Computer Project

1 Introduction

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Abstract- The project is based on a part of the transmission of Leonardo AW609 VTOL aircraft. This gear-train is used for taking off a part of the input power to one rotor and delivering to the other rotor. The analysis is idealised to be solvable using our current knowledge of vibration systems.
In the extra credit section, we have also considered the gear transmission between gear 2 and gear 3 to be non ideal, which when solved with the relevant values according to actual aircraft can provide us with a more accurate and precise analysis.

2 Solution

Problem 1: Tiltrotor Gearbox

(1) Identify the degrees of freedom and develop the nonlinear equations of motion for this system in the state space form. Use $q_1=\Omega_1$, $q_2=\theta_1-\theta_2$, and $q_3=\Omega_3$

Solution:These are the equations obtained after drawing the FBD.

$J_1\ddot{\theta}_1 = T_m + k_1(\theta_2-\theta_1) - B_1\dot{\theta}_1,$
$J_2\ddot{\theta}_2 = -k_1(\theta_2-\theta_1) - FR_2,$
$J_3\ddot{\theta}_3 = FR_3 - \left(1.5\dot{\theta}_3 + \beta\ddot{\theta}_3\right)\dot{\theta}_3$
$R_2\dot{\theta}_2 = R_3\dot{\theta}_3$
Here, F is the reaction force acting between gear 2 and gear 3

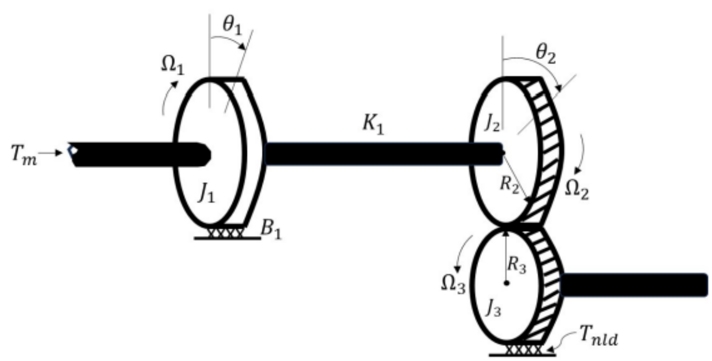


Figure 1: Free-body diagram of the tiltrotor gearbox system

$$J_1\ddot{\theta}_1 = T_m + k_1(\theta_2 - \theta_1) - B_1\dot{\theta}_1 \tag{1}$$

$$J_2\ddot{\theta}_2 = -k_1(\theta_2 - \theta_1) - FR_2 \tag{2}$$

$$J_3\ddot{\theta}_3 = FR_3 - \left(1.5\dot{\theta}_3 + \beta\ddot{\theta}_3\right)\dot{\theta}_3 \tag{3}$$

$$R_2\ddot{\theta}_2 = R_3\ddot{\theta}_3 \tag{4}$$

Adding the second and third equations,

$$J_1\ddot{\theta}_1 = T_m + k_1(\theta_2 - \theta_1) - B_1\dot{\theta}_1 \tag{5}$$

Defining state space variables,

$$q_1 = \dot{\theta}_1, \quad q_2 = \theta_1 - \theta_2, \quad q_3 = \dot{\theta}_3 \tag{6}$$

$$\dot{q}_2 = q_1 - \dot{\theta}_2 \tag{7}$$

$$\dot{q}_1 = \frac{T_m - k_1q_2 - B_1q_1}{J_1} \tag{8}$$

$$\dot{q}_2 = q_1 - \frac{R_3}{R_2}q_3 \tag{9}$$

$$\dot{q}_3 = \frac{k_1q_2 - 1.5q_3^2}{J_2\frac{R_3}{R_2} + J_3 + \beta q_3} \tag{10}$$

Now let's find out the equilibrium conditions by putting in T_m= 1 N-m and putting in $\dot{q}_1, \dot{q}_2, \dot{q}_3$ all equal to 0.
We get the initial values of q_1, q_2, q_3 as: $q_1 = 0.09rad/s, q_2 = 0rad, q_3 = 0.19rad/s$

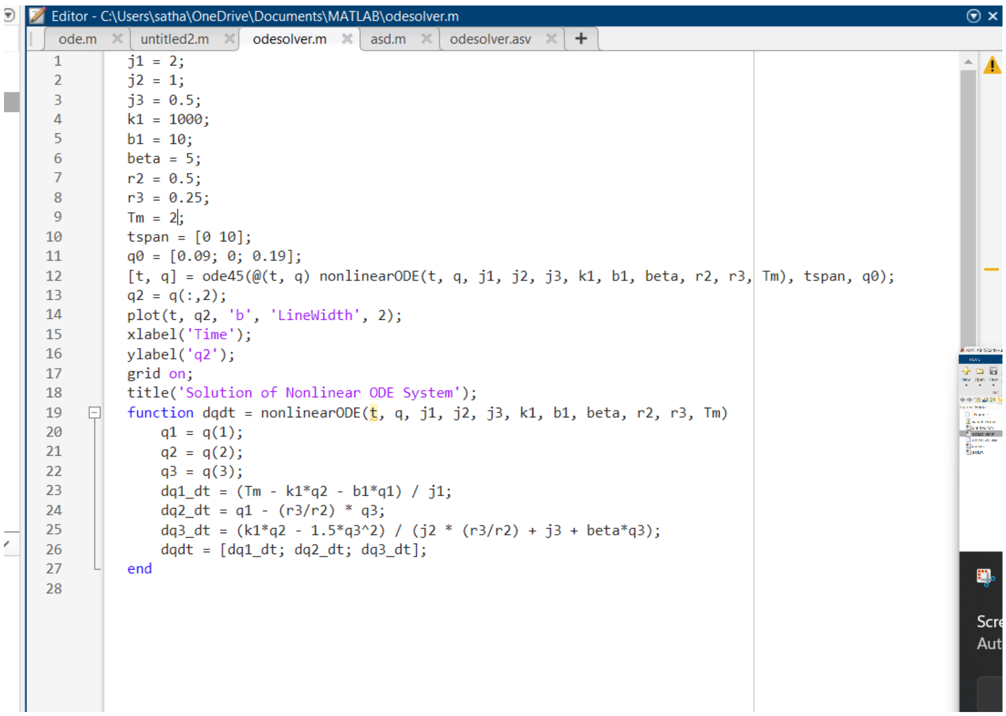
```matlab

j1 = 2;
j2 = 1;
j3 = 0.5;
k1 = 1000;
b1 = 10;
beta = 5;
r2 = 0.5;
r3 = 0.25;


q0 = [0; 0; 0];


solution = fsolve(@(q) myEquations(q, j1, j2, j3, k1, b1, beta, r2, r3), q0);


disp('Solution for q1, q2, q3:')
disp(q0)
disp(solution)


function F = myEquations(q, j1, j2, j3, k1, b1, beta, r2, r3)
    q1 = q(1);
    q2 = q(2);
    q3 = q(3);


    F(1) = (1 - k1 * q2 - b1 * q1) / j1;
    F(2) = q1 - (r3 * q3) / r2;
    F(3) = (k1 * q2 - 1.5 * q3 * q3) / ((j2 * r3) / r2 + j3 + beta * q3);
end

```

Figure 2: Matlab Script for finding conditions.

Problem 2: MATLAB Code

Use the MATLAB program ode23/ode45 to calculate and plot the response $(\theta_1 - \theta_2)$ of the system to a step torque (T_m) which changes from magnitude 1 N-m to 2 N-m at t = 0. Repeat the same with Tm changing magnitude from 1 N-m to 30 N-m at t = 0. For both the inputs, determine the steady state (equilibrium value), maximum overshoot, and period of oscillations of rotational displacement $(\theta_1 - \theta_2)$.

To solve this, we used MATLAB ode45 function to solve the obtained set of equations.




```matlab
j1 = 2;
j2 = 1;
j3 = 0.5;
k1 = 1000;
b1 = 10;
beta = 5;
r2 = 0.5;
r3 = 0.25;
Tm = 2;
tspan = [0 10];
q0 = [0.09; 0; 0.19];
[t, q] = ode45(@(t, q) nonlinearODE(t, q, j1, j2, j3, k1, b1, beta, r2, r3, Tm), tspan, q0);
q2 = q(:,2);
plot(t, q2, 'b', 'LineWidth', 2);
xlabel('Time');
ylabel('q2');
grid on;
title('Solution of Nonlinear ODE System');
function dqdt = nonlinearODE(t, q, j1, j2, j3, k1, b1, beta, r2, r3, Tm)
    q1 = q(1);
    q2 = q(2);
    q3 = q(3);
    dq1_dt = (Tm - k1*q2 - b1*q1) / j1;
    dq2_dt = q1 - (r3/r2) * q3;
    dq3_dt = (k1*q2 - 1.5*q3^2) / (j2 * (r3/r2) + j3 + beta*q3);
    dqdt = [dq1_dt; dq2_dt; dq3_dt];
end
```

Figure 3: ODE45

Here, the value of T_m is set to 2 N-m. The graph obtained is as follows:

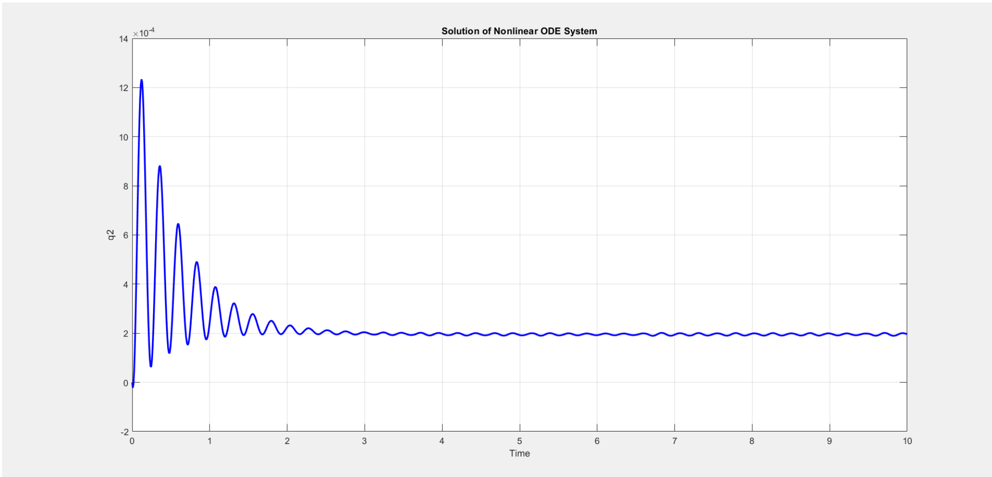


Figure 4: $T_m = 2N - m$

The maximum overshoot of q_2 is 12.33×10^{-4} radians. The equilibrium value of q_2 is 1.95×10^{-4} radians. The time period of oscillations in steady state is around 0.25 seconds. This completely describes the system response of q_2 against time.

Similarly, for $T_m = 30Nm$, following is the graph obtained:

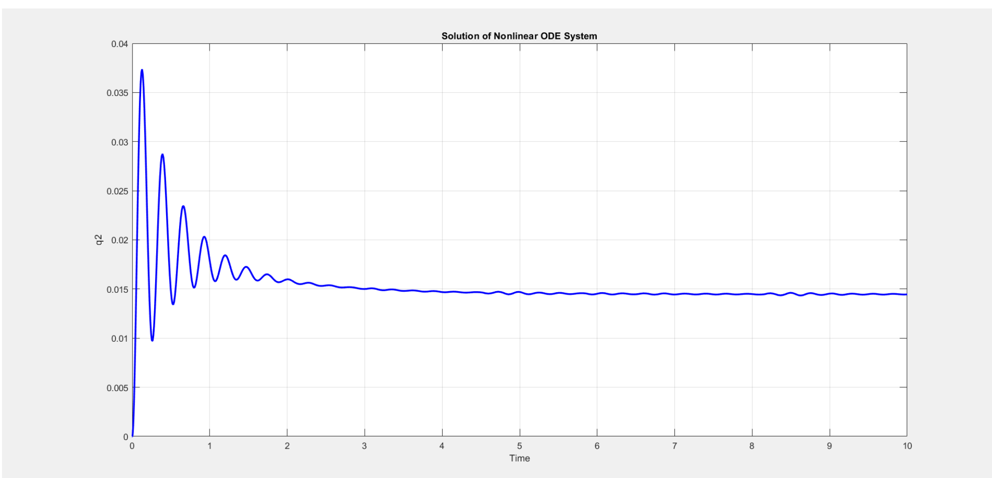


Figure 5: $T_m = 30Nm$ response

The maximum overshoot of $q_2 = 3.73 \times 10^{-2}$ radians. The equilibrium value is 1.45×10^{-2} radians. The time period is around 0.25 seconds.

Problem 3: Linearizing

Linearize the system model in the vicinity of the equilibrium operating point, $T_m = 1$N m. Obtain step responses of the linearized model for the two step inputs in part (2).Again for both inputs determine steady-state value, maximum overshoot, and period of oscillations. Compare these values with the non-linear system and comment on the validity of the linearized model.

The following is our code for linearizing the system model in the around T_m=1Nm using Jacobian:

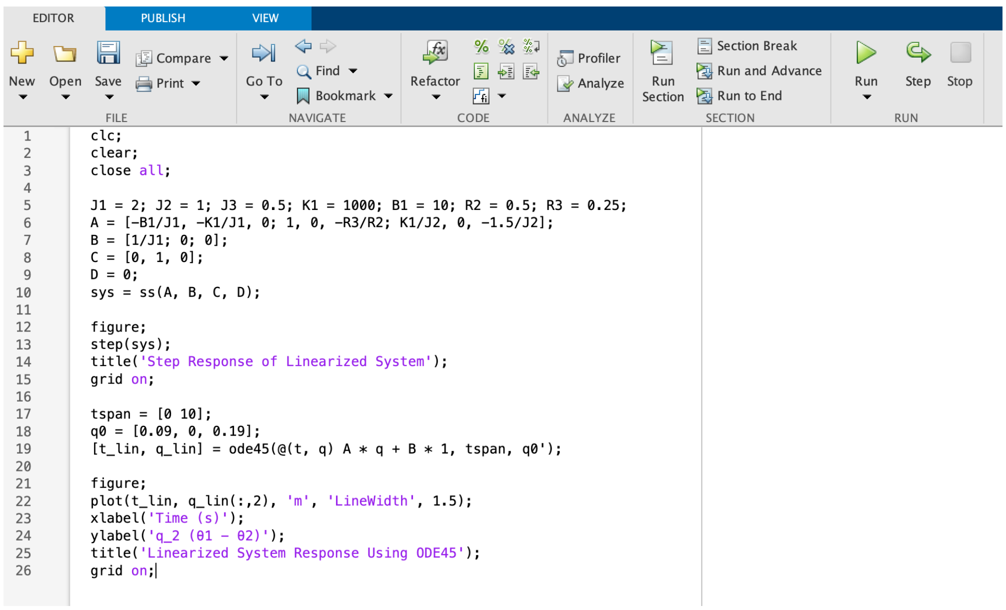


Figure 6: ODE45

This is our code for the linear solver:

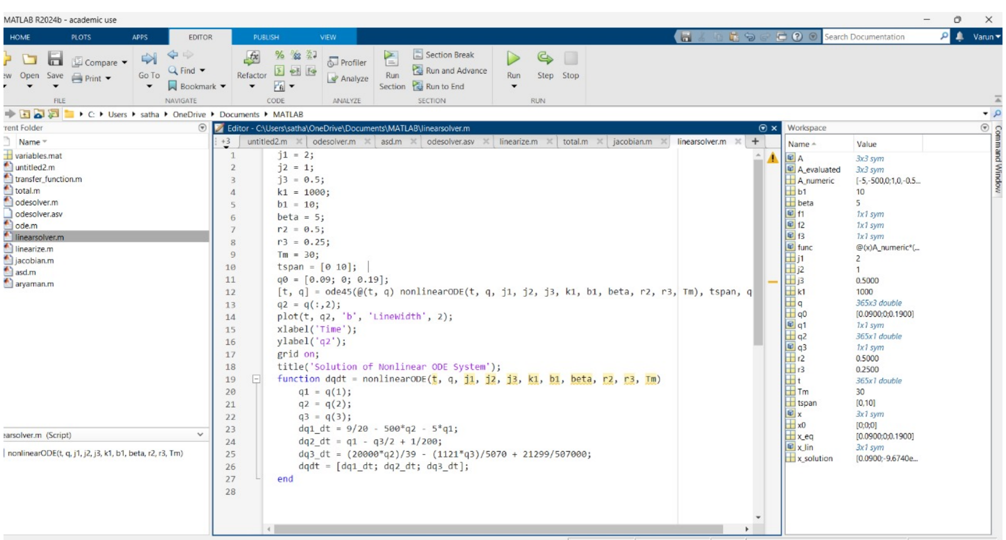


Figure 7: ODE45

The Non Linear system response is as follows:-

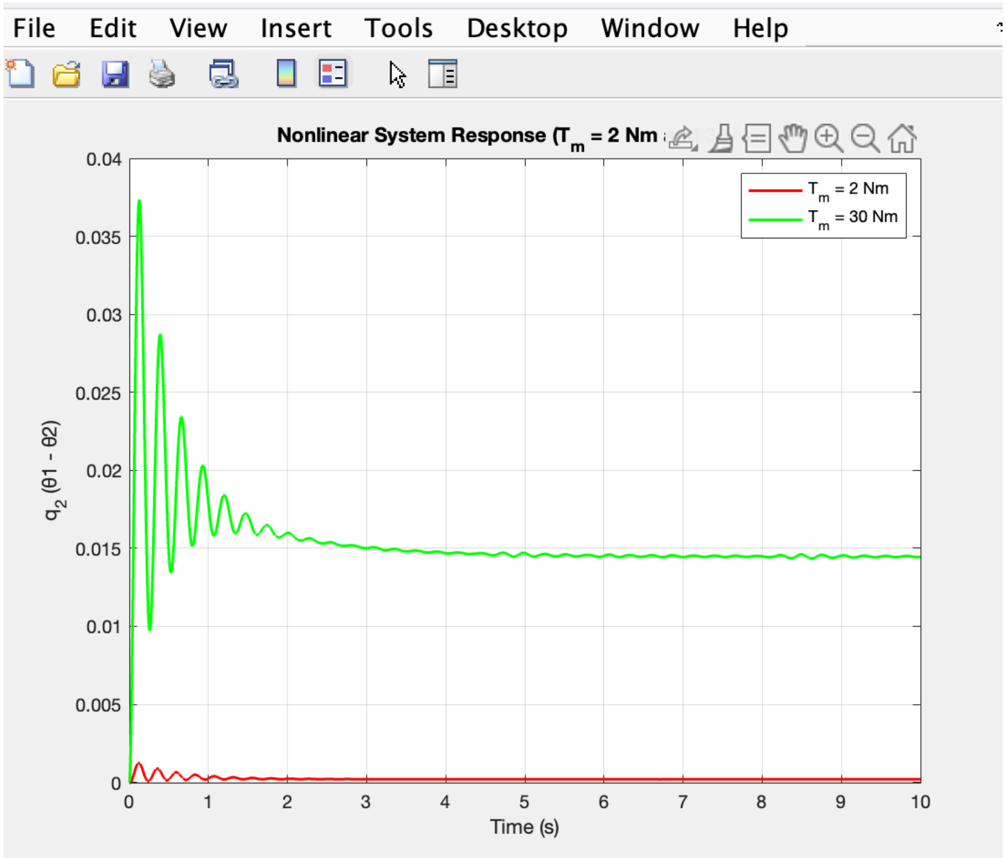


Figure 8: ODE45

The following is the step response of the linearized system:-

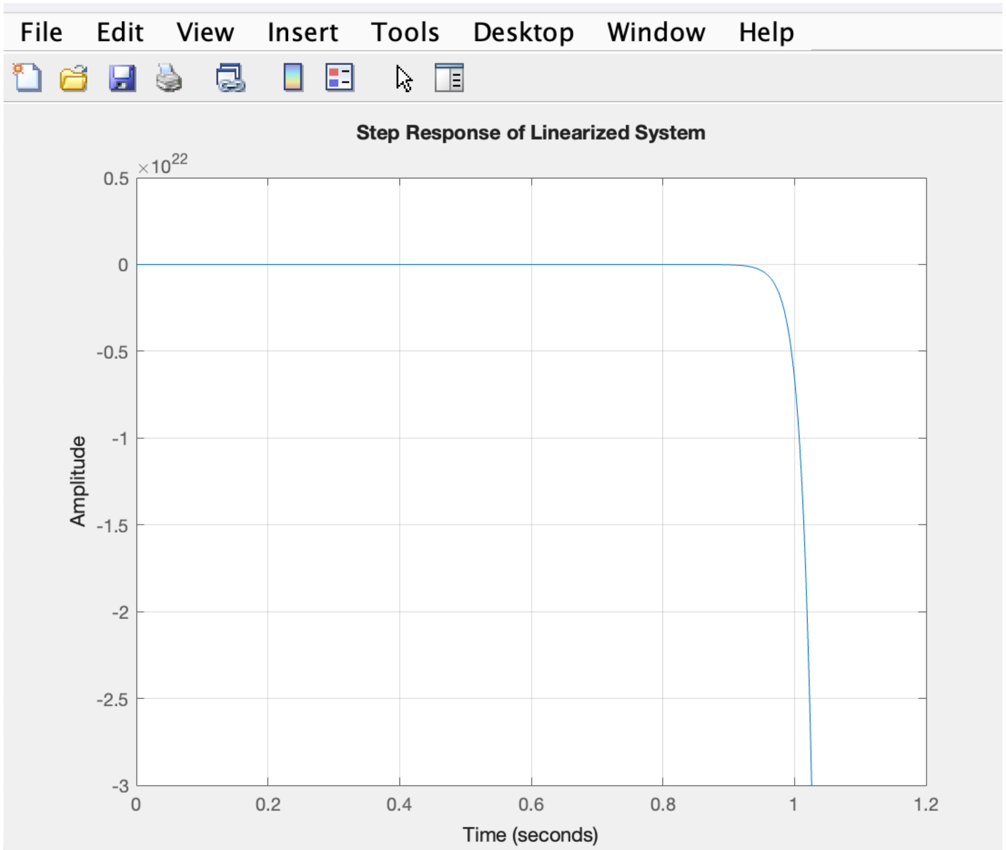


Figure 9: ODE45

The maximum overshoot of $q_2 = 2.89 \times 10^{-2}$ radians. The equilibrium value is 1.42×10^{-2} radians. The time period is around 0.24 seconds.We can comment that the linearized system slightly underestimates the maximum overshoot but closely approximates the steady-state value and time period compared to the nonlinear system.

> **Problem 4: Transfer Function**
>
> For the linearized model with $(\theta_1 - \theta_2)$ as the output, find the transfer function, poles and zeros using MATLAB. Find natural frequency and damping ratio on the basis of dominant poles.

The following is our code for calculating the transfer function:

```
clc;
clear;
close all;

J1 = 2; J2 = 1; J3 = 0.5; K1 = 1000; B1 = 10; R2 = 0.5; R3 = 0.25;
A = [-B1/J1, -K1/J1, 0; 1, 0, -R3/R2; K1/J2, 0, -1.5/J2];
B = [1/J1; 0; 0];
C = [0, 1, 0];
D = 0;
sys = ss(A, B, C, D);
sys_tf = tf(sys);

figure;
bode(sys_tf);
title('Bode Plot of Linearized System');
grid on;

figure;
subplot(1,2,1);
rlocus(sys_tf);
title('Root Locus');
grid on;

subplot(1,2,2);
pzmap(sys_tf);
title('Pole-Zero Map');
grid on;

poles = pole(sys_tf);
wn = abs(poles(1)); % Natural frequency
zeta = -real(poles(1)) / wn; % Damping ratio

disp('Transfer Function:');
disp(sys_tf);
disp('Poles:');
disp(poles);
disp('Zeros:');
disp(zero(sys_tf));
disp(['Natural Frequency (ω_n): ', num2str(wn), ' rad/s']);
disp(['Damping Ratio (ζ): ', num2str(zeta)]);
```

Figure 10: ODE45

The bode plot of the linearized system is as follows:

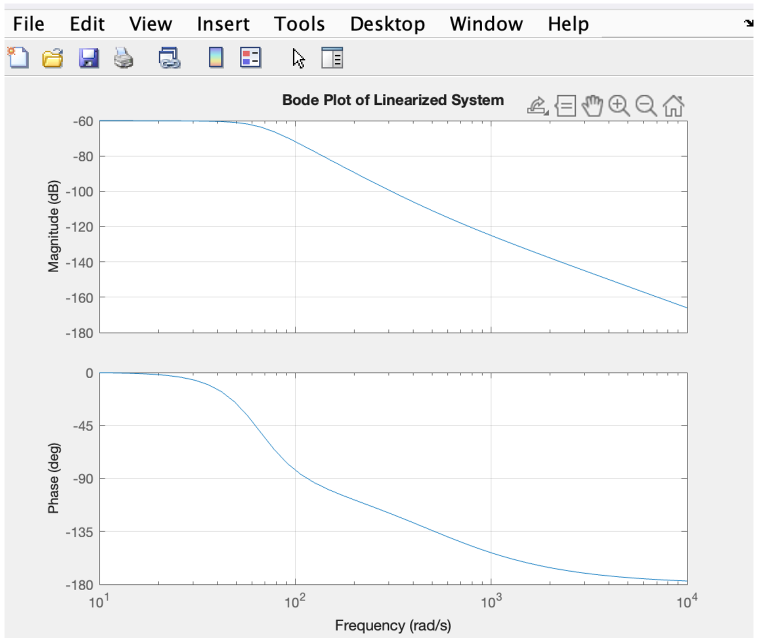


Figure 11: ODE45

From our calculations we have received the following results for poles,zeroes,natural frequency and the damping ratio:

```
Poles of the system:
 -32.3744 +56.8414i
 -32.3744 -56.8414i
  58.2489 + 0.0000i

Zeros of the system:
  498.5000

Natural Frequency: 65.4145
Damping Ratio: 0.49491
```

Figure 12: ODE45

> **Problem 5: Extra credit**
>
> Comment on how would your analysis change if there is a contact stiffness between the gears, that is, gear teeth are no longer rigid and can deform.

The given question is a system with 2 DOFs(degree of freedoms). If we assume deformation in the gear teeth, we introduce a complexity by increasing the DOF of the system to 3. This causes a delay in the response of θ_3 to θ_2. So to model this change, we add a spring and damper between two gear teeth in contact. The new Equations of Motion are as follow:

$$\mathrm{J}_1\ddot{\theta}_1 = T_m + k_1(\theta_2 - \theta_1) - B_1\dot{\theta}_1$$

$$\mathrm{J}_2\ddot{\theta}_2 = -k_1(\theta_2 - \theta_1) + k_0(R_3\theta_3 - R_2\theta_2) - c_0(k_2\dot{\theta}_2 - k_3\dot{\theta}_3)$$

$$\mathrm{J}_3\ddot{\theta}_3 = -k_0(R_3\theta_3 - R_2\theta_2) - \left(1.5\dot{\theta}_3 + \beta\dot{\theta}_3^3\right)\dot{\theta}_3 + c_0(R_2\dot{\theta}_2 - R_3\dot{\theta}_3)$$

Where k_0 is the equivalent spring constant and c_0 is the damping constant of the damper between the teeth.

For a given set of input of step torque i.e. its value being changed once the system reaches equilibrium, the steady state value of q_2 will be relatively high. This is because the value of θ_2 is relatively less at equilibrium owing to the non conservative work done by the newly introduced damper that is between teeth, and the value of θ_1 will be higher since for less value of θ_2, the negative torsional effects of the torsional spring will reduce on J_1. **This clearly indicates a higher equilibrium value of q_2.**

3 Project Report End

Work Distribution:

Problem 1: Arul Gupta, Yug Vora- Used Newtonian Mechanics to write down the Non-linear Equation Of Motions in the required space state form.
Problem 2: Varun Sathaye- Developed the MATLAB code using StepFunction
Problem 3: Aryamann Srivastava, Arul Gupta,Varun Sathaye- Linearised the system and developed the required MATLAB code.
Problem 4: Yug Vora, Aryamann Srivastava : Calculated the transfer function, and developed the MATLAB code for required Bode Plot.
Extra Credit Problem: Qualitative Analysis and modified Equations of Motion developed through Group Discussion.