A SPECIAL CASE OF LONGEST PATH PROBLEM

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1. Definition and claim

Definition 1. Long Path Problem:
Input: An undirected graph $G = (V, E)$
Output: Yes if there exists a path of length at least $\frac{|V|}{4}$ in G, otherwise No

Idea: Reduction from Hamiltonian path. Try to think how many vertices should be added to a special instance $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ of longest path problem so that if there is a path of length at least $\frac{|\mathcal{V}|}{4}$ in $\mathcal{G}$ then there is a Hamiltonian path in G.

It is easy to see that requiring the graph G to be connected does not decrease the hardness of our problem here. So, in what follows, we always assume (and try to maintain) that G is connected.

Claim 2. *We have that* Hamiltonian Path $\leq_p$ Long Path Problem

2. Reducing Hamiltonian Path to Long Path Problem

In this section, we prove the claim 2.

Proof. Reduce from Hamiltonian Path between 2 specified vertices, namely $s, t \in V(G)$

As mentioned above, we want to add new vertices to the graph while keeping it connected.

Now, do **some arithmetics**:

- A Hamiltonian path between s and t is of length $n - 1$, where $n = |V|$
- If we attach some paths of length (at most) k to s and t, then we can increase the length of a path up to $n - 1 + 2k$.

We should have $4(n - 1 + 2k) = |\mathcal{V}|$ in the new graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$.

So $|\mathcal{V}| - |V| = 3n - 4 + 8k$. Take $k = \sqrt{n}$. Do this in an alternative manner, attach to s and then to $latext$ and again to $latexs$, etc. a new short path of length (at most) k (i.e. k new vertices).

The point of setting $k = \sqrt{n}$ is to prevent one from concatenating 2 short paths attached to the same vertex (either s or t). So, one must take one path attached to each vertex among s and t. And the middle part of the path is a Hamiltonian path between s and t. □

Key words and phrases. graph, path, hamiltonian.
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3. Conclusion

Garey and Johnson [1] shape their theory based on previous primal works of Cook, Levin and Karp. Johnson [2] moves on with the guide to this theory. As long as we study a mathematical conjecture, we should encourage ourselves to have enough labour hours on popular mathematics books like these. Then, reading some articles on theory of computing like [3] is a good practice. Only after that, could we think of the ultimate final for all mathematical sciences.

References

1. Michael R. Garey, David S. Johnson, *Computers and Intractability: A Guide to the Theory of* **NP**-*Completeness*
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3. Phan Dinh Dieu, Le Cong Thanh, Le Tuan Hoa, *Average Polynomial Time Complexity of Some NP-Complete Problems*, Theor. Comput. Sci. 46(3): pp.219-237 (1986)

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