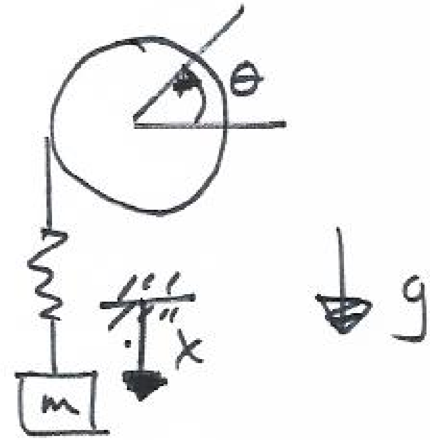
$$k_h = \frac{EA}{L} \qquad k_v = \frac{EA}{L/2}$$

Disc

$$I = \frac{Mr^2}{2}$$

$$T = \frac{1}{2}\left(\frac{mr_1^2}{2}\right)\dot{\theta}^2 + \frac{1}{2}\left(\frac{mr_2^2}{2}\right)\dot{\theta}^2 + \frac{1}{2}m\dot{x}^2$$

$$U = \frac{1}{2}k_h(r_1\theta)^2 + \frac{1}{2}k_v(x - r_2\theta)^2 - mgx$$




$L = T - U$ Lagrangian

$$① \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0$$

$$② \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$$

2 DOF

$$\frac{d}{dt}\left(\frac{1}{2}mr_1^2\dot{\theta} + \frac{1}{2}mr_2^2\dot{\theta}\right) + k_h r_1^2\theta + k_v(-r_2)(x - r_2\theta) = 0$$

$$\frac{d}{dt}(m\dot{x}) + k_v(x - r_2\theta) - mg = 0$$

$$\frac{m}{2} r_1^2 \ddot{\theta} + \frac{m}{2} r_2^2 \ddot{\theta} + k_h r_1^2 \theta + k_v r_2 (x - r_2 \theta) = 0$$

$$m\ddot{x} + k_v x - k_v r_2 \theta - mg = 0$$

$M$

$$\begin{bmatrix} \frac{m}{2}(r_1^2 + r_2^2) & 0 \\ 0 & m \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{x} \end{bmatrix} + \begin{bmatrix} k_h r_1^2 + k_v r_2^2 & \\ & \end{bmatrix} \begin{bmatrix} \theta \\ x \end{bmatrix} = \begin{bmatrix} 0 \\ mg \end{bmatrix}$$

$$K \quad \begin{bmatrix} k_h r_1^2 + k_v r_2^2 & -k_v r_2 \\ -k_v r_2 & k_v \end{bmatrix}$$

$x = L^{-1} \underset{\sim}{q}$ $\qquad M = L L^T$

$\uparrow$ cholesky decomp.

$$L = \begin{bmatrix} \sqrt{m/2 (r_1^2 + r_2^2)} & 0 \\ 0 & \sqrt{m} \end{bmatrix} \qquad L^{-1} = \begin{bmatrix} \frac{1}{\sqrt{m/2 (r_1^2 + r_2^2)}} & 0 \\ 0 & \frac{1}{\sqrt{m}} \end{bmatrix}$$

$$\begin{vmatrix} \dfrac{k_h r_1^2 + k_v r_2^2}{m^*} - \lambda & -\dfrac{k_v r_2}{\sqrt{m^*}\sqrt{m}} \\ -\dfrac{k_v r_2}{\sqrt{m^*}\sqrt{m}} & \dfrac{k_v}{m} - \lambda \end{vmatrix} = 0$$

$$\left(\frac{k_h r_1^2 + k_v r_2^2}{m^*} - \lambda\right)\left(\frac{k_v}{m} - \lambda\right) - \frac{k_v^2 r_2^2}{m^* m} = 0$$

$$\lambda^2 - \left(\frac{k_h r_1^2 + k_v r_2^2}{m^*} + \frac{k_v}{m}\right)\lambda + \frac{k_v k_h r_1^2 + \cancel{k_v^2 r_2^2}}{m m^*} - \cancel{\frac{k_v^2 r_2^2}{m^* m}} = 0$$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2}$$

$$\lambda^2 - \left(\frac{k_h r_1^2 + k_v r_2^2}{m^*} + \frac{k_v}{m}\right)\lambda + \frac{k_v k_h r_1^2}{m m^*} = 0$$

$a = 1$

$$b = -\left(\frac{k_h r_1^2 + k_v r_2^2}{m^*} + \frac{k_v}{m}\right)$$

$$c = \frac{k_v k_h r_1^2}{m m^*}$$

$$\tilde{K} = L^{-1} K L^{-1}$$

$$\begin{bmatrix} \dfrac{1}{\sqrt{\underbrace{m/2\,(r_1^2 + r_2^2)}_{m^*}}} & 0 \\ 0 & \dfrac{1}{\sqrt{m}} \end{bmatrix} \begin{bmatrix} k_h r_1^2 + k_v r_2^2 & -k_v r_2 \\ -k_v r_2 & k_v \end{bmatrix}$$

$$\begin{bmatrix} \dfrac{k_h r_1^2 + k_v r_2^2}{\sqrt{m^*}} & -\dfrac{k_v r_2}{\sqrt{m^*}} \\ \dfrac{-k_v r_2}{\sqrt{m}} & \dfrac{k_v}{\sqrt{m}} \end{bmatrix} \begin{bmatrix} \dfrac{1}{\sqrt{m^*}} & 0 \\ 0 & \dfrac{1}{\sqrt{m}} \end{bmatrix}$$

$$\begin{matrix} \dfrac{k_h r_1^2 + k_v r_2^2}{m^*} & \cancel{0} & -\dfrac{k_v r_2}{\sqrt{m^*}\sqrt{m}} \\ -\dfrac{k_v r_2}{\sqrt{m^*}\sqrt{m}} & \cancel{-\dfrac{k_v r_2}{\sqrt{m^*}\sqrt{m}}} & \dfrac{k_v}{m} \end{matrix}$$

$$\tilde{K} = \begin{bmatrix} \dfrac{k_n r_1^2 + k_v r_2^2}{m^*} & -\dfrac{k_v r_2}{\sqrt{m^*}\sqrt{m}} \\ & \\ -\dfrac{k_v r_2}{\sqrt{m^*}\sqrt{m}} & \dfrac{k_v}{m} \end{bmatrix}$$

$$\ddot{\bar{q}} + \tilde{K}\bar{q} = L^{-1}\begin{bmatrix} 0 \\ mg \end{bmatrix}$$

$$\ddot{\bar{q}} + \tilde{K}\bar{q} = \begin{bmatrix} 0 \\ \frac{mg}{\sqrt{m}} \end{bmatrix}$$

$$\lambda \tilde{K} = \tilde{K}\bar{v}$$

$\rightarrow$ orthogonal vectors

$$\det\left[\tilde{K} - \lambda I\right] = 0$$

$$\lambda = \frac{\frac{k_h r_1^2 + k_v r_2^2}{m^*} + \frac{k_v}{m} \pm \sqrt{\left(\frac{k_h r_1^2 + k_v r_2^2}{m^*} + \frac{k}{m}\right)^2 - \frac{4 k_v k_h r_1^2}{m m^*}}}{2}$$

$$\lambda = \omega^2$$

$$\lambda \tilde{K} = \bar{v} \tilde{K}$$

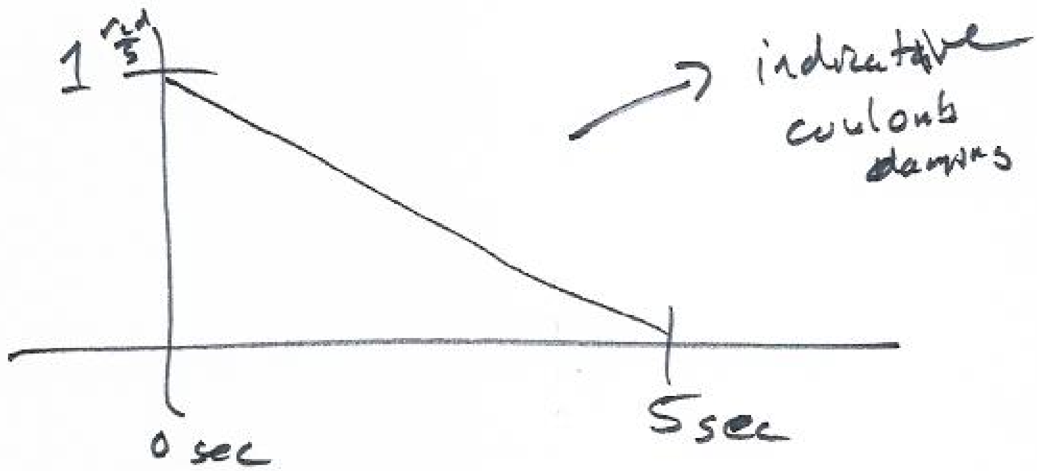
$K_h \quad K_v$

$$k_h = \frac{EA}{L} \qquad k_v = \frac{EA}{L/2}$$

$$k_v = \frac{k_h}{1/2} \Rightarrow 2k_h = k_v$$

$\cancel{2k_v = k_h}$

$$\lambda = \frac{k_h r_1^2 + 2k_h r_2^2}{m^*} + \frac{2k_h}{m} \pm \sqrt{\phantom{x}}$$




$$m\ddot{x} + c\dot{x} + kx = a_0 + \sum a_n \cos\theta + \sum b_n \sin\theta$$

$$X_p = X$$

$$m\ddot{x} + c\dot{x} + kx = \frac{a_0}{2} \Rightarrow x(t) = \frac{a_0}{2k}$$

$$m\ddot{x}_{cn} + c\dot{x}_{cn} + kx_{cn} = a_n \cos n\omega_T t$$

$$m\ddot{x}_{sn} + c\dot{x}_{sn} + kx_{sn} = b_n \sin n\omega_T t$$

$$X_{cn}(t) = \frac{a_n/m}{\left[\left[\omega_n^2 - (n\omega_T)^2\right]^2 + (2\zeta\omega_n n\omega_T)^2\right]^{1/2}} \cos(n\omega_T - \theta_n)$$

$$X(t) = Ae^{-\zeta\omega_n t}\sin(\omega_d t + \phi)$$

$$+ \frac{a_0}{2k} + \sum_{n=1}^{\infty}\left[x_{cn}(t) + x_{sn}(t)\right]$$

$$\theta_n = \tan^{-1}\frac{2\zeta\omega_n n\omega_T}{\omega_n^2 - (n\omega_T)^2}$$

$A, \phi \Rightarrow$ initial conditions