Assignment
Title of Course: Calculus and Statistical Analysis
Course Code: 22AS001
Topic Name: Applications of Partial Derivative and Multiple Integral

1. The total weekly revenue (in Rupee) that Acrosonic realizes in producing and selling its bookshelf loudspeaker systems is given by

$$R(x,y) = -\frac{1}{4}x^2 - \frac{3}{8}y^2 - \frac{1}{4}xy + 300x + 240y$$

where x denotes the number of fully assembled units and y denotes the number of kits produced and sold each week. The total weekly cost is given by

$$C(x,y) = 180x + 140y + 5000$$

Determine how many assembled units and how many kits Acrosonic should produce per week to maximize its profit.

2. An aquarium with rectangular sides and bottom (no top) is to hold 32 litres. Find its dimensions so that it will use the least amount of material for its construction.
3. A manufacturer can produce three distinct products in quantities q_1,q_2 and q_3, respectively, and there by derive a profit $p(q_1,q_2, q_3) = 2q_1+8q_2+24q_3$. Find q_1, q_2, q_3 that maximize profit if production is subject to the constraint $q_1^2+ 2q_2^2+4q_3^2= 450000$.
4. The profit for some software company is given by $PR(x,y) = -100 + 80x - 0.1x^2 + 100y - 0.2y^2$, where x & y represents the levels of output of two products produced by the company. If the company knows its maximum combined feasible production to be 325. How can the company maximize its profit?
5. If temperature is given by $T(x,y) = (x+y)e^{-x^2-y^2}$ .Find the maximum and minimum temperatures for the given surface.
6. Evaluate $\iint_D (x+2y)\,dA$ where D is the region bounded by the parabolas $y = 2x^2$ and $y = 1 + x^2$.
7. Evaluate $\int_0^1\int_0^1 x\max(x,y)\,dydx$.
8. Evaluate $\iint\limits_R (x-y+1)\,dxdy$, where R is the region inside the unit square in which, $x+y \geq 0.5$.
9. Evaluate the integral $\int_0^2\int_x^2 e^{-y^2}\,dydx$.
10. Find the area enclosed by one loop of the curve $r = \cos 2\theta$.

11. An engineer is interested to design a ring shape pond which is bounded by circles $r = 4\cos\theta$ and $r = 8\cos\theta$. Find the area of top of the pond.
12. Find by double integration, the area lying inside the circle $r = a\sin\theta$ & outside the cardiod $r = a(1-\cos\theta)$.
13. Verify the formula $V = \frac{4}{3}\pi a^3$ for the volume of a sphere of radius a.
14. Find the volume of ellipsoid $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1$.
15. A thin plate covers a triangular region D of the xy-plane bounded by the x-axis and the lines x = 1 and y = 2x in the first quadrant. The plate's density at the point (x, y) ∈ D is δ(x, y) = 6x + 6y + 6. Find the plate's Mass, First Moments and Center-of-Mass about the coordinate axes.
16. A plane lamina of non-uniform density ia in the form of a quadrant of the ellipse $x^2/a^2+y^2/b^2=1$. If density at any point (x,y) be kxy, where k is a constant ,find the co-ordinates of the centroid of lamina.
17. Suppose the solid bounded by $x = y^2$, z = x, z = 0, y = 0, x = 1 has uniform density ρ = 3. Find the mass of the solid.
18. Suppose the solid bounded by $x = y^2$, z = x, z = 0, y = 0, x = 1 has density is variable:
    ρ (x, y, z) = x + y + z then what is the mass of the region?
19. Find the volume of the solid S that is bounded by the elliptic paraboloid $x^2 + 2y^2 + z = 16$. The planes x = 2 , y = 2 and the three coordinate p l a n s .
20. Suppose we have a lamina which is a triangle with vertices (0, 0),(0, 1),(1, 0), and density function ρ(x, y) = y. What is the mass of the lamina?
21. Suppose that the temperature (in degrees Celsius) at a point (x, y) on a flat metal plate is

    $T(x,y) = 10 - 8x^2 - 2y^2$, where x and y are in meters. Find the average temperature of the rectangular portion of the plate for which $0 \le x \le 1$ and $0 \le y \le 2$, where Average Temperature = $\frac{1}{Area\ of\ rectangle}\iint_D T(x,y)dA$

22. If the cylinder $x^2 + y^2 = 1$ cut out the sphere $x^2 + y^2 + z^2 \le 4$, then find out the volume of the remaining solid left.
23. If the depth of water provided by a water sprinkler in a given unit of time is 2^{-r} feet at a distance of r feet from the sprinkler, find the total volume of water within a distance of a feet from the sprinkler after one unit of time.

24. Find the volume of the tetrahedron bounded by the coordinate axes and the plane $3x + 6y + 4z = 12$.
25. A thin plate covers a triangle region D of the xy-plane bounded by x-axis and the lines x=1 and y=2x in the first quadrant .The plate's density at the point $(x,y) \in D$ is δ(x,y)=6x+6y+6.Find the plate's Mass= $\iint_D \delta(x,y)dA$ and first moments $M_x = \iint_D y\delta(x,y)dA$ and $M_y = \iint_D x\delta(x,y)dA$.