

## Fundamental nucleus calculation:

1) Two point gauss quadrature has been taken for the axis of the beam (y)

$$\text{points} = (0.57735, -0.57735) \quad \text{weights} = 1, 1$$

2) Two point gauss quadrature is also considered for the cross section.

$$\text{point 1} = (0.57735, 0.57735, -0.57735, -0.57735)$$

$$\text{point 2} = (0.57735, -0.57735, 0.57735, -0.57735)$$

$$\text{weight} = [(1, 1), (1, 1), (1, 1), (1, 1)]$$

3) After that I have taken derivative of the lagrange polynomials with respect to alpha ( $\alpha$ ) and beta ( $\beta$ )

4) Then I have found  $x_\alpha, x_\beta, y_\alpha, y_\beta$  using the formula

$$\cancel{x_\alpha = F_1}$$

$$x = F_1 x_1 + F_2 x_2 + F_3 x_3 + F_4 x_4$$

$$x_{,\alpha} = F_{1,\alpha} x_1 + F_{2,\alpha} x_2 + F_{3,\alpha} x_3 + F_{4,\alpha} x_4$$

$\vdots$

$\vdots$

$\vdots$

5) After that I have found Jacobian of the cross section using

$$J = (Z_{\beta} * x, \alpha - Z, \alpha * x, \beta)$$

For my case it is 0.01.

6) Then my code goes into the loop.

for  $n$  in (beam nodes)  $\rightarrow$  ②

for  $m$  in (beam nodes):

7) Then it goes into the cross section loop.

for  $z$  in cross section nodes  $\rightarrow$  ④

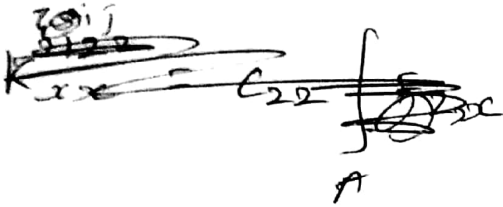
for  $s$  in cross section nodes:

8) In the loop first, I evaluate the values for  $F_{z,x}$ ,  $F_{z,z}$ ,  $F_{s,x}$ ,  $F_{s,z}$ .

9) After that ~~FA~~ components of fundamental nucleus are calculated.

For example:

(2)



$$k_{xxx}^{IIII} = c_{22} \int_A F_{1,x} F_{1,x} dx dz \int_1 N_1 N_1 dy$$

$$+ c_{66} \int_A F_{1,z} F_{1,z} dx dz \int_1 N_1 N_1 dy$$

$$+ c_{44} \int_A F_1 F_1 dx dz \int_1 N_{1,y} N_{1,y} dy$$

$$J = 0.01 \quad x, \alpha = 0.1 \quad z, \alpha = 0 \quad z, \beta = 0.1 \quad x, \beta = 0$$

$$F_{1,x} = \frac{1}{|J|} [z, \beta F_{1,\alpha} - z, \alpha F_{1,\beta}]$$

$$F_{1,x} = -2.5 [1 - \beta] \quad N_1 = \frac{1}{2} (1 - \xi)$$

Similarly,

$$F_{1,z} = 2.5 [1 - \beta]$$

∴ the eqn becomes,

$$k_{xxx}^{IIII} = c_{22} \int_A -2.5 [1 - \beta] * -2.5 [1 - \beta] dx dz \int_1 \frac{1}{2} (1 - \xi) * \frac{1}{2} (1 - \xi) d\xi$$

$$\rightarrow + C_{26} \int_A 2.5(1-\beta) \times 2.5(1-\beta) dx dz \int_1 \frac{1}{2}(1-\xi) \times \frac{1}{2}(1-\xi) d\xi$$

$$+ C_{44} \int_A \frac{1}{4}(1-\alpha)(1-\beta) \times \frac{1}{4}(1-\alpha)(1-\beta) dx dz$$

$$\int_1 -\frac{1}{2} \times -\frac{1}{2} d\xi$$

$$\left[ \begin{matrix} \vdots \\ N_1 \\ \vdots \end{matrix} \right] \uparrow$$

~~•  $\beta$  has~~

$\alpha$  and  $\beta$  has 4 values in gauss quadrature  
my code: (First component of  $K_{xx}$ )

$$K_{xx} = C_{22} * np.sum(w\_cs * F_{1,x} * F_{1,x} * J\_cs)$$

$$* np.sum(w\_length * shapefunc[1] * shapefunc[1] * J\_length).$$

since  $\alpha$  and  $\beta$  has 4 values  $F_{1,x}$  and  $F$  will have four values. therefore  $F_{1,x}$  will be an array.

$$np.sum \left[ \begin{matrix} \left[ \begin{matrix} w\_length \end{matrix} \right] * \left[ \begin{matrix} F_{1,x} \end{matrix} \right] * \left[ \begin{matrix} F_{1,x} \end{matrix} \right] * \left[ \begin{matrix} J\_cs \end{matrix} \right] \end{matrix} \right]$$

So my code will multiply corresponding element of each column and add all of them (ie first element of each column and the second element of each column soon)

$$w\_length[1] * F_{1,x}[1] * F_{1,x}[1] * J\_cs[1]$$

③

10) After that the ~~element~~ components ( $k_{xx}, \dots$ ) are then stacked in  $3 \times 3$  array to form a fundamental nuclei.

ii) Then the Fundamental nuclei is stacked into the nodal-stiffness matrix.

