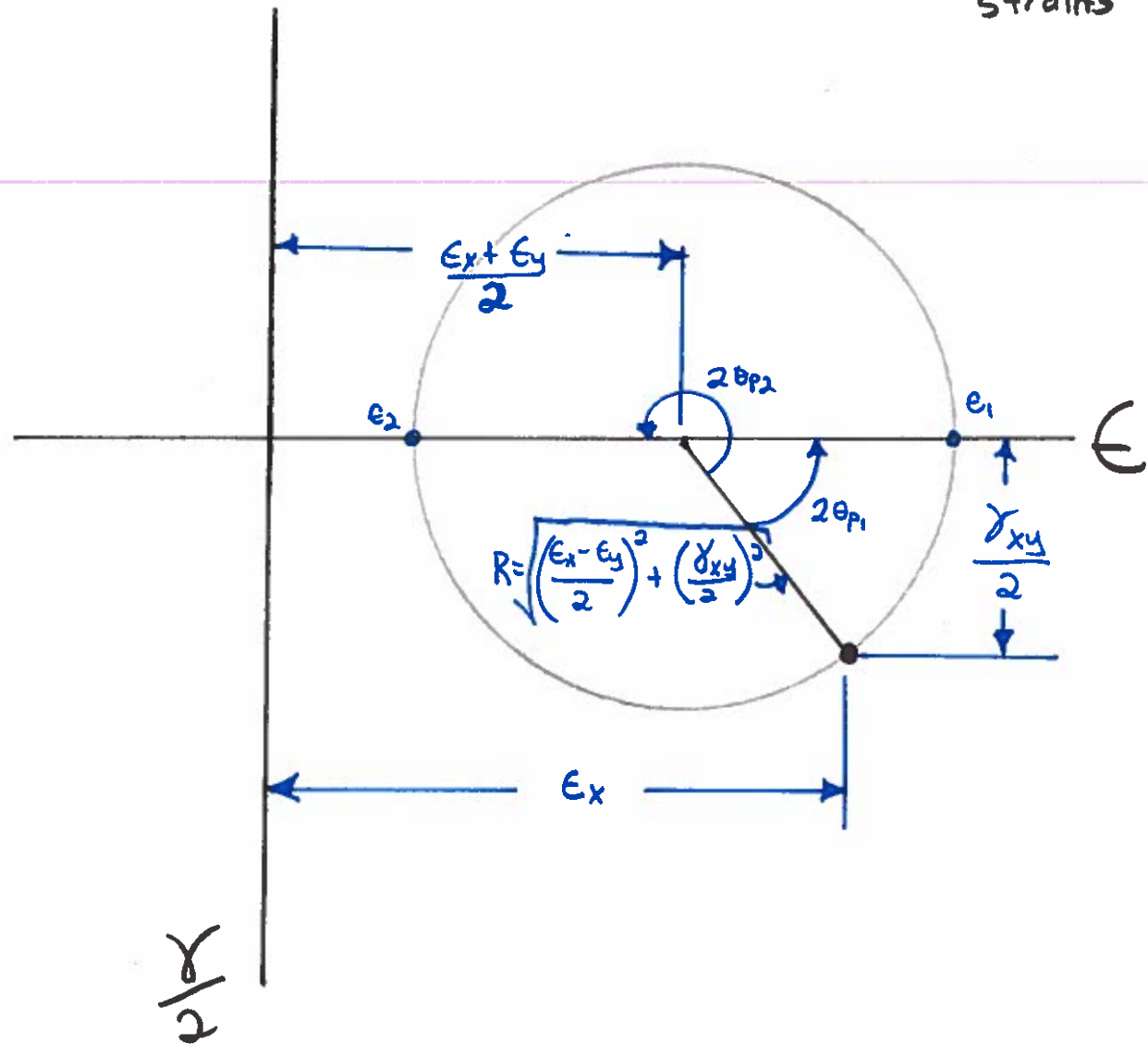


Mohr's Circle For Plane Strain

$\epsilon_1, \epsilon_2 \Rightarrow$ principal strains



Curved Beams in ~~Bents~~ Bending

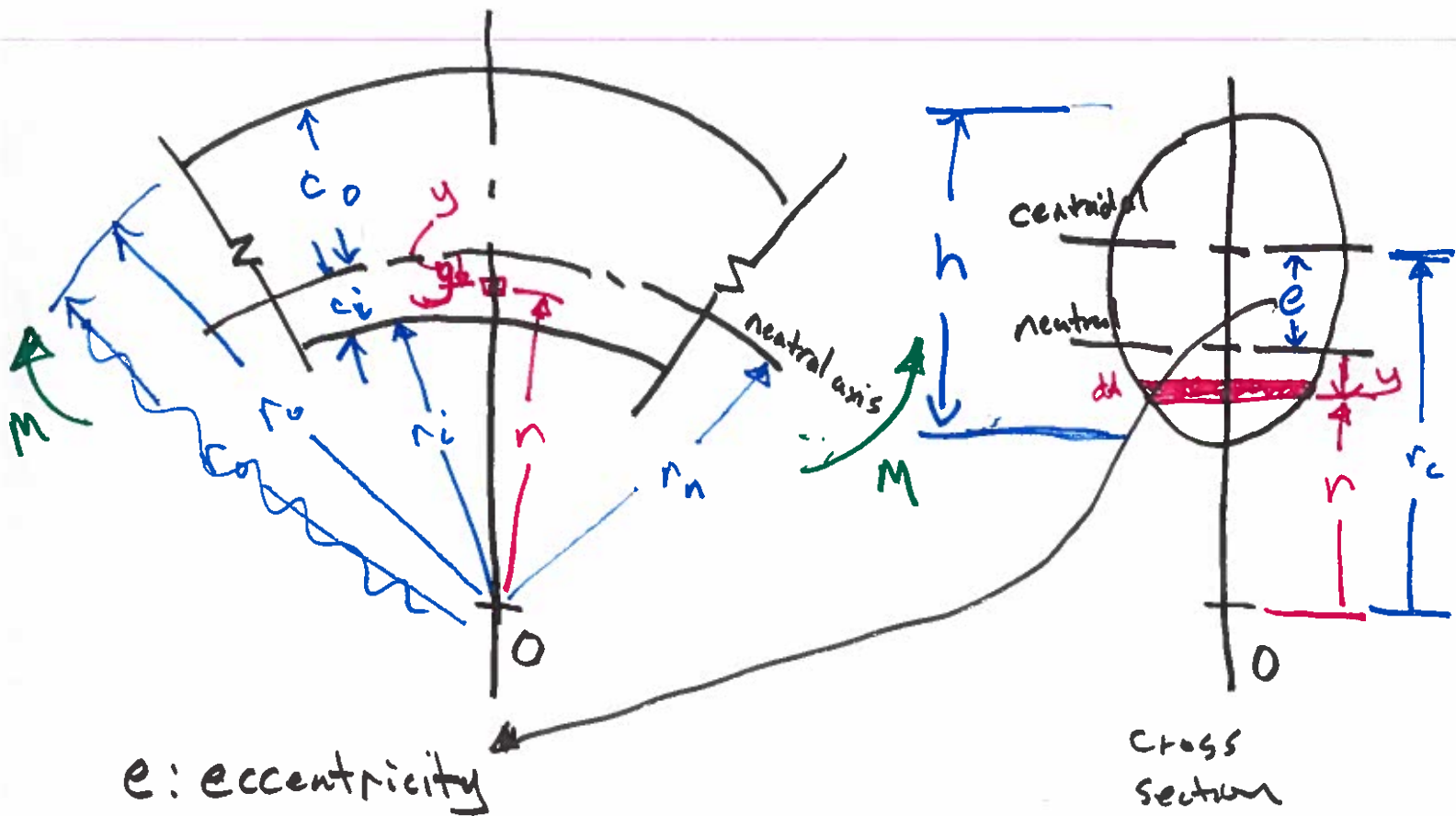
Can't use same analysis as straight beams!

Centroidal axis \neq neutral axis

Assumptions

- cross section has axis of symmetry in the plane of bending
- plane cross sections remain plane
- modulus of elasticity is same tension and compression

M : decrease the curvature



Neutral axis

$$r_n = \frac{A}{\int_{r_i}^{r_o} \frac{dA}{r}}$$

Centroidal axis

$$r_c = \frac{\int_{r_i}^{r_o} r dA}{A}$$

Normal Stress Distributions

$$\sigma(y) = \frac{M y}{A e (r_n - y)} \quad \left. \vphantom{\frac{M y}{A e (r_n - y)}} \right\} \begin{array}{l} \text{hyperbolic function} \\ \text{in } y \end{array}$$

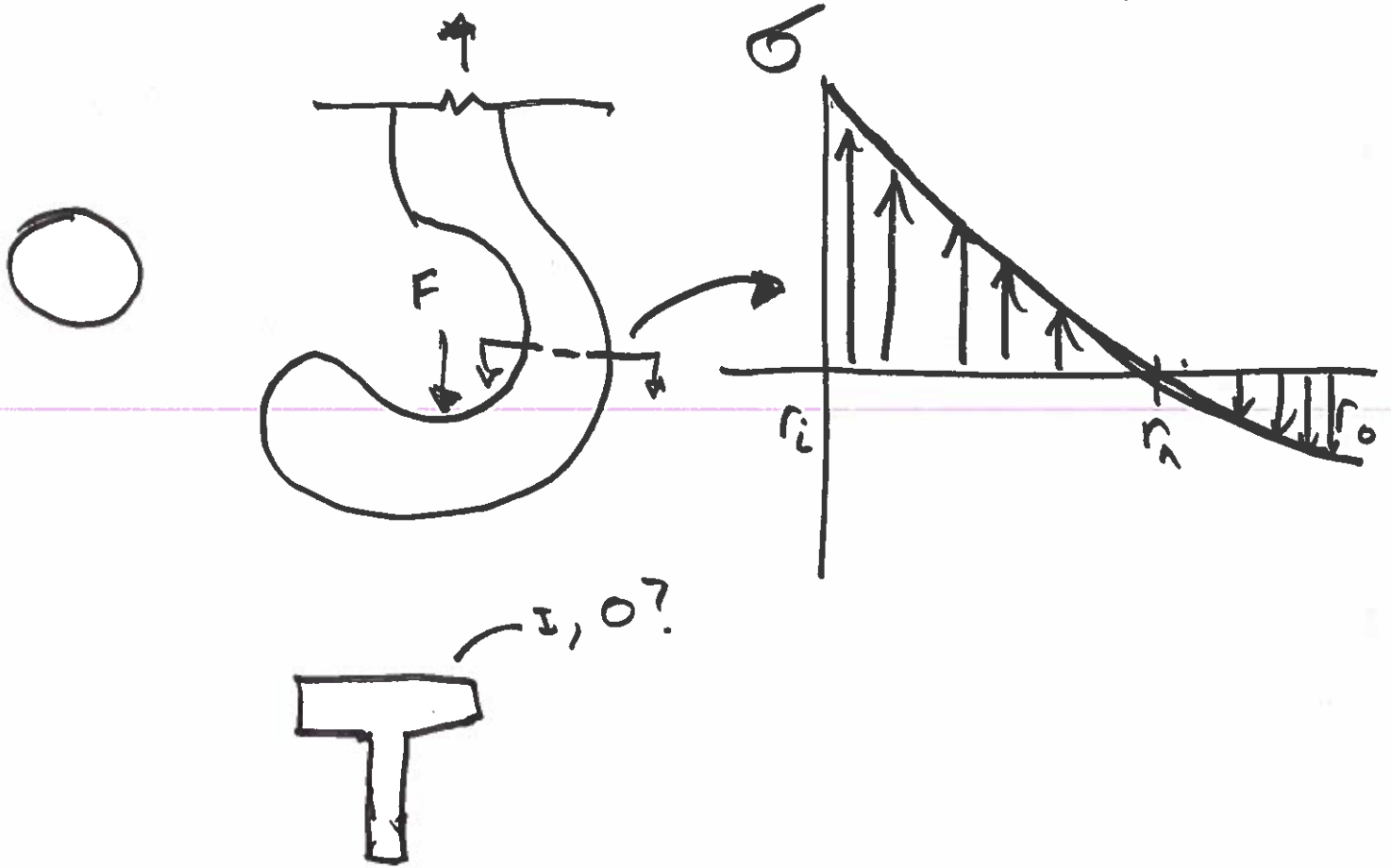
$$\sigma_i = \frac{\boxed{M C_i} M e_i}{A e r_i}$$

$$\sigma_o = - \frac{M C_o}{A e r_o}$$

+ : tension
- : compression

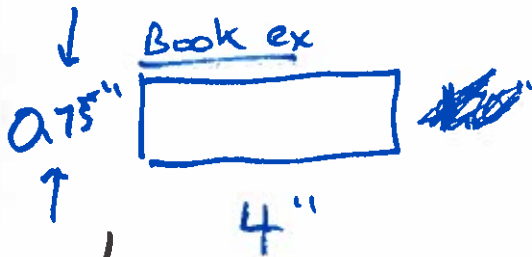
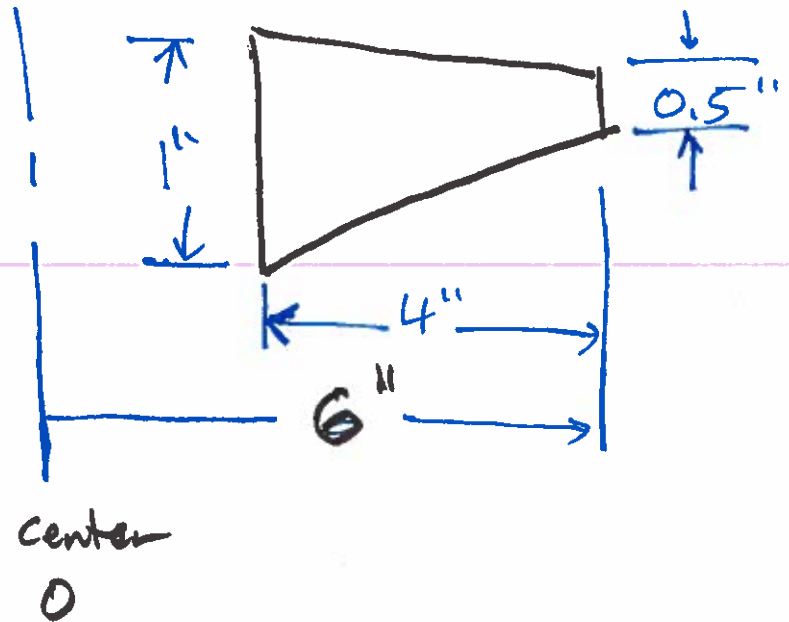
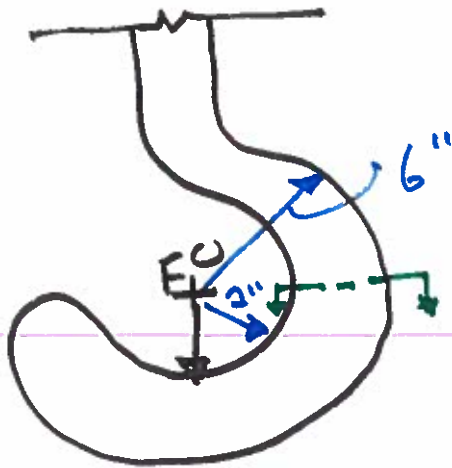
Fig 3.35

$\sigma_i > \sigma_o$



Example

$$F = 5000 \text{ lb}$$



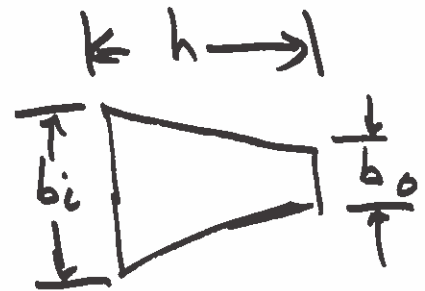
$$\sigma_i = 16.9 \text{ Kpsi}$$

$$\sigma_o = -5.63 \text{ Kpsi}$$

Table 3-4

$$r_c = r_i + \frac{h}{3} \frac{b_i + 2b_o}{b_i + b_o}$$

$$r_n = \frac{A}{b_o - b_i + [(b_i r_o - b_o r_i) / h] \ln \left(\frac{r_o}{r_i} \right)}$$



$$\sigma_i = \frac{M c_i}{A e r_i}$$

$$A = b_i h - \frac{(b_i - b_o)}{2} h$$

$$A \approx 3 \text{ in}^2$$

$$r_i = 2''$$

$$h = 4''$$

$$b_i = 1''$$

$$b_o = 0.5''$$

$$r_o = 6''$$

$$c_i = r_n - r_i$$

$$r_n \approx 3.44''$$

$$c_i \approx 1.44''$$

$$e = 0.34''$$

$$\sigma_i = \frac{(20000 \text{ in} \cdot \text{lb}) (1.44'')}{(3 \text{ in}^2) (0.34'') (2'')}$$

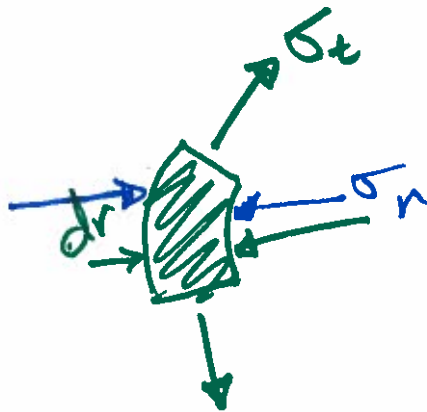
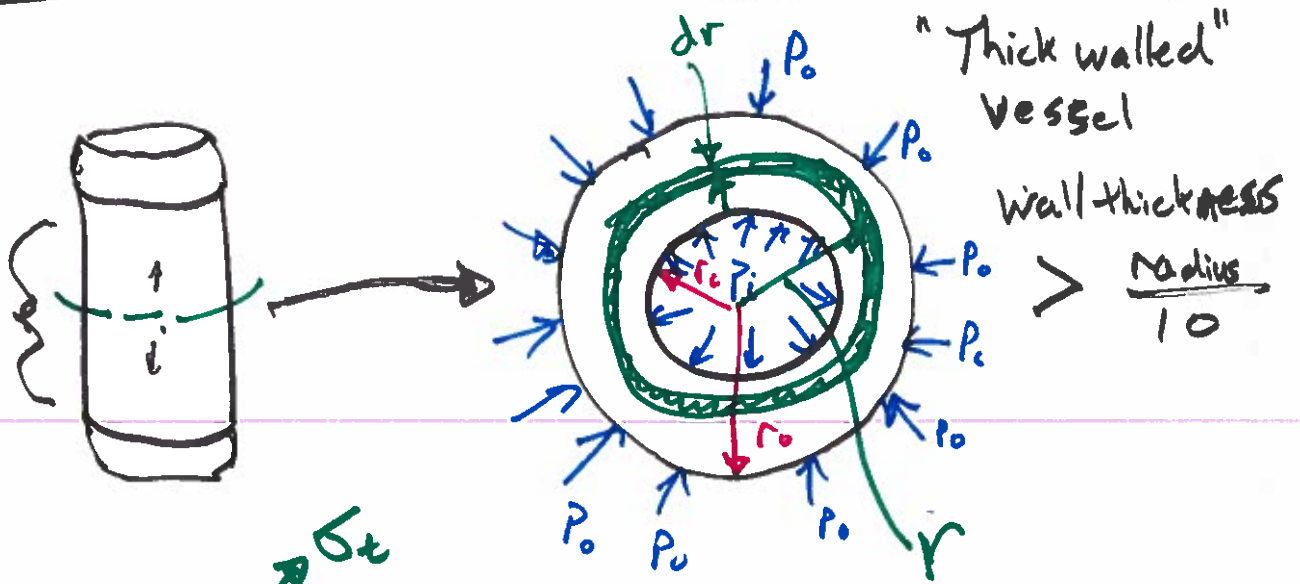
$$\sigma_i \approx 14.1 \text{ Kpsi} < 16.9 \text{ Kpsi}$$

$$\sigma_o = -7.3 \text{ Kpsi}$$

$$\sigma_{o_{\text{rect}}} \Rightarrow -5.63 \text{ Kpsi}$$

increased
compression
stress

Stress in Pressure Vessels



σ_r = radial stress

σ_t = tangential

$$\sigma_t = \frac{P_i r_i^2 - P_o r_o^2 - r_i^2 r_o^2 (P_o - P_i) / r^2}{r_o^2 - r_i^2}$$

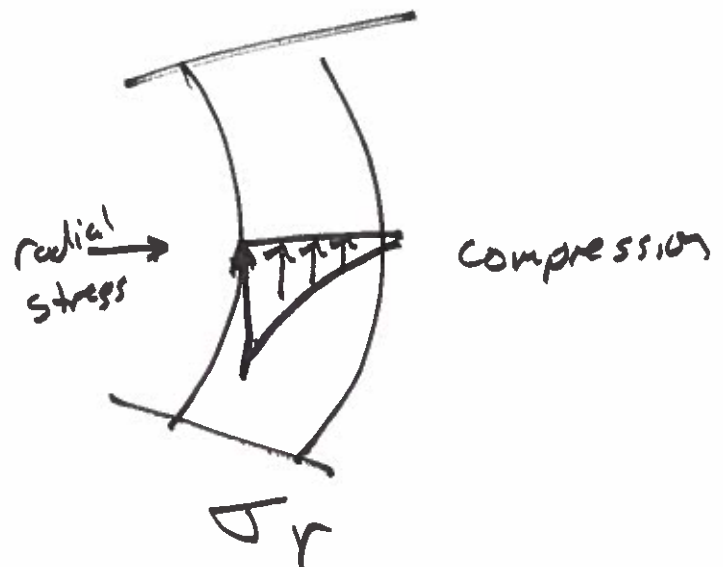
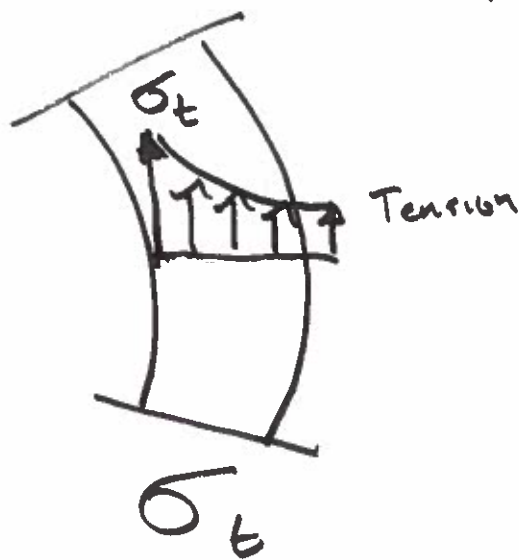
$$\sigma_r = \frac{P_i r_i^2 - P_o r_o^2 + r_i^2 r_o^2 \frac{(P_o - P_i)}{r^2}}{r_o^2 - r_i^2}$$

$$P_o = 0$$

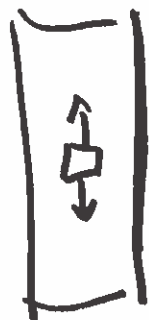
Special Case

$$\sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2} \left(1 + \frac{r_i^2}{r^2} \right)$$

$$\sigma_r = \frac{r_i^2 p_i}{r_o^2 - r_i^2} \left(1 - \frac{r_o^2}{r^2} \right)$$



$$\underline{\underline{P_o = 0}}$$



Longitudinal stress

$$\sigma_l = \frac{p_i r_i^2}{r_o^2 - r_i^2}$$