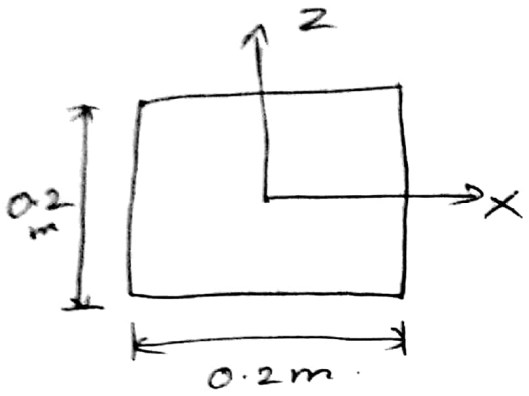
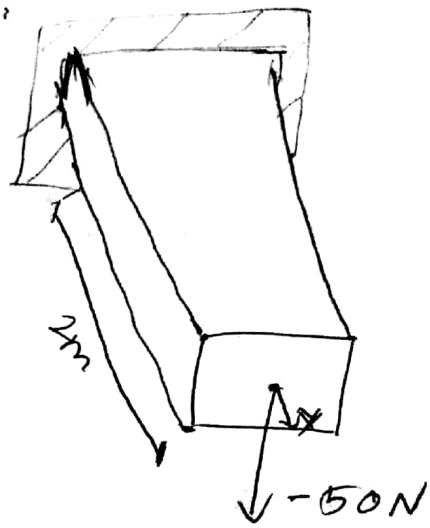
Loading vector:





$E = 75\,GPa$; $\nu = 0.33$; $a = 0.2m$; $L = 2m$;

A -50N Load is applied at the centre of the free tip $(0, L, 0)$. For 1 element case

$N_1 = \frac{1}{2}(1-\xi_1)$ $N_2 = \frac{1}{2}(1+\xi_1)$, In normalised geometry and in the cross section the Lagrange polynomials are

$$F_1 = \frac{1}{4}(1-\alpha)(1-\beta) \qquad F_3 = \frac{1}{4}(1+\alpha)(1+\beta)$$

$$F_2 = \frac{1}{4}(1+\alpha)(1-\beta) \qquad F_4 = \frac{1}{4}(1-\alpha)(1+\beta)$$

$\therefore \quad I, J = 1, 2 \qquad I, S = 1, 2, 3, 4.$

According to Principle of virtual displacement

$$\delta L_{ext} = P \delta u = P F_I N_i U_{Ii}$$

$UJ = 1, 2$ $\quad r, s = 1, 2, 3, 4$. Load is applied in 2 direction

$$\delta L_{ext} = P F_1 N_1 U_{z11} + P F_2 N_1 U_{z21} + P F_3 N_1 U_{z31}$$

$$+ P F_4 N_1 U_{z41} + P F_1 N_2 U_{z12} + P F_2 N_2 U_{z22}$$

$$+ P F_3 N_2 U_{z32} + P F_4 N_2 U_{z42}.$$

A

$$= P \frac{1}{4} (1-\alpha)(1-\beta)$$

At load application point $N_1 = 0$, $N_2 = 1$.

$$\therefore \delta L_{ext} = P F_1 N_2 U_{z12} + P F_2 N_2 U_{z22}$$

$$+ P F_3 N_2 U_{z32} + P F_4 N_2 U_{z42}$$

$\therefore N_2 = 1$

$$\delta L_{ext} = P \frac{1}{4} (1-\alpha)(1-\beta) U_{z12} + P \frac{1}{4} (1+\alpha)(1-\beta) U_{z22}$$

$$+ P \frac{1}{4} (1+\alpha)(1+\beta) U_{z32} + P \frac{1}{4} (1-\alpha)(1+\beta) U_{z42}.$$

At the centre of the free tip, $\boxed{\alpha = 0 \text{ and } \beta = 0}$ in the normalised geometry

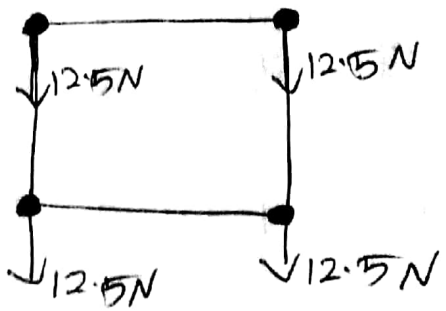
$P = -50\,N$

$$\delta L_{ext} = -\frac{1}{4} \times 50 \times UZ_{12} - \frac{1}{4} \times 50 \times U_{Z22} - \frac{1}{4} \times 50 \times U_{Z32} - \frac{1}{4} \times 50 \times U_{Z42}.$$

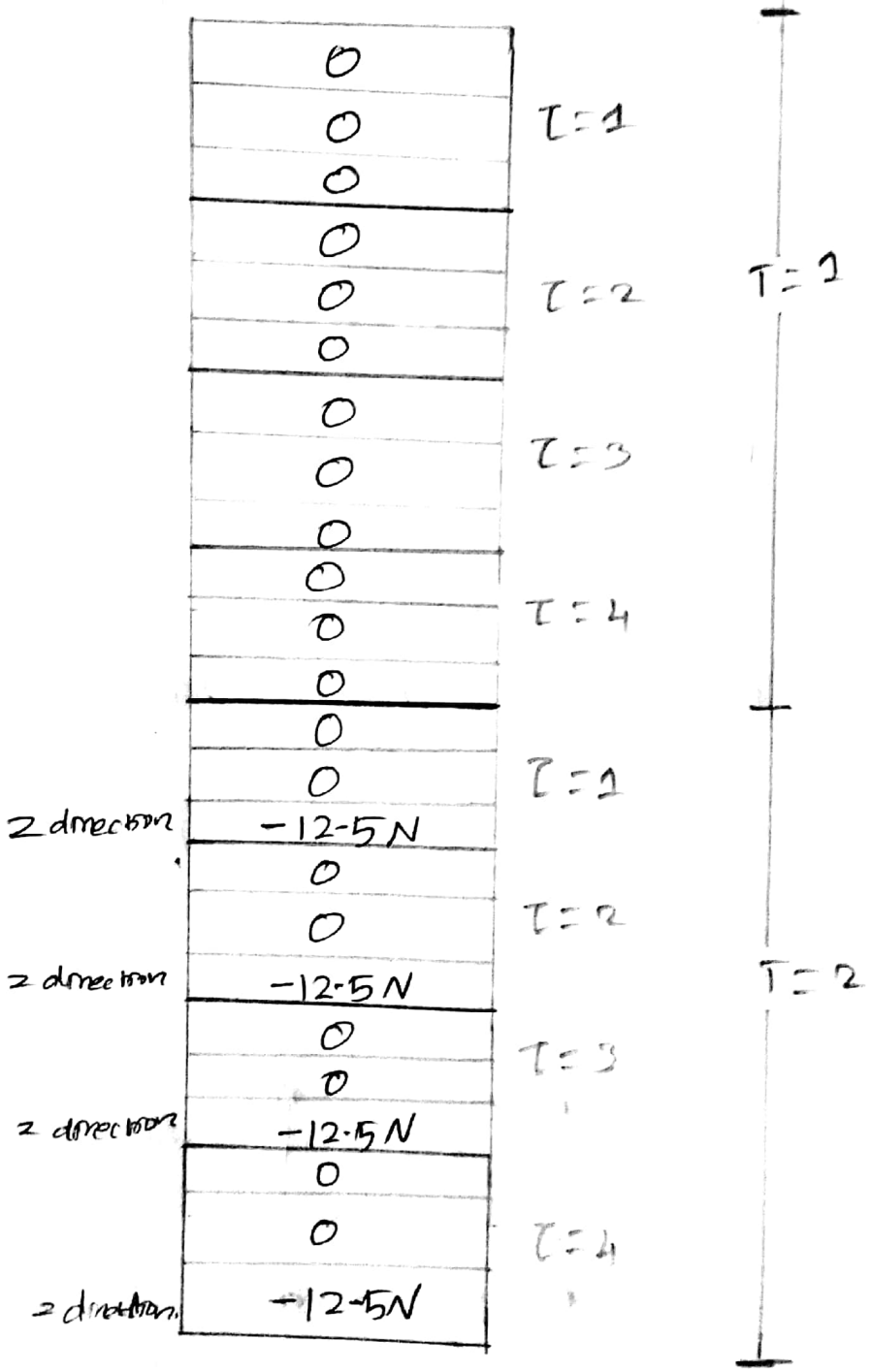
$$\delta L_{ext} = -12.5\,UZ_{12} - 12.5\,UZ_{22} - 12.5\,U_{Z32} - 12.5\,UZ_{42}.$$

∴

Therefore $-12.5\,N$ Load is applied at each Lagrange nodes at the cross section of the free tip.



∴ Load vector for 1 element case with 8 nodes (4 nodes at the fixed tip & 4 nodes at free tip)



Load vector.