

## Question Bank

1. Find the rank of the following matrices:

$$(a) \begin{bmatrix} 4 & 2 & 3 \\ 8 & 5 & 2 \\ 12 & -4 & 5 \end{bmatrix} \quad (b) \begin{bmatrix} 2 & -1 & 0 & 5 \\ 0 & 3 & 1 & 4 \end{bmatrix} \quad (c) \begin{bmatrix} 2 & 1 & -3 \\ 3 & -3 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad (d) \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

2. Find inverse of the following matrices by Gauss Jordan method :

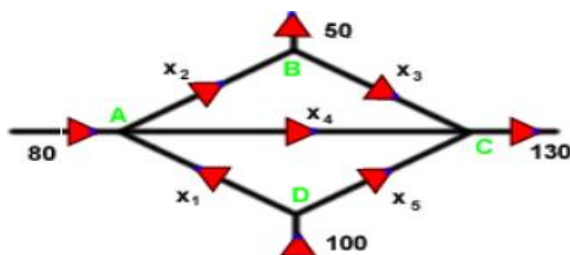
$$(a) \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \quad (b) \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix} \quad (c) \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$$

3. Find the value of k such that the rank of  $\begin{bmatrix} 1 & 2 & 3 \\ 2 & k & 7 \\ 3 & 6 & 10 \end{bmatrix}$  is 2.

4. Find the rank of the following matrices using normal form

$$(a) \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix} \quad (b) \begin{bmatrix} 2 & -4 & 3 & 1 & 0 \\ 1 & -2 & 1 & -4 & 2 \\ 0 & 1 & -1 & 3 & 1 \\ 4 & -7 & 4 & -4 & 5 \end{bmatrix} \quad (c) \begin{bmatrix} 0 & 1 & -3 & -1 \\ 0 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

5. Find all the  $x_i$ 's in the following traffic network



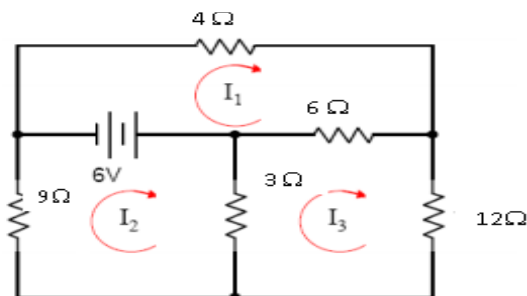
6. Find the currents in network ,if current flow is given by the following equations using Kirchoff's current and voltage law:

$$5I_1 + 10I_3 = 220$$

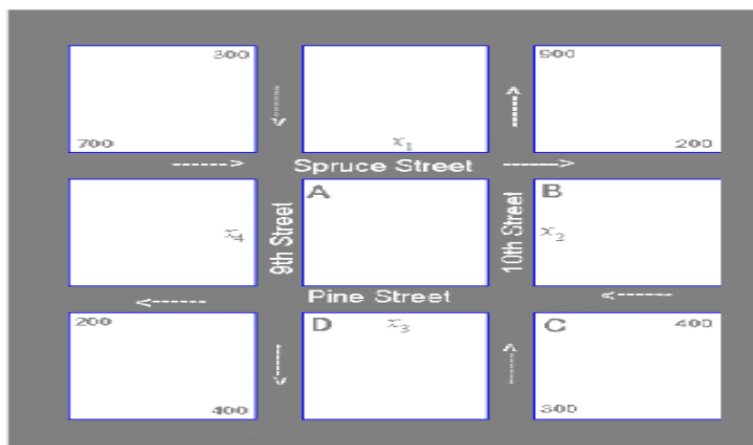
$$20I_2 + 10I_3 = 240$$

$$I_1 + I_2 - I_3 = 0$$

7. Calculate the current in each branch of the circuit given below



8. A florist is making 5 identical bouquets for a wedding. She has Rs 610 to spend (including tax) and wants 24 flowers for each bouquet. Roses cost Rs 6 each, tulips cost Rs 4 each, and lilies cost Rs 3 each. She wants to have twice as many roses as the other 2 flowers combined in each bouquet. How many roses, tulips, and lilies are in each bouquet?
9. At rush hours, traffic congestion is encountered at the street intersections shown in the figure below. The city wishes to improve the traffic signals at these corners to improve the traffic flow. All streets are one-way and the directions are indicated by the arrows. Form linear system of equation to avoid congestion and hence find the solution.



10. A nut distributor wants to know the nutritional content of various mixtures of almonds, cashews, and peanuts. Her supplier has provided the following nutrition information:

	Almonds	Cashews	Peanuts
Protein(g/cup)	26.2	21	10.1
Sugar(g/cup)	40.2	44.8	14.3
Fats(g/cup)	71.9	63.5	82.8

Her first mixture, a protein blend, consists of 6 cups of almonds, 3 cups of cashews, and 1 cup of peanuts. Her second mixture, a low fat mix, consists of 3 cups of almonds, 6 cups of cashews, and 1 cup of peanuts. Her third mixture, a low sugar mix consists of 3 cups of almonds, 1 cup of cashews, and 6 cups of peanuts. Determine the amount of protein, sugar, and fats in a 1 cup serving of each of the mixtures.

11. A nutritious drink consists of whole egg, milk, and orange juice. The food energy and protein for each of the ingredients are given by the table:

	Food Energy (Calories)	Protein (Grams)
1 Egg	80	6
1 cup Milk	160	9
1 cup Orange juice	110	2

How much of each ingredient is needed to produce a drink of 540 calories and 25 grams of protein?

12. A company produces three types of alloys; soft (S), medium (M) and hard (H). They are produced as blends of two basic components C and K. To produce one ton of S, 0.75 ton of C and 0.25 ton of K are needed. The requirements for M and H are 0.55, 0.45 and 0.3, 0.7 respectively. The company wants to know what amounts to the components are needed to fulfill an order of 10 tons of S, 12 tons of M and 20 tons of H?
13. The atomic weights of three compounds are shown. Use a linear system to find the atomic weights of carbon(C ), hydrogen(H), and oxygen(O). Determine atomic weight of Carbon, Hydrogen and Oxygen.

Compound	Formula	Atomic weight
Methane	$\text{CH}_4$	16
Glycerol	$\text{C}_3\text{H}_8\text{O}_3$	92
Water	$\text{H}_2\text{O}$	18

14. Find the value of  $\lambda$  for which  $x + y + 4z = 1$ ,  $x + 2y - 2z = 1$  and  $\lambda x + y + z = 1$  has unique solution.

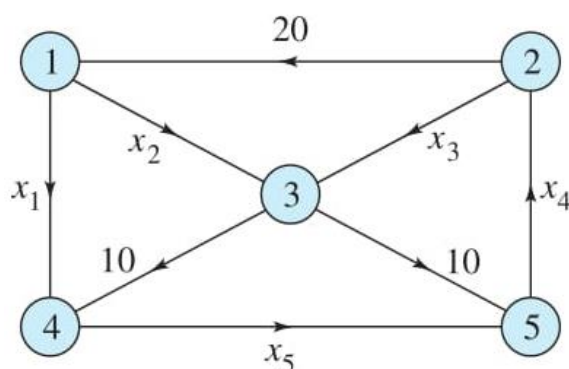
15. Find the eigen values and eigen vector of the matrix  $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$  and  $\begin{bmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$

16. Check the consistency of a system of equations  $3x + y + 2z = 3$ ,  $2x - 3y - z = -3$ ,  $x + 2y + z = 4$ .

17. For which value's of 'k' the system has  $kx + y = 1$  and  $6x + (k+1)y = 3$  has infinite number of solutions?

18. The standard equation of a circle is  $x^2 + y^2 + Ax + By + C = 0$ . Find the equation of the circle that passes through the points  $(3, -1)$ ,  $(-2, 4)$  and  $(6, 8)$ .

19. Set up a system of linear equations to represent the network shown in Figure. Then solve the system.



20. On average, Seema spends 3 hours on homework, 4 hours watching TV and 8 hours sleeping each day. Reena watch 1 hour of TV, sleeps 6 hours and spends 5 hours doing homework. Rahul works on homework for an hour and sleeps for 9 hours. Is Cayley Hamilton theorem applicable for this problem, if yes then verify it and find inverse using the same.

21. Diagonalize the matrices:

(a)  $\begin{bmatrix} 1 & 2 & -2 \\ 2 & 1 & -4 \\ 1 & -1 & -2 \end{bmatrix}$

(b)  $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

22. Find the quadratic form of the matrix:

(a)  $\begin{bmatrix} 2 & 4 & 5 \\ 4 & 3 & 1 \\ 5 & 1 & 1 \end{bmatrix}$

(b)  $\begin{bmatrix} 1 & 1 & -2 & 0 \\ 1 & -4 & 0 & 0 \\ -2 & 0 & 6 & -3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$

23. (a) Find the matrix of the quadratic form.

$$2x^2 + 5y^2 - 6z^2 - 2xy - yz + 8zx$$

(b) Reduce the quadratic form into canonical form:

$$3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$$

24. For what values of a & b do the equations  $x + 2y + 3z = 6$ ,  $x + 3y + 5z = 9$ ,  $2x + 5y + az = b$  have

- (i) No solution                      (ii) a unique solution                      (iii) more than one solution?

25. Using the loop current method on a circuit, the following equations were obtained

$$7i_1 - 4i_2 = 12, -4i_1 + 12i_2 - 6i_3 = 0, -6i_2 + 14i_3 = 0. \text{ Solve for } i_1, i_2 \text{ \& } i_3 \text{ by obtaining matrix from}$$

these equations. Also verify the Cayley Hamilton Theorem for the matrix.