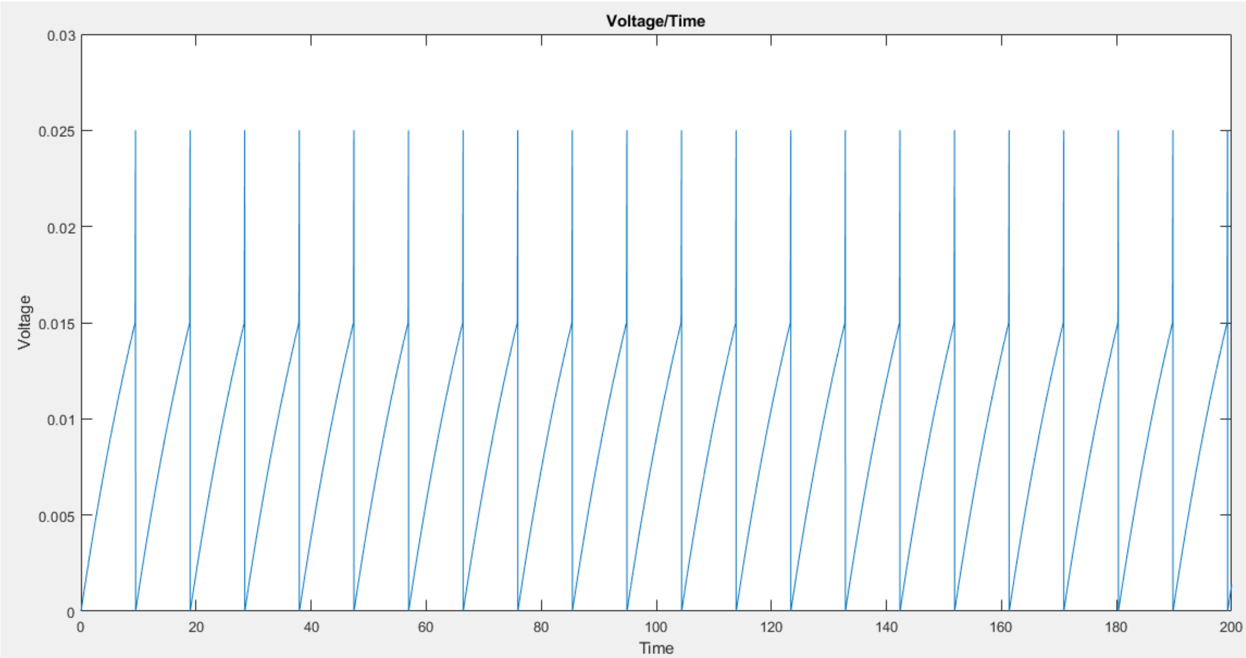
2 - Leaky Integrate and Fire Neuron Model:

1 – **Voltage of the neuron:**

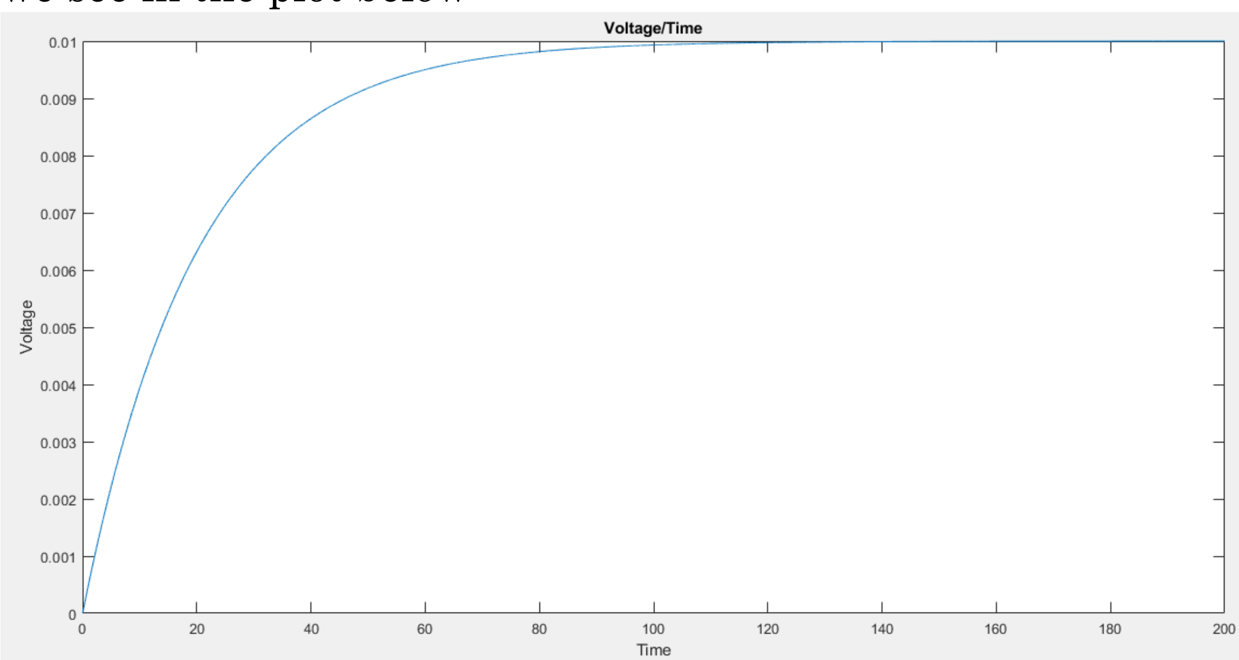




If we increase the RI, the spiking frequency will increase as we see in the plot below:



If we decrease the RI, the spiking frequency will decrease as we see in the plot below:



As we see, neuron won`t spike when we decrease the RI value. (when lower than Vth = 15mV)

When v(t) reaches the threshold voltage

2 – Solving the diffrential equation:

v(t) = $ce^{-t/\tau_m} + RI$

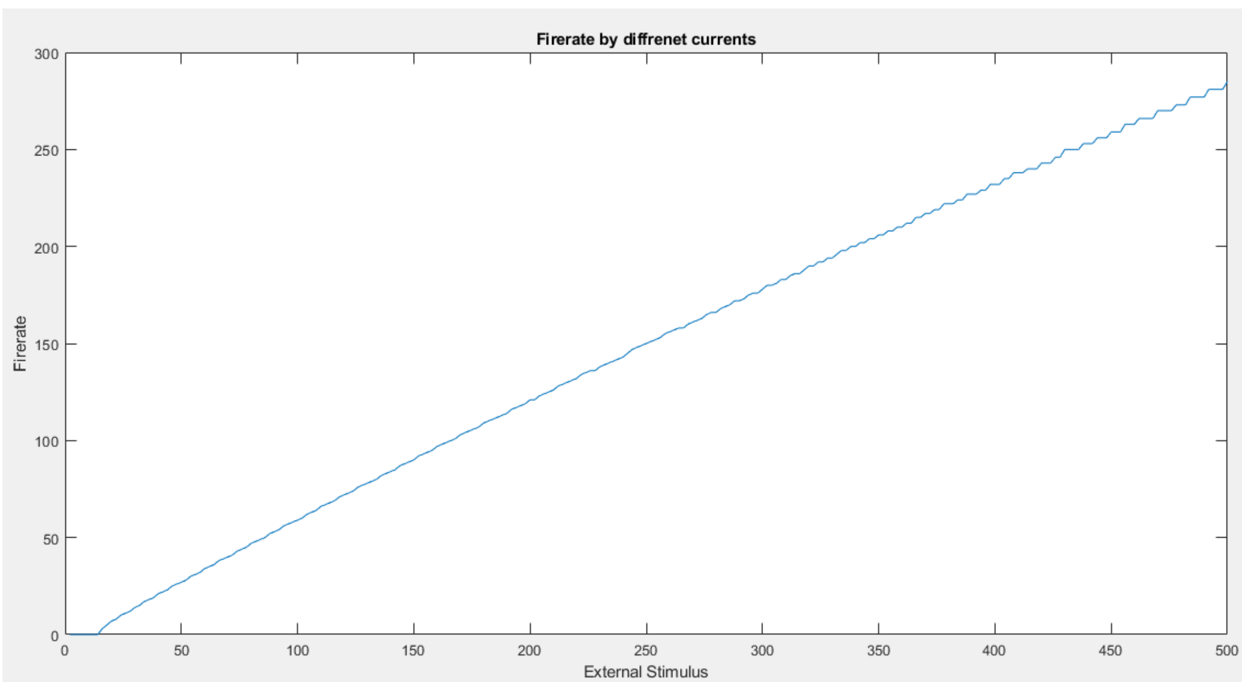
v(t) = v_{th} $\Rightarrow$ $-RIe^{-t/\tau_m} + RI = v_{th}$ $\Rightarrow$ $\boxed{t = -\tau_m \ln(\frac{v_{th}-RI}{c})}$

$$T = -\tau_m \ln\left(\frac{RI-v_{th}}{RI}\right) + \tau_r$$

So f = 1 / T

As we can see, when RI increases, T decreases and so f increases and when RI decreases, T increases and f decreases as we saw in part 1.

٣ – I-F plot: (for 250 diffrenet currents):



As we expected, by increasing I and so increasing RI, frequency of spikes increases and fire rate increases. And spiking start when RI passes about 15mV as we saw in the part_1.

When RI gets bigger than Vth, by increasing it more, frequency and fire rate increases as we saw in part_2.

4 – **A vector of 0,1 with Poisson Distribution:**

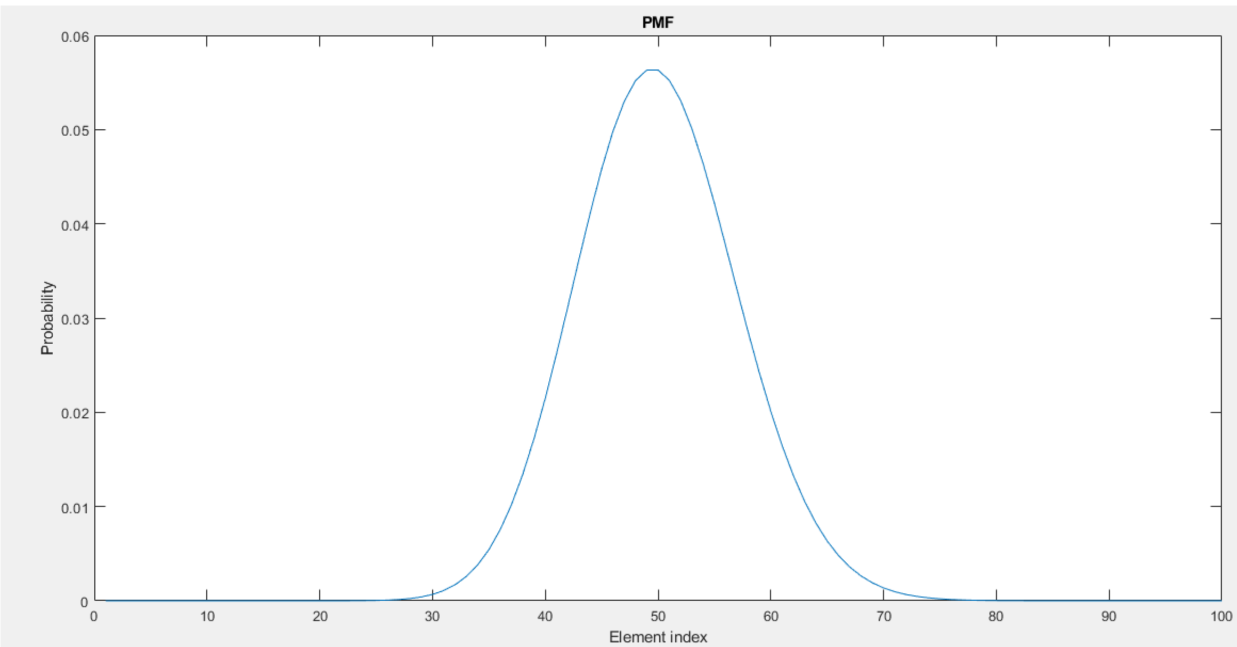
The probability of being 1 for each element of our vector follow the pmf of the poisson distribution with parameter lambda which user gives us in input. We can have the pmf of this distribution with the function below:

```
probabilities = poisspdf(1:T,lambda);
```

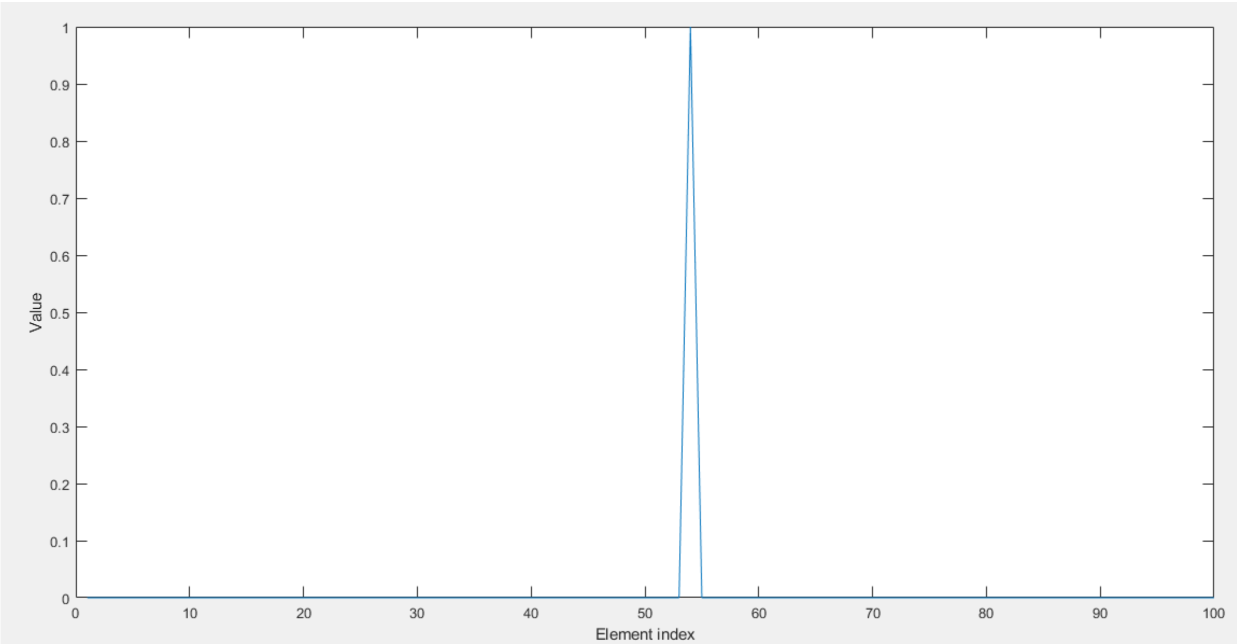
then by the probabilities we have for each element of the vector, we change the number of elements like Bernoulli distribution.

```
vector(i) = binornd(1,probabilities(i));
```

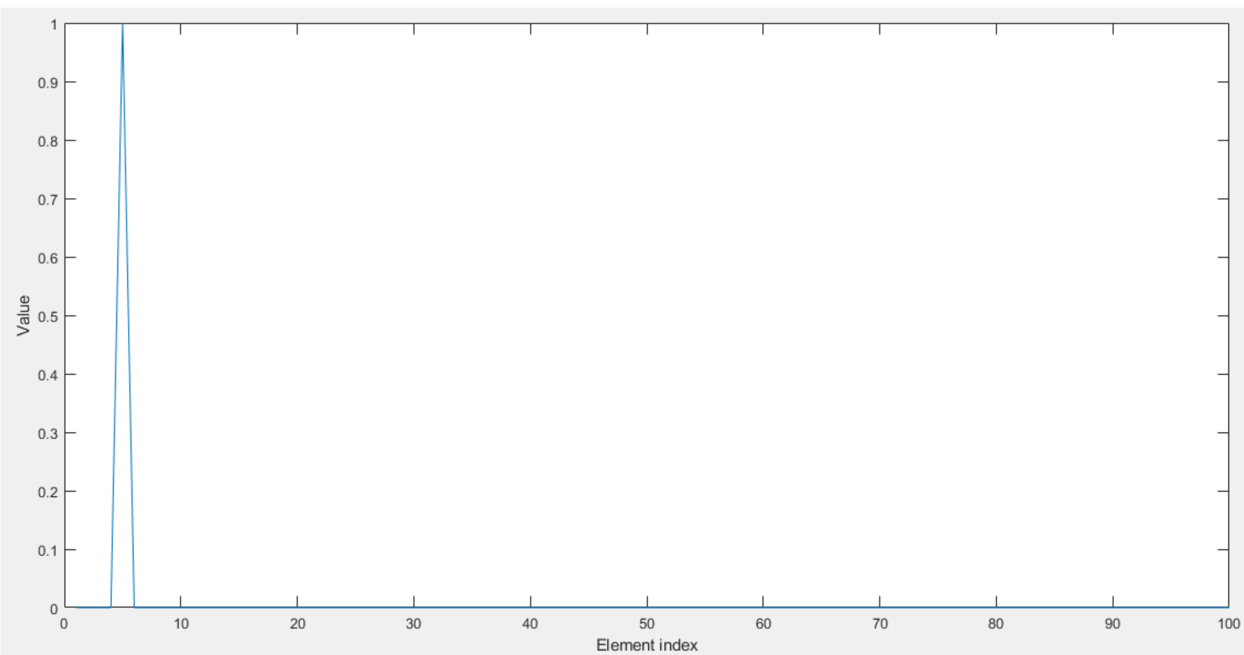
PMF of the probabilities:(for lambda = 50)



Our vector:(for lambda = 50)

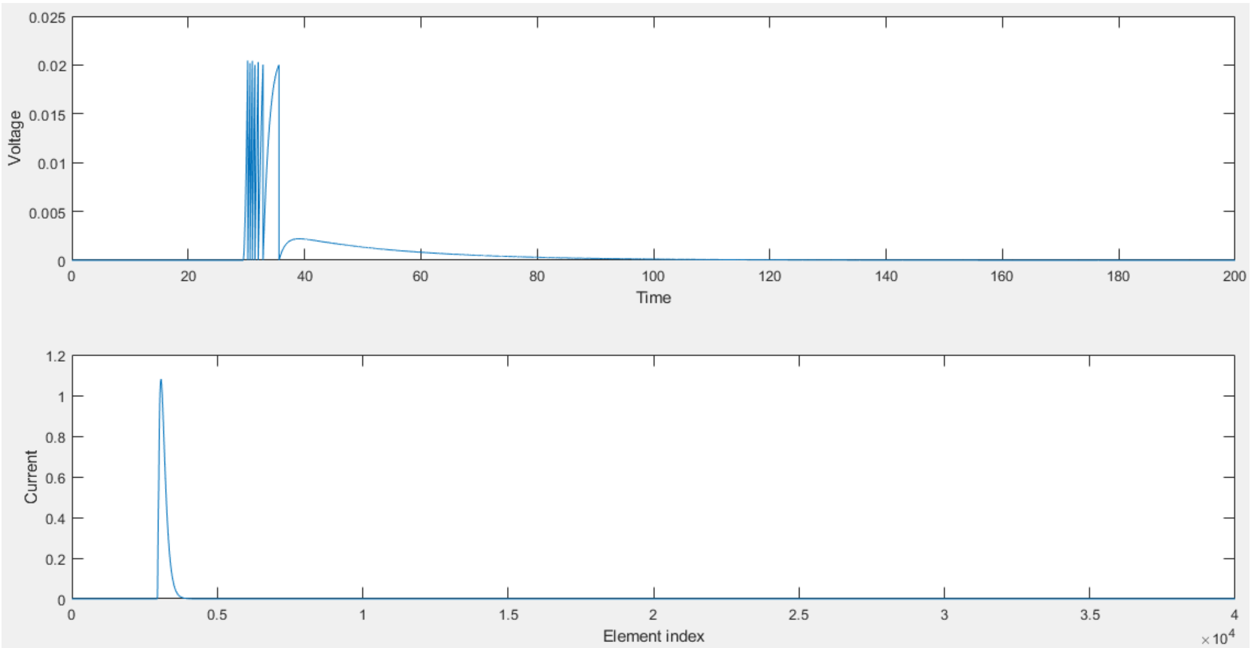


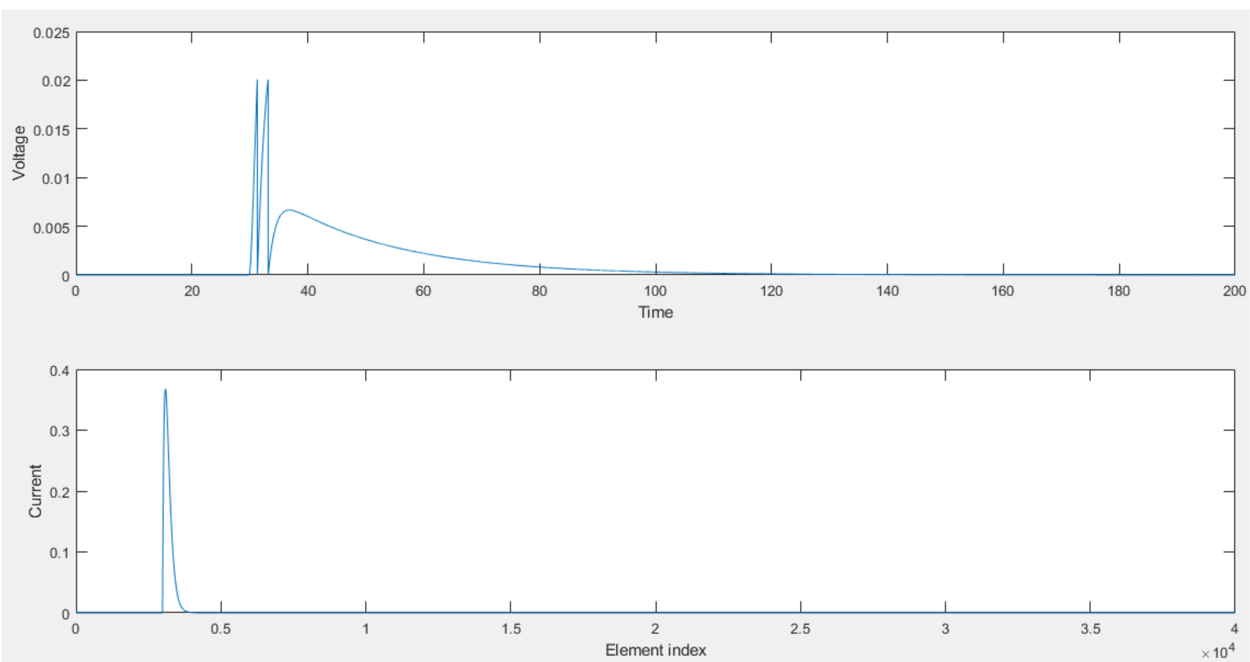
Our vector:(for lambda = 5)



5 – **LIF real model:**

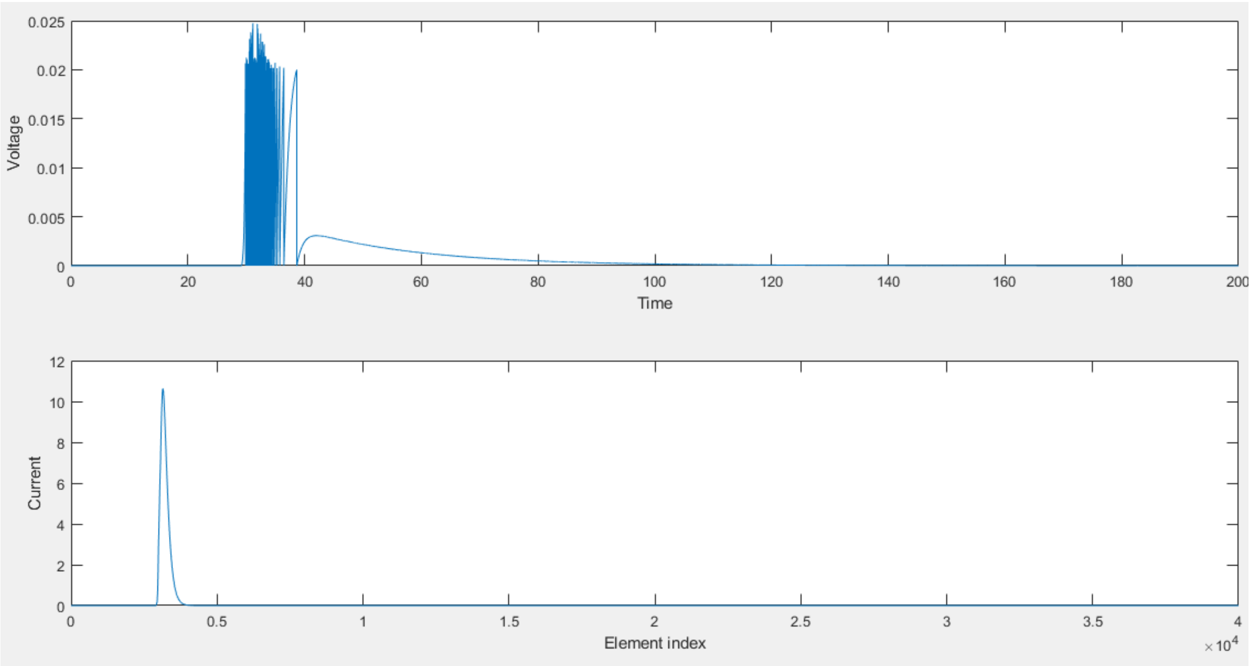
We use <u>conv</u> function for convolution and calculate I and then with Euler method, we calculate v.



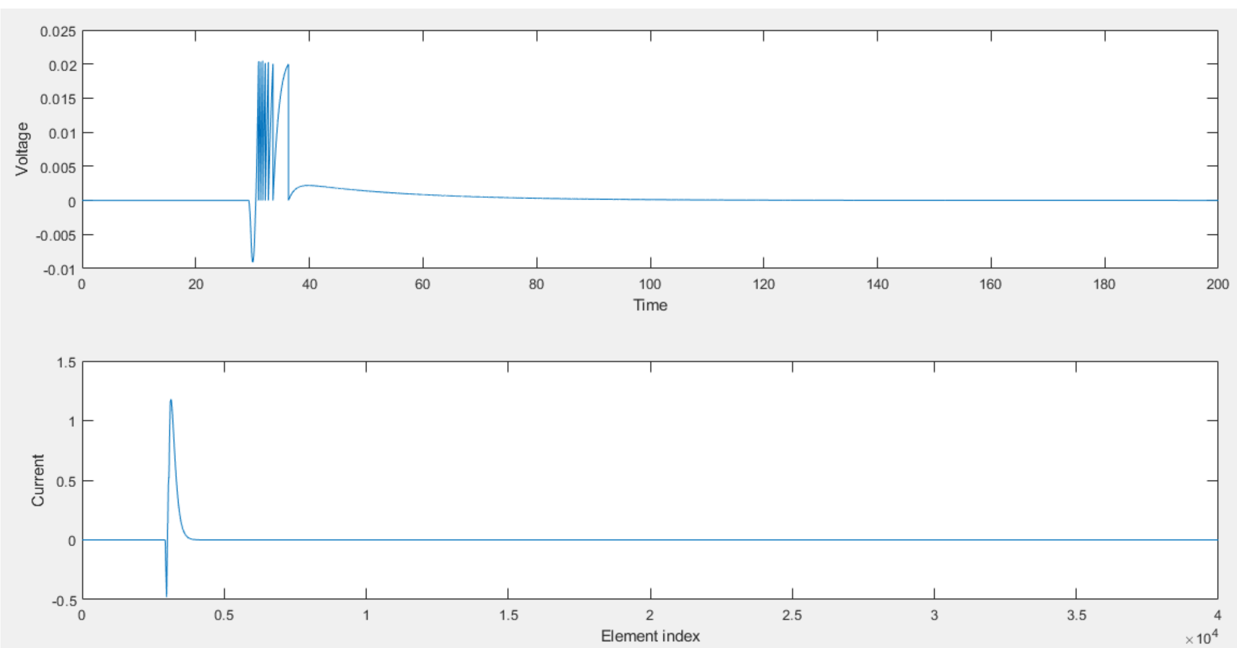


Poisson distributions are used to understand independent events that occur in a time. here, a stimulation happens to neurons around lambda. so poisson distribution seems good enough for this modeling.

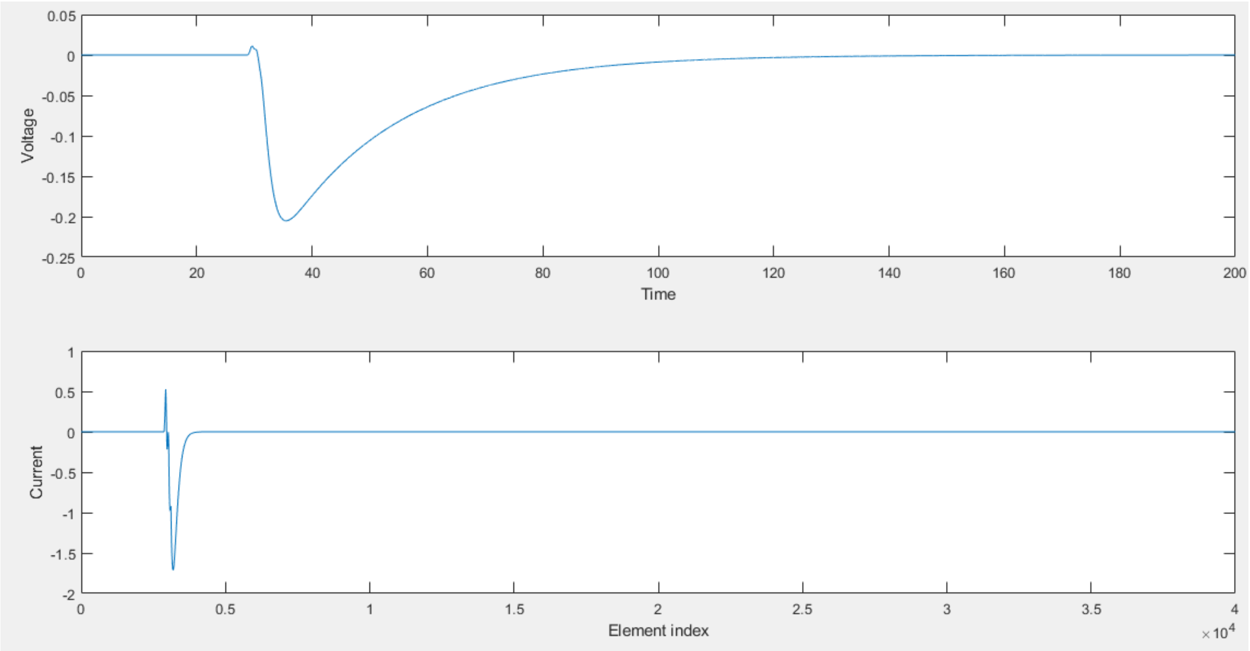
6 – Inhibitory and Excitatory:(sum of 50 diffrenet currents)



Twenty-six(26) percent of the currents are inhibitory and as we can see, neuron spikes because the inhibitory currents are not that much and the sum of the 50 currents is like a excitatory.

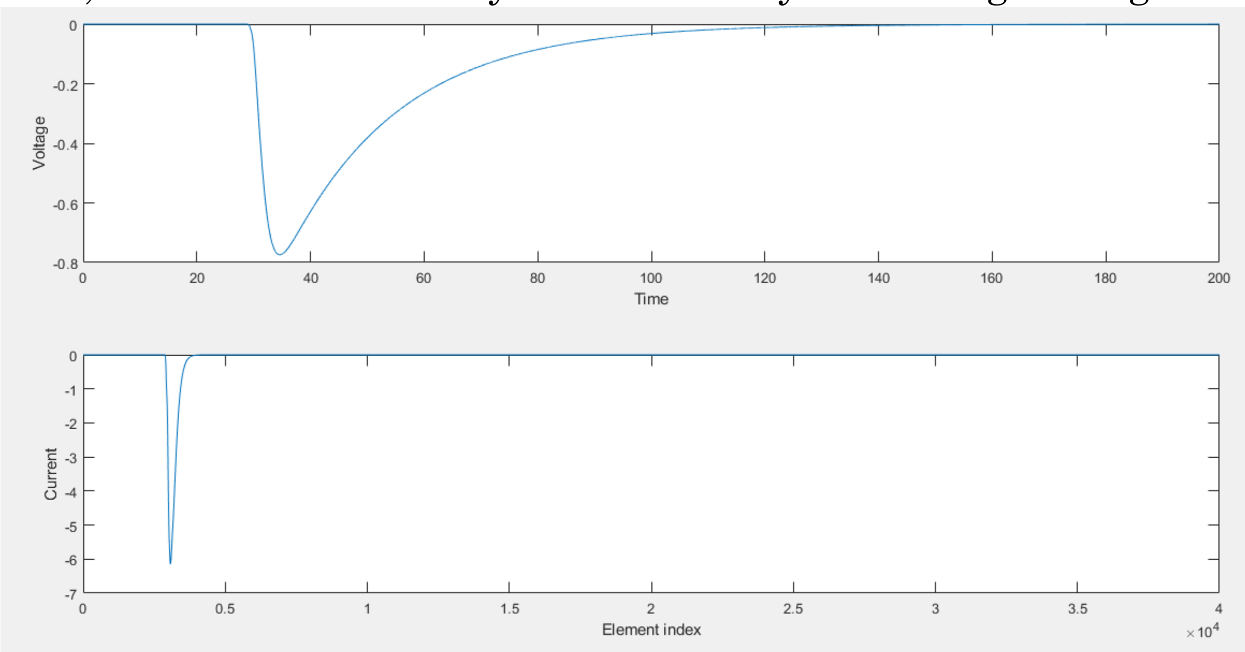


Thirty-eight(38) percent of the currents are inhibitory and as we can see, in a period of time, sum of the currents will be negative



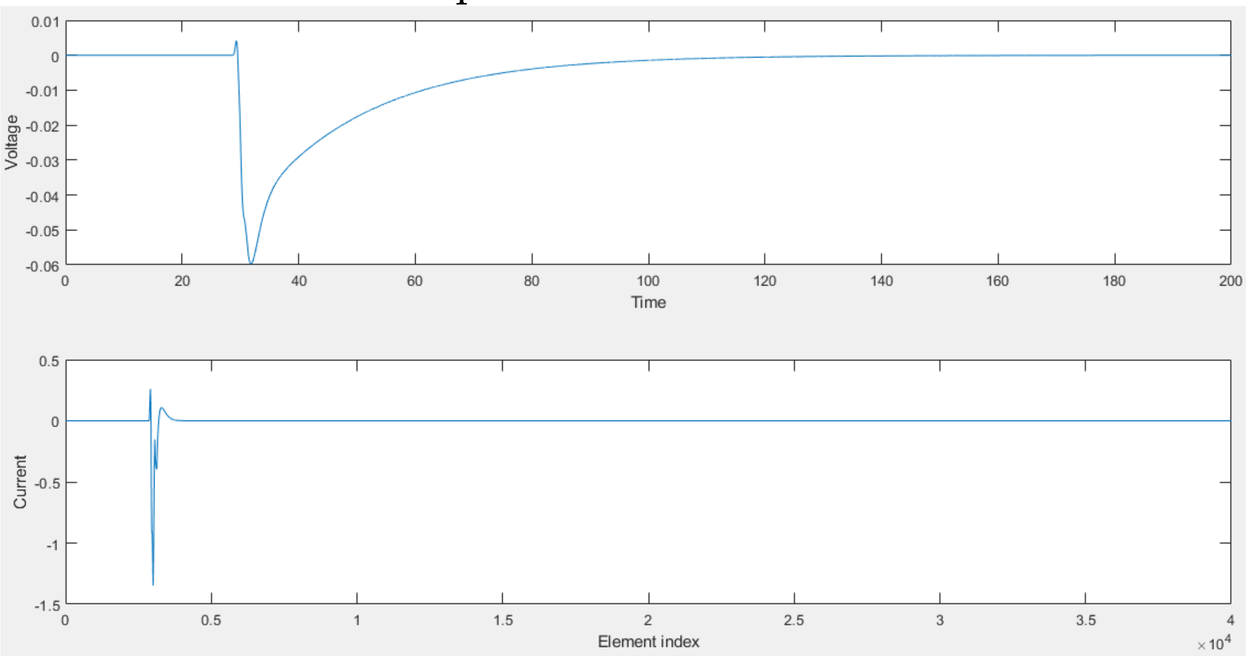
and the voltage goes negative but after that currents are excitatory and neuron spikes.

Sixty(60) percent of the currents are inhibitory and as we can see current goes up a little at first and so on the voltage but after that, sum of the inhibitory and excitatory currents goes negative

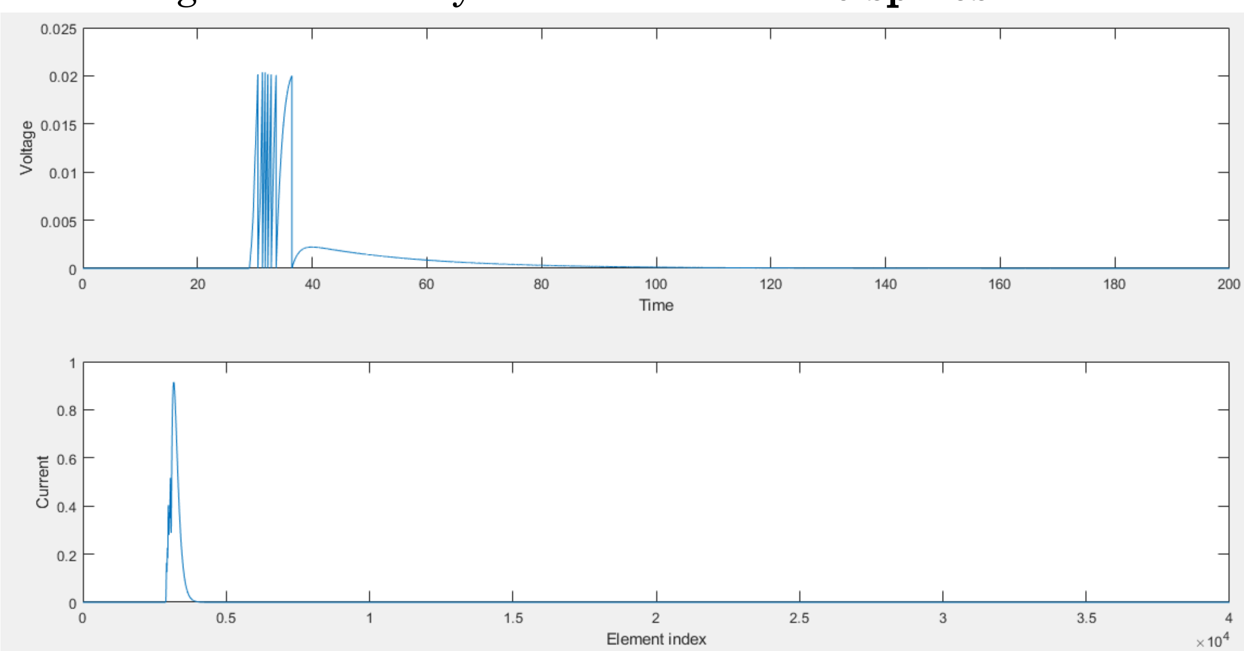


and the voltage goes negative and so we can`t see no spikes with this percentage.

And as we expected, if the percentage of the inhibitory currents increases(for example here it is 70%), the voltage goes down and we have no spikes.



Percentage of inhibitory currents: 44% - **No spikes**



Percentage of inhibitory currents: 46% - **Spikes**

As we can see the maximum percentage that neuron will spike is something about 45-50% or maybe a little more.

We can`t find a very exact percent because for example as we can see in the last page we have spiking for 46% but not for 44,

Because distribution of the inhibitory currents can be different and if they`re close to each other, sum of them can be big and make voltage goes to negative numbers but if they`re not close they can`t. It`s all based on possibilities.