A **discrete probability distribution** is a probability distribution that can take on a countable number of values.

Well-known discrete probability distributions used in statistical modeling include the Poisson distribution, the Bernoulli distribution, the binomial distribution, the geometric distribution, and the negative binomial distribution. Additionally, the discrete uniform distribution is commonly used in computer programs that make equal-probability random selections between a number of choices. When a sample (a set of observations) is drawn from a larger population, the sample points have an empirical distribution that is discrete and that provides information about the population distribution.

Poisson distribution:

named after French mathematician Siméon Denis Poisson, is a discrete probability distribution that expresses the probability of a given number of events occurring in a fixed interval of time or space if these events occur with a known constant rate and independently of the time since the last event.

Probability of events for a Poisson distribution [edit]

An event can occur 0, 1, 2, ... times in an interval. The average number of events in an interval is designated λ (lambda). λ is the event rate, also called the rate parameter. The probability of observing k events in an interval is given by the equation

$$P(k \text{ events in interval}) = \frac{\lambda^k e^{-\lambda}}{k!}$$

where

- λ is the average number of events per interval
- e is the number 2.71828... (Euler's number) the base of the natural logarithms
- k takes values 0, 1, 2, ...
- $k! = k \times (k-1) \times (k-2) \times \ldots \times 2 \times 1$ is the factorial of k.

This equation is the probability mass function (PMF) for a Poisson distribution.

This equation can be adapted if, instead of the average number of events λ, we are given a time rate r for the events to happen. Then $\lambda = rt$ (with r in units of 1/time), and

$$P(k \text{ events in interval } t) = \frac{(rt)^k e^{-rt}}{k!}$$

Ugarte and colleagues report that the average number of goals in a World Cup soccer match is approximately 2.5 and the Poisson model is appropriate. Because the average event rate is 2.5 goals per match, $\lambda = 2.5$.

$$P(k \text{ goals in a match}) = \frac{2.5^k e^{-2.5}}{k!}$$

$$P(k = 0 \text{ goals in a match}) = \frac{2.5^0 e^{-2.5}}{0!} = \frac{e^{-2.5}}{1} \approx 0.082$$

$$P(k = 1 \text{ goal in a match}) = \frac{2.5^1 e^{-2.5}}{1!} = \frac{2.5e^{-2.5}}{1} \approx 0.205$$

$$P(k = 2 \text{ goals in a match}) = \frac{2.5^2 e^{-2.5}}{2!} = \frac{6.25e^{-2.5}}{2} \approx 0.257$$

Examples that Poisson distribution does not work:

The number of students who arrive at the student union per minute will likely not follow a Poisson distribution, because the rate is not constant (low rate during class time, high rate between class times) and the arrivals of individual students are not independent (students tend to come in groups).

The number of magnitude 5 earthquakes per year in a country may not follow a Poisson distribution if one large earthquake increases the probability of aftershocks of similar magnitude.

Bernoulli distribution

In probability theory and statistics, the **Bernoulli distribution**, named after Swiss mathematician Jacob Bernoulli,[1] is the discrete probability distribution of a random variable which takes the value 1 with probability p and the value 0 with probability $q = 1 - p$, that is, the probability distribution of any single experiment that asks a yes–no question; the question results in a boolean-valued outcome, a single bit whose value is success/yes/true/one with probability p and failure/no/false/zero with probability q. It can be used to represent a (possibly biased) coin toss where 1 and 0 would represent "heads" and "tails" (or vice versa), respectively, and p would be the probability of the coin landing on heads or tails, respectively. In particular, unfair coins would have $p \neq 1/2$.

The probability mass function f of this distribution, over possible outcomes k, is

$$f(k;p) = \begin{cases} p & \text{if } k = 1, \\ q = 1 - p & \text{if } k = 0. \end{cases}$$ [2]

Binomial distribution

In probability theory and statistics, the **binomial distribution** with parameters n and p is the discrete probability distribution of the number of successes in a sequence of n independent experiments, each asking a yes–no question, and each with its own boolean-valued outcome: success/yes/true/one (with probability p) or failure/no/false/zero (with probability $q = 1 - p$). A single success/failure experiment is also called a Bernoulli trial or Bernoulli experiment and a sequence of outcomes is called a Bernoulli process; for a single trial, i.e., $n = 1$, the binomial distribution is a Bernoulli distribution. The binomial distribution is the basis for the popular binomial test of statistical significance.

The binomial distribution is frequently used to model the number of successes in a sample of size n drawn with replacement from a population of size N. If the sampling is carried out without replacement, the draws are not independent and so the resulting distribution is a hypergeometric distribution, not a binomial one. However, for N much larger than n, the binomial distribution remains a good approximation, and is widely used.

Probability mass function [edit]

In general, if the random variable X follows the binomial distribution with parameters $n \in \mathbb{N}$ and $p \in [0,1]$, we write $X \sim B(n, p)$. The probability of getting exactly k successes in n *independent Bernoulli* trials is given by the probability mass function:

$$f(k,n,p) = \Pr(k;n,p) = \Pr(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

for $k = 0, 1, 2, ..., n$, where

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

is the binomial coefficient, hence the name of the distribution. The formula can be understood as follows. k successes occur with probability p^k and $n - k$ failures occur with probability $(1 - p)^{n-k}$. However, the k successes can occur anywhere among the n trials, and there are $\binom{n}{k}$ different ways of distributing k successes in a sequence of n trials.

Continuous probability distribution [edit]

See also: Probability density function

A **continuous probability distribution** is a probability distribution with a cumulative distribution function that is absolutely continuous. Equivalently, it is a probability distribution on the real numbers that is absolutely continuous with respect to Lebesgue measure. Such distributions can be represented by their probability density functions. If the distribution of X is continuous, then X is called a **continuous random variable**. There are many examples of continuous probability distributions: normal, uniform, chi-squared, and others.

Formally, if X is a continuous random variable, then it has a probability density function $f(x)$, and therefore its probability of falling into a given interval, say $[a, b]$, is given by the integral

$$\mathrm{P}[a \leq X \leq b] = \int_a^b f(x)\,dx$$

In particular, the probability for X to take any single value a (that is $a \leq X \leq a$) is zero, because an integral with coinciding upper and lower limits is always equal to zero.

Exponential distribution

In probability theory and statistics, the **exponential distribution** is the probability distribution of the time between events in a Poisson point process, i.e., a process in which events occur continuously and independently at a constant average rate. It is a particular case of the gamma distribution. It is the continuous analogue of the geometric distribution, and it has the key property of being memoryless. In addition to being used for the analysis of Poisson point processes it is found in various other contexts.

The exponential distribution is not the same as the class of exponential families of distributions, which is a large class of probability distributions that includes the exponential distribution as one of its members, but also includes the normal distribution, binomial distribution, gamma distribution, Poisson, and many others.

Probability density function [edit]

The probability density function (pdf) of an exponential distribution is

$$f(x;\lambda) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0, \\ 0 & x < 0. \end{cases}$$

Here $\lambda > 0$ is the parameter of the distribution, often called the *rate parameter*. The distribution is supported on the interval $[0, \infty)$. If a random variable X has this distribution, we write $X \sim \mathrm{Exp}(\lambda)$.

The exponential distribution exhibits infinite divisibility.

Cumulative distribution function [edit]

The cumulative distribution function is given by

$$F(x;\lambda) = \begin{cases} 1 - e^{-\lambda x} & x \geq 0, \\ 0 & x < 0. \end{cases}$$

Logistic distribution

In probability theory and statistics, the **logistic distribution** is a continuous probability distribution. Its cumulative distribution function is the logistic function, which appears in logistic regression and feedforward neural networks. It resembles the normal distribution in shape but has heavier tails (higher kurtosis). The logistic distribution is a special case of the Tukey lambda distribution.

Probability density function [edit]

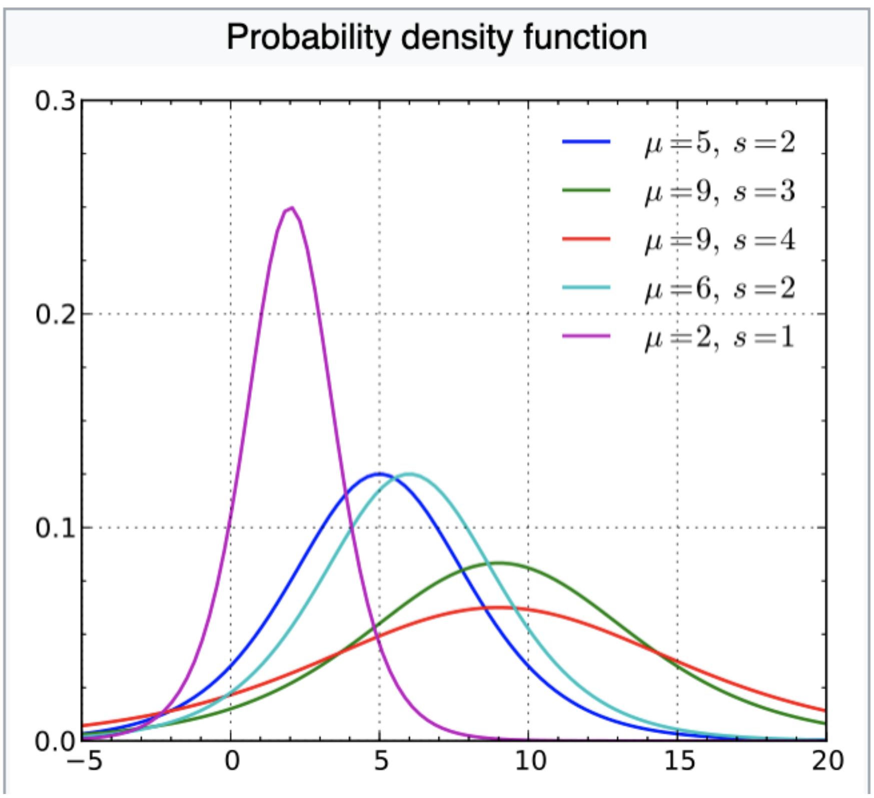
When the location parameter μ is 0 and the scale parameter s is 1, then the probability density function of the logistic distribution is given by

$$f(x;0,1) = \frac{e^{-x}}{(1+e^{-x})^2} = \frac{1}{(e^{x/2}+e^{-x/2})^2} = \frac{1}{4}\operatorname{sech}^2\left(\frac{x}{2}\right).$$

Thus in general the density is:

$$f(x;\mu,s) = \frac{e^{-(x-\mu)/s}}{s\left(1+e^{-(x-\mu)/s}\right)^2} = \frac{1}{s\left(e^{(x-\mu)/(2s)}+e^{-(x-\mu)/(2s)}\right)^2} = \frac{1}{4s}\operatorname{sech}^2\left(\frac{x-\mu}{2s}\right).$$

Logistic



Weibull distribution

In probability theory and statistics, the **Weibull distribution** /ˈveɪbʊl/ is a continuous probability distribution. It is named after Swedish mathematician Waloddi Weibull, who described it in detail in 1951, although it was first identified by Fréchet (1927) and first applied by Rosin & Rammler (1933) to describe a particle size distribution.

Standard parameterization [edit]

The probability density function of a Weibull random variable is:[1]

$$f(x;\lambda,k)=\begin{cases}\frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1}e^{-(x/\lambda)^k} & x\geq 0,\\ 0 & x<0,\end{cases}$$

where $k > 0$ is the *shape parameter* and $\lambda > 0$ is the *scale parameter* of the distribution. Its complementary cumulative distribution function is a stretched exponential function. The Weibull distribution is related to a number of other probability distributions; in particular, it interpolates between the exponential distribution ($k = 1$) and the Rayleigh distribution ($k = 2$ and $\lambda = \sqrt{2}\sigma$[2]).

If the quantity X is a "time-to-failure", the Weibull distribution gives a distribution for which the failure rate is proportional to a power of time. The *shape* parameter, k, is that power plus one, and so this parameter can be interpreted directly as follows:[3]

- A value of $k<1$ indicates that the failure rate decreases over time (Lindy effect). This happens if there is significant "infant mortality", or defective items failing early and the failure rate decreasing over time as the defective items are weeded out of the population. In the context of the diffusion of innovations, this means negative word of mouth: the hazard function is a monotonically decreasing function of the proportion of adopters;
- A value of $k=1$ indicates that the failure rate is constant over time. This might suggest random external events are causing mortality, or failure. The Weibull distribution reduces to an exponential distribution;
- A value of $k>1$ indicates that the failure rate increases with time. This happens if there is an "aging" process, or parts that are more likely to fail as time goes on. In the context of the diffusion of innovations, this means positive word of mouth: the hazard function is a monotonically increasing function of the proportion of adopters. The function is first convex, then concave with an inflexion point at $(e^{1/k}-1)/e^{1/k}, k>1$.

In the field of materials science, the shape parameter k of a distribution of strengths is known as the Weibull modulus. In the context of diffusion of innovations, the Weibull distribution is a "pure" imitation/rejection model.

Moments:

$$\mu=\int_{-\infty}^{+\infty} z\,p(z)\,dz=E(z)$$

$$\bar{z}=\frac{1}{n}\sum_{i=1}^{n} z_i$$

$$\sigma^2=\int_{-\infty}^{+\infty}(z-\mu)^2\,p(z)\,dz=E\left[(z-\mu)^2\right]$$

$$\sigma^2=E(z^2-2z\mu+\mu^2)=E(z^2)-2\mu E(z)+\mu^2=E(z^2)-\mu^2$$

$$E(x+y) = E(x) + E(y)$$
$$E(c\,x) = c\,E(x)$$

$$\mathrm{Var}(z) = \frac{n\,(\,\overline{z^2} - \bar{z}^{\,2}\,)}{n-1}$$

$$\begin{aligned}
\mu_3 &= \int_{-\infty}^{+\infty} (z-\mu)^3\, p(z)\, dz = E\left[(z-\mu)^3\right] \\
&= E(z^3) - 3\mu E(z^2) + 3\mu[E(z)]^2 - [E(z)]^3 \\
&= E(z^3) - 3\mu E(z^2) + 2\mu^3
\end{aligned}$$

$$\mathrm{Skw}(z) = \frac{n^2\,(\,\overline{z^3} - 3\,\overline{z^2}\,\bar{z} + 2\,\bar{z}^3\,)}{(n-1)(n-2)}$$

Programming excise Nov 22, 2019.
The data file can be downloaded from:
https://drive.google.com/file/d/1TaRkEqV96oC4iSZPqDVCTfI6k4fcEDZr/view?usp=sharing

Given stress strain curves that from a 2D discrete dislocation dynamics simulation. Explore the statistical information from the curve.

1, Plot the stress strain curves.
   1.1 what the Young's modulus is
   1.2 where the initial yielding happens?

2, Define a strain range where we want to explore the statistical information.
   2.1 define the flow stress, plot flow stress as function of the sample size
   2.2 check the fluctuations: standard deviation.
   2.3 histogram of the fluctuations.
   2.4 probability density distribution of the fluctuations.
   2.2, 2.3, 2.4, do it in our own way and compare with the results using python library.