Generalized Eigenvalue Problem

The generalized eigenvalue problem of two symmetric matrices $\mathbf{K} = \mathbf{K}^T$ and $\mathbf{M} = \mathbf{M}^T$:

$$\mathbf{K}\Phi = \lambda \mathbf{M}\Phi \tag{1}$$

Cholesky factorization leads to:

$$\mathbf{M} = \mathbf{L}\mathbf{L}^T \tag{2}$$

The generalized eigenvalue problem can be reduced to:

$$\mathbf{C}y = \lambda y \tag{3}$$

where:

$$\mathbf{C} = \mathbf{L}^{-1}\mathbf{K}\mathbf{L}^{-T} \tag{4}$$

Given that $\mathbf{C}$ is a symmetric matrix, its eigenvalues can be solved for using the Exact Jacobi Method. This is done by reducing the norm of the off diagonal elements:

$$F(\mathbf{C}) = \sqrt{\sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} c_{ij}^2} \tag{5}$$

This is achomplished by a sequence of orhogonal similarity transformations:

$$\mathbf{C}^{(K+1)} = \mathbf{J}_{pq}^T \mathbf{C}^K \mathbf{J}_{pq}, \qquad k = 0, 1, 2, ... \tag{6}$$

where:

$$\mathbf{C}^0 = \mathbf{C} \tag{7}$$