**Homework Cover Page BC**

Name Achintya Rajan

Date 03/19/20 HW # 9 Period 2

**Assignment List**

Monday 11.4 (3...35), WS: Series (22...27)

Score: 10 /10 Stamped? Y / N

Tuesday 11.5 (5...41)

Score: 10 /10 Stamped? Y / N

Wednesday 11.5 (6... 45), WS Series (29, 32)

Score: 10 /10 Stamped? Y / N

Thursday 11.5 (7..39), WS Series: 33, AP100

Score: 10 /10 Stamped? Y / N

Friday 11.5 (8...40), 11.11 (6...38)

Score: 10 /10 Stamped? N/A Y / N

**Total Self-Assessed Score** 50 / 50

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11.4: #3-35(4x-1)
WS: Series: #22, 26def, 27

10/10

## 11.4

3) $\sum_{n=1}^{\infty} \frac{5^n}{n(3^{n+1})} \to \frac{5^{n+1}}{(n+1)(3^{n+2})} \cdot \frac{n(3^{n+1})}{5^n}$

$= \frac{5 \cdot \cancel{5^n}}{3(n+1)\cancel{(3^{n+1})}} \cdot \frac{n\cancel{(3^{n+1})}}{\cancel{5^n}} = \frac{5n}{3(n+1)}$

$\lim_{n\to\infty} \frac{5n}{3n+3} = \frac{5}{3} > 1 \to \boxed{\text{diverges}}$

7) $\sum_{n=1}^{\infty} \frac{n+3}{n^2+2n+5}$ $\quad b_n = \frac{1}{n}$

$\lim_{n\to\infty} \frac{n+3}{n^2+2n+5} \cdot \frac{n}{1} = 1 \to \boxed{\text{diverges}}$

11) $\sum_{n=1}^{\infty} \frac{1}{n^n}$ $\quad \lim_{n\to\infty} \sqrt[n]{\frac{1}{n^n}} = \frac{1}{n} = 0 < 1$

$\boxed{\text{converges}}$

15) $\sum_{n=1}^{\infty} \frac{n}{3^n} \to \frac{n+1}{3^{n+1}} = \frac{3^n}{n}$ ; $\boxed{\text{converges}}$

$= \frac{n+1}{3 \cdot \cancel{3^n}} \cdot \frac{\cancel{3^n}}{n} = \frac{n+1}{3n} \to \frac{1}{3}$

19) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$ $\quad b_n = \frac{\sqrt{n}}{n^2} \leftarrow \boxed{\text{converges}}$

$\lim_{n\to\infty} \frac{\cancel{\sqrt{n}}}{n^2+1} \cdot \frac{n^2}{\cancel{\sqrt{n}}} = \frac{n^2}{n^2+1} = 1$

23) $\sum_{n=1}^{\infty} \frac{2}{n^3+e^n}$ $\quad \frac{1}{e^n} > \frac{2}{n^3+e^n}$

geo $\to$ converges $\quad \hookrightarrow \boxed{\text{converges}}$

27) $\sum_{n=1}^{\infty} \frac{n^n}{10^{n+1}}$ $\quad \frac{n^{n+1}}{10^{n+2}} \cdot \frac{10^{n+1}}{n^n}$

$= \frac{n \cdot \cancel{n^n}}{10 \cdot \cancel{10^{n+1}}} \cdot \frac{\cancel{10^{n+1}}}{\cancel{n^n}} = \frac{n}{10} \to \boxed{\text{diverges}}$

31) $\sum_{n=2}^{\infty} \frac{1}{n\sqrt[3]{\ln n}} \to$ cont, pos, decrease

$\int^{\infty} \frac{1}{n\sqrt[3]{\ln n}}$ $\quad u = \ln n$, $du = 1/n \, dn$ $\Big\}$ $\int_2^{\infty} \frac{\cancel{n}}{\cancel{n}\sqrt[3]{u}} du$

$= \lim_{t\to\infty} \left[\frac{3}{2} \ln x^{\frac{2}{3}}\right]_2^t = \infty \to \boxed{\text{diverges}}$

35) $\sum_{n=1}^{\infty} n \tan\frac{1}{n} \to$ cont, pos, decrease

$\lim_{n\to\infty} n\tan\frac{1}{n} \to \infty \cdot 0 \Big\}$ $\lim_{n\to\infty} \frac{\tan(1/n)}{1/n} \Big\}$ L

$= \lim_{n\to\infty} \frac{\cancel{(-1/n^2)}\sec^2(1/n)}{\cancel{(-1/n^2)}} = 1 \neq 0 \to \boxed{\text{diverges}}$

## WS: Series

22) $0 = 0 + 0 + 0 + \cdots$

$0 = (1-1) + (1-1) + (1-1) \cdots$ $\Big\}$ $\boxed{\text{always pairs}}$

$0 = 1 - 1 + 1 - 1 + 1 - 1$

26) d) $\sum_{n=1}^{\infty} \frac{(n^4+1)2^{2n}}{5^{n+3}} = \frac{((n+1)^4+1)2^{2(n+1)}}{5^{n+4}} \cdot \frac{1}{a_n}$

$\lim_{n\to\infty} \frac{4}{5}\left(\frac{(n+1)^4+1}{(n+1)^4}\right) = \frac{4}{5} < 1 \to \boxed{\text{converges}}$

e) $\sum_{n=1}^{\infty} \left(\frac{n-1}{n+1}\right)^{n^2} = \left(\frac{n}{n+2}\right)^{(n+1)^2} \cdot \frac{1}{(n-1/n+1)^{n^2}}$

$\boxed{\text{converges}}$

f) $\sum_{n=1}^{\infty} \frac{2^n n!}{n^n} \to \frac{2^{n+1}(n+1)!}{n^{n+1}} \cdot \frac{n^n}{2^n n!}$ $\quad \boxed{\text{converge}}$

$= \frac{2 \cdot \cancel{2^n}(n+1)\cancel{n!}}{(n+1)^n} \cdot \frac{n^n}{\cancel{2^n}\cancel{n!}}$ $\quad \lim_{n\to\infty} \frac{2(n+1)}{(n+1)^n} = 0$

27) $p(n) = n^l$ $\quad q(n) = n^k$

$\sum \frac{n^l}{n^k} \to \frac{(n+1)^l}{(n+1)^k} \cdot \frac{n^k}{n^l} = (n+1)^{l-k} \cdot n^{k-l}$

$\frac{n^{k-l}}{(n+1)^{k-l}}$ $\Big\}$ $\boxed{\begin{array}{l} k-l \leq 1 \to \text{diverge} \\ k-l > 1 \to \text{converge} \end{array}}$

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11.5 : # 5, 9, 13, 17, 21, 29, 33, 37, 41

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## 11.5

5) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{\sqrt{2n+1}}$ $\quad |a_n| = \frac{1}{\sqrt{2n+1}}$

BST $\frac{1}{n} < \frac{1}{\sqrt{2n+1}} \rightarrow$ diverges

$\frac{a_{n+1}}{a_n} = \frac{1}{\sqrt{2(n+1)+1}} \cdot \frac{\sqrt{2n+1}}{1} = \frac{\sqrt{2n+1}}{\sqrt{2n+3}} < 1$

$\boxed{\text{conditionally convergent}}$

9) $\sum_{n=2}^{\infty} (-1)^n \frac{n}{\ln(n)}$ $\quad \lim_{n\to\infty} \frac{n}{\ln(n)} = \infty \rightarrow D$

$\boxed{\text{diverges}}$ $\quad \infty > 0$

13) $\sum_{n=1}^{\infty} \frac{(-10)^n}{n!} \rightarrow \lim_{n\to\infty} \left| \frac{(-10)^{n+1}}{(n+1)!} \cdot \frac{n!}{(-10)^n} \right|$

$\lim_{n\to\infty} \left| \frac{(-10)\cancel{(-10)^n}}{(n+1)\cancel{n!}} \cdot \frac{\cancel{n!}}{\cancel{(-10)^n}} \right| \Rightarrow \frac{+10}{n+1} = 0$

$\boxed{\text{absolutely convergent}}$

17) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{\sqrt[3]{n}}{n+1}$ $\quad \frac{\sqrt[3]{n+1}}{n+2} \cdot \frac{n+1}{\sqrt[3]{n}} < 1$

converges (AST)

$\frac{\sqrt[3]{n}}{n+1} > \frac{1}{n^{0.1}} \rightarrow$ diverges

$\boxed{\text{conditionally convergent}}$

21) $\sum_{n=1}^{\infty} (-1)^n n \sin\frac{1}{n} \rightarrow \frac{\sin(1/n)}{1/n} \Big\} L$

$\lim_{n\to\infty} \frac{\sin(1/n)}{(1/n)} = \frac{\cancel{(-1/n^2)} \cos(1/n)}{\cancel{(-1/n^2)}} = 1$

$\boxed{\text{diverges}}$

29) $\sum_{n=1}^{\infty} \frac{(-1)^n \cos \pi n}{n}$ cancel each other??

$a_n = 1 + \frac{1}{2} + \frac{1}{3} + \dots \rightarrow$ harmonic $\boxed{\text{diverges}}$

33) $\sum_{n=0}^{\infty} (-1)^n \frac{1}{n!}$ $\quad \frac{1}{n!} < 0.0005 \rightarrow n! < 2000$

$n \geq 7$

$S_6 = \boxed{0.368}$

37) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n+1}{5^n} \Big\} \frac{n+1}{5^n} < 0.0005$

$2000(n+1) < 5^n \rightarrow n \geq 6$

$S_6 \approx \boxed{0.306}$

41) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^n}$ $\quad \frac{1}{n^n} < 0.00005 \rightarrow (n+1)^{n+1} > 20k$

$n+1 \geq 6$

$n \geq 5$

$\boxed{5}$

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11.5: #6, 10, 14, 22, 34, 38, 45
WS: Series #29, 32

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## 11.5

6) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n^{2/3}} \to |a_n| = \frac{1}{n^{2/3}}$ ↳ divergent p-series

$\frac{1}{(n+1)^{2/3}} < \frac{1}{n^{2/3}}$

$a_{k+1} < a_k \to$ $\boxed{\text{conditionally convergent}}$

10) $\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n} \to |a_n| = \frac{\ln(n)}{n}$

$b_n = \frac{1}{n}, \frac{1}{n} < \frac{\ln(n)}{n}$ } diverges (BCT)

$\frac{\ln(n+1)}{n+1} < \frac{\ln(n)}{n}$ } $a_{k+1} < a_n$ $\boxed{\text{conditionally converg}}$

14) $\sum_{n=1}^{\infty} \frac{n!}{(-5)^n} \to |a_n| = \frac{n!}{5^n}$

$\lim_{n\to\infty} \frac{n!}{5^n} = \infty \to$ $\boxed{\text{divergent}}$

22) $\sum_{n=1}^{\infty} (-1)^n \frac{\tan^{-1}(n)}{n^2} \to |a_n| = \frac{\tan^{-1}(n)}{n^2}$

$b_n = \frac{\pi}{n^2}, \frac{\pi}{n^2} > \frac{\tan^{-1}(n)}{n^2}$ } converges

$a_{k+1} < a_k \to$ $\boxed{\text{absolute convergence}}$

34) $\sum_{n=0}^{\infty} (-1)^{n+1} \frac{1}{(2n)!} \to |a_n| = \frac{1}{2n!}$

$b_n = \frac{1}{n^2}, \frac{1}{n^2} > \frac{1}{2n!} \to$ $\boxed{\text{absolute converges}}$

38) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n}\left(\frac{1}{2}\right)^n \to \frac{1}{n}\left(\frac{1}{2}\right)^n < 0.0005$

$2000 < n2^n \to n \geq 8$

$S_7 \approx$ $\boxed{-0.4058}$

45) $\sum a_n \to$ converges, $\sum b_n$ converges

$\boxed{\text{yes}}$ → product of 2 convergent ↳ terms → smaller

## WS

29) a) $\sum a_n$ converges, but $\sum a_n^2$ diverges

b) $\sum a_n$ diverges, but $\sum a_n^2$ converges

$a_n = 1/n$

32) a) $\sum a_n$ abs. converg, $\sum a_n^2$ con → $\boxed{\text{true}}$

b) $\sum a_n$ $\sum b_n$ conv. → $\sum a_n b_n$ = conv.

$\boxed{\text{true?}}$ → terms get smaller

c) $\sum |a_n|$ d, $\sum a_n$ d $\boxed{\text{true}}$

d) $\sum |a_n|$ C, $\sum a_n$ C → $\boxed{\text{false}}$ $(-1)^{n-1}\left(\frac{1}{n}\right)$

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11.5: # 7, 11, 15, 19, 23, 27, 35, 39

WS: Series # 33

AP 100

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## 11.5

7) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\ln(n+1)} \to |a_n| = \frac{1}{\ln(n+1)}$

$b_n = \frac{1}{n} \Big\} \frac{1}{n} < \frac{1}{\ln(n+1)} \to$ diverges

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} \ni a_{k+1} < a_k \to \boxed{\text{conditionally convergent}}$

11) $\sum_{n=1}^{\infty} (-1)^n \frac{5}{n^3+1} \quad |a_n| = \frac{5}{n^3+1}$

$b_n = \frac{1}{n^2} \Big\} \frac{1}{n^2} > \frac{5}{n^3+1} \to \boxed{\text{absolutely converges}}$

15) $\sum_{n=1}^{\infty} (-1)^n \frac{n^2+3}{(2n-5)^2} \to |a_n| = \frac{n^2+3}{(2n-5)^2}$

$\lim_{n\to\infty} |a_n| = \frac{1}{4} \to \boxed{\text{divergent}}$

19) $\sum_{n=1}^{\infty} \frac{\cos \frac{1}{6}\pi n}{n^2} \quad |a_n| = \frac{|\cos \frac{\pi}{6} n|}{n^2}$

$b_n = \frac{1}{n^2}, \quad \frac{1}{n^2} \geq \frac{\cos \frac{\pi}{6} n}{n^2} \to \boxed{\text{absolute convergence}}$

23) $\sum_{n=2}^{\infty} (-1)^n \frac{1}{n\sqrt{\ln n}} \to |a_n| = \frac{1}{n\sqrt{\ln n}}$

decrease, cont, pos $\to u = \ln x, \; du = \frac{1}{x} dx$

$\lim_{t\to\infty} \int_2^t \frac{x}{x\sqrt{u}} du = \lim_{t\to\infty} \left[2\sqrt{\ln x}\right]_2^t$

$\ln(\infty) = \infty \to$ diverges

$a_k > a_{k+1} \to \boxed{\text{conditionally converges}}$

27) $\sum_{n=1}^{\infty} (-1)^n \frac{1+4^n}{1+3^n} \quad |a_n| = \frac{1+4^n}{1+3^n}$

$\lim_{n\to\infty} \frac{1+4^n}{1+3^n} = \infty \to \boxed{\text{diverges}}$

35) $\frac{1}{n^3} < 0.0005 \qquad n^3 > 2000$

$n \geq 13$

$S_{12} \approx \boxed{0.901}$

39) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^2} \qquad \frac{1}{n^2} < 0.00005 \qquad (n+1)^2 > 20k$

$n > \sqrt{20k} - 1$

$n \geq \boxed{141}$

## WS

33) false because $L = 1$ means no conclusion

b) false because ratio test needs to show divergence ($>1$)

c) true $\to$ each term is getting bigger

d) true $\to$ hypothesis of AST

## AP100

16) I) $\sum_{k=3}^{\infty} \frac{2}{k^2+1} \Big\} 2\sum_{k=3}^{\infty} \frac{1}{k^2} \to$ converges

II) $\sum_{k=1}^{\infty} \left(\frac{6}{7}\right)^n \Big\}$ geometric $< 1 \to$ converges

III) $\sum_{k=2}^{\infty} (-1)^k \frac{1}{k} \to$ converges $(a_k < a_{k+1})$ $\quad \boxed{A}$

14) I ✓ II ✗ III ✓ $\boxed{C}$

44) I) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{2n+1} \to$ converges $\quad \boxed{A}$

II) $\sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{3}{2}\right)^n$ ? III) $\sum_{n=2}^{\infty} \frac{1}{n \ln n} \to$ diverges

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11.5: # 8, 16, 32, 40
11.11: # 6, 14, 22, 30, 38

corona virus

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## 11.5

8) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^2+4}$ $\quad |a_n| = \frac{n}{n^2+4}$

$b_n = \frac{1}{n} \to \frac{n}{n^2} \Big\} \frac{n}{n^2} \leq \frac{n^2}{n^2+4} \to$ diverges

↑ diverges

$f'(x) = \frac{(n^2+4) - n(2n)}{(n^2+4)^2} = \frac{n^2+4-2n^2}{(n^2+4)^2} = (-)$

$\boxed{\text{conditionally convergent}}$

16) $\sum_{n=1}^{\infty} \frac{\sin\sqrt{n}}{\sqrt{n^3+4}}$ $\quad |a_n| \to \frac{[0,1]}{\sqrt{n^3+4}}$

$b_n = \frac{1}{n^{3/2}} \Big\} \frac{1}{n^{3/2}} > \frac{[0,1]}{\sqrt{n^3+4}}$

p-series → converges ——→ converges

$\boxed{\text{absolutely converges}}$

32) $\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n^{1/3}} \to |a_n| = \frac{\ln n}{n^{1/3}}$

$b_n = \frac{1}{n^{1/3}} \Big\} \frac{1}{n^{1/3}} < \frac{\ln n}{n^{1/3}} \to$ diverges

↳ divergent p-series

$f'(x) = \frac{3 - \ln x}{3x^{4/3}}$ $\quad 3 - \ln x < 0$, $\ln x < 3 \to x > e^3$ AST ✓

$\boxed{\text{conditionally converges}}$

40) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}} \to a_{n+1} = \frac{1}{\sqrt{n+1}} < 0.00005$

$\sqrt{(n+1)} > 20{,}000 \to \boxed{n \geq 4 \cdot 10^8}$

## 11.11

6) $\left\{\left(1+\frac{2}{n}\right)^{2n}\right\}$ $\quad \lim_{n\to\infty} \left(1+\frac{2}{n}\right)^{2n} = 1$

$\boxed{\text{diverges}}$

14) $\sum_{n=1}^{\infty} \frac{n^2-1}{n^2+1}$ $\quad \lim_{n\to\infty} \frac{n^2-1}{n^2+1} = 1 > 0$

$\boxed{\text{diverges}}$

22) $\sum_{n=2}^{\infty} (-1)^n \frac{\sqrt[3]{n-1}}{n^2-1} \to |a_n| = \frac{\sqrt[3]{n-1}}{n^2-1}$

$b_n = \frac{1}{n^{5/3}} \to$ convergent p-series

$\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{\sqrt[3]{n-1}}{n^2-1} \cdot \frac{n^{5/3}}{1} = 1 > 0$

$\boxed{\text{absolutely convergent}}$

30) $\sum_{n=1}^{\infty} \left(\frac{1}{3^n} - \frac{5}{\sqrt{n}}\right) \to a_n = \frac{1}{3^n}$, $b_n = \frac{5}{\sqrt{n}}$

$a_n$ converges (geometric where $r < 1$)

$c_n = \frac{1}{n^{1/2}} \Big\} \frac{1}{n^{1/2}} < \frac{5}{\sqrt{n}} \Big\}$ $c_n$ = divergent p-series, $b_n$ = diverges

$a_n - b_n = \boxed{\text{diverges}}$

38) $\sum_{n=5}^{\infty} \frac{1}{n^2-4n}$ $\quad f'(x) = -\frac{(2n-4)}{(n^2-4n)^2}$ = decreasing

↳ positive for $[5,\infty]$, continuous (composition of con. funcs)

$\lim_{t\to\infty} \int_5^t \frac{1}{x^2-4x} dx = \lim_{t\to\infty} \int_5^t \frac{1}{x(x-4)} dx$

$A(x-4) + Bx = 1 \to B = 1/4,\ A = 1/4$

$\lim_{t\to\infty} \int_5^t \frac{1/4}{x-4} - \frac{1/4}{x} = \frac{1}{4} \lim_{t\to\infty} \int_5^t \frac{1}{x-4} - \frac{1}{x} dx$

$\lim_{t\to\infty} \left[\frac{1}{4}\ln|x-4| - \frac{1}{4}\ln|x|\right]_5^t = \left[\frac{1}{4}\ln\left|\frac{x-4}{x}\right|\right]_5^t$

$\frac{1}{4}\left(\ln 1 - \ln\frac{1}{5}\right) = c \to \boxed{\text{converges}}$