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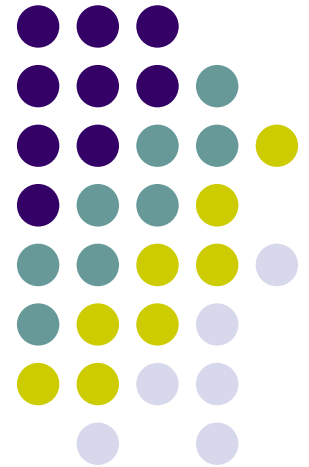


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Efficient Algorithms for the Longest Path Problem

Ryuhei UEHARA (JAIST)

Yushi UNO (Osaka Prefecture University)





The Longest Path Problem

- Finding a longest (vertex disjoint) path in a given graph
- Motivation (comparing to Hamiltonian path):
 - ... Approx. Algorithm, Parameterized Complexity
 - ... More practical/natural
 - ... More difficult(?)

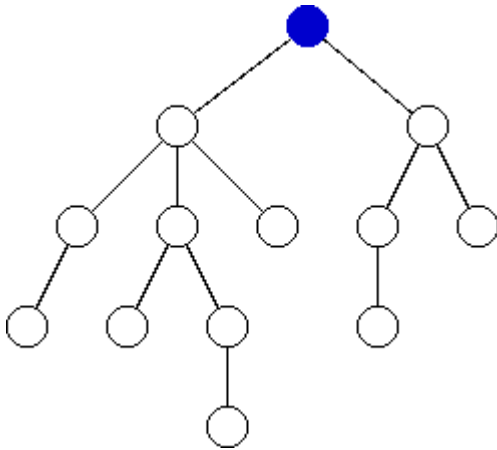


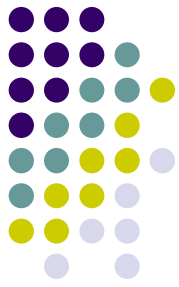
The Longest Path Problem

- Known (hardness) results;
 - We cannot find a path of length $n - n^\epsilon$ in a given Hamiltonian graph in poly-time unless $P=NP$ [Karger, Motwani, Ramkumar; 1997]
 - We can find $O(\log n)$ length path [Alon, Yuster, Zwick; 1995]
($\Rightarrow O((\log n / \log \log n)^2)$ [Björklund, Husfeldt; 2003])
 - Approx. Alg. achieves $O(n / \log n)$ [AYZ95]
($\Rightarrow O(n(\log \log n / \log n)^2)$ [BH03])
- Exponential algorithm [Monien 1985]

The Longest Path Problem

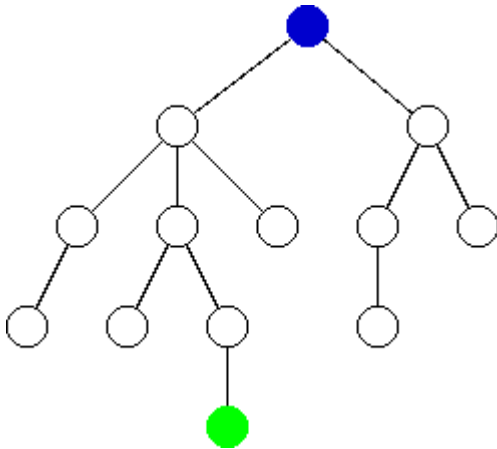
- Known polynomial time algorithm;
- Dijkstra's Alg.(196?): Linear alg. for finding a longest path in a tree;





The Longest Path Problem

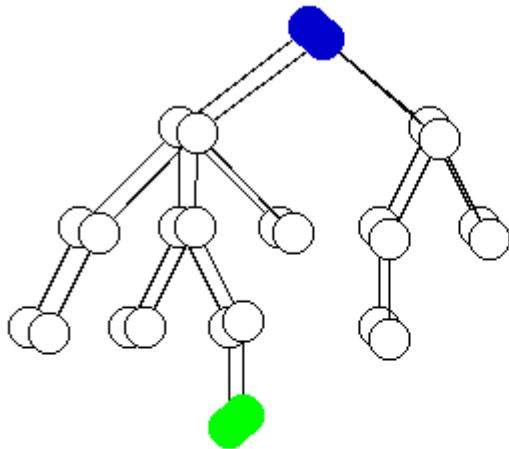
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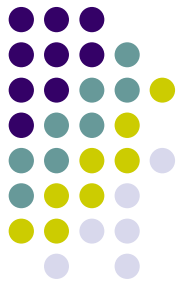




The Longest Path Problem

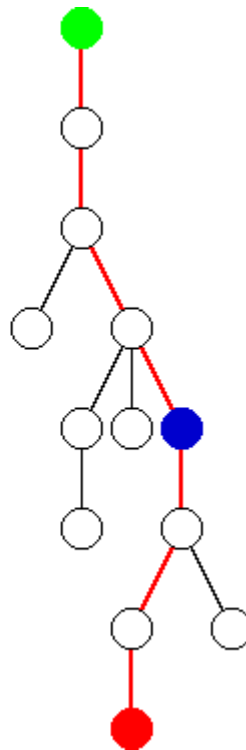
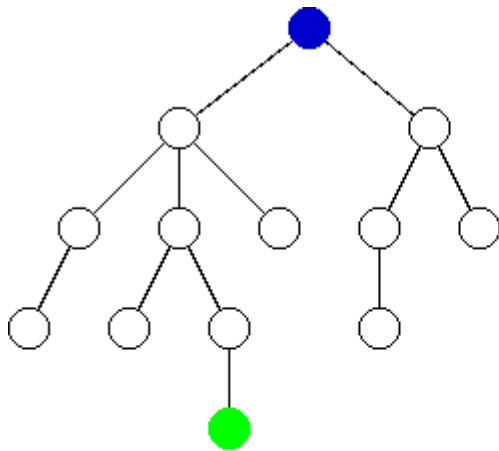
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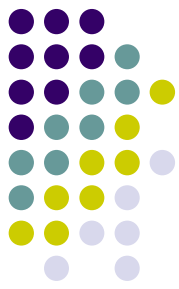


The Longest Path Problem

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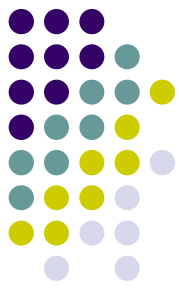


Approaches to the Efficient Algs to Longest Path Problem



1. Extension of the Dijkstra's algorithm
 - Weighted trees (linear), block graphs (linear), cacti ($O(n^2)$).
(ISAAC 2004)
2. Graph classes s.t. Hamiltonian Path can be found in poly time
 - Some graph classes having interval representations (bipartite permutation, interval biconvex graphs)
(ISAAC 2004)
3. Dynamic programming to the graph classes that have tree representations (on going)
 - Cacti(linear), ...

Approaches to the Efficient Algs to Longest Path Problem



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1. Ex of Dijkstra's Alg

Bulterman et.al. (*IPL*,2002) showed that the correctness of Dijkstra's alg stands for;

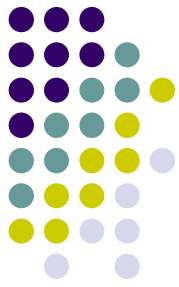
1. For each u, v ,
length of the **shortest** path between u and v
= length of the **longest** path between u and v
2. For each u, v, w ,
 $d(u, v) \leq d(u, w) + d(w, v)$
3. For each u, v, w ,
 $d(u, v) = d(u, w) + d(w, v)$ if and only if
 w is on the **unique** path between u and v



1. Ex of Dijkstra's Alg

Construct $G'=(V',E')$ from $G=(V,E)$ s.t.:

- $V \subseteq V'$
- For each $u, v \in V$,
length of the **shortest path** between u, v on G'
= length of the **longest path** between u, v on G
- For each $u, v \in V$,
the **shortest path** between u, v on G' is **unique**

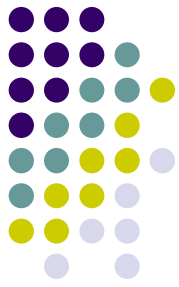


1. Ex of Dijkstra's Alg

Theorem: ExDijkstra finds a longest path if G and G' satisfy the conditions.

ExDijkstra: $G=(V,E)$ and $G'=(V',E')$

1. pick any vertex w in V ;
2. find $x \in V$ with $\max\{d(w,x)\}$ on G' ;
3. find $y \in V$ with $\max\{d(x,y)\}$ on G' ;
4. x and y are the endpoints of the longest path in G , and $d(x,y)$ on G' is its length.



1. Ex of Dijkstra's Alg (Summary)

Theorem: Vertex/edge weighted tree (linear)

Theorem: Block graph ($O(|V|+|E|)$)

Theorem: **Cactus** ($O(|V|^2)$)

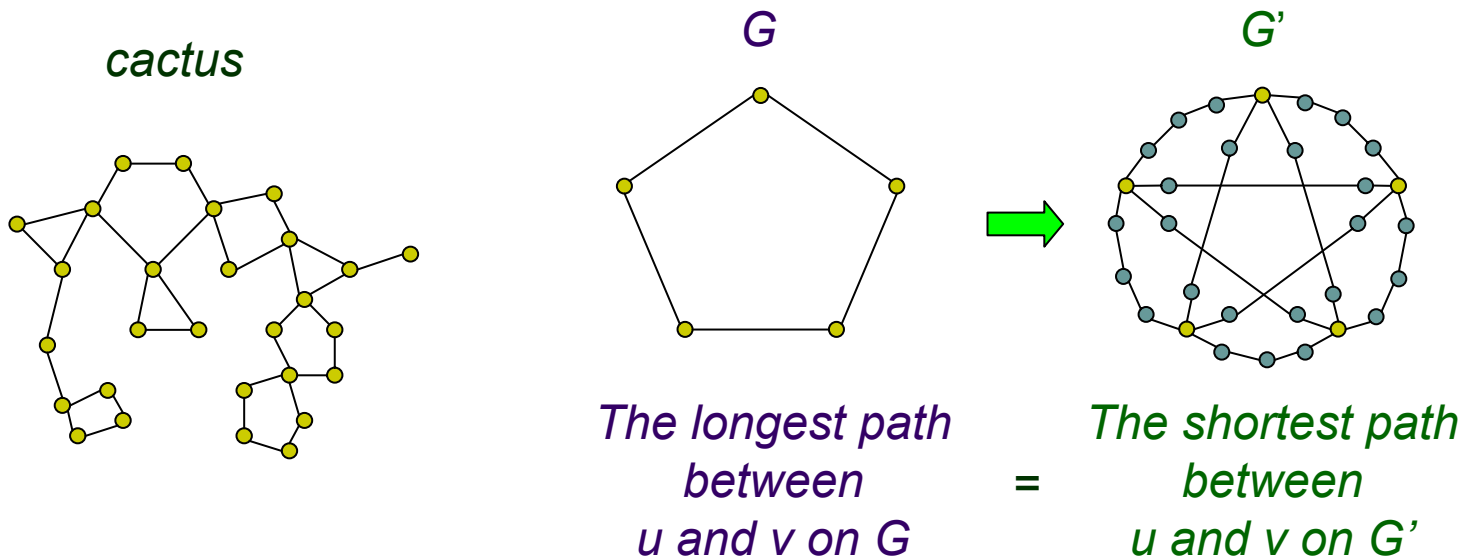


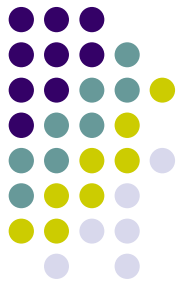
1. Ex of Dijkstra's Alg (Cacti)

Cactus:

Each block is a *cycle*

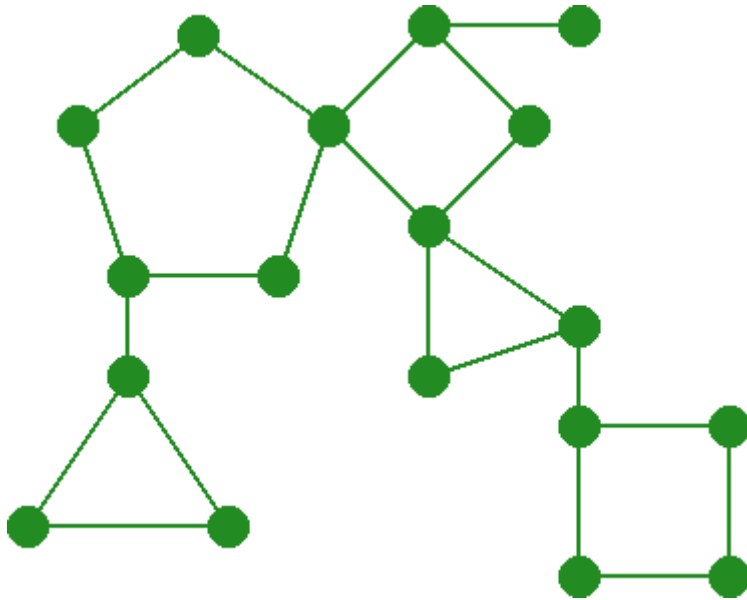
Two cycle share at most one vertex *which is a separator*

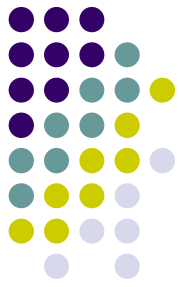




1. Ex of Dijkstra's Alg (Cacti)

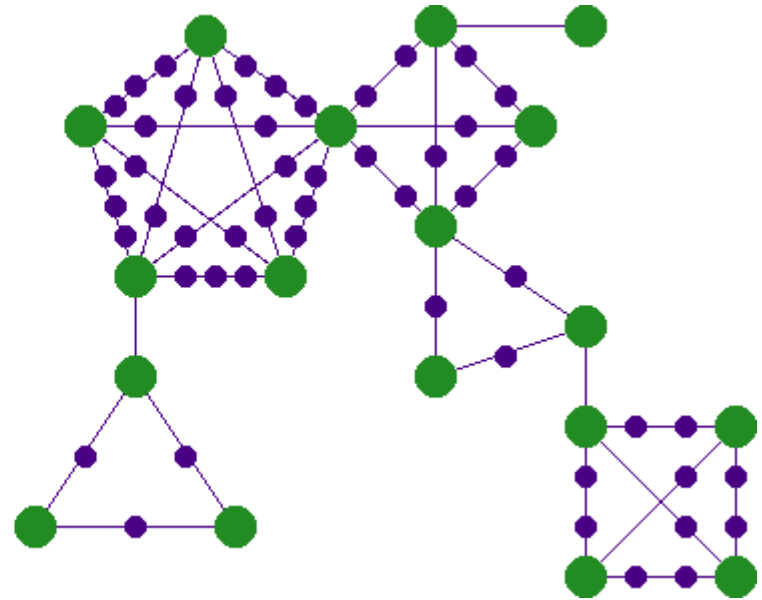
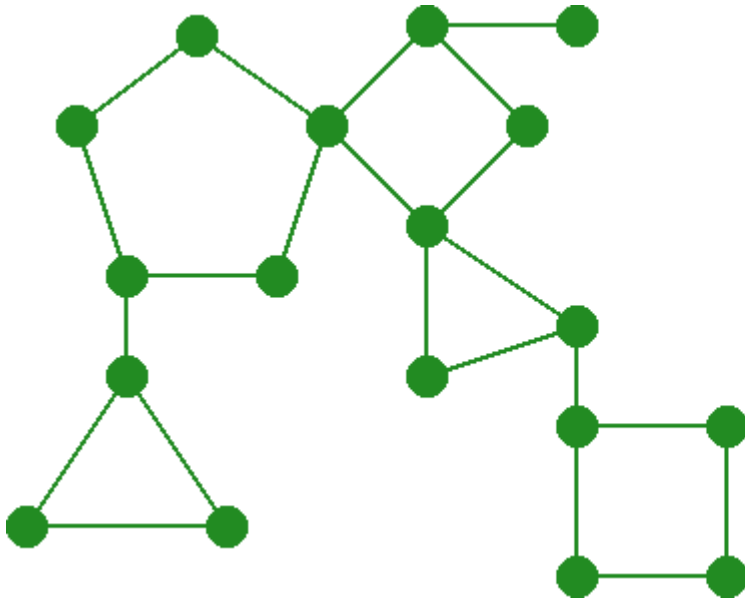
Sample





1. Ex of Dijkstra's Alg (Cacti)

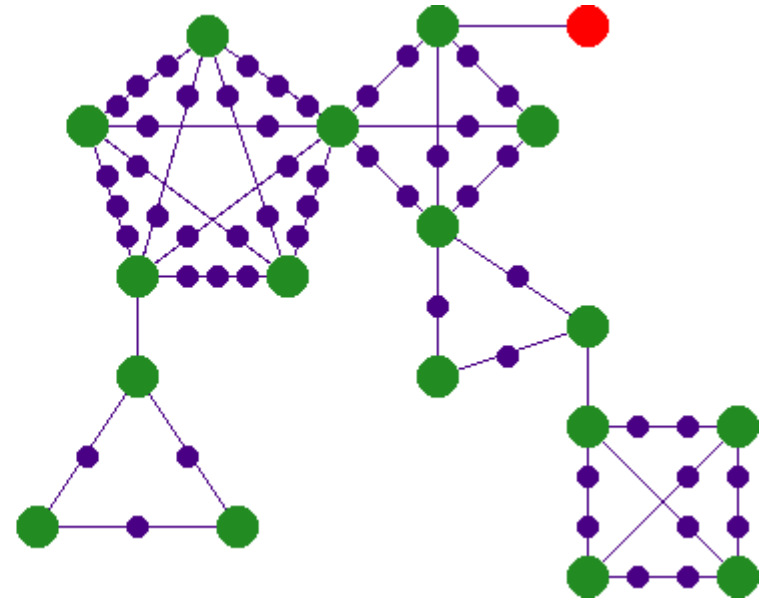
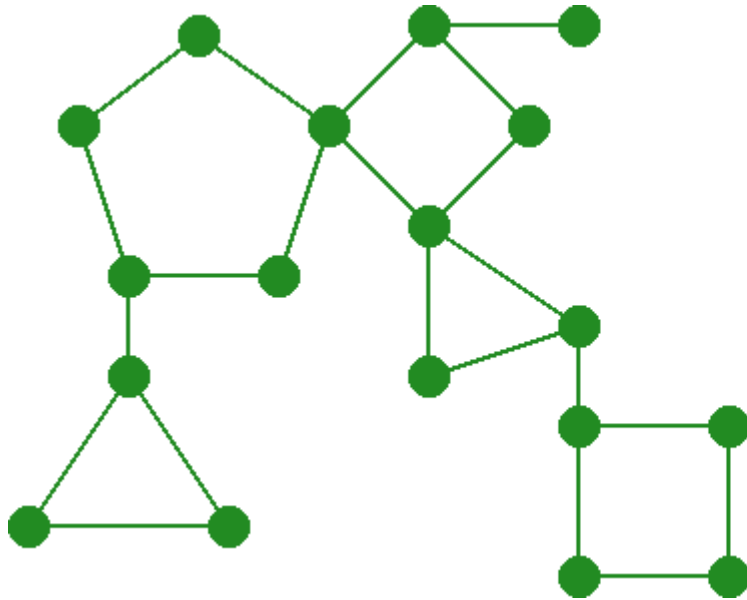
Sample





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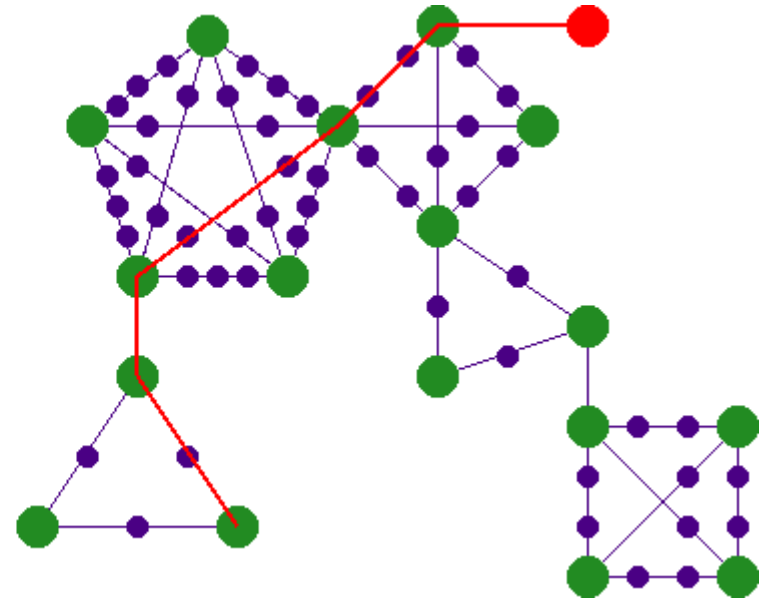
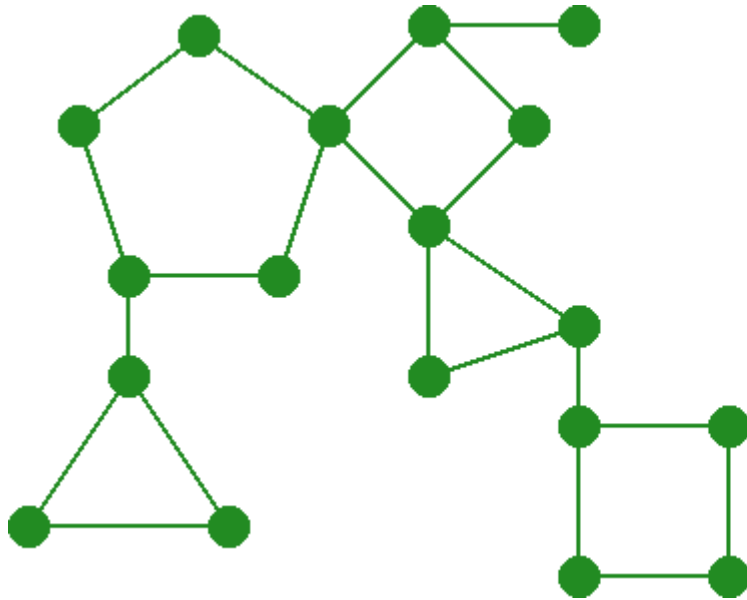
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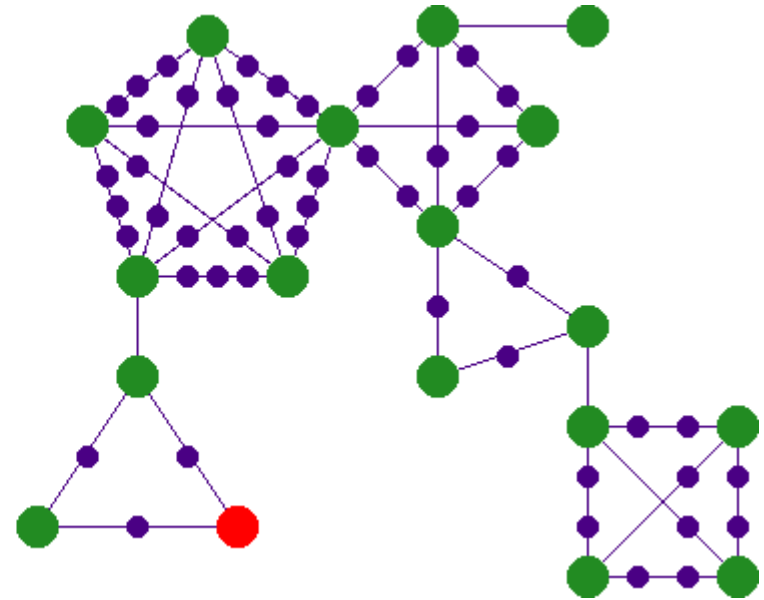
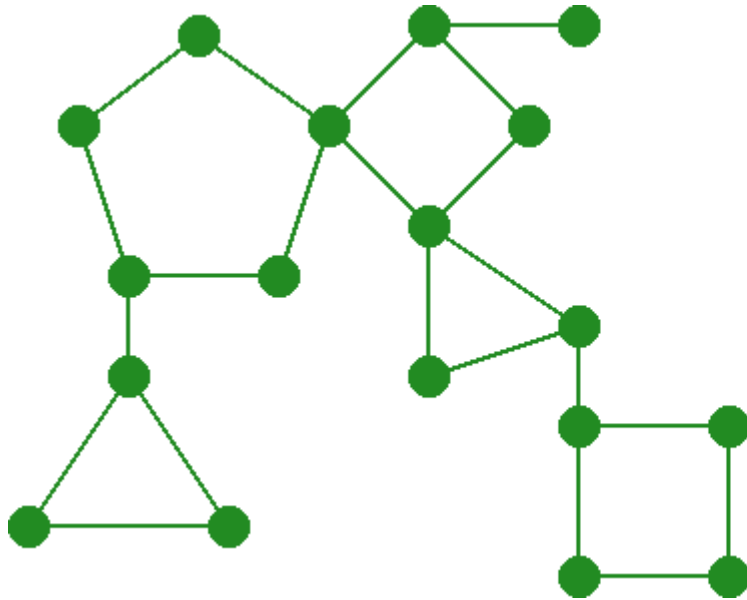
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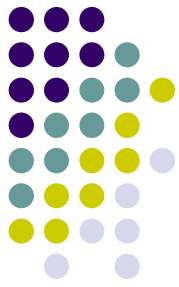




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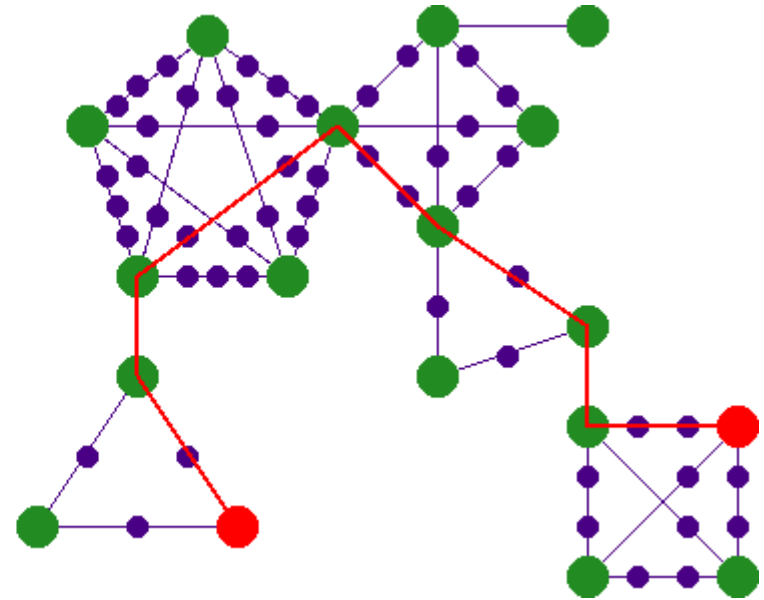
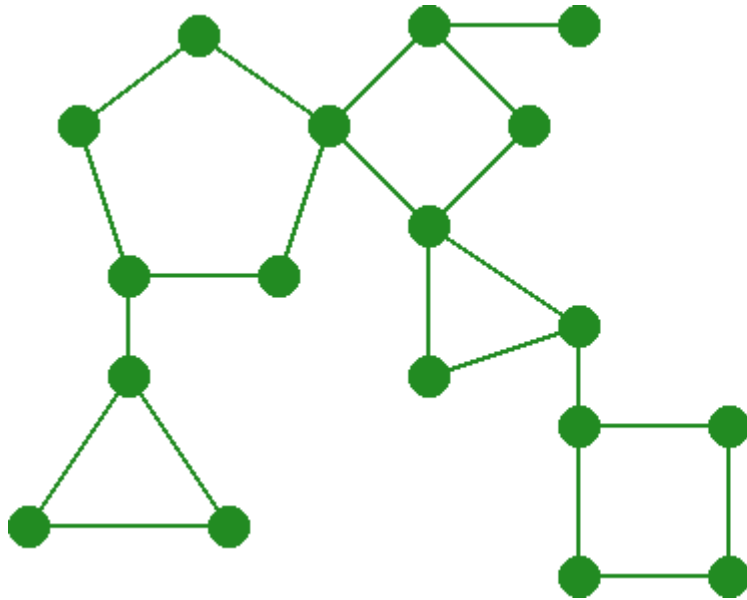
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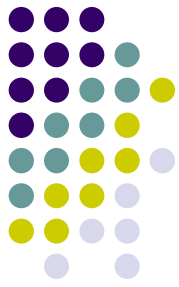




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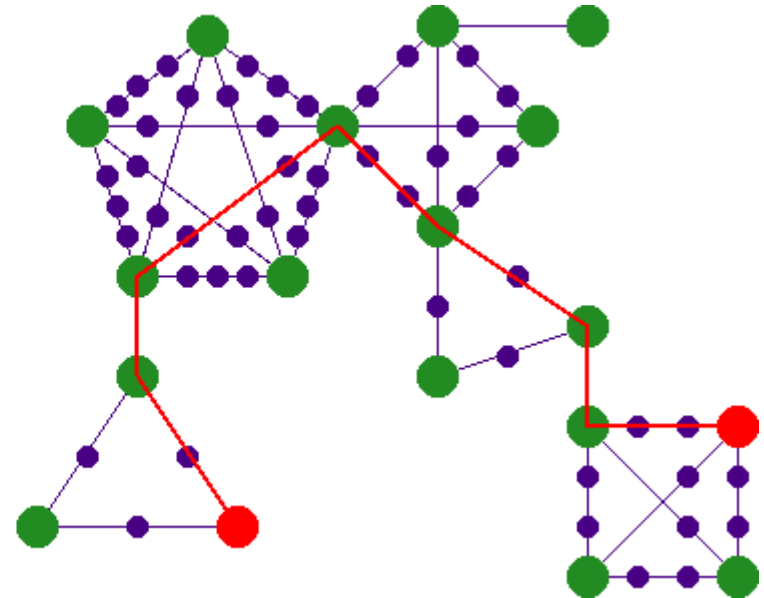
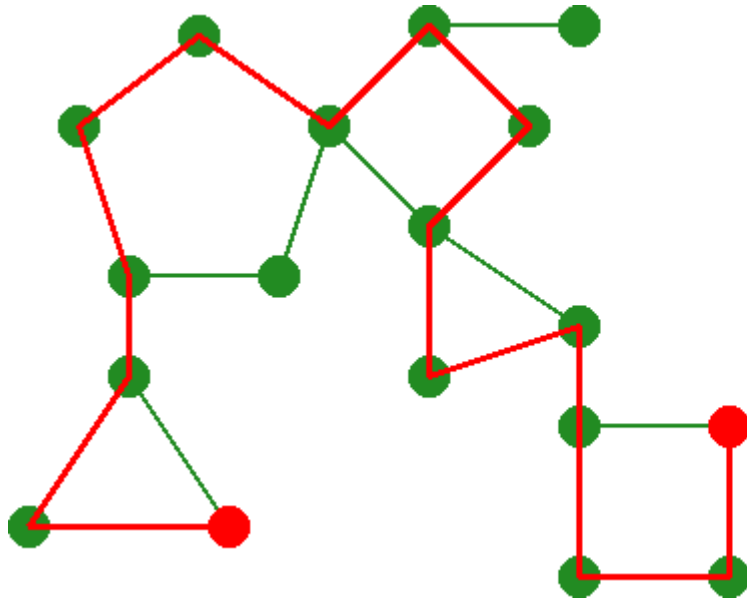
Sample



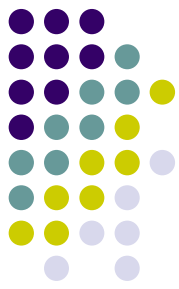


1. Ex of Dijkstra's Alg (Cacti)

Sample



Graph classes s.t. Hamiltonian Path can be found in poly time



Fact 1:

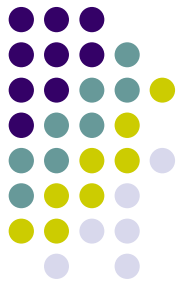
Hamiltonian Path is NP-hard on a **chordal graph**.
(In fact, strongly chordal split graph[Müller,1997].)

Fact 2:

Hamiltonian Path is solvable on an **interval graph** in linear time. [Damaschke, 1993].

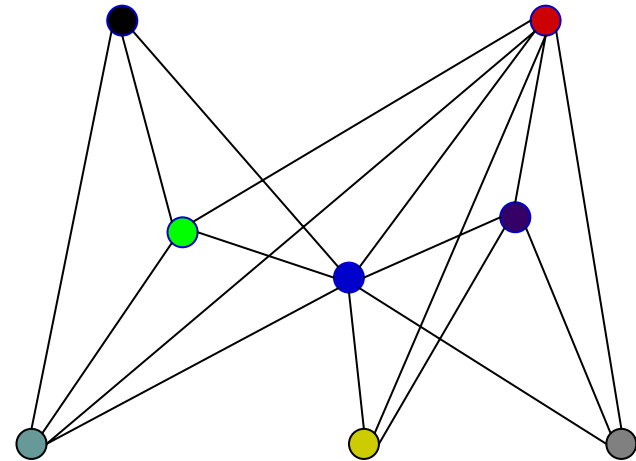
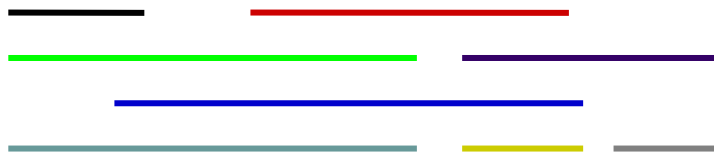
Our goal:

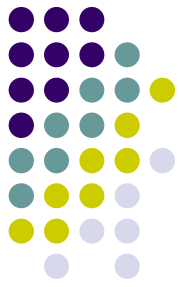
Poly-time algorithm for Longest Path on an **interval graph**.



Interval Graphs

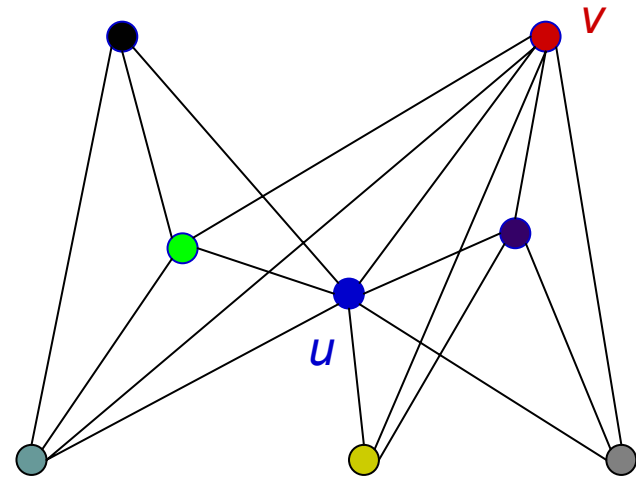
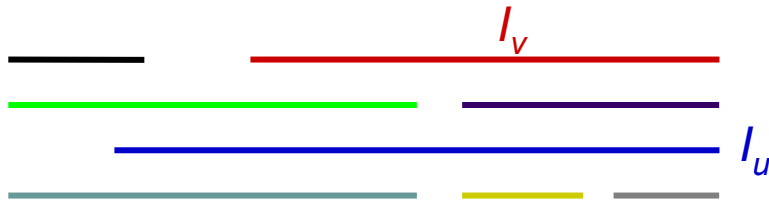
- An interval graph $G=(V,E)$ has an interval representation s.t. $\{u,v\} \in E$ iff $I_u \cap I_v \neq \emptyset$





Interval Graphs

- An interval graph $G=(V,E)$ has an interval representation s.t. $\{u,v\} \in E$ iff $I_u \cap I_v \neq \emptyset$



Hamiltonian Path: linear time solvable.

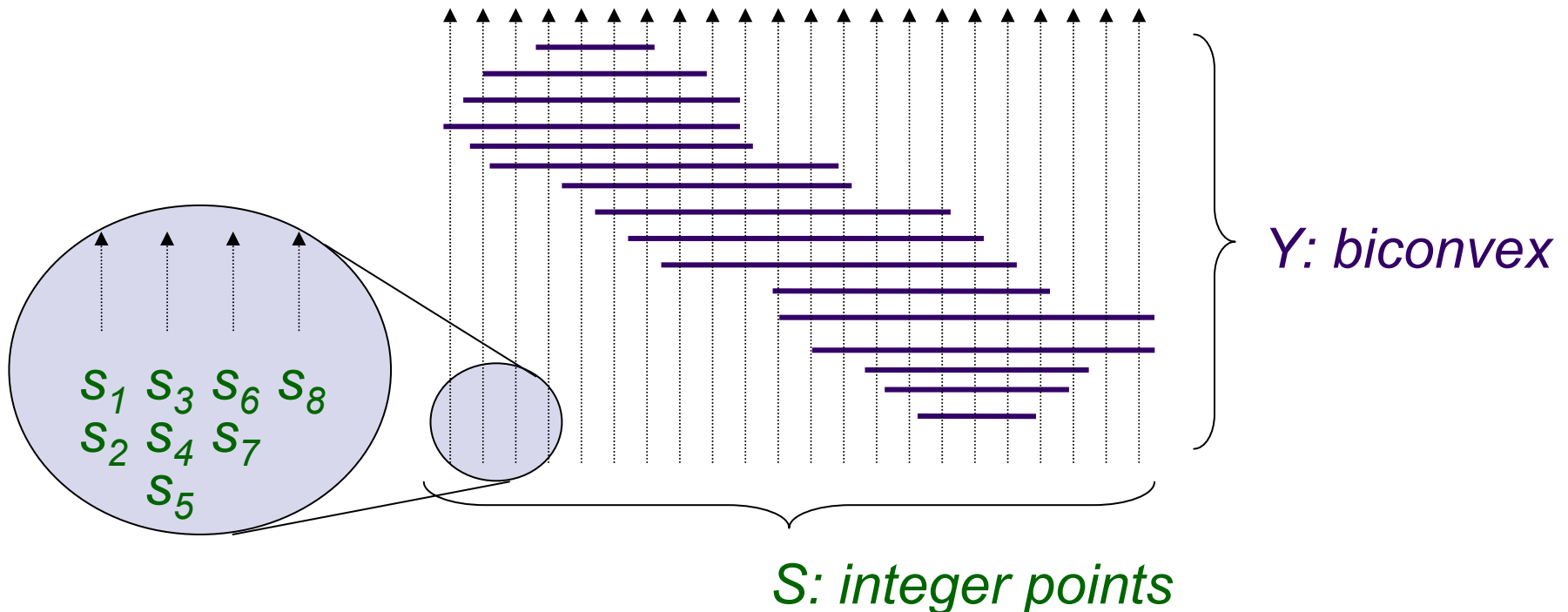
Longest Path: ????

\Rightarrow Restricted interval graphs...



Restricted Interval Graphs

- An interval **biconvex** graph $G=(S \cup Y, E)$ has an interval representation s.t...

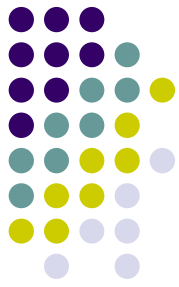




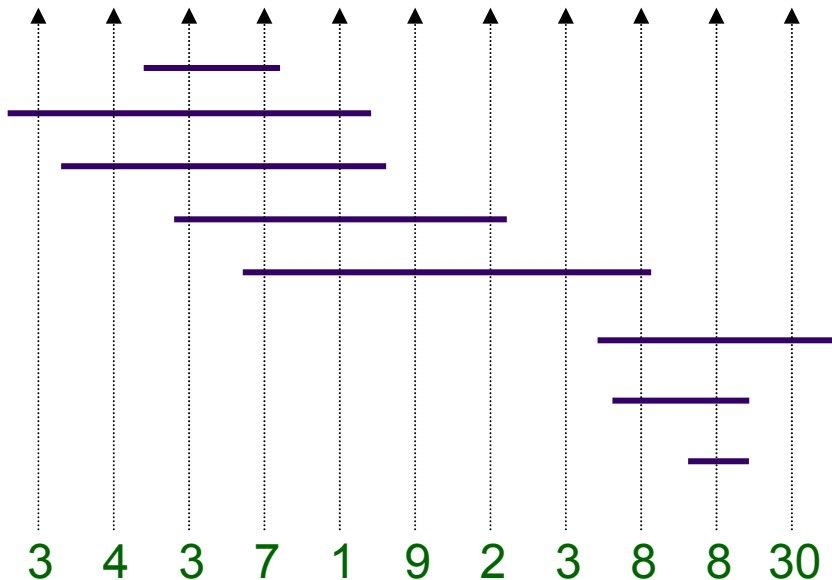
Restricted Interval Graphs

- Interval **biconvex** graph $G=(S \cup Y, E)$ is introduced [Uehara, Uno; 2004] from graph theoretical viewpoints;
 - ✓ *Natural analogy of biconvex graphs (bipartite graph class)*
 - ✓ *Generalization of proper interval graphs*
 - ✓ *Generalization of threshold graphs*
 - ✓ *Best possible class longest path can be found in poly time...*

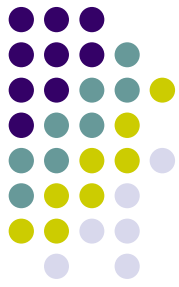
Poly-time alg for longest path on an interval biconvex graph (idea)



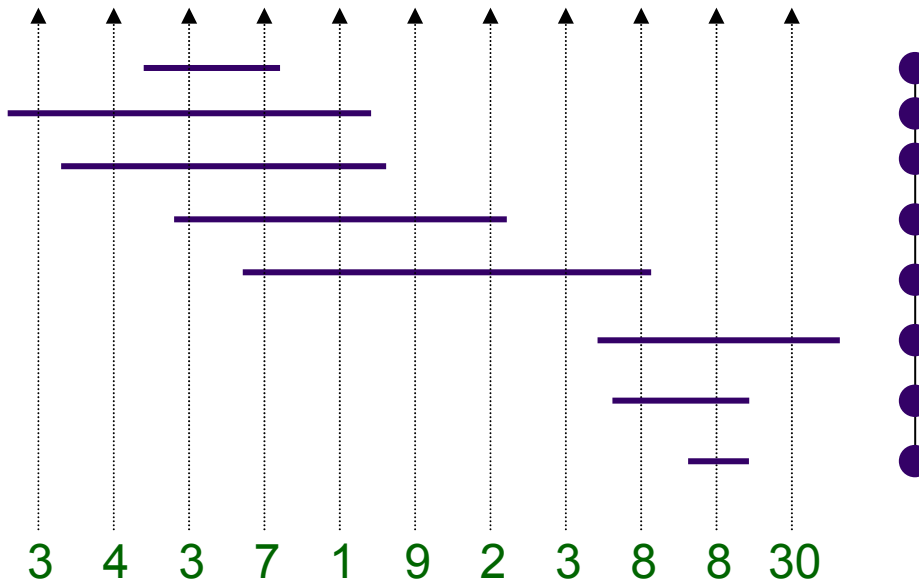
- Find the trivial longest path P on $G[Y]$;
- Embed the vertices in S into P as possible;
- Adjust endpoints if necessary.



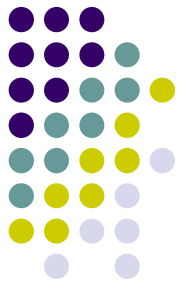
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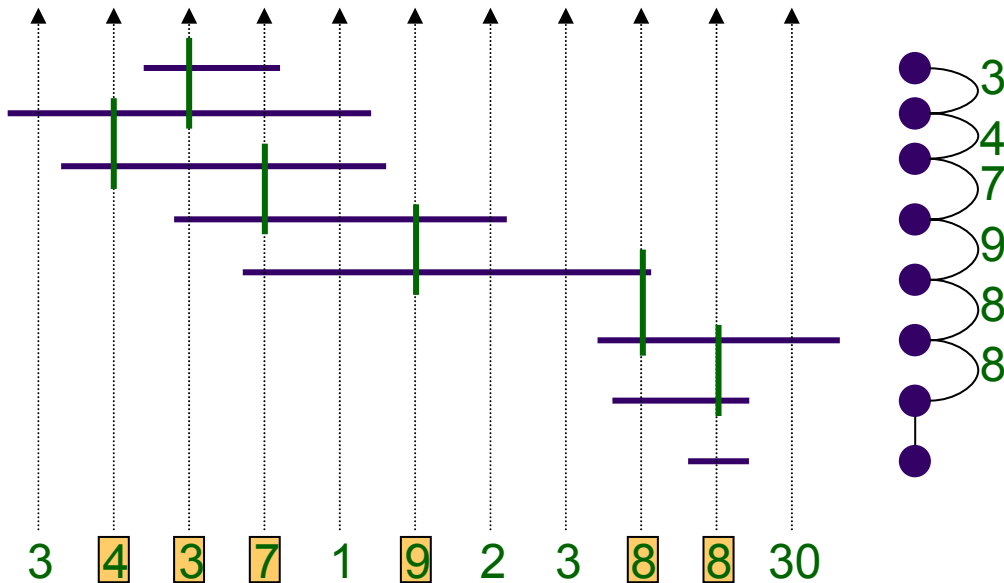
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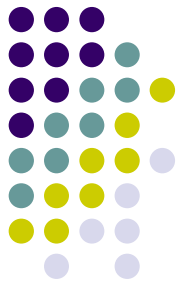
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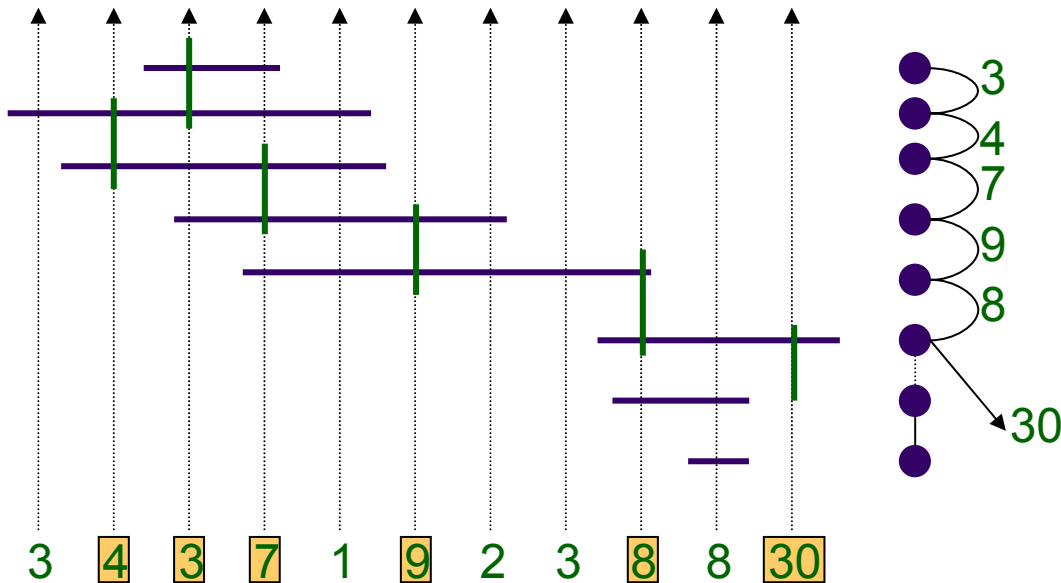
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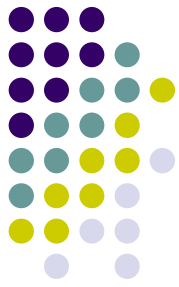
Poly-time alg for longest path on an interval biconvex graph (idea)



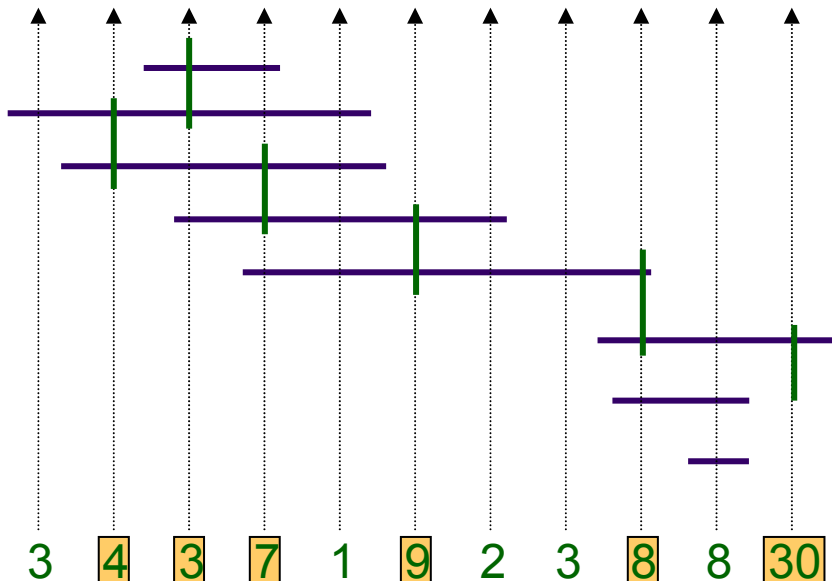
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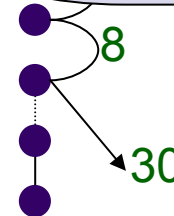
Poly-time alg for longest path on an interval biconvex graph (idea)



- Find the trivial longest path P on $G[Y]$;
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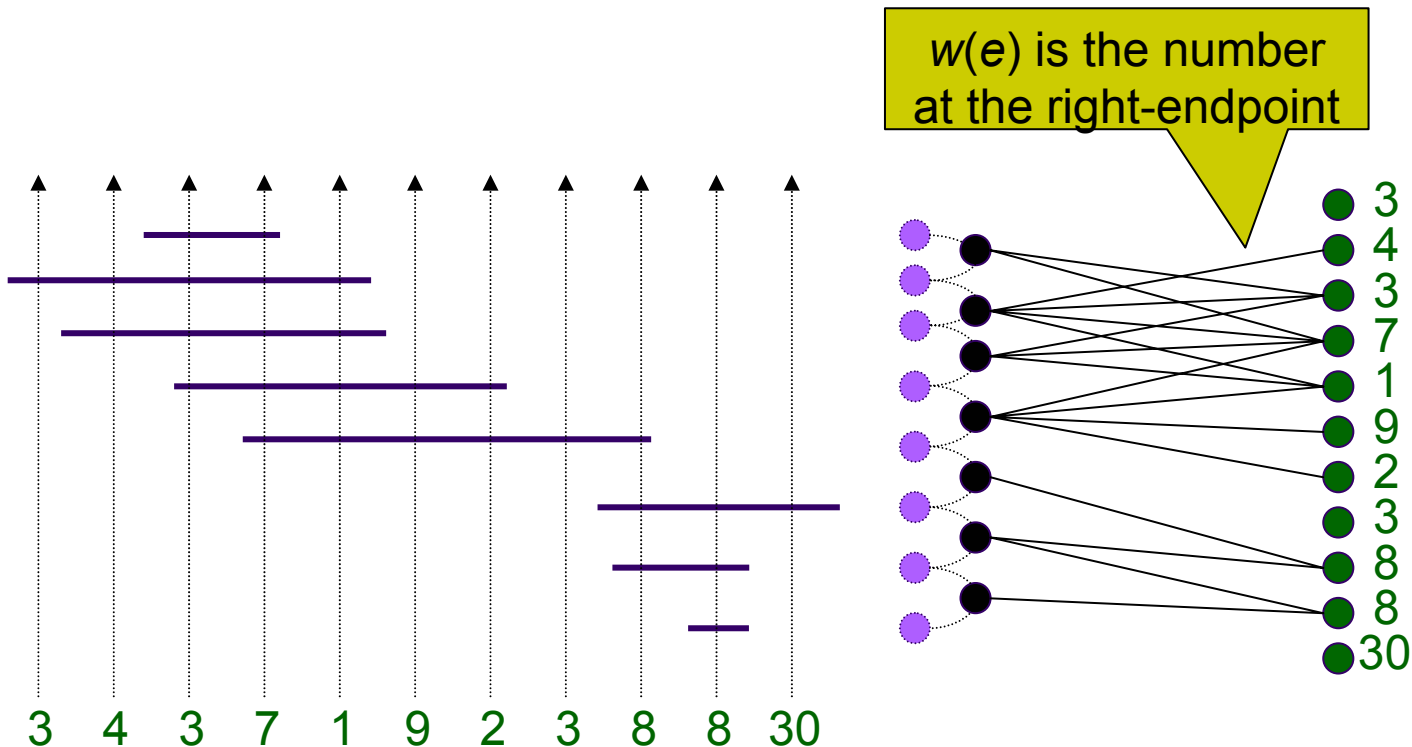
- ✓ How can we determine the vertices in S ?
- ✓ Where do we embed them?



Poly-time alg for longest path on an interval biconvex graph (idea)



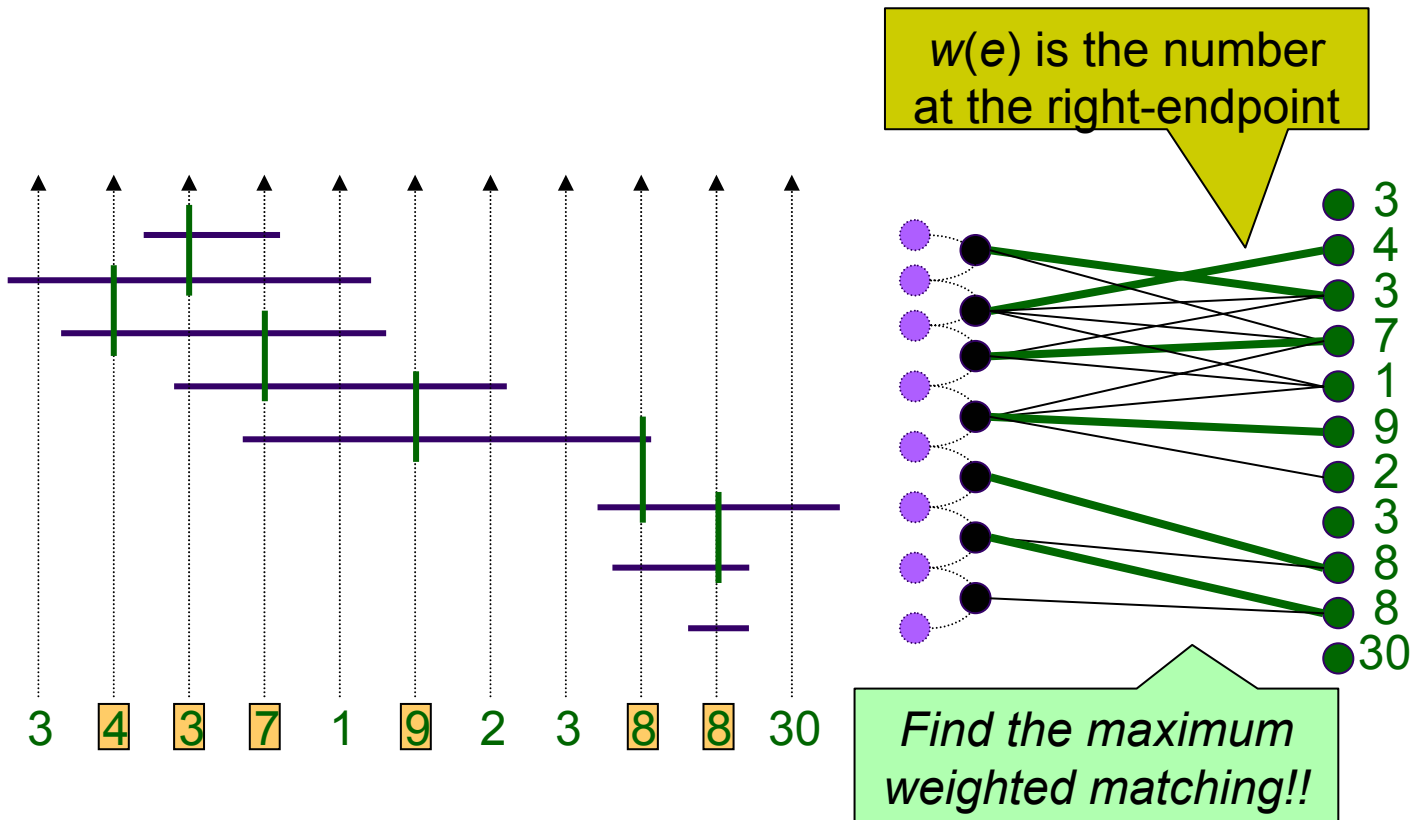
- Embed the vertices in S into P as possible;



Poly-time alg for longest path on an interval biconvex graph (idea)



- Embed the vertices in S into P as possible;





Open Problems

- Longest Path on an interval graph??
 - Combination of *DP/Dijkstra* and *weighted maximum matching* on MPQ-tree representation?
 - Related to the following open problem?
Hamiltonian Path *with a start point* on an interval graph?
[Damaschke, 1993].
- Extension to
 - Longest *cycle* on some graph classes
 - *Hamiltonian cycle/path* on some graph classes