Summary of Main Equations in Mechanical Vibrations

The general response for the **undamped** case has the form:

$$x(t) = A\sin(\omega_n t + \phi) + X\cos(\omega t)$$

where:

free response:

$$A = \frac{\sqrt{\omega_n^2 x_0^2 + v_0^2}}{\omega_n}, \quad \phi = \tan^{-1}\left(\frac{x_0\omega_n}{v_0}\right), \text{ and } X = 0$$

forced response:

$$A = \sqrt{\left(\frac{v_0}{\omega_n}\right)^2 + (x_0 - X)^2}, \quad \phi = \tan^{-1}\left(\frac{\omega_n(x_0 - X)}{v_0}\right), \text{ and } X = \frac{f_0}{\omega_n^2 - \omega^2}$$

The general response for the **underdamped** case has the form:

$$x(t) = Ae^{-\zeta\omega_n t}\sin(\omega_d t + \phi) + X\cos(\omega t - \phi_p)$$

where:

free response:

$$A = \frac{\sqrt{(v_0 + \zeta\omega_n x_0)^2 + (x_0\omega_d)^2}}{\omega_d}, \quad \phi = \tan^{-1}\left(\frac{x_0\omega_d}{v_0 + \zeta\omega_n x_0}\right), \text{ and } X = 0$$

forced response:

$$A = \frac{x_0 - X\cos(\phi_p)}{\sin(\phi)}, \quad \phi = \tan^{-1}\left(\frac{\omega_d(x_0 - X\cos(\phi_p))}{v_0 + (x_0 - X\cos(\phi_p))\zeta\omega_n - \omega X\sin(\phi_p)}\right),$$

$$X = \frac{f_0}{\sqrt{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega_n\omega)^2}}, \text{ and } \phi_p = \tan^{-1}\left(\frac{2\zeta\omega_n\omega}{\omega_n^2 - \omega^2}\right)$$