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Theoretical previous study.

1. SOURCE CALL MODEL.

Question 1. Obtain analytically $F_\tau(t)$ from $f_\tau(t)$.

$$F_\tau(t) = P[\tau \le t] = \int_0^t f(\tau)\,d\tau = \int_0^t \lambda \cdot e^{-\lambda\tau} d\tau = -\int_0^t -\lambda \cdot e^{-\lambda\tau} d\tau = -\left[e^{-\lambda\tau}\right]_0^t = -\left[e^{-\lambda t} - e^{0}\right] = 1 - e^{-\lambda t}$$

Question 3.a. Demonstrate the equation: $t = -\dfrac{\ln(1 - F_\tau(t))}{\lambda}$.

$$F_\tau(t) = 1 - e^{-\lambda t} \rightarrow e^{-\lambda t} = 1 - F_\tau(t) \rightarrow -\lambda t = \ln(1 - F_\tau(t))$$

2. QUEUING MODEL SYSTEM.

2.1. LOSSY SYSTEM OR ERLANG-B.

Question 13: Mathematically show that the Erlang-B function:

$$P_B = PP = Er_1(C, A_O) = \frac{A_O^C}{C!} \frac{1}{\sum_{i=0}^{C} \frac{A_O^i}{i!}}$$

can be iteratively written as:

$$\frac{1}{Er_1(C, A_O)} = 1 + \frac{C}{A_O} \cdot \frac{1}{Er_1(C-1, A_O)}, \text{ si } C \geq 1$$

$$Er_1(0, A_O) = 1$$

$$\text{Si } C=0 \rightarrow \underline{\underline{E_{r}(0, A_0)}} = \frac{A_0^0}{0!} \frac{1}{\sum_{i=0}^{0} \frac{A_0^i}{i!}} = \frac{1}{1} \cdot \frac{1}{\frac{A_0^0}{0!}} = \frac{1}{1} \cdot \frac{1}{1} = \underline{\underline{1}}$$

$$\text{Si } C \geq 1 \rightarrow E_{r_1}(C, A_0) = \frac{A_0^C}{C!} \frac{1}{\sum_{i=0}^{C} \frac{A_0^i}{i!}} = \frac{A_0^C}{C!} \cdot \left(\frac{1}{\frac{A_0^C}{C!} + \sum_{i=0}^{C-1} \frac{A_0^i}{i!}} \right) = \frac{A_0^C}{\cancel{C!}\frac{A_0^C}{\cancel{C!}} + C! \sum_{i=0}^{C-1} \frac{A_0^i}{i!}} \qquad \frac{1}{E_{r_1}(C-1, A_0)}$$

$$\underline{\underline{\frac{1}{E_{r_1}(C, A_0)}}} = \frac{A_0^C + C! \sum_{i=0}^{C-1} \frac{A_0^i}{i!}}{A_0^C} = 1 + \frac{C! \sum_{i=0}^{C-1} \frac{A_0^i}{i!}}{A_0^C} = 1 + \frac{C \cdot (C-1)! \sum_{i=0}^{C-1} \frac{A_0^i}{i!}}{A_0 \cdot A_0^{C-1}} = 1 + \frac{C}{A_0} \cdot \underbrace{\frac{(C-1)! \sum_{i=0}^{C-1} \frac{A_0^i}{i!}}{A_0^{C-1}}}_{} =$$

$$= \underline{\underline{1 + \frac{C}{A_0} \cdot \frac{1}{E_{r_1}(C-1, A_0)}}}$$

2.2. WAITING SYSTEM OR ERLANG-C.

Question 16. Mathematically show that the Erlang-C function:

$$P_{ESP} = Er_2(C, A_O) = \frac{\frac{A_O^C}{C!}\frac{C}{C-A_O}}{\sum_{k=0}^{C}\frac{A_O^k}{k!} + \frac{A_O^{C+1}}{C!}\frac{1}{C-A_O}}, \textbf{ if } A_O < C$$

$$P_{ESP} = Er_2(C, A_O) = 1, \textbf{ if } A_O \geq C$$

can be written using the Erlang-B function as:

$$P_{ESP} = Er_2(C, A_O) = \frac{C \cdot Er_1(C, A_O)}{A_O \cdot Er_1(C, A_O) + C - A_O}, \textbf{ if } A_O < C$$

$$P_{ESP} = Er_2(C, A_O) = 1, \textbf{ if } A_O \geq C$$

$$\underline{Er_2(C,A_0)} = \frac{\frac{A_0^C \cdot C}{C!(C-A_0)}}{\sum_{K=0}^{C}\frac{A_0^K}{K!} + \frac{A_0^{C+1}}{C!}\frac{1}{C-A_0}} = \frac{C}{\frac{C!(C-A_0)}{A_0^C}\left(\sum_{K=0}^{C}\frac{A_0^K}{K!} + \frac{A_0^{C+1}}{C!}\frac{1}{C-A_0}\right)} =$$

$$= \frac{C}{(C-A_0)\cdot \underbrace{\frac{C!\sum_{K=0}^{C}\frac{A_0^K}{K!}}{A_0^C}}_{\frac{1}{E_{R1}(C,A_0)}} + \frac{\cancel{C!}\cancel{(C-A_0)}\cdot A_0^{C+1}}{A_0^C\cdot\cancel{C!}\cdot\cancel{(C-A_0)}}} = \frac{C}{\frac{(C-A_0)}{E_{R1}(C,A_0)} + A_0} \cdot \frac{\overbrace{E_{R1}(C,A_0)}^{\text{añadimos}}}{E_{R1}(C,A_0)} =$$

$$= \frac{C \cdot E_{R1}(C,A_0)}{\frac{(C-A_0)\cdot \cancel{E_{R1}(C,A_0)}}{\cancel{E_{R1}(C,A_0)}} + A_0 \cdot E_{R1}(C,A_0)} = \underline{\underline{\frac{C \cdot E_{R1}(C,A_0)}{(C-A_0) + A_0 \cdot E_{R1}(C,A_0)}}}$$

NOTA: El estudio teórico previo de la práctica 2 se encuentra detallado en el boletín de prácticas. Este formulario es solamente para completar las soluciones y poder entregarlo al profesor al inicio de la práctica.