EME 150A FALL 2015 LECTURE 26 NOV 30

Compound Loading

axial + bending + torsion (all at one point)

fluctuating stress

von mises

$$\sigma_a' = \left\{ \left[(K_f)_{bend} (\sigma_{a0})_{bend} + (K_f)_{axial} \frac{(\sigma_{a0})_{axial}}{0.85} \right]^2 + 3\left[(K_{fs})_{tor} (\tau_{a0})_{tor} \right]^2 \right\}^{1/2}$$

$$\sigma_m' = \left\{ \left[(K_f)_{bend} (\sigma_{m0})_{bend} + (K_f)_{ax} (\sigma_{m0})_{ax} \right]^2 + 3\left[(K_{fs})_{tor} (\tau_{m0})_{tor} \right]^2 \right\}^{1/2}$$

Load factor k_c

Don't use it!

yield $\Rightarrow$ $\sigma_a' + \sigma_m' = \dfrac{S_y}{n}$

$$S_e = k_a k_b k_d k_e k_f S_e'$$

$\sigma_{max} = K_f \sigma_0$

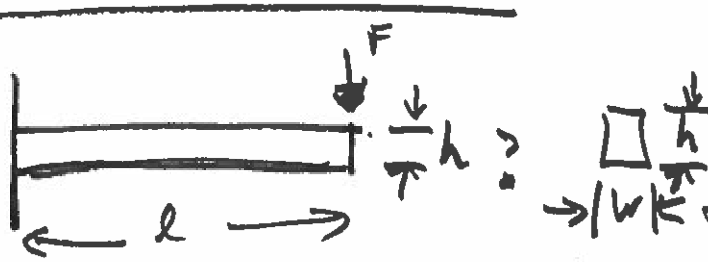
Don't include K_f in k_f (misc factor) tension

$$n = \frac{S_{yt}}{\underbrace{\sigma_a' + \sigma_m'}_{\text{von mises}}} \qquad (\sigma_m + \sigma_a) \qquad \tau_a + \tau_m$$

shear

$$n = \frac{-S_{yc}}{\sigma_m' - \sigma_a'}$$ compression

HW 7 Problem 4



factor of safety
$n = 2$

fully reversed

$$\sigma_a = \frac{6Fl}{h^2 w} \qquad \sigma_m = 0$$

Hot rolled

$$S_e = \underset{\text{surface}}{K_a}\ \underset{\text{size factor}}{K_b}\ \overbrace{\frac{S_{ut}}{2}}^{S_e'}$$

$K_b =$ < 51mm

$1.51 d_e^{-0.157}$ > 51mm

$d_e = 0.808\sqrt{hw}$

$$n = \frac{(S_f)_{10^4}}{\sigma_a}$$

$$(S_f) = a N^b$$

$$f = 0.9$$

$$a = (f S_{ut})^2 / S_{e}$$

$$b = -\tfrac{1}{3}\log\left(\frac{f S_{ut}}{S_e}\right)$$

$$(S_f) = 2.62E{+}6\, N^{\frac{-0.145\log\left(S_{ut}^{0.718}(hw)^{0.0785}\right)}{+1.85}} \cdot S_{ut}^{1.72}(hw)^{0.0785}$$

$$n = \frac{S_y}{\sigma_a}$$

$$2 = n = 512 h^{1.9738}$$

$$\boxed{h = \left(\frac{2}{512}\right)^{\frac{1}{1.9738}} = 0.06 \text{ m}}$$

1. guess at h
2. implicit function $\Rightarrow$ solve iteratively: matlab fsolve( )
3. analytial solution

if $d_e < 51$! $\Rightarrow$ change the equation for K_b and recalc

Cumlative Loading

Element that loaded: $n < N$

σ_1' for n_1 cycles for which N_1 cycles would produce failure.

$\vdots$

σ_2' for n_2 cycles ... N_2 cycles

σ_3' n_3 N_3 cycles

$\vdots$

$$\underbrace{\sum \frac{n_i}{N_i} = C \quad \begin{array}{l} \text{if } C<1 \text{ failure will not occur} \\ \text{if } C \geq 1 \text{ failure will occur} \end{array}}_{\text{Miner's Rule}}$$

What I want to know remaining # of cycles after various various cumlative loads?

If $C<1$, remaining life:

$$n_r = \left[C - \sum_{i=1}^{m} \frac{n_i}{N_i} \right] N_r$$

$C = 1$ unless empirically derived

N_r: number of finite cycles to failure for the last applied stress

n_r: remaing number of cycles for cumalitive load

<u>Example</u> A machined part is cycled $(\sigma_a)_1 = \pm 350$ MPa for $n = 5 \times 10^3$ cycles, then $(\sigma_a)_2 = \pm 260$ MPa applied for $n_2 = 5 \times 10^4$ cycles, finally $(\sigma_a)_3 = \pm 225$ MPa is applied. How many cycles remain before failure?

$S_{ut} = 530$ MPa, $f = 0.9$, $S_e = 210$ MPa

$a = 1083.47 \qquad b = -.11876$

$$N_3 = \left[\frac{(\sigma_a)_3}{a}\right]^{1/b} = 559{,}400$$

$$N_2 = \left[\frac{(\sigma_a)_2}{a}\right]^{1/b} = 13{,}580$$

$N_1 = 165{,}600$

$$n_3 = \left[C - \sum_{i=1}^{2} \frac{n_i}{N_i} \right] N_3$$

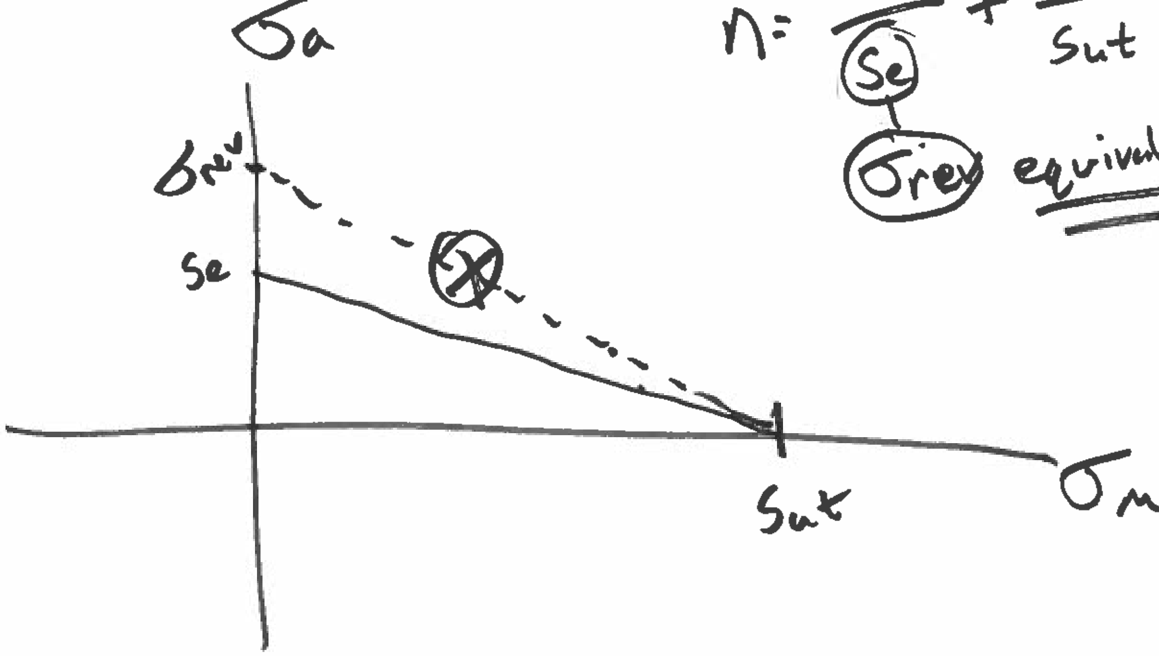
$C = 1$

$$n_3 = \left[1 - \frac{5000}{13550} - \frac{50\,000}{165606} \right] 559400$$

$n_3 = 184{,}000$ cycles @ $(\sigma_a)_3$

$$n = \frac{\sigma_m}{S_e} + \frac{\sigma_a}{S_{ut}}$$

S_e — σ_{rev} equivalent



Compound Fluctuating Stress Example

Nov 25, 2015

A part has compound loading of bending, axial, and torsion with the following stresses:

- fully reversed bending where $\sigma_{max} = 60\,MPa$
- constant axial stress where $\sigma = 20\,MPa$
- repeated torsional stress where $\tau_{max} = 50\,MPa$

All stresses are in phase with each other.
The part has a notch such that the stress concentrations are:

$$(K_f)_{bending} = 1.4, \quad (K_f)_{axial} = 1.1, \quad (K_f)_{torsion} = 2.0$$

Find the factors of safety for Goodman infinite life and yielding if the material properties are: $S_y = 300\,MPa$, $S_{ut} = 400\,MPa$, and $S_e = 200\,MPa$.

Solution

Bending: $\sigma_m = 0$, $\sigma_a = 60\,MPa$

Axial: $\sigma_m = 20\,MPa$, $\sigma_a = 0\,MPa$

Torsion: $\sigma_m = 25\,MPa$, $\sigma_a = 25\,MPa$

$$\sigma_a' = \{[1.4 \cdot 60 + 1.1 \cdot 0]^2 + 3[2 \cdot 25]^2\}^{1/2} = 120.6\,MPa$$

$$\sigma_m' = \{[1.4 \cdot 0 + 1.1 \cdot 20]^2 + 3[2 \cdot 25]^2\}^{1/2} = 89.35\,MPa$$

Goodman

$$n = \left[\frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}}\right]^{-1} = \boxed{1.21}$$

Yielding

$$n = \frac{S_y}{\sigma_a' + \sigma_m'} = \boxed{1.43}$$