Exercise 4: Implementation of a three-dimensional von Mises plasticity model with linear hardening at small strains

Problem 5.1

The stress-update algorithm for a von Mises plasticity model with linear isotropic and kinematic hardening is given in the algorithmic box 1

Alg. 1 Radial return stress-update algorithm for J_2-plasticity with linear hardening

... Given: history information $\{\boldsymbol{\varepsilon}^{\mathrm{p}}_n, \boldsymbol{\alpha}_n, \alpha_n\}$ and current total strain $\boldsymbol{\varepsilon}_{n+1}$ ...

... Compute trial elastic state: (elastic prediction) ...

$\operatorname{dev}(\boldsymbol{\varepsilon}_{n+1}) = \boldsymbol{\varepsilon}_{n+1} - \frac{1}{3}\operatorname{tr}(\boldsymbol{\varepsilon}_{n+1})\,\mathbf{I}$

$\operatorname{dev}\left(\boldsymbol{\sigma}^{\text{trial}}_{n+1}\right) = 2\,\mu\operatorname{dev}\left(\boldsymbol{\varepsilon}^{\text{e,trial}}_{n+1}\right) = 2\,\mu\left[\operatorname{dev}(\boldsymbol{\varepsilon}_{n+1}) - \boldsymbol{\varepsilon}^{\mathrm{p}}_n\right]$

$\boldsymbol{\beta}^{\text{trial}}_{n+1} = \mathrm{H}\boldsymbol{\alpha}_n, \quad \beta^{\text{trial}}_{n+1} = \mathrm{h}\alpha_n$

$\boldsymbol{\xi}^{\text{trial}}_{n+1} = \operatorname{dev}\left(\boldsymbol{\sigma}^{\text{trial}}_{n+1}\right) - \boldsymbol{\beta}^{\text{trial}}_{n+1}$

$\mathbf{n}^{\text{trial}}_{n+1} = \frac{\boldsymbol{\xi}^{\text{trial}}_{n+1}}{\|\boldsymbol{\xi}^{\text{trial}}_{n+1}\|}$

$\phi^{\text{trial}}_{n+1} = \|\boldsymbol{\xi}^{\text{trial}}_{n+1}\| - \sqrt{\frac{2}{3}}\left[\sigma_{\mathrm{y}_0} + \beta^{\text{trial}}_{n+1}\right]$

... Check for elastic-plastic loading ...

if $\phi^{\text{trial}}_{n+1} < 0$ **then**

... elastic step ...

$\operatorname{dev}(\boldsymbol{\sigma}_{n+1}) = \operatorname{dev}\left(\boldsymbol{\sigma}^{\text{trial}}_{n+1}\right)$

$\mathbb{C}^{\text{dev}}_{n+1} = 2\,\mu\,\mathbb{P}_{\text{sym}}$

$\boldsymbol{\varepsilon}^{\mathrm{p}}_{n+1} = \boldsymbol{\varepsilon}^{\mathrm{p}}_n, \quad \boldsymbol{\alpha}_{n+1} = \boldsymbol{\alpha}_n, \quad \alpha_{n+1} = \alpha_n$

else

... Radial return (plastic correction) ...

$\gamma_{n+1} = \frac{\phi^{\text{trial}}_{n+1}}{2\,\mu + \mathrm{H} + \frac{2}{3}\mathrm{h}}$

$\operatorname{dev}(\boldsymbol{\sigma}_{n+1}) = \operatorname{dev}\left(\boldsymbol{\sigma}^{\text{trial}}_{n+1}\right) - 2\,\mu\,\gamma_{n+1}\mathbf{n}^{\text{trial}}_{n+1}$

$\boldsymbol{\varepsilon}^{\mathrm{p}}_{n+1} = \boldsymbol{\varepsilon}^{\mathrm{p}}_n + \gamma_{n+1}\mathbf{n}^{\text{trial}}_{n+1}, \quad \boldsymbol{\alpha}_{n+1} = \boldsymbol{\alpha}_n + \gamma_{n+1}\mathbf{n}^{\text{trial}}_{n+1}, \quad \alpha_{n+1} = \alpha_n + \sqrt{\frac{2}{3}}\gamma_{n+1}$

$\beta_1 = 1 - \frac{\phi^{\text{trial}}_{n+1}}{\|\boldsymbol{\xi}^{\text{trial}}_{n+1}\|}\frac{1}{1 + \frac{\mathrm{H}}{2\,\mu} + \frac{\mathrm{h}}{3\,\mu}} \qquad \beta_2 = \left[1 - \frac{\phi^{\text{trial}}_{n+1}}{\|\boldsymbol{\xi}^{\text{trial}}_{n+1}\|}\right]\frac{1}{1 + \frac{\mathrm{H}}{2\,\mu} + \frac{\mathrm{h}}{3\,\mu}}$

$\mathbb{C}^{\text{dev}}_{n+1} = 2\,\mu\,\beta_1\mathbb{P}_{\text{sym}} - 2\,\mu\,\beta_2\mathbf{n}^{\text{trial}}_{n+1}\otimes\mathbf{n}^{\text{trial}}_{n+1}$

end if

... Add spherical contribution to stress and moduli: ...

$\boldsymbol{\sigma}_{n+1} = \operatorname{dev}(\boldsymbol{\sigma}_{n+1}) + \kappa\operatorname{tr}(\boldsymbol{\varepsilon}_{n+1})\,\mathbf{I}$

$\mathbb{C}_{n+1} = \mathbb{C}^{\text{dev}}_{n+1} + \kappa\,\mathbf{I}\otimes\mathbf{I}$

(a) Implement the stress-update algorithm in the function `vmises.m`, test your implementation for different hardening parameters by running the isochoric tensile test given in `test_suite_isochoric_tension.m`. Compare your results with the analytic solution.

(b) Extent your function `vmises.m` to include the algorithmic tangent. Run the uniaxial tensile test, implemented in `drive.m`, check the convergence and compare to the analytic solution.