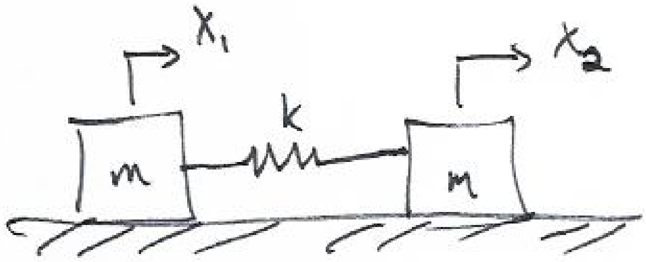
What if K is singular? (M can never be singular)

Example



EOM

$$m\ddot{x}_1 = -k(x_1 - x_2)$$
$$m\ddot{x}_2 = -k(x_2 - x_1)$$

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} k & -k \\ -k & k \end{bmatrix}}_{K}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\det(K) = k^2 - k^2 = 0 \implies$ singulor matrix

Find eigenvalus

$$M^{-1}K = \begin{bmatrix} \frac{1}{m} & 0 \\ 0 & \frac{1}{m} \end{bmatrix}\begin{bmatrix} k & -k \\ -k & k \end{bmatrix}$$

$$K' = \begin{bmatrix} \frac{k}{m} & -\frac{k}{m} \\ -\frac{k}{m} & \frac{k}{m} \end{bmatrix}$$

$$\det(I\omega^2 - K') = 0$$

$$\begin{vmatrix} \omega^2 - \frac{k}{m} & \frac{k}{m} \\ \frac{k}{m} & \omega^2 - \frac{k}{m} \end{vmatrix} = 0$$

$$(\omega^2)^2 - 2\frac{k}{m}\omega^2 + \cancelto{0}{\left(\frac{k}{m}\right)^2 - \left(\frac{k}{m}\right)^2} =$$

$$\omega^4 - 2\frac{k}{m}\omega^2 = 0$$

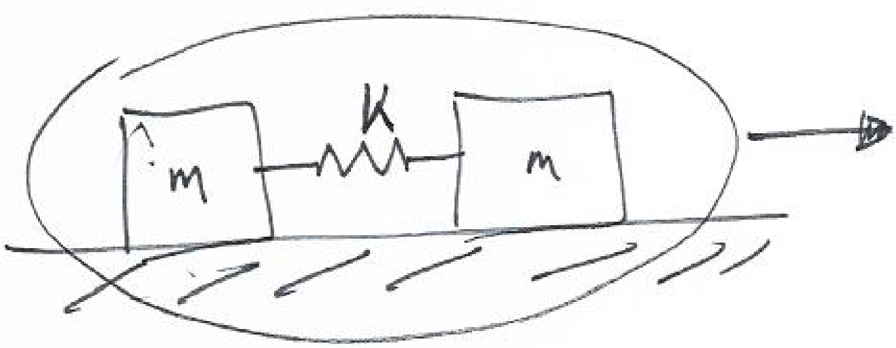
$$\omega^2 = 0 \qquad \text{or} \qquad \omega^2 = 2\frac{k}{m}$$

$$\omega_1 = 0 \qquad\qquad \omega_2 = \sqrt{2\frac{k}{m}}$$

$\uparrow$

zero frequency

$\boxed{\text{Rigid Body Mode}}$



Multi-DoF with Damping (free vibration)

$$M\ddot{x} + C\dot{x} + Kx = 0$$

M, C, K $n \times n$ matrices $\quad x$ is column vector of n generalized coordinates

The modal analysis transforms $x \to r$ and decouples the M and K matrices.

In general, if you have damping, i.e. C matrix, the equations cannot be decoupled with the modal transform.

Special case: proportional damping

If $\quad C = \underbrace{\alpha M + \beta K}_{\text{scalars}}$

$$M\ddot{x} + (\alpha M + \beta K)\dot{x} + Kx = 0$$

$M \to I$, $K \to \Lambda$ with **S** diagonilizing ~~M and K~~ M and K

$$\ddot{r} + \underbrace{(\alpha I + \beta \Lambda)}_{\text{modal damping}}\dot{r} + \Lambda r = 0 \quad \text{M and K}$$

$2\zeta_i\omega_i$

$\zeta_i \to$ modal damping ratio

$$\zeta_i = \frac{\alpha}{2\omega_i} + \frac{\beta \omega_i}{2}$$

$$r_i = A_i e^{-\zeta_i \omega_i t} \sin\left(\omega_{di} t + \phi_i\right) \qquad i = 1, \ldots, n$$

Any damping that is not proportional will require numerical tools to find eignenvalues and mode shapes.

$$\left.\begin{matrix} A_i \\ \phi_i \end{matrix}\right\} \text{ from initial conditions}$$