TELECOMUNICATION SYSTEMS AND SERVICES

SESSION 2:

Traffic

LABORATORIO TSC-3

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Description:

In the lesson 3 in *Telecommunication Systems and Service,* the fundamental concepts of telecommunication traffic are presented. This session allows students to deepen the concepts of traffic and to understand the nature of the mathematical models used to model traffic Telecommunications (for example their statistical distributions etc.).

The content of this practice corresponds to practice 1 in Telecommunication Systems course taught at the Escuela Politécnica Superior in Gandía, entitled "Statistics and Telegraph" and facilitated by professors and Marta Cabedo Fabres and Toni Canós.

Objectives:

- Understand the basis of the model used in the theory of traffic, such as distributions for generating calls and their duration etc.
- Generating Erlang-B and Erlang-C tables for planning telecommunication systems.

Previos work

Read the document and prepare Questions 1, 3a, 13 and 16 at home before coming to the session.

1. SOURCE CALL MODEL.

1.1. POISSION CALL SOURCE.

The first step in modeling a network is the analysis of the source that generates service requests over time. Assuming an infinite population that generates requests for occupation of a communications link (normally calls) at $t_1, t_2, \ldots, t_n$.

You can define the random variable *time between arrivals,* τ_i, as the time between the arrival of a call *i* and the arrival of the next call *(i + 1),* i.e.

$$\tau_i = t_{i+1} - t_i$$

An important parameter of the system is the arrival rate, λ, which can be obtained by:

$$\lambda = \frac{1}{\overline{\tau}}$$

During a large observation time, *T,* the arrival rate can be computed as:

$$\lambda = \frac{\text{nº llamadas}}{\text{T}}$$

In most systems, it can be assumed that:

1) In a small time interval, *dt*, only one call can be generated, i.e. it is not possible to receive two calls at the same time.

2) The probability that a call appears in *dt* is proportional to the length of the interval (i.e. probability of an arrival = $\lambda \cdot$ dt).

3) The probability that a call comes in any particular interval is independent of what has happened in other intervals (no memory).

Under these assumptions, it can be shown that different τ_i are independent random variables whose values follow a negative exponential distribution, i.e. its probability density function and distribution function are respectively:

$$f_\tau(t) = \lambda \cdot e^{-\lambda \cdot t}$$

$$F_\tau(t) = \mathrm{P}[\tau \le t] = 1 - e^{-\lambda \cdot t}$$

Question 1. Obtain analytically $F_\tau(t)$ from $f_\tau(t)$

Question 2. Draw using MATLAB functions $F_\tau(t)$ and $f_\tau(t)$ for a value of $\lambda = 0.3\,\text{calls/s}$ between 0 and 20 sec. For constructing time domain use the command *linspace (1,20,100)* in order to create a vector with equally spaced time samples.

A source that generates calls following this statistical law is call *Poisson source.*

To simulate call requests from a Poisson source, use the distribution $F_\tau(t) = 1 - e^{-\lambda \cdot t}$. Therefore, the inverse function of distribution is

$$t = -\frac{\ln(1 - F_\tau(t))}{\lambda}$$

Question 3a. Demonstrate the previous equation.

In order to implement it in MATLAB, use a uniform variable for $F_\tau(t)$ *(Rand).*

Question 3b. Simulate a Poisson source with a calling rate of $\lambda = 0.3\,\text{calls/s}$, observed during a time interval 20000 of seconds. Show the time between arrivals.

<u>Help:</u> Create a *while* loop that will generate random values of *t*, until the accumulated sum is a maximum time of observation.

The times between calls, τ_i, in theory follow a negative exponential distribution.

Question 4: To check, represent in a bar diagram the histogram of the values *(hist* (x, 51))

The histogram indicates the absolute frequencies of occurrence of a value at different intervals. With the above statement, the value range of the variable is divided into 51 equal intervals.

Question 5. Compute the call arrival rate λ from the τ_i obtained and compare it with the source call rate (the inverse of the average value). Interpret the results.

	T (s)	λ obtained (Calls / s)
Series 1	20000	
Series 2	20000	
Series 3	200	
Series 4	200	

In these systems, it can also be shown that the probability of exactly *k* calls arriving in a time interval *t*, $\mathrm{P}_k(t)$, can be determined by

$$\mathrm{P}_k(t) = \frac{(\lambda \cdot t)^k}{k!} \cdot e^{-\lambda \cdot t};\ k = 0, 1, 2, \ldots$$

This is an alternative characterization of these processes, which are called *Poisson processes*. In this case, the random variable k represents the 'number of received calls during a time interval of t', follows a Poisson distribution with an average value of $\lambda' = \lambda \cdot t$.

Note that the Poisson distribution refers to a discrete random variable ('number of calls in a time slot'), whereas the exponential distribution characterized a continuous random variable (' time between calls').

For example, let us consider the variable 'number of calls received during 10 seconds'. This variable, in theory, will follow this distribution

$$\mathrm{P}_k(10) = \frac{(10 \cdot \lambda)^k}{k!} \cdot e^{-(10 \cdot \lambda)};\ k = 0, 1, 2, \ldots$$

That is, a Poisson process with an average value of $\lambda' = 10 \cdot \lambda$.

To verify this theoretical result, let us generate intervals between calls again from a Poisson source with a calling rate of $\lambda = 0.3$ calls/s observed during a time interval of T = 20,000 seconds (we can use the same variable used before).

Question 6.

We divide the observation interval [0, T], in slots of 10 seconds and we count the calls that have arrived in each of them.

tt=10;centros=[tt/2:tt:T];m=hist(ti,centros);

The vector m contains the number of calls that have arrived in each slot (m = 0, 1, 2, ...). Let us study the probability of each value of m (being natural numbers). To do this we plot the histogram of m, defining previously their slot.

k=[0:1:max(m)];

contadas=hist(m,k);

Then the probability of having k arriving calls in a 10 second interval is obtained by:

pk=contadas/sum(contadas);

Then we adjust the values of the obtained values of P_k to a Poisson distribution, whose λ' parameter is obtained by

$$\lambda' = \mathrm{E}[k] = \sum_{k=0}^{\infty} k \cdot P_k$$

This summation can be calculated by:

lambda_prima=k*pk.'

Compare the value obtained with the practical procedure with the theoretical one and write both values in the table.

Theoretical λ'	Obtained λ'

Now we compare both results graphically.

Question 7. Get the theoretical values of the distribution by the Poisson distribution. To do this run:

for ii=1:length(k)

*pkfitted(ii)=lambda_prima^k(ii)*exp(-lambda_prima)/factorial(k(ii));*

end

Represent jointly both results by using a bar representation for the values of P_k from the simulation and with line the theoretical ones.

bar(k,pk); hold on;

plot(k,pkfitted,'r'); plot(k,pkfitted,'.r');

grid; hold off

1.2. EXPONENTIAL SERVICE TIME. OFFERED TRAFFIC.

Another parameter that characterizes calls coming into a system is the duration of the service. Service time of calls, s_i, are independent random variables whose values also follow a negative exponential distribution :

$$F_s(t) = \mathrm{P}[s \le t] = 1 - e^{-\mu \cdot t}$$

where μ is the rate or speed of service which can be obtained by:

$$\mu = \frac{1}{\bar{s}}$$

being $\bar{s} = \mathrm{E}[s] \equiv$ average service time .

Then it comes the definition of offered traffic to a system, A_O , which is defined by:

$$A_O = \frac{\lambda}{\mu}$$

Then, we will simulate the arrival of calls from a Poisson source with exponential services, and we will monitor the instantaneous traffic offered to the system using during a short interval of time.

To do this we write the following code:

```
T=50; %Tiempo de observación
lambda=0.3;
mu=0.15;
```

Question 8: Generate calls, as before, during 50 seconds, and at the same time anther variable *s* (service time), for each call.

Then, it is possible to obtain the time when a call is finished (to),

```
to=ti+s; to=sort(to);
```

Question 9: A graphical representation of the starting and ending times of each call can be done by:

```
stem(ti,ones(size(ti)),'.b'); hold on
stem(to,-ones(size(to)),'.r');
grid; hold off;
```

From the arrival times, t_i, and output time , t_o , it is possible to obtain the instantaneous traffic offered to the system, $A_{O,inst}$ By:

```
tiempoaux=[ti,to];
signosaux=[ones(size(ti)),-ones(size(to))];
[tiempos,indexes]=sort(tiempoaux);
signos=signosaux(indexes);
Ainstant=cumsum(signos);
tiempos=[0 tiempos];    % Origen de tiempos en t=0
```

Ainstant=[0 Ainstant]; % Se considera que antes de llegar la primera llamada, el trafico es cero.

figure

stairs(tiempos,Ainstant);

grid;

xlabel ('Tiempo (s)');

ylabel ('Trafico (E)')

title ('Intensidad de Trafico Instantanea');

Question 10: Observe figure where the traffic offered to the system ($A_{O,inst}$) is depicted and relate the results obtained with T.

If all offered traffic would be done instantaneously, the figure would indicate directly the channel occupancy. The average offered traffic is the **weighted** average of the instantaneous data traffic offered. On a weighted average values not all values have the same weight in obtaining the mean value.

In that case, in order to obtain an approximation of the offered traffic:

taus=diff(tiempos);

*Ao_aprox=Ainstant(1:end-1)*taus.'/tiempos(end*)

And the theoretical offered traffic by:

Ao=lambda/mu

Question 11: Compare both values and interpret the results.

Question 12: Generate several series of calls with observation times, *T*, and fill the following table. Interpret the results.

	T (s)	Theoretical A_O	Obtained A_O
Serie 1	50		
Serie 2	50		
Serie 3	20000		
Serie 4	20000		

2. QUEUING MODEL SYSTEM

2.1. LOSSY SYSTEM OR ERLANG-B.

The Erlang-B (or Er_1) function provides the blocking probability, P_B, of a lossy system (no queue) with C servers where A_o traffic is offered. It is considered that there is b locking when all C servers are in use.

This function can be expressed as:

$$P_B = PP = Er_1(C, A_O) = \frac{A_O^C}{C!} \frac{1}{\sum_{i=0}^{C} \frac{A_O^i}{i!}}$$

Question 13: Mathematically show that this function can be iteratively written as:

$$\frac{1}{Er_1(C, A_O)} = 1 + \frac{C}{A_O} \cdot \frac{1}{Er_1(C-1, A_O)}, \text{si } C \geq 1$$

$$Er_1(0, A_O) = 1$$

Question 14: Program the Erlang-B function.

With the previous expression, it is easy to get the blocking probability from traffic and channels, but no with other combination. For this reason, we facilitate the function *erlangbinv*.

Question 15: Construct a traffic table for 1-10 channels and for blocking probabilities of 1.5, 2.5, 7.5 and 15%. To do that you can use this loop.

```
CPb=[1.5 2.5 7.5 15]/100;
for n=1:10,
for p=1:length(Pb),
E(n,p)=findrhob(n,Pb(p));
end
end
```

Compare these values with those presented in the problems book.

2 .2. WAITING SYSTEM OR ERLANG-C.

The Erlang-C (or *Er₂)* function gives the probability that a call is waiting, $\boldsymbol{P_{ESP}}$, in a queuing system with and infinite queue with C servers where a A_o traffic is offered. It is considered that you have to wait when all C channels are busy.

This function can be expressed as:

$$P_{ESP} = Er_2(C, A_O) = \frac{\frac{A_O^C}{C!}\frac{C}{C - A_O}}{\sum_{k=0}^{C}\frac{A_O^k}{k!} + \frac{A_O^{C+1}}{C!}\frac{1}{C - A_O}} \text{, if } A_O < C$$

$$P_{ESP} = Er_2(C, A_O) = 1 \text{, if } A_O \geq C$$

Question 16. Show that this function can be written using the Erlang-B function as:

$$P_{ESP} = Er_2(C, A_O) = \frac{C \cdot Er_1(C, A_O)}{A_O \cdot Er_1(C, A_O) + C - A_O} \text{, if } A_O < C$$

$$P_{ESP} = Er_2(C, A_O) = 1 \text{, if } A_O \geq C$$

Question 17: Program the function **<u>Help Erlang-C.</u>** You can do it from the Erlang-B formula.

With the previous formula, it is easy to obtain the probability of expected traffic from the channels, but not vice versa. For this, the feature is provided *erlangcinv*

Question 18: Construct a traffic table for 1-10 channels and for waiting probabilities of 1.5, 2.5, 7.5 and 15%. To do that you can use this loop.

```
CPb=[1.5 2.5 7.5 15]/100;
for n=1:10,
for p=1:length(Pb),
E(n,p)=findrhoc(n,Pb(p));
end
end
```

Compare these values with those presented in the problems book.