Stanford-LBNL MPC Optimization Model Mathematical Formulation

Fred Fan

December 20, 2024

Decision Variables

$\mathrm{T}_{indoor,t} \geq 0 \quad \forall t$ — Indoor temperature [°C]

$\mathrm{B\_SOC}_t \in [0,1] \quad \forall t$ — Battery state of charge

$\mathrm{PV2H}_t \geq 0 \quad \forall t$ — Electrical energy transfer from PV to home [kWh]

$\mathrm{PV2G}_t \geq 0 \quad \forall t$ — Electrical energy transfer from PV to ground (curtailment) [kWh]

$\mathrm{PV2B}_t \geq 0 \quad \forall t$ — Electrical energy transfer from PV to battery [kWh]

$\mathrm{B2H}_t \geq 0 \quad \forall t$ — Electrical energy transfer from battery to home [kWh]

$\mathrm{G2H}_t \geq 0 \quad \forall t$ — Electrical energy transfer from grid to home (loss of load) [kWh]

$\mathrm{HP2H}_{i,t} \geq 0 \quad \forall i \in \{1,2\}, \forall t$ — Thermal energy delivered to home from HP [kWh] $\begin{cases} i=1: & \text{Heating} \\ i=2: & \text{Cooling} \end{cases}$

$\mathrm{PCM\_charging}_{i,t} \geq 0 \quad \forall i \in \{1,2\}, \forall t$ — Thermal energy charged to PCM [kWh] $\begin{cases} i=1: & \text{Hot PCM} \\ i=2: & \text{Cold PCM} \end{cases}$

$\mathrm{HP\_state}_{i,t} \in \{0,1\} \quad \forall i \in \{1,2,3\}, \forall t$ — Heat pump operating modes $\begin{cases} i=1: & \text{Off Mode} \\ i=2: & \text{Heating Mode} \\ i=3: & \text{Cooling Mode} \end{cases}$

$\mathrm{PCM\_state}_{i,t} \in \{0,1\} \quad \forall i \in \{1,\ldots,5\}, \forall t$ — PCM modes $\begin{cases} i=1: & \text{Off Mode} \\ i=2: & \text{Charge Hot PCM} \\ i=3: & \text{Charge Cold PCM} \\ i=4: & \text{Discharge Hot PCM} \\ i=5: & \text{Discharge Cold PCM} \end{cases}$

$\mathrm{PCM\_SOC}_{i,t} \in [-0.05, 1.05] \quad \forall i \in \{1,2\}, \forall t$ — State of charge of PCM $\begin{cases} i=1: & \text{Hot PCM} \\ i=2: & \text{Cold PCM} \end{cases}$

$\mathrm{ON\_Ratio}_{i,t} \geq 0 \in \mathbb{Z} \quad \forall i \in \{1,\ldots,4\},\ t \quad \begin{cases} i=1: & \text{Heat pump heating mode} \\ i=2: & \text{Heat pump cooling mode} \\ i=3: & \text{Discharging Hot PCM} \\ i=4: & \text{Discharging Cold PCM} \end{cases}$

Due to the increasing time length of time step, the decision variable "ON_Ratio" is used to represent how many base intervals (5 mins) out of the a large time step are used to execute a certain action. For example, if a certain future time step in the optimization model is 2 hours long, when there is a call for heating, instead of choosing to discharge the Hot PCM for two hours straight or not discharge at all, the model has an option to choose how many 5 minute base time steps out of the 2 hours will the Hot PCM be discharged.

Objective Function

Minimize:

$$\underbrace{\sum_{t=1}^{T} \mathrm{G2H}_t}_{\text{Loss of load}} + \underbrace{0.01 \sum_{t=1}^{T} (1 - \mathrm{B\_SOC}_t)}_{\text{Penalty for empty battery}} + \underbrace{0.01 \sum_{i=1}^{2} \sum_{t=1}^{T} (1.05 - \mathrm{PCM\_SOC}_{i,t})}_{\text{Penalty for empty PCM}}$$

Constraints

Initialization Constraints:

$$\begin{aligned}
&\mathrm{T}_{indoor,1} = \mathrm{T}_{indoor,initial} \\
&\mathrm{B_SOC}_1 = \mathrm{B_SOC}_{initial} \\
&\mathrm{PCM_SOC}_{i,1} = \mathrm{PCM_SOC}_{i,initial} \quad \forall i
\end{aligned}$$

The initial indoor temperature and PCM SOCs are read from the output of the Modelica from last iteration. The initial indoor battery is solved elsewhere after knowing the operations of the system in the last iteration.

Heat Pump Operational Constraints:

$$\sum_{i=1}^{3} \mathrm{HP\_state}_{i,t} = 1 \quad \forall t$$

Thermal Storage Operational Constraints:

$$\sum_{i=1}^{5} \mathrm{PCM\_state}_{i,t} = 1 \quad \forall t$$

Indoor Temperature Balance:

$$\mathrm{T}_{indoor,t+1} = \mathrm{T}_{indoor,t} + \frac{1}{\mathrm{C}} \left(Q_{\mathrm{Passive}}[t] + Q_{\mathrm{Electrical}}[t] + \mathrm{Heat\_delivered}_t \right) \quad \forall t$$

C is the total thermal capacitance of the structure, and Q_{Passive} is the total passive heat gain (conduction, convection, radiation), and $Q_{\mathrm{Electrical}}$ is the total heat gain from other electrical appliances in the structure (lighting, plugs...)

PV Energy Balance:

$$\mathrm{PVGeneration}_t = \mathrm{PV2B}_t + \mathrm{PV2H}_t + \mathrm{PV2G}_t \quad \forall t$$

House Load Balance Constraint:

$$E_t + \mathrm{H2HP}_{1,t} + \mathrm{H2HP}_{2,t} = \mathrm{PV2H}_t \cdot \eta_{\mathrm{PVIV}} + \mathrm{B2H}_t \cdot \eta + \mathrm{G2H}_t \quad \forall t \quad (\mathrm{kWh})$$

E_t is the total electrical consumption from other electrical appliances bar heat pump, η_{PVIV} is the PV inverter efficiency, and η is the battery efficiency.

Battery Storage Balance:

$$\text{B\_SOC}_{t+1} = \text{B\_SOC}_t \cdot (1 - \delta t_t \cdot \text{BatteryLoss}) + \frac{\text{PV2B}_t \cdot \eta - \text{B2H}_t}{\text{Battery\_Size}} \quad \forall t$$

δt_t is the length of the time step (in hour) in time index t.

Heating and Cooling Logic: Heating and cooling states of the system (heating_t and cooling_t) follow bounds determined by hysteresis values on temperature thresholds.

Heating and Cooling Mode Constraints:

- **Heating Mode:** If heating mode is ON ($\text{heating}_t = 1$), at least one between HP heating and PCM hot discharge must heat the room:

$$\text{HP\_state}_{2,t} + \text{PCM\_state}_{4,t} \leq 2 \cdot \text{heating}_t \quad \forall t$$
$$\text{HP\_state}_{2,t} + \text{PCM\_state}_{4,t} \geq \text{heating}_t \quad \forall t$$

- **Cooling Mode:** If cooling mode is ON ($\text{cooling}_t = 1$), at least one between HP cooling and PCM cold discharge must cool the room:

$$\text{HP\_state}_{3,t} + \text{PCM\_state}_{5,t} \leq 2 \cdot \text{cooling}_t \quad \forall t$$
$$\text{HP\_state}_{3,t} + \text{PCM\_state}_{5,t} \geq \text{cooling}_t \quad \forall t$$

Electric Energy Consumed by Heat Pump:

$$\text{H2HP}_{1,t} = \text{HP\_state}_{2,t} \cdot \text{HP\_power\_H} \cdot \text{ON\_Ratio}_{1,t} \cdot \delta t_t \quad \forall t \quad \text{(kWh)}$$
$$\text{H2HP}_{2,t} = \text{HP\_state}_{3,t} \cdot \text{HP\_power\_C} \cdot \text{ON\_Ratio}_{2,t} \cdot \delta t_t \quad \forall t \quad \text{(kWh)}$$

Thermal Energy Delivered to the Room:

$$\text{Heat\_delivered}_t = \text{HP2H}_{1,t} - \text{HP2H}_{2,t} + \text{PCM\_discharge}_{1,t} - \text{PCM\_discharge}_{2,t} \quad \forall t \quad \text{(kWh)}$$

Heat Pump Thermal Power Balance Constraints:

$$\text{HP2H}_{i,t} = \text{H2HP}_{i,t} \cdot \text{COP}_{i,t} - \text{PCM\_charge}_{i,t} \quad \forall t \quad \text{(kWh)}$$

Charging and Discharging of PCM Storages:

$$\text{PCM\_charge}_{1,t} = \text{PCM\_state}_{2,t} \cdot \text{PCM\_charging}_{1,t} \quad \forall t \quad \text{(kWh)}$$
$$\text{PCM\_charge}_{2,t} = \text{PCM\_state}_{3,t} \cdot \text{PCM\_charging}_{2,t} \quad \forall t \quad \text{(kWh)}$$
$$\text{PCM\_discharge}_{1,t} = \text{PCM\_state}_{4,t} \cdot \text{PCM\_H\_discharge\_rate} \cdot \text{ON\_Ratio}_{3,t} \cdot \delta t_t \quad \forall t \quad \text{(kWh)}$$
$$\text{PCM\_discharge}_{2,t} = \text{PCM\_state}_{5,t} \cdot \text{PCM\_C\_discharge\_rate} \cdot \text{ON\_Ratio}_{4,t} \cdot \delta t_t \quad \forall t \quad \text{(kWh)}$$

$PCM\_discharge_{i,t}$ (for both Hot and Cold PCMs discharge rate) are assumed constant values.

PCM SOC Balance:

$$\text{PCM\_SOC}_{i,t+1} = \text{PCM\_SOC}_{i,t} + \frac{\text{PCM\_charge}_{i,t} - \text{PCM\_discharge}_{i,t}}{\text{PCM\_Size}} \quad \forall i, t$$

ON_Ratio Constraint:

$$\text{ON\_Ratio}[t] \leq \left\lfloor \frac{\text{steps}[t]}{\text{base\_stepsize}} \right\rfloor \quad \forall t$$