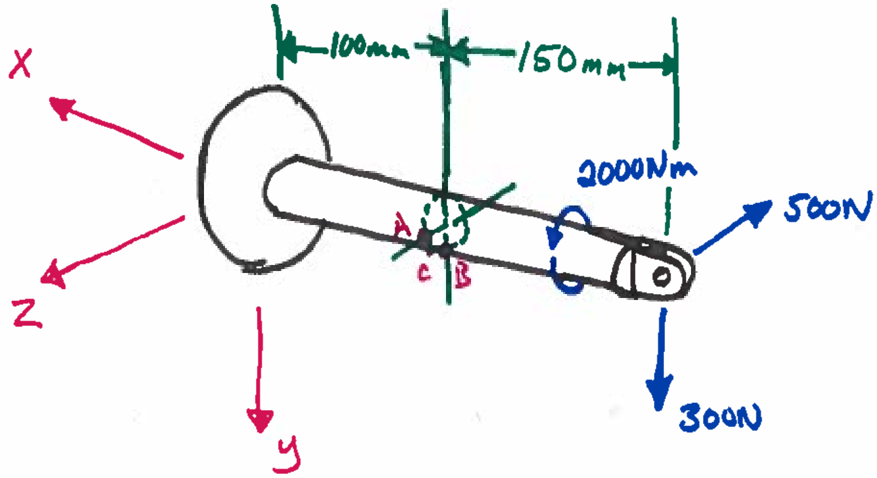
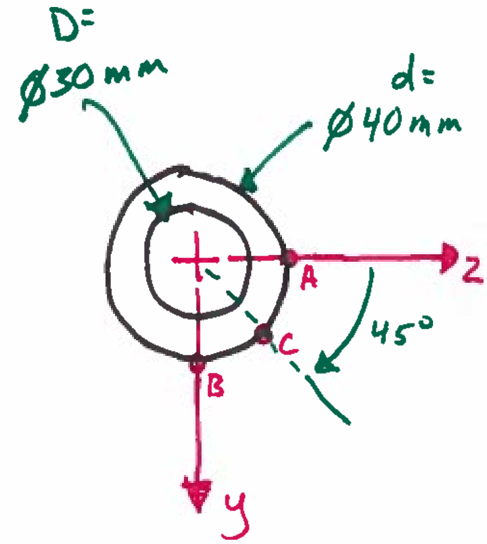
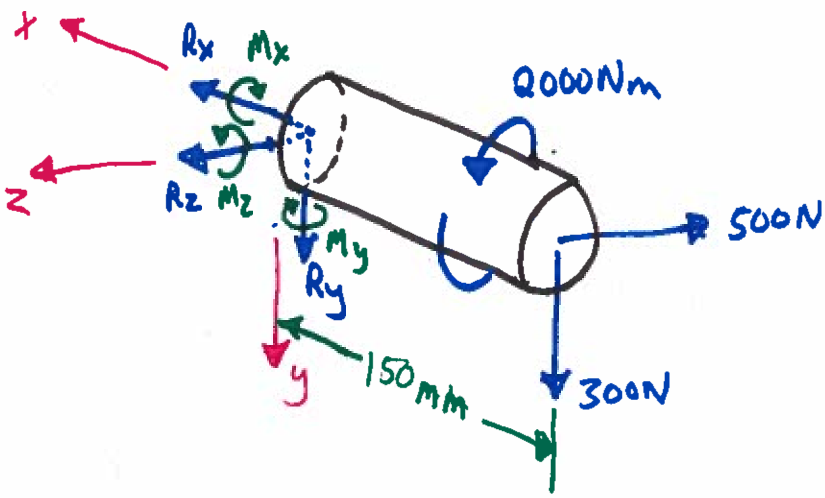
Example: multi-axial stress





Free body diagram



All reaction loads are defined in the positive sense wrt to the xyz coordinates.

$\Sigma F_x = 0$

$R_x = 0$ axial (normal)

$R_y + 300N = 0$

$R_y = -300N$ shear

$R_z - 500N = 0$

$R_z = 500N$ Shear

$\Sigma M = 0$

about the reaction side centroid

$M_x - 2000Nm = 0$

$M_x = 2000Nm$ shear from torsion

$M_y - (500N)(0.15m) = 0$

$M_y = 75Nm$ transverse shear + bending normal

$M_z - (300N)(0.15m) = 0$

$M_z = 45Nm$ transverse shear + bending normal

axial load

no axial load ($R_x=0$), so no contribution to stress state

$\sigma_x^A = 0$

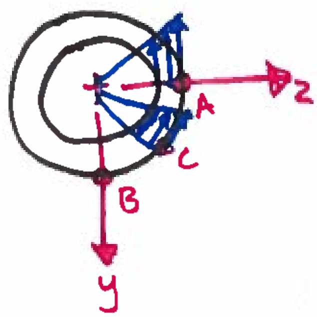
$\sigma_x^B = 0$

$\sigma_x^C = 0$

torsion about x

$M_x = 2000\ \text{Nm}$

$J = \frac{\pi}{32}(D^4 - d^4)$

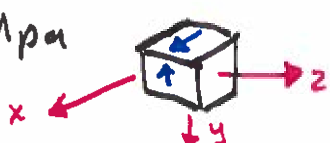


Point A

$$\tau_{xy}^A = -\frac{M_x\ D/2}{J} = -\frac{M_x\ D/2}{\frac{\pi}{32}(D^4-d^4)}$$

$$\tau_{xy}^A = -\frac{(2000\text{Nm})(0.04\text{m})(16)}{(0.04^2 - 0.03^2)}$$

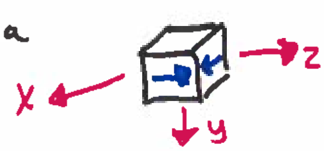
$$\tau_{xy}^A = -1.83\ \text{Mpa}$$



Point B

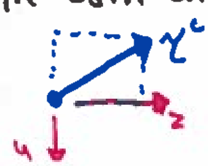
Point B has equivalent magnitude as A but is in the z direction

$$\tau_{xz} = 1.83\ \text{Mpa}$$



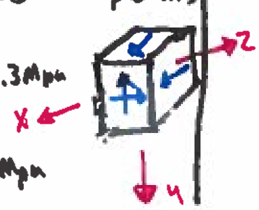
Point C

Has same magnitude but has components in both directions.



$$\tau_{xy}^c = -\frac{\tau^c}{\sqrt{2}} = -1.3\text{Mpa}$$

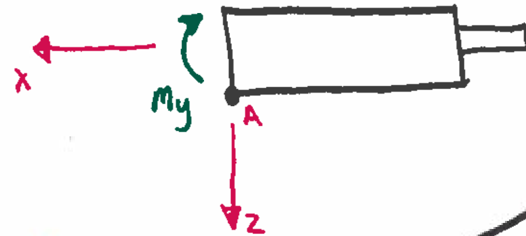
$$\tau_{xz}^c = \frac{\tau^c}{\sqrt{2}} = 1.3\text{Mpa}$$

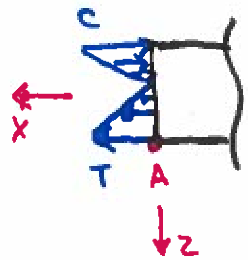


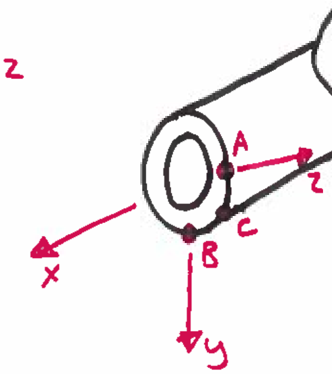
bending about y

normal stress

$M_y = 75\ \text{Nm}$







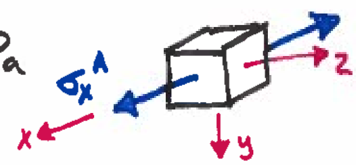
Point A

A is in normal tension due to bending about y.

$$\sigma_x^A = \frac{M_y\ D/2}{I_y} = \frac{M_y\ D}{\frac{2\pi}{64}(D^4-d^4)}$$

$$\sigma_x^A = \frac{32 M_y D}{\pi(D^4-d^4)} = \frac{32(75\text{Nm})(0.04\text{m})}{\pi(0.04\text{m}^4 - 0.03\text{m}^4)}$$

$$\sigma_x^A = 44\ kPa$$



Point B

Point B is on the neutral axis.

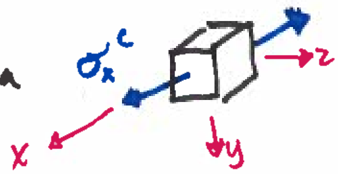
$$\sigma_x^B = 0$$

Point C

Point C is also in normal tension but the magnitude is less than A.

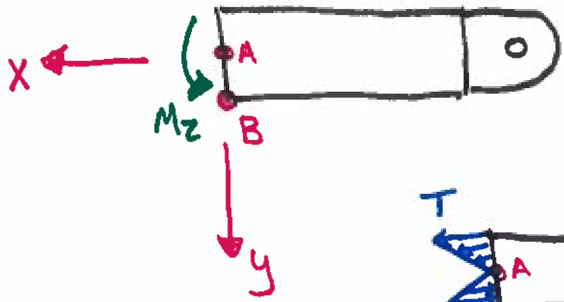
$$\sigma_x^c = \frac{M_y\ D/2\sqrt{2}}{I_y} = \frac{M_y D 32}{\sqrt{2}(D^4-d^4)\pi}$$

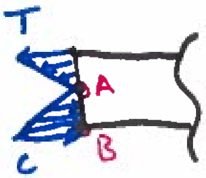
$$\sigma_x^c = 31\ KPa$$



bending about z

$M_z = 45 Nm$





Point A

Point A is on the neutral axis.

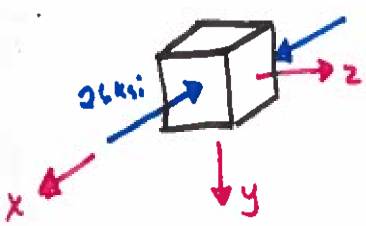
$$\sigma_x^A = 0$$

Point B

Point B is in maximum compression.

$$\sigma_x^B = -\frac{M_z\ D/2}{\frac{\pi}{64}(D^4-d^4)} = -\frac{32\, M_z\, D}{\pi(D^4-d^4)}$$

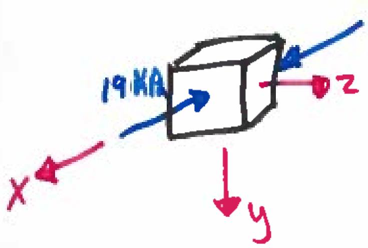
$$\sigma_x^B = -\frac{32(45Nm)(0.04m)}{\pi(0.04^4-0.03^4)} = -26\ KPa$$



Point C

Also in compression.

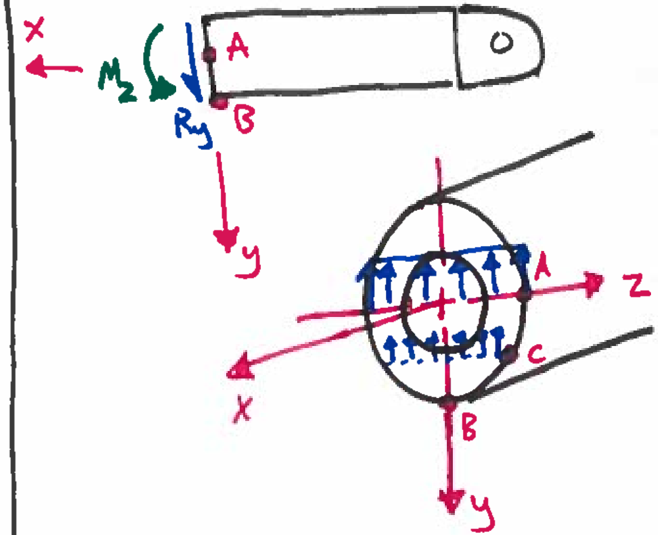
$$\sigma_x^C = -\frac{M_z\, D\, 32}{\sqrt{2}(D^4-d^4)\pi} = -19\ KPa$$



transverse shear along y

a moment is present so the transverse shear eq $\frac{VQ}{Ib}$ must be used

$R_y = -300N$



Point A

$$\tau_{xy}^A = \frac{VQ}{Ib} = \frac{V\bar{y}'A'}{Ib}$$

$V = R_y = -300N$

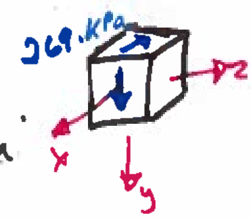
$I = \frac{\pi}{64}(D^4-d^4)$

$b^A = D$

$$\bar{y}' = \frac{4}{3\pi}\frac{(R^3-r^3)}{(R^2-r^2)}$$

$$A' = \frac{\pi(R^2-r^2)}{2}$$

$$\tau_{xy}^A = -\ 269\ KPa$$



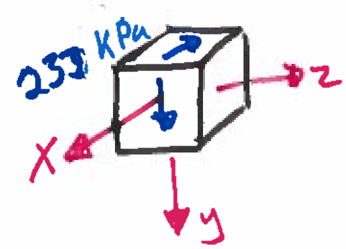
Point B

$$\tau_{xy}^B = 0$$

Point C

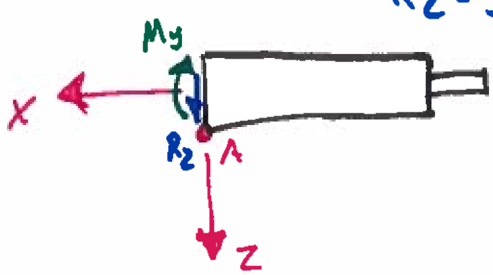
$b^c = \frac{D}{\sqrt{2}}$ $\quad A' = \frac{R^2}{4}(\pi-2)$ $\quad \bar{y}' = \frac{4R\sin^3(\frac{\pi}{4})}{3(\frac{\pi}{2}-1)}$

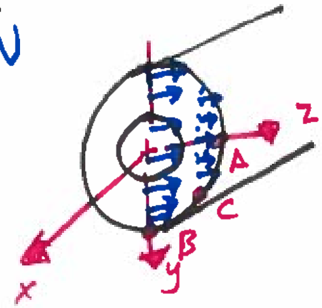
$$\tau_{xy}^C = -\ 233\ KPa$$



transverse along z

$R_z = 500N$





Point A

$\tau_{xz}^{A} = 0$

Point B

$$\tau_{xz}^{B} = \frac{V\bar{y}'A'}{I\,b} \qquad V = 500N \qquad b^{B} = D \qquad I = \frac{\pi}{64}(D^4 - d^4)$$

$$\bar{y}' = \frac{4}{3\pi}\frac{(R^3 - r^3)}{(R^2 - r^2)} \qquad A' = \frac{\pi(R^2 - r^2)}{2}$$

$\tau_{xz}^{B} = 449\ kPa$

Point C

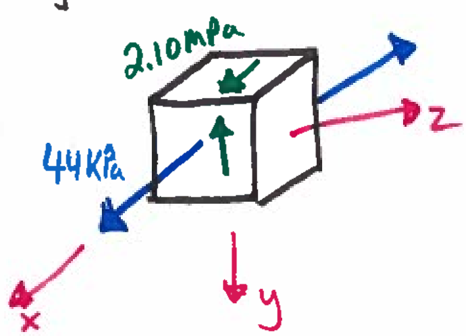
$\tau_{xz}^{C} = 388\ KPa$ follow same pattern in shear along y section.

Point A

$\sigma_x^A = 44\,kPa$

$\tau_{xy}^{A} = -1.83\ MPa - 269\,KPa$

$\tau_{xy}^{A} = -2.10\,MPa$

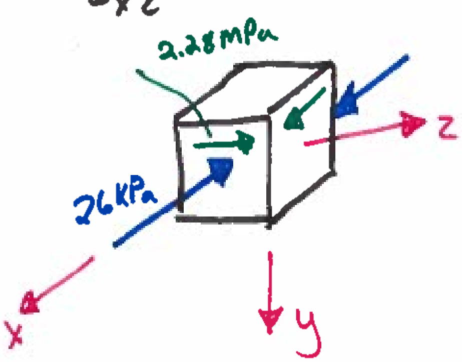


Point B

$\sigma_x^B = -26\,KPa$

$\tau_{xz}^{B} = 1.83MPa + 449\,kPa$

$\tau_{xz}^{B} = 2.28\ MPa$



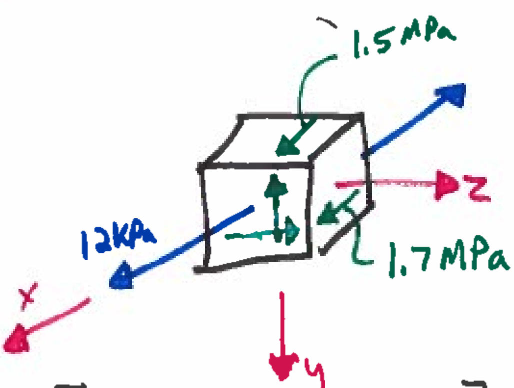
Point C

$\sigma_x^C = 31KPa - 19KPa$

$\sigma_x^C = 12\ KPa$

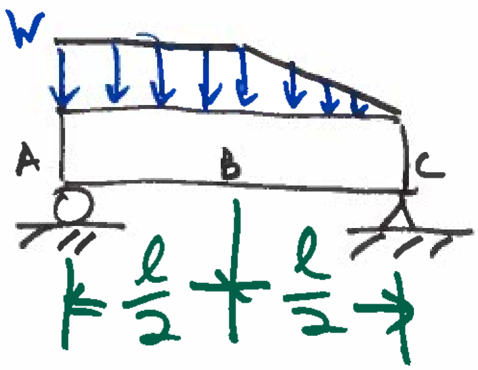
$\tau_{xy}^{C} = -1.3Mpa - 233KPa$
$= -1.53Mpa$

$\tau_{xz}^{C} = 1.3Mpa + 388KPa$
$= 1.7\ Mpa$

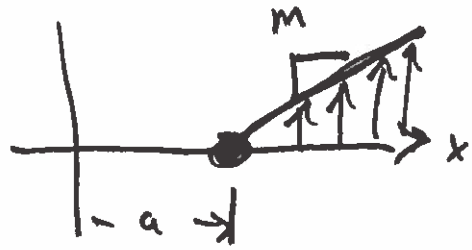


$$\sigma^A = \begin{bmatrix} 44KPa & -2.1MPa & 0 \\ -2.1MPa & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \sigma^B = \begin{bmatrix} -26KPa & 0 & 2.28MPa \\ 0 & 0 & 0 \\ 2.28MPa & 0 & 0 \end{bmatrix}$$

$$\sigma^C = \begin{bmatrix} 12KPa & -1.53MPa & 1.7MPa \\ -1.53MPa & 0 & 0 \\ 1.7MPa & 0 & 0 \end{bmatrix}$$

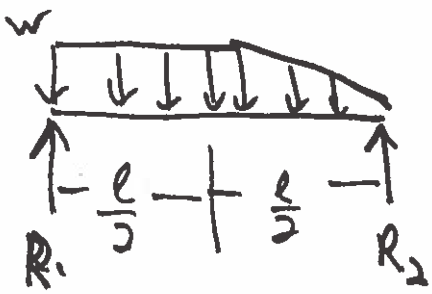


Find singularity functions for $q(x), V(x), M(x), \theta, y$.



$m\langle x-a\rangle^{1}$

FBD



$$q(x) = R_1\langle x\rangle^{-1} - w\langle x\rangle^{0} + \frac{2w}{\ell}\langle x-\tfrac{\ell}{2}\rangle^{1} + R_2\langle x-\ell\rangle^{-1}$$

$$V(x) = R_1\langle x\rangle^{0} - w\langle x\rangle^{1} + \frac{2w}{2\ell}\langle x-\tfrac{\ell}{2}\rangle^{2} + R_2\langle x-\ell\rangle^{0}$$

$$M(x) = R_1\langle x\rangle^{1} - \frac{w}{2}\langle x^2\rangle + \frac{w}{3\ell}\langle x-\tfrac{\ell}{2}\rangle^{3} + R_2\langle x-\ell\rangle^{1}$$

$$V(\ell^+)=0, \quad M(\ell^+)=0$$

$$V(\ell^+)=0= R_1 - w\ell + \frac{w\ell}{4} + R_2$$

$$M(\ell^+)=0= R_1\ell - \frac{w}{2}\ell^2 + \frac{w}{3\ell}\left(\frac{\ell}{2}\right)^3 + 0$$

$$\boxed{R_1 = \frac{11}{24}w\ell \qquad R_2 = \frac{7}{24}w\ell}$$

$$EI\,\theta(x) = \frac{11}{224} w\ell x^2 + \frac{W}{6}x^3 + \frac{W}{12\ell}\langle x - \frac{\ell}{2}\rangle^4 + C_1$$

$$EI\,y(x) = \frac{11}{144} w\ell x^3 - \frac{W}{24}x^4 + \frac{W}{60\ell}\langle x - \frac{\ell}{2}\rangle^5 + C_1 x + C_2$$

$y(0) = 0 \qquad y(\ell) = 0$

$C_2 = 0 \qquad C_1 = -\frac{11}{144} w\ell^3 + \frac{W}{24}\ell^3 - \frac{w\ell^3}{3840}$

$y_{AB} = f(x) \quad 0 < x < \frac{\ell}{2}$

$y_{BC} = f(x) \quad \frac{\ell}{2} < x < \ell$

$y_{max_{AB}} \Rightarrow \theta_{AB}(x) = 0$