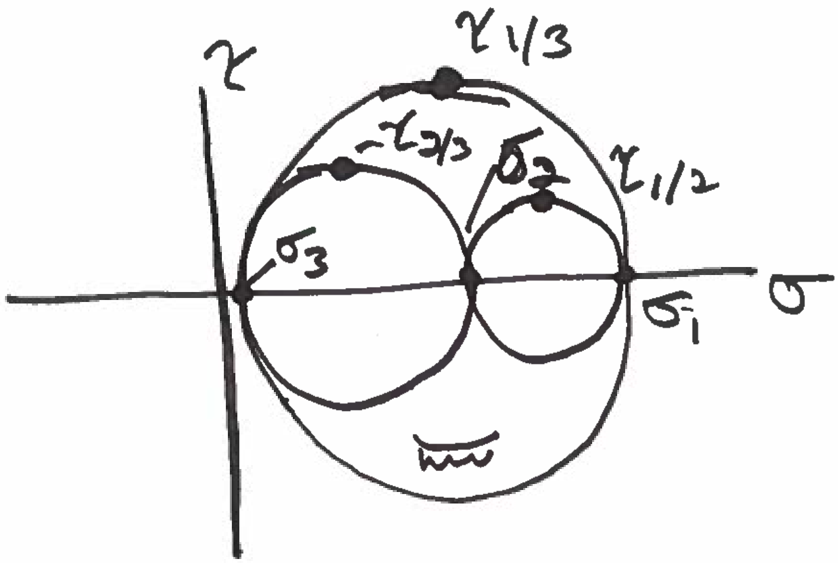
EME 150A Lecture notes Oct 12, 2015

Triaxial State of Stress

$$\underline{\sigma}^3 - (\sigma_x + \sigma_y + \sigma_z)\underline{\sigma}^2 + (\sigma_x\sigma_y + \sigma_x\sigma_z + \sigma_y\sigma_z - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{xz}^2)\underline{\sigma} - (\sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{yz}\tau_{xz} - \sigma_x\tau_{xz}^2 - \sigma_y\tau_{xz}^2 - \sigma_z\sigma_{xy}^2) = 0$$

3D Mohr's Circle

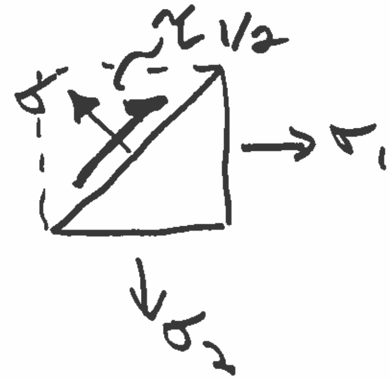


$$\tau_{max} = \tau_{1/3}$$

$$\tau_{1/3} = \frac{\sigma_1 - \sigma_3}{2}$$

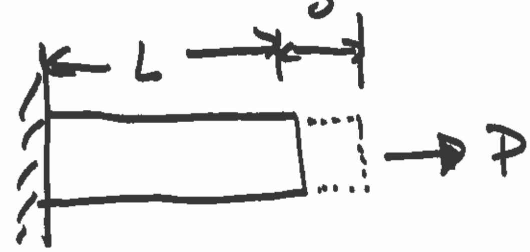
$$\tau_{1/2} = \frac{\sigma_1 - \sigma_2}{2}$$

$$\tau_{2/3} = \frac{\sigma_2 - \sigma_3}{2}$$



Elastic Strain

Under an applied load a structural member will deform.



$$\epsilon = \frac{\delta}{L} = \frac{\text{change in length}}{\text{original length}}$$

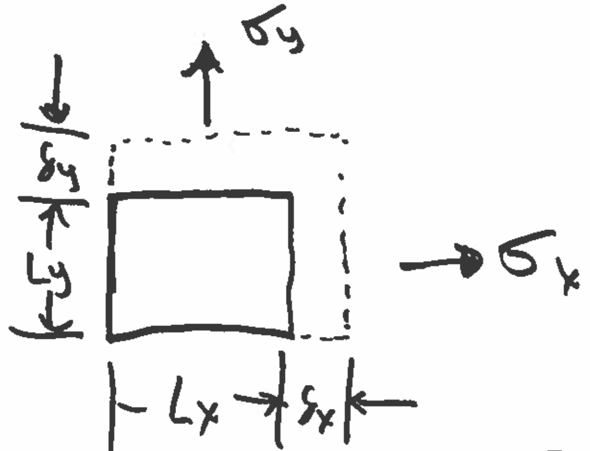
stress ⇒ nearly impossible to measure

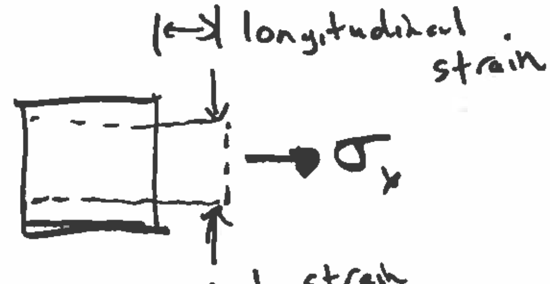
strain ⇒ measurable

↳ direction and intensity of deformation

Normal strain

$$\epsilon_x = \frac{\delta_x}{L_x}, \quad \epsilon_y = \frac{\delta_x}{L_x}$$



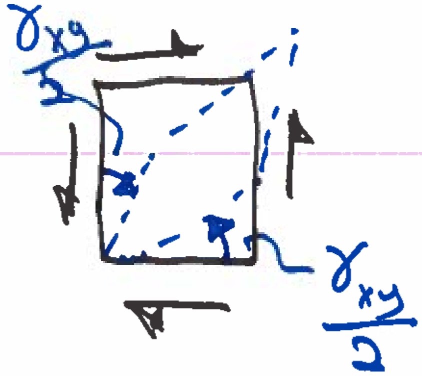


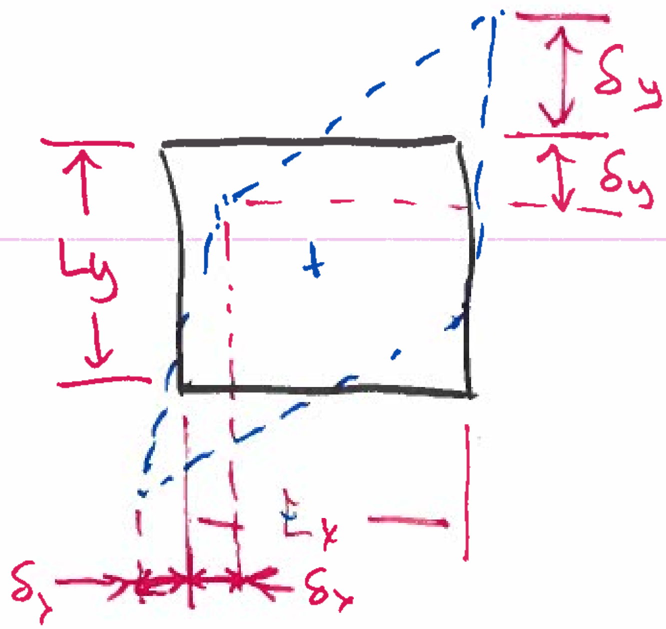
Poisson's ratio

$$\nu = \frac{\text{lat. strain}}{\text{long. strain}}$$

~~Shear~~

Shear strain





$$\gamma_{xy} = \frac{2\delta_x}{L_y} + \frac{2\delta_y}{L_x} \Rightarrow \frac{\gamma_{xy}}{2} = \frac{\delta_x}{L_y} + \frac{\delta_y}{L_x}$$

Angular distortion due to shear stress. Assume small angles.

Strain Tensor

$$\epsilon_{ij} = \begin{bmatrix} \epsilon_x & \frac{\gamma_{xy}}{2} & \frac{\gamma_{xz}}{2} \\ \frac{\gamma_{xy}}{2} & \epsilon_y & \frac{\gamma_{yz}}{2} \\ \frac{\gamma_{zx}}{2} & \frac{\gamma_{zy}}{2} & \epsilon_z \end{bmatrix}$$

Principal strains (plane strain)

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left[\frac{(\epsilon_x - \epsilon_y)}{2}\right]^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\left(\frac{\gamma}{2}\right)_{max} = \frac{\epsilon_1 - \epsilon_2}{2}$$

Hooke's Law

relates stress & strain

simple case : $\sigma = E \epsilon$

Elasticity : material recovers its original shape once load is removed

E: modulus of elasticity (Young's modulus)

G: shear modulus of elasticity

ν: poisson's ratio

$$E = 2G(1+\nu)$$

General Form of Hooke's

$$\epsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$$

$$\epsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]$$

$$\epsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]$$

$$\gamma_{xy} = \frac{1}{G}\tau_{xy} \qquad \gamma_{xz} = \frac{1}{G}\tau_{xz}$$

$$\gamma_{yz} = \frac{1}{G}\tau_{yz}$$

Stress Type	Principal Strains	Principal Stresses
Uniaxial	$\epsilon_1 = \frac{\sigma_1}{E}$ $\epsilon_2 = -\nu\epsilon_1$ $\epsilon_3 = -\nu\epsilon_1$	$\sigma_1 = E\epsilon_1$ $\sigma_2 = 0$ $\sigma_3 = 0$
Biaxial	$\epsilon_1 = \frac{\sigma_1}{E} - \frac{\nu\sigma_2}{E}$ $\epsilon_2 = \frac{\sigma_2}{E} - \frac{\nu\sigma_1}{E}$ $\epsilon_3 = \frac{-\nu\sigma_1}{E} - \frac{\nu\sigma_2}{E}$	$\sigma_1 = \frac{E(\epsilon_1 + \epsilon_2)}{1-\nu^2}$ $\sigma_2 = \frac{E(\epsilon_2 + \nu\epsilon_1)}{1-\nu^2}$ $\sigma_3 = 0$
Triaxial	$\epsilon_1 = \frac{\sigma_1}{E} - \frac{\nu\sigma_2}{E} - \frac{\nu\sigma_3}{E}$ $\epsilon_2 = \frac{\sigma_2}{E} - \frac{\nu\sigma_1}{E} - \frac{\nu\sigma_3}{E}$ $\epsilon_3 = \frac{\sigma_3}{E} - \frac{\nu\sigma_1}{E} - \frac{\nu\sigma_2}{E}$	$\sigma_1 = \frac{E\epsilon_1(1-\nu) + \nu E(\epsilon_2 + \epsilon_3)}{1-\nu-2\nu^2}$ $\sigma_2 = \frac{E\epsilon_2(1-\nu) + \nu E(\epsilon_1 + \epsilon_3)}{1-\nu-2\nu^2}$ $\sigma_3 = \frac{E\epsilon_3(1-\nu) + \nu E(\epsilon_1 + \epsilon_2}{1-\nu-2\nu^2}$

General Form of Hooke's Law

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix}$$

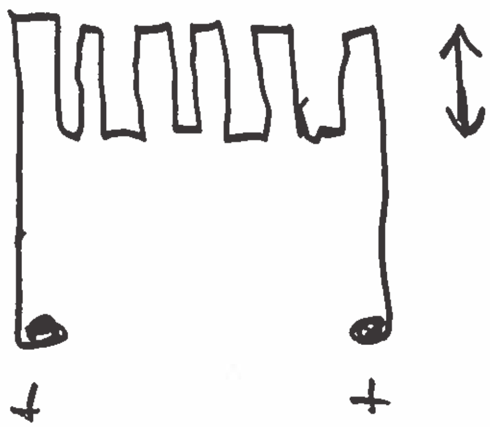
$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & -\nu & 0 & 0 & 0 \\ -\nu & -\nu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(1+\nu) & 0 & 0 \\ 0 & 0 & 0 & 0 & 2(1+\nu) & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(1+\nu) \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{bmatrix}$$

Show old table 2-1

Strain Gauges

- Change in electrical resistance to measure strain.

  change in resistance proportional to change in strain



Gauge Factor

$$GF = \frac{\Delta R / R_G}{\epsilon}$$

ΔR: change in resistance

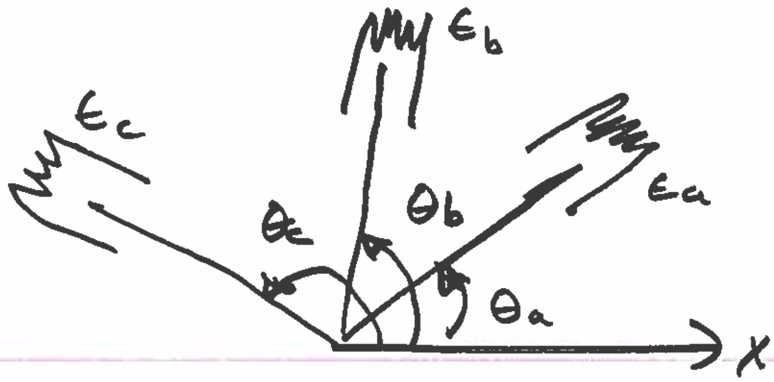
R_G: nominal resistance

ϵ: strain

Plane Strain Measurement

Strain

Rosettes



$$\epsilon_a = \epsilon_x \cos^2\theta_a + \epsilon_y \sin^2\theta_a + \gamma_{xy} \sin\theta_a \cos\theta_a$$

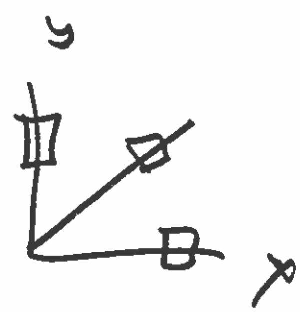
$$\vdots$$

$$\underbrace{\epsilon_c = \epsilon_x \cos^2\theta_c + \epsilon_y \sin^2\theta_c + \gamma_{xy} \sin\theta_c \cos\theta_c}$$

solve for

$\epsilon_x, \epsilon_y, \gamma_{xy}$

$\theta_a = 0, \ \theta_b = 45°, \ \theta_c = 90°$

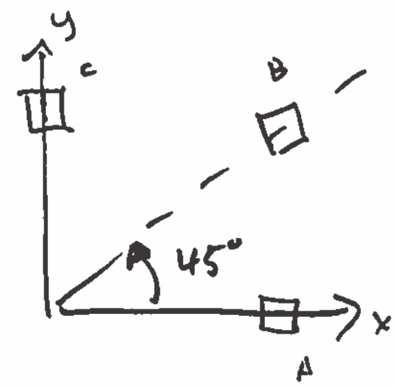


$\epsilon_x = \epsilon_a$

$\epsilon_y = \epsilon_c$

$\gamma_{xy} = 2\epsilon_b - (\epsilon_a + \epsilon_c)$

Example



$E = 200\,GPa$

$\nu = 0.3$

$\epsilon_x = \epsilon_a$

$\epsilon_y = \epsilon_c$

$\gamma_{xy} = 2\epsilon_b - (\epsilon_a + \epsilon_c)$

What are the principal ~~strains~~ stresses if strain gauge readings are $\epsilon_a = 60E\text{-}6$, $\epsilon_b = -75E\text{-}6$, $\epsilon_c = 232E\text{-}6$?

State of strain

$\epsilon_x = 60E\text{-}6$

$\epsilon_y = 232E\text{-}6$

$\gamma_{xy} = 2(-75E\text{-}6) - (60E\text{-}6 + 232E\text{-}6) = -0.000442$

$= -442E\text{-}6$

Principal Strains

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$\epsilon_{1,2} =$

$\epsilon_1 = 383E\text{-}6$

$\epsilon_2 = -91E\text{-}6$

Principal stresses

$$\sigma_1 = \frac{E(\epsilon_1 + \nu\epsilon_2)}{1-\nu^2} = 78\,MPa$$

$\boxed{78\,MPa}$

$$\sigma_2 = \frac{E(\epsilon_2 + \nu\epsilon_1)}{1-\nu^2} = \boxed{5\,MPa}$$