Achintya Rajan, 03/30/20, 2°

11.8: # 2, 6, 10, 12, 14, 18, 20, 22, 26, 30, 37, 45

AP # 104

11.8

2) $f(x) = e^{-2x} \longrightarrow 1$

$f'(x) = -2e^{-2x} \longrightarrow -2$

$f''(x) = (-2)^2 e^{-2x} \rightarrow 4$

$f^{(n)}(x) = (-2)^n$

$$a_n = \frac{f^{(n)}(0)}{n!} = \frac{(-2)^n}{n!} \rightarrow \boxed{(-1)^n \frac{2^n}{n!}}$$

6) $f(x) = \dfrac{1}{1-2x} \quad f(1) = \dfrac{1}{1-2} = -1$

$f'(x) = \dfrac{2}{(1-2x)^2} \rightarrow f'(1) = 2$

$f''(x) = \dfrac{8}{(1-2x)^3} \rightarrow f''(1) = 8$

$f^{(n)} = ???\ (-1)^n n!\,(-2)^n (1-2x)^{-n-1}$

$f^{(n)}(0) = n!\, 2^n$

$$a_n = \frac{f^{(n)}(0)}{n!} = \boxed{2^n}$$

10) $f(x) = x^2 \sin x$

$$x^2 \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \Rightarrow (-1)^n \frac{1}{(2n+1)!} x^{2n+3}$$

$$= \boxed{\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+3}}{(2n+1)!}}$$

12) $f(x) = \cos(x^2)$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{(x^2)^{2n}}{(2n)!} \rightarrow \boxed{\sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n)!} x^{4n}}$$

14) $f(x) = \sin x \cdot \sin x$?

$f(x) = 1 - \cos^2 x$

$$1 - \left(1 + \sum_{n=1}^{\infty} (-1)^n \frac{2^{2n-1}}{(2n)!} x^{2n}\right)$$

$$= \sum_{n=1}^{\infty} (-1)^{n+1} \frac{2^{2n-1}}{(2n)!} x^{2n} \ ??$$

18) $f(x) = \cos(x), \quad c = \pi/3 \rightarrow 1/2$

$f'(x) = -\sin(x) \rightarrow f'(1) = -\sqrt{3}/2$

$f''(x) = -\cos(x) \rightarrow f''(1) = -1/2$

$f'''(x) = \sin(x) \rightarrow f'''(1) = \sqrt{3}/2$

$$\cos x = \frac{1}{2} - \frac{\sqrt{3}}{2}(x - \pi/3) - \frac{1}{4}\left(x - \frac{\pi}{3}\right)^2 + \dots$$

$$= \boxed{\sum_{n=0}^{\infty} (-1)^n \frac{1}{2(2n)!}\left(x - \frac{\pi}{3}\right)^{2n} + \sum_{n=0}^{\infty} (-1)^{n+1} \frac{\sqrt{3}}{2(2n+1)!}\left(x - \frac{\pi}{3}\right)^{2n+1}}$$

20) $f(x) = e^x, \quad c = -3 \longrightarrow e^{-3}$

$f'(x) = e^x \longrightarrow f'(-3) = e^{-3}$

$f''(x) = e^x \longrightarrow f''(x) = e^{-3}$

$$e^x = e^{-3} + e^{-3}(x+3) + e^{-3}(2!)^{-1}(x+3)^2 + \dots$$

$$= \boxed{\sum_{n=0}^{\infty} \frac{1}{e^3 n!}(x+3)^n}$$

22) $\ln(x)$, powers of $x - 1 \rightarrow c = 1$

$f'(x) = x^{-1} \longrightarrow f'(1) = 0$

$f''(x) = -x^{-2} \rightarrow f'(1) = -1$

$f'''(x) = 2x^{-3} \rightarrow f'(1) = 2$

$$\ln x = 0 + 1(x-1) - \frac{1}{2!}(x-1)^2 + \dots$$

$$= \boxed{\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}(x-1)^n}$$

26) $f(x) = \tan^{-1}(x), \quad c = 1 \longrightarrow \pi/4$

$f'(x) = \dfrac{1}{1+x^2} \longrightarrow \dfrac{1}{2}$

$f''(x) = -\dfrac{2x}{(1+x^2)^2} \longrightarrow -\dfrac{1}{2}$

$$f(x) = \boxed{\frac{\pi}{4} - \frac{1}{2}(x-1) - \frac{1}{2}(x-1)^2 + \dots}$$

30) $\dfrac{1}{e} \rightarrow e^{-x} = \displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^n}{n!}$

$$e^{-1} = 1 - \left(\frac{1}{2}\right) + \frac{(1/2)^2}{2!} = \boxed{0.5}$$

$$E \frac{(1/2)^2}{2!} = \frac{1}{8} = \boxed{0.125}$$

6) $\int_0^{0.5} \cos(x^2)\,dx \rightarrow \cos(x^2) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n}}{(2n)!}$

$\left[\sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+1}}{(4n+1)(2n)!}\right]_0^{0.5}$?

$= \frac{(0.5)^1}{1(0!)} - \frac{(0.5)^5}{5(2!)} + \frac{(0.5)^9}{9(4!)} = \boxed{0.496}$

$E = \frac{(0.5)^9}{9(4!)} \approx \boxed{9.04 \cdot 10^{-6}}$

45) $f(x) = \ln\left(\frac{1+x}{1-x}\right) = \ln(1+x) - \ln(1-x)$

$1+(-x)$

$\ln(1+(-x)) = \sum_{n=0}^{\infty} (-1)^n \frac{(-x)^{n+1}}{n+1} = -\sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1}$

$\rightarrow \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1} + \sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1}$

$= \left(x - \frac{x^2}{2} + \frac{x^3}{3} + \ldots\right) + \left(x + \frac{x^2}{2} + \frac{x^3}{3} + \ldots\right)$

$= \boxed{2\sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1}}$

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17) $f(x) = \ln(3-x) \longrightarrow f(2) = \ln(1) = 0$

$f'(x) = -\frac{1}{3-x} \longrightarrow f'(2) = -\frac{1}{1} = -1$

$f''(x) = -\frac{1}{(3-x)^2} \longrightarrow f''(2) = -1$

$\boxed{B}$

16) $\frac{\sin(t)}{t} = \frac{1}{t} \cdot \sin(t) \rightarrow \frac{1}{t}\sum$

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WS: Series #43-47
AP #105

WS

$\frac{1}{2}$3) $f(x)=e^{-x^2}$

$$e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}\rightarrow e^{-x^2}=\sum_{n=0}^{\infty}\frac{(-x^2)^n}{n!}$$

$$=\boxed{\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{n!}}\quad |u_n|=\frac{x^{2n}}{n!}$$

$$\lim_{n\to\infty}\frac{(x+1)^{2n}}{(n+1)!}\cdot\frac{n!}{x^{2n}}=\frac{(x+1)^{2n}}{n\cdot\cancel{n!}}\cdot\frac{\cancel{n!}}{x^{2n}}\ \times$$

$$=\lim_{n\to\infty}\frac{1}{n}\cdot\frac{(x+1)^{2n}}{x^{2n}}\rightarrow \text{diverges except @ }\boxed{x=0}$$

$r=\infty$

b) $f(x)=\cos\sqrt{x}$

$$\cos x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}\rightarrow(-1)^n\frac{\sqrt{x}^{2n}}{(2n)!}$$

✓ $$\cos\sqrt{x}=\boxed{\sum_{n=0}^{\infty}(-1)^n\frac{x^n}{(2n)!}}\quad |u_n|=\frac{x^n}{(2n)!}$$

$$\lim_{n\to\infty}\frac{x^{n+1}}{(2n+2)!}\cdot\frac{(2n)!}{x^n}$$

$$=\lim_{n\to\infty}\frac{x\cdot\cancel{x^n}}{(2n+2)(2n+1)\cancel{(2n)!}}\cdot\frac{\cancel{(2n)!}}{\cancel{x^n}}$$

$$=\lim_{n\to\infty}\frac{1}{(2n+2)(2n+1)}\cdot x\rightarrow\boxed{r=\infty}$$

44) $$\lim_{x\to0}\frac{x-\arctan(x)}{x^2}\rightarrow f(x)=\frac{x-\arctan(x)}{x^2}$$

$$f'(x)=\frac{x^2(1-1/(1+x^2))-2x(x-\arctan x)}{x^4}\ ?$$

45) a) $$\sum_{n=0}^{\infty}\frac{x^n}{(n+1)!}=\frac{1}{1}+\frac{x}{2!}+\frac{x^2}{3!}+\frac{x^3}{4!}+\dots$$

$$=1+\frac{1}{2!}x+\frac{1}{3!}x^2+\frac{1}{4!}x^3+\dots+\frac{1}{n!}x^n$$

$$\rightarrow\sum_{n=1}^{\infty}\frac{x^{n-1}}{n!}\rightarrow\boxed{f(x)=\frac{e^x-1}{x}}$$

b)

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WS: Lagrange #4, 5 + 1, 2, 3
11.11: # 32, 43, 45, 50, 53, 61
~~AP #107~~

2hr!

WS

4) $f(2.2) \approx 3 + (x-2) - \frac{2(x-2)^2}{3}$

$|R_n(x)| \le \frac{M|x-a|^{n+1}}{(n+1)!}$, $x = 2.2$

$R_n \le \frac{3.3|2.2-2|^3}{3!} \rightarrow \frac{3.3(0.2)^3}{3!}$

$= \boxed{0.0044} \longrightarrow \boxed{D}$

5) $R_n \le \frac{M|x-a|^{n+1}}{(n+1)!} = \frac{4(0.3)^2}{2} = 2(0.3)^2$

$R_n \le \boxed{0.18} \rightarrow \boxed{D}$

11.11

32) $\sum_{n=1}^{\infty} \frac{\tan^{-1}(n)}{\sqrt{1+n^2}} \rightarrow \ge \frac{\pi/4}{\sqrt{2n^2}} \ge \frac{\pi}{4\sqrt{2}} \sum_{n=1}^{\infty} \frac{1}{n}$

$\rightarrow \boxed{\text{diverges}}$

43) $\sum_{n=1}^{\infty} \frac{1}{n 2^n}(x+10)^n \quad u_n = \frac{(x+10)^n}{n \cdot 2^n}$

$\lim_{n\to\infty} \frac{(x+10)\cancel{(x+10)^n}}{(n+1)\cdot 2 \cdot \cancel{2^n}} \cdot \frac{n \cdot \cancel{2^n}}{\cancel{(x+10)^n}}$

$= \lim_{n\to\infty} \frac{n}{n+1} \cdot \frac{1}{2}(x+10) \rightarrow \begin{matrix} |x+10| < 2 \\ -12 < x < -8 \end{matrix}$

$x = -8$: $\sum_{n=1}^{\infty} \frac{1}{n} \rightarrow$ diverges

$x = -12$: $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n} \rightarrow$ converges

$\boxed{[-12, -8)}$

45) $\sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2} x^n = \frac{2\cancel{n!}}{n!^{\cancel{2}}} x^n = \frac{2}{n!} x^n$

$\lim_{n\to\infty} \frac{2x \cdot \cancel{x^n}}{(n+1)\cancel{n!}} \cdot \frac{\cancel{n!}}{\cancel{2x^n}} = \frac{1}{n+1} \cdot x$

$-1 < x < 1 \rightarrow r = \boxed{\cancel{1}}$ 1/4

50) $f(x) = \ln(2+x)$

$f'(x) = (2+x)^{-1}$

$f''(x) = -(2+x)^{-2}$

$f'''(x) = 2(2+x)^{-3}$

$\ln(2+x) - \ln(2+0)$

$\int_0^x \frac{1}{2+t} dt = \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{2^{n+1}(n+1)}$???

$\frac{1}{2} \sum_{n=0}^{\infty} (-1)^n \left(\frac{x}{2}\right)^n \quad \left|\frac{x}{2}\right| < 2$

$\int_0^x \frac{1}{2+t} dt = \sum_{n=0}^{\infty} (-1)^{n-1} \frac{x^n}{2^n n}$ $+\ln(2)$

53) $f(x) = e^{-x}$, $c = -2 \rightarrow e^2$

$f'(x) = -e^{-x} \longrightarrow -e^2$

$f''(x) = e^{-x} \longrightarrow e^2$

$\boxed{\sum_{n=0}^{\infty} (-1)^n \frac{1}{n!}(x+2)^n}$

61) $f(x) = \ln \cos x$, $c = \pi/6$, $n = 3$ $\quad \ln(\sqrt{3}/2)$

$\frac{1}{\cos x} \cdot -\sin x = -\tan x \rightarrow -\frac{\sqrt{3}}{3}$

$-\sec^2 x \longrightarrow -\frac{4}{3}$

$-2\sec^2 x \tan x \longrightarrow -\frac{8\sqrt{3}}{9}$

$-2\sec^4 x - 4\sec^2 x \tan^2 x$

$\ln\left(\frac{\sqrt{3}}{2}\right) - \frac{\sqrt{3}}{3}\left(x - \frac{\pi}{6}\right) - \frac{2}{3}$ ......

WS

1) $f(0.8) \approx 1 + x + \frac{x^2}{2} + \frac{x^3}{6} \quad R_n = \frac{M|x-a|^{n+1}}{(n+1)!}$

$R_n = \frac{(1.4)|0.8-0|^4}{4!} = \boxed{0.024} \longrightarrow \boxed{C}$

2) $f(0.5) \approx e^x \quad R_n = \frac{M|x-a|^{n+1}}{(n+1)!} = \frac{12|0.5|^3}{3!}$

$\boxed{0.25}$

$\boxed{A}$

3) $f(0.6) \approx (x-1) - \frac{(x-1)^3}{3!} + \frac{(x-1)^5}{5!} \quad R_n = \frac{M|x-a|^{n+1}}{(n+1)!}$

$R_n = \frac{(2)|0.6-1|^6}{6!} = 1.138 \cdot 10^{-5}$?