Monday, September 26
2016

History of Vibrations

Galileo (Italian) Father of experimental science
(1564-1642)
- simple pendulum
- resonance frequencies
- telescope : moons jupiter

(Fr) Joseph Sauveur (1653-1716)
- vibrating string mode shapes
- frequency vs length
- harmonics & beats

Newton (En) (1642 -1727)
- calculus, gravational equations
- Laws of motion ⇒ F=ma
- Principia 1687

Brook Taylor (En) 1685-1731
- Taylor's thereom
- Taylor Series
- theorectical solution vibrating string

Daniel Bernoulli (Swiss) 1700-1782
Leonhard Euler 1707-1783
Bernoulli-Euler Beam Theory
- Newton-Euler equations

Joseph-Louis Lagrange It (1736 -1813)
- generalized coordinates
- alternative way to form equations

Charles-Augustin de Coulomb (Fr)

(1736-1806) - Friction, torsional oscillations

E.F.F Chladni (German) 1756-1827

- vibrating plates

Sophie Germain

(1776-1831) . plate theory (1809)

Kirchoff - corrected plate boundary conditions

(1824-1887)

Poisson (Fr) - rectangular membrane vibration

(1781-1840)

Frahm (1909) vibration absorber.

Stephen Timoshenko (Ukraine)

(1878-1972) - Improved beam vibration theory significantly

- Father applied mechanics

J.P. Den Hartog

(1901-1989) - The primary authority on applied vibrations.

Mechanical Vibrations

Three essential factors:

① Inertia of the oscillatory motion ($m, I, ..$)
(Kinetic energy storage)

② Restoring forces (elastic)
(Potential energy storage)

③ Dissapative mechanism (Energy loss)

Degrees of Freedom: the number of independent coordinates to describe the motion of a system

$\boxed{\text{DoF}}$

Classify diff. vibraton systems

I	Discrete finite # of DoF	VS	Continous systems Infinite # of DoF
II	Free of external forces	VS	Excited by external Forces
III	Damped has energy disappation	VS	Undamped doesn't

IV Linear vs non-linear
ODE ODE

IV Determinstic vs Random/stochastic
System System

F=ma are enough to predict motion

predictable in the statistical sense

Analysis Procedure

1) Mathematical model of reality
   ODE/PDE that determine the system state
2) Derive the Equations of Motion
3) Seek solutions to the equations
4) Interpret results

Mathematical Models of Reality

$F = ma$ Second order ordinary differential equation

Prescribe F, m, a or T, I, α to precisely describe a system's motion.

Concepts:

- Rigid bodies (lumped masses with inertia, not flexible)
- dof
- reference frames (need inertial frame for EoM derivation)
- coordinates describe config of sys.
  - Generalized coordinates
    ↳ are minimum set of coordinates that uniquely define the config.

Derive the EoM of a simple pendulum

① Direct Method (F.B.D + Newton-Euler Equations)

② Indirect Method (Lagrange, Hamilton's, Kane' method, etc)

Lagranges Method

"Energy method"

kinetic energy: $T = \sum_{i=1}^{N} \frac{1}{2} m_i v_i^2$

$= \sum_{i=1}^{N} \frac{1}{2} m_i \dot{x}_i^2$

Momentum of particle

$$\frac{\partial T}{\partial \dot{x}_i} = m_i v_i = p_i$$

$\frac{dp}{dt} = \Sigma F$

Newton's Law

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_i}\right) = m_i \ddot{x}_i$$

For conservative forces

$$F_i = -\frac{\partial U}{\partial x_i}$$ — potential energy

From $\vec{F} = m\vec{a}$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_i}\right) = -\frac{\partial U}{\partial x_i} \qquad (1)$$

"ma" "F"

$$\frac{\partial T}{\partial x_i} = 0 \quad , \quad \frac{\partial U}{\partial \dot{x}_i} = 0 \qquad (2)$$

Define <u>The</u> <u>Lagrangian</u>

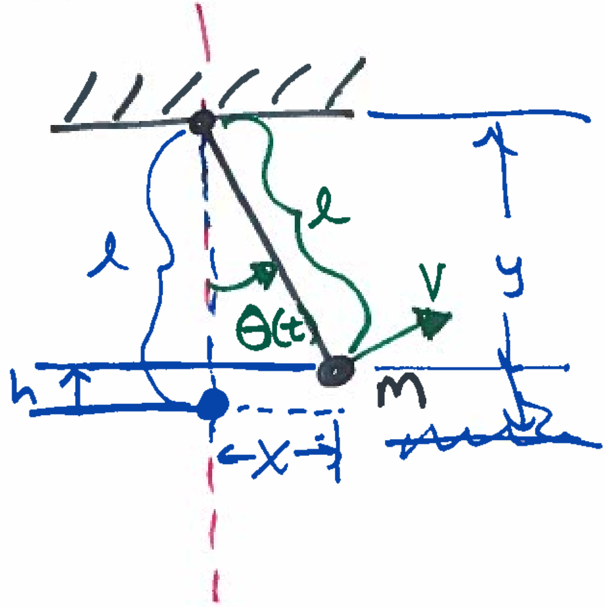
$$L = T - U$$

$$\frac{d}{dt}\left(\frac{\partial (T-U)}{\partial \dot{x}_i}\right) = \frac{\partial (T-U)}{\partial x_i}$$

$$\boxed{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_i}\right) = \frac{\partial L}{\partial x_i}}$$

Lagrange Equation (of the second kind)

Example



$$\dot{\theta}\,\ell = v$$

$$h = \ell - \ell\cos\theta) = \ell(1-\cos\theta)$$

$$T = \frac{1}{2} m v^2 = \frac{1}{2} m (\dot{\theta}\ell)^2$$

$$U = mgh = mg\left[\ell(1-\cos\theta)\right]$$

$$L = T - U = \frac{1}{2} m \dot{\theta}^2 \ell^2 - mg\ell(1-\cos\theta)$$

$$\frac{\partial L}{\partial \dot{\theta}} = m\ell^2\dot{\theta} \qquad \frac{\partial L}{\partial \theta} = -mg\ell\sin\theta$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = m\ell^2\ddot{\theta}$$

$$\boxed{\cancel{m}\ell^{\cancel{2}}\ddot{\theta} = -\cancel{m}g\cancel{\ell}\sin\theta}$$ EoM

Second order ODE in $\underline{\underline{\theta}}$

$$\ddot{\Theta} = -\frac{g}{l}\sin\Theta$$ non-linear

$$\sin\Theta \approx \Theta$$

$$\ddot{\Theta} = -\frac{g}{l}\Theta$$ linearized