Geometric Deep Learning Self-Study Group

Session 5

November 3, 2025

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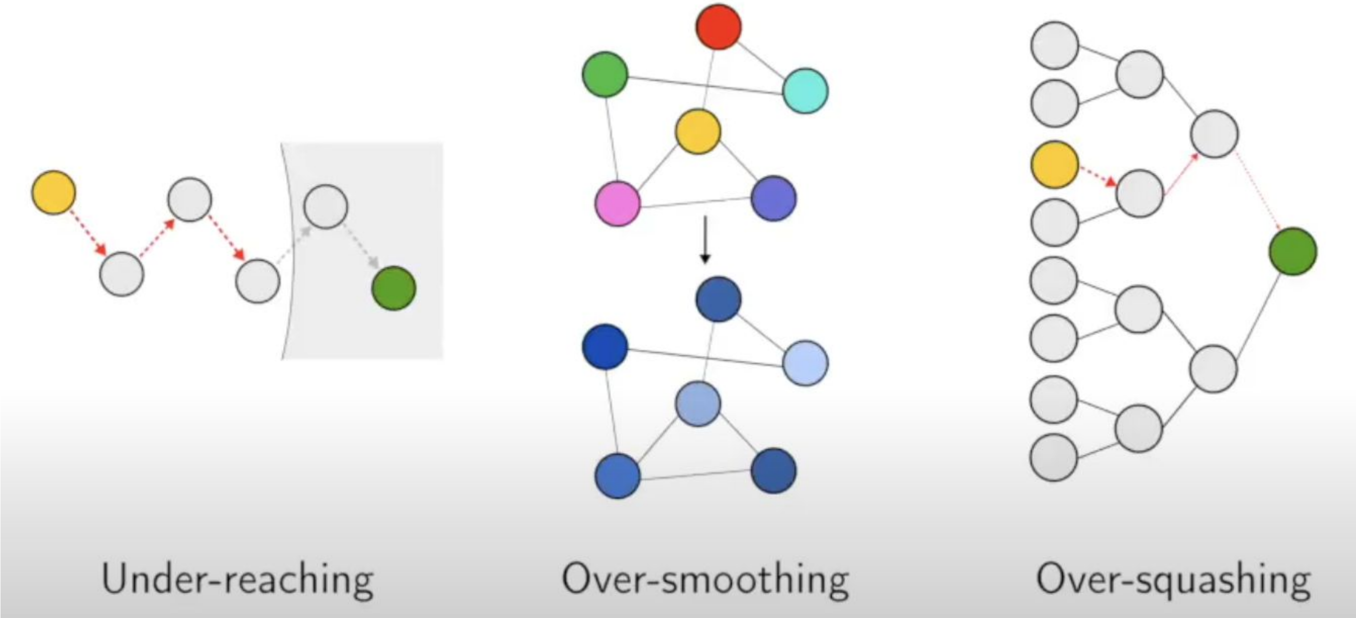
https://armargolis.github.io/

Session Outline

Expressivity/
Under-reaching

Tutorials



Remaining Sessions:

Oct 13: Expressivity

Nov 3: Over-smoothing, Over-squashing and Curvature

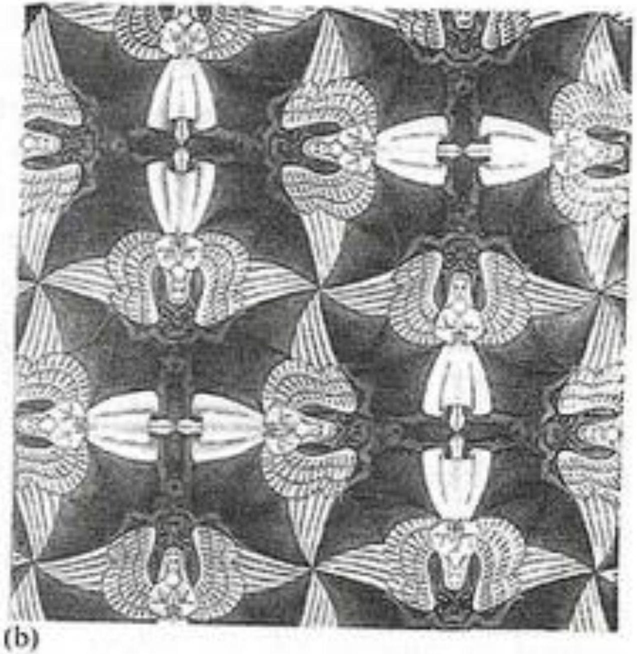
Nov 17: Equivariance

Dec 1: Manifolds

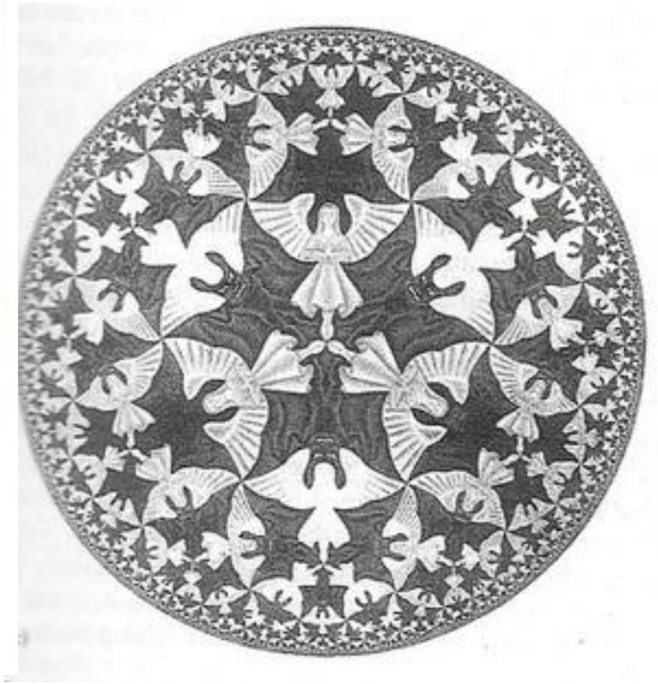
Curvature

Spherical (Positive)

Flat (Zero)



Hyperbolic (Negative)

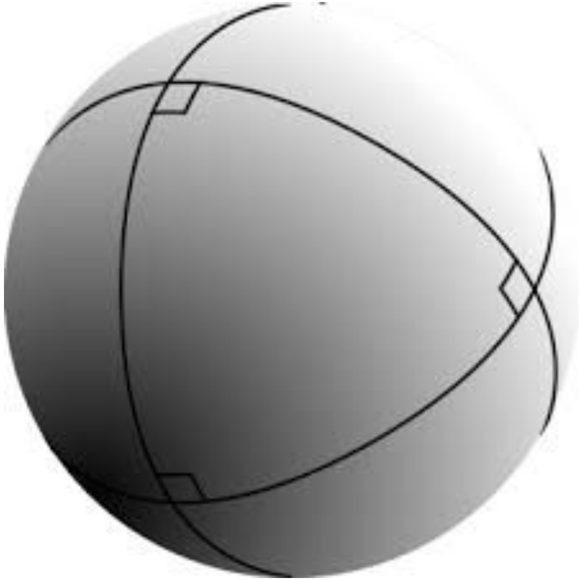


Source: “Road to Reality” by Roger Penrose

Source:
Jan Gaska (2014), "Geodesics on Some Surfaces"
Wolfram Demonstrations Project.
demonstrations.wolfram.com/GeodesicsOnSomeSurfaces/

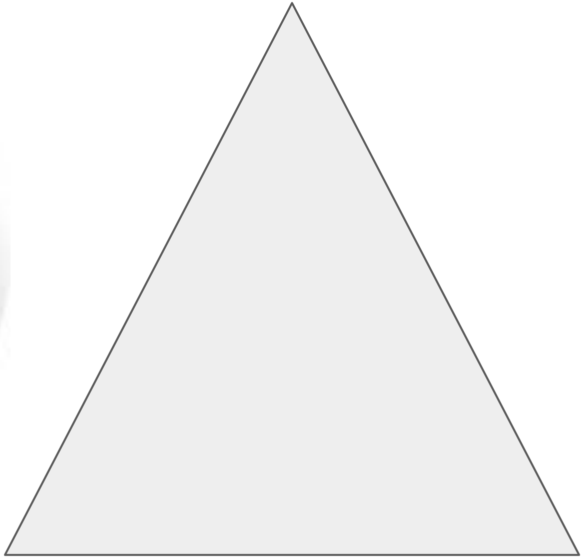
Parallel Transport

Spherical (Positive)



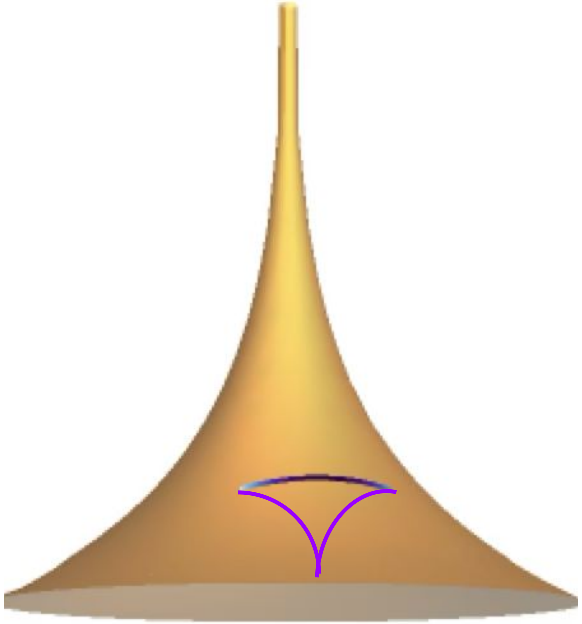
Sum of Triangle Angles:
Over 180 degrees

Flat (Zero)



Sum of Triangle Angles:
180 degrees

Hyperbolic (Negative)

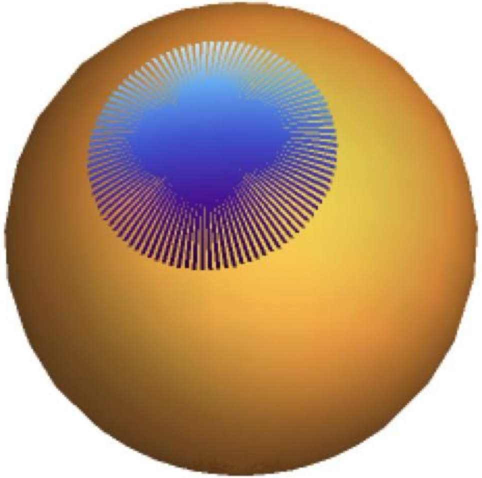


Sum of Triangle Angles:
Under 180 degrees

Source:
Jan Gaska (2014), "Geodesics on Some Surfaces"
Wolfram Demonstrations Project.
demonstrations.wolfram.com/GeodesicsOnSomeSurfaces/

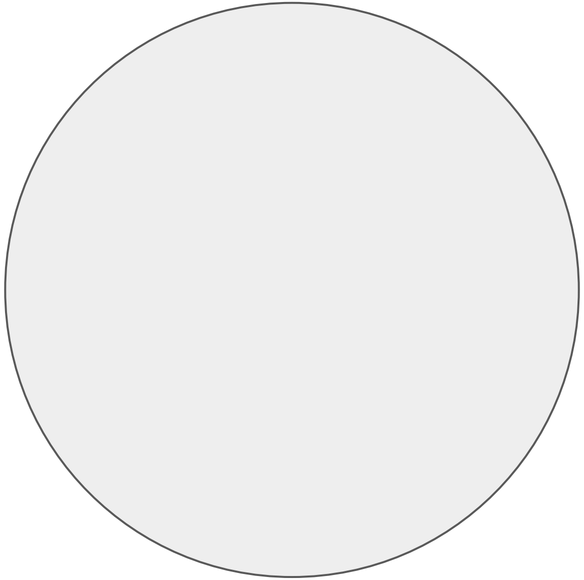
Parallel Transport

Spherical (Positive)



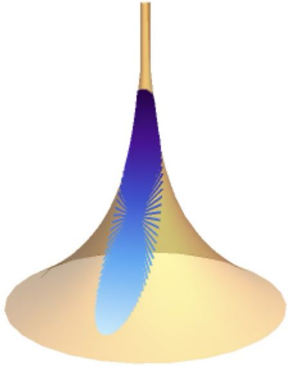
Circumference over Radius:
Less than 2π

Flat (Zero)



Circumference over Radius:
2π

Hyperbolic (Negative)



Circumference over Radius:
Greater than 2π

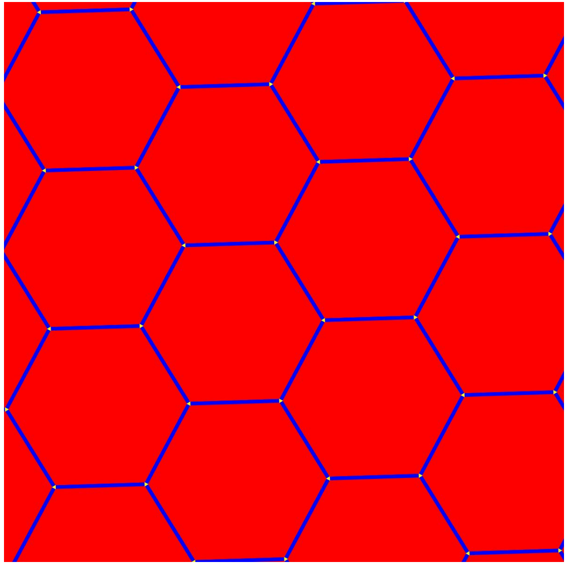
Expected Angles

Source:
https://web.colby.edu/thegeometricviewpoint/2016/12/21/tessellations-of-the-hyperbolic-plane-and-m-c-escher/

Spherical (Positive)

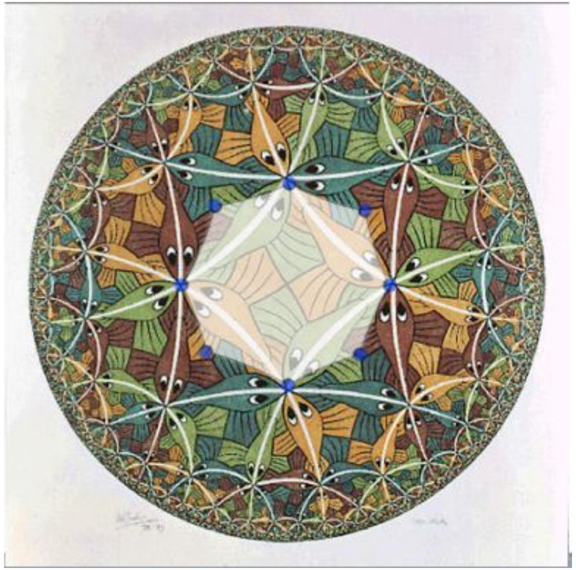
One pentagon ($\frac{2}{5}\pi$) and two hexagons ($\frac{2}{3}\pi$) < 2π

Flat (Zero)



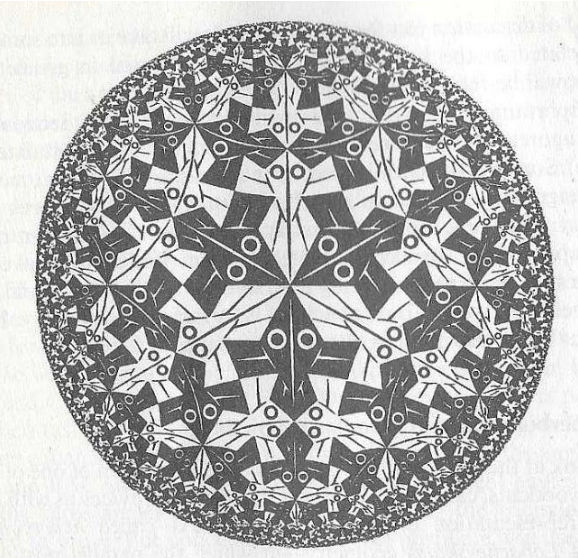
Three hexagons ($\frac{2}{3}\pi$) = 2π

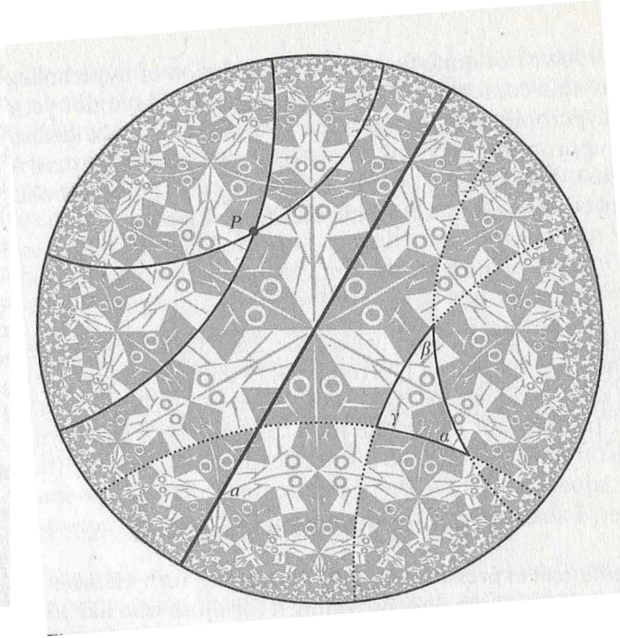
Hyperbolic (Negative)

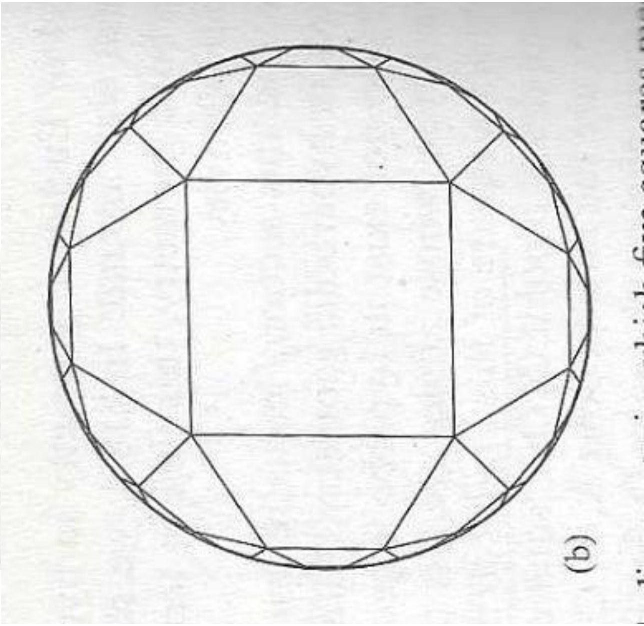


Three square ($\frac{1}{2}\pi$) and three triangles ($\frac{1}{3}\pi$) > 2π

Hyperbolic Curvature (From manifolds to graphs)

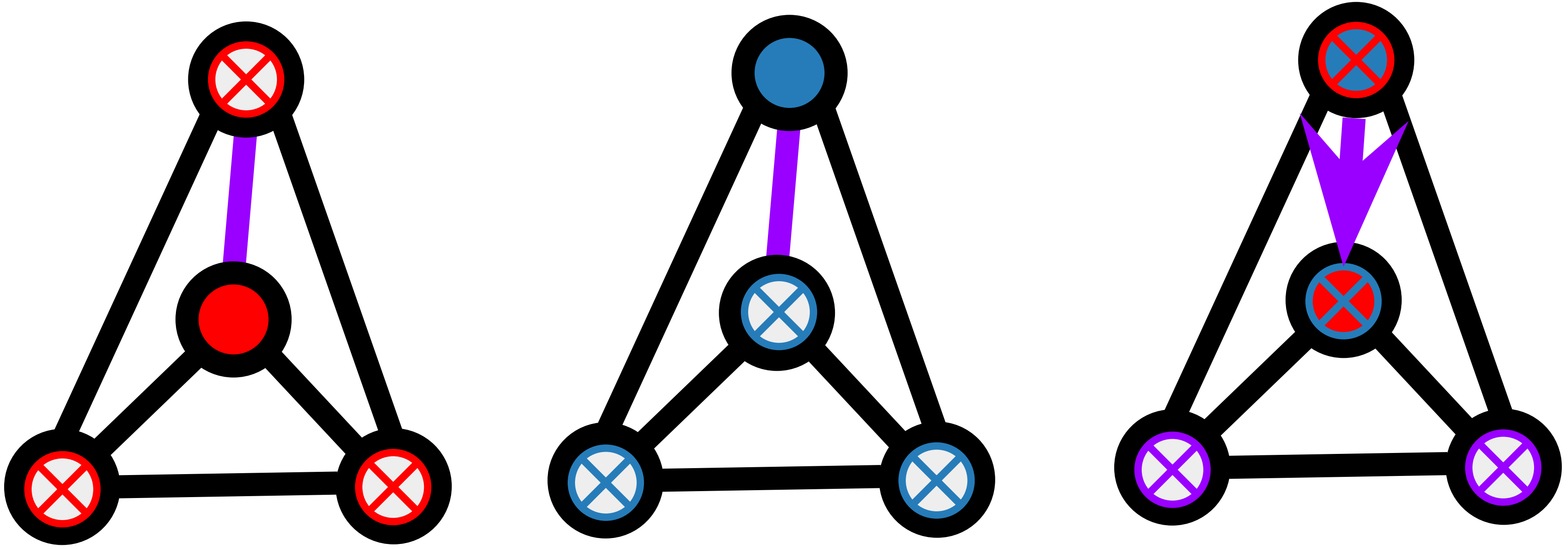






Source: “Road to Reality” by Roger Penrose

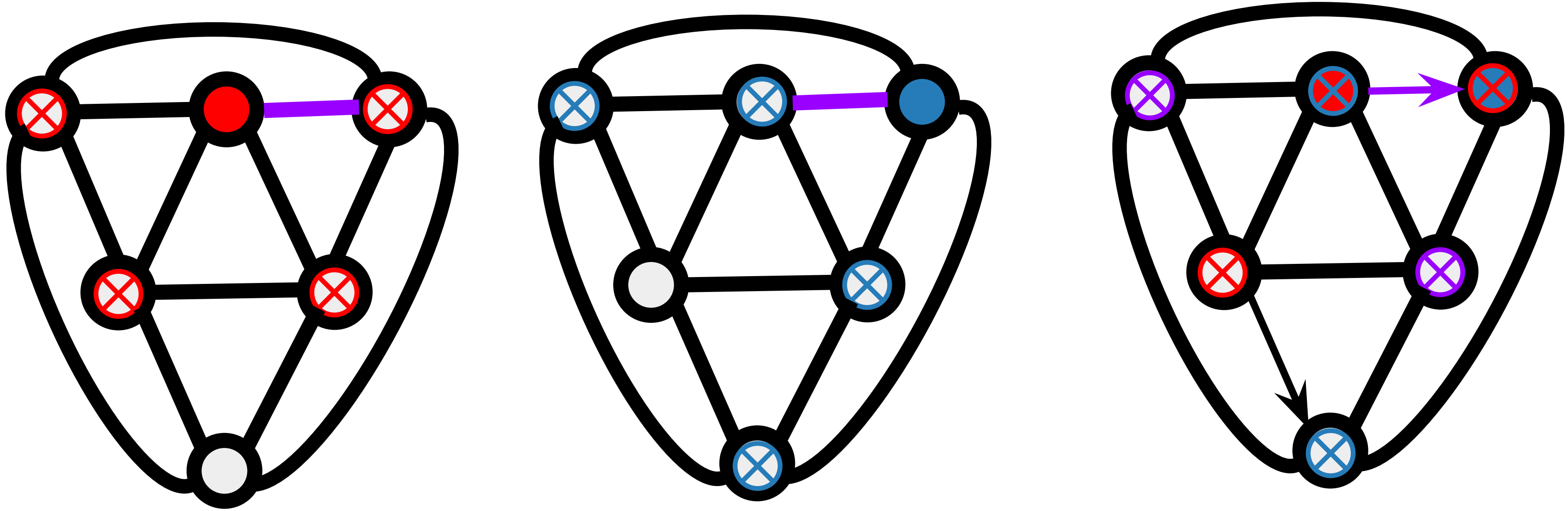
Graph Curvature on the Edge (Tetrahedron)



1 Arrow = Move 1 node to adjacent node
Curvature= 1 - #Arrows/#Neighbors

$$1 - \frac{1}{3} = \frac{2}{3}$$

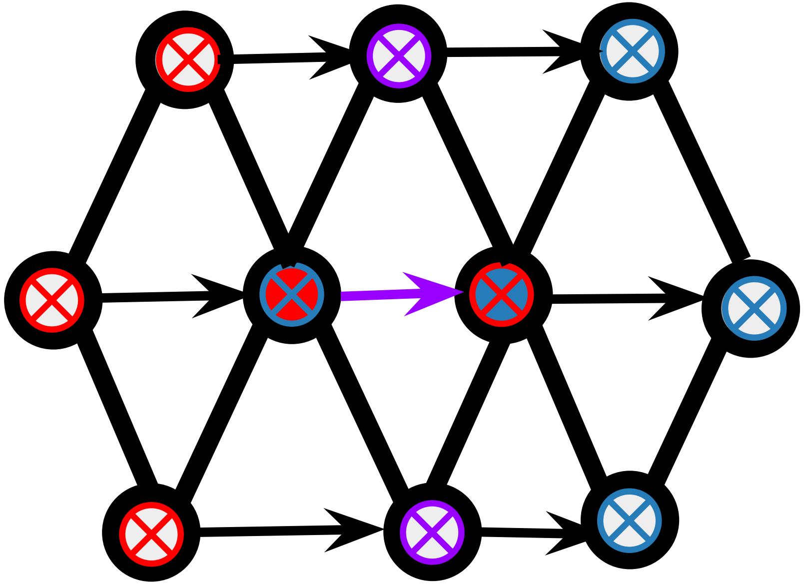
Graph Curvature on the Edge (Octahedron)



1 Arrow = Move 1 node to adjacent node
Curvature= 1 - #Arrows/#Neighbors

$$1 - \frac{2}{4} = \frac{1}{2}$$

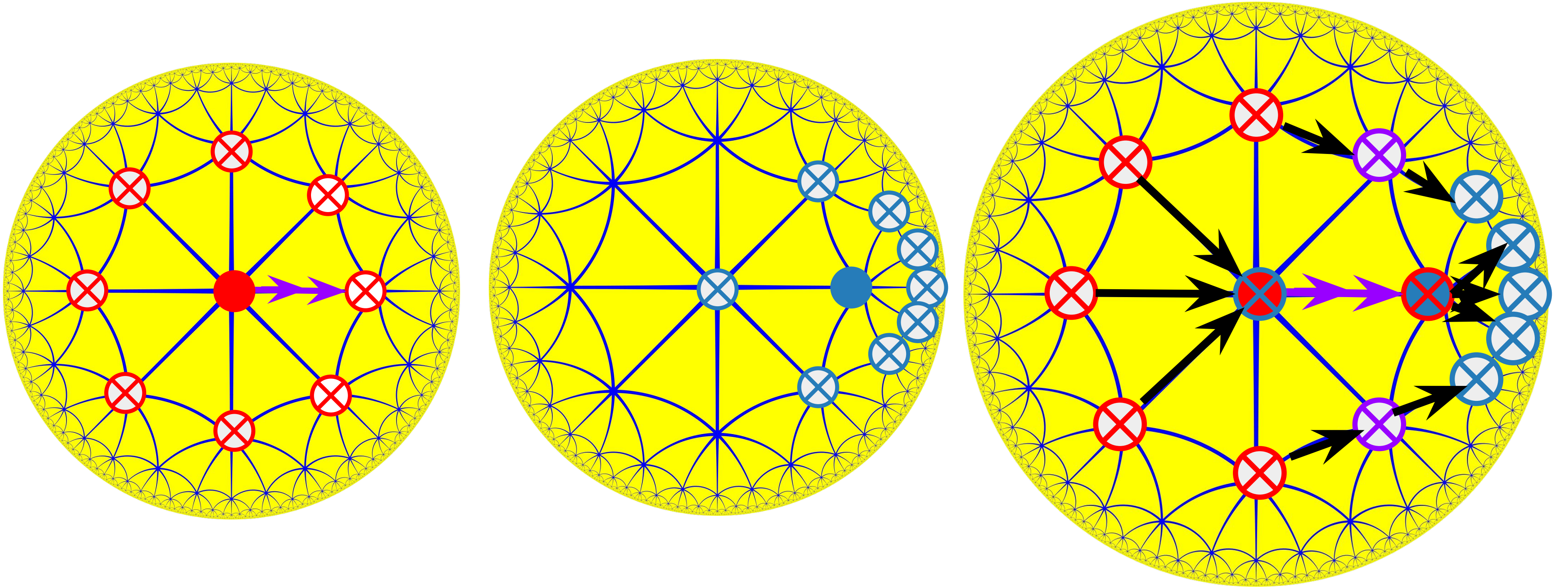
Graph Curvature on the Edge (Grid)



1 Arrow = Move 1 node to adjacent node
Curvature= 1 - #Arrows/#Neighbors

$$1 - \frac{6}{6} = 0$$

Graph Curvature on the Edge (Hyperbolic)

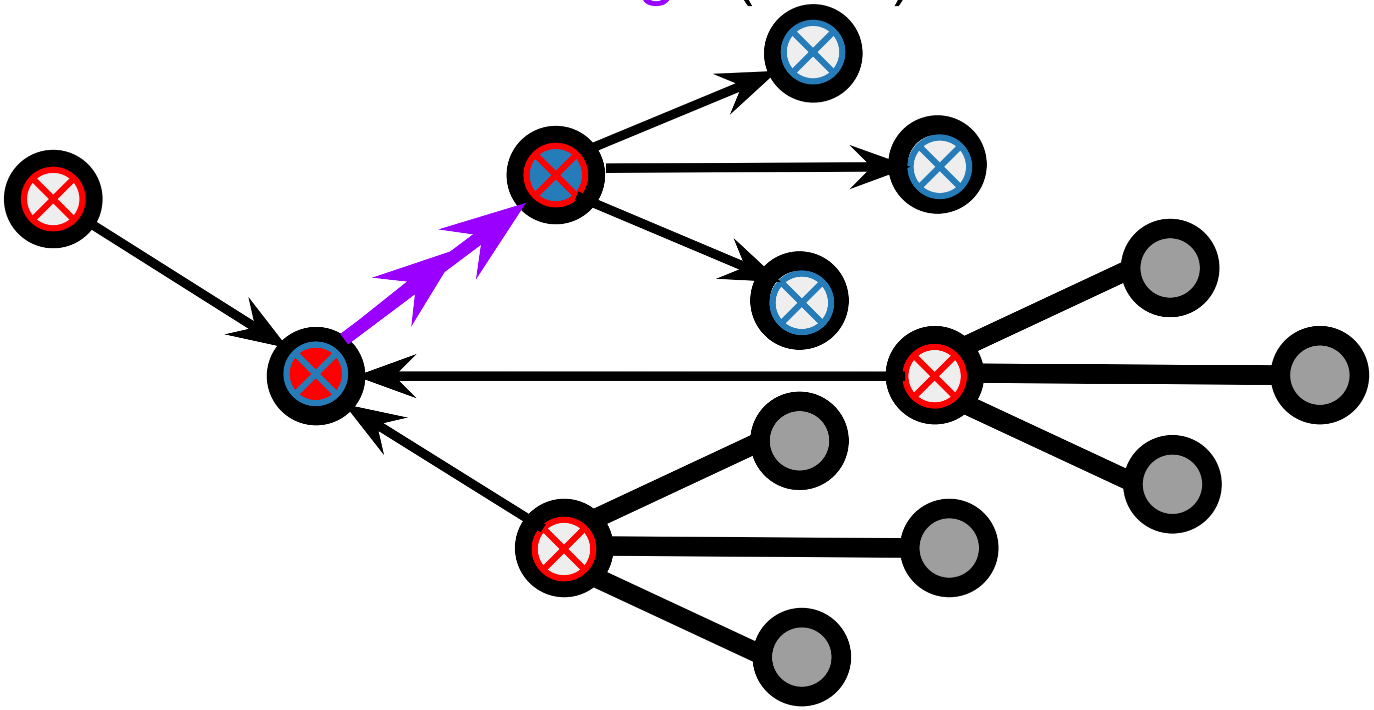


1 Arrow = Move 1 node to adjacent node

Curvature= 1 - #Arrows/#Neighbors

$$1 - \frac{12}{8} = -\frac{1}{2}$$

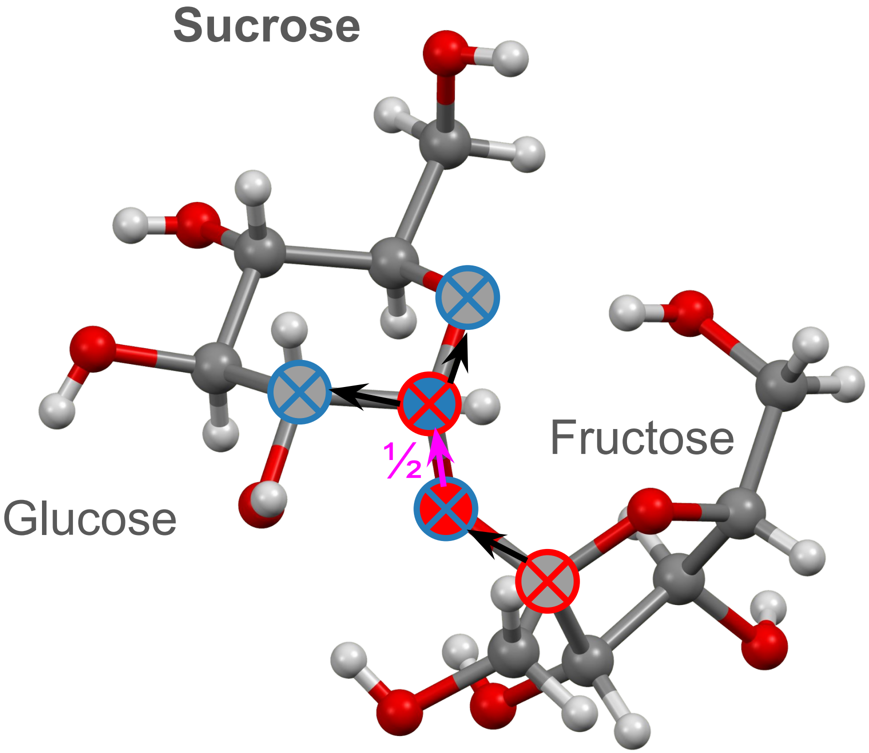
Graph Curvature on the Edge (Tree)



1 Arrow = Move 1 node to adjacent node
Curvature= 1 - #Arrows/#Neighbors

$$1 - \frac{8}{4} = -1$$

Trees as Inputs: Real World Example

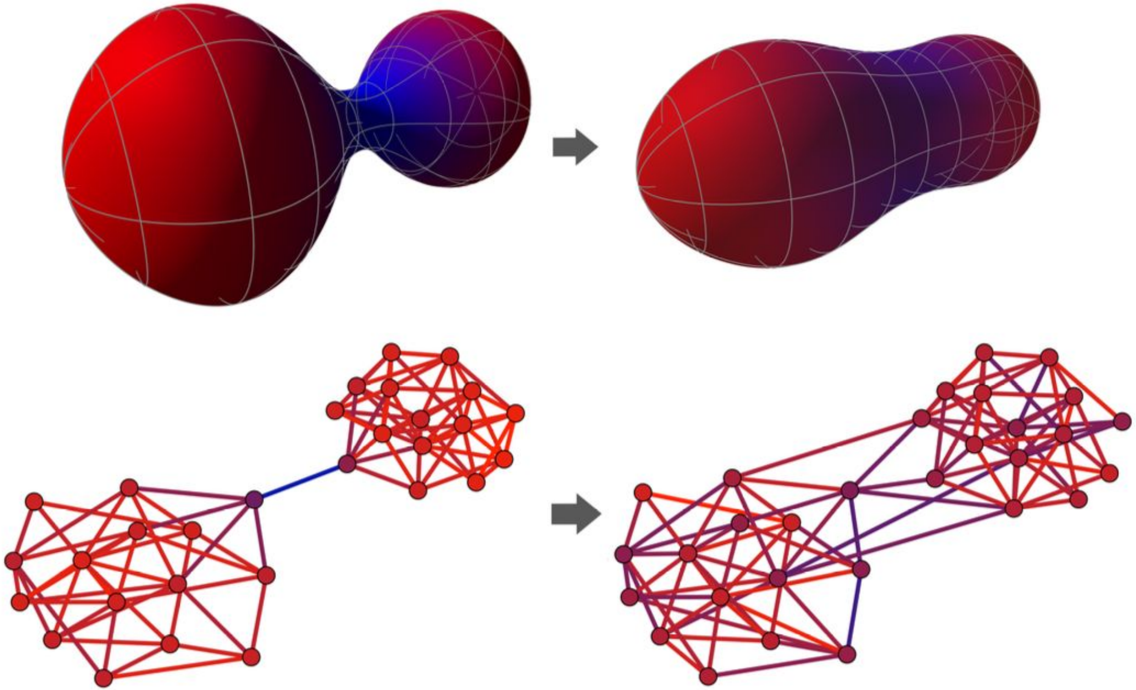


There are 2 $\otimes$ and 3 $\otimes$

Thus, each $\otimes$ is worth 1.5 $\otimes$

$$1 - \frac{3.5}{3} = -\frac{1}{6}$$

Solution? Add a fast lane to eliminate bottlenecks



By allow message-passing between non-connected nodes, we can eliminate bottlenecks (edges with negative curvature).

Disadvantage: connecting nodes nearby in space may yield better results

Source:
Jake Topping , Francesco Di Giovanni, Benjamin P. Chamberlain, Xiaowen Dong , and Michael M. Bronstein (2021), UNDERSTANDING OVER-SQUASHING AND BOTTLENECKS ON GRAPHS VIA CURVATURE
https://arxiv.org/pdf/2111.14522

Deep Learning on Other Geometries

The Eight Three-Dimensional Geometries as hypothesized by Thurston and proved by Perelman

Source: Gabriel Khan (2022), An Illustrated Introduction to the Ricci Flow https://arxiv.org/pdf/2201.04923

References and Resources

Main Reference:
Geometric Deep Learning: Grids, Groups, Graphs, Geodesics, and Gauges
Michael M. Bronstein, Joan Bruna, Taco Cohen, Petar Veličković
https://geometricdeeplearning.com/book/ We'll use the chapters rather than the big pdf

Additional References:
Mathematical Foundations of Geometric Deep Learning by Borde and Bronstein
https://www.arxiv.org/abs/2508.02723
Introduction to Geometric Deep Learning by Patrick Nicolas
https://patricknicolas.substack.com/p/introduction-to-geometric-deep-learning

Further Reading:
Valence Labs
https://portal.valencelabs.com

Graph Learning on Wednesdays
https://sites.google.com/view/graph-learning-on-weds