Homework problems:

Problem 1.4

A one-dimensional, phenomenological plasticity model with nonlinear isotropic and and linear kinematic hardening is defined by the following set of equations:

elastic law:	$\sigma = \mathrm{E}\,\varepsilon_{\mathrm{e}}$
strain decomposition:	$\varepsilon = \varepsilon_{\mathrm{e}} + \varepsilon_{\mathrm{p}}$
yield surface and KKT conditions:	$\Phi(\xi, R) = \lvert \underbrace{\sigma - X}_{\xi} \rvert - \sigma_0 - R \leq 0, \quad \dot{\lambda} \geq 0, \quad \dot{\lambda}\Phi = 0$
evolution equations of internal variables:	$\dot{\alpha} = -\dot{\lambda}\,\frac{\partial \Phi}{\partial R} \qquad \dot{\varepsilon}_{\mathrm{p}} = \dot{\lambda}\,\frac{\partial \Phi}{\partial \xi}$
increase in yield stress:	$R = \mathrm{h}\,\alpha + \Delta\mathrm{Y}\,[1 - \exp(-\eta\alpha)]$
evolution of back-stress:	$X = \mathrm{H}\,\varepsilon_{\mathrm{p}}$

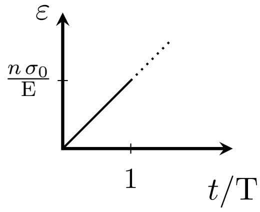
(a) In the case of elastic-plastic loading, i.e. $\Phi = 0$, the Lagrange multiplier $\dot{\lambda}$ is obtained from the consistency condition $\dot{\Phi} = 0$. Verify that for the model given above, the evaluation of the consistency condition yields the ordinary differential equation

$$\dot{\lambda}\,[\mathrm{E} + \mathrm{H} + \mathrm{h} + \Delta\mathrm{Y}\eta\exp(-\eta\alpha)] = \mathrm{sign}(\xi)\,\mathrm{E}\,\dot{\varepsilon}\ .$$

(b) Determine the time t^*, at which initial yielding occurs. The initial conditions for the internal variables read as $\alpha(t=0) = \varepsilon_{\mathrm{p}}(t=0) = 0$.

(c) Integrate the ordinary differential equation (a) in closed form for the linear increasing strain loading defined as,

$$\varepsilon(t) = \frac{n\,\sigma_0}{\mathrm{E}}\,\frac{t}{\mathrm{T}} \qquad 0 \leq t \leq \mathrm{T}$$



employing the initial conditions $\lambda(t=0) = 0$.