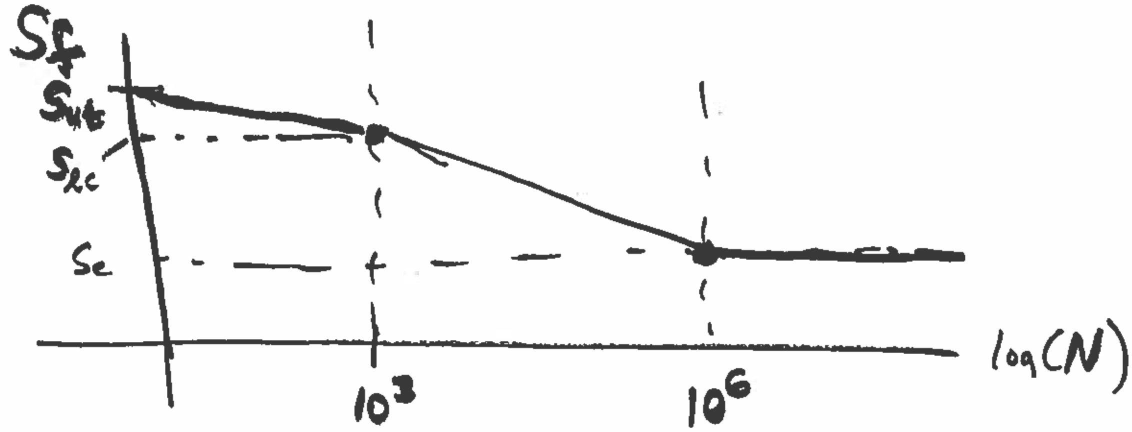
S-N (Ferrous)



① find low cycle line

$$S_{lc} = f\, S_{ut}$$

f ← Fig 6-18

② Endurance limit

$$S_e = \underbrace{\qquad}_{\text{Modifying param}} S_e'$$

Ferrous

$$S_e' = \begin{cases} 0.5\, S_{ut} & S_{ut} < 200 \text{ kpsi} \\ 100 \text{ kpsi}, \; 700 \text{ MPa} & S_{ut} > 200 \text{ kpsi} \end{cases}$$

③ High Cycle fatigue

$$S_f = a N^b$$

$$a = \frac{(f\, S_{ut})^2}{S_e}$$

$$b = -\frac{1}{3}\log\left(\frac{f\, S_{ut}}{S_e}\right)$$

Stress Concentration Factors in Fatigue

- Static ⟹ stress concentration factors (for brittle materials)

  K_t, K_{ts}

- Fatigue: brittle and ductile materials

- Different Materials have different sensitives to notches (stress con.)

- Fatigue stress concentration

  $$\sigma_{max} = K_f \sigma_0 \quad \text{and} \quad \tau_{max} = K_{fs} \tau_0$$

- Fatigue concentration factor are smaller than K_t, K_{ts}

- $$K_f = \frac{\text{max stress in a notched specimen}}{\text{stress in the notch-free specimen}}$$

- $$K_f = 1 + q(K_t - 1)$$

  q: notch sensitivity

Figs 6-20, 6-21 $\Rightarrow q$

- You can always use K_t and be conservative

- Cast irons $\Rightarrow q = 0.2$ for all grades

- For infinite life you can use $k_f = \dfrac{1}{K_f}$ equivalent to $K_f \sigma_o$

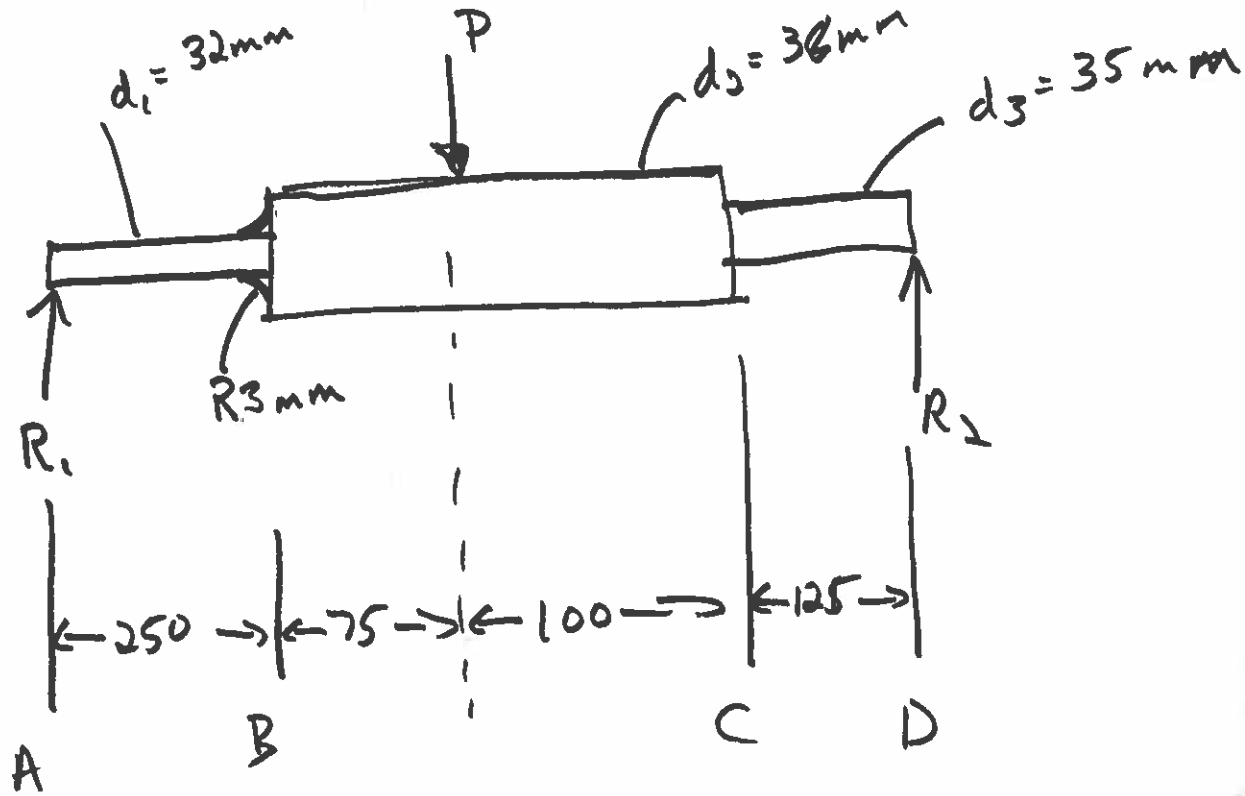
k_f • Marin • misc par

* Does not apply for finite life **

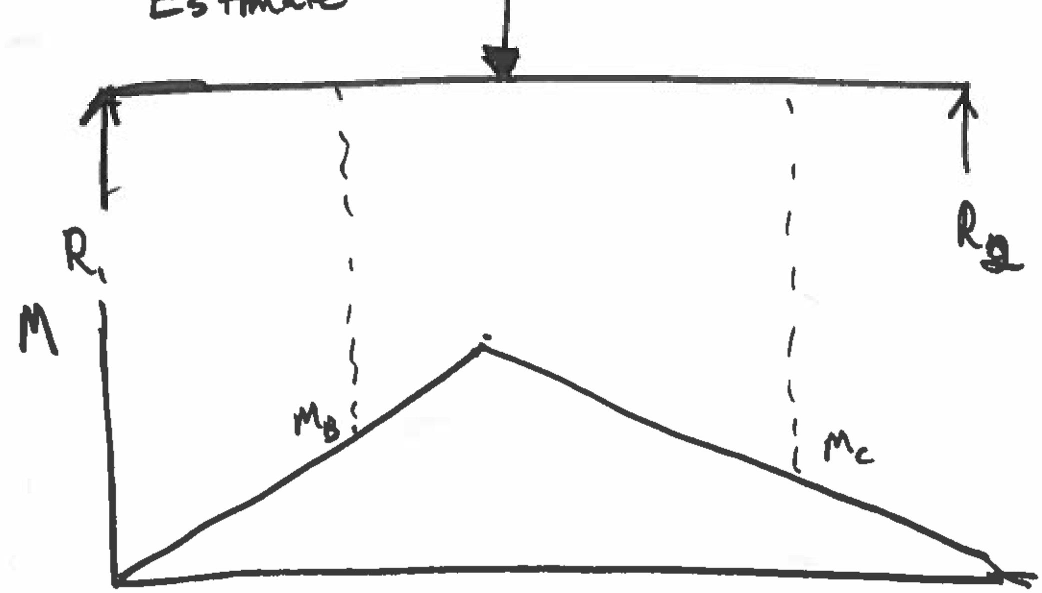
$$\underbrace{k_f = \frac{1}{K_f}}_{\text{Only for infinite life}} \qquad \underbrace{\sigma_{max} = K_f \sigma_o}_{\text{best to use this all the time}}$$

Example Rotating Beam simply supported at A + D with P = 6.8 kN

Material: CD 1050 steel



Goal: Estimate the life of the part.



Solution

$$\Sigma F = 0 = R_1 + P + R_2$$
$$\Sigma M_A = 0 = (550)R_2 - (325)P$$

$R_1 = 2.782\ kN$

$R_2 = 4.018\ kN$

$$M_B = 250 \cdot R_1 \Rightarrow M_B = 695.5\ Nm$$

$S_e' = 0.5\, S_{ut}$ $\qquad S_{ut} < 1400\ MPa$

$S_{ut} = 690\ MPa$, $S_y = 580\ MPa$, $\Rightarrow$ Table A-20

$$\boxed{S_e' = 345\ MPa}$$

Marin Parameters

Surface finish: K_a $\qquad$ Table-6-2

$$K_a = a\, S_{ut}^{\,b} = (4.51)(690)^{-.265} = K_a = .798$$

Size k_b

$d < 51\ mm$

$$K_b = \left(\frac{d_1}{7.62}\right)^{-0.107} = 0.858$$

other

$$K_c = K_d = K_e = K_f = 1$$

$$\boxed{S_e = K_a\, k_b\, S_e' = 236\ MPa}$$

Stress Concentration

$$\sigma_B = K_f \sigma_{B0}$$

$$K_f = 1 + q(K_t - 1)$$

Chart A-15-9

$$r/d = 3/32 = .093$$

$$D/d = \frac{38}{32} = 1.19$$

From Fig 6-20

$$q \approx 0.85$$

$$K_t \approx 1.65$$

$$\sigma_B = \left(1 + (0.85)[1.65 - 1]\right) 216.2 \text{ MPa}$$

$$\boxed{\sigma_B = 335.1 \text{ MPa}}$$

$$\sigma_B > S_e \quad + \quad \sigma_B < S_y$$

Fully reversed life High cycle regime.

$$\sigma_B = a N^b \Rightarrow N = \left(\frac{\sigma_B}{a}\right)^{1/b}$$

$$a = \frac{(f S_{ut})^2}{S_e} \qquad b = -\frac{1}{3}\log\left(\frac{f S_{ut}}{S_e}\right)$$

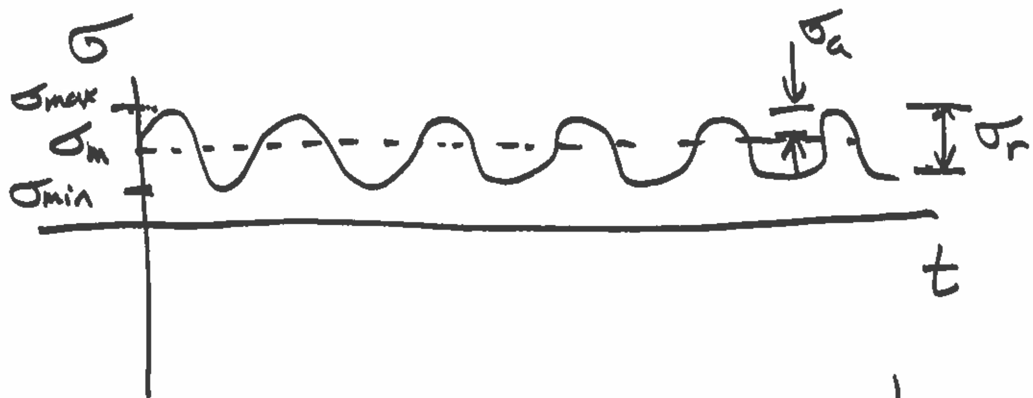
$$S_{ec} = f S_{ut} = (.85)(690 \text{ MPa}) = 582.4 \text{ MPa}$$

$$a = 1437 \text{ MPa} \qquad b = -0.1308$$

$$\boxed{N = 68 \times 10^3 \text{ cycles}}$$

Fluctuating Stress

When the mean stress is non-zero.



$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}, \quad \sigma_a = \left| \frac{\sigma_{max} - \sigma_{min}}{2} \right|$$

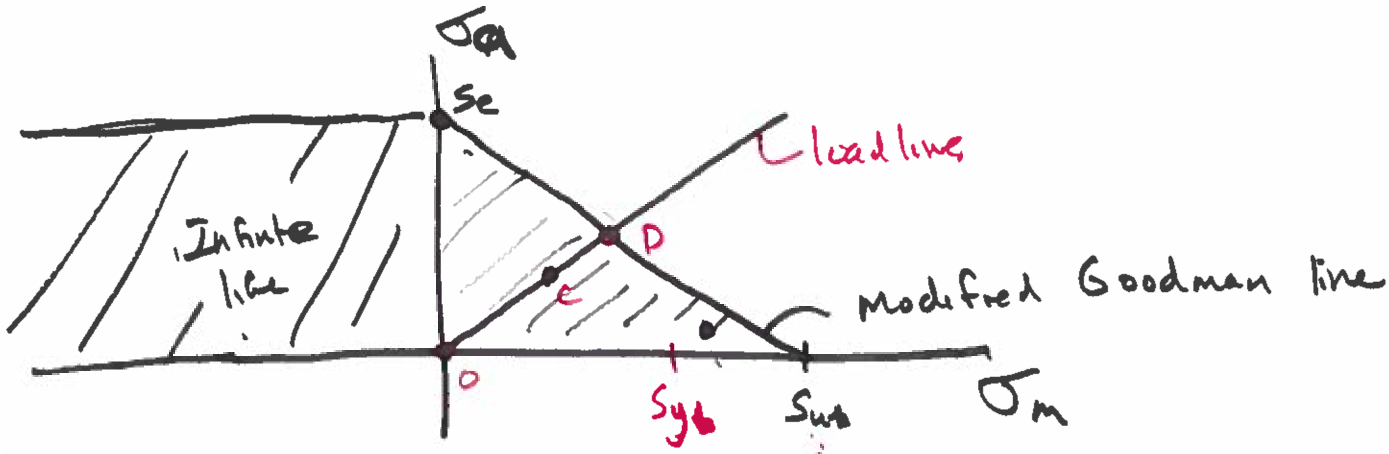
$$\sigma_r = |\sigma_{max} - \sigma_{min}| \qquad A = \frac{\sigma_a}{\sigma_m} \quad \left(\text{form load line}\right)$$

If $\sigma_m < 0$ (compressive): S_f is the same as the fully reversed case (but check yielding)

If $\sigma_m > 0$ (tensile): S_f is <u>less</u> than fully reversed case.

If $\sigma_a = 0 \Rightarrow$ the part fails ~~at~~ $\sigma_m = S_y$ or S_{ut}

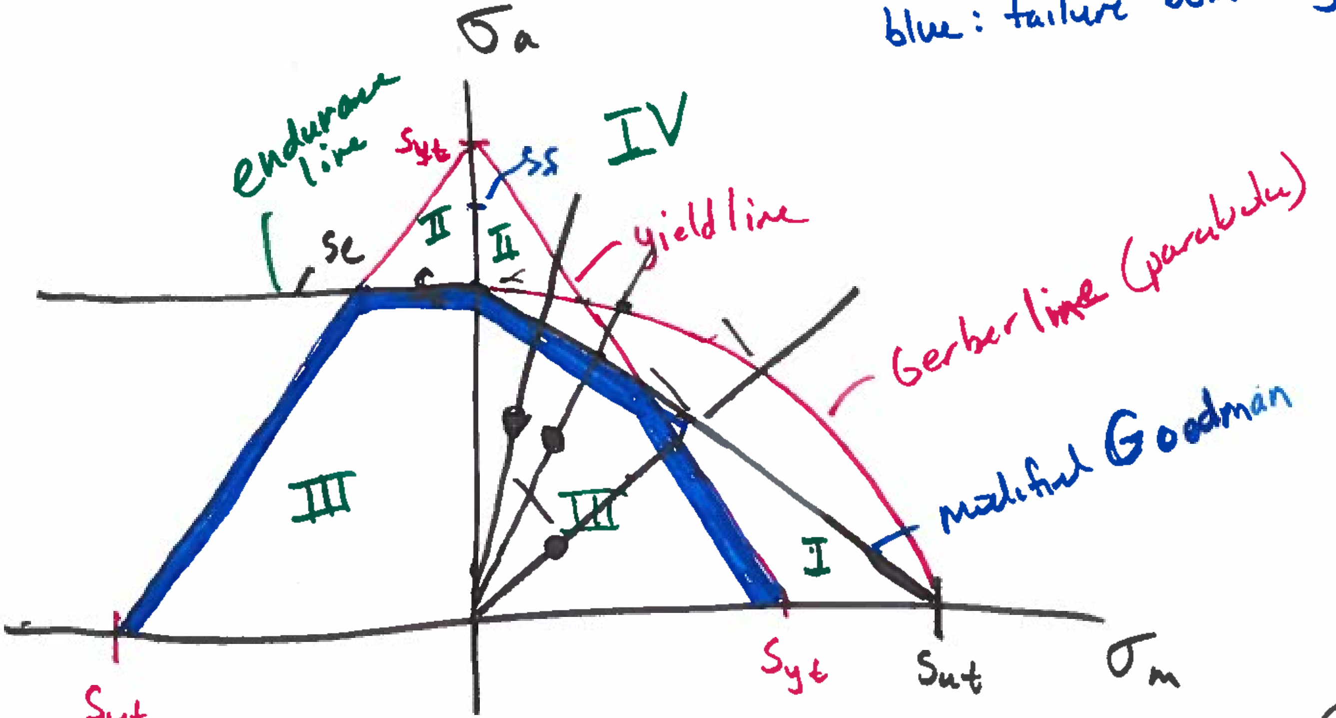
If $\sigma_m = 0 \Rightarrow$ fully reversed case $\Rightarrow$ failure $\sigma_a = S_e$ (S_e or S_f)



Design Relations

(1) $\dfrac{\sigma_a}{S_e} + \dfrac{\sigma_m}{S_{ut}} < 1$ for $\sigma_m > 0$

(2) $\dfrac{\sigma_a}{S_e} < 1$ for $\sigma_m < 0$



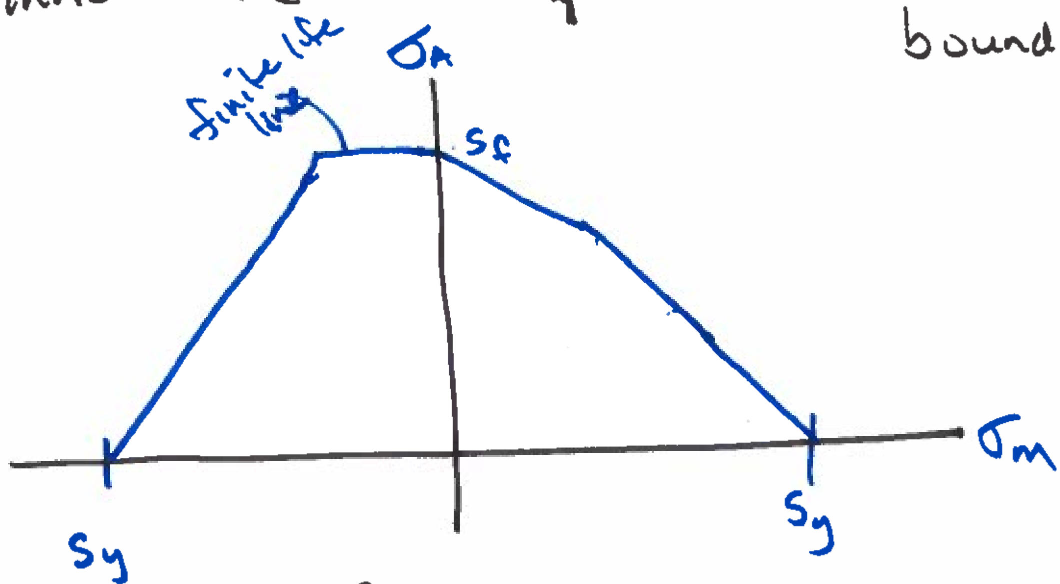
I - immediate failure due to yielding

II - finite life (fail by fatigue)

III - infinite life

IV - failure due to yielding, fracture, fatigue

Finite Life use S_f instead of S_e for specific boundary in zone II.



Gerber line

Relation: $\frac{\sigma_a}{S_e} + \left(\frac{\sigma_m}{S_{ut}}\right)^2 < 1$ for $\sigma_m > 0$

Factor of Safety

Static: $n = \frac{\text{load capacity}}{\text{actual load}} = \frac{S}{\sigma}$

σ_a and σ_m increase proportionally along the load line.

$$n = \frac{OC}{OP} = \frac{S_a}{\sigma_a} = \frac{S_m}{\sigma_m}$$

Goodman

$$n = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}\right)^{-1}$$

fatigue

yielding

$$n = \frac{S_y}{\sigma_a + \sigma_m}$$

Gerber

$$\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_{ut}}\right)^2 = 1$$

Example

1.5" diameter bar, CD 1050 steel

fluctuating load from 0 – 16000 lbs (tensile)

Assume $K_f = 1.85$ for 10^6 or longer life

Find n_{inf} using Goodman line

Solution

$S_e' = 0.5\, S_{ut}$

$S_{ut} = 100\ Kpsi$

$S_y = 84\ Kpsi$

$S_e' = 50\ Kpsi$

$S_e = K_a K_b K_c S_e'$

$K_d = K_e = K_f = 1$

$K_a = 0.797$

$K_b = 1$

$K_c = 0.85$

$S_e = 33.87\ Kpsi$

$$(\sigma_m)_o = \frac{F_m}{A} = \frac{8000}{\frac{\pi (1.5)^2}{4}} = 4.53\ Kpsi$$

$(\sigma_a)_o = \sigma_m = 4.53\ Kpsi$

$\sigma_a = K_f \sigma_{a_o} = 8.38\ Kpsi = \sigma_m$

Goodman line

$$n_{inf} = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}\right)^{-1}$$

$$\boxed{\begin{aligned} n_{inf} &= 3.02 \\ n_{yielding} &= \frac{S_y}{\sigma_a + \sigma_m} \Rightarrow 5.01 \end{aligned}}$$