Mackey-Glass time series

Mackey-Glass system is a nonlinear time-delay dynamic system which has been studied as a model of deterministic chaos. The equation is described as

$$\frac{dx}{dt} = \beta \frac{x(t-\tau)}{1+x^{10}(t-\tau)} - \gamma x \tag{1}$$

with $\gamma, \beta > 0$. The equation modulates the control value x with the feedback term $\frac{x(t-\tau)}{1+x(t-\tau)^n}$. The time lag τ describes the lag between the sensing of the parameter under control and mounting the of an appropriate response. Depending of the parameters selected, the Mackey-Glass equation displays a range of periodic and chaotic dynamics. If $\gamma > \beta > 0$, the oscillations decays to zero no matter the initial conditions. In case of $\beta > \gamma > 0$, as the delay increases, the solution of the system periodic solutions to chaotic motions.

The differential equation of Mackey-Glass can be written as an approximate $m+1$ dimensional map with a discrete step of $\Delta t = \tau/m$ as [2]

$$x_{n+1} = \frac{1}{2m+\gamma\tau} \left((2m - \gamma\tau)\, x_n + \tau\beta \left(\frac{x_{n-m}}{1+x_{n-m}^{10}} + \frac{x_{n-m+1}}{1+x_{n-m+1}^{10}} \right) \right) \tag{2}$$

This map can be used to create chaotic time series of different dimension. The delay parameter τ can be used to vary the dimension to an arbitrary value. If the parameters of the Mackey-Glass equation changes slowly over time as the system is evolving, the a non-stationary system with clear bifurcation points over time is observable.

Fig. 1(a-e) shows the evolution of the dynamics form a periodic to a chaotic solution of the system for $\gamma = 0.1$ and $\beta = 0.2$ as the delay value increases. With these parameters, the system has an equilibrium point at $x = 1$. For $\tau < 5$, the equilibrium is a fixed point and the the values moves slowly towards the equilibrium point as the time goes on with a periodic motion (Fig. 1(a)). As the τ increases ($\tau > 5$), the fixed point loses stability and turns into a limit cycle (Fig. 1(b)). By further increasing the τ, the system experiences a series of period doubling solutions (Fig. 1(c-d)). The behavior of the system is chaotic for $\tau > 16.8$ (Fig. 1(e)) [1].

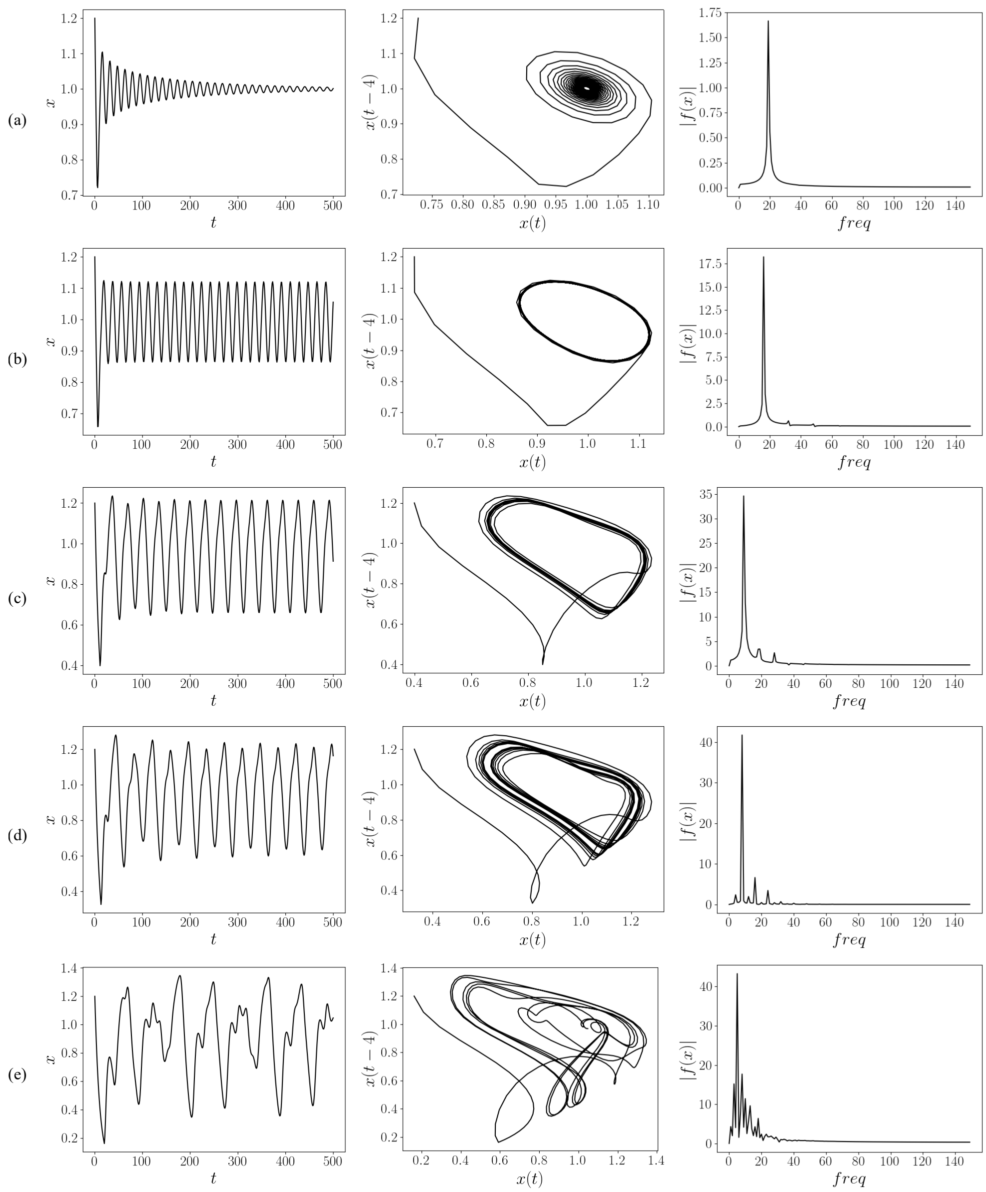

Figure 1: Mackey-Glass dynamics solution for $gamma = 0.1$ and $\beta = 0.2$ as the delay parameter changes from (a) $\tau = 4$, (b) $\tau = 6$, (c) $\tau = 11$, (d) $\tau = 13$, and (d) $\tau = 20$.

References

[1] Kyongmin Yeo and Igor Melnyk. Deep learning algorithm for data-driven simulation of noisy dynamical system. *Journal of Computational Physics*, 376:1212–1231, jan 2019.

[2] Thomas Schreiber Holger Kantz. *Nonlinear Time Series Analysis*. Cambridge University Press, 2006.