# 3D Bin Packing Problem

We define a 3D bin packing problem with the following parameters:
$B$: the bin with dimensions $(L_B, W_B, H_B)$
$I$: a set of items, each with dimensions $(l_i, w_i, h_i)$, and attributes such as stackability and fragility
$x_{i,dx,dy,dz}$: binary decision variable, equal to 1 if item $i$ is placed at position $(dx, dy, dz)$, and 0 otherwise

## Objective Function

Maximize the placement of items, with a slight preference for placing items towards the end of the bin:

$$\text{maximize} \quad \sum_{i \in I} \sum_{dx=0}^{L_B - l_i} \sum_{dy=0}^{W_B - w_i} \sum_{dz=0}^{H_B - h_i} x_{i,dx,dy,dz} \left(1 + 0.01 \frac{dx}{L_B}\right)$$

## Constraints

## 1. Item Placement Constraint

Each item can be placed in the bin at most once, ensuring that no duplicate items are considered in the solution. This is enforced by the constraint:

$$\sum_{dx=0}^{L_B - l_i} \sum_{dy=0}^{W_B - w_i} \sum_{dz=0}^{H_B - h_i} x_{i,dx,dy,dz} \leq 1 \quad \forall i \in I \tag{1}$$

where $x_{i,dx,dy,dz}$ is a binary variable indicating whether item $i$ is placed at position $(dx, dy, dz)$.

## 2. No Overlapping Constraint

To prevent items from overlapping, each position in the bin can be occupied by at most one item. This is represented as:

$$\sum_{i \in I} \sum_{dx=\max(0, px - l_i + 1)}^{\min(px+1, L_B - l_i + 1)} \sum_{dy=\max(0, py - w_i + 1)}^{\min(py+1, W_B - w_i + 1)} \sum_{dz=\max(0, pz - h_i + 1)}^{\min(pz+1, H_B - h_i + 1)} x_{i,dx,dy,dz} \leq 1 \tag{2}$$

for all positions $(px, py, pz)$ in the bin.

## 3. Support Constraint

Items must be placed on a solid base, either on the floor of the bin or supported by other items beneath them. This prevents items from floating in mid-air. The constraint is given by:

$$x_{i,dx,dy,dz} \leq \sum_{j \neq i} \sum_{dx1=\max(0,dx-l_j+1)}^{\min(dx+l_i,L_B-l_j+1)} \sum_{dy1=\max(0,dy-w_j+1)}^{\min(dy+w_i,W_B-w_j+1)} x_{j,dx1,dy1,dz-1} \tag{3}$$

for all $i \in I$, where $dz > 0$.

## 4. Stackable Items Constraint

Stackable items can be placed on top of other items that can support their dimensions. This is expressed by:

$$x_{i,dx,dy,dz} \leq \sum_{j \neq i \text{ and } j \text{ stackable}} \sum_{dx1=\max(0,dx-l_j+l_i)}^{\min(dx+1,L_B-l_j+1)} \sum_{dy1=\max(0,dy-w_j+w_i)}^{\min(dy+1,W_B-w_j+1)} x_{j,dx1,dy1,dz-1} \tag{4}$$

for all $i \in I$ and where $dz > 0$.

## 5. Fragile Items Placement Constraint

Fragile items must either be placed directly on the floor or fully supported by other items. The constraint is:

$$x_{i,dx,dy,dz} \leq \sum_{j \neq i \text{ and } j \text{ supports } i} \sum_{dx1=dx}^{dx+l_i-1} \sum_{dy1=dy}^{dy+w_i-1} x_{j,dx1,dy1,dz-1} \tag{5}$$

for all $i \in I$ that are fragile and where $dz > 0$.

## 6. No Items on Fragile Items Constraint

To protect fragile items, no other items can be placed on top of them. This is enforced with:

$$\sum_{j \neq i} \sum_{dx1=\max(0,dx-l_j+1)}^{\min(dx+l_i,L_B-l_j+1)} \sum_{dy1=\max(0,dy-w_j+1)}^{\min(dy+w_i,W_B-w_j+1)} \sum_{dz1=dz+h_i}^{\min(dz+h_i+h_j,H_B)} x_{j,dx1,dy1,dz1} \leq 1000(1-x_{i,dx,dy,dz}) \tag{6}$$

for all $i \in I$ that are fragile, and for all positions $(dx, dy, dz)$ in the bin.

# Conclusion

These constraints collectively ensure the optimal and safe placement of items within the bin, taking into account practical considerations such as support, stackability, and fragility. The goal is to maximize the number of items packed while adhering to these constraints.

# References


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