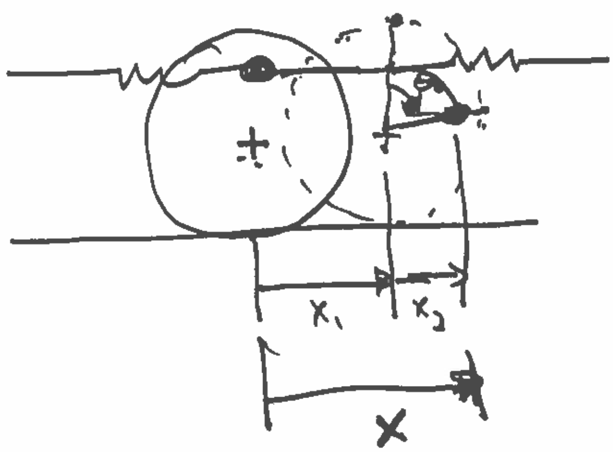
ω_n f_n T_n

$\frac{\text{Rad}}{s}$ $\frac{\text{cycles}}{s}$ $\frac{s}{\text{cycle}}$

Hz $\frac{1}{\text{Hz}}$

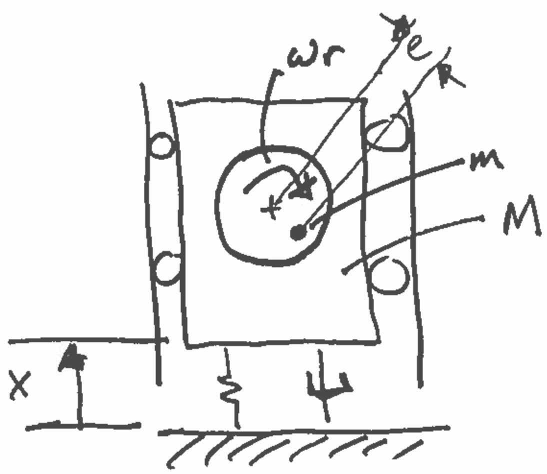
$2\pi f_n = \omega_n$ $T = \frac{1}{f_n}$

P1.80



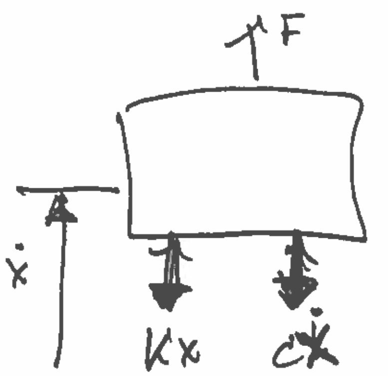
make sure to sum distances

Mass Unbalance (rotating machinery)

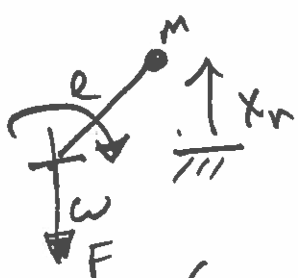


M: total mass (includes m)

m: offset mass on the rotor



$$(M-m)\ddot{x} + c\dot{x} + kx = F$$



$$m\ddot{x}_r = -F$$

$$(M-m)\ddot{x} + c\dot{x} + kx + m\ddot{x}_r = 0$$

$$x_r = x + e\sin\omega_r t$$

$$\dot{x}_r = \dot{x} + e\omega_r\cos\omega_r t$$

$$\ddot{x}_r = \ddot{x} - e\omega_r^2\sin\omega_r t$$

$$M\ddot{x} + c\dot{x} + kx = \underbrace{\underbrace{me\,\omega_r^2}_{\text{amp}} \sin \omega_r t}_{\text{harmonic force due to unbalanced mass}}$$

Steady ~~state~~ solution

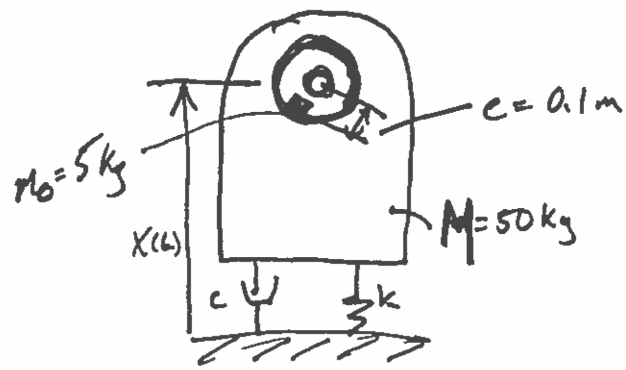
$$x_p(t) = \bar{X} \sin(\omega t - \phi)$$

$\bar{X}$ $$\bar{X} = \frac{me}{M} \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

ϕ $$\phi = \arctan\left(\frac{2\zeta r}{1-r^2}\right)$$

$$\frac{\bar{X} M}{me} = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$ non-dimensional

2.61 LATHE



$\zeta = 0.06$ (viscous)

$f_n = 7.5\ Hz$

$f_p = 30\ Hz$

$X_{ss} = ?$

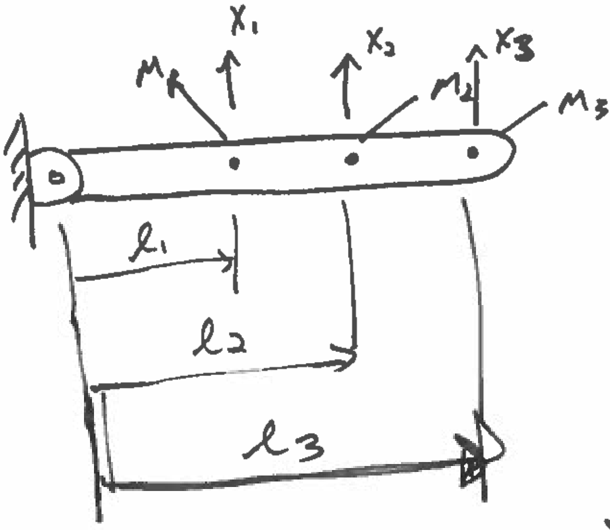
$$X_{ss} = \frac{m_0 e}{M} \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

$$r = \frac{30\,Hz}{7.5\,Hz} = 4$$

$$\boxed{X_{ss} = 0.0106\ m}$$

1 cm

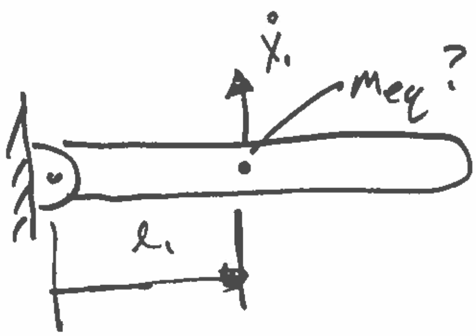
Equivalent Mass



Assume small angles

$$\frac{\dot{x}_2}{\dot{x}_1} = \frac{l_2}{l_1} \qquad \frac{\dot{x}_3}{\dot{x}_1} = \frac{l_3}{l_1}$$

$$T = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 + \frac{1}{2} m_3 \dot{x}_3^2$$

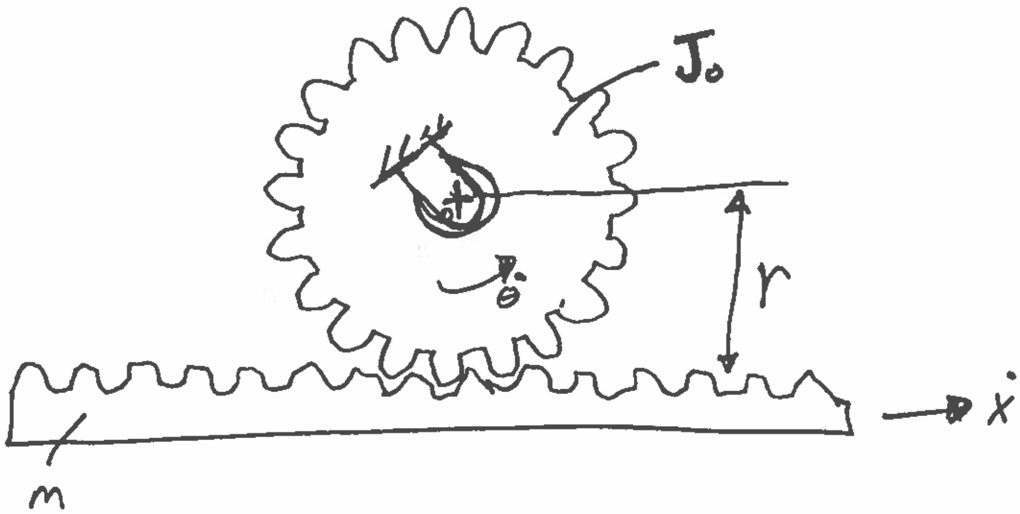


$$T_{eq} = \frac{1}{2} m_{eq} \dot{x}_1^2$$

$$m_{eq} = m_1 + \left(\frac{l_2}{l_1}\right)^2 m_2 + \left(\frac{l_3}{l_1}\right)^2 m_3$$

$$m_{eq} \ddot{x}_1 + c \dot{x}_1 + k x_1 = \cdots$$

Equivalent Mass



1. What is the equivalent rotational inertia of the rack?

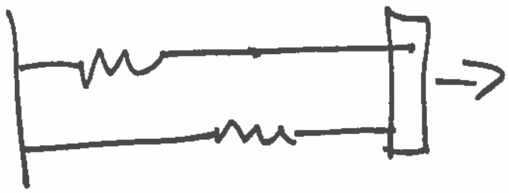
2. What is the equivalent translational mass of the pinion?

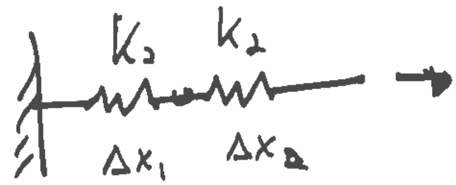
1. $m_{eq} = m + \frac{J_o}{r^2}$ $\qquad\qquad \dot{\theta} r = \dot{x}$

$$\left.\begin{aligned} T &= \tfrac{1}{2} m \dot{x}^2 + \tfrac{1}{2} J_o \dot{\theta}^2 \\ T_{eq} &= \tfrac{1}{2} m_{eq} \dot{x}_{eq}^2 \end{aligned}\right\} \quad m_{eq} = m + J_o/r^2$$

2. $T_{eq} = \frac{1}{2} J_{eq} \dot{\theta}_{eq}^2$ $\qquad\qquad J_{eq} = J_o + m r^2$

Equivalent Stiffness





$$U = \tfrac{1}{2} k_1 \Delta x_1^2 + \tfrac{1}{2} k_2 \Delta x_2^2$$

$$U_{eq} = \frac{1}{2} k_{eq} (\Delta x_1 + \Delta x_2)^2$$

solve $\underline{k_{eq}}$

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$$

Equivalent Damping

equivalency to viscous damping

How much energy does a viscous damper lose per cycle? $\quad c\dot{x}$

$$\Delta E = \int_{cycle} \underset{\text{damping}}{F_d}\, dx = \int_{cycle} c\dot{x}\, dx = \int_0^{\frac{2\pi}{\omega}} c \frac{dx}{dt} \cdot \frac{dx}{dt}\, dt$$

$$\Delta E = \int_0^{\frac{2\pi}{\omega}} c \dot{x}^2 dt$$

$$\dot{x} = \omega \bar{X} \cos \omega t$$

$$\Delta E = \int_0^{\frac{2\pi}{\omega}} c(\omega X \cos\omega t)^2\, dt = c\int_0^{\frac{2\pi}{\omega}} \omega^2 X^2 \cos^2\omega t\, dt$$

$$= c\omega X^2\left(\frac{1}{2}\omega t + \frac{1}{4}\sin 2\omega t\right)\Bigg|_0^{\frac{2\pi}{\omega}}$$

$$\boxed{\Delta E = \pi c \omega X^2}$$

energy loss per cycle for simple viscous damping

<u>Coulomb Damping</u>

$F_0 \gg \mu N$ approximate equivalency to viscous damping

$$\Delta E = \int F_d \frac{dx}{dt}\, dt$$

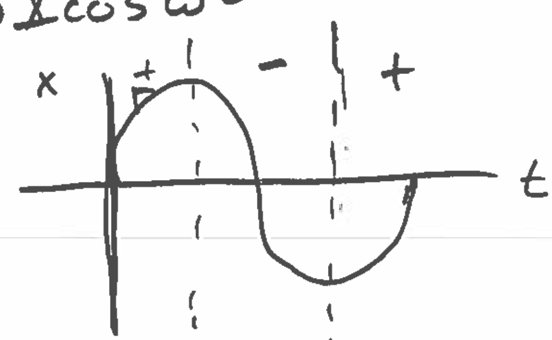
$N=mg$

$$m\ddot{x} + \underbrace{mg\mu\, \mathrm{sgn}(\dot{x})}\,\dot{x} + kx = F\sin\omega t$$

$$= \int_0^{\frac{2\pi}{\omega}} \mu N\, \mathrm{sgn}(\dot{x})\,\dot{x}\, dt$$

$x = X \sin \omega t$

$\dot{x} = \omega X \cos \omega t$



$$= \mu N X\left(\int_0^{\frac{\pi}{2}} \cos(\omega t)\, d(\omega t) - \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos(\omega t)\, d(\omega t) + \int_{\frac{3\pi}{2}}^{2\pi} \cos\omega t\, d(\omega t)\right)$$

$$\Delta E_c = \mu N X\left[1-(-1-1)+1\right] = 4\mu N X$$

$$\Delta E_c = 4\mu N X \qquad \Delta E = \pi c \omega X^2$$

$$\Delta E_c = \Delta E \implies \boxed{c_{eq} = \frac{4\mu N}{\pi \omega X}}$$

$$\zeta_{eq} = \frac{2\mu g}{\pi \omega_n \omega X}$$

$$X = \frac{F_0/k}{\sqrt{(1-r^2)^2 + (2\zeta_{eq} r)^2}} = \frac{F_0}{k}\frac{\sqrt{1-(4\mu m g/\pi F_0)^2}}{|(1-r^2)|}$$

$$\Theta = \arctan\left(\frac{2\zeta_{eq} r}{1-r^2}\right) = \frac{\pm 4\mu m g}{\pi F_0\sqrt{1-(4mg/\pi F_0)^2}}$$

X pos if $r<1$

$-$ if $r>1$

Θ is constant wrt frequency

$r=1$ $\quad X \Rightarrow \infty$ and phase is discontinuous

Specific damping capacity: $\frac{\Delta E}{U} = \frac{\text{energy loss per cycle}}{\text{peak potential energy}}$

loss factor $\eta = \frac{\Delta E}{2\pi U_{max}} = \frac{\text{energy loss per cycle}}{\text{potential energy at max disp.}}$

@ $r=1$

$$\eta = 2\zeta$$

<u>Example</u> Aerodynamic damping

$$F_{drag} = a V^2 \operatorname{sgn}(v)$$

EOM $m\ddot{x} + a\dot{x}^2 \operatorname{sgn}(\dot{x}) + kx = F_0 \cos \omega t$

$$\Delta E_a = \int_0^{\frac{2\pi}{\omega}} F_d \cdot \dot{x}\, dx = \frac{8}{3} a \omega^2 X^3$$

$$\underbrace{\Delta E = \pi c_{eq} \omega X^2 = \Delta E_a}$$

$$c_{eq} = \frac{8}{3\pi} a \omega X$$

Example "Hysteretic Damping"

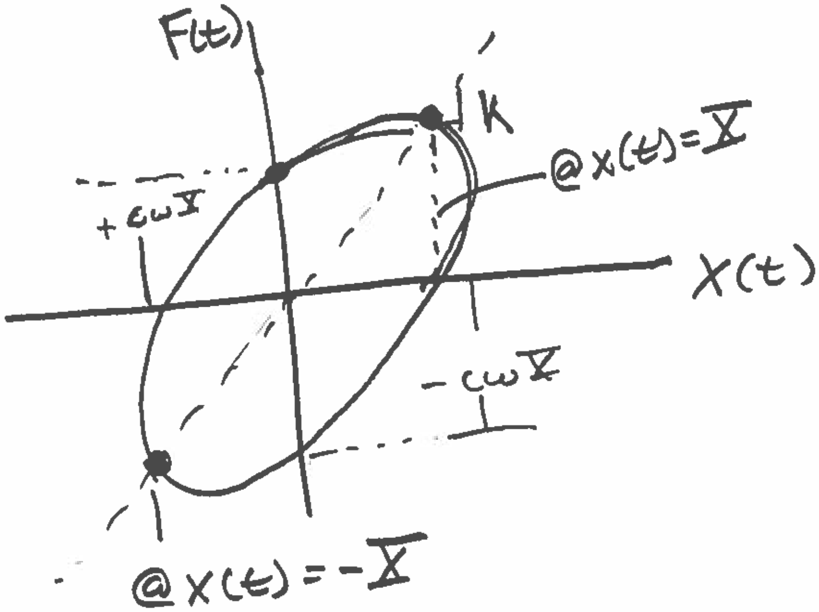
$$F(t) = c\,\dot{x}(t) + k\,x(t)$$

$F(t) = c\omega X \cos\omega t + kX \sin\omega t$

$F(t) = kx \pm c\omega\sqrt{X^2 - x^2}$

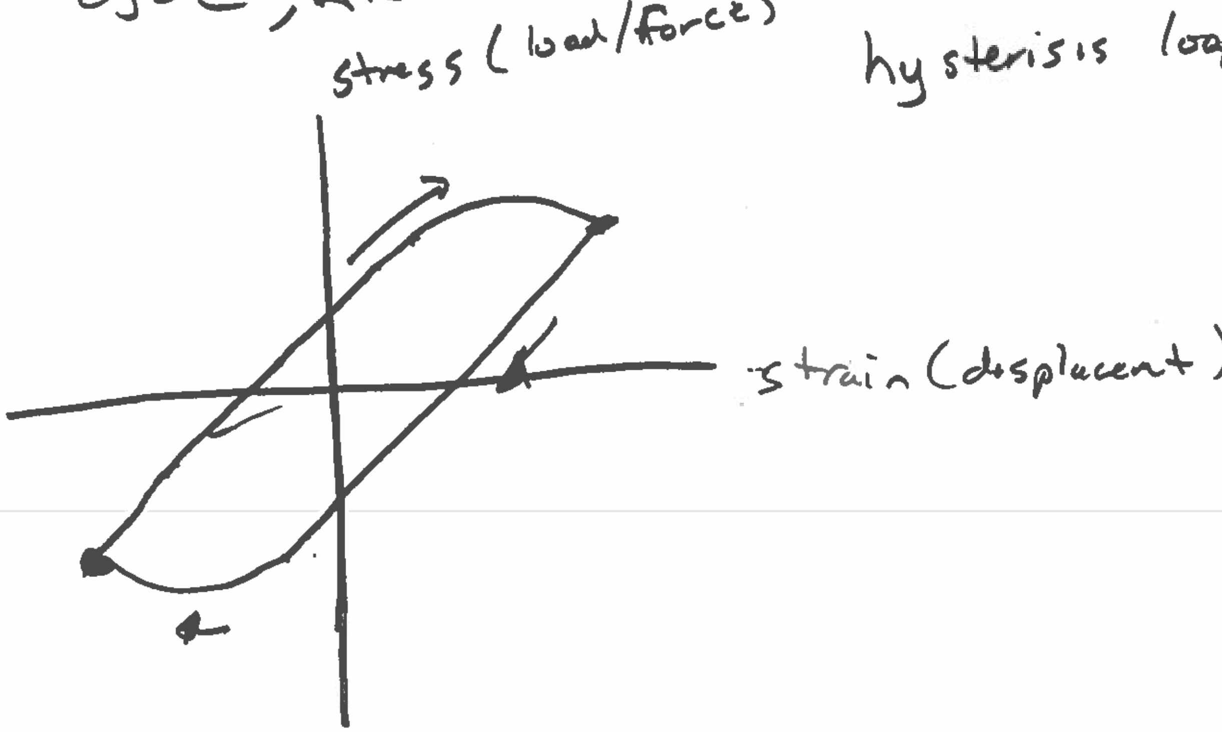
for $x = 0$

$F = \pm c\omega X$



area enclosed is the energy loss per cycle, ΔE

hysterisis loop



area in the ellipse is πab

a, b : vertex lengths

$$\Delta E = \pi \underbrace{c\omega} X^2$$

hysteretic damping coefficient

$$\Delta E_h = \pi h X^2 = \Delta E_c \Rightarrow C_{eq} = \frac{h}{\omega}$$

goal to find h for abnormal hysteresis loops

Forced Vibrations with Nonlinear Damping

Type	ΔE	C_{eq}
Viscous	$\pi c\omega X^2$	c
Coulomb	$4\mu N X$	$\frac{4\mu N}{\pi\omega X}$
Aerodynamic	$\frac{8}{3}a\omega^2 X^3$	$\frac{8}{3}a\omega X$
Hysteretic	$\pi h X^2$	$\frac{h}{\omega}$