University of Ghana
Department of Mathematics
Math126: Algebra and Geometry
Semester 2 2020/21
(Submit: 1b, 2c, 5, 6, 7, 8b, 11b, 12, 13c)

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1. Matrices A and B are defined as follows

$$A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}, \qquad B = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}.$$

Find:

(a) AB

(b) BA

2. Suppose

$$A = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix} \quad \text{and } B = \begin{bmatrix} -1 & 1 \\ -2 & 0 \end{bmatrix}.$$

Compute the following matrices:

(a) $A + B$

(b) $A - B$

(c) $2B - 3A$

(d) $2(A + B) - A$

3. Suppose A and B are as given in question 2 and let

$$C = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \end{bmatrix}; \quad D = \begin{bmatrix} 0 & -2 & 1 \\ 2 & 2 & 0 \end{bmatrix}.$$

Compute the following matrices:

(a) AB

(b) CD^T

(c) $C^T D$

(d) AC

(e) $(A + B)(C + D)$

(f) $\left(C^T D\right)^2$

(g) $D^T \left(A^T + B\right)^T C$

4. Let $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$. Compute A^{1000}. [Hint: Start by computing A^2, A^3,... and see if you notive a pattern.]

5. Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

   (a) Compute A^2.

   (b) Compute A^3.

   (c) Find a general formula for A^k (where $k \geq 0$ is an integer).

6. Suppose $A_1, A_2, \ldots A_n$ are 2×2 matrices. Prove that $(A_1 A_2 \ldots A_n)^T = A_n^T \ldots A_2^T A_1^T$.[Hint: Use induction].

7. Let A be a 2×2 matrix. Expand and simplify $(A+I)^2 (A-I)$.

8. Suppose that A, B, C are 3×3 matrices with $\det(A) = 2$, $\det(B) = 3$ and $\det(C) = 5$. Compute the following determinants:

   (a) $\det(AB)$

   (b) $\det(3AB^{-2}C^2)$

   (c) $\det\left(A^2 C^T B^{-1}\right)$

9. Compute the determinant of the following matrices:

   (a) $\begin{bmatrix} 2 & 2 \\ 4 & 5 \end{bmatrix}$

   (b) $\begin{bmatrix} 0 & 2 \\ 3 & 4 \end{bmatrix}$

   (c) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix}$

10. Compute all minors and cofactors of the following matrices:

    (a) $\begin{bmatrix} 0 & 2 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & 0 \end{bmatrix}$

    (b) $\begin{bmatrix} 2 & 2 \\ 4 & 5 \end{bmatrix}$

11. Compute the determinant of the following matrices along the given row or column:

    (a) $\begin{bmatrix} 1 & 3 & 0 \\ 0 & -2 & 2 \\ -1 & 0 & 1 \end{bmatrix}$ column 2

(b) $\begin{bmatrix} 1 & 1 & 3 \\ -4 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix}$ row 3

12. Compute the determinant of the matrix

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}.$$

13. Suppose that

$$\det\left(\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\right) = 4.$$

Compute the determinant of the followong matrices:

(a) $\begin{bmatrix} g & h & i \\ d & e & f \\ a & b & c \end{bmatrix}$

(b) $\begin{bmatrix} a & b & c \\ 2d & 2e & 2f \\ 3g & 3h & 3i \end{bmatrix}$

(c) $\begin{bmatrix} a & b+a & 2c \\ d & e+d & 2f \\ g & h+g & 2i \end{bmatrix}$