A LEVEL (P3) PARTIAL FRACTIONS QUESTION'S

1 Let $f(x) = \dfrac{6+7x}{(2-x)(1+x^2)}$.

(i) Express $f(x)$ in partial fractions. [4]

2 An appropriate form for expressing $\dfrac{3x}{(x+1)(x-2)}$ in partial fractions is

$$\frac{A}{x+1} + \frac{B}{x-2},$$

where A and B are constants.

(a) Without evaluating any constants, state appropriate forms for expressing the following in partial fractions:

(i) $\dfrac{4x}{(x+4)(x^2+3)}$, [1]

(ii) $\dfrac{2x+1}{(x-2)(x+2)^2}$. [2]

3 **(i)** Express $\dfrac{3x^2+x}{(x+2)(x^2+1)}$ in partial fractions. [5]

4 **(i)** Express $\dfrac{10}{(2-x)(1+x^2)}$ in partial fractions. [5]

5 **(i)** Simplify $\left(\sqrt{(1+x)}+\sqrt{(1-x)}\right)\left(\sqrt{(1+x)}-\sqrt{(1-x)}\right)$, showing your working, and deduce that

$$\frac{1}{\sqrt{(1+x)}+\sqrt{(1-x)}} = \frac{\sqrt{(1+x)}-\sqrt{(1-x)}}{2x}.$$ [2]

6 Let $f(x) = \dfrac{7x+4}{(2x+1)(x+1)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence show that $\displaystyle\int_0^2 f(x)\,dx = 2 + \ln\tfrac{5}{3}$. [5]

7 **(i)** Express $\dfrac{2-x+8x^2}{(1-x)(1+2x)(2+x)}$ in partial fractions. [5]

8 **(i)** Express $\dfrac{100}{x^2(10-x)}$ in partial fractions. [4]

9 **(i)** Express $\dfrac{5x+3}{(x+1)^2(3x+2)}$ in partial fractions. [5]

10 **(i)** Express $\dfrac{2}{(x+1)(x+3)}$ in partial fractions. [2]

11 **(i)** Find the values of the constants A, B, C and D such that

$$\frac{2x^3-1}{x^2(2x-1)} \equiv A + \frac{B}{x} + \frac{C}{x^2} + \frac{D}{2x-1}.$$ [5]

12 **(i)** Express $\dfrac{4+5x-x^2}{(1-2x)(2+x)^2}$ in partial fractions. [5]

13 Let $\mathrm{f}(x) = \dfrac{3x}{(1+x)(1+2x^2)}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

14 **(i)** Express $\dfrac{5x-x^2}{(1+x)(2+x^2)}$ in partial fractions. [5]

15 Let $\mathrm{f}(x) = \dfrac{12+8x-x^2}{(2-x)(4+x^2)}$.

(i) Express $\mathrm{f}(x)$ in the form $\dfrac{A}{2-x} + \dfrac{Bx+C}{4+x^2}$. [4]

16 Let $\mathrm{f}(x) = \dfrac{4x^2-7x-1}{(x+1)(2x-3)}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

17 **(i)** Express $\dfrac{9-7x+8x^2}{(3-x)(1+x^2)}$ in partial fractions. [5]

18 Express $\dfrac{7x^2 - 3x + 2}{x(x^2 + 1)}$ in partial fractions. [5]

19 **(i)** Express $\dfrac{1}{\boldsymbol{x}^2(2\boldsymbol{x} + 1)}$ in the form $\dfrac{\boldsymbol{A}}{\boldsymbol{x}^2} + \dfrac{\boldsymbol{B}}{\boldsymbol{x}} + \dfrac{\boldsymbol{C}}{2\boldsymbol{x} + 1}$. [4]

20 Let $f(x) = \dfrac{2x^2 - 7x - 1}{(x - 2)(x^2 + 3)}$.

(i) Express $f(x)$ in partial fractions. [5]

21 **(i)** Express $\dfrac{7x^2 + 8}{(1 + x)^2(2 - 3x)}$ in partial fractions. [5]

22 **(i)** Express $\dfrac{4 + 12x + x^2}{(3 - x)(1 + 2x)^2}$ in partial fractions. [5]

23 Let $f(\boldsymbol{x}) = \dfrac{6 + 6\boldsymbol{x}}{(2 - \boldsymbol{x})(2 + \boldsymbol{x}^2)}$.

(i) Express $f(\boldsymbol{x})$ in the form $\dfrac{\boldsymbol{A}}{2 - \boldsymbol{x}} + \dfrac{\boldsymbol{B}\boldsymbol{x} + \boldsymbol{C}}{2 + \boldsymbol{x}^2}$. [4]

24 Let $f(x) = \dfrac{x^2 - 8x + 9}{(1 - x)(2 - x)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

25 Let $f(x) = \dfrac{4x}{(3x + 1)(x + 1)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

26 Let $f(x) = \dfrac{9x^2 + 4}{(2x + 1)(x - 2)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

27 Let $f(x) = \dfrac{x^3 - x - 2}{(x-1)(x^2+1)}$.

(i) Express $f(x)$ in the form

$$A + \frac{B}{x-1} + \frac{Cx+D}{x^2+1},$$

where A, B, C and D are constants. [5]

28 Let $f(x) = \dfrac{x^2 + 7x - 6}{(x-1)(x-2)(x+1)}$.

(i) Express $f(x)$ in partial fractions. [4]

29 Let $f(x) \equiv \dfrac{x^2 + 3x + 3}{(x+1)(x+3)}$.

(i) Express $f(x)$ in partial fractions. [5]

30 Let $f(x) = \dfrac{5x^2 + x + 6}{(3-2x)(x^2+4)}$.

(i) Express $f(x)$ in partial fractions. [5]

31 Let $f(\boldsymbol{x}) = \dfrac{11\boldsymbol{x} + 7}{(2\boldsymbol{x}-1)(\boldsymbol{x}+2)^2}$.

(i) Express $f(\boldsymbol{x})$ in partial fractions. [5]