Number of Subsequences (DP exercise)

Problem

You are given a string s consisting of lowercase letters latin letters (‘a’ to ‘z’) and some ‘?’. Your task is to find the number of subsequences ‘abc’ in all the strings. ‘?’ should be replaced by either of {‘a’, ‘b’, ‘c’}.

Example

ac?b?c

Possible Strings	Number of Subsequences ‘abc’
acabac	2
acabbc	4
acabcc	4
acbbac	2
acbbbc	3
acbbcc	4
accbac	1
accbbc	2
accbcc	2
Total	**24**

Brute force approach

1. Compute all the strings and then count subsequences (“abc”) in each string.

Time complexity: $O(2^n)$

where n is the length of string.

Optimal Approach (Using dynamic programming)

Declare 4 variables,

A. e - denoting count of all possible strings upto current element.
B. a - denoting count of subsequences (‘a’) in all the strings upto current elements.
C. ab - denoting count of subsequences (‘ab’) in all the strings upto current element.
D. abc - denoting count of subsequences (‘abc’) in all the strings upto current element.

According to this table, update the above variables while iterating in the string.

Current Element	e	a	ab	abc
“a”	(e)	(a) + e	(ab)	(abc)
“b”	(e)	(a)	(ab) + (a)	(abc)
“c”	(e)	(a)	(ab)	(abc) + (ab)
“d-z”	(e)	(a)	(ab)	(abc)
“?”	3*(e)	3*(a)+e	3*(ab)+(a)	3*(abc)+(ab)

Dry Run

Variables	Current element	‘a’	‘c’	‘?’	‘b’	‘?’	‘c’
e	1	1	1	3	3	9	9
a	0	1	1	4	4	15	15
ab	0	0	0	1	5	19	19
abc	0	0	0	0	0	5	**24**