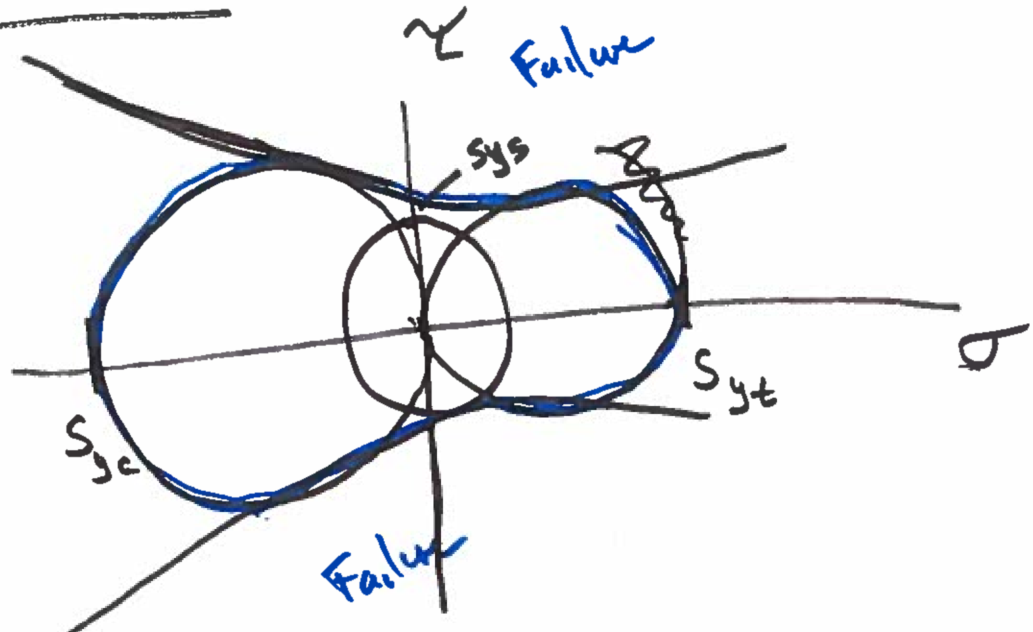
EME 150A FALL 2015 LECTURE 20

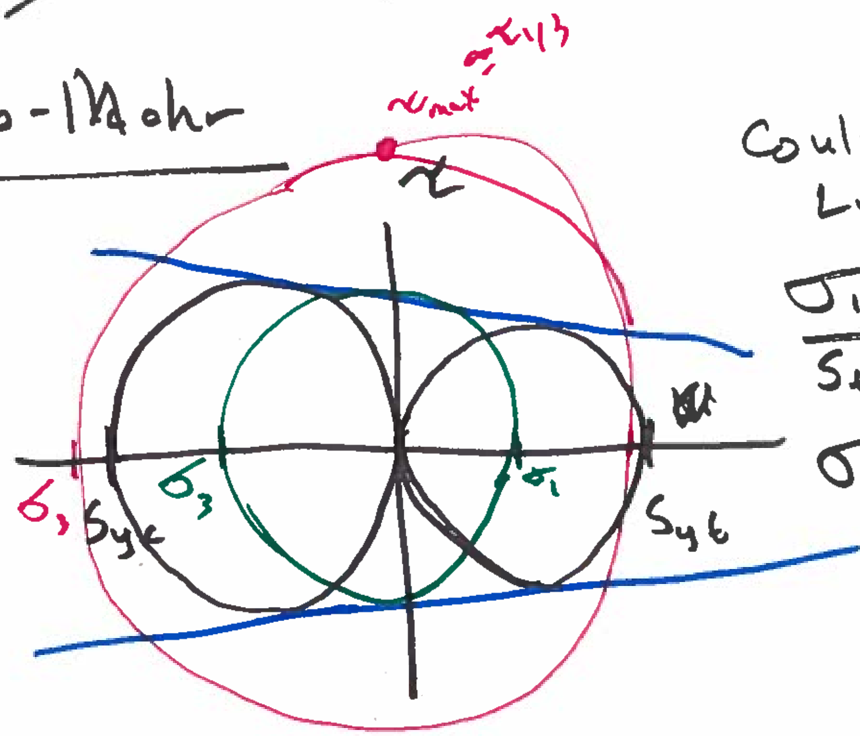
Coulomb-Mohr and Mohr Ductile

$$|S_{yt}| \neq |S_{yc}|$$

Mohr Theory

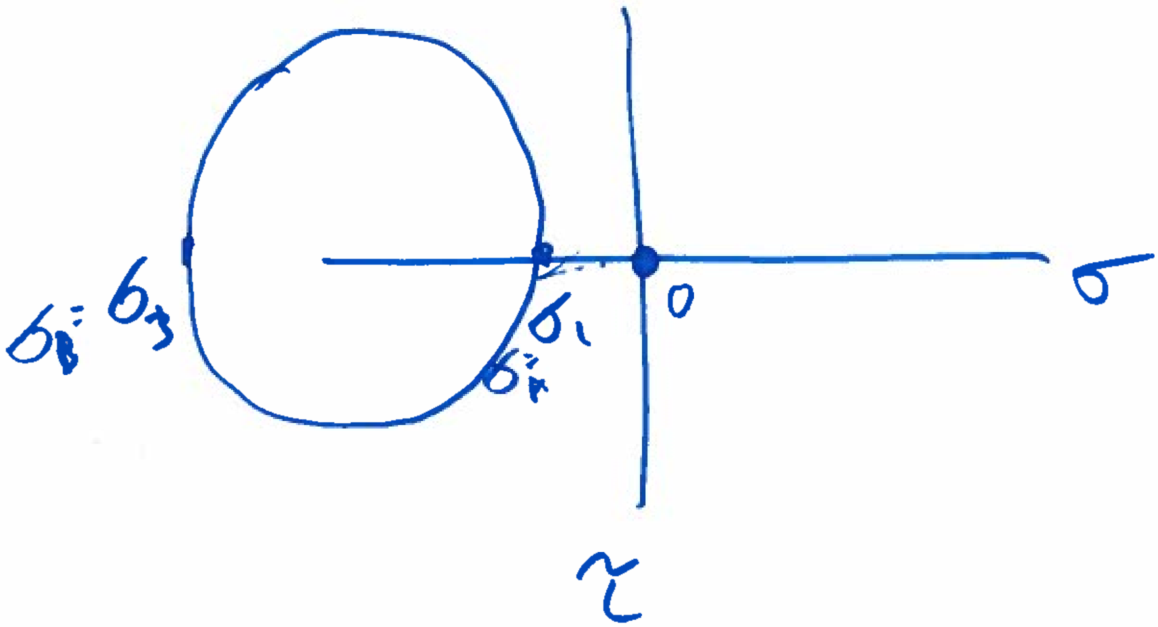


Coulomb-Mohr



Coulomb-Mohr Line

$$\frac{\sigma_1}{S_t} - \frac{\sigma_3}{S_c} = 1$$



$\sigma_B = \sigma_3$
σ_1
$\sigma_A =$
0
σ
τ

Example Steel element $S_y = 295$ MPa

Ductile

Planar $\sigma_x = -80$ MPa, $\sigma_y = 30$ MPa, $\tau_{xy} = 10$ MPa

What are the factors of safety for MSS and DE?

Solution

$$\sigma_1 = \sigma_A = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 30.9 \text{ MPa}$$

$$\sigma_3 = \sigma_B = \quad '' \quad - \quad '' \quad = -80.9 \text{ MPa}$$

$$\overset{\sigma_1}{\sigma_A} \geq 0 \geq \overset{\sigma_3}{\sigma_B}$$

MSS

$$n_{mss} = \frac{S_y}{\sigma_1 - \sigma_3} = \frac{295 \text{ MPa}}{30.9 - (-80.9)} = 2.64$$

DE

$$\sigma' = \left[\frac{\sigma_1^2 + (-\sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}\right]^{1/2}$$

$$\sigma' = 99.7 \text{ MPa} \qquad n_{de} = \frac{295}{99.7} = 2.95$$

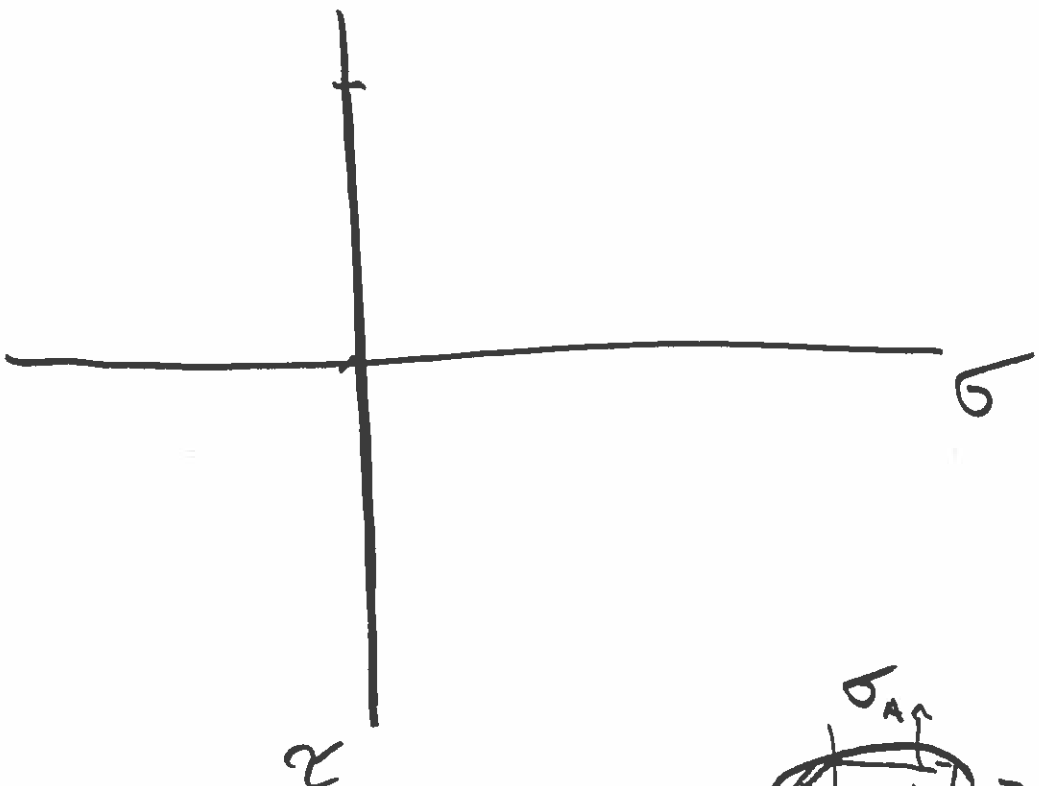
What if $S_{yt} = 295\,MPa$, $S_{yc} = 320\,MPa$?

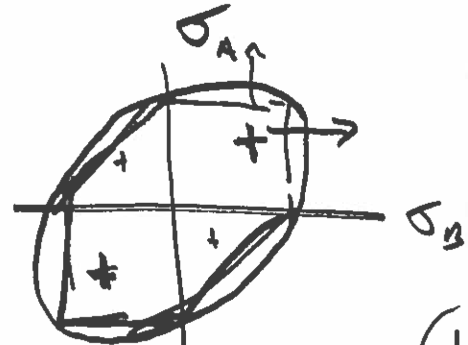
<u>Ductile</u>

Coulomb-Mohr Theory:

$$n_{dcm} = \left[\frac{\sigma_A}{S_{yt}} - \frac{\sigma_B}{S_{yc}}\right]^{-1} = \left[\frac{30.9\,MPa}{295\,MPa} - \frac{-80.9\,MPa}{320\,MPa}\right]^{-1}$$

$$n_{dcm} = 2.79$$





Failure Brittle Materials

- Brittle fails due to fracture instead of yielding, in general
- Compressive ultimate strength > tensile ultimate strength
- Empirical studies:
  - tension failure due to normal stresses
  - compression " " compressive and shear stresses

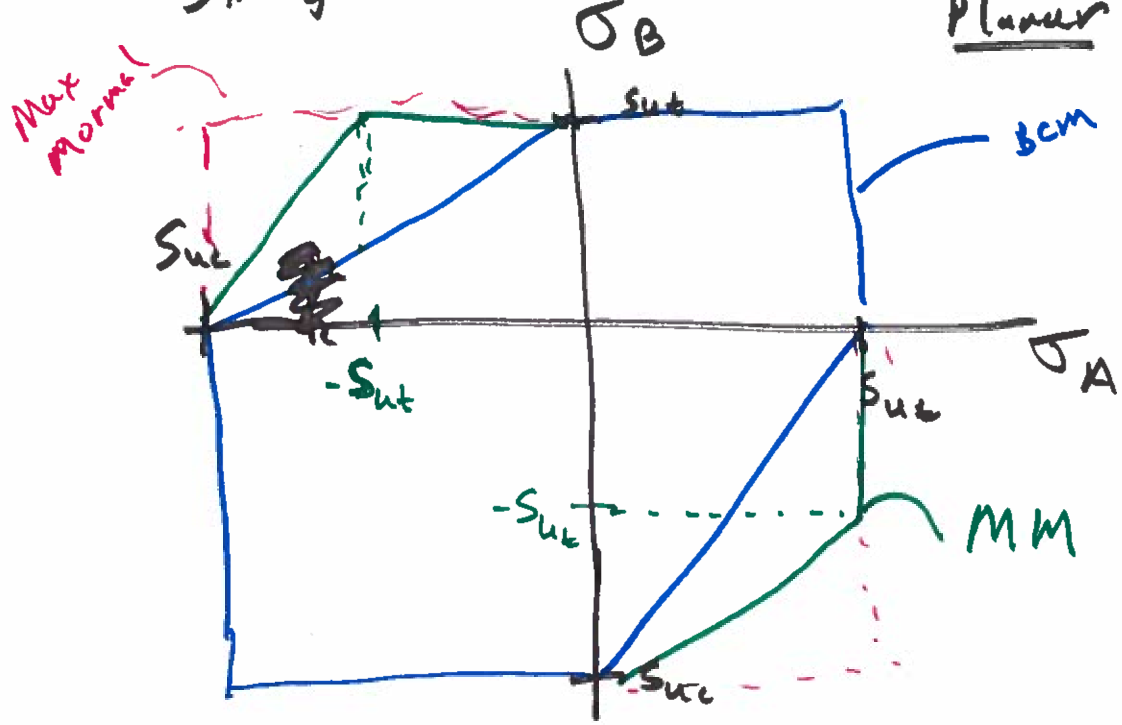
Theories for static failure

- Max normal stress ⇒ ignore
- Brittle ~~Mohr~~ Coulomb-Mohr (BCM)
- Modified Mohr (MM)

BCM

Analagous to DCM except use ultimate strength

Planar



Modified Mohr

Less conservative than the BCM

$$\sigma_A = \frac{S_{ut}}{n_{mm}} \qquad \sigma_A \geq \sigma_B \geq 0$$

$$\text{"} \qquad \sigma_A \geq 0 \geq \sigma_B \text{ and } \left|\frac{\sigma_B}{\sigma_A}\right| \leq 1$$

$$\frac{(S_{uc} - S_{ut})\sigma_A}{S_{uc} S_{ut}} - \frac{\sigma_B}{S_{uc}} = \frac{1}{n_{mm}} \qquad \sigma_A \geq 0 \geq \sigma_B \text{ and } \left|\frac{\sigma_B}{\sigma_A}\right| > 1$$

$$\sigma_B = -\frac{S_{uc}}{n_{mm}} \qquad 0 \geq \sigma_A \geq \sigma_B$$