INDETERMINACIONES

* "$\dfrac{\infty}{\infty}$"

ej: $\displaystyle \lim_{x \to \infty} \frac{x^2+7}{x^3-5} = \lim_{x \to +\infty} \frac{\cancel{x^2}\left(1 + \overbrace{7/x^2}^{\to 0}\right)}{\cancel{x^3}\left(1 - \underbrace{5/x^3}_{\to 0}\right)} = 0$

ej: $\displaystyle \lim_{x \to \infty} \frac{x^2+7}{3x^2-5x-10} = \lim_{x \to \infty} \frac{\cancel{x^2}\left(1 + 7/x^2\right)}{\cancel{x^2}\left(3 - 5/x - 10/x^2\right)} = \frac{1}{3}$

ej: $\displaystyle \lim_{x \to \infty} \frac{2x^5-10}{x^4+3x^3} = \lim_{x \to \infty} \frac{\cancel{x^5}\left(2 - 10/x^5\right)}{\cancel{x^4}\left(1 + 3/x\right)} = \infty$

$*$ "$\frac{0}{0}$"

$x^2-4 \rightarrow (x-2)(x+2)$

eg: $\lim\limits_{x\to 2} \frac{x^2-4}{x^2-3x+2} = \lim\limits_{x\to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(x-1)} = 4$

$x^2-3x+2 \rightarrow (x-2)(x-1)$

$*$ "$\infty - \infty$"

eg: $\lim\limits_{x\to\infty} \sqrt{3x+2} - \sqrt{x+5} =$

$\rightarrow \left(\sqrt{3x+2} - \sqrt{x+5}\right) \cdot \frac{\left(\sqrt{3x+2} + \sqrt{x+5}\right)}{\left(\sqrt{3x+2} + \sqrt{x+5}\right)}$

$= \frac{\overset{2x+7}{3x+2 - x+5}}{\sqrt{3x+2} + \sqrt{x+5}}$

$= \lim\limits_{x\to\infty} \frac{2x+7}{\sqrt{3x+2} + \sqrt{x+5}} = \lim\limits_{x\to\infty} \frac{2\sqrt{x} + \overset{\rightarrow 0}{7/\sqrt{x}}}{\underset{\rightarrow \sqrt{3}}{\sqrt{3+2/x}} + \underset{\rightarrow 1}{\sqrt{1+5/x}}} = \infty$

* "1^{∞}"

$$\underline{\underline{ej}}: \lim_{x\to+\infty} \left(1+\frac{1}{x}\right)^{x} = e \approx 2{,}718\ldots$$

$$a^{b\cdot c} = (a^{b})^{c}$$

CONSECUENCIAS:

$$\text{Si } \lim_{x\to a} f(x) = +\infty \Rightarrow \lim_{x\to a}\left(1+\frac{1}{f(x)}\right)^{f(x)} = e$$

$$\text{Si } \lim_{x\to a} f(x) = 0 \Rightarrow \lim_{x\to a}\left(1+f(x)\right)^{1/f(x)} = e$$

$$\underline{ej}: \lim_{x\to+\infty}\left(1-\frac{1}{3x}\right)^{x} = \lim_{x\to+\infty}\left(1+\frac{1}{(-3x)}\right)^{\frac{(-3x)}{(-3x)}\cdot x} = \lim_{x\to+\infty}\left[\left(1+\frac{1}{(-3x)}\right)^{(-3x)}\right]^{\frac{\cancel{x}}{-3\cancel{x}}} = \frac{1}{\sqrt[3]{e}}$$

$$1+\frac{1}{-3x}$$

$\rightarrow e$

ej: $\lim_{x\to+\infty}\left(\frac{x+5}{x+2}\right)^{x^3} = \lim_{x\to\infty}\left(1+\frac{x+5}{x+2}-1\right)^{x^3} =$

$\frac{x+5-x-2}{x+2} = \frac{3}{x+2}$

$= \lim_{x\to+\infty}\left(1+\frac{3}{x+2}\right)^{x^3} = \lim_{x\to+\infty}\left(1+\frac{3}{x+2}\right)^{\frac{x+2}{3}\cdot\frac{3}{x+2}\cdot x^3} =$

$= \lim_{x\to+\infty}\underbrace{\left[\left(1+\frac{3}{x+2}\right)^{\frac{x+2}{3}}\right]}_{\to e}^{\frac{3x^3}{x+2}\to+\infty} = \boxed{+\infty}$

$\text{“}\frac{\infty}{0}\text{”} \to$ No ES INDET

$\text{“}0^{\infty}\text{”} \to$ " " "

TAREA: $\lim_{x\to1}\left(\frac{1}{x}\right)^{\frac{1}{1-x^2}}$

INFINITÉSIMOS:

$f(x)$ es infinitésimo en $x=a \Longleftrightarrow \lim_{x\to a} f(x) = 0$

* $f(x)$ y $g(x)$ son infinitésimos del mismo orden en $x=a \Leftrightarrow \lim_{x\to a} \frac{f(x)}{g(x)} = L \in \mathbb{R}-\{0\}$

ej: $f(x)=x^2-4$ y $g(x)=x^2-3x+2$

$\Rightarrow$ son infinitésimos en $x=2$

Además, son del mismo orden, pues $\lim_{x\to 2} \frac{f(x)}{g(x)} = 4$

* $f(x)$ es de orden mayor q' $g(x)$ si $\lim_{x \to a} \frac{f(x)}{g(x)} = 0$

ej: $f(x) = (x-2)^2$; $g(x) = x^2 - 3x + 2$ → son infinitésimos en $x=2$

$$\lim_{x \to 2} \frac{f(x)}{g(x)} = \lim_{x \to 2} \frac{(x-2)^{\cancel{2}}}{\cancel{(x-2)}(x-1)} = 0$$

$f(x) \nsim g(x)$

$f(x)$ y $g(x)$ son equivalentes

Si $\lim_{x \to a} \frac{f(x)}{g(x)} = 1$

CASOS IMPORTANTES (para $x \to 0$):

- $\operatorname{sen} x \sim x \sim \operatorname{tg}(x) \sim \operatorname{arctg}(x) \sim \operatorname{arcsen}(x)$
- $1 - \cos x \sim \frac{x^2}{2}$
- $e^x - 1 \sim x$
- $\ln(1+x) \sim x$

ej: $\lim\limits_{x \to 0} \dfrac{\operatorname{sen}^2(x)}{x} = \lim\limits_{x \to 0} \underbrace{\dfrac{\operatorname{sen}(x)}{x}}_{\to 1} \operatorname{sen}(x) = 0$