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Final Project

Signals and Systems

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Semester 99-2

1 Sampling and DFT

Functions:

- HalfBandFFT.m
- CTFourierTransform

HalfBandFFT :

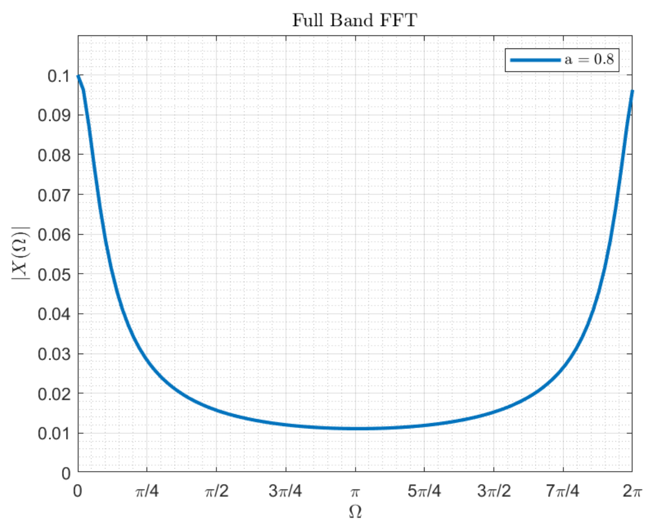
This functions receives a discrete signal as an input and in the output it generates the DFT of signal; as we know the DFT of a discrete signal with length N is computed as follows:

$$X[k] = X(\Omega)\big|_{\Omega=\frac{2k\pi}{N}}$$

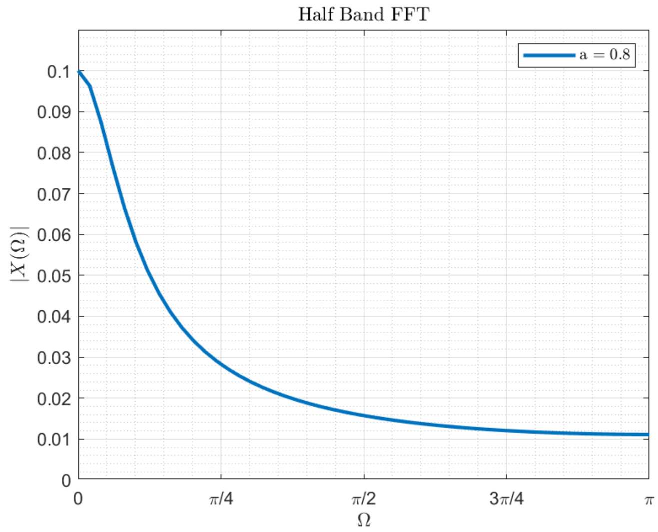
So, to compute the DTFT of a signal, which we know is periodic with period 2π and also symmetric for real signals, we use the DFT samples; which was computed with the FFT algorithm for fast implementation. Also we have normalized the magnitude of fourier transform for better visualization by the factor $\frac{2}{N}$.

Example:

$$x[n] = a^n u[n] \leftrightarrow X(\Omega) = \frac{1}{1 - ae^{-j\Omega}} \qquad |a| < 1$$



As we observe, the DTFT is symmetric with respect to $\Omega = \pi$; so we plot only half band of DTFT:



Nyquist Frequency:

As we know, to be able to reconstruct the signal from the down-sampled signal, which is shifted versions of continuous time Fourier Transform of signal, the following condition should be met:

$(f_s \text{ is sampling frequency and } f_{\max} \text{ is the maximum frequency of signal})$

$$f_{\max} < f_s - f_{\max} \rightarrow \boxed{f_s > 2f_{\max}}$$

Aliasing:

We know that, from what we have learned during the course, if the condition of Nyquist frequency is not met for a signal, its fourier transform, and hence the fourier inverse of that signal, is interfered and that is the phenomenon which we call *Aliasing*. So, the reconstructed signal would not be the same as original signal:

$$\hat{x}[n] \neq x[n]$$

The example below demonstrates the above explanations:

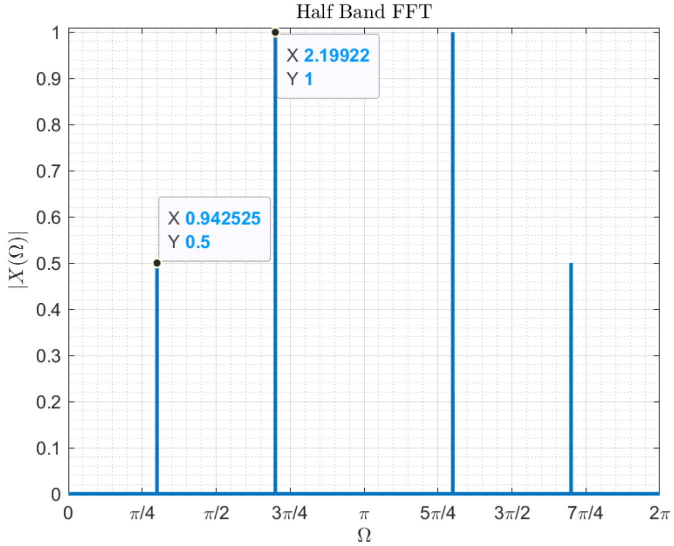
Example:

Consider x(t) = $\frac{1}{2}\sin(2\pi.30t) + \cos\,(2\pi.70t)$

If we sample x(t) with $F_s = 200Hz$, the Nyquist condition is met, because:

$$F_s = 200 > 2f_{max} = 140\ Hz$$

So, if x[n] = x($\frac{n}{Fs}$) , the DTFT of x[n] has no aliasing, as we see below:



We know that in the discrete signal, each mapped frequency is computed as follows:

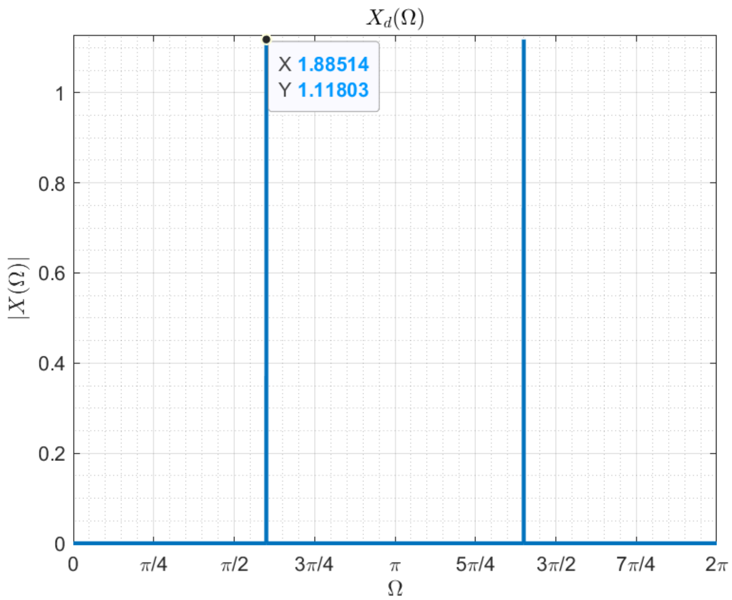
$$\Omega = \frac{w}{Fs} \rightarrow$$

$$\Omega_1 = \frac{2\pi.30}{200} = \frac{3\pi}{10} \cong 0.942 \qquad\qquad \Omega_2 = \frac{2\pi.70}{200} = \frac{7\pi}{10} \cong 2.199$$

Which agrees with the above figure; However, if we down-sample x[n] with rate 2, in fact we are sampling x(t) with $F_s' = \frac{F_s}{2} = 100\ Hz$; hence Nyquist condition is not met:

$$F_s' = 100Hz < 2f_{max} = 140\ Hz$$

So, we expect aliasing to happen; which can be seen in the next page:



As it is obvious from the above figure, the frequency in which the DTFT has a peak is not the corresponding continuous frequency. In addition, we have lost the other component frequency and also the value of the peak is not true. So we are not able to reconstruct the original signal due to the aliasing.

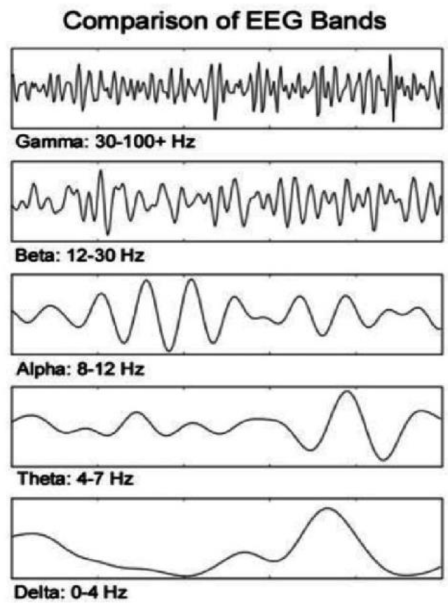
+ CTFourierTransform

This function computes the fourier transform for a continuous-time signals(i.e. the frequency axis contains all frequencies in Hz) and plots the one-sided amplitude spectrum. This function is required too much in the following parts, so we write that in this part.

2 Introduction to EEG signals

ERPs and P300

- **Frequency bands:**



Although there are different classifications for EEG signals frequency bands, one of the classification is the one you see in the image in front. According to this definition, there are 5 frequency bands as you see in the image. Delta, Theta, Alpha, Beta and Gamma. Usually **Delta band**, tends to be the highest in the amplitude and the slowest waves. Normally this frequency band is normal as the dominant rhythm in the infants up to one year old and adults` slow-wave sleep. Also, it has been found during some continues-attention tasks. **Theta band**, is classified as “slow” activity. It may be seen in drowsiness or arousal in older children and adults. It is seen when we`re idling, too. This frequency band is completely normal in children up to 13 years old and in sleep but it`s abnormal in awake adults. **Alpha band**, is usually seen in posterior regions of head in both sides, being higher in amplitude on the dominant side and in central cites(c3, c4) at rest. It appears when closing eyes and relaxing and disappears when opening eyes or getting alerted by any mechanism like thinking or calculating. It is the dominant rhythm seen in normal relaxed adults. This band is presented during most of the life specially after thirteen. **Beta band**, has a low amplitude and its activity classifies as “fast” activities and it is closely linked to the motor behavior. It has been founds on both side of the head with a symmetric distribution and it is mostly evident in the front. It happens during active thinking, focusing, being anxious, have eyes open and when highly alerted. Also, it is accentuated by sedative-hypnotic drugs. **Gamma band**, is found in Somatosensory cortex and shows rest state of motor neurons. Gammas are thought to represent binding of different populations of neurons together into a network for carrying out a motor or cognitive function.

- **Sampling frequency:**

EEG signals are usually recorded at sampling rates between 250 and 2000 Hz in clinical and research things. Why? As you saw in the previous part, maximum frequencies are in Gamma band which they`re about 100 Hz. So according to the Nyquist, sampling rate should be at least twice or 2.5 times bigger than the biggest frequency available which tells us the frequency rate should be at least around 200-250 Hz.

- **Sampling rate of the given data, “SubjectData1”:**

```
SamplingFreq = 1/(Subject1.train(1,3) - Subject1.train(1,2))
```

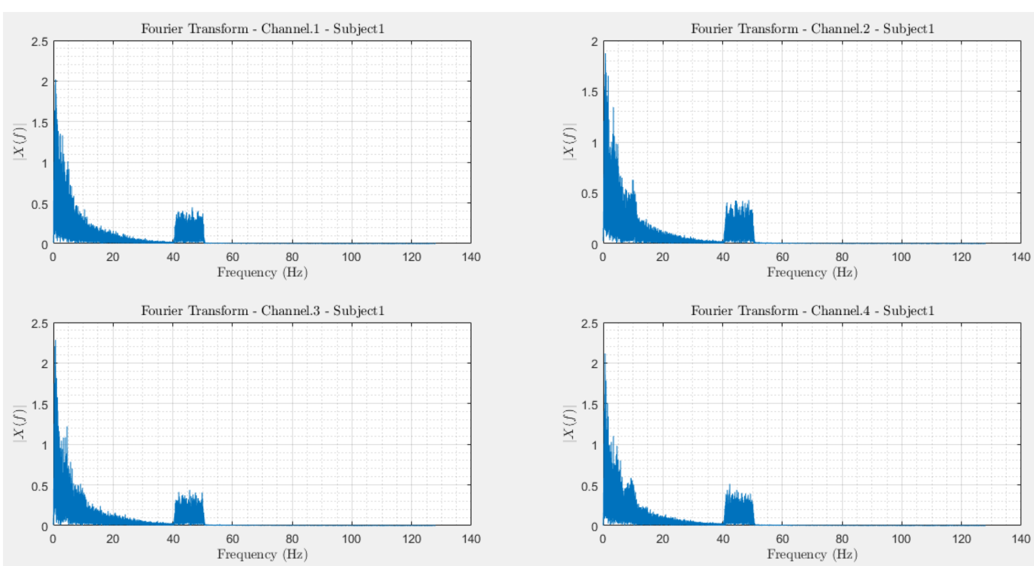
```
SamplingFreq =

   256
```

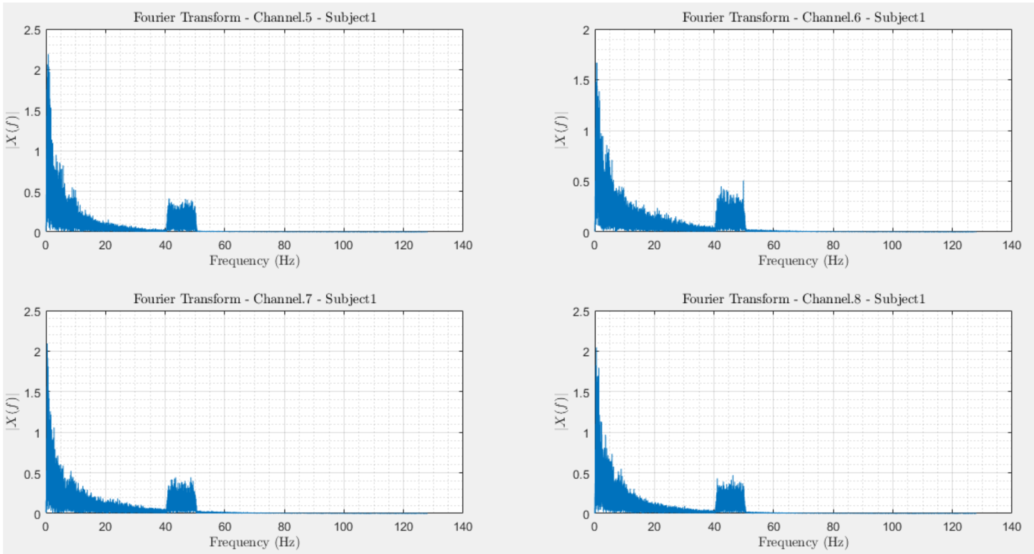
- **Guessing cut off frequency Depend on frequency bands information:**

Depend on the information gathered about frequency bands, cut off frequency could be for example 70 Hz. We`re not sure until we check the Fourier transform of the data.

- **Finding cut off frequency using Fourier transform:**

Here, Fourier transforms of each 8 channels are plotted using CTFourierTransform function which we saw in previous parts:





As the plots show, our data highest frequency is about 50 Hz. In range of 40 to 50 Hz, a noise(containing power line noise around 49-50 Hz) is on the data which it has to be removed. So the best cut off frequency is **40 Hz**.

- **Frequency range with the highest energy:**

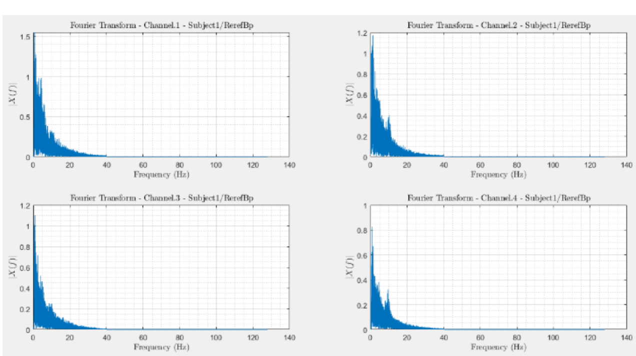
As you can see in the plots in the previous part, our signal`s energy is mostly between 0.5 to 40 Hz.

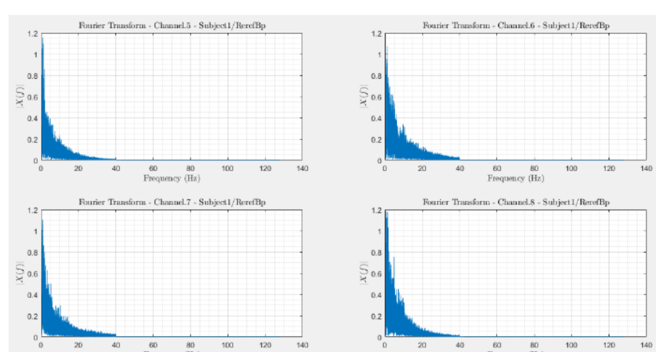
- **Low Pass filter cut off frequency:**

Low pass filter cut off frequency would be 0.5 Hz according to the previous part.

- **Re-referencing and Band Pass filter:**

To remove the DC component of the signal, re-referencing to the average of all 8 channels is done. Also, instead of applying a low pass and a high pass filter for filtering, we can apply a band pass filter with this cut off frequencies: [0.5 40] Hz using Matlab **bandpass** function:





- **Is removing average enough for removing DC component?**

Actually frequencies between 0 to 0.5 or 1 should be removed using a high pass filter, too. Why? Because in EEG processing there is a method called ICA and this method is very sensitive to big amplitudes which this can cause problems finding components. So this part of the signal should be removed. What cause these big amplitudes? For example EEG electrodes were not ideally(correctly) placed on the head or Conduction gel between the head and electrodes were too much(more than enough).

- **Down Sampling:**

According to the previous parts` results, our data sampling rate is 256 Hz and the highest frequency after band pass filtering is 40 Hz. Nyquist says sampling rate should be at least 80 Hz. So we are just capable of applying a down sampling with the rate of 2 or 3 which 2 is better because 256 is not dividable to three. So we apply a down sample with the rate of 2 to our channels` data using Matlab function **down-sample**. The new sampling rate is 128 Hz.

- **Epoching:**

We epoch our data to a 3d matrix which the first dimension of it is the number of channels which is 8, the second dimension is time of the data captured and the doc has said 200ms before giving the stimulus up to 800ms after giving the stimulus which with the sampling rate of 128 Hz it would be 128 samples and the third dimension is the number of times stimulus were given to the subject (number of trials). Epoching is done using the function below:

```
function dataEpoched =
epoch(inputSignal,backwardSamples,forwardSamples,StimuliTimes,Fs)
```

at the end, the output is a 8×128×10800 matrix which is our epoched data:

- **Why do we do filtering before down sampling?**

Without filtering, we can`t know what is the lowest sampling which Nyquist tells us. If we down sample before filtering, some useless frequencies in our signal increase the sampling rate limit. But after filtering we have our useful data without any noise or useless info, So down sampling can be done according to the highest frequency we have.

- **Example:**

Filtering has to be done before epoching because after epoching we just have a part of the data (1s for each stimuli) and filtering would be done depend on just a part of the data which would have a wrong result.

- **Why do we have to wait a little to do the experiment after the EEG starts capturing signals?**

We wait a little for the EEG device circuits to get stable.

3 Clustering based on Correlation

For two discrete-time signals, the value of correlation function at $\tau = 0$ is computed as follows:

$$R_{XY}[0] = \sum_{n=-\infty}^{+\infty} X[n]Y[n] = X.Y$$

which is simply the inner product of them.

Correlation Coefficient is defined as:

$$r_{XY} = \frac{\left(\sum_{n=-\infty}^{+\infty} X[n]Y[n]\right)}{\sqrt{\left(\sum_{n=-\infty}^{+\infty} X^2[n]\right)\left(\sum_{n=-\infty}^{+\infty} Y^2[n]\right)}} = \frac{R_{XY}[0]}{\sqrt{R_{XX}[0]\, R_{YY}[0]}}$$

Based on the definition of inner product we can simplify the above equation:

$$r_{XY} = \frac{X.Y}{\sqrt{(X.X)(Y.Y)}} = \frac{X.Y}{\sqrt{|X|^2|Y|^2}} = \frac{X.Y}{|X||Y|} = \cos(X,Y)$$

In fact, correlation coefficient of two signals is an indicator of the angle between them, where we have considered each signal as a vector with its values as elements of this vector. So, this coefficient shows how much two signals are aligned with each other.

As we show that $r_{XY} = \cos(X,Y)$, we conclude:

$$-1 \le r_{XY} \le 1$$

Earlier in this part, we stated that correlation coefficient shows how much two signals (i.e. two vectors) are parallel or in the same direction. So the magnitude of r_{XY} is 1 if and only if X and Y are parallel and hence the angle between them would be 0 or π :

$$|r_{XY}| = 1 \leftrightarrow \cos(X,Y) = \pm 1 \leftrightarrow X[n] = \alpha Y[n]\, , \alpha \epsilon \mathbb{R}$$

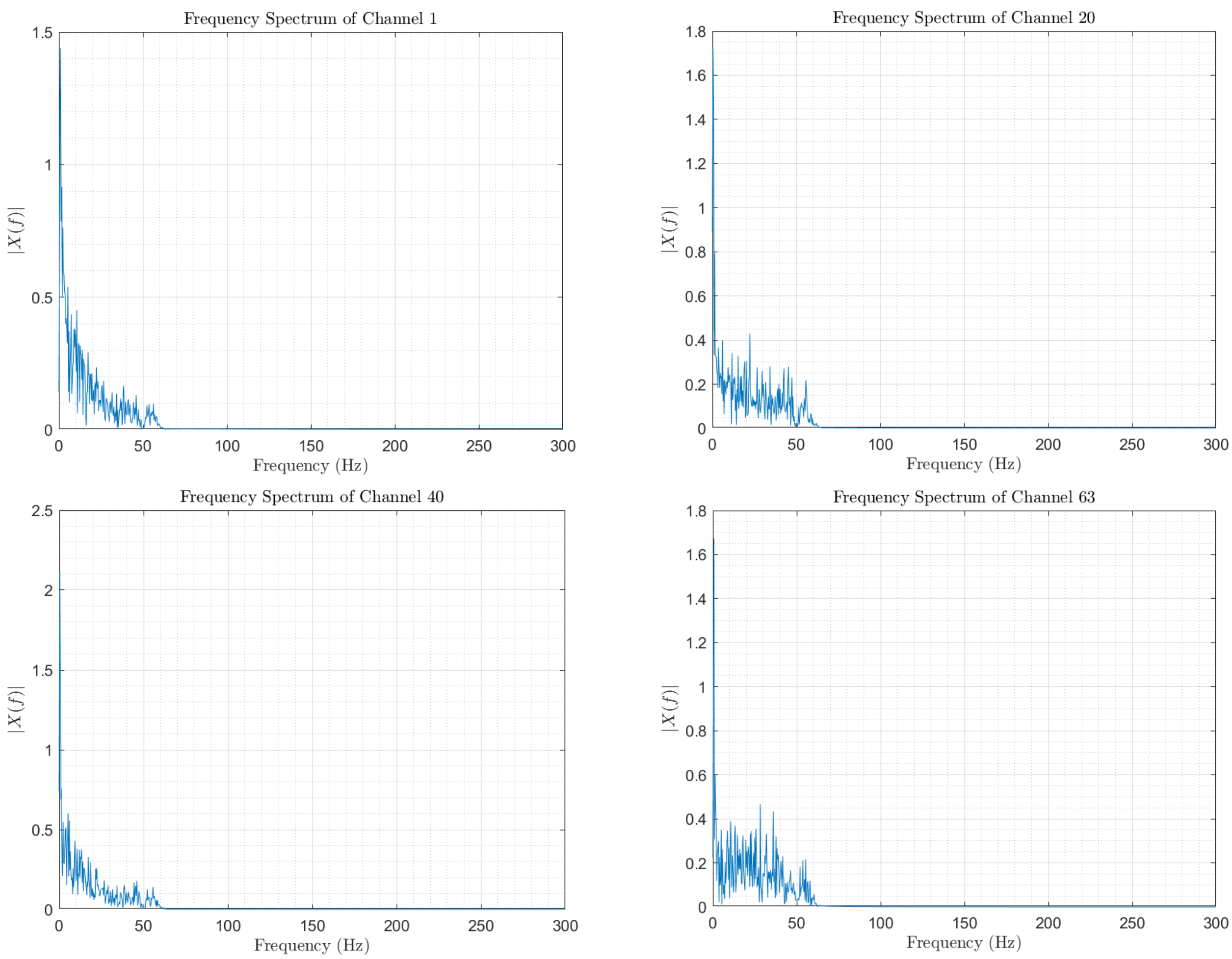
+ In the continuous form, the above results remain the same; because the inner product of two continuous signals is defined as:

$$< X, Y > = \int_{-\infty}^{+\infty} X(t)Y(t)dt$$

As we mentioned before, correlation coefficient is an indicator of the alignment of two signals. So this coefficient can be used to measure the similarity between them and is good a parameter for our purpose of clustering.

Correlation Clustering on 63-channel Data form BCI experiment:

By plotting different channels' frequency responses, we observe no filtering is needed and even the 50-Hz noise is filtered; for example:



First of all, we average signals over trials; so that the probable noise of experiment is removed and therefore we have a 63×1800 data.

From the channels' frequency responses, we observe that maximum frequency for each channel is less than 70 or 65 Hz. So to decrease the sampling frequency, the Nyquist condition would be:

$$F_s' > 2 \times 70\ Hz = 140\ Hz$$

So we can down-sample the data by rate 4 (i.e. choose 1 sample out of 4 consecutive samples) and still have the same frequency response. Now our data would be 63×450 and we now compute the *Correlation Coefficient* matrix which is simply a matrix that contains correlation coefficient of each pair of channels:

$$\text{Correlation Coefficient Matrix } \boldsymbol{R} \therefore \begin{bmatrix} r_{X_1X_2} & \cdots & r_{X_1X_n} \\ \vdots & \ddots & \vdots \\ r_{X_nX_1} & \cdots & r_{X_nX_n} \end{bmatrix}$$

CorrCoefMat.m

This function computes the correlation coefficient matrix of a 2D data (like channels of EEG signals) based on the formula:

$$r_{X_iX_j} = \frac{X_i.X_j}{|X_i||X_j|}$$

So defining three matrixes as below

$$R_1 = \begin{bmatrix} X_1X_1 & \cdots & X_nX_1 \\ \vdots & \ddots & \vdots \\ X_nX_1 & \cdots & X_nX_n \end{bmatrix} \qquad R_2 = \begin{bmatrix} |X_1|^2 & \cdots & |X_1|^2 \\ \vdots & \ddots & \vdots \\ |X_n|^2 & \cdots & |X_n|^2 \end{bmatrix} \qquad R_3 = \begin{bmatrix} |X_1|^2 & \cdots & |X_n|^2 \\ \vdots & \ddots & \vdots \\ |X_1|^2 & \cdots & |X_n|^2 \end{bmatrix}$$

We can find the correlation matrix as:

$$\boldsymbol{R} = R_1./\sqrt{R_2.* R_3}$$

This kind of implementation is much faster than the simple loops for each pair channel computation due to the high-speed matrix computations of matlab. Our function does this and in the output, we have the correlation coefficient matrix.