

Longest Bitonic Subsequence

Problem

Given an array $a[]$ of size n , our task is to find the longest bitonic subsequence

Bitonic: Unlike LIS, it can first increase and then decrease.

Possible bitonic subsequences nature

1. First increases then decreases
2. Only increases
3. Only decreases

Example

4	1	3	6	8	7	12	10	5
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Few subsequences which are valid

$\{4\}$, $\{4,1,3\}$, $\{8,7,5\}$, $\{1,3,6,8\}$, $\{1,3,6,8,12,10,5\}$, $\{1,3,6,7,12,10,5\}$

Maximum Length = 7.

Brute force approach

1. Compute all subsequences
2. Choose the one with maximum length and following the property of bitonic subsequences.

Time Complexity: $O(2^n)$

Optimal Solution (Using dynamic programming)

1. Similar to the longest increasing subsequence question.
2. Make two tables of LIS,
 - a. One from starting (LIS1)
 - b. Other from end (LIS2)

3. Iterate over both the tables/ arrays and store the maximum of $(lis1[i] + lis2[i]-1)$.
4. Output this maximum value.