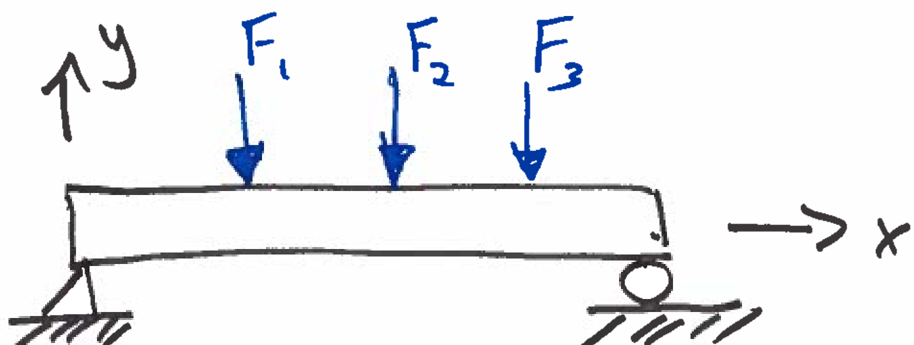
~~Transer~~

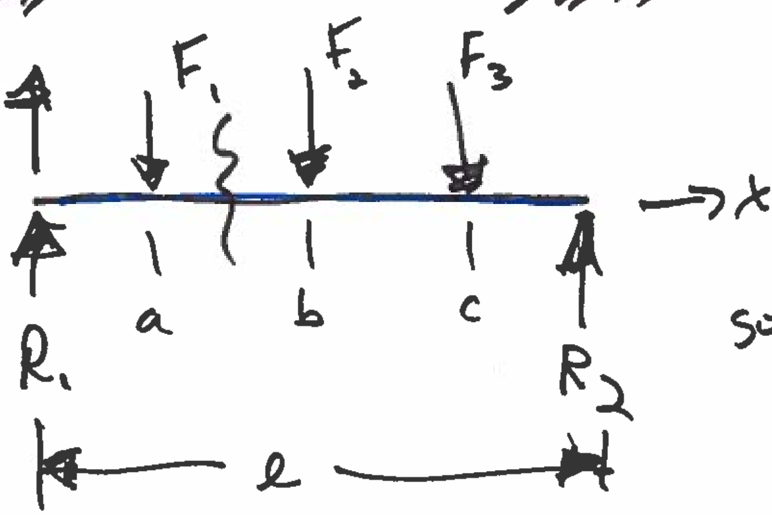
Transverse Loading of Long Slender Beams

Beam: slender element ~~width~~ with a $\frac{\text{length}}{\text{width}} \gtrsim 10$

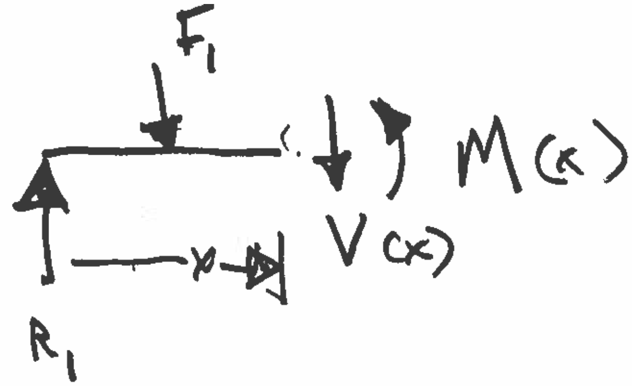
↑ approximation

in general they ~~so~~ support transverse loads.



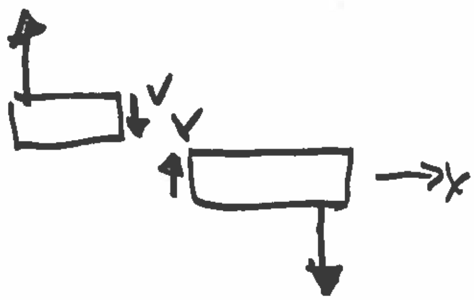


solve for R_1, R_2

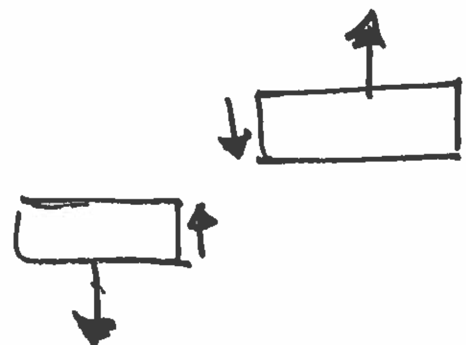


Sign Conventions

Shear

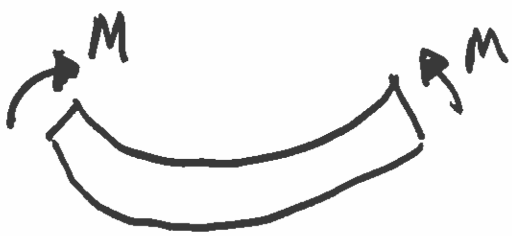


positive

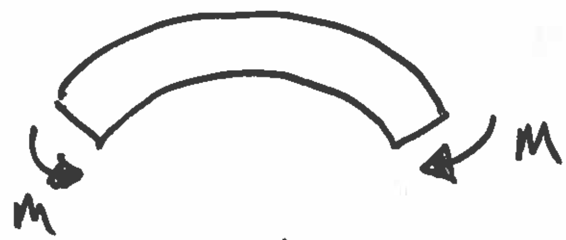


negative

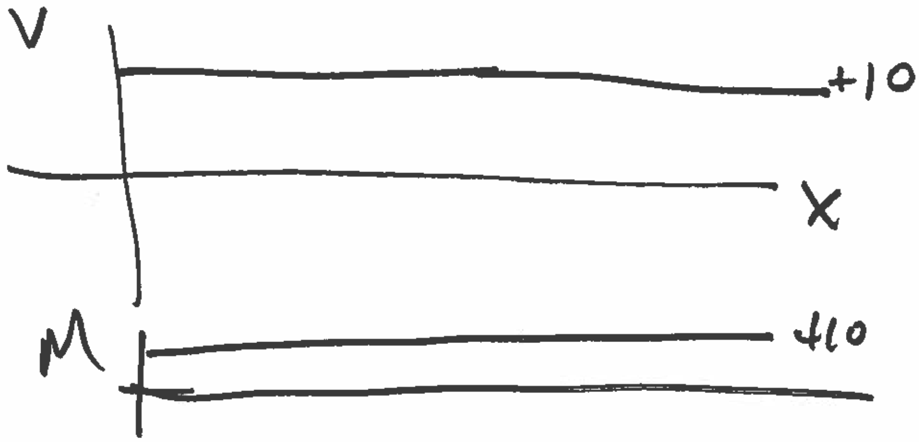
Moments



positive bending



negative

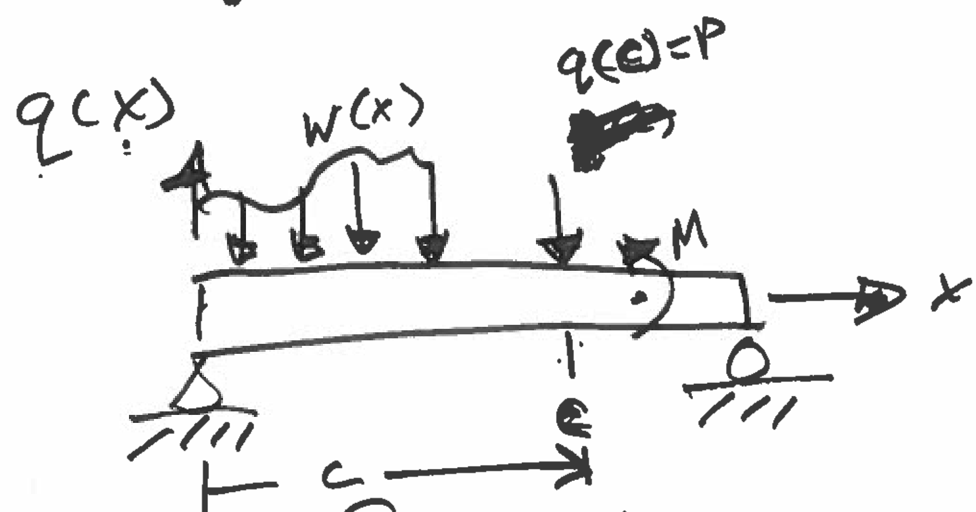


For equilibrium at any tranverse cross section the shear reaction, $V(x)$, and bending moment reaction, $M(x)$, will be present.

$$V(x) = \frac{d\,M(x)}{dx}$$

The load, $q(x)$

$$-q(x) = \frac{d\,V(x)}{dx} = \frac{d^2 M(x)}{dx^2}$$



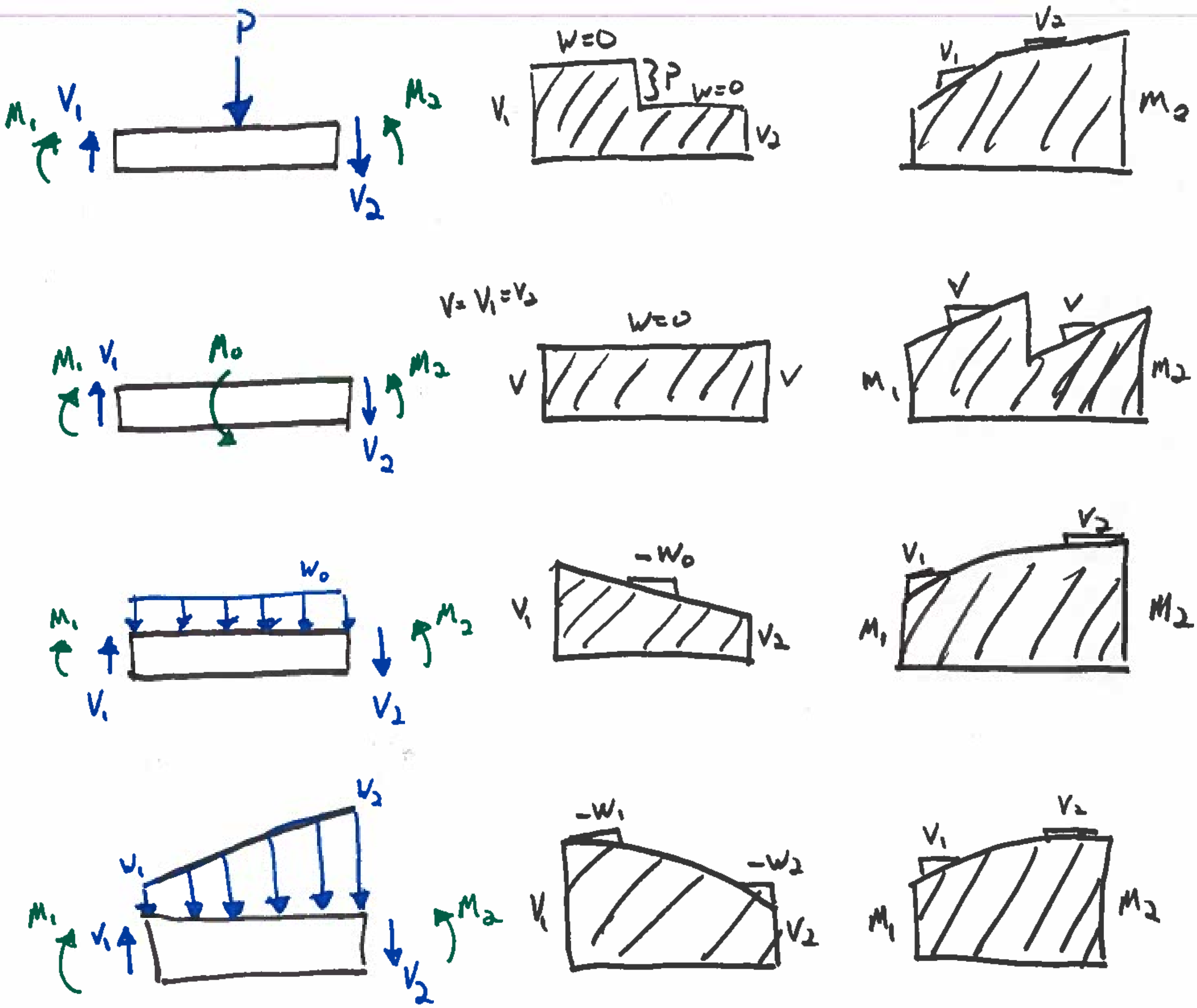
$$V(x) = -\int q(x)\,dx$$

$$M(x) = \int V(x)\,dx$$

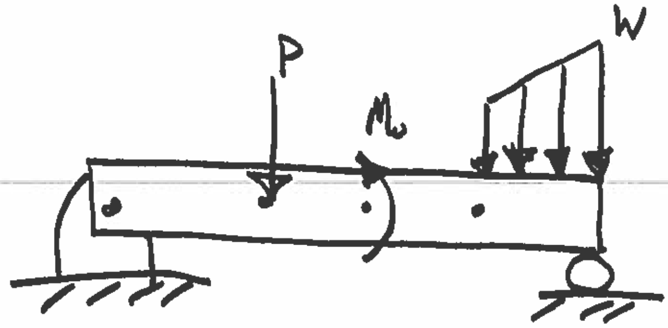
Sketch the shear and bending diagrams $[V(x), M(x)]$ for the following loads using your knowledge of:

$$V(x) = -\int q(x)\,dx$$

$$M(x) = \int V(x)\,dx$$



Q: How many free body diagrams would be required to find $M(x)$ and $V(x)$ of this:



5 FBDs

8 integration constants

Singularity Functions

- allows us to use a single expression for the whole beam
- the functions are made up of dirac delta, unit doublets, Heaviside

Distributed Loads

$$\langle x-a \rangle^n = \begin{cases} 0 & x < a \\ (x-a)^n & x \geq a \end{cases}$$

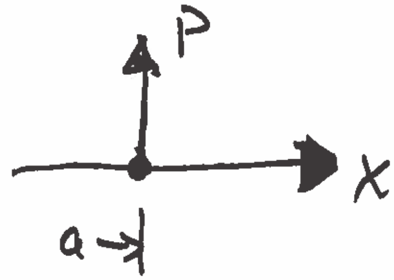
coordinate position

location of the discontinuity

$n \geq 0$

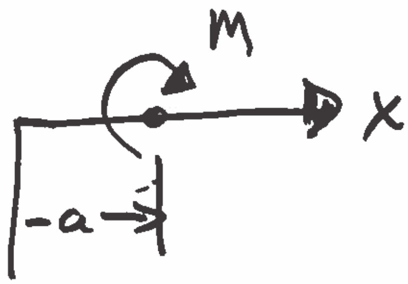
$$\int \langle x-a \rangle^n dx = \frac{\langle x-a \rangle^{n+1}}{n+1} + C$$

Concentrated Loads



$$q(x) = P\langle x-a\rangle^{-1}$$

$$= \begin{cases} 0 & \text{for } x \neq a \\ +\infty & \text{for } x = a \end{cases}$$



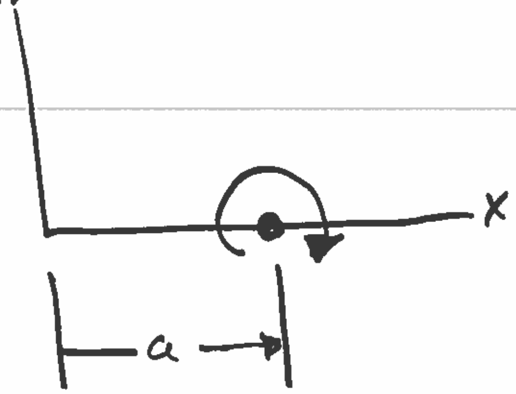
$$q(x) = M\langle x-a\rangle^{-2}$$

$$= \begin{cases} 0 & \text{for } x \neq a \\ \pm\infty & \text{for } x = a \end{cases}$$

$$\int \langle x-a\rangle^{n} dx = \langle x-a\rangle^{n+1} \quad \text{for } n = -1, -2$$

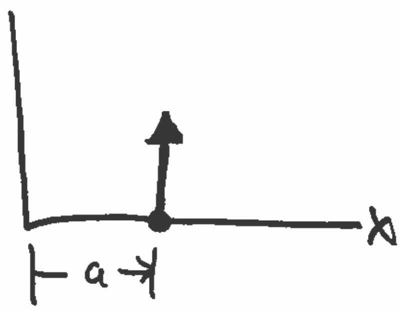
For singularity functions see Table 3-1 on page 90 of the text book or see Wikipedia: "singularity function"

Concentrated Moment $\langle x-a \rangle^{-2}$



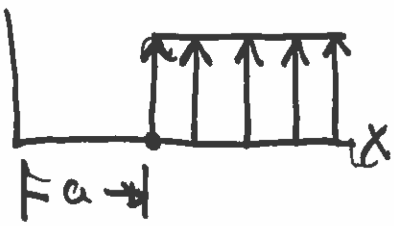
$$\langle x-a \rangle^{-2} = 0 \quad x \neq a$$
$$\langle x-a \rangle^{-2} = \pm\infty \quad x = a$$
$$\int \langle x-a \rangle^{-2} dx = \langle x-a \rangle^{-1}$$

Concentrated force $\langle x-a \rangle^{-1}$



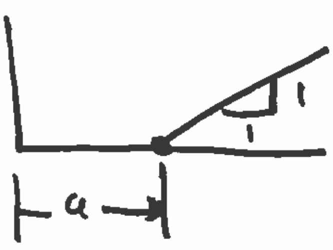
$$\langle x-a \rangle^{-1} = 0 \quad x \neq a$$
$$\langle x-a \rangle^{-1} = +\infty \quad x = a$$
$$\int \langle x-a \rangle^{-1} dx = \langle x-a \rangle^{0}$$

Unit Step $\langle x-a \rangle^{0}$



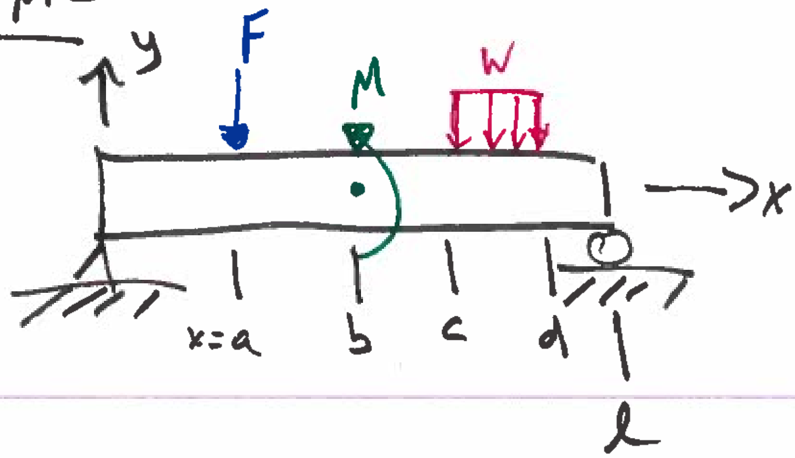
$$\langle x-a \rangle^{0} = \begin{cases} 0 & x < a \\ 1 & x \geq a \end{cases}$$
$$\int \langle x-a \rangle^{0} dx = \langle x-a \rangle^{1}$$

Ramp $\langle x-a \rangle^{1}$



$$\langle x-a \rangle^{1} = \begin{cases} 0 & x < a \\ x-a & x \geq a \end{cases}$$
$$\int \langle x-a \rangle^{1} dx = \frac{\langle x-a \rangle^{2}}{2}$$

Example

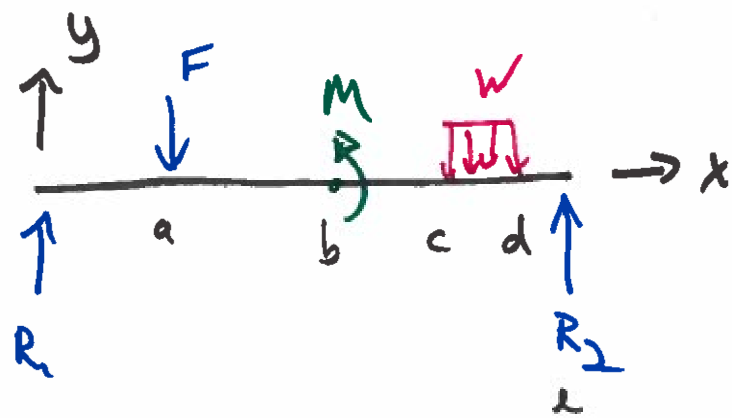


Known: F, M, W, a, b, c, d, l

Find: - reactions @ x=0, x=l

- shear and bending moment diagrams

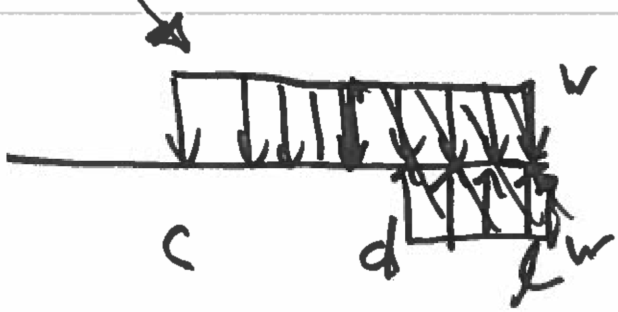
FBD



$$q(x) = R_1\langle x-0\rangle^{-1} - F\langle x-a\rangle^{-1}$$

$$-M\langle x-b\rangle^{-2} \cancel{+} - W\langle x-c\rangle^{0}$$

$$+ W\langle x-d\rangle^{0} + R_2\langle x-\ell\rangle^{-1}$$



$$V(x) = \int q(x)\,dx$$

$$= R_1\langle x\rangle^{0} - F\langle x-a\rangle^{0} - M\langle x-b\rangle^{-1}$$

$$-W\langle x-c\rangle^{1} + w\langle x-d\rangle^{1} + R_2\langle x-\ell\rangle^{0}$$

$$M(x) = \int V(x)\,dx$$

$$= R_1\langle x\rangle^{1} - F\langle x-a\rangle^{1} - M\langle x-b\rangle^{0}$$

$$\frac{-W\langle x-c\rangle^{2}}{2} + \frac{W}{2}\langle x-d\rangle^{2} + R_2\langle x-\ell\rangle^{1}$$

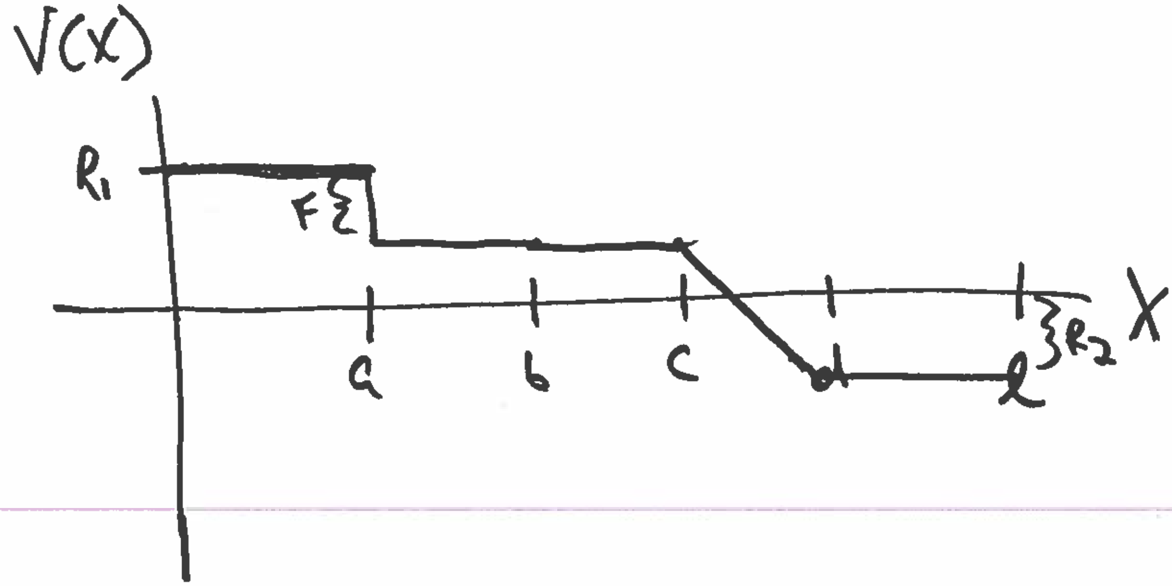
$$\left.\begin{matrix} V(x) \\ M(x) \end{matrix}\right\} = 0 \quad \left\{\begin{matrix} x < 0 \\ x > \ell \end{matrix}\right.$$

$$x = \ell^+ , \quad \underset{\underset{0}{\parallel}}{V(\ell^+)} , \quad \underset{\underset{0}{\parallel}}{M(\ell^+)}$$

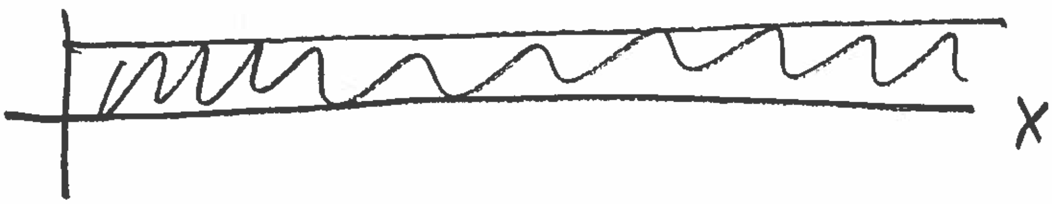
$$0 = V(\ell^+) = R_1\langle \ell^+ \rangle^0 - F\langle \ell - a \rangle^0 - M\langle \ell - b \rangle^{-1}$$
$$- w\langle \ell - c \rangle^1 + w\langle \ell - d \rangle^1 + R_2\langle \ell - \ell \rangle^0$$

$$0 = R_1 - F - w(\ell - c) + w(\ell - d) + R_2$$

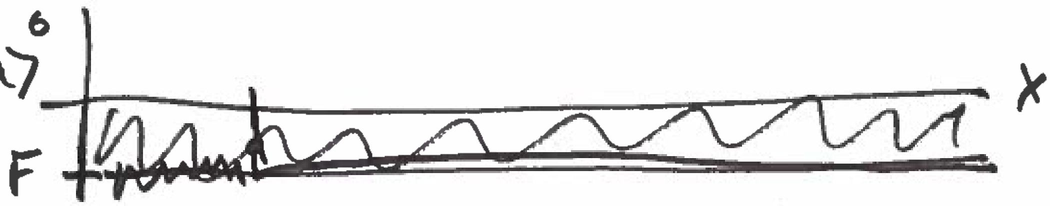
$$0 = M(\ell^+) = R_1 \ell - F(\ell - a) - M - \frac{w}{2}(\ell - c)^2$$
$$+ \frac{w}{2}(\ell - d)^2 + 0$$



$R_1\langle x\rangle^0$



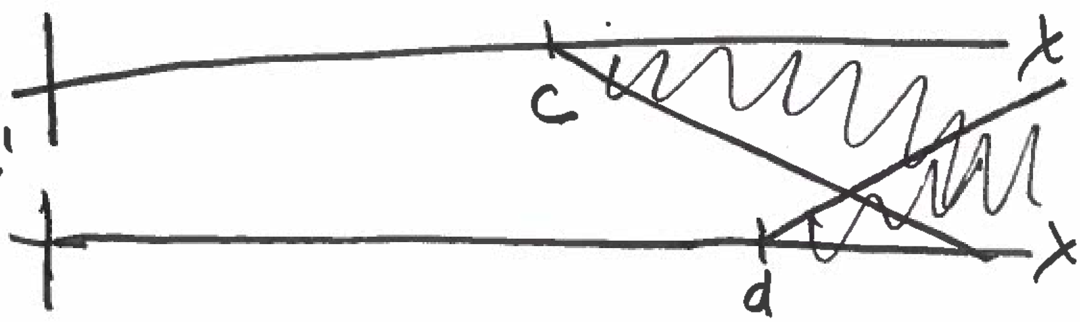
$-\cancel{R}F\langle x-a\rangle^0$



$-M\langle x-b\rangle^0$

$-w\langle x-c\rangle^1$

$w\langle x-c\rangle^1$



+ add with superposition