Cholesky-Jacobi Eigenvalue Decomposition

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Computing Eigenvalues on a FPGA

- Needed to develop a generalized eigenvalue solver that could run on an FPGA
  - Could not find anything in the literature for generalized eigenvalues solved on a FPGA
  - The core concept is to keep all matrices as symmetric (as possible)
  - Investigated several methodologies, the proposed solution is a two step generalized eigenvalue solver.
    - λ 1: Cholesky decomposition to create a symmetric reduction of the M and K matrix
    - λ 2: Jacobi eigenvalue algorithm to solve the reduced problem.

The generalized eigenvalue problem of two symmetric matrices $\mathbf{A} = \mathbf{A}^T$ and $\mathbf{B} = \mathbf{B}^T$

$$\mathbf{A}\Phi = \lambda \mathbf{B}\Phi \tag{1}$$

Cholesky factorization leads to

$$\mathbf{B} = \mathbf{L}\mathbf{L}^T \tag{2}$$

The generalized eigenvalue problem can be reduced to

$$\mathbf{C}y = \lambda y \tag{3}$$

where:

$$\mathbf{C} = \mathbf{L}^{-1}\mathbf{A}\mathbf{L}^{-T} \tag{4}$$

Given that $\mathbf{C}$ is a symmetric matrix, its eigenvalues can be solved for using the Exact Jacobi Method. This is done by reducing the norm of the off diagonal elements:

$$F(\mathbf{A}) = \sqrt{\sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} a_{ij}^2} \tag{5}$$

This is achomplished by a sequence of orhogonal similarity transformations:

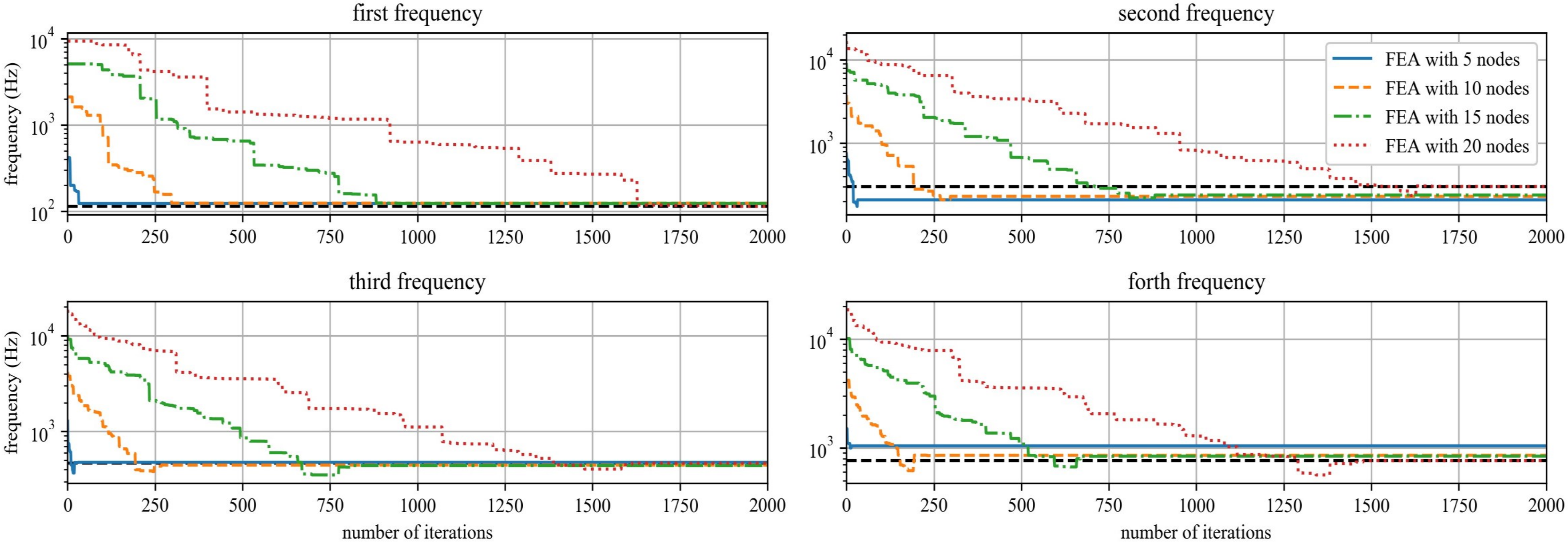
$$\mathbf{A}^{(K+1)} = \mathbf{J}_{pq}^T \mathbf{A}^K \mathbf{J}_{pq}, \qquad k = 0, 1, 2, \ldots \tag{6}$$

where:

$$\mathbf{A}^0 = \mathbf{A} \tag{7}$$

Cholesky-Jacobi Simulations

- Simulated the proposed Cholesky-Jacobi general eigenvalue solver
  - The code is not fast on a CPU (order of seconds).
  - Code is concise and should be straightforward to implement in LabVIEW.



Thank You for Your Time

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