UNIVERSITY OF GHANA

MATH 126: Algebra and Geometry
(3 credits)

Matrices and Systems of Linear Equations
(Module 1)

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1. Show that
   (a) If A and B are 2×2 matrices then the trace of $AB - BA$ is zero.
   (b) If E is a 2×2 matrix and the trace of E

   is zero then $E^2 = \lambda I_2$ for some scalar λ.

2. If A is a square matrix such that $A^2 = A$ and $(A - A^T)^2 = O$, prove that $(AA^T)^2 = AA^T$

3. Let A, B be square matrices with A symmetric and B skew-symmetric. Determine which of the following are symmetric or skew-symmetric.
   (a) $AB + BA$
   (b) $AB - BA$
   (c) $A^p B^q A^p$

4. Solve the following systems of linear equations using Gaussian elimination or Gauss-Jordan elimination.

   (a) $$\begin{array}{rcrcrcr} 3x & - & 2y & + & 2z & = & 6 \\ 7x & - & y & + & 2z & = & -1 \\ 2x & - & 3y & + & 4z & = & 0 \end{array}$$

   (b) $$\begin{array}{rcrcrcl} 2x & - & 3y & - & z & = & 0 \\ 3x & + & 2y & + & 2z & = & 2 \\ x & + & 5y & + & 3z & = & 2 \end{array}$$

5. Solve for x

   (a) $$\begin{vmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{vmatrix} = 0$$

6. Determine conditions for which the linear system

$$\begin{array}{rcrcrcl} 3x & - & y & + & z & = & 3 \\ ax & + & y & + & z & = & 4 \\ 8x & - & y & + & 3z & = & b \end{array}$$

has

(a) a unique solution

(b) infinitely many solutions

(c) no solution

7. Calculate the inverse of the matrix $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 3 & 6 \\ 1 & 2 & 3 \end{pmatrix}$ using

   (a) elementary row operations

   (b) cofactors
   Hence, solve the system of linear equations

$$\begin{array}{rcrcrcr} x & + & y & + & z & = & 1 \\ x & + & 3y & + & 6z & = & 2 \\ x & + & 2y & + & 3z & = & 1 \end{array}$$

8. Using Cramer's rule, solve the system

$$\begin{array}{rcrcrcr} x & + & 2y & - & z & = & -3 \\ 2x & - & 4y & + & z & = & -7 \\ -2x & + & 2y & - & 3z & = & 4 \end{array}$$