$K$
$\theta \in R^d$
$K$
$\mathbf{x}(t) = [x_1(t), \ldots, x_K(t)]^T$

$$\dot{\mathbf{x}}(t) = \frac{d\mathbf{x}(t)}{dt} = \mathbf{f}(\mathbf{x}(t), \theta) \quad (1)$$

$\mathbf{y}(t)$
$K$
$\mathbf{E} \sim \mathcal{N}(\mathbf{E}; \mathbf{0}, \mathbf{D})$
$\mathbf{D}_{ik} = \sigma_k^2 \delta_{ik}$
$N$

$$\mathbf{Y} = \mathbf{X} + \mathbf{E}$$

$$\mathbf{X} = [\mathbf{x}(t_1), \ldots, \mathbf{x}(t_N)] = [\mathbf{x}_1, \ldots, \mathbf{x}_K]^T$$

$$\mathbf{Y} = [\mathbf{y}(t_1), \ldots, \mathbf{y}(t_N)] = [\mathbf{y}_1, \ldots, \mathbf{y}_K]^T$$

$\mathbf{x}_k = [x_k(t_1), \ldots, x_k(t_N)]^T$
$k$
$\mathbf{y}_k = [y_k(t_1), \ldots, y_k(t_N)]^T$
$\mathbf{Y}$
$\mathbf{X}$
$\theta$
$\theta$
$\mathbf{x}_u$

$$f_k(\mathbf{x}(t), \theta) = \sum_{i=1} \theta_{ki} \prod_{j \in \mathcal{M}_{ki}} x_j \quad (2)$$

$\mathcal{M}_{ki} \subseteq \{1, \ldots, K\}$

$$\begin{pmatrix} \mathbf{X} \\ \dot{\mathbf{X}} \end{pmatrix} \sim \mathcal{N}\left(\begin{matrix} \mathbf{X} \\ \dot{\mathbf{X}} \end{matrix}; \begin{matrix} \mathbf{0} \\ \mathbf{0} \end{matrix}, \begin{matrix} \mathbf{C}_\phi & \mathbf{C}'_\phi \\ {}'\mathbf{C}_\phi & \mathbf{C}''_\phi \end{matrix}\right) \quad (3)$$

$$\mathrm{cov}(x_k(t), x_k(t)) = C_{\phi_k}(t, t')$$

$$\mathrm{cov}(\dot{x}_k(t), x_k(t)) = \frac{\partial C_{\phi_k}(t,t')}{\partial t} =: C'_{\phi_k}(t, t')$$

$$\mathrm{cov}(x_k(t), \dot{x}_k(t)) = \frac{\partial C_{\phi_k}(t,t')}{\partial t'} =: {}'C_{\phi_k}(t, t')$$

$$\mathrm{cov}(\dot{x}_k(t), \dot{x}_k(t)) = \frac{\partial C_{\phi_k}(t,t')}{\partial t \partial t'} =: C''_{\phi_k}(t, t')$$

$$\dot{\mathbf{X}} = \mathbf{F} + \epsilon_1, \epsilon_1 \sim \mathcal{N}(\epsilon_1; \mathbf{0}, \mathbf{I}\gamma)$$

$$\dot{\mathbf{X}} = {}'\mathbf{C}_\phi \mathbf{C}_\phi^{-1} \mathbf{X} + \epsilon_2, \epsilon_2 \sim \mathcal{N}(\epsilon_2; \mathbf{0}, \mathbf{A})$$

$\mathbf{F} := \mathbf{f}(\mathbf{X}, \theta)$
$\mathbf{A} := \mathbf{C}''_\phi - {}'\mathbf{C}_\phi \mathbf{C}_\phi^{-1} \mathbf{C}'_\phi$
$\gamma$
$\mathbf{F}$
$\dot{\mathbf{X}}$
$\dot{\mathbf{X}}$
$\dot{\mathbf{X}}$
$\mathbf{X}$

$$\mathbf{F} = {}'\mathbf{C}_\phi \mathbf{C}_\phi^{-1} \mathbf{X} + \epsilon_0 \quad (4)$$

$\epsilon_0 := \epsilon_2 - \epsilon_1$
$\theta$
$\theta =$