

## //Tabulation

int[][] dp[n+1][m+1] = {{0}};

for(int i=0;i<n+1) dp[i][0] = i

for(int j=0;j<m+1) dp[0][j] = j

for(int i=1; i<=N; i++) {

for(int j=1; j<=M; j++) {

if (s1[i-1] == s2[j-1])

dp[i][j] = dp[i-1][j-1]

else {

ins = dp[i][j-1]

rep = dp[i-1][j-1]

del = dp[i-1][j]

dp[i][j] = min (ins, rep, del)

}

(100)

K = 13

(0,0) (N-1, M-1)

SRC  $\rightarrow$  dest

21 29 29 29

21 5 5 5

13 21 21 21

6 8 21 21

find unique paths

from src to dest

where  $x \text{ XOR } k$

$(i, j) \rightarrow (i+1, j)$

$(i, j) \rightarrow (i, j+1)$

$f(i, j, \text{mat}, \text{xor}, n, m, K) \{$

if ( $i \geq n \text{ or } j \geq m$ ) ret 0;

if ( $i == n-1 \text{ and } j == m-1$ ) ret ( $\text{xor} \wedge \text{mat}(i, j) == K$ );

$x = f(i+1, j, \text{mat}, \text{xor} \wedge \text{mat}(i, j), n, m, K);$

$y = f(i, j+1, \text{mat}, \text{xor} \wedge \text{mat}(i, j), n, m, K);$

ret  $x+y;$

}

/ memoization, (i,j) aim -> (i,j) qb

$f(i, j, \text{mat}, \text{xor}, n, m, K, \text{int}[\text{int}][\text{int}][\text{int}][\text{int}] \text{ dp}) \{$

if ( $i \geq m \text{ or } j \geq m$ ) ret 0;

if ( $i == n-1 \text{ and } j == m-1$ ) ret ( $\text{xor} \wedge \text{mat}(i, j) == K$ );

if ( $\text{dp}[i][j][\text{xor}] != -1$ ) ret  $\text{dp}[i][j][\text{xor}];$

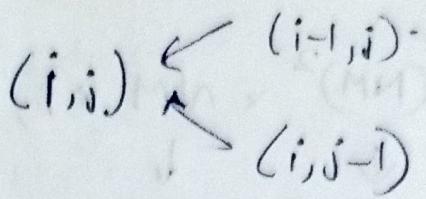
$x = f(i+1, j, \text{mat}, \text{xor} \wedge \text{mat}(i, j), n, m, K, \text{dp});$

$y = f(i, j+1, \text{mat}, \text{xor} \wedge \text{mat}(i, j), n, m, K, \text{dp});$

ret  $\text{dp}[i][j][\text{xor}] = (x+y);$

(10)

## // Tabulation.



int  $dp[n+1][M+1][120] = \{0\}$

$dp[1][1][mat[0][0]] = \{1\}$

for (int  $i=1; i \leq N; i++$ ) {

    for (int  $j=1; j \leq M; j++$ ) {

        for (int curr=0; curr <= 80; curr++) {

$$x = dp[i-1][j][curr + mat[i-1][j-1]]$$

$$y = dp[i][j-1][curr + mat[i][j-1]]$$

$$dp[i][j][curr] = x + y$$

}  
3  
3

mat

10 1 0 1 1

1 1 1 1 2 6

1 1 1 5 3 1

0 2 3 4 3 2

0 1 1 0 1 0

1 1 1 0 0 0

$k = 20$  (student)

1 (instructor)

Matrix is the capacities of  $N \times M$  labs. There are  $k$  students and 1 instructor. We have to place them such

that distance between farthest student and instructor should be minimum.

$$1. \left( (NM)^2 \times NM \times 1, 1 \right)$$

$\downarrow$   
to generate  
sub matrices

$\downarrow$   
to check  
sum capability

## 2. BS + DP

$N \times M$  +  $\log(\text{high-low+1}) \times N \times M$ ,  $N \times M$   
 (sum computation) (BS on answer) (validity of answer)  $\downarrow$   
 in dp table  $\downarrow$  space  
 $\text{low} = 0$

$$\text{high} = \max(N, M)$$

while ( $\text{low} \leq \text{high}$ )

$$\text{mid} = (l+b)/2$$

if (valid(mat, n, m, mid, dp)) {

$$\text{ans} = \text{mid};$$

$$h = \text{mid}-1;$$

} else {

$$l = \text{mid}+1; }$$

}

valid (mat, n, m, mid, dp) {

for (int i=0; i<n; i++) {

for (int j=0; j<m; j++) {

x = compute (i, j, mid, dp);

if ( $x \geq k+1$ )

} } ret True;

ret False;

}



bottom-left bottom-right

middle-left middle-right

top-left top-right

right-left right-right

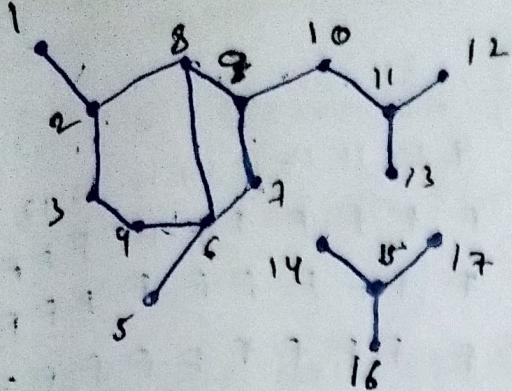
left-left left-right

middle-left middle-right

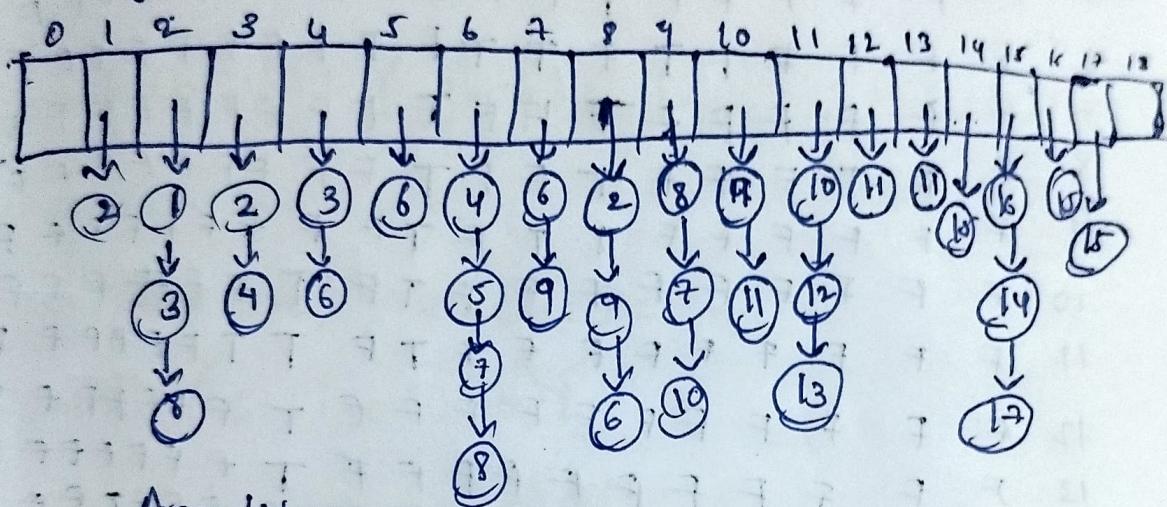
Check if there exists

a path from src to dest.

<u>src</u>	<u>dest</u>
2	4
13	16



## 1. Adjacency List



ArrayList<ArrayList<Integer>> graph;

Undirected weighted

graph.get(u).add((v,w));

graph.get(v).add((u,w)) // Avoid if directed.

Undirected unweighted

graph.get(u).add(v);

graph.get(v).add(u) // Avoid if directed

## Adjacency matrix

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
1	F	F	T	F	F	F	F	F	F	F	F	F	F	F	F
2	F	T	F	T	F	F	F	T	F	F	F	F	F	F	F
3	F	T	F	F	T	F	F	F	F	F	F	F	F	F	F
4	F	F	F	T	F	F	T	F	F	F	F	F	F	F	F
5	F	F	F	F	F	T	F	F	F	F	F	F	F	F	F
6	F	F	F	F	T	T	F	T	F	F	F	F	F	F	F
7	F	F	F	F	F	F	T	F	E	F	F	F	F	F	F
8	F	F	T	F	F	F	T	F	F	F	F	F	F	F	F
9	F	F	F	F	F	T	T	E	T	F	F	F	F	F	F
10	F	F	F	F	F	F	F	E	T	P	T	F	F	F	F
11	F	F	F	F	F	F	F	T	F	T	T	F	F	F	F
12	F	F	F	F	F	F	F	F	T	F	P	F	F	F	F
13	F	F	F	F	F	F	F	F	F	T	F	F	F	F	F
14	F	F	F	F	F	F	F	F	F	F	F	E	F	F	F
15	F	F	F	F	F	F	F	F	F	F	F	F	T	T	F
16	F	F	F	F	F	F	F	F	F	F	F	F	F	T	F
17	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
18	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F

list

- Space efficient (Adv)
- Checking an edge existence is an expensive operation (Dis-Adv).
- Checking an edge is  $O(1)$  (Adv)
- Space is too much  $(N^2) \times (N^2)$  (Dis-Adv)

## Graph Traversal

BFS :- Breadth first search

DFS - Depth first search

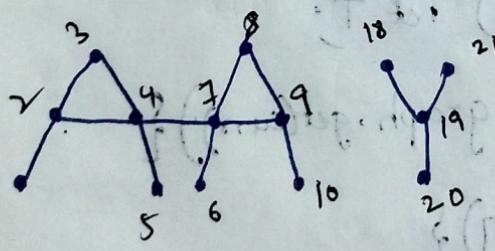
BFS : We will go to all neighbours of a node. (We can use a queue for it)

```
boolean bfs (graph, int src, int dest){  
    queue<int> q; // boolean vis[]  
    q.add(src); vis[src] = true;  
    while (q.size() > 0) {  
        int curr = q.poll();  
        if (curr == dest) ret T;  
        for (int n: graph.get(curr)) {  
            if (!vis[n]) {  
                q.add(n);  
                vis[n] = true;  
            }  
        }  
    }  
    return false;  
}
```

DFS: for a node we will explore all possible nodes of each neighbour.

```
boolean dfs(graph, int src, int dest, boolean[] vis){  
    if(src == dest) return true;  
    vis[src] = true;  
    for(int ele : graph[src]) {  
        if(!vis[ele] && dfs(graph, ele, dest, vis))  
            return true;  
    }  
    return false;  
}
```

④ Given a graph, find # connected components.

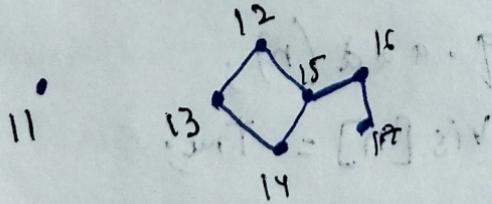


Undirected

Unweighted

Simple

weakly  
connected



boolean vis[N+1] = {F}

int cnt = 0;

$\forall_{i=1}^N$  if (!vis[i])  $\begin{cases} \text{cnt}++ \\ \text{bfs or dfs,} \\ (\text{graph}, i, \text{vis}) \end{cases}$

void bfs(graph, int src, vis){

vis[src] = T;

queue > q;

q.add(src);

while (q.size() > 0) {

int curr = q.poll();

for (int n : graph.get(curr)) {

if (!vis[n]) {

vis[n] = T;

q.add(n)

}

}

void dfs(graph, int src, vis){

vis[src] = T;

for (int n : graph[src]) {

if (!vis[n])

dfs(graph, n, vis);

3 3

(110)

Check which graph is also a Tree.



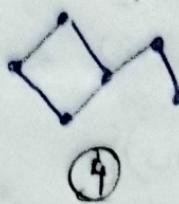
①



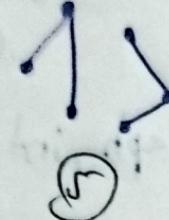
②

$N \Rightarrow$  No. of nodes

$M \Rightarrow$  No. of edges



③



④

Properties of a graph which is Tree

① Acyclic ( $M = N - 1$ )

②  $\underbrace{\text{Connected components}}_{(cc)} = 1$

if  $M = N - cc$  (it's a tree)

Find the len of shortest path from src to dest.

```
void short(graph, src, dist[])
```

```
{ queue<int> q;
```

```
q.add(src);
```

```
dist[src] = 0;
```

```
while (q.size() > 0) {
```

```
    int curr = q.poll();
```

```
    for (int n : graph.get(curr)) {
```

```
        if (dist[n] == -1) {
```

```
            q.add(n);
```

```
            dist[n] = dist[curr] + 1;
```

```
}
```

```
3
```

① Path in a graph

② No. of connected components

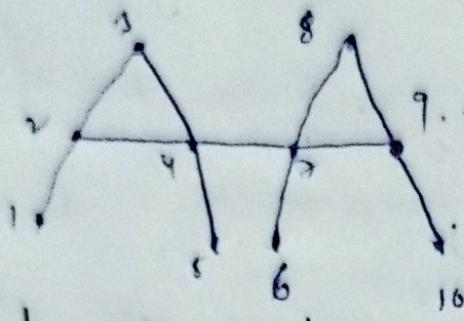
③ Check forest

④ Shortest path [Unweighted]

⑤ 1D 2D 3D DP SI primary.

(112)

10/4/25



Largest path length?

Solutions

1)  $N^2, N$

for every node find distance to all other nodes

2)  $N+N, N$

From any random node, if we do bfs the farthest node is a src or dest node. From src or dest node we will find longest path.

MAIN

int src = bfs(graph, n, dis, true);  
int ans = bfs(graph, n, dis, false);

## MANN

```
int src = bfs(graph, n, 1, true);
int ans = bfs(graph, n, src, false);

int bfs(... graph, int n, int src, bool flag) {
    int dist[n+1] = {-1};
    dist[src] = 0;
    int curr = 0;
    Queue<int> q;
    q.add(src);
    while (q.size() > 0) {
        curr = q.poll();
        for (int n: graph.get(curr)) {
            if (dist[n] == -1) {
                q.add(n);
                dist[n] = dist[curr] + 1;
            }
        }
    }
    if (flag) return curr;
    return dist[curr];
```

1 0 0 1 1  
 . . . .  
 0 → water

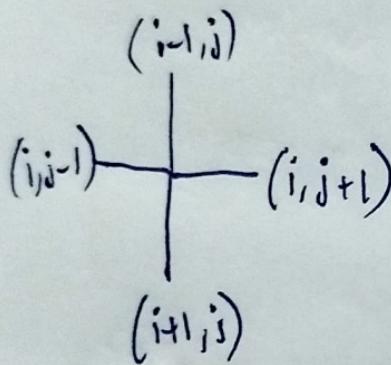
1 1 1 1 1  
 . . . .  
 1 → land

0 1 0 0 0  
 find no. of  
 # islands

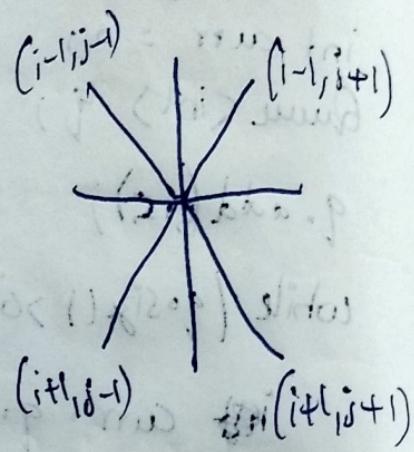
0 0 1 1 0

1 1 0 0 1

N-4 connectivity



N-8 connectivity



```

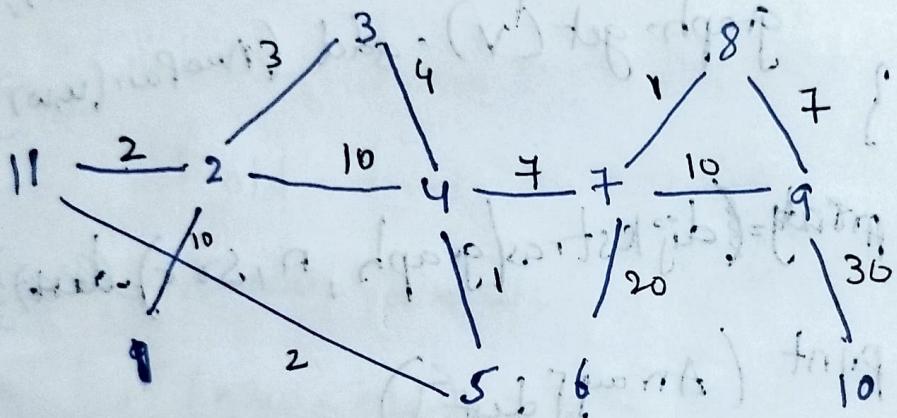
void dfs(mat, i, j, n, m) {
  if(i < 0 || i > n || j < 0 || j > m || mat(i, j) == 0) return;
  mat(i, j) = 0;
  dfs(mat, i + 1, j, n, m);
  dfs(mat, i - 1, j, n, m);
  dfs(mat, i, j - 1, n, m);
  dfs(mat, i, j + 1, n, m);
}
  
```

```

void des (mat, i, j, n, m) {
    int dx = {1, -1, 0, 0}; {-1, 1, 1, -1};
    int dy = {0, 0, 1, -1}; {1, -1, -1, 1};

    if (i < collision || j > m || j < 0 || mat(i, j) == 0)
        mat(i, j) = 0;
    for (int k = 0; k < 4 or 8; k++)
        ddfs (mat, i + dx[k], j + dy[k], n, m);
}

```



Find the min weighted path from 2 to 9. (Dijkstras)

1. Greedy BFS.
2. Source to every other node. (min weighted path)

Read N, M;

ArrayList<ArrayList<Pair>> graph = new ArrayList<ArrayList<Pair>>();  
for (int i = 0; i < N; i++) graph.add(new ArrayList<Pair>());

loop M {

    Read u, v, w;

    graph.get(u).add(new Pair(v, w));

    graph.get(v).add(new Pair(u, w));

}

List<Integer> priority = dijkstra(graph, n, src);

Print (Array[dist])

List<Integer> dijkstra (graph, int n, int src) {

PriorityQueue<Pair> pq = new PQ<Pair> (p1, p2)

    → p1.first - p2.first)

ArrayList<Integer> dist = new List<Integer>(n + 1);

dist.fill (Integer.MAX\_VALUE);

pq.add (new Pair(0, src));

```
while (pq.size() > 0) {
```

```
    Pair cur = pq.poll();
```

```
    for (Pair n : graph.get(cur)) {
```

```
        int d = cur.first;
```

```
        int u = cur.second;
```

```
        for (Pair n : graph[u]) {
```

```
            int v = n.first;
```

```
            int w = n.second;
```

```
            int old = dist[v];
```

```
            int new = d + w;
```

```
            if (new < old) {
```

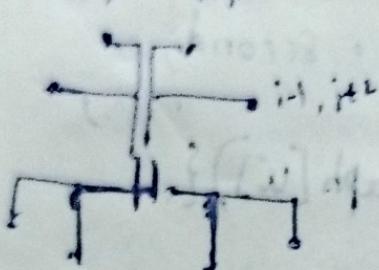
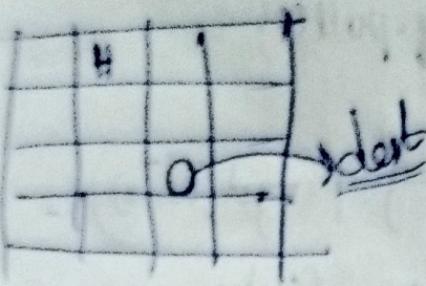
```
                pq.add(new, v);
```

```
                dist[v] = new;
```

```
}
```

```
return dist;
```

# ④ Knight on a chess Board



```
int solve (int a, int b, int c, int d, int e, int f) {
```

```
    int dx = {
```

```
    int dy = {
```

```
    int dist = new [a+1][b+1]
```

```
    dist = {-1};
```

```
    dist[c][d] = 0;
```

```
    q.add((c, d));
```

```
    while (q.size() > 0) {
```

```
        Pair curr = q.poll();
```

```
        int x = curr.first;
```

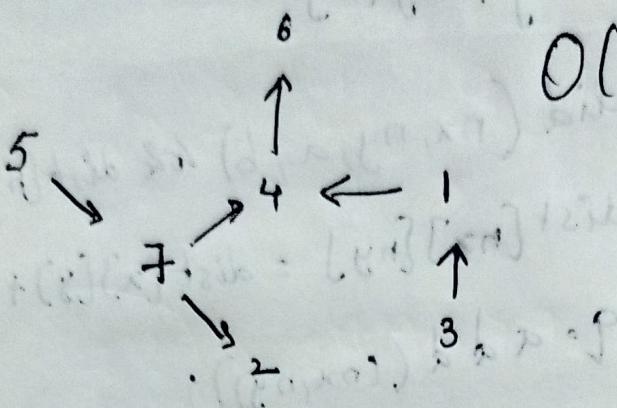
```
        int y = curr.second;
```

24/04/25

# Topological Sort

Kahn's Algorithm

$O(n \log n)$



Paths to solve the tasks

5 + 2 3 1 4 6

[ 3 1 5 7 2 4 6,

↳ (lexicographically shortest path)

We can directly complt a task if it is independent. If it dependent, first we hv to solve dep those tasks to complt the dependent task

(eg: 4 can be solved only if 1,7 are solved)

so the path we choose shld be like take the smallest number from possible moves we have.

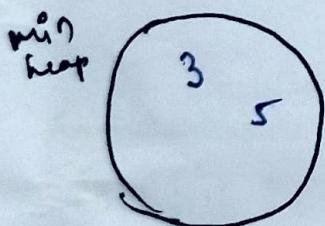
3 is the least independent task.

Then possible tasks to be solved are 1, 5, so we chose 1.

Then only possible task to be completed is 5, then 7.

After it we have 4, 2 as possible tasks so we chose 2 then 4 and 6.

$$\frac{1}{1} \frac{1}{2} \frac{0}{3} \frac{2}{4} \frac{0}{5} \frac{1}{6} \frac{1}{7} \quad (\text{indeg arr})$$



add elements with indeg 0

get Min  $\rightarrow 3$  (neigh : 1)

$$\frac{0}{1} \cdot \frac{1}{2} \cdot \frac{0}{3} \cdot \frac{2}{4}, \frac{0}{5} \cdot \frac{1}{6} \cdot \frac{1}{7}$$



$$\frac{0}{1} \cdot \frac{1}{2} \cdot \frac{0}{3} \cdot \frac{1}{4} \cdot \frac{0}{5} \cdot \frac{1}{6} \cdot \frac{1}{7}$$

(122)

None of the ~~remaining~~ nodes  
indeg is 0

5

getMin  $\rightarrow$  5

$$\begin{array}{c} 0 \quad 1 \quad 0 \quad 0 \quad 0 \\ \underline{-} \quad \underline{1} \quad \underline{2} \quad \underline{3} \quad \underline{4} \\ 1 \quad 2 \quad 3 \quad 4 \quad 5+6, 7, 8, 9 \end{array}$$

7

getMin  $\rightarrow$  7

$$\begin{array}{c} 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \\ \underline{-} \quad \underline{2} \quad \underline{3} \quad \underline{4} \quad \underline{5} \quad \underline{6} \quad \underline{7} \\ 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \end{array}$$

2

getMin  $\rightarrow$  2

8

$$\begin{array}{c} 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \\ \underline{-} \quad \underline{2} \quad \underline{3} \quad \underline{4} \quad \underline{5} \quad \underline{6} \quad \underline{7} \\ 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \end{array}$$

1

getMin  $\rightarrow$  1

$$\begin{array}{c} 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \\ \underline{-} \quad \underline{2} \quad \underline{3} \quad \underline{4} \quad \underline{5} \quad \underline{6} \quad \underline{7} \\ 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \end{array}$$

6

getMin  $\rightarrow$  6.

```

main() {
    int ind[N+1] = {0};
    loop(M) {
        Read u, v;
        graph.get(u).add(v);
        ind[v]++;
    }
    ArrayList<Integer> ans = Kahn's(graph, N, ind);
}

```

ArrayList<Integer> Kahn's(graph, N, ind) {

PriorityQueue<Integer> pq; { } 26

```

    = new PriorityQueue<>((p1, p2) >
        ArrayList<Integer> res = new ArrayList<>();
        for (int i : ind) {
            if (i == 0) pq.add(i); } N
        while (pq.size() > 0) {
            int x = pq.getMin(); } N log N
            res.add(x);
            ind[x] = -1;
            for (int i : graph.get(x)) {
                ind[i] -= 1;
                if (ind[i] == 0) pq.add(i); } N log N
        }
        return res;
    }
}

```

```

for (int k=0; k<8; k++)
    int nx = x + dx[k]
    int ny = y + dy[k]
    if (valid(nx, ny, a, b) && dist[nx][ny] == -1)
        dist[nx][ny] = dist[x][y] + 1;
        q.add((nx, ny));
    }
    set dist[e][f];
}

```

## Lab Agenda

1. Longest path in a graph
2. No. of Islands
3. Shortest path (weighted)
4. Knight in a chessboard [unweighted]
5. " " [weighted].

03/4/2025

\* N Floors

K eggs

There exists a floor  $f$  till where  
the egg doesn't break

(if  $f=4$  from 0 to 4 egg doesn't break)

Find min egg drops required to  
find  $f$ .

(DP + BS)

## GRAPHS

1. Undirected / Directed

2. Unweighted / Weighted

3. Simple / Complex

4. Cyclic / Acyclic

5. Strongly / Weakly

From any  
node we  
can reach  
all other nodes

