MECHANICS OF MATERIAL

The study of mechanics of materials reveals that the behavior of structures under load is not just a matter of material properties but also of the interplay between forces, geometry, and constraints

LET'S GET STARTED

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CONTENT

MECHANICS OF MATERIALS

Rigid Body – Centre of mass – Rotational Energy - Moment of inertia (M.I)- Moment of Inertia for uniform objects with various geometrical shapes. Elasticity –Hooke's law - Poisson's ratio - stress-strain diagram for ductile and brittle materials – uses- Bending of beams – Cantilever - Simply supported beams - uniform and non-uniform bending - Young's modulus determination - I shaped girders –Twisting couple – Shafts. Viscosity – Viscous drag – Surface Tension.

TEXT BOOKS:

1. Raymond A. Serway, John W. Jewett, Physics for Scientists and Engineers, Thomson Brooks/Cole, 2013.
2. D. Halliday, R. Resnick and J. Walker, Principles of Physics. John Wiley & Sons, 10^{th} Edition, 2015.
3. N. Garcia, A. Damask and S. Schwarz, Physics for Computer Science Students, Springer-Verlag, 2012.
4. Alan Giambattista, Betty McCarthy Richardson and Robert C. Richardson, College Physics, McGraw-Hill Higher Education, 2012.

RIGID BODY

DEFINITION:

A rigid body in physics is an idealized object that doesn't deform or change shape under the influence of forces. In other words, the distances between any two points within a rigid body remain constant, regardless of the external forces applied.

CONDITION:

- No Deformation: The object must not undergo any deformation, which means the distances between all pairs of points within the body remain fixed.
- Uniform Material: The material of the body is typically assumed to be uniform, although this is a simplifying assumption rather than a strict requirement.
- External Forces: External forces and moments applied to the body can cause it to translate or rotate, but they do not alter its shape

RIGID BODY

EXAMPLES

- Solid Bar or Beam: In engineering, a steel bar used in construction can be approximated as a rigid body for many calculations, assuming it doesn't bend or stretch significantly.

- Planetary Bodies: Planets and moons are often treated as rigid bodies in astrophysics, especially when analyzing their rotational dynamics, even though they can experience slight deformations.

- Simple Mechanical Systems: Components like gears, wheels, or levers in mechanical systems are often modeled as rigid bodies to simplify the analysis of their motion and interactions.

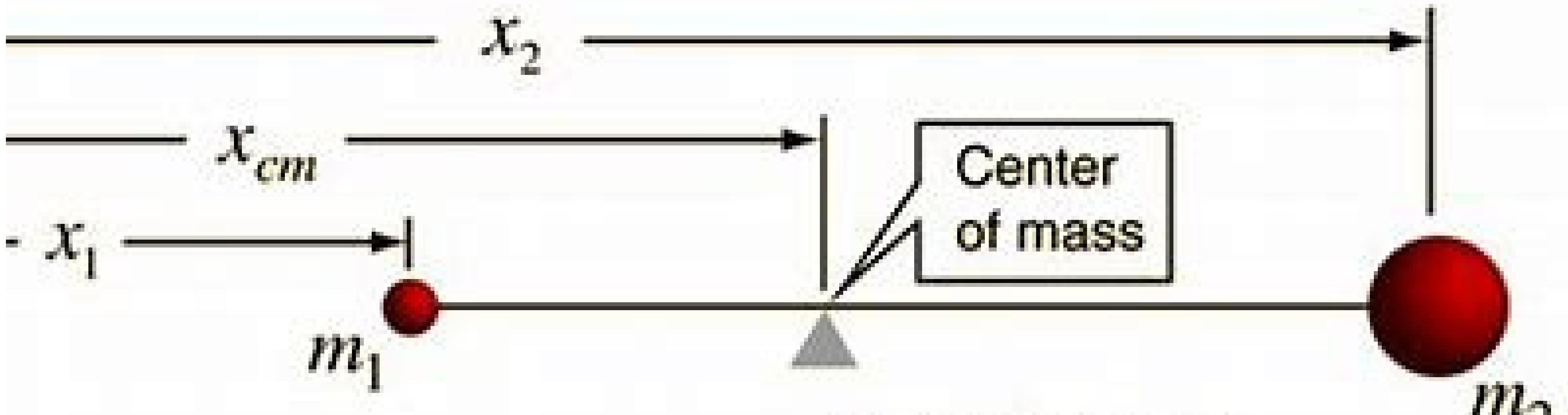
NOTE : Discuss some real examples of rigid bodies.....

CENTRE OF MASS

DEFINITION:

The center of mass of a system of particles or a continuous mass distribution is the point where the mass-weighted position averages out.

In other words, The center of mass is a point representing the average position of all the mass in a system or object. It's the point where you can balance the object perfectly if it were supported at that location.



For two masses: $x_{cm} = \dfrac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$

CENTRE OF MASS
PROPERTIES

- Balance Point: The center of mass is the point where you can balance an object perfectly. For symmetrical and evenly dense objects, this point is usually in the middle.

- Motion Analysis: The center of mass helps us understand how objects move. It's like the average location of all the mass, and it moves predictably based on the forces acting on the object.

- Stability: How stable an object is depends on where its center of mass is. If the center of mass is low and centered over its base, the object is more stable and less likely to tip over.

ROTATIONAL ENERGY

DEFINITION:

Rotational energy [rotational kinetic energy], is the kinetic energy associated with the rotation of an object around an axis.

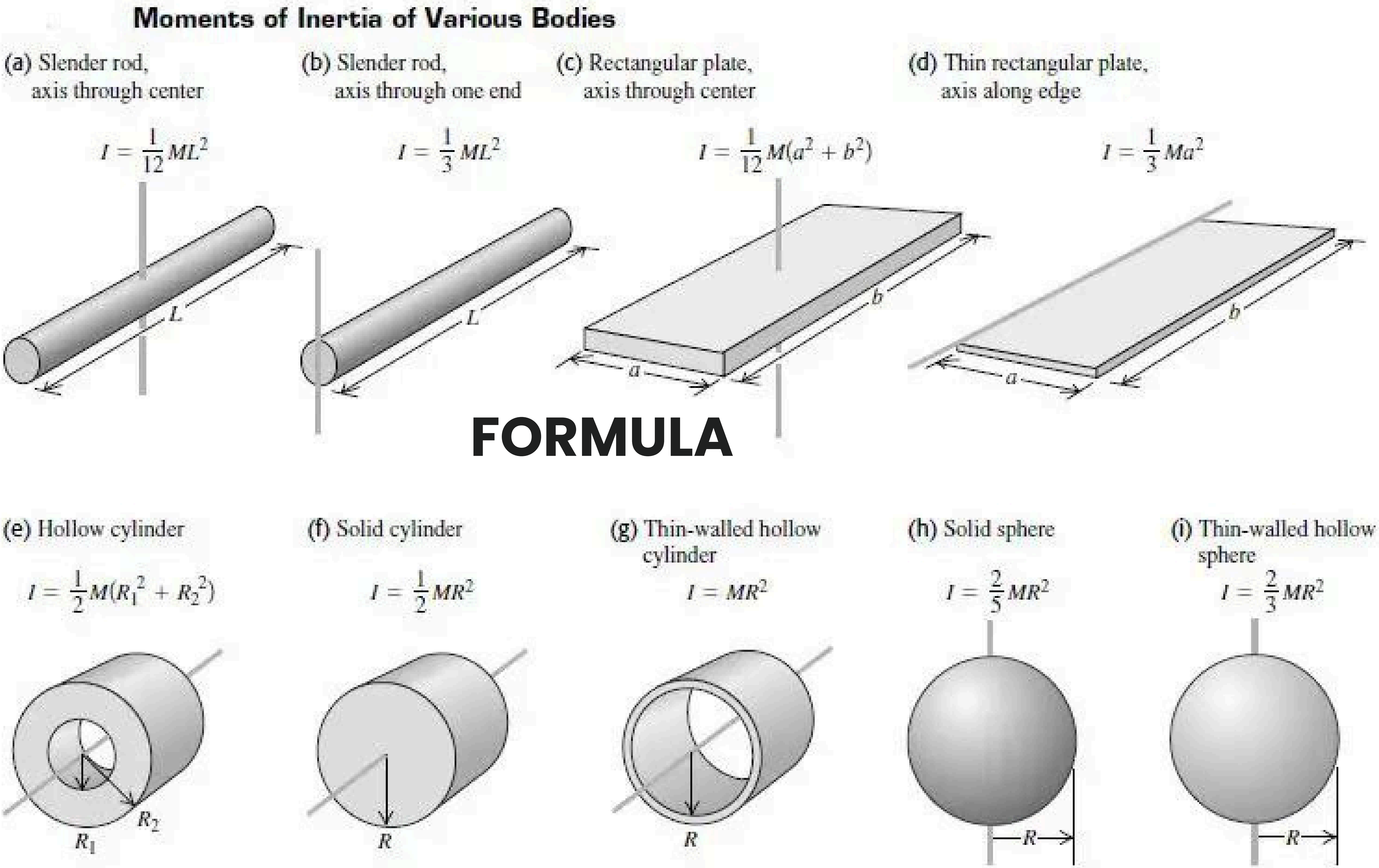
It is a form of energy that depends on the object's rotational motion, similar to how linear kinetic energy depends on translational motion.

CONDITION:

$$K = \frac{1}{2} I \omega^2$$

- Rigid Body Rotation: The object should be treated as a rigid body, meaning it doesn't deform under rotational motion. The distribution of mass remains constant.
- Constant Axis of Rotation: The axis of rotation should be fixed and constant. If the axis changes, the moment of inertia can vary.
- Non-Slipping Condition: For rolling objects, like wheels, the point of contact with the surface should not slip, ensuring that the rotational and translational motions are consistent.

ROTATIONAL ENERGY



FORMULA

ROTATIONAL ENERGY
PROBLEMS

1) A 100-kg solid sphere with a radius equal to 2.0 m is rotating at 10.0 radians/s, what is its rotational kinetic energy?

$$KE_R = (1/2)\ I\ \omega^2 = (1/2)\ (2/5)\ m\ r^2\ .\ \omega^2 = (1/2)\ (2/5)\ 100\ 2^2\ .\ 10^2 = 800\ J$$

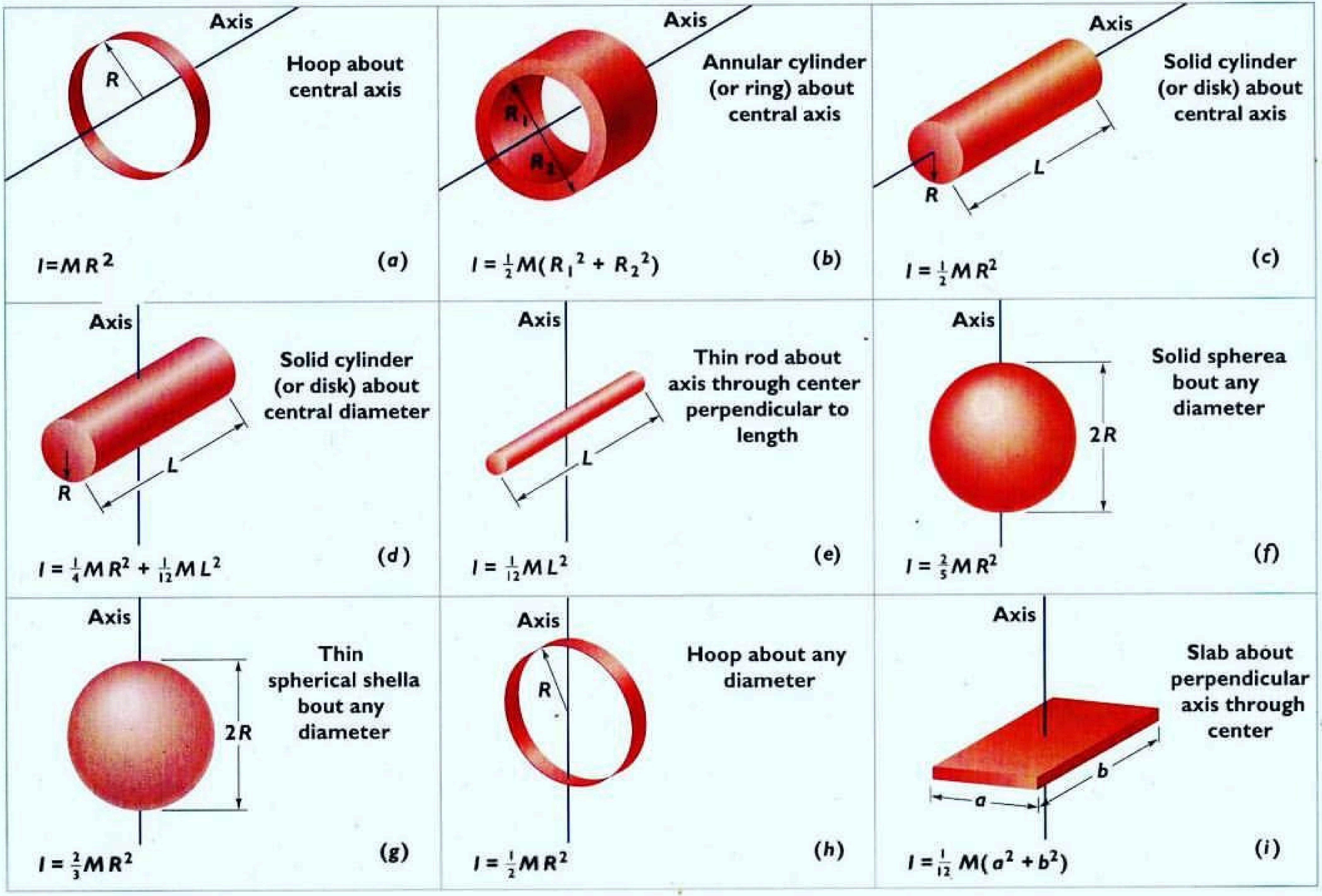
2) How much rotational kinetic energy does a spinning tire of mass 10.0 kg and radius 0.50 m have if it's spinning at 40.0 rotations/s?

$$KE_R = (1/2)\ I\ \omega^2 = (1/2)\ m\ r^2\ .\ \omega^2 = (1/2)\ 10\ (0.5)^2\ .\ (80\pi)^2 = 78.8 \times 10^3\ J$$

3) How much rotational kinetic energy does a spinning tire of mass 12 kg and radius 0.80 m have if it's spinning at 200.0 radians/s?

4)How much work do you do to spin a tire, which has a mass of 5.0 kg and a radius of 0.40 m, from 0.0 radians/s to 100.0 radians/s?

5)How much work do you do to spin a hollow sphere, which has a mass of 10.0 kg and a radius of 0.50 m, from 0.0 radians/s to 200.0 radians/s?


Axis
R
Hoop about central axis
$I = MR^2$
(a)
Axis
R_1
R_2
Annular cylinder (or ring) about central axis
$I = \frac{1}{2}M(R_1^2 + R_2^2)$
(b)
Axis
Solid cylinder (or disk) about central axis
L
R
$I = \frac{1}{2}MR^2$
(c)
Axis
Solid cylinder (or disk) about central diameter
L
R
$I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$
(d)
Axis
Thin rod about axis through center perpendicular to length
L
$I = \frac{1}{12}ML^2$
(e)
Axis
Solid spherea bout any diameter
2R
$I = \frac{2}{5}MR^2$
(f)
Axis
Thin spherical shella bout any diameter
2R
$I = \frac{2}{3}MR^2$
(g)
Axis
R
Hoop about any diameter
$I = \frac{1}{2}MR^2$
(h)
Axis
Slab about perpendicular axis through center
b
a
$I = \frac{1}{12}M(a^2 + b^2)$
(i)

ELASTICITY

DEFINITION:

Elasticity refers to the ability of a material to return to its original shape and size after the removal of an external force that caused deformation.

Essentially, an elastic material can "bounce back" to its original form after being stretched, compressed, or otherwise deformed.

CONDITION:

- Reversible Deformation: The material goes back to its original shape after the force is taken away. The change is temporary
- Linear Relationship: For many materials, if you double the force, the change in shape also doubles, as long as the force is within the material's elastic range.
- Elastic Limit: There's a maximum stretch or squeeze a material can handle before it starts to permanently change shape or break.

HOOK'S LAW

STATEMENT:

1. The strain of a material is proportional to the applied stress within the elastic limit of that material.

2 . For small displacements or deformations, the displacement or deformation is directly proportional to the applied force or load.

CONDITION:

1. $\sigma = E\,\varepsilon$
2. $F = -kx.$

Stress/Elastic limit:

It is the resistance offered by the body to any deformation.

$$\sigma = F / A$$

Strain

Deformation per unit length in the direction of deformation is known as strain.

$$\varepsilon = \Delta L / L$$

PROBLEMS

Question 1] *A spring is stretched 10 mm (0.01 m) by a weight of 2.0 N. Calculate: (a) the force constant k, and (b) the weight W of an object that causes an extension of 80 mm (0.08 m).*

Ans: *k = F/x = 2/0.01 N/m = 200 N/m (answer)*

W = 200 x 0.08 N = 16 N

Question: How much force is needed to pull a spring with a spring constant of 20 N/m a distance of 25 cm?

Ans: A force of 5 Newtons is needed to pull this spring a distance of 25 cm.

Question 3] The elastic limit of brass is 379 MPa. What should be the minimum diameter of a brass rod if it to support a 400 N load without exceeding its elastic limit?

Ans: d = 1.15 mm

Question 4] A steel wire having a radius of 2.0 mm, carrying a load of 4 kg, is hanging from a ceiling. Given that g = 3.1πm/s2, what will be the tensile stress that would be developed in the wire?

Ans: Tensile stress = 3.1 x 10^6 Nm-2

POISSON’S RATIO

STATEMENT:

Poisson’s ratio is “the ratio of transverse contraction strain to longitudinal extension strain in the direction of the stretching force. Here,

- Compressive deformation is considered negative
- Tensile deformation is considered positive.

SIGNIFICANCE:

Symbol	Greek letter ‘nu’,v
Formula	Poisson’s ratio = − Lateral strain / Longitudinal strain
Range	-1.0 to +0.5
Units	Unitless quantity
Scalar / Vector	Scalar quantity

PROBLEMS

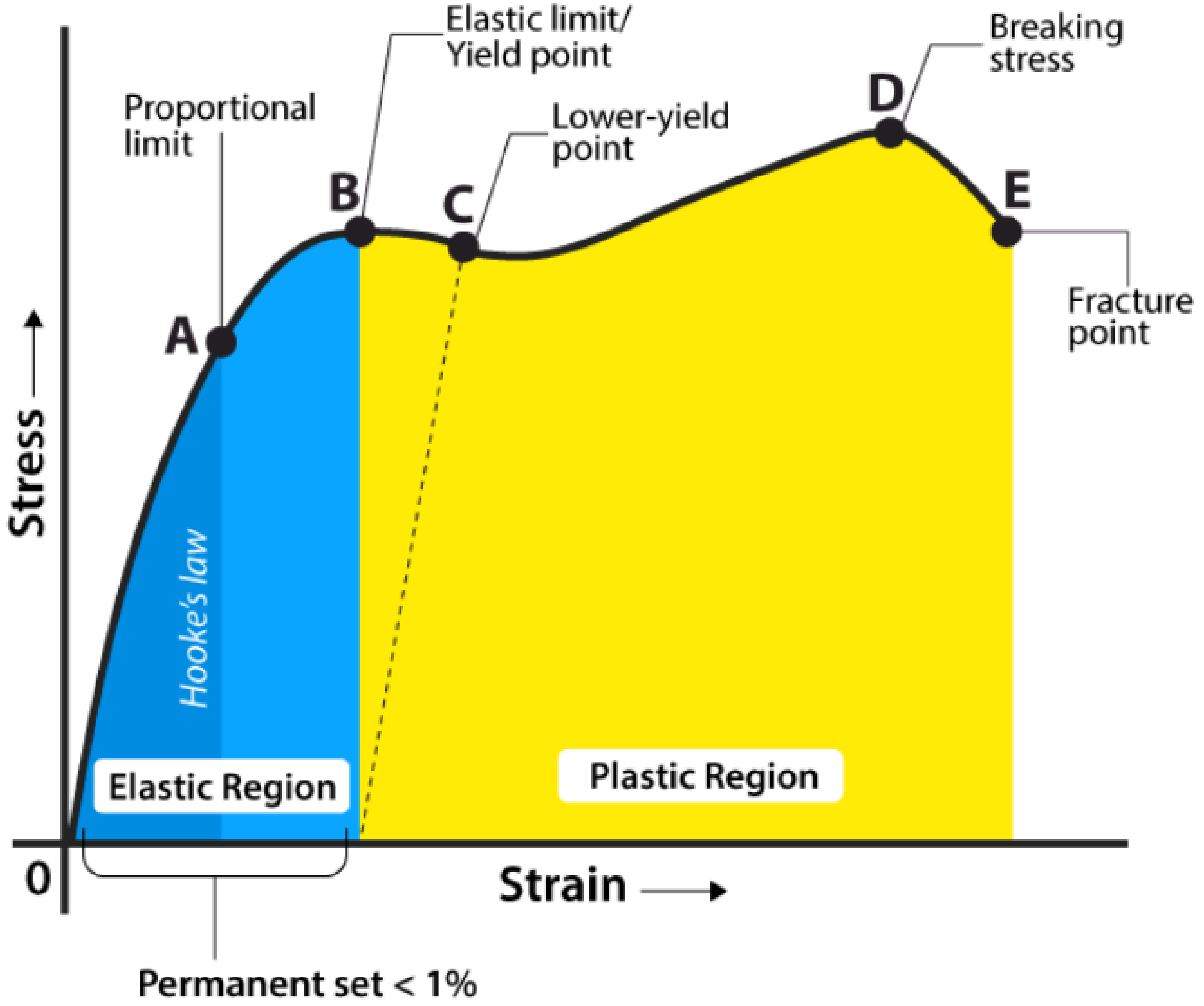
When a brass rod of diameter 6 mm is subjected to a tension of 5 $\times 10^3$ N, the diameter changes by 3.6×10^{-4} cm. Calculate the longitudinal strain and Poisson's ratio for brass given that Y for the brass is 9×10^{10} N/m².

A metal wire of length 1.5 m is loaded and an elongation of 2 mm is produced. If the diameter of the wire is 1 mm, find the change in the diameter of the wire when elongated. $\sigma = 0.24$.

A metallic wire ($Y = 20 \times 10^{10}$ N/m². and $\sigma = 0.26$) of length 3 m and diameter 0.1 cm is stretched by a load of 10 kg. Calculate the decrease in diameter of the wire.

A copper wire 3m long and 1 mm² in cross-section is fixed at one end and a weight of 10 kg is attached at the free end. If Y for copper is 12.5×10^{10} N/m² and $\sigma = 0.25$ find the extension, lateral strain and the lateral compression produced in the wire.

STRESS-STRAIN CURVE



STRESS-STRAIN CURVE

- Proportional Limit: The point on the graph where the material follows Hooke's Law. Here, stress and strain are directly related, and you can calculate Young's modulus. It's where the line starts bending on the graph.
- Elastic Limit: The maximum point where the material still returns to its original shape after the force is removed. Beyond this point, the material starts to permanently change shape.
- Yield Point: The point where the material begins to deform permanently. There are two yield points:
- Upper Yield Point: The highest stress level before the material starts to permanently deform.
- Lower Yield Point: The stress level after which the material continues to deform permanently.
- Ultimate Stress Point: The highest stress the material can handle before it fails or breaks. It's the peak on the graph.
- Fracture or Breaking Point: The point where the material actually breaks or fails. This is where it can no longer withstand the stress and breaks apart.

STRESS-STRAIN CURVE

APPLICATION:

Stress-strain diagram is used to:

- Determine material properties such as Young's modulus, yield strength, ultimate strength, and fracture toughness.
- Guide design and safety by selecting appropriate materials and ensuring they operate within safe limits.
- Predict material behavior under different stresses, both elastic and plastic.
- Analyze failure modes to understand how and why materials break.
- Maintain quality control by ensuring materials meet performance standards.

Practice Questions

1. What is meant by stress?
2. Define the stress-strain curve.
3. What is the formula to calculate the strain?
4. Define elasticity.
5. What is meant by tensile stress?