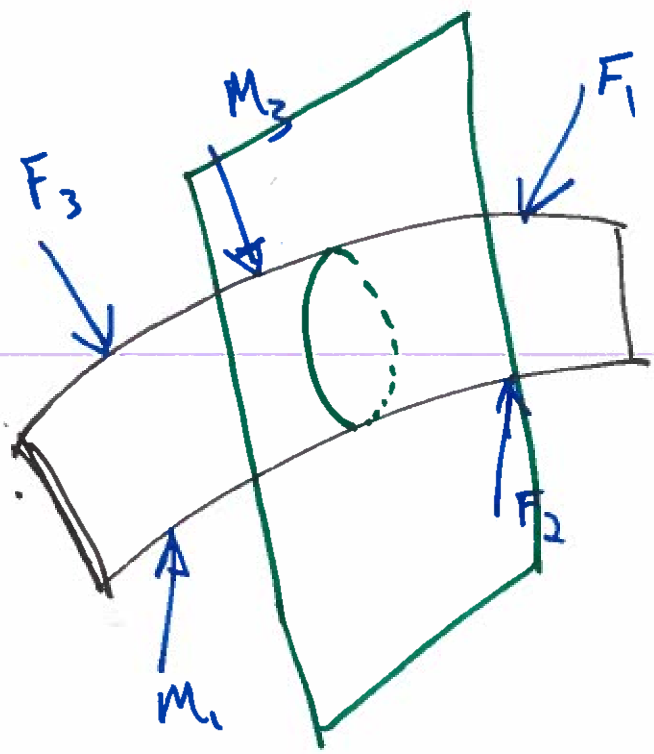
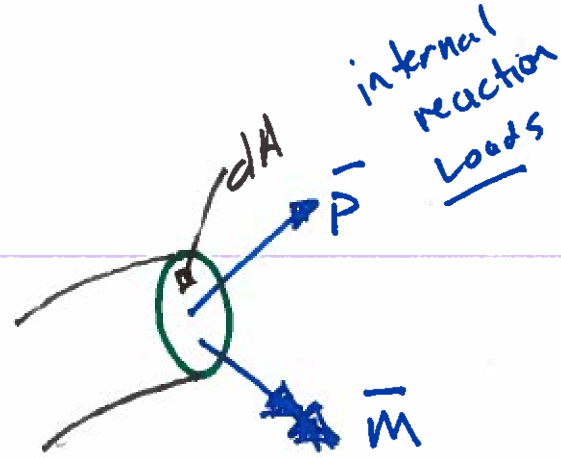
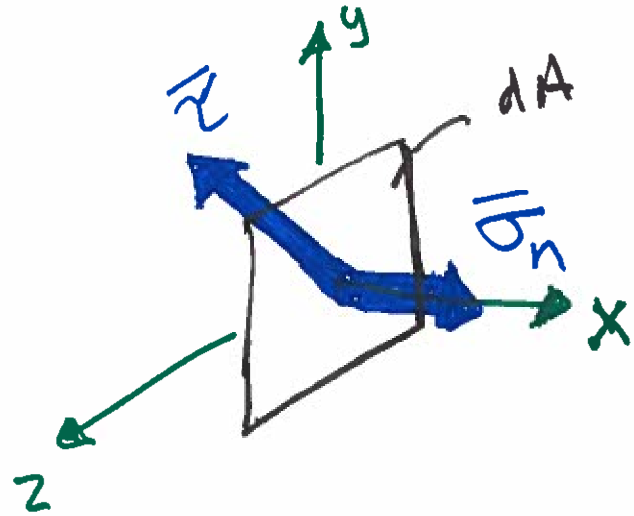
Monday, October 5, 2015 EME 150A Fall 2015
Lecture #5

Multi-Axial Stress







Infintesimal volumetric element $d\forall$ must equilibrium.

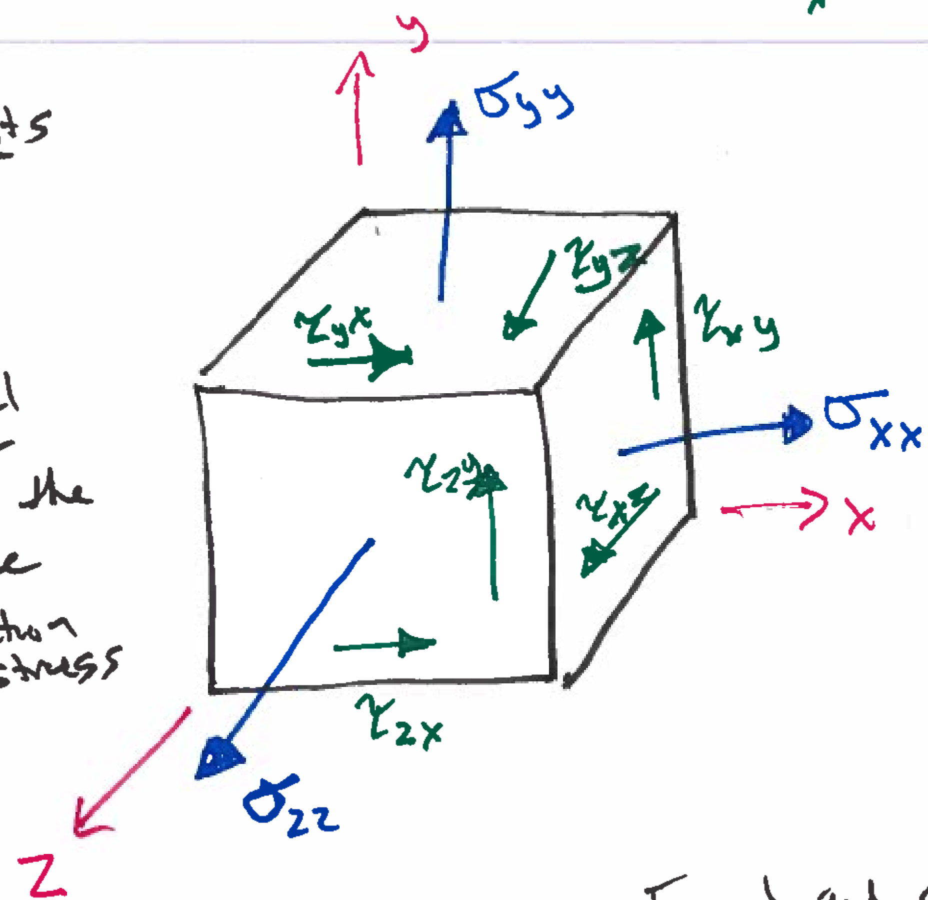
$$\bar{\tau}_x = \tau_{xy}\hat{j} + \tau_{xz}\hat{k}$$

Subscripts

σ/τ_{ij}

i: normal vector of the face

j: direction of stress



9 components

Equal and opposite forces ⇒ on the hidden sides

6 unique values

$$\tau_{ij} = \tau_{ji}$$

Cauchy Stress Tensor

2nd order tensor

$$\sigma = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

defines a state of stress at a particular point

$$\bar{\sigma} = \begin{bmatrix} \bar{\sigma}_x \\ \bar{\sigma}_y \\ \bar{\sigma}_z \end{bmatrix} = \sigma \begin{bmatrix} \hat{i} \\ \hat{j} \\ \hat{k} \end{bmatrix}$$

$$\bar{\sigma}_x = \underbrace{\sigma_{xx}}_{\bar{\sigma}_{nx}} \hat{i} + \underbrace{\tau_{xy} \hat{j} + \tau_{xz} \hat{k}}_{\bar{\tau}_x}$$

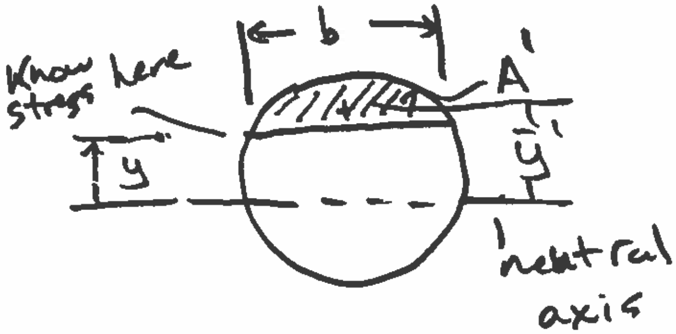
$$\bar{\sigma}_x = \bar{\sigma}_{nx} + \bar{\tau}_x$$

Transverse Shear Stress

Pure shear stress $\left(\frac{V}{A}\right)$ only occurs when there is no bending moment

If bending is present:

$$\tau = \frac{VQ}{Ib}$$



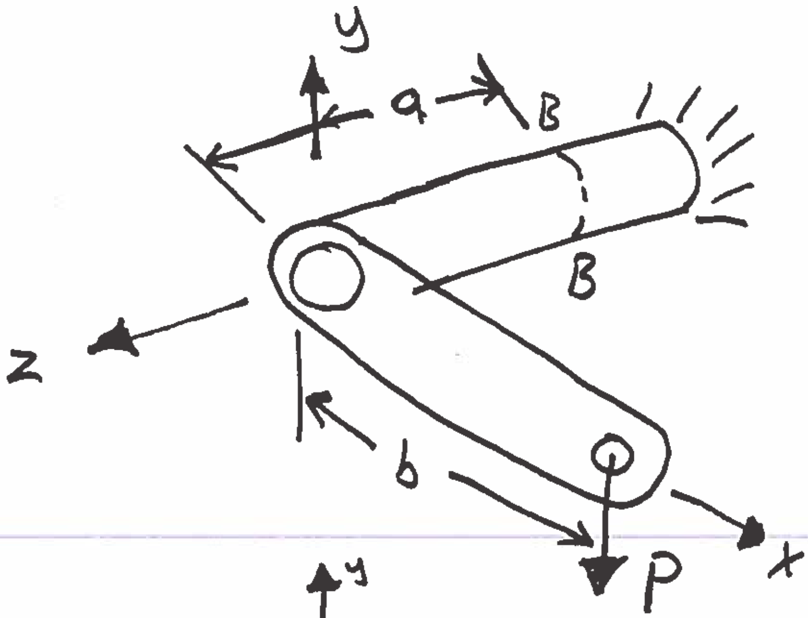
V: shear force

I: second moment of area of entire cross section

b: width of point of concern

Q: $\int y \, dA' = \bar{y}' A'$

$\bar{y}'$: distance from neutral axis to centroid of A'



Shear

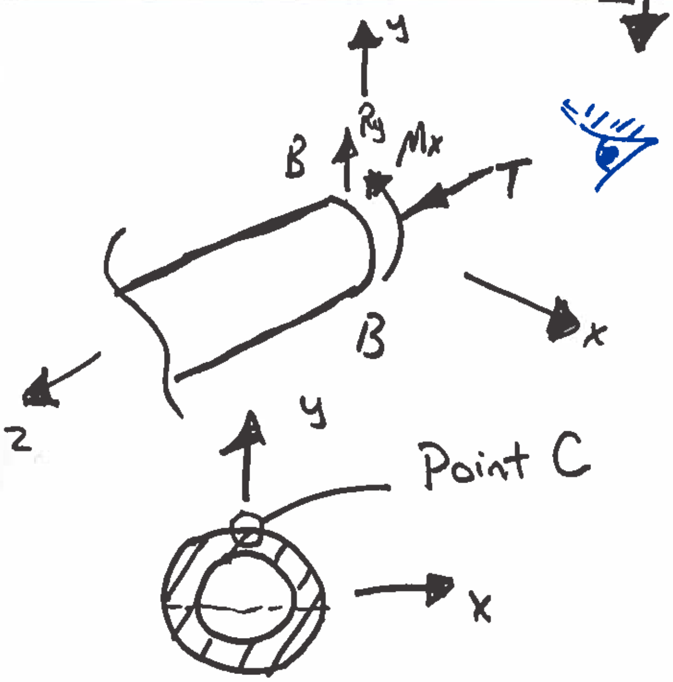
$R_y = P$

Bending

$M_x = -Pa$

Torsion

$T = Pb$



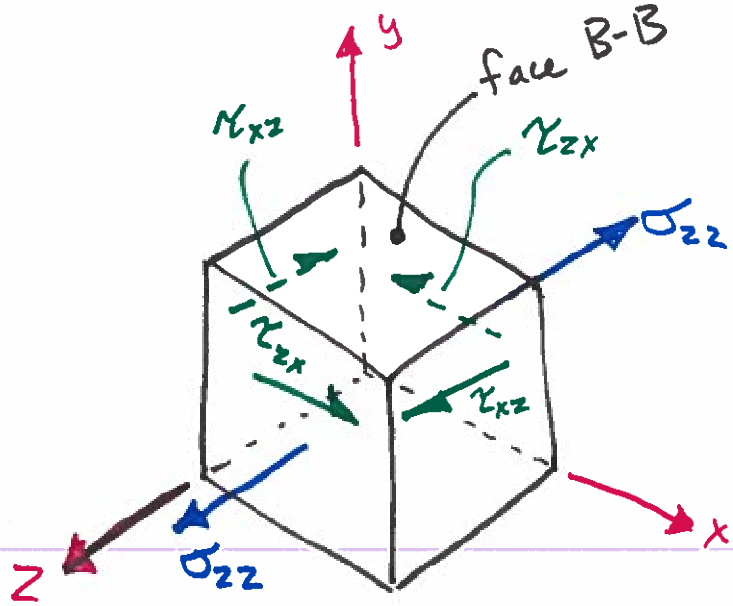
I: second moment of area

A: area of cross section

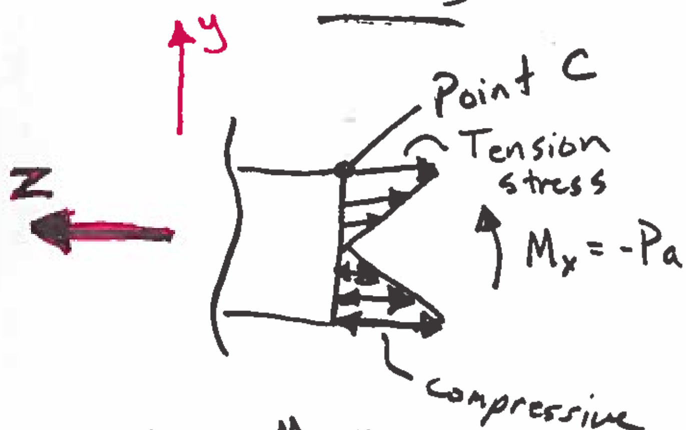
J: polar moment of area

r: outer radius

If point C is on the left half of the section, what does the stress tensor look like for that point?

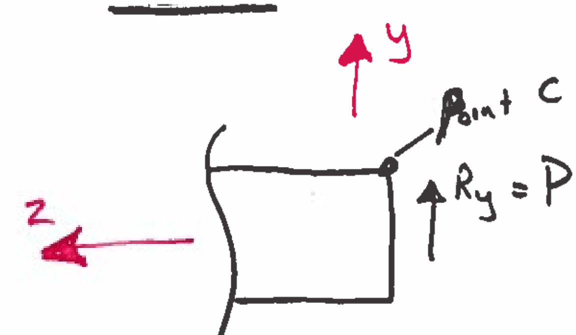


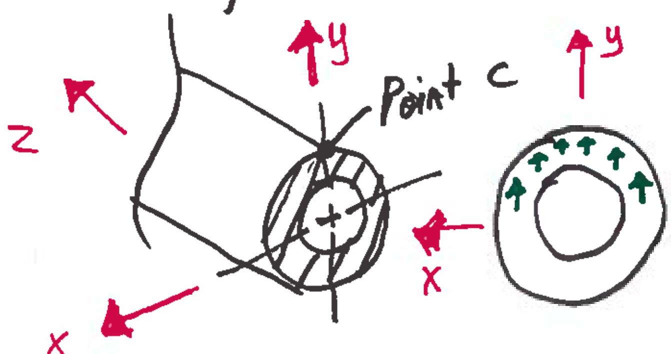
Bending



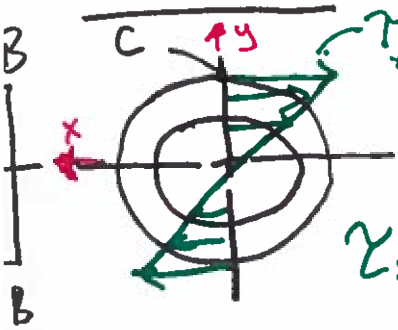
$$\sigma_{zz} = -\frac{M_x r}{I}$$

Shear





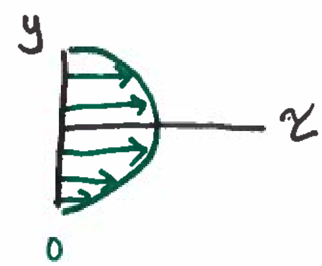
Torsional



$$\tau_{torsional} = \frac{Tr}{J} = \frac{Pbr}{J}$$

$$\tau_{zx} = \frac{Tr}{J} = \tau_{xz}$$

$$\tau_{shear}(r) = 0$$



$$\sigma_c = \begin{bmatrix} 0 & 0 & \frac{Pbr}{J} \\ 0 & 0 & 0 \\ \frac{Pbr}{J} & 0 & \frac{Par}{I} \end{bmatrix}$$