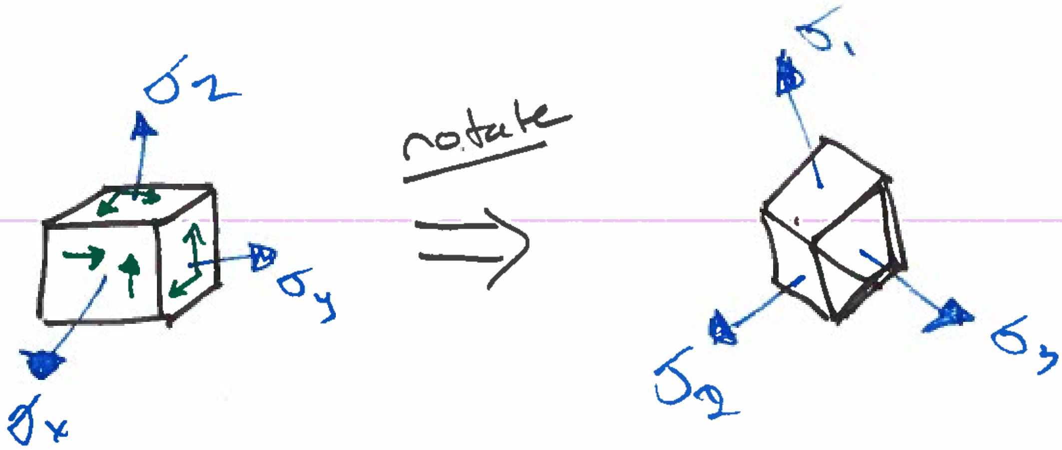
Principal Stresses



$$\begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix} \Rightarrow \begin{bmatrix} \sigma_1 & & 0 \\ & \sigma_2 & \\ 0 & & \sigma_3 \end{bmatrix}$$

$$\bar{\sigma}_n = \sigma \hat{n} = \underset{\approx}{\sigma}\left(\underline{l}\,\hat{i} + \underline{m}\,\hat{j} + \underline{n}\,\hat{k}\right) \quad (1)$$

$$\bar{\sigma}_n = \underline{l}\,\bar{\sigma}_x + \underline{m}\,\bar{\sigma}_y + \underline{n}\,\bar{\sigma}_z \quad (2)$$

$$\underbrace{(A - \lambda I)}\bar{v} = 0$$

$$\det\left|\cancel{A^*}\right| = 0$$

$$\det\left|A^*\right| = 0$$

$$\sigma^3 - I_1\sigma^2 + I_2\sigma - I_3 = 0$$

characteristic equation

Stress invariants

$$\begin{cases} I_1 = \sigma_x + \sigma_y + \sigma_z \\ I_2 = \det\begin{vmatrix} \sigma_y & \tau_{yz} \\ \tau_{yz} & \sigma_z \end{vmatrix} + \det\begin{vmatrix} \sigma_x & \tau_{xz} \\ \tau_{xz} & \sigma_z \end{vmatrix} + \det\begin{vmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{vmatrix} \\ I_3 = \det\left|A\right| = \det\left|\sigma_{ij}\right| \end{cases}$$

↳ initial stress tensor

$$= l\left(\sigma_x \hat{i} + \tau_{xy} \hat{j} + \tau_{xz} \hat{k}\right)$$

$$+ \; m\left(\tau_{xy} \hat{i} + \sigma_y \hat{j} + \tau_{yz} \hat{k}\right)$$

$$+ \; n\left(\tau_{zx} \hat{i} + \tau_{zy} \hat{j} + \sigma_z \hat{k}\right)$$

$$\sigma l = l\sigma_x + m\tau_{xy} + n\tau_{zx}$$

$$\sigma m = l\tau_{xy} + m\sigma_y + n\tau_{zy}$$

$$\sigma n = l\tau_{xz} + m\tau_{yz} + n\sigma_z$$

$$\Downarrow$$

$$\begin{bmatrix} \sigma_x - \sigma & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y - \sigma & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z - \sigma \end{bmatrix} \begin{bmatrix} l \\ m \\ n \end{bmatrix} = 0$$

$$A\bar{v} = \lambda \bar{v}$$

$$[A - \lambda I]\bar{v} = 0$$

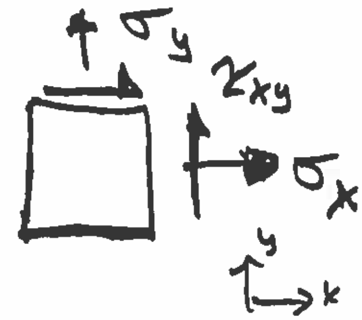
Roots of characteristic Eq:

$$\sigma_1 \geq \sigma_2 \geq \sigma_3$$

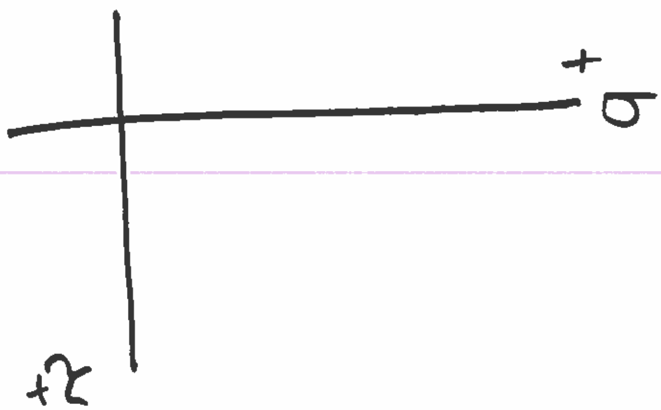
Finding the eigenvectors $\Rightarrow$ ~~dir~~ principal directions

$$\left.\begin{aligned} \left[\sigma_{ij} - \sigma_1 \mathbf{I}\right] \overset{\bar{V}_1}{\begin{bmatrix} l_1 \\ m_1 \\ n_1 \end{bmatrix}} &= 0 \\ l_1^2 + m_1^2 + n_1^2 &= 1 \end{aligned}\right\} \text{give } l_1, m_1, n_1$$

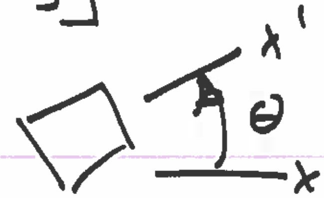
Mohr's Circle For Plane Stress



1. Establish a coordinate system:

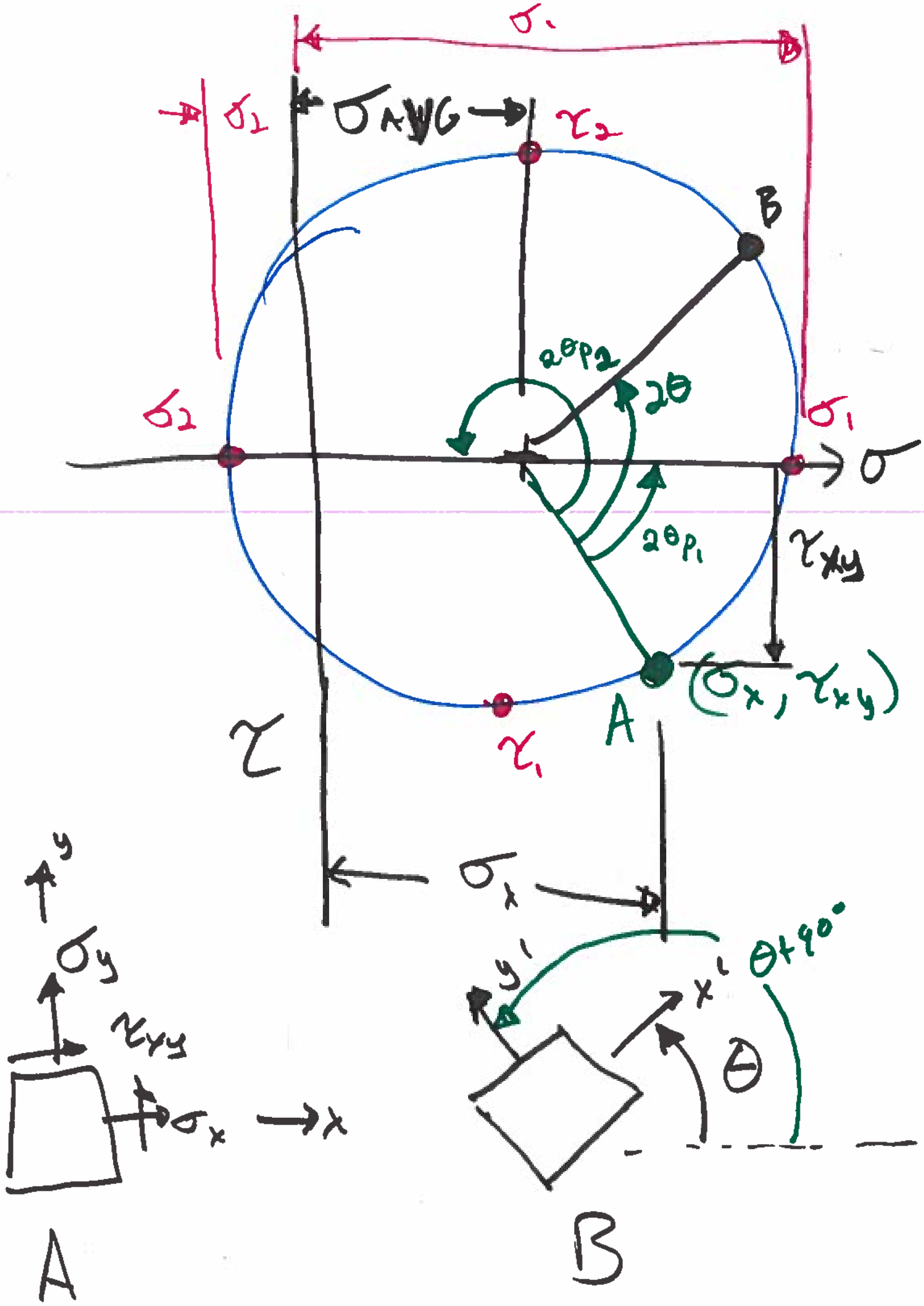


$$\begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix}$$



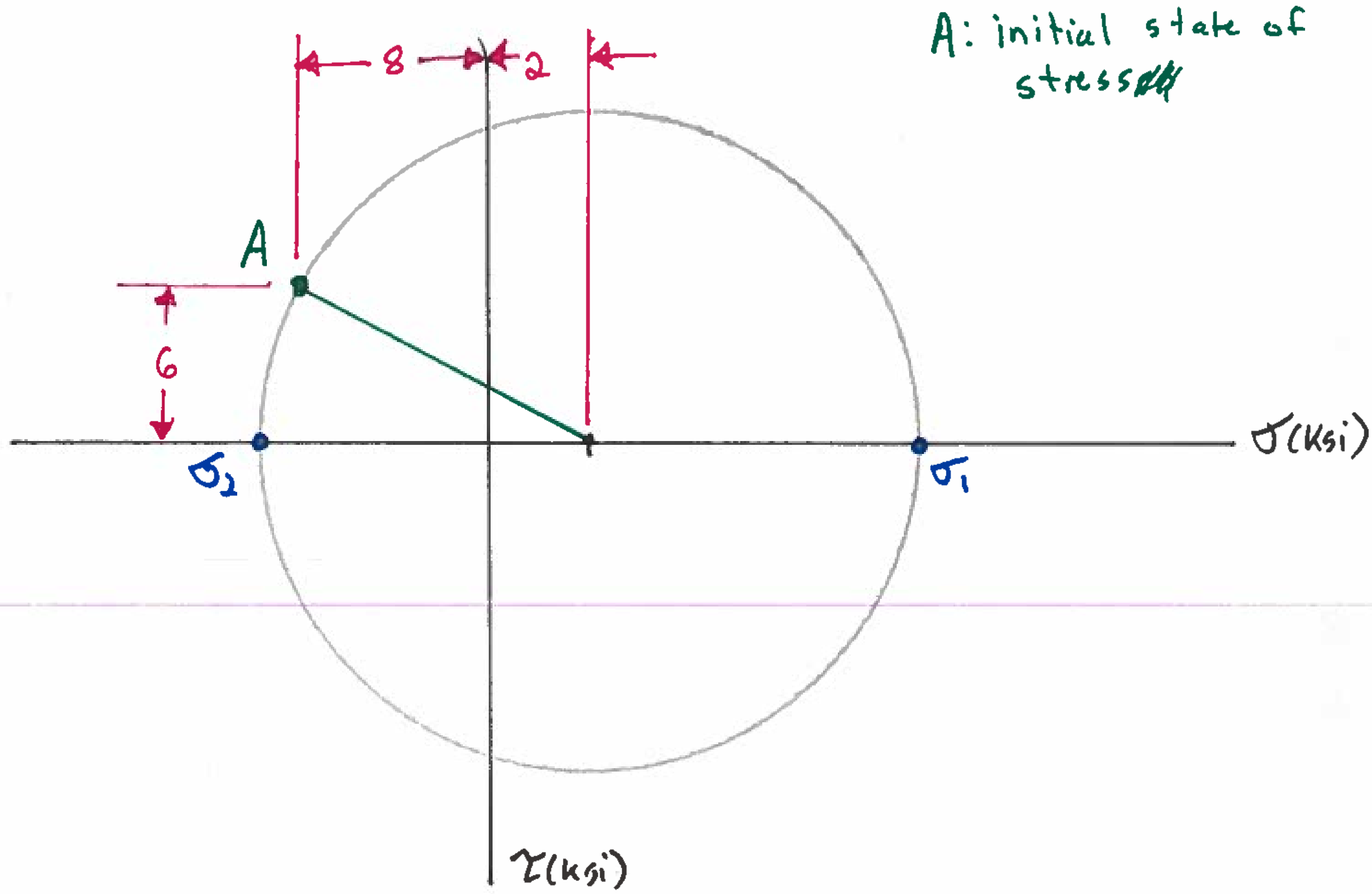
2. Plot center of the circle at the average stress on the abscissa: $\sigma_{AVG} = \dfrac{\sigma_x + \sigma_y}{2}$
3. Plot a reference point for the given state of stress at $\sigma = \sigma_x$, $\tau = \tau_{xy}$
4. Connect the reference point to the center of the circle. Can compute the radius of circle.
5. Sketch the circle
6. Identify the prinicpal stresses/directions and max/min shear stresses. $\sigma_1 \geq \sigma_2$

7. Find the state of stress at any other orientation.



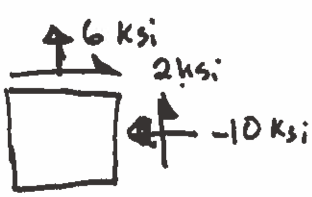
σ_1
σ_{AVG}
τ_2
σ_2
B
$2\theta_{p2}$
2θ
σ_1
σ
τ_{xy}
$2\theta_{p1}$
(σ_x, τ_{xy})
A
τ
τ_1
σ_x
y
σ_y
τ_{xy}
σ_x
x
y'
x'
$\theta+90°$
θ
A
B

Eqs 3-8
to
3-14

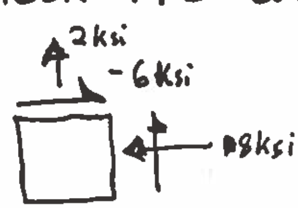


1) What does the stress look like at point A?

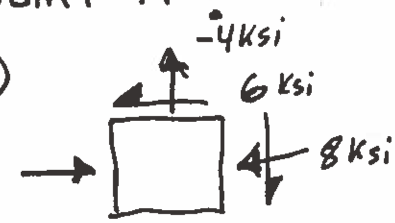
A)



B)



C)



2) What is the most likely angle one needs to orient the stress square to the second principal stress (σ_2)?

A) 31° B) 15° C) 375° D) -149°

3) Write out the expression for the radius of the circle and compute the value (estimate is fine).