Evaluation of Human Steering Performance of Powered Wheelchairs

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1 Kinematics

Following chapter 2 in [1] robot pose (x_R, y_R, φ_R) and goal pose (x_G, y_G, φ_G) can be transformed to egocentric polar coordinates (see Figure 2).

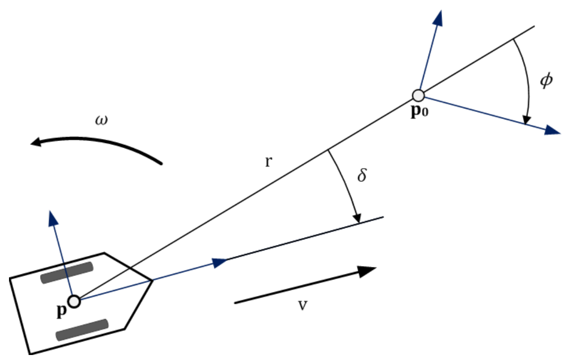

Figure 1: egocentric polar coordinate system (here, θ and δ have negative values)

The kinematics can be written as:

$$\begin{pmatrix} \dot{r} \\ \dot{\phi} \end{pmatrix} = \begin{pmatrix} -v\cos\delta \\ \frac{v}{r}\sin\delta \end{pmatrix} \tag{1}$$

$$\dot{\delta} = \frac{v}{r}\sin\delta + \omega \tag{2}$$

This is a two time-scale decomposition:

- Eq. (1) describes the vehicle dynamics in position subspace (slow subsystem).
- Eq. (2) describes the dynamics of the heading (steering) subspace (fast subsystem)

Note: The heading δ is a (virtual) control for eq. (1). The idea [2] is to find

1. virtual control δ (vehicle heading) which steers the subsystem (1) (vehicle position) to the origin, and
2. real control ω which render the dynamics of the subsystem (2) sufficiently faster than the subsystem (1) and stabilizes δ quickly to a desired virtual control, such that (1) becomes a slow subsystem and (2) becomes a fast subsystem in a singularly perturbed form.

Note that this process is analogous to a human driver controlling the steering wheel (fast subsystem) to drive the vehicle (slow subsystem) to a desired pose in space [2].

2 Non-Holonomic Distance Measure

In position subspace, the following weighted norm [3] is used to measure how far a point $(r, \phi)^T$ in polar coordinates is to the origin (the target pose):

$$l(r, \phi) = \sqrt{r^2 + k_\phi^2 \phi^2} \tag{3}$$

Note: The orientation of the target pose is incorporated in ϕ.

3 Feedback Control

3.1 Slow Subsystem

The origin is Lyapunov-stable under the following feedback examples:
(Proof in [1], assuming non-zero positive velocity v)

$$\delta_s^*(\phi) = \mathrm{atan}(-k_\phi \phi) \tag{4}$$

$$\delta_g^*(r, \phi) = \mathrm{atan}(-k_\phi^2 \phi / r^2) \tag{5}$$

Eq. (4) is the heading that reduces r and ϕ very smoothly at ratio $\dot{\phi}/\phi = k_\phi \dot{r}/r$ [3] Eq. (5) is the gradient of (3) along (1)., generating curves that quickly approach the target pose and then align to the target orientation [1]

3.2 Fast Subsystem

Derivation of ω for δ (see [3]):

$$\omega = -\frac{v}{r}[k_2(\delta - \arctan(-k_1\phi)) + (1 + \frac{k_1}{1 + (k_1\phi)^2})\sin(\delta)] \tag{6}$$

4 Vector Field

Each of these feedback control laws for the heading, eq. (4) and eq. (5), specifies a heading vector at every point in the position space by construction. Each control law (eq. (4) and eq. (4)) describes a stabilizing vector field.

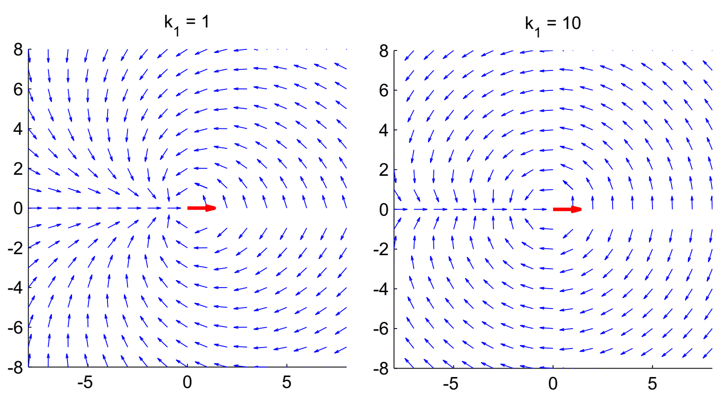

Figure 2: vector field defined by $\delta = atan(-k_1 * \phi)$ [3]

5 Evaluation

Assumptions:

- The described vector field defines a motion the human user would like to follow.
- The human wants to align the wheelchair orientation with the vector field.

If the wheelchair trajectory does not follow the vector field, the user tries to correct the heading.

5.1 Solution-Approach

Eq. (4) and eq. (5) are defining δ_{ref} as a function of ϕ or r and ϕ. For every sample-time, the current heading δ can be computed and then compared to the reference value from the vector field:

$$e = \delta - \delta_{ref} \tag{7}$$

The evaluation is done on slow (heading) subsystem. Note: The reference vector field is a geometrical definition of the heading reference to some goal pose.

References

[1] J. J. Park and B. Kuipers, "Feedback motion planning via non-holonomic RRT* for mobile robots," in *2015 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*. Hamburg, Germany: IEEE, Sep. 2015, pp. 4035–4040.

[2] J. J. Park, "Graceful Navigation for Mobile Robots in Dynamic and Uncertain Environments," p. 110.

[3] J. J. Park and B. Kuipers, "A smooth control law for graceful motion of differential wheeled mobile robots in 2D environment," in *2011 IEEE International Conference on Robotics and Automation*. Shanghai, China: IEEE, May 2011, pp. 4896–4902.