Hyperspectral Inverse Skinning

Team 21

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1 INTRODUCTION

Linear blend skinning (LBS) is a popular method for efficient and realistic animation, where a vertex's position is computed as a weighted average of transformation matrices, simplifying animation data handling. In inverse skinning, given only observed deformations, the goal is to retrieve skinning weights and transformations. Traditional methods for this can struggle with sparsity and rigidity constraints. This work reframes inverse skinning as a high-dimensional optimization problem, where transformations are vertices in a simplex. By fitting the smallest-volume simplex to the observed data, our method minimizes error, enabling high-fidelity skinning solutions with applications in areas like hyperspectral imaging.

2 LITERATURE REVIEW

2.1 Skinning Decomposition

The early research on inverse Linear Blend Skinning (LBS) began with a study by Wang and Phillips [10] to address artifacts like joint collapse and "candy-wrapper" distortions. They improved LBS by associating unique weights for each transformation matrix entry. [6] advanced the study of inverse LBS by introducing an approach that extracted rotations from mesh triangles and used clustering to estimate initial bone transformations. Further developments in inverse LBS research included determining bone skeleton hierarchies through clustering of transformations (e.g., [9]). The authors highlight a distinct approach by using convex geometric structures in transformation matrices as points in a high-dimensional space, which does not involve skeleton hierarchies or rigid transformations. This method uses the vertices of a convex hull (simplex) as handles, optimizing error with fewer bones than previous clustering-based approaches. [7] explored inverse skinning through matrix factorization, optimizing transformations, skinning weights, and rest pose positions efficiently by assuming weight sparsity. Le and Deng's work addressed skinning decomposition, weight compression, and automatic rigging with a bone skeleton. In comparison, this method avoids rigid transformations and achieves faster results with lower errors.

2.2 Surface Registration

Both our approach and the aforementioned inverse LBS approaches assume vertex correspondences across all mesh poses. However, some related work does not require this assumption. [3] presented a series of works on surface registration for articulated shapes, employing LBS with binary weights to segment the surface at joints and restrict transformations to rigid motions. Iterative Closest Point (ICP) has been extensively studied in surface registration, with [1] extending ICP to nonrigid registration using adjustable stiffness regularization. For 3D shape registration, deformable registration with known mesh topology is of interest, with works utilizing spin-image methods for reliable correspondences and as-rigid-as-possible deformation.

2.3 Subspace Clustering

Subspace clustering studies, such as [11] and [5], address the clustering of points onto flats or closest-point clusters, though they do not consider cases where both input and output data are flats. Our approach, in contrast,

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addresses clustering within an LBS subspace, where all transformations lie on the same flat. The flat optimization problem involves finding the optimal flat distance between deformed and undeformed vertex positions.

2.4 Hyperspectral Unmixing

Hyperspectral unmixing, or unsupervised hyperspectral unmixing, aims to recover spectral signatures (endmembers) and their abundances from hyperspectral images. Methods developed by [4] and others seek to solve this problem by finding the minimum-volume simplex enclosing observed points, despite challenges of local minima. Algorithms now efficiently address this problem even with outliers [2]. Nonnegative Matrix Factorization (NMF), an equivalent problem to hyperspectral unmixing, is NP-complete in general, though separable NMF problems are polynomially solvable. In our case, we avoid the pure-pixel assumption and nonnegative constraints.

2.5 Hyperspectral Inverse Skinning

Inverse skinning can be reformulated as a problem in high-dimensional Euclidean space. The transformation matrices applied to a vertex across all poses can be conceptualized as points in this high-dimensional space. Liu et al. approach the inverse linear blend skinning (LBS) problem by finding a tight-fitting simplex around these points, a problem that has been well-studied in hyperspectral imaging. Although transformation matrices are not directly observed, the 3D positions of a vertex across all poses define an affine subspace, or flat. This leads to the formulation of a “closest flat” optimization problem to identify points on these flats. Subsequently, they compute a minimum-volume enclosing simplex, with vertices corresponding to the transformation matrices and barycentric coordinates representing the skinning weights. This method enables the creation of LBS rigs that achieve state-of-the-art reconstruction error and compression ratios for mesh animation sequences. Their proposed solution does not account for weight sparsity or the rigidity of the recovered transformations. They provide observations and insights into the closest flat problem, noting that its ideal solution and optimal LBS reconstruction error remain open questions [8].

3 MILESTONES

The following milestones were established to guide the implementation of the Hyperspectral Inverse Skinning project:

S. No.	Milestone	Member
	Interim Evaluation	
1	Framework design	Arpan Verma
2	Implementing flat optimization	Arpan Verma
3	Hyperspectral data collection	Harshit Gupta
4	Implementing the Initial Guess Code	Harshit Gupta
	Final evaluation	
5	Handle Transformation Estimation	Arpan Verma
6	Minimum Volume Enclosing Simplex Algorithm	Arpan Verma
7	Skinning weights estimation	Harshit Gupta
8	Results Analysis and Testing	Harshit Gupta
9	Exploring Future Works	Arpan Verma and Harshit Gupta

4 MILESTONES ACHIEVED FOR PROJECT EVALUATION-II:

We have completed the following Milestones:

- Framework Design.
- Data Collection and visualization.
- Initial Guess Code.
- Apply Per Vertex Transformation/ Transformation Estimation.
- Implement Flat Optimization. ‘

We have completed all the set milestones for this deadline and also moved forward and completed the first milestone of the Final Evaluation.

5 ALGORITHMS IMPLEMENTED:

The algorithms implemented were mainly of

Stage 1: Per-Vertex Transformation

The goal of this stage is to estimate a point $x \in \mathbb{R}^{12\cdot\#\text{poses}}$ for each vertex that minimizes the distance between its associated flat and a guessed handle flat.

Formulation. Each vertex i defines a flat as:

$$V_i x = v_i',$$

where:

- $V_i = I_{3\cdot\#\text{poses}} \otimes v_i^\top$,
- $\otimes$ denotes the Kronecker product,
- v_i is the undeformed vertex in homogeneous coordinates,
- v_i' is the stacked positions of v_i across all poses.

We compute an initial estimate x_i by solving the following optimization problem:

$$x_i =_x \sum_{j\in\{i\}\cup N(i)} \left\| \frac{1}{\|v_j\|^2} V_j^\top V_j (x - t_j) \right\|^2,$$

where:

- $N(i)$ denotes the one-ring neighborhood of vertex i,
- t_j is any valid transformation matrix in j's flat,
- The divisor $\|v_j\|^2$ normalizes the rows of V_j.

For computational simplicity, this can be rewritten as:

$$x_i =_x \sum_{j\in\{i\}\cup N(i)} \|V_j x - v_j'\|^2.$$

Stage 2: Flat Optimization

This stage aims to find the $(h-1)$-dimensional handle flat L that minimizes the squared Euclidean distance to all vertex flats.

Formulation. The handle flat L is parameterized as:

$$L = \{p + Bz : z \in \mathbb{R}^{h-1}\},$$

where:

- $p \in \mathbb{R}^{12\cdot\#\text{poses}}$ is a point on the flat,
- $B \in \mathbb{R}^{(12\cdot\#\text{poses})\times(h-1)}$ is a matrix whose columns span directions parallel to the flat.

We solve the following optimization problem:

$$ {}_{p,B} \sum_{i} \min_{z_i} \|V_i(p + Bz_i) - v_i'\|^2, $$

where $z_i \in \mathbb{R}^{h-1}$ are the parameters for the closest point on L.

The solution for z_i is:

$$ z_i = -(B^\top V_i^\top V_i B)^{-1} B^\top V_i^\top (V_i p - v_i'). $$

Substituting z_i back, the problem becomes:

$$ {}_{p,B} \sum_{i} \|V_i(p + Bz_i) - v_i'\|^2, $$

subject to the constraints:

$$ \begin{aligned} 1^\top w_i &= 1, \quad \forall i, \\ w_i &\geq 0, \quad \forall i. \end{aligned} $$

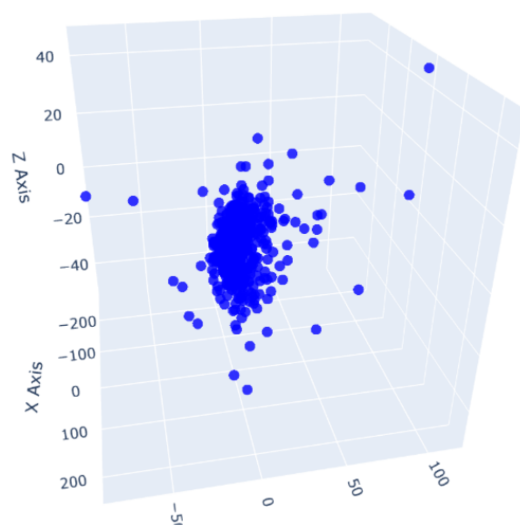

Fig. 1. Initial Guess for Flat Optimization

6 STAGE 3: MINIMUM VOLUME ENCLOSING SIMPLEX

The goal of this stage is to compute the smallest volume simplex that encloses all points in the handle flat, representing the transformation matrices across all poses.

Formulation. Given a set of points in $\mathbb{R}^{12\cdot\#\text{poses}}$, all lying on an $(h-1)$-dimensional flat, the objective is to find the simplex vertices $\{T_j\}_{j=1}^{h}$ that minimize the volume of the simplex subject to the constraint that all points are within the simplex.

The volume of the simplex is proportional to the determinant of the matrix formed by the simplex vertices:

$$ \text{Volume} \propto \left|\det([T_1 - T_h, T_2 - T_h, \ldots, T_{h-1} - T_h])\right|. $$

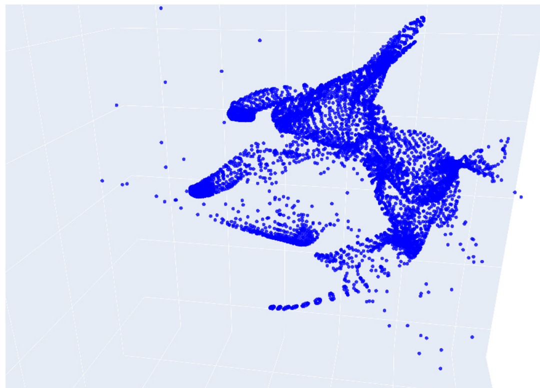

Fig. 2. Converged Flat

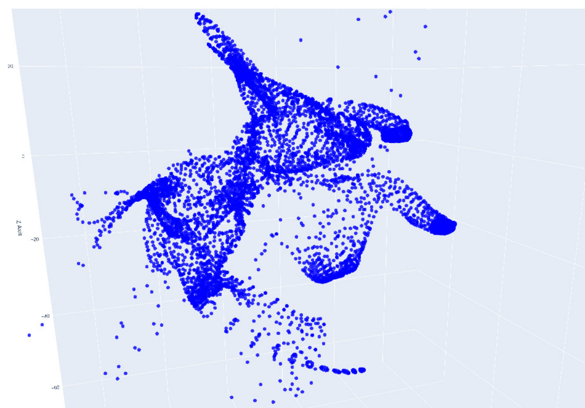

Fig. 3. Converged Flat

Optimization Approach. This optimization problem can be solved using Sequential Quadratic Programming (SQP) or other constrained optimization techniques. Constraints ensure that:

- All observed points are within the simplex (convexity constraints).
- The simplex satisfies affine constraints (sum-to-one for barycentric coordinates).

Results. The vertices of the simplex correspond to the transformation matrices (T_j), while the barycentric coordinates represent the skinning weights (w_i) for each vertex. This provides a complete LBS rig.

7 STAGE 4: SKINNING WEIGHTS ESTIMATION

Once the handle transformations ($\{T_j\}$) are determined, the skinning weights for each vertex can be computed.

Formulation. Given the observed deformed positions v_i' of a vertex and the handle transformations $\{T_j\}$, the weights w_i are computed as the barycentric coordinates of v_i' with respect to the simplex formed by $\{T_j\}$:

$$w_i =_{w\geq 0, \sum_j w_j = 1} \|v_i' - \sum_j w_j T_j v_i\|^2.$$

Optimization Approach. This is a simple constrained least squares problem, which can be solved efficiently using standard numerical solvers.

8 EVALUATION OF RESULTS

The proposed algorithm was tested on a cat with 1 rest pose and 8 deformed poses. The inverse skinning pipeline was able to render the cat correctly, as the visual representation closely matheched the given input poses.

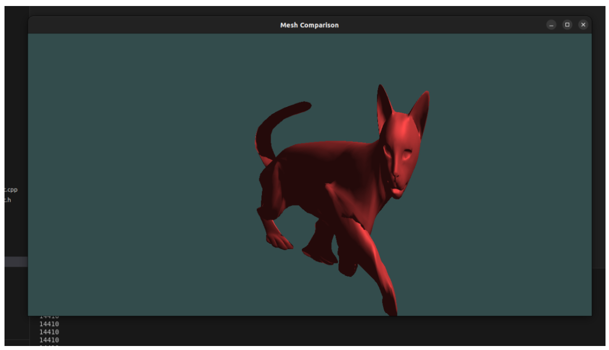

Fig. 4. Cat after skinning pose

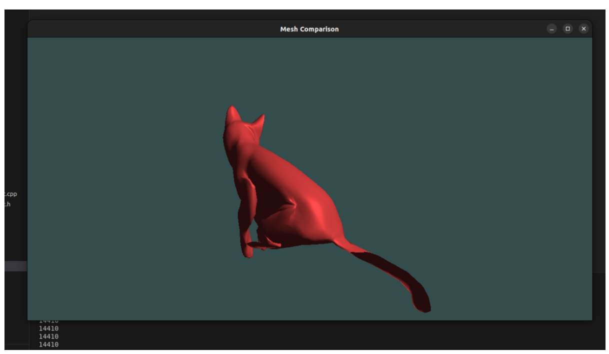

Fig. 5. Cat after skinning pose

The initial pose that was given to me:

9 CHALLENGES FACED

- **Optimization Challenges:** Solving the minimum volume enclosing simplex problem involved handling local minima and ensuring numerical stability, Mosek not being integrated so had to revert to using Python, then tried our best and hence got delayed to submit by translating from python to Cpp.

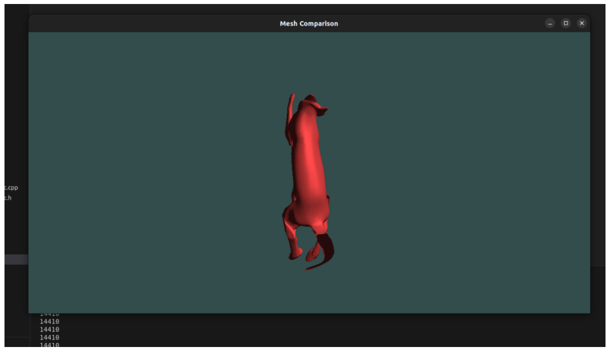

Fig. 6. Cat after skinning pose

Fig. 7. Initial pose given

- **High-Dimensional Data:** Efficiently managing and processing the high-dimensional transformation matrices required careful optimization and data handling, also we had to go through various rigorous mathematics and also derive a few equations to make our solving faster.
- **Visualization that the code written till now is correct:** Visualizing the Flat and determining the convergence by plotting the points from R12p to R3. This was important as we were not getting good results and the cat was highly distorted, so we needed mechanism to check the pipeline at interim stages rather than just checking at the very end.

10 FUTURE WORK

Several avenues for future work were identified:

- Extending the approach to handle sparse weights and rigid transformations.
- Investigating alternative optimization techniques for flat and simplex computation.
- Applying the method to additional applications, such as character animation and 3D shape reconstruction.

SUMMARY

Implemented the complete pipeline of first finding the Pervertex Flat transformation, Computing the Initial Guess, and finding the Optimial Flat, Computing the Minimum Volume Enclosing Simplex and then computing the Skinning Weights and Handle Trasnformations. This provided the correct implementation for the Hyper Spectral Inverse Skinning Approach to the Inverse Skinning Problem.

- Shows the flat converging into the true shape.
- Maintains numerical stability.
- Renders the results at par with the original poses.

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