

**A LEVEL (P3)**  
**PARTIAL FRACTIONS**  
**MARK SCHEME**

1 (i) State or imply  $f(x) = \frac{A}{(2-x)} + \frac{Bx+C}{(x^2+1)}$

B1\*

State or obtain  $A = 4$ 

B1(dep\*)

Use any relevant method to find  $B$  or  $C$ 

M1

Obtain both  $B = 4$  and  $C = 1$ 

A1 4

2 (a)(i) State answer  $\frac{A}{x+4} + \frac{Bx+C}{x^2+3}$

B1

1

(ii) State answer  $\frac{A}{x-2} + \frac{Bx+C}{(x+2)^2}$  or  $\frac{A}{x-2} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$

B2

2

[Award B1 if the  $B$  term is omitted or for the form  $\frac{A}{x-2} + \frac{B}{x+2} + \frac{Cx+D}{(x+2)^2}$ .]

3 (i) State or imply partial fractions are of the form  $\frac{A}{x+2} + \frac{Bx+C}{x^2+1}$

B1

Use any relevant method to obtain a constant

M1

Obtain  $A = 2$ 

A1

Obtain  $B = 1$ 

A1

Obtain  $C = -1$ 

A1

[5]

4 (i) State or imply partial fractions are of the form  $\frac{A}{2-x} + \frac{Bx+C}{1+x^2}$

B1

Use any relevant method to obtain a constant

M1

Obtain one of the values  $A = 2$ ,  $B = 2$ ,  $C = 4$ 

A1

Obtain a second value

A1

Obtain the third value

A1

5

5 (i) Simplify product and obtain  $(1+x) - (1-x)$   
Complete the proof of the given result with no errors seen

B1

B1

2

6 (i) EITHER: State or imply  $f(x) = \frac{A}{2x+1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$

B1

Use any relevant method to obtain a constant

M1

Obtain one of the values  $A = 2$ ,  $B = -1$ ,  $C = 3$ 

A1

Obtain the remaining two values

A1 + A1

[A correct solution starting with third term  $\frac{Cx}{(x+1)^2}$  or  $\frac{Cx+D}{(x+1)^2}$  is also possible.]

OR: State or imply  $f(x) = \frac{A}{2x+1} + \frac{Dx+E}{(x+1)^2}$

B1

Use any relevant method to obtain a constant

M1

Obtain one of the values  $A = 2$ ,  $D = -1$ ,  $E = 2$ 

A1

Obtain the remaining two values

A1 + A1

5

- 7 (i) State or imply the form  $\frac{A}{1-x} + \frac{B}{1+2x} + \frac{C}{2+x}$  B1  
 Use any relevant method to determine a constant M1  
 Obtain  $A = 1, B = 2$  and  $C = -4$  A1 + A1 + A1 [5]
- 8 (i) State or imply the form  $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{10-x}$  B1  
 Use any relevant method to determine a constant M1  
 Obtain one of the values  $A = 1, B = 10, C = 1$  A1  
 Obtain the remaining two values A1 4  
 [The form  $\frac{Dx+E}{x^2} + \frac{C}{10-x}$  is acceptable and leads to  $D = 1, E = 10, C = 1$ ]
- 9 (i) State or imply partial fractions are of the form  $\frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{3x+2}$  B1  
 Use any relevant method to obtain a constant M1  
 Obtain one of the values  $A = 1, B = 2, C = -3$  A1  
 Obtain a second value A1  
 Obtain the third value A1 [5]
- 10 (i) State or imply the form  $\frac{A}{x+1} + \frac{B}{x+3}$  and use a relevant method to find  $A$  or  $B$  M1  
 Obtain  $A = 1, B = -1$  A1 [2]
- 11 (i) EITHER: Divide by denominator and obtain quadratic remainder M1  
 Obtain  $A = 1$  A1  
 Use any relevant method to obtain  $B, C$  or  $D$  M1  
 Obtain one correct answer A1  
 Obtain  $B = 2, C = 1$  and  $D = -3$  A1  
 OR: Reduce RHS to a single fraction and equate numerators, or equivalent M1  
 Obtain  $A = 1$  A1  
 Use any relevant method to obtain  $B, C$  or  $D$  M1  
 Obtain one correct answer A1  
 Obtain  $B = 2, C = 1$  and  $D = -3$  A1 [5]  
 [SR: If  $A = 1$  stated without working give B1.]
- 12 (i) State or imply partial fractions of the form  $\frac{A}{1-2x} + \frac{B}{2+x} + \frac{C}{(2+x)^2}$  B1  
 Use any relevant method to determine a constant M1  
 Obtain one of the values  $A = 1, B = 1, C = -2$  A1  
 Obtain a second value A1  
 Obtain the third value A1 [5]  
 [The form  $\frac{A}{1-2x} + \frac{Dx+E}{(2+x)^2}$ , where  $A = 1, D = 1, E = 0$ , is acceptable  
 scoring B1M1A1A1A1 as above.]

- 13 (i) State or imply the form  $\frac{A}{1+x} + \frac{Bx+C}{1+2x^2}$  B1  
 Use any relevant method to evaluate a constant M1  
 Obtain one of  $A = -1, B = 2, C = 1$  A1  
 Obtain a second value A1  
 Obtain the third value A1 [5]
- 14 (i) State or imply partial fractions are of the form  $\frac{A}{1+x} + \frac{Bx+C}{2+x^2}$  B1  
 Use a relevant method to determine a constant M1  
 Obtain one of the values  $A = -2, B = 1, C = 4$  A1  
 Obtain a second value A1  
 Obtain the third value A1 [5]
- 15 (i) Use any relevant method to determine a constant M1  
 Obtain one of the values  $A = 3, B = 4, C = 0$  A1  
 Obtain a second value A1  
 Obtain the third value A1 [4]
- 16 (i) State or imply the form  $A + \frac{B}{x+1} + \frac{C}{2x-3}$  B1  
 State or obtain  $A = 2$  B1  
 Use a correct method for finding a constant M1  
 Obtain  $B = -2$  A1  
 Obtain  $C = -1$  A1 [5]
- 17 (i) State or imply form  $\frac{A}{3-x} + \frac{Bx+C}{1+x^2}$  B1  
 Use relevant method to determine a constant M1  
 Obtain  $A = 6$  A1  
 Obtain  $B = -2$  A1  
 Obtain  $C = 1$  A1 [5]
- 18 State or imply correct form  $\frac{A}{x} + \frac{Bx+C}{x^2+1}$  B1  
 Use any relevant method to find at least one constant M1  
 Obtain  $A = 2$  A1  
 Obtain  $B = 5$  A1  
 Obtain  $C = -3$  A1 [5]
- 19 (i) Use any relevant method to determine a constant M1  
 Obtain one of the values  $A = 1, B = -2, C = 4$  A1  
 Obtain a second value A1  
 Obtain the third value A1 [4]  
 [If  $A$  and  $C$  are found by the cover up rule, give B1 + B1 then M1A1 for finding  $B$ . If only one is found by the rule, give B1M1A1A1.]

- 20 (i) State or imply partial fractions are of the form  $\frac{A}{x-2} + \frac{Bx+C}{x^2+3}$  B1  
 Use a relevant method to determine a constant M1  
 Obtain one of the values  $A = -1, B = 3, C = -1$  A1  
 Obtain a second value A1  
 Obtain the third value A1 [5]
- 21 (i) Either State or imply form  $\frac{A}{1+x} + \frac{B}{(1+x)^2} + \frac{C}{2-3x}$  B1  
 Use any relevant method to find at least one constant M1  
 Obtain  $A = -1$  A1  
 Obtain  $B = 3$  A1  
 Obtain  $C = 4$  A1  
Or State or imply form  $\frac{A}{1+x} + \frac{Bx}{(1+x)^2} + \frac{C}{2-3x}$  B1  
 Use any relevant method to find at least one constant M1  
 Obtain  $A = 2$  A1  
 Obtain  $B = -3$  A1  
 Obtain  $C = 4$  A1  
Or State or imply form  $\frac{Dx+E}{(1+x)^2} + \frac{F}{2-3x}$  B1  
 Use any relevant method to find at least one constant M1  
 Obtain  $D = -1$  A1  
 Obtain  $E = 2$  A1  
 Obtain  $F = 4$  A1 [5]
- 22 (i) Either State or imply partial fractions are of form  $\frac{A}{3-x} + \frac{B}{1+2x} + \frac{C}{(1+2x)^2}$  B1  
 Use any relevant method to obtain a constant M1  
 Obtain  $A = 1$  A1  
 Obtain  $B = \frac{3}{2}$  A1  
 Obtain  $C = -\frac{1}{2}$  A1 [5]  
Or State or imply partial fractions are of form  $\frac{A}{3-x} + \frac{Dx+E}{(1+2x)^2}$  B1  
 Use any relevant method to obtain a constant M1  
 Obtain  $A = 1$  A1  
 Obtain  $D = 3$  A1  
 Obtain  $E = 1$  A1 [5]
- 23 (i) Use a correct method for finding a constant M1  
 Obtain one of  $A = 3, B = 3, C = 0$  A1  
 Obtain a second value A1  
 Obtain a third value A1 4

- 24 (i) State or imply the form  $\frac{A}{1-x} + \frac{B}{2-x} + \frac{C}{(2-x)^2}$  B1
- Use a correct method to determine a constant M1
- Obtain one of  $A = 2, B = -1, C = 3$  A1
- Obtain a second value A1
- Obtain a third value A1 [5]
- [The alternative form  $\frac{A}{1-x} + \frac{Dx+E}{(2-x)^2}$ , where  $A = 2, D = 1, E = 1$  is marked B1M1A1A1A1 as above.]
- 25 (i) State or imply  $f(x) \equiv \frac{A}{(3x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)}$  B1
- State or obtain  $A = -3$  B1
- State or obtain  $B = 2$  B1
- Use any relevant method to find  $C$  M1
- Obtain  $C = 1$  A1 5
- [Special case: allow the form  $\frac{A}{(3x+1)} + \frac{Dx+E}{(x+1)^2}$  and apply the above scheme ( $A = -3, D = 1, E = 3$ ).]
- {SR: if  $f(x)$  is given an incomplete form of partial fractions, give B1 for a form equivalent to the omission of  $C$ , or  $E$ , or  $B$  in the above, and M1 for finding one coefficient.]
- 26 (i) EITHER State or imply  $f(x) \equiv \frac{A}{2x+1} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$  B1
- State or obtain  $A = 1$  B1
- State or obtain  $C = 8$  B1
- Use any relevant method to find  $B$  M1
- Obtain value  $B = 4$  A1
- OR State or imply  $f(x) \equiv \frac{A}{2x+1} + \frac{Dx+E}{(x-2)^2}$  B1
- State or obtain  $A = 1$  B1
- Use any relevant method to find  $D$  or  $E$  M1
- Obtain value  $D = 4$  A1
- Obtain value  $E = 0$  A1
- [5]
- 27 (i) EITHER: Divide by denominator and obtain a quadratic remainder M1
- Obtain  $A = 1$  A1
- Use any relevant method to obtain  $B, C$  or  $D$  M1
- Obtain one correct answer A1
- Obtain  $B = -1, C = 2, D = 0$  A1
- OR: Reduce  $RHS$  to a single fraction and identify numerator with that of  $f(x)$  M1
- Obtain  $A = 1$  A1
- Use any relevant method to obtain  $B, C$  or  $D$  M1
- Obtain one correct answer A1
- Obtain  $B = -1, C = 2, D = 0$  A1
- [5]

${}_{28}\overrightarrow{OA} = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}, \overrightarrow{OB} = \begin{pmatrix} 3 \\ -1 \\ 3 \end{pmatrix}, \overrightarrow{OC} = \begin{pmatrix} 4 \\ 2 \\ p \end{pmatrix}, \overrightarrow{OD} = \begin{pmatrix} -1 \\ 0 \\ q \end{pmatrix}$		<p>Condone notation throughout.</p> <p>Allow column vectors or <b>i,j,k</b> throughout</p>
<p>(i) <math>\overrightarrow{AB} = \mathbf{b} - \mathbf{a} = 2\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}</math></p> <p>Unit vector = <math>(2\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}) / \sqrt{(2^2 + 4^2 + 4^2)}</math></p> <p><math>= \pm (2\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}) / 6</math></p>	<p>M1</p> <p>M1</p> <p>A1</p>	<p>Use of <b>b-a</b>, rather than <b>b+a</b> or <b>a-b</b></p> <p>Dividing by the modulus of "his" <math>\overrightarrow{AB}</math></p> <p>Co (allow – for candidates using <b>a-b</b>)</p>
	[3]	

- 29 (i) State or imply the form  $A + \frac{B}{x+1} + \frac{C}{x+3}$  B1
- State or obtain  $A = 1$  B1
- Use correct method for finding  $B$  or  $C$  M1
- Obtain  $B = \frac{1}{2}$  A1
- Obtain  $C = -\frac{3}{2}$  A1 [5]

- 30 (i) State or imply the form  $\frac{A}{3-2x} + \frac{Bx+C}{x^2+4}$  B1
- Use a relevant method to determine a constant M1
- Obtain one of the values  $A = 3, B = -1, C = -2$  A1
- Obtain a second value A1
- Obtain the third value A1 [5]

- 31 (i) State or imply  $f(x) \equiv \frac{A}{2x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$  B1
- Use a relevant method to determine a constant M1
- Obtain one of the values  $A = 2, B = -1, C = 3$  A1
- Obtain the remaining values A1 + A1 5
- [Apply an analogous scheme to the form  $\frac{A}{2x-1} + \frac{Dx+E}{(x+2)^2}$ ; the values being  $A = 2,$
- $D = -1, E = 1.$ ]