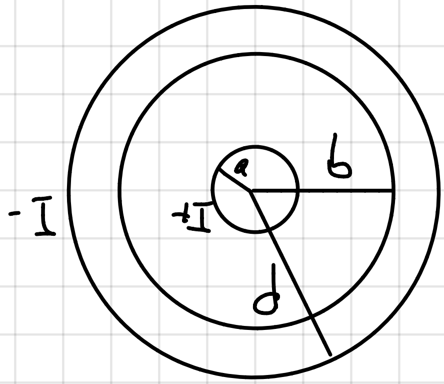
Práctica 2



$$I_{ins} = \oint_c \bar{H} \cdot \bar{dl}$$

$$\bar{H} = H_\varphi(\rho)\,\hat{\varphi}$$

$$\bar{J} = \frac{I}{\pi a^2}\hat{z} \quad \left(\frac{A}{m^2}\right)$$

Tomamos $\rho < a$

$$\oint_c \bar{H} \cdot \bar{dl} = H_\varphi(\rho) \cdot \hat{\varphi} \cdot \hat{\varphi} \cdot 2\pi\rho = H_\varphi 2\pi\rho$$

$$I_{ins} = \bar{J} \cdot S_{circulo} = \frac{I}{\pi a^2} \cdot \pi\rho^2 = I\frac{\rho^2}{a^2}$$

$$\left\{ H_\varphi 2\pi\rho = I\frac{\rho^2}{a^2} \rightarrow \right.$$

$$\rightarrow H_\varphi = I\frac{\rho}{a^2 2\pi} \rightarrow$$

$$\rightarrow H_\varphi = \frac{I\rho}{2\pi a^2} \rightarrow \boxed{\bar{H} = \frac{I\rho}{2\pi a^2}\hat{\varphi}}$$

Tomamos $a < \rho < b$

$$\oint_c \bar{H} \cdot \bar{dl} = H_\varphi 2\pi\rho$$

$$I_{ins} = \bar{J} \cdot S_{circulo} = \frac{I}{\cancel{\pi a^2}} \cdot \cancel{\pi a^2}$$

$$\left\{ H_\varphi 2\pi\rho = I \rightarrow H_\varphi = \frac{I}{2\pi\rho} \rightarrow \right.$$

$$\rightarrow \boxed{\bar{H} = \frac{I}{2\pi\rho}\hat{\varphi}}$$

Tomamos $b < \rho < d$

$$\oint_c \bar{H} \cdot \bar{dl} = H_\varphi 2\pi\rho$$

$$I_{ins} = I - \frac{I}{\cancel{\pi}(d^2 - b^2)} \cdot (\rho^2 - b^2) \cdot \cancel{\pi} = I\left(1 - \frac{(\rho^2 - b^2)}{d^2 - b^2}\right)$$

$$H_\varphi 2\pi\rho = I\left(1 - \frac{\rho^2 - b^2}{d^2 - b^2}\right) \rightarrow H_\varphi = \frac{I}{2\pi\rho}\left(1 - \frac{\rho^2 - b^2}{d^2 - b^2}\right) \rightarrow \boxed{\bar{H} = \frac{I}{2\pi\rho}\left(1 - \frac{\rho^2 - b^2}{d^2 - b^2}\right)\hat{\varphi}}$$

Para $\rho > d$

$$\oint_C \bar{H} \cdot \bar{dl} = H_\varphi 2\pi\rho$$

$$I_{ins} = I_{ins_1} - I_{ins_3} = 0$$

No hay campo magnético en el exterior del cable coaxial

Calculamos W_{m_2}

$$W_{m_2} = \frac{\mu}{2} 2\pi l \cdot \int |\bar{H}|^2 \cdot \rho \, d\rho = \frac{\mu}{2} 2\pi l \cdot \int \frac{I^2}{2^2\pi^2\rho^2} \cdot \rho \, d\rho \rightarrow$$

$$\rightarrow \frac{\mu}{4\pi} \cdot I^2 \cdot l \cdot \int_a^b \frac{1}{\rho} \, d\rho = \cancel{\frac{\mu}{4\pi} I^2 l \cdot \ln|\rho| \, (J)}$$

$$\boxed{\frac{L_2}{l} = \frac{\mu}{2\pi} \cdot \ln\left(\frac{b}{a}\right) \left(\frac{H}{m}\right)}$$

← Inductancia por unidad de longitud en la zona comprendida entre a y b.

Calculamos W_{m_3}

$$W_{m_3} = \frac{\mu}{2} 2\pi l \int_b^d \left| \frac{I}{2\pi\rho} \cdot \left(\frac{d^2 - b^2}{d^2 - \rho^2}\right) \right|^2 \cdot \rho \, d\rho = \frac{\mu \cdot l}{4\pi} \cdot I^2 \cdot \int_b^d \frac{1}{\rho} \cdot \frac{(d^2 - \rho^2)^2}{(d^2 - b^2)^2} \, d\rho \rightarrow$$

$$\rightarrow \frac{\mu \cdot l}{4\pi} \cdot \frac{I^2}{(d^2 - b^2)^2} \int_b^d \frac{(d^2 - \rho^2)^2}{\rho} \, d\rho = \frac{\mu \cdot l}{4\pi} \cdot \frac{I^2}{(d^2 - b^2)^2} \left[\int_b^d \frac{d^4}{\rho} \, d\rho - \int_b^d 2d^2 \rho \, d\rho + \int_b^d \rho^4 \, d\rho \right]$$

$$\Rightarrow \frac{\mu \cdot l}{4\pi} \cdot \frac{I}{(d^2 - b^2)^2} \left[d^4 \ln\left(\frac{d}{b}\right) - 2d^2 \cdot \left(\frac{d^2}{2} - \frac{b^2}{2}\right) + \left(\frac{d^5}{5} - \frac{b^5}{5}\right) \right] (J)$$

$$\frac{L_3}{l} = \frac{\mu}{4\pi (d^2 - b^2)^2} \cdot \left[d^4 \ln\left(\frac{d}{b}\right) - 2d^2 \left(\frac{d^2}{2} - \frac{b^2}{2}\right) + \left(\frac{d^5}{5} - \frac{b^5}{5}\right) \right] \left(\frac{H}{m}\right)$$