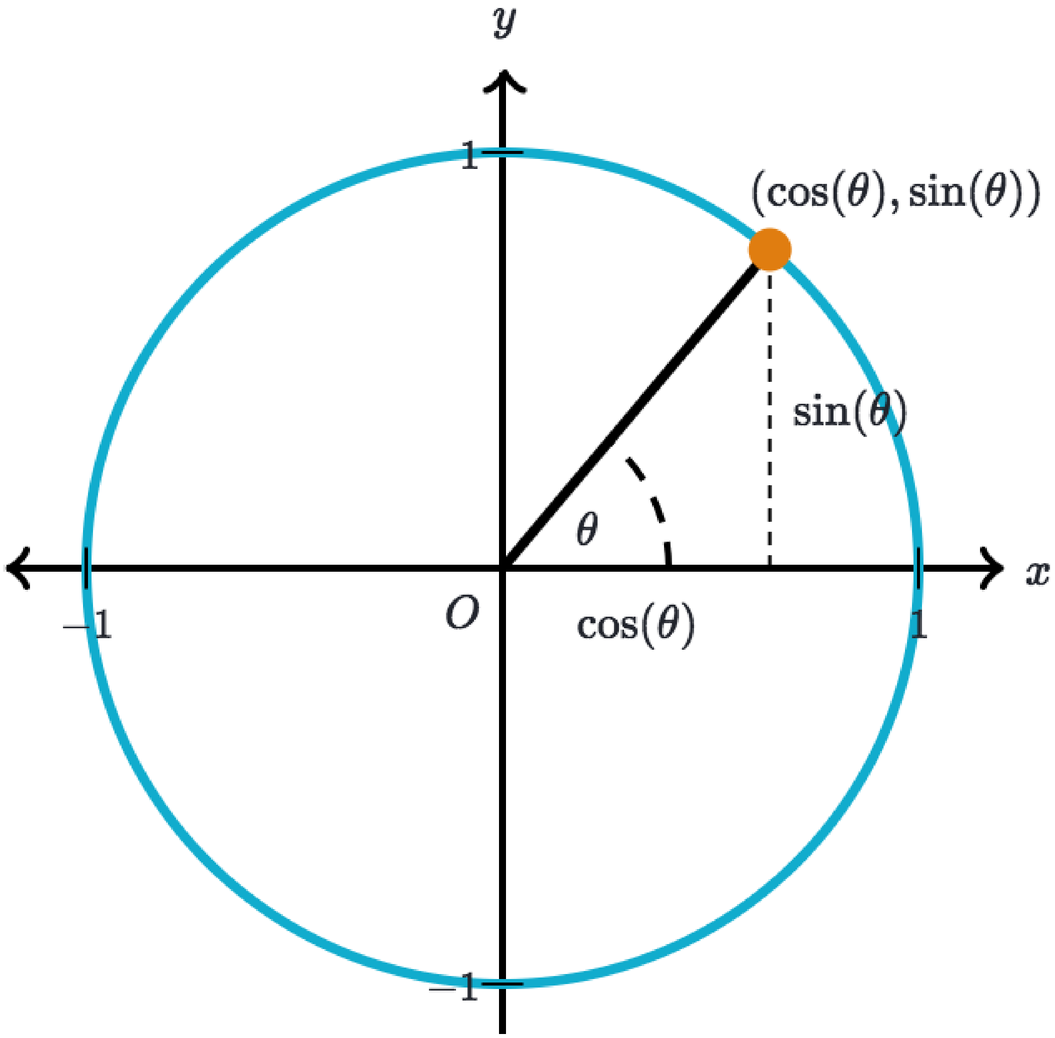
- **Funcions hiperbòliques**

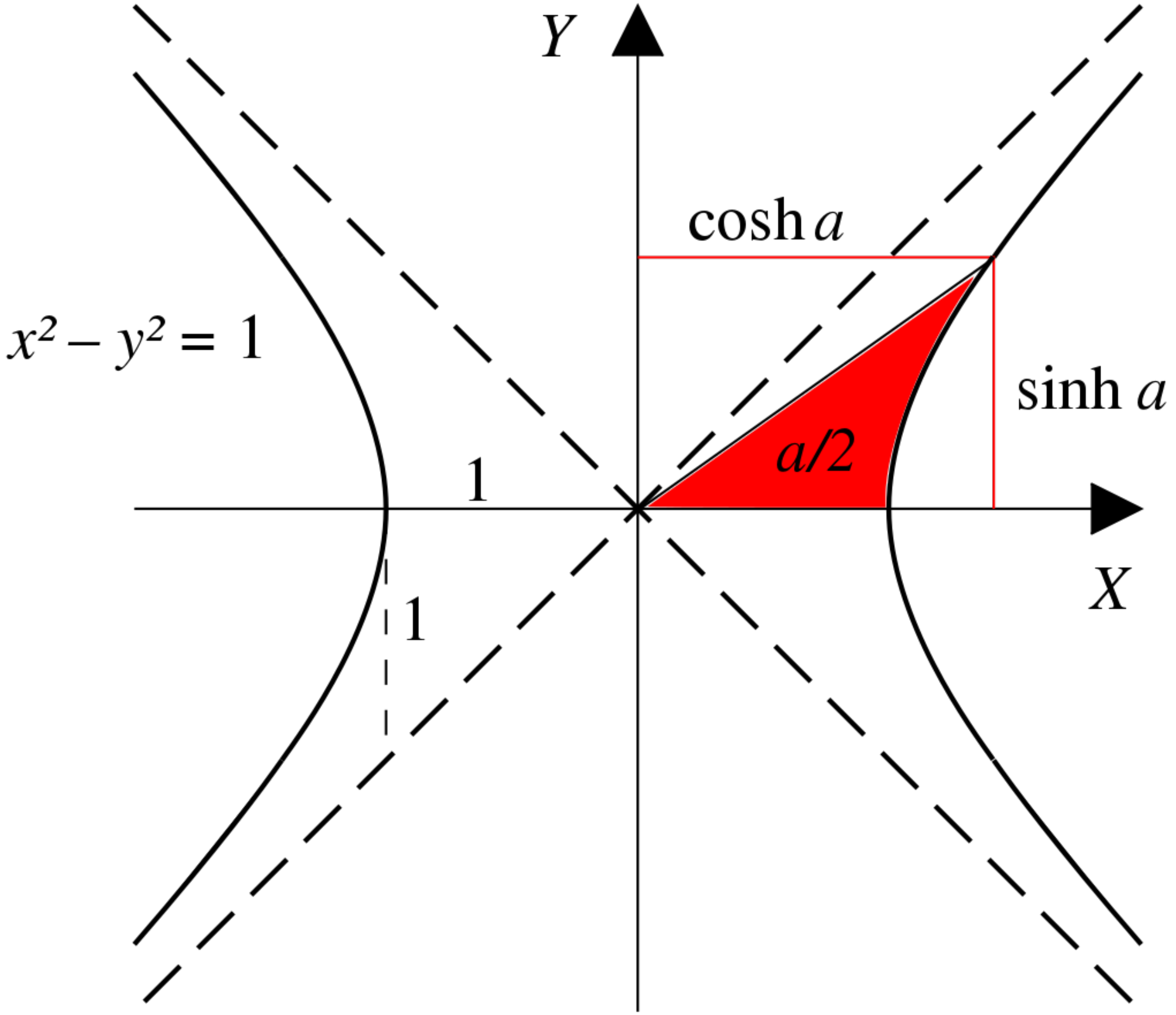
Igual que $(\cos\theta, \sin\theta)$ parametritza la circumferència de radi 1,

$$x^2 + y^2 = 1 \rightarrow cos^2\theta + sin^2\theta = 1$$



$(\cosh a, \sinh a)$ parametritza la hipèrbola:

$$x^2 - y^2 = 1 \rightarrow cosh^2a - sinh^2a = 1$$



Recordem les fórmules de Euler:

$$e^{jx} = \cos x + j\sin x \quad (1)$$
$$e^{-jx} = \cos x - j\sin x \quad (2)$$

$$e^{-jx} = \cos(-x) + j\sin(-x)$$

(1)+(2) $\quad e^{jx} + e^{-jx} = 2\cos x \Rightarrow \cos x = \dfrac{e^{jx} + e^{-jx}}{2}$

(1)-(2) $\quad e^{jx} - e^{-jx} = 2j\sin x \;\Rightarrow \sin x = \dfrac{e^{jx} - e^{-jx}}{2j}$

De forma similar:

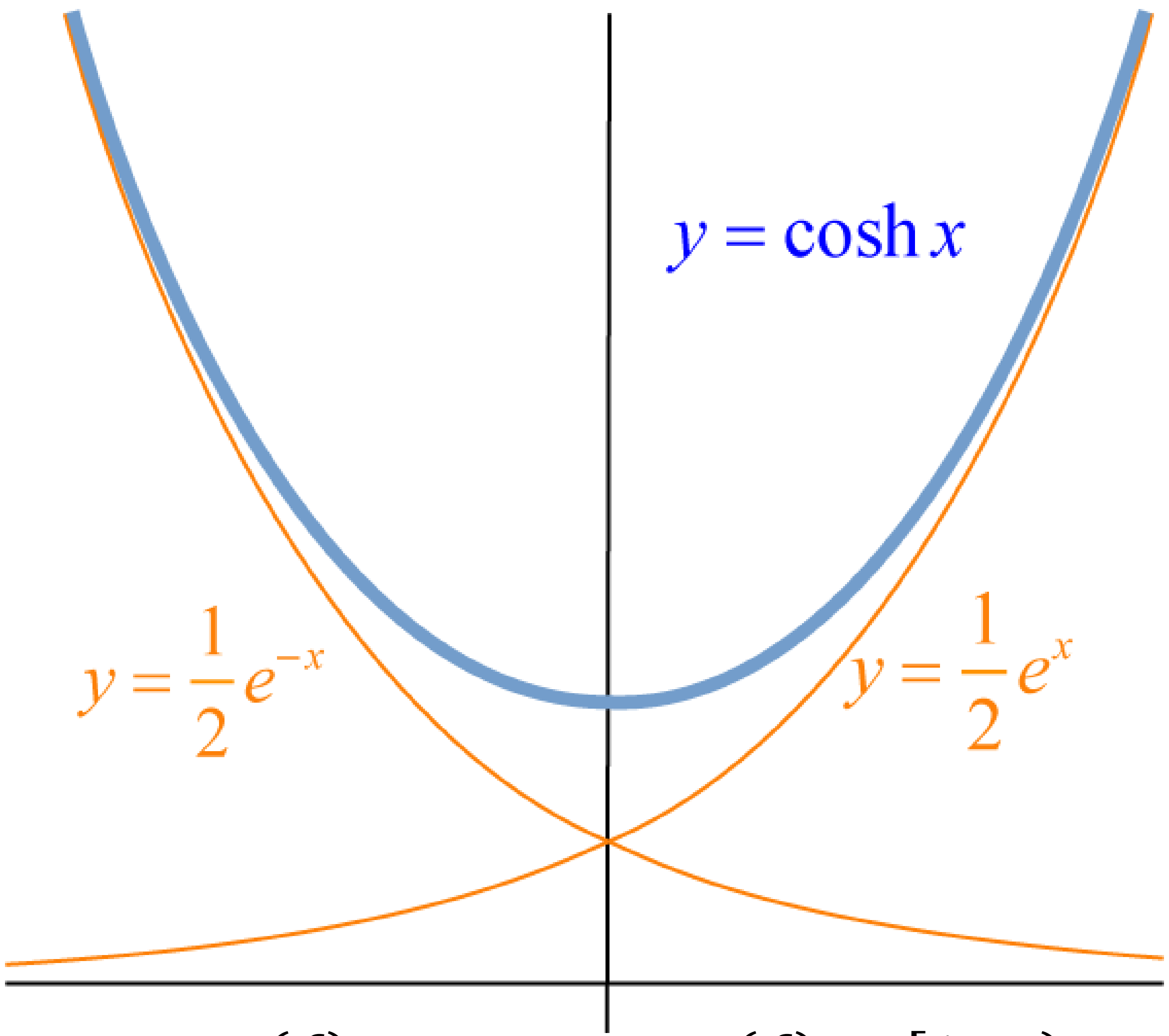
$$e^{x} = \cosh x + \sinh x \quad (1)$$
$$e^{-x} = \cosh x - \sinh x \;(2)$$

$$e^{-x} = \cosh(-x) + \sinh(-x)$$

(1)+(2) $\quad e^{x} + e^{-x} = 2\cosh x \Rightarrow \cosh x = \dfrac{e^{x} + e^{-x}}{2}$

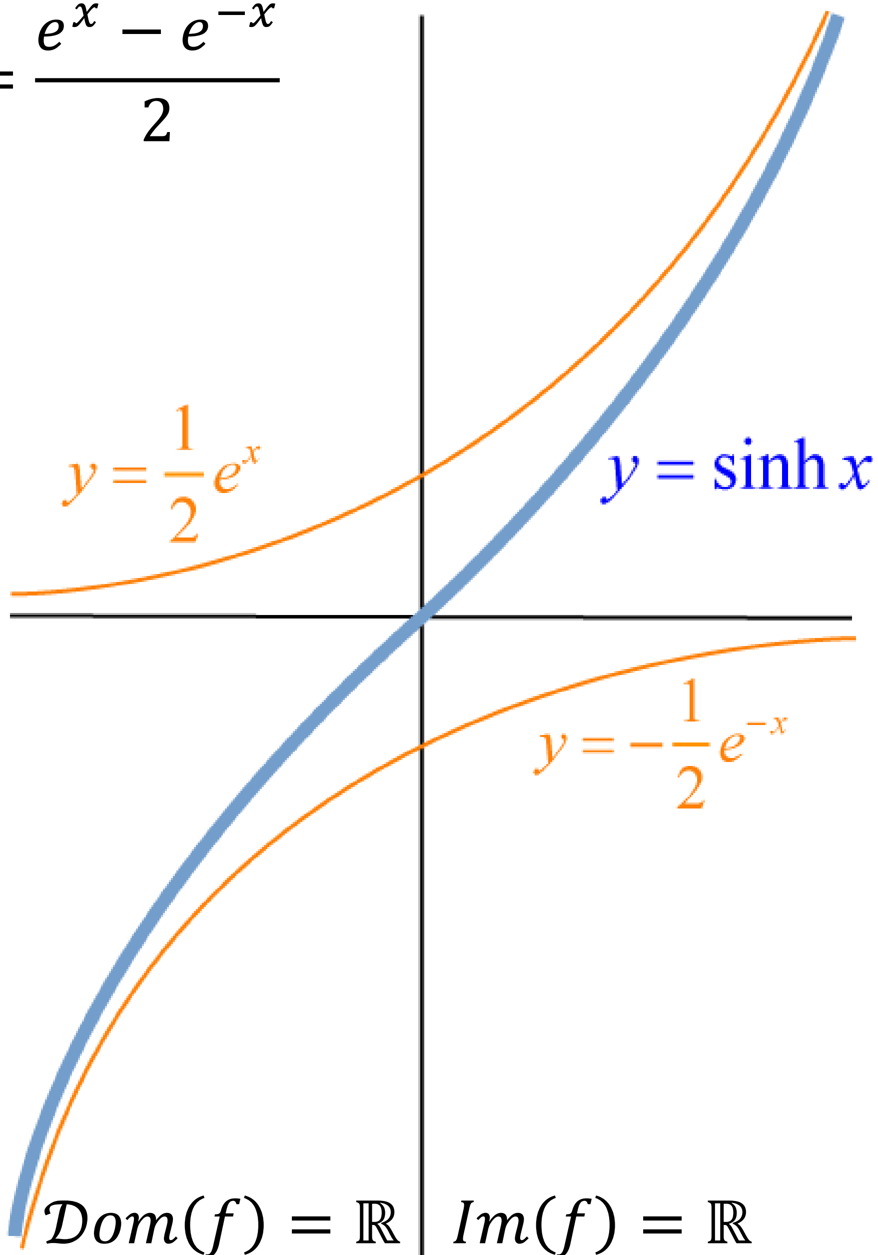
(1)-(2) $\quad e^{x} - e^{-x} = 2\sinh x \quad \Rightarrow \sinh x = \dfrac{e^{x} - e^{-x}}{2}$

$$\cosh x = \frac{e^{x} + e^{-x}}{2}$$

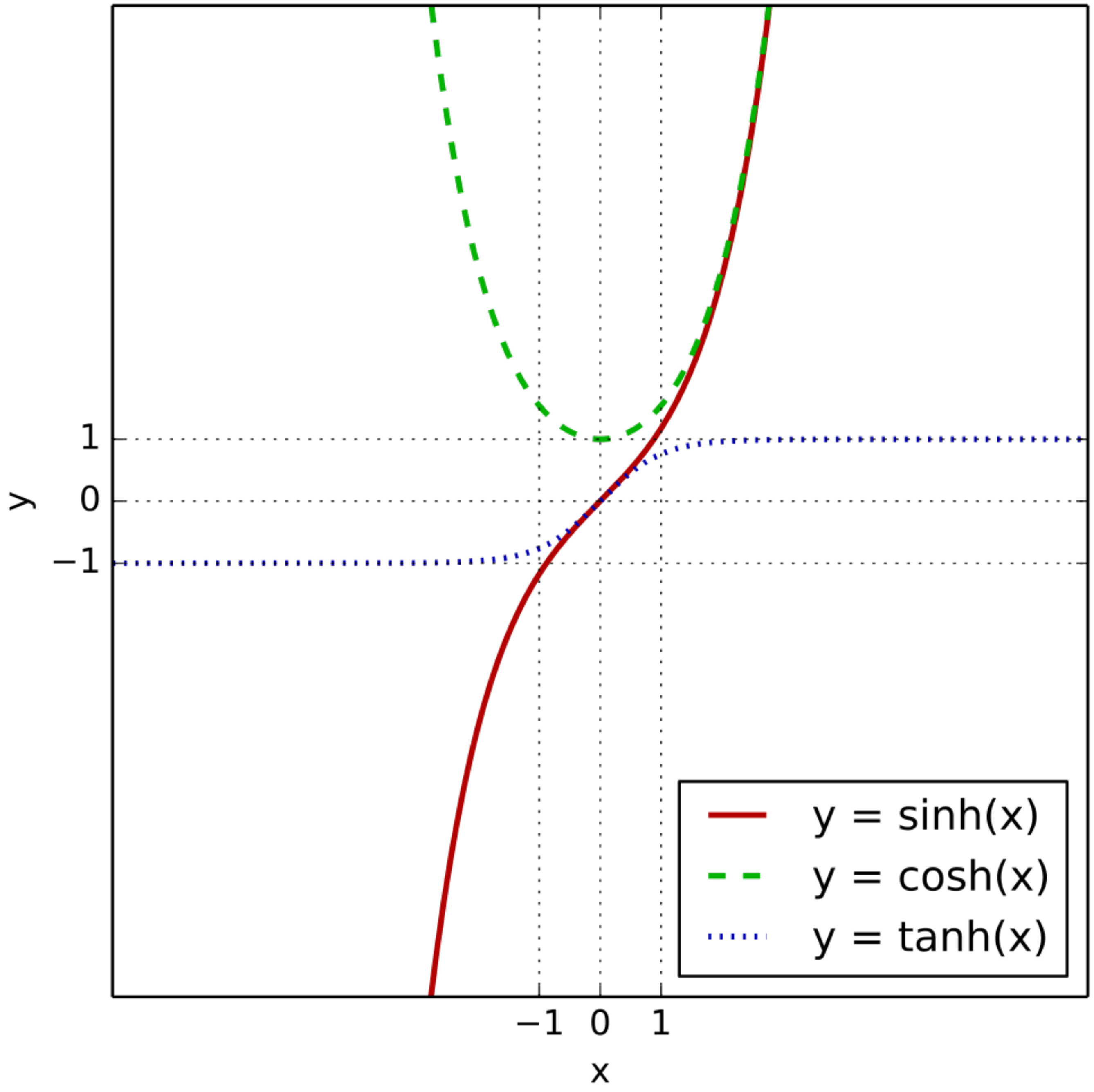


$\mathcal{D}om(f) = \mathbb{R}$ $\quad$ $Im(f) = [1, \infty)$

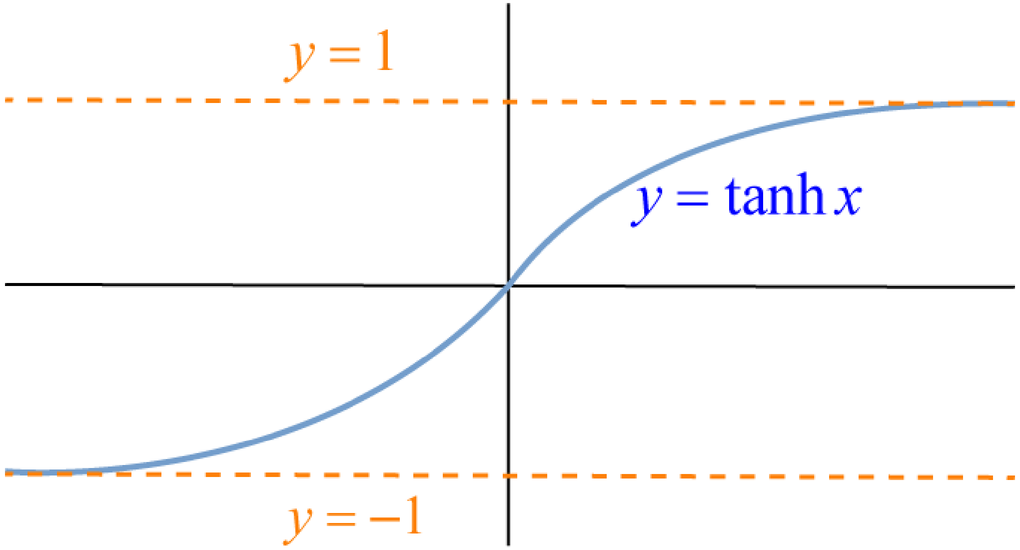
$$\sinh x = \frac{e^{x} - e^{-x}}{2}$$



$\mathcal{D}om(f) = \mathbb{R}$ $\quad$ $Im(f) = \mathbb{R}$

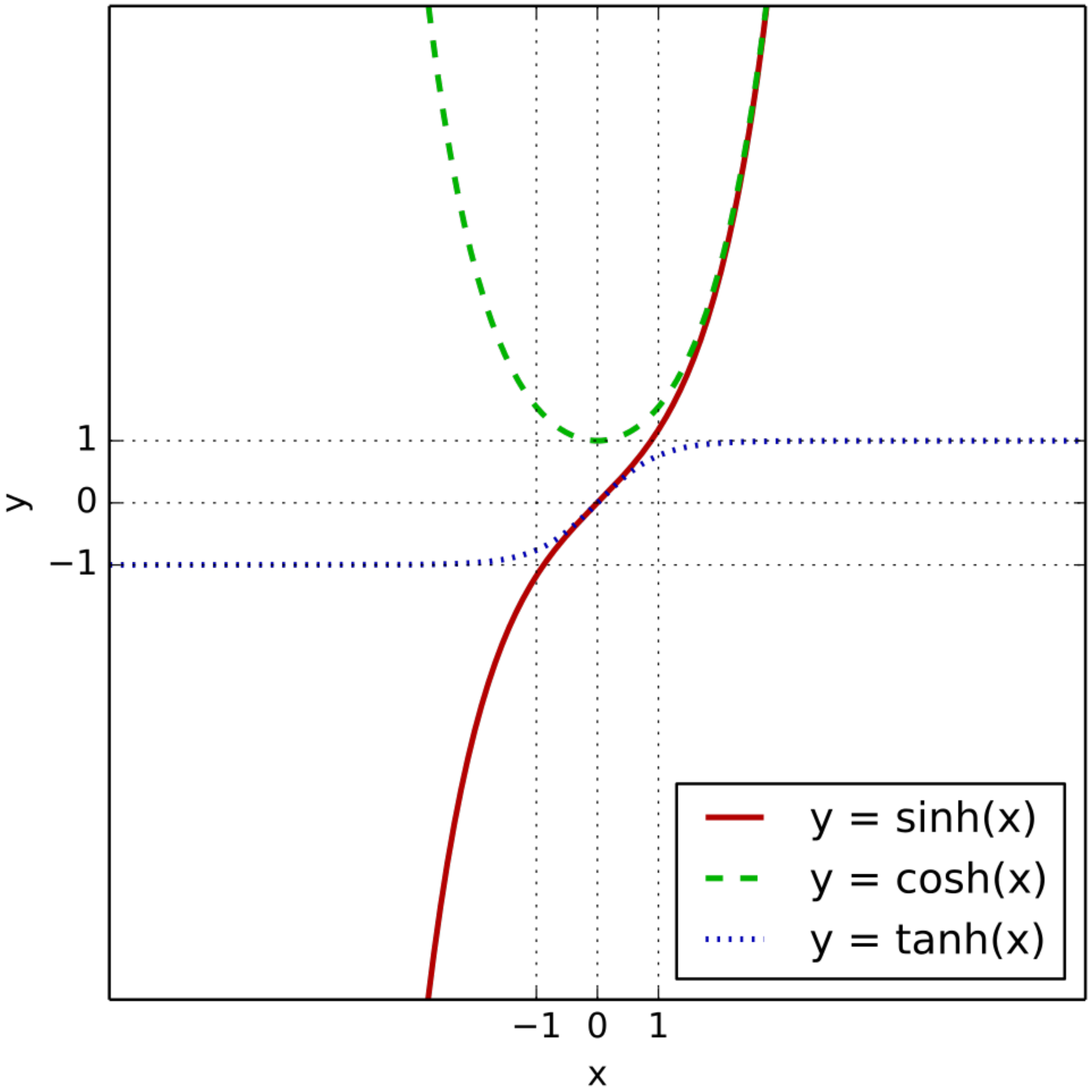


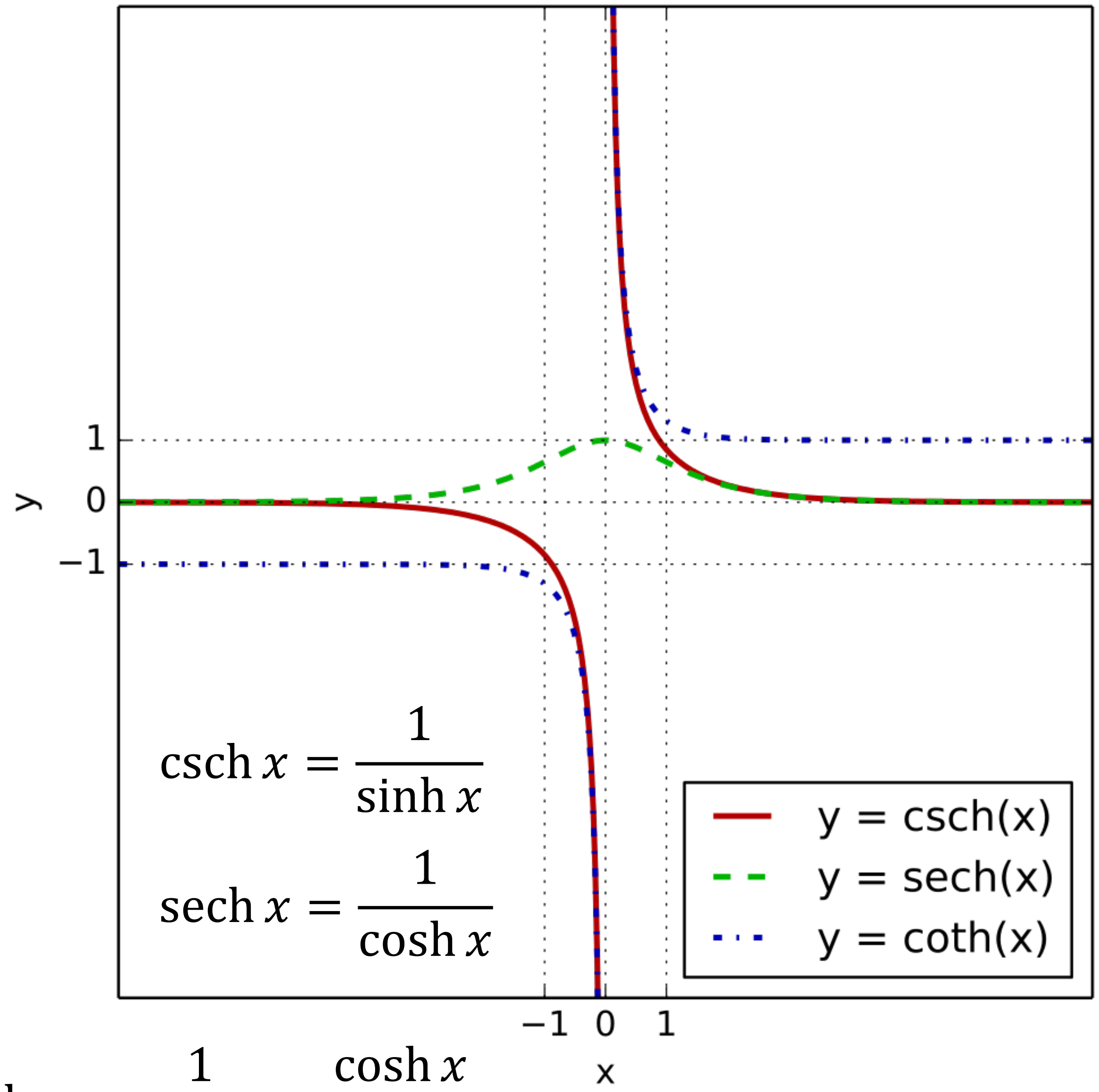
$$\tanh x = \frac{\sinh x}{\cosh x} \qquad \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$



$$\mathcal{D}om(f) = \mathbb{R} \qquad Im(f) = (-1,1)$$

- **Funcions hiperbòliques**





$$\operatorname{csch} x = \frac{1}{\sinh x}$$

$$\operatorname{sech} x = \frac{1}{\cosh x}$$

$$\coth x = \frac{1}{\tanh x} = \frac{\cosh x}{\sinh x}$$

- **Funcions inverses hiperbòliques**

Prenem com a exemple $y = \sinh x$

Com $\sinh x$ és injectiva, podem calcular la seva inversa en tot el seu domini.

Pas 1: $x \leftrightarrow y \quad \Rightarrow \boxed{x = \sinh y}$

Pas 2: hem d'aïllar y, $y = \operatorname{arcsinh} x = \sinh^{-1} x$,

per a fer-ho, utilitzarem que $e^y = \sinh y + \cosh y$, aplicant $\ln$ en ambdós costats:

$$\ln(e^y) = \ln(\sinh y + \cosh y)$$

$$y = \operatorname{arcsinh} x = \ln(\boxed{\sinh y} + \boxed{\cosh y})$$

$$\cosh^2 y - \sinh^2 y = 1$$

$$\cosh^2 y = 1 + \sinh^2 y$$

$$\boxed{\cosh y = \sqrt{1 + \sinh^2 y}}$$

$$\boxed{x = \sinh y}$$

$$\operatorname{arcsinh} x = \ln\left(\boxed{\sinh y} + \boxed{\sqrt{1 + \boxed{\sinh^2 x}}}\right)$$

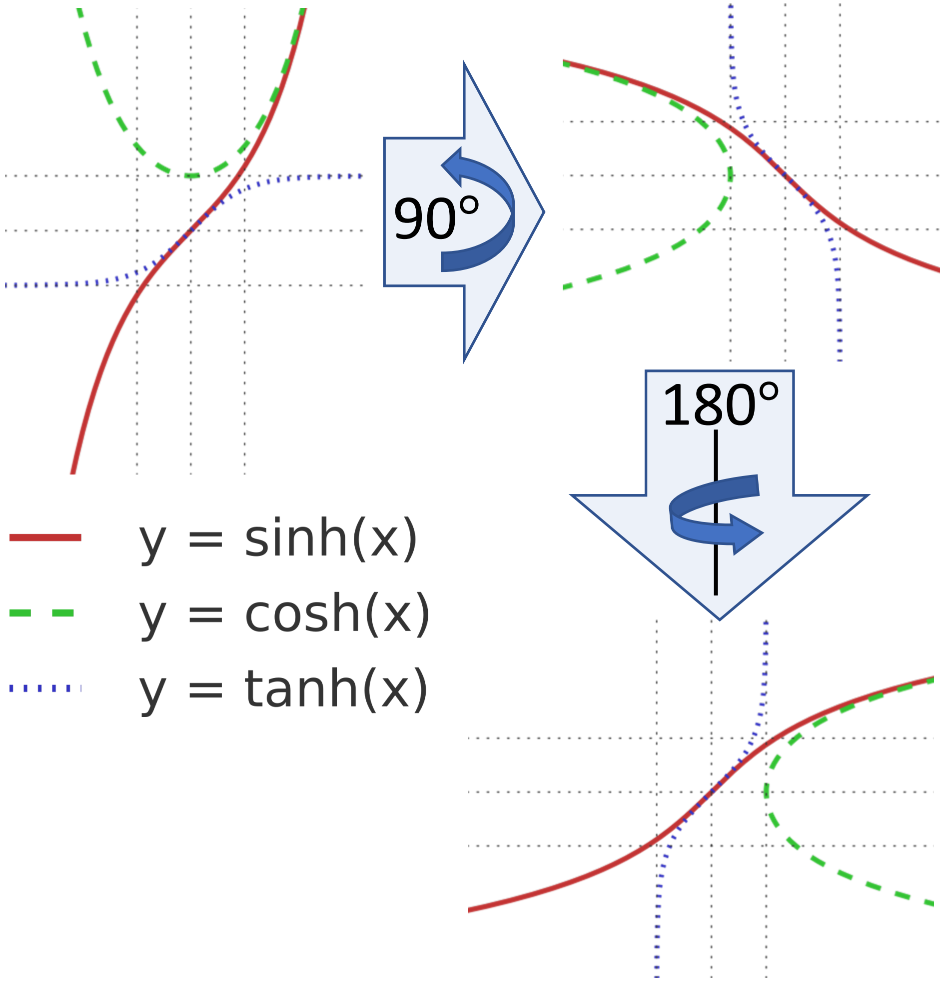
$$\operatorname{arcsinh} x = \ln\left(x + \sqrt{1 + x^2}\right)$$

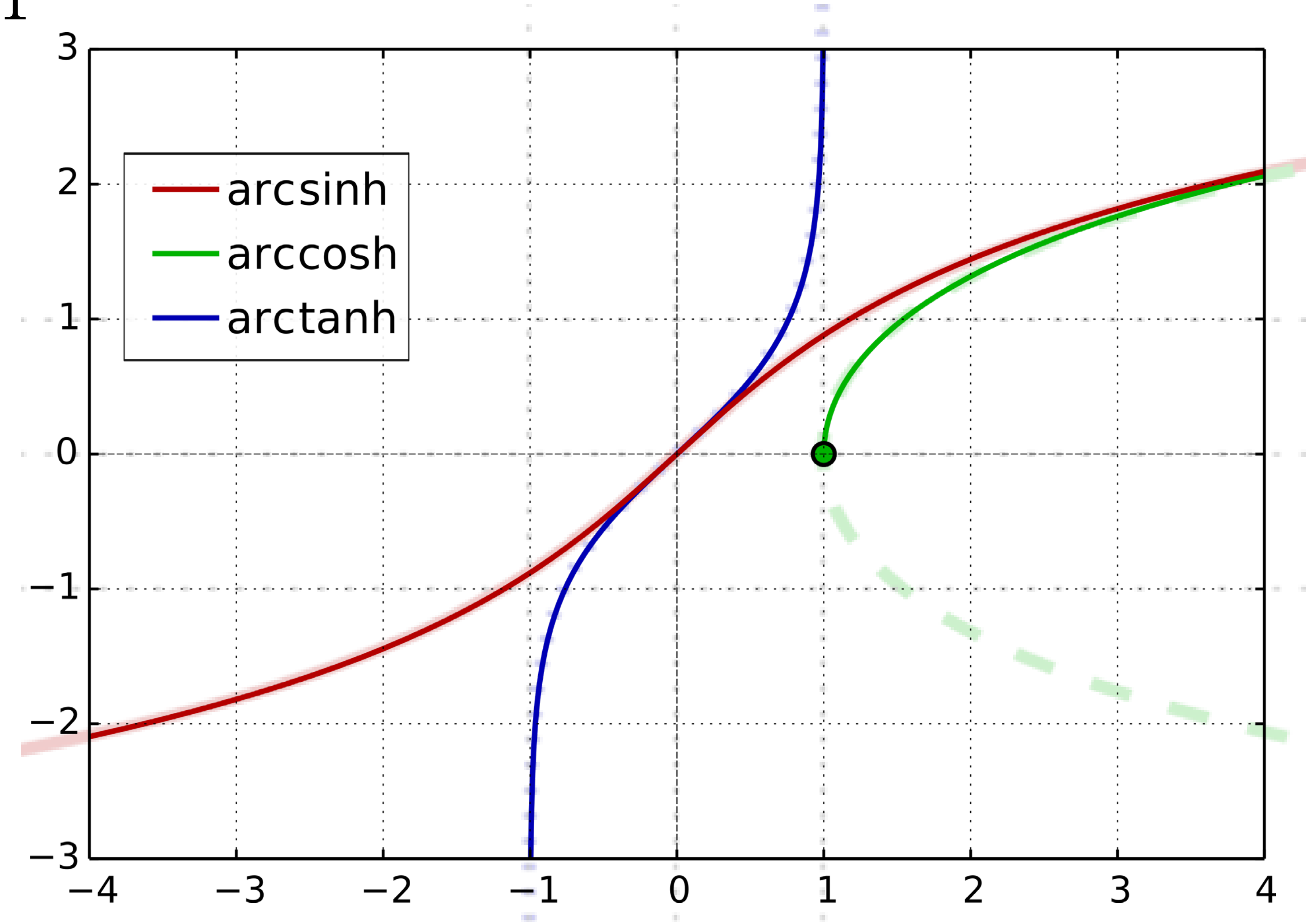
- **Funcions inverses hiperbòliques**

$$\operatorname{arcsinh} x = \ln(x + \sqrt{x^2 + 1})$$

$$\operatorname{arccosh} x = \ln\left(x + \sqrt{x^2 - 1}\right), x \geq 1$$

$$\operatorname{arctgh} x = \frac{1}{2}\ln\left(\frac{1 + x}{1 - x}\right), |x| < 1$$





- **Propietats de les funcions hiperbòliques**

$$\sinh(\alpha+\beta) = \sinh\alpha\cosh\beta + \cosh\alpha\sinh\beta$$

$$\sinh(2\alpha) = 2\sinh\alpha\cosh\alpha$$

$$\cosh(\alpha+\beta) = \cosh\alpha\cosh\beta \boxed{+} \sinh\alpha\sinh\beta$$

$$\cosh(2\alpha) = cosh^2\alpha \boxed{+} sinh^2\alpha$$

$$\tanh(\alpha+\beta) = \frac{\tanh\alpha + \tanh\beta}{1 \boxed{+} \tanh\alpha\tanh\beta}$$

$$\tanh(2\alpha) = \frac{2\tanh\alpha}{1 \boxed{+} tanh^2\alpha}$$

He marcat aquells signes que canvien respecte las raons trigonomètriques (revisar slide del tema 1 de repàs de trigonometria).