CHOLESKY – JACOBI EIGENVALUE ALGORITHM

A METHOD FOR SOLVING THE GENERALIZED EIGENVALUE PROBLEM ON FPGAS

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Technical Update: Computing Eigenvalues on a FPGA

- Needed to develop a generalized eigenvalue solver that could run on an FPGA
- Could not find anything in the literature for generalized eigenvalues solved on a FPGA The core concept is to keep all matrices as symmetric (as possible)
- Investigated several methodologies, the proposed solution is a two-step generalized eigenvalue solver.
  - Cholesky decomposition to create a symmetric reduction of the M and K matrix
  - Jacobi eigenvalue algorithm to solve the reduced problem.

The generalized eigenvalue problem of two symmetric matrices $\mathbf{A} = \mathbf{A}^T$ and $\mathbf{B} = \mathbf{B}^T$

$$\mathbf{A}\Phi = \lambda \mathbf{B}\Phi \quad (1)$$

Cholesky factorization leads to

$$\mathbf{B} = \mathbf{L}\mathbf{L}^T \quad (2)$$

The generalized eigenvalue problem can be reduced to

$$\mathbf{C}y = \lambda y \quad (3)$$

where:

$$\mathbf{C} = \mathbf{L}^{-1}\mathbf{A}\mathbf{L}^{-T} \quad (4)$$

Given that $\mathbf{C}$ is a symmetric matrix, its eigenvalues can be solved for using the Exact Jacobi Method. This is done by reducing the norm of the off diagonal elements:

$$F(\mathbf{A}) = \sqrt{\sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} a_{ij}^2} \quad (5)$$

This is achomplished by a sequence of orhogonal similarity transformations:

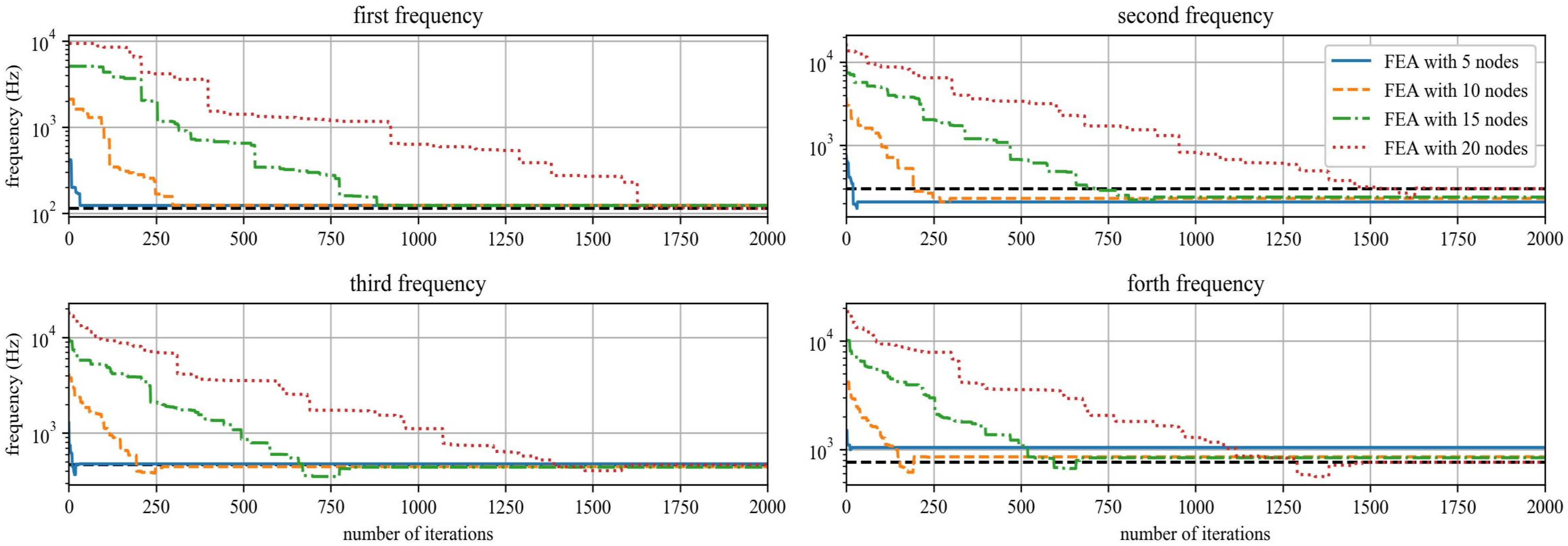
$$\mathbf{A}^{(K+1)} = \mathbf{J}_{pq}^T \mathbf{A}^K \mathbf{J}_{pq}, \qquad k = 0, 1, 2, \ldots \quad (6)$$

where:

$$\mathbf{A}^0 = \mathbf{A} \quad (7)$$

Technical Update: Cholesky-Jacobi Simulations

- Simulated the proposed Cholesky-Jacobi general eigenvalue solver
  - The code is not fast on a CPU (order of seconds).
  - Code is concise and should be straightforward to implement in LabVIEW.



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Next Steps: Hardware Implementation and Updating Probability Density Functions Cholesky-Jacobi

Implement proposed Cholesky-Jacobi algorithm on a FPGA

- Need to consider hardware space and data sharing between FPGA and CPU Potential of just

using FPGA for the rotations of the Jacobi eigenvalue solver

- Implementing a reduced order model technique may be a key step in accelerating calculations

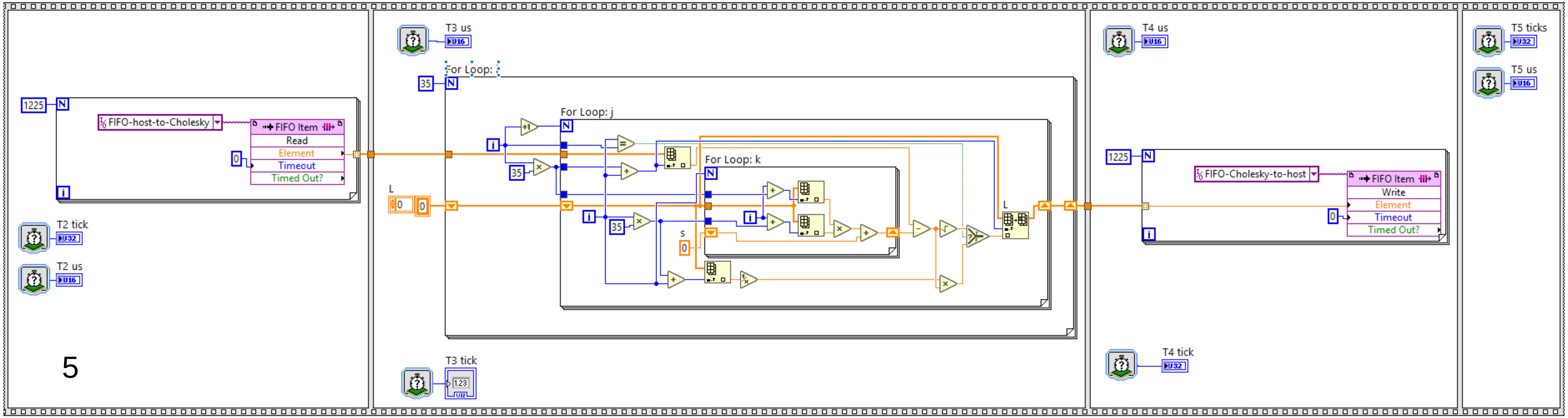
Investigate updating the PDFs based on prior knowledge and upcoming events.

- Look into existing mathematical tools for quantifying uncertainty (DRAM: Efficient adaptive MCMC)
- Use precalculated uncertainty to limit search space (variances of PDF)
- Update PDFs based on prior error in the predictions.

Technical Update: FPGA Update

Implemented Cholesky algorithms on FPGA

- In the single precision floating point form, the CPU is 2x faster than the FPGA.
- Fixed point numbers allow for single cycle timed loops (increases speed, decreases hardware requirements).
- Implementing a usable fixed point method will take some optimization, currently have a 64-bit version working.



THANKS!