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Unique Properties of Lateral-Directional Stability and Control in Birds

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ABSTRACT

There are specific features in birds which lead to unique properties of lateral-directional stability and control. One is the lacking of the vertical tail. It is shown that this primarily has an impact on yaw stability and damping. In order to achieve adequate levels in yaw stability and damping, the wing plays an important role. Another property concerns the size of birds. It is shown that their size with respect to their aerodynamic configuration is such that there is a very fast behavior concerning the rotary dynamics. This particularly holds for the rotary dynamics in the roll axis. Significant factors in this respect are identified and dealt with.

Nomenclature

b	wing span
C_L	lift coefficient
C_l	rolling moment coefficient, $C_l = 2L/(\rho V^2 Ss)$
$C_{li} = \partial C_l / \partial i, \quad i = ps/V, rs/V, \beta, \delta_a, \delta_r$	
C_n	yawing moment coefficient, $C_n = 2N/(\rho V^2 Ss)$
$C_{ni} = \partial C_n / \partial i, \quad i = ps/V, rs/V, \beta, \delta_a, \delta_r$	
g	acceleration due to gravity
i_i	radius of gyration, $i = x, z$
L	lift, rolling moment
m	mass
N	yawing moment
p	roll rate
r	yaw rate
S	reference area
s	half wing span
V	speed
T_r	roll mode time constant
A	aspect ratio, $A = b^2/S$
β	sideslip angle
δ_a	roll control
δ_r	yaw control

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ρ	air density
σ_d	dutch roll damping ratio
ω_{nd}	undamped natural frequency of dutch roll

I. INTRODUCTION

The aerodynamic configuration of birds is unique when compared with technical air vehicles since it is lacking a vertical tail. They have in common the other aerodynamic components which are essential for the ability of flight, i.e. the wing, the body and the horizontal tail. Lacking of the vertical tail has a substantial impact on the lateral-directional stability and control properties. A fundamental issue is the stability and the damping as well as the controllability in the yaw axis. This is subject of recent research involving theoretical investigations and experimental studies (Refs. 1-7).

Another issue of comparable significance are the properties of rotary dynamics and control in the roll axis. These are important for initiating and performing turns and, in more general terms, for flight maneuvers involving lateral-directional dynamics. For the roll properties, the lacking of the vertical tail does not have such a great impact as on directional stability and control. However, birds show unique properties in roll dynamics and control in terms of a very fast behavior. This is due to their size when related to the aerodynamic configuration.

Subject of this paper are the addressed issues. It will be shown that there are unique properties in the lateral-directional dynamics of birds, both due to lacking a vertical tail and due to the smallness in size related to the aerodynamic configuration.

II. BASIC CONSIDERATIONS ON LATERAL-DIRECTIONAL MOMENT PROPERTIES IN BIRDS

Lateral-directional dynamics concern the rotary motions in the roll and yaw axes as well as the translatory motion in the lateral direction. The lateral-directional stability and damping properties are determined by the aerodynamic moments generated by the motion and the control deflections and by the inertia features with regard to the roll and yaw axes. In this respect, the dynamics and control characteristics related to the roll axis can be considered dominant for the lateral-directional dynamics properties in birds, i.e. for the type, magnitude and rapidity of the response due to disturbances and control inputs and for the maneuverability as a whole. This is because the generation of aerodynamic moments in the roll axis is more pronounced than in the yaw axis. A primary reason for the reduction in generating yawing moments is due to the lacking of the vertical tail in birds. By contrast, the aerodynamic moment capability in the roll axis is primarily due to the wing, with little or negligible influence of the vertical tail. For an aerodynamic configuration without a vertical tail, the wing is the most important component for generating moments with respect to the lateral-directional motion.

The addressed relationships between the rolling and yawing moment properties manifest in the contributions which the wing and the vertical tail provide. An overview is given in Table 1 which presents the entire set of stability and control moment derivatives of the lateral-directional motion. It is shown which component of the aerodynamic configuration provides a primary contribution.

	Roll Stability and Control Derivatives	Yaw Stability and Control Derivatives
β	$C_{l\beta}$: wing	$C_{n\beta}$: vertical tail
r	C_{lr} : wing	C_{nr} : vertical tail
p	C_{lp} : wing	C_{np} : wing
δ_a	$C_{l\delta_a}$: wing	$C_{n\delta_a}$: wing
δ_r	$C_{l\delta_r}$: vertical tail	$C_{n\delta_r}$: vertical tail

Table 1 Primary contributions to lateral-directional moment derivatives

With regard to Table 1, the following considerations apply:

- Lacking of the vertical tail means that the moment derivatives $C_{n\beta}$, C_{nr}, $C_{l\delta r}$ and $C_{n\delta r}$ are significantly reduced in magnitude.
- The other moment derivatives, $C_{l\beta}$, C_{lr}, C_{lp}, C_{np}, $C_{l\delta a}$ and $C_{n\delta a}$, are primarily influenced by the wing. Since this influence is basically due to the wing form, it is independent of the size. As a result, these moment derivatives show the same numerical values for small wings as for large ones.
- There is no dedicated yaw control surface in birds δ_r. By contrast, the roll control power can be assumed to be high. This is because birds have the capability for roll control moments by rotating one wing half or both, by partially retracting one wing half and/or by twisting the horizontal tail.
- Flight tasks requiring a yaw control capability (in terms of a rudder in aerodynamic configurations with a vertical tail) can be performed by birds using the roll control. These tasks include take-off and landing in a cross wind as well as flight with an asymmetric configuration concerning drag, lift and/or external load (Ref. 7).
- It is important to note that the yawing moment derivatives $C_{n\beta}$ and C_{nr} can fulfill their functions with respect to static and dynamic yaw stability as well as to yaw damping of birds though they are substantially reduced in magnitude because of lacking of the vertical tail (Refs. 6, 7). This will be considered in more detail in a subsequent section.

From the above considerations, it can be concluded that the rolling moments gain an increased significance, due to lacking of the vertical tail. This is an indication for the dominant role of the roll dynamics as part of the whole lateral-directional motion.

III. YAW STABILITY AND DAMPING

General Aspects

Yaw stability, also termed directional or weathercock stability, means that a restoring yawing moment is generated when a flying object (bird, aircraft) is disturbed by a sideslip angle β from the symmetric flight condition ($\beta = 0$). There are two points in this respect: static and dynamic yaw stability.

Static yaw stability is related to the change in the aerodynamic yawing moment N with the sideslip angle β, yielding

$$N(\beta) = C_{n\beta}\beta(\rho/2)V^2 Ss \tag{1}$$

There is yaw stability if $\partial N/\partial\beta > 0$, or

$$C_{n\beta} > 0 \tag{2}$$

Dynamic yaw stability refers to the dynamic action following a sideslip disturbance. It can be related to the natural frequency of the dutch roll mode of motion, ω_{nd}:

$$\omega_{nd} \approx \frac{s}{i_z^2}\sqrt{\frac{g}{s}\frac{C_{n\beta}}{C_L}} \tag{3}$$

Comparison of Eqs. (2) and (3) shows that static yaw stability is a part of dynamic yaw stability. Therefore, the following treatment is concerned with the dynamic yaw stability problem, thus covering also the static stability aspects.

For a required fast response capability, it is necessary that ω_{nd} is equal or larger than a minimum value $\omega_{nd,\min}$ which is considered to be sufficient. With $\omega_{nd,\min}$ specified, a relation can be derived for the minimum of the yaw stability derivative, $C_{n\beta,\min}$, yielding

$$C_{n\beta,\min} = \frac{s}{g}\left(\frac{i_z}{s}\right)^2 C_L \omega_{nd,\min}^2 \tag{4}$$

An evaluation of Eq. (4) for describing the size effects in birds is possible with appropriate data. For his purpose, the following relations are applied:

– The dependence of the half span s on the size is described in terms of a relationship between s and the mass m. With reference to Ref. 8, the relation $s = 0.5825\, m^{0.394}$ is used.
– The relationship between the half span s and the radius of gyration is assumed to be independent of the size. As a conservative estimate, $(i_z / s)^2 = 0.05$ is applied (Ref. 6).
– A value of $\omega_{nd,\min} = 1.0\ \mathrm{rad/s}$ is used as a minimum for the natural frequency of the dutch roll, considered a conservative value. In specifying this value, reference is made to flying qualities requirements of aircraft (Refs. 9, 10). A minimum value of 1.0 rad/s relates to rapid maneuvering, precision tracking, or precise flight path control.

Results from an evaluation of Eq. (4) with the described data are presented in Fig. 1. From these results it basically follows that $C_{n\beta,\min}$ is very small, being progressively reduced with a decrease in the size. The most important result from Fig. 1 is that $C_{n\beta,\min}$ can be achieved with the wing alone.

Wing Effects

The yaw stability and damping properties of birds have been dealt with in detail in recent papers (Refs. 6, 7). With reference to these papers, a short description is given in the following.

The wing plays a fundamental role for yaw stability in birds. This is because it can generate stabilizing yawing moments large enough for an adequate level of dynamic yaw stability in terms of the natural frequency of the dutch roll. A wing property for efficiently generating stabilizing yawing moments is sweep. There is sweep in wings with pointed tips, advantageously at the outer parts. Other wings have slotted tips, showing significant sweep in the separated feathers.

An example for a wing with pointed tips is presented in Fig. 2 which depicts the planform and the profile. The wing which is from a gull employs considerable sweep which is at its outer part. The $C_{n\beta}$ properties of this wing are presented in Fig. 3. Basically, the wing provides a stabilizing yawing moment because $C_{n\beta}$ is positive. Further, the stability level becomes increasingly significant in the lift coefficient region greater than $C_L = 0.8$, growing progressively with C_L. This region which is related to flight conditions at minimum glide angle or minimum sink rate can be considered particularly relevant for gliding birds (Ref. 6).

Also shown in Fig. 3 is $C_{n\beta,\min}$ for the gull, using $\omega_{nd,\min} = 1.0\ \mathrm{rad/s}$. Comparison of $C_{n\beta}$ and $C_{n\beta,\min}$ shows that the required minimum is met, with the surplus of $C_{n\beta}$ considerably increasing in the region above $C_L = 0.8$.

The yawing moment derivative $C_{n\beta}$ of the gull wing, presented in Fig. 3, was determined using the FLM-Eu Code which is a computer program developed at the Institute of Fluid Mechanics of the Technische Universität München, Refs. 12 and 13. The FLM-Eu Code is a program for modelling fluid flow around complex two- and three-dimensional geometries and to obtain results of high numerical precision. It provides comprehensive modelling capabilities for a wide range of steady and unsteady flows of inviscid, rotational and/or compressible nature. The calculations are based on a finite-volume approximation to the integral form of the unsteady Euler equations.

Yaw damping refers to an aerodynamic moment which acts in a direction opposite to the rotary rate in the yaw axis. The damping moment can be expressed as

$$N(r) = C_{nr} r (s/V)(\rho/2) V^2 S s \tag{5}$$

A wing basically produces an aerodynamic damping moment when it experiences a yaw rate. This is shown in Fig. 4 which presents results on the yaw damping derivative of wings, C_{nr}, for an aspect ratio and the lift coefficient range relevant for birds. The size of C_{nr} increases with a reduction of A and an increase of C_L.

For achieving an adequate damping of yawing motions (i.e. of the dutch roll mode), a minimum level in the aerodynamic damping derivative C_{nr} is required. The following relation for the minimum amount of the aerodynamic damping derivative, denoted by $|C_{nr}|_{\min}$, can be derived

$$|C_{nr}|_{\min} \approx 2\left(\frac{i_z}{s}\right)^2 \sqrt{\frac{2C_L m}{g\rho S}} |\sigma_d|_{\min} \tag{6}$$

where $|\sigma_d|_{\min}$ is the required damping of the dutch roll mode.

A graphical evaluation of this relation is shown in Fig. 5 for $|\sigma_d|_{\min} = 0{,}1\ \mathrm{s}^{-1}$. For s and i_z/s, the same relations are applied as before. The dependence of the reference area S on the size is described using $S = 0.1576\, m^{0.722}$, with reference made to Ref. 8. The results presented in Fig. 4 show that the required aerodynamic damping derivative is rather small. This particularly holds with regard to the data given in Fig. 3.

IV. ROLL DYNAMICS AND CONTROL

While the ability to generate aerodynamic yawing moments is strongly dependent on having or lacking a vertical tail, there is no such effect for the rolling moments. This is because the aerodynamic rolling moments are primarily determined by the wing (as addressed in Table 1). Therefore, the aerodynamics with regard to roll dynamics and control show basically comparable properties for configurations with or without a vertical tail. This also holds for birds. Nevertheless, there are unique properties of roll dynamics and control in birds. For the following treatment, it is assumed that the bird can be dealt with as a rigid body.

For the rotary motion in the roll axis, the following topics are important:

– roll performance
– roll mode of motion
– roll response rapidity
– roll control moments

Roll Performance

Roll performance denotes the achievable roll rate in a stationary roll. For determining the achievable roll rate, designated as p_{stat}, a stationary rolling motion for a given roll control input δ_a is considered:

$$\frac{\partial L}{\partial p} p_{stat} + \frac{\partial L}{\partial \delta_a} \delta_a = 0 \tag{7}$$

Solving for p_{stat} and using non-dimensional derivatives (C_{lp}, $C_{l\delta a}$) yields

$$p_{stat} = -\frac{V}{s} \frac{C_{l\delta a}}{C_{lp}} \delta_a \tag{8}$$

With reference to a 1-g flight condition, given by $L = mg$ or

$$C_L(\rho/2)V^2S = mg \tag{9}$$

the following relation is obtained

$$p_{stat} = -\sqrt{\frac{2gm}{C_L\rho S}}\frac{1}{s}\frac{C_{l\delta a}}{C_{lp}}\delta_a \tag{10}$$

This relation can be evaluated to show the effect of size on the roll performance and, thus, the unique properties of small flying objects.

For this purpose, it is assumed that C_{lp} is independent of the size. It is basically determined by the wing, and here it is the aspect ratio A that plays a primary role. This manifests in the following relation for unswept wings (Ref. 14)

$$C_{lp} = -\frac{1}{4}\frac{\pi A}{\sqrt{A^2/4+4}+2} \tag{11}$$

Furthermore, the roll control derivative $C_{l\delta a}$ can be regarded as constant with respect to the size effect for the following cases. The constancy of $C_{l\delta a}$ holds for conventional ailerons if the geometric relationship relative to the wing is not changed. It can also be considered to apply for birds if the movable wing part for generating roll control moments shows a constant relation with regard to the entire wing.

The remaining quantities that change with the size are the mass m, the wing area S and the half span s, Eq. (10). For describing the changes in these quantities, the following scaling law relationships are applied

$$\begin{aligned} S/S_{ref} &= (m/m_{ref})^{2/3} \\ s/s_{ref} &= (m/m_{ref})^{1/3} \end{aligned} \tag{12}$$

where S_{ref}, m_{ref} and s_{ref} are introduced as quantities of a reference case.

With Eqs. (10) and (12), the relation between the roll performance and the size can be expressed as

$$p_{stat} = (m_{ref}/m)^{1/6}\,p_{stat,ref} \tag{13}$$

where $p_{stat,ref}$ relates to the reference case. This relation shows that the roll performance increases with a decrease in the size. A quantitative evaluation of Eq. (13) is presented in Fig. 6, providing an insight into the magnitude of the changes.

The effect of the size on the achievable roll performance means that small flying objects, like birds, basically have a high roll performance. If a given level of roll performance can be regarded to be sufficient, it follows from Eq. (13) that the roll control power, in terms of aileron size or movable wing size, can be reduced with a reduction in the size of a flying object.

For aircraft, there are flying qualities requirements on the achievable roll performance (Refs. 9, 10). It may be of interest that the greatest level required for airplanes of high maneuverability is at about 150 deg/s. For comparison, roll rates of a bat are reported, showing an average value of 1800-2250 deg/s for a roll through 180 deg (Ref. 15).

Roll Mode

The roll mode, also termed roll subsidence, is determinative for the roll dynamics. It is basically a 1-degree-of-freedom rotary motion in the roll axis, showing the bank angle, or the roll rate, as its main modal component. The sideslip angle and the yaw rate are of secondary or negligible significance.

According to the 1-degree-of-freedom model for the roll mode, the following relation holds

$$I_x \frac{\mathrm{d}p}{\mathrm{d}t} = \frac{\partial L}{\partial p} p + \frac{\partial L}{\partial \delta_a} \delta_a \tag{14}$$

Solving for the roll mode time constant which describes the time behavior of this aperiodic mode of motion yields the following expression for a 1-g reference flight condition, using non-dimensional derivatives:

$$T_r = -\sqrt{\frac{2C_L m}{g\rho S}} \frac{(i_x / s)^2}{C_{lp}} \tag{15}$$

For showing the effect of size, it is assumed that i_x / s is constant. Applying the scaling relationship given in Eq. (12) yields

$$T_r = (m / m_{ref})^{1/6} T_{r,ref} \tag{16}$$

where $T_{r,ref}$ relates to a reference case. From this relation it follows that T_r decreases with a reduction in the size. As a result, the dynamics in the roll axis become faster for smaller flying objects.

An evaluation of Eq. (16) is presented in Fig. 7, using data of a gull and applying Eq. (11) for estimating C_{lp}. There is a significant effect of the lift coefficient, yielding an increase of T_r with C_L.

It is important to note with respect to the size effect on the roll mode time constant that the only aerodynamic quantity which may have an influence is the roll damping derivative C_{lp} (the lift coefficient can be considered to be at a constant level). This quantity can be treated as independent of the size because it is determined by the wing geometry, Eq. (11).

It may be of interest to compare the level of roll mode time constants possible for birds with values of aircraft of high maneuverability. There are flying qualities requirements in this respect (Refs. 13, 14). For aircraft for which a very fast roll mode behavior is required, the maximum value of the roll mode time constant amounts to $T_r = 1.0$ s.

Roll Response Rapidity

An issue of roll dynamics and control is the rapidity with which the initial response to a roll control input develops. This can be judged when considering the roll response to step control input. The following relation holds for this case

$$p = (1 - e^{-t/T_r}) p_{stat} \tag{17}$$

where p_{stat} is given by Eq. (8).

The initial response characteristics can be related to the roll acceleration at the beginning ($t = 0$):

$$\dot{p}(0) = \left(\frac{s}{i_x}\right)^2 \frac{g}{s} \frac{C_{l\delta_a} \delta_a}{C_L} \tag{18}$$

Using the scaling relations Eq. (12) and assuming again i_x / s constant, the effect of size can be expressed as

$$\dot{p}(0) = (m_{ref} / m)^{1/3} \dot{p}_{ref}(0) \tag{19}$$

where $\dot{p}_{ref}(0)$ relates to a reference case. This expression shows that the roll response rapidity is even more influenced by the size than the roll performance and the roll mode time constant, Eqs. (13) and (16).

A graphical interpretation is presented in Fig. 5. Two cases are shown, a reference case with m_{ref} and a decreased size case with m. The size effect can be decomposed into two parts, Fig. 5. One relates to the increase of p_{stat}, and the other to the decrease of the roll mode time constant T_r.

Roll Control Moments

Basically, roll control moments can be produced by generating an asymmetric lift distribution at the wing. This can be achieved by increasing the lift on one wing half and/or by decreasing the lift on the other.

Birds have the ability for generating roll control moments by various means (e.g., Refs. 15-21). One concerns the wing. An asymmetry of the lift distribution can be generated by an appropriate control action, yielding different lift forces on the left and right sides. This may be combined with a change in the points of action of the different lift forces on the left and right sides such that their effect on the roll moment is increased. A roll control moment can be achieved by differential twisting or flexing of the wing on the left and right. The left and right wing halves can be stretched out to different extents. For achieving a high roll rate, one wing half may be partially retracted. This yields a greater lift difference between the left and right sides and a decrease in the distance between the center of gravity and the lift point of action at the partially retracted wing half, contributing to an increase of the roll moment.

The tail can also be used to generate roll control moments. For this purpose, the tail attitude is controlled such that it is twisted. Thus, an asymmetry of the lift distribution at the tail can be achieved. As a result, a roll control moment is generated.

V. CONCLUSIONS

Flight mechanical characteristics of birds are identified and dealt with which cause unique properties of lateral-directional stability and control. One point is the lacking of the vertical tail which has a large impact on yaw dynamics. It is shown that the wing can play a major role in order do achieve an adequate level in yaw stability and damping. Another point relates to the size of birds. It is shown that the rotary dynamic behavior in birds is basically fast due to their smallness. This particularly concerns the rotary properties in the roll axis.

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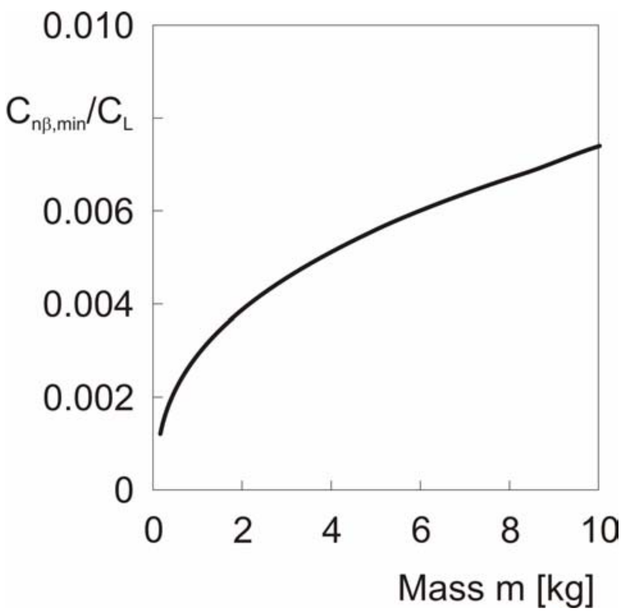

Figure 1 Minimum static yaw stability $C_{n\beta,\min}$ for birds

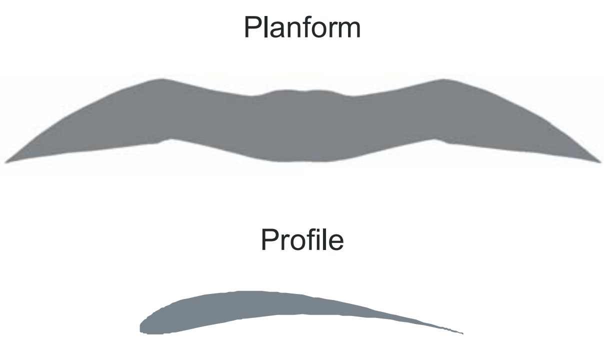

Figure 2 Gull wing (from Ref. 11)

Data: $S = 0.206\ \mathrm{m}^2 \quad s = 0.78\ \mathrm{m}$ **,** $A = 11.81$

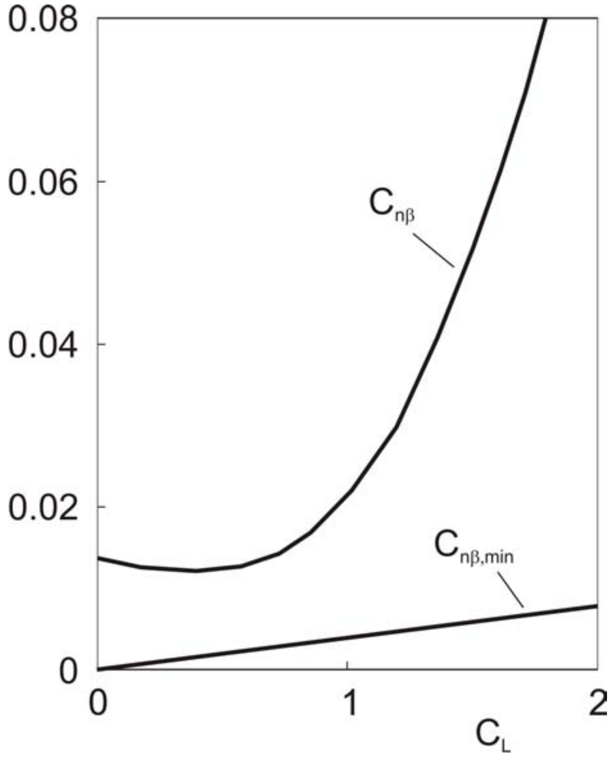

Figure 3 Yawing moment characteristics of gull
Data of gull wing: $S = 0.206\ \mathrm{m}^2 \quad s = 0.78\ \mathrm{m}$ **(from Ref. 11)**

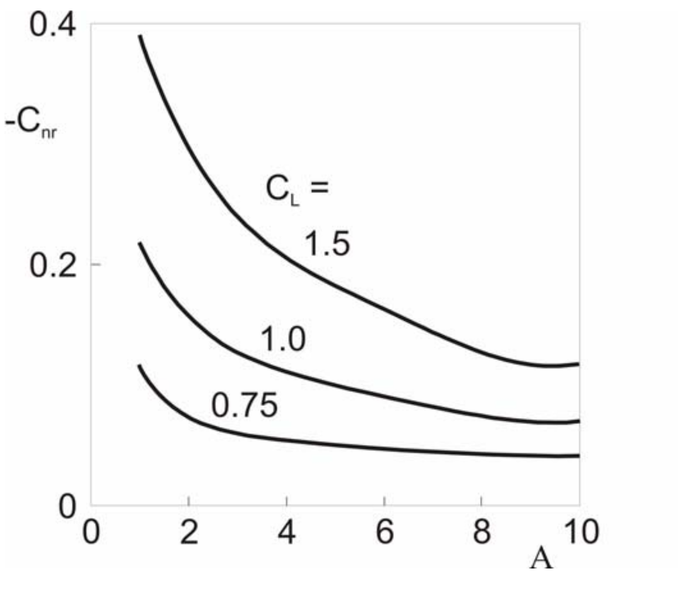

Figure 4 Estimated yaw damping derivative of unswept wings

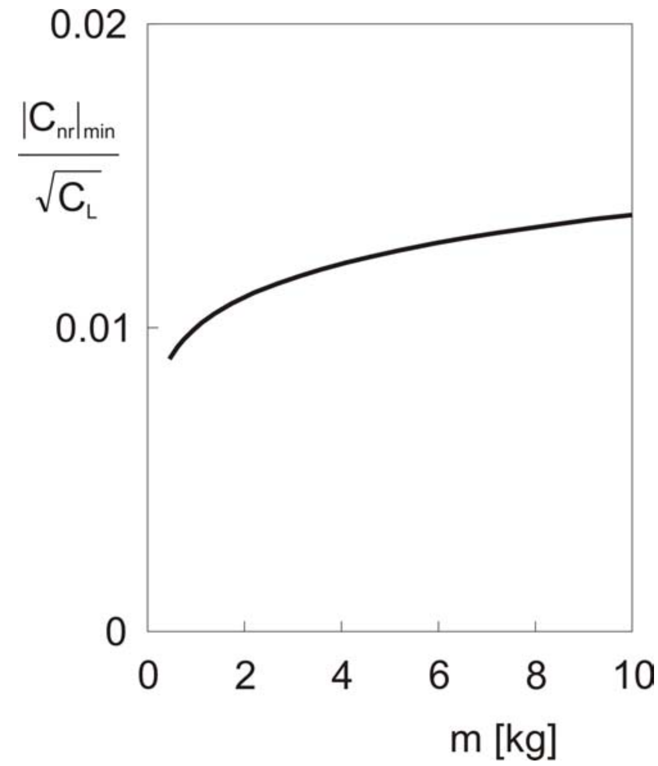

Figure 5 Required yaw damping derivative

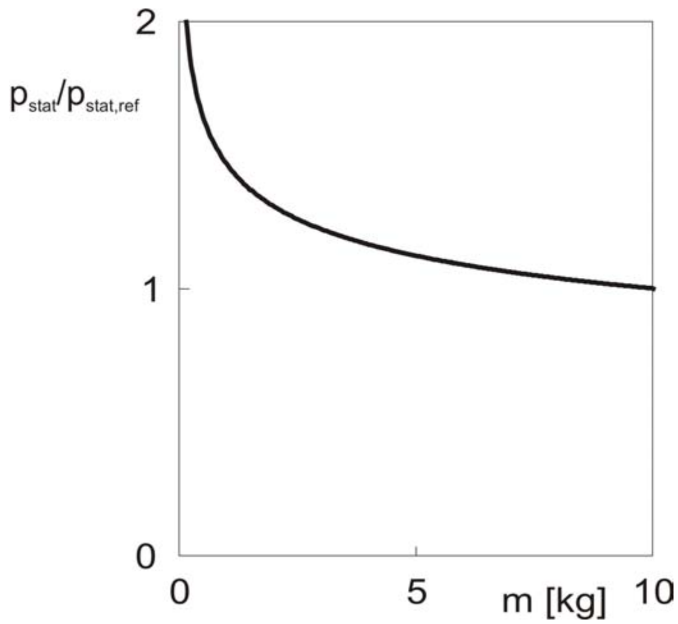

Figure 6 Effect of size on roll performance ($p_{stat,ref}$ related to $m_{ref} = 10$ kg)

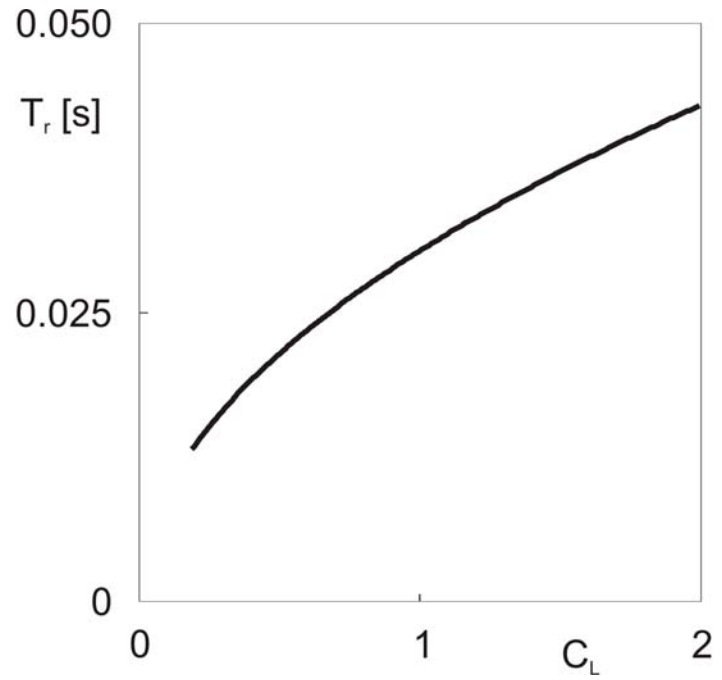

Figure 7 Roll mode time constant of gull
Data: $S = 0.206\ \mathrm{m}^2$ **,** $m = 1.607\ \mathrm{kg}$ **(from Ref. 11),** $(i_x / s)^2 = 0.03$ **(from Ref. 6)**

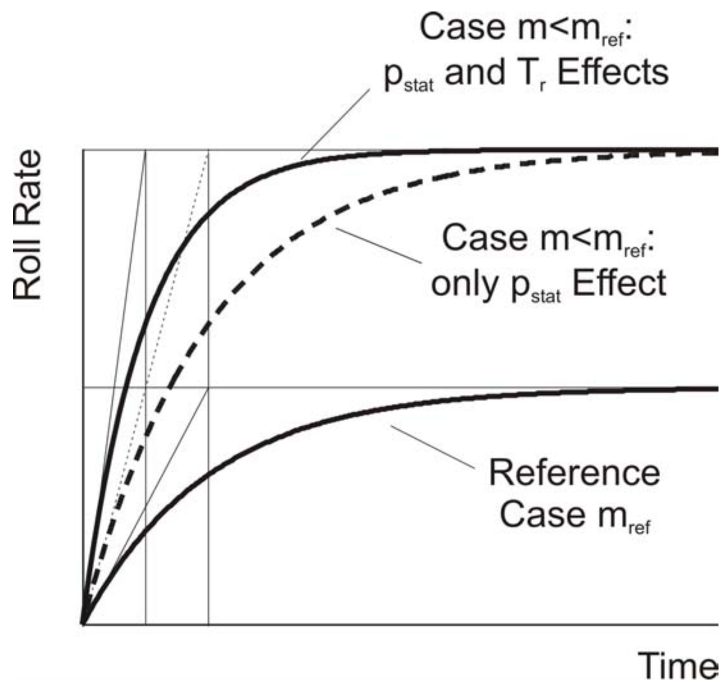

Figure 8 Effect of size on roll response to step roll control input